Optimization Models: Homework #4

Due on October 14, 2025 at 11:59pm

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Problem 1

Find all constant coefficients a_0, a_1, a_2, a_3 for which the following problem is a convex optimization:

$$\min_{x \in \mathbb{R}^2} \quad \left(\frac{1}{12}x_1^4 + a_3 x_1^3 + a_2 x_1^2 + a_1 x_1 + a_0\right) + x_2^{18} \tag{1a}$$

$$\text{s.t.} \quad a_2 x_1^2 + a_1 x_1 + a_0 \leq -10 x_2 + 5 \tag{1b}$$

$$a_1 x_1 + a_0 = x_2 + 1 \tag{1c}$$

Problem 2

Consider the optimization problem:

$$\min_{x \in \mathbb{R}^3} \quad e^{-x_1 - x_2 - x_3} + (x_1 + 2x_2 + 3x_3)^4 \tag{2a}$$

$$\text{s.t.} \quad x_1 - x_2 - x_3 = 1 \tag{2b}$$

$$x_1^2 + x_2^2 + x_3^2 + x_1 x_2 + x_2 x_3 + x_1 x_3 \leq 2 \tag{2c}$$

$$0 \leq x_1 \leq 1 \tag{2d}$$

$$0 \leq x_2 \leq 1 \tag{2e}$$

$$0 \leq x_3 \leq 1 \tag{2f}$$

i) Prove that $x = [1 \;\; 0 \;\; 0]^T$ is a feasible point.

ii) Using the Weierstrass theorem, prove that the above optimization problem has a solution.

iii) Prove that every local minimum of the above optimization problem is a global minimum.

Problem 3

Consider a convex function $f(x_1, x_2) : \mathbb{R}^2 \to \mathbb{R}$ such that:

- The points $(2, 2)$ and $(-2, 4)$ are both global minima of the function $f(x_1, x_2)$ with the property that $f(2, 2) = 5$.
- $f(0, 0)$ is equal to 10.

Answer the following questions:

i) Prove that the point $(0, 3)$ is a global minimum of the function $f(x_1, x_2)$.

ii) Prove that the point $(0, -1)$ is not a local minimum of the function $f(x_1, x_2)$.

Problem 4

Answer the following questions about coercive functions:

i) Find all coefficients $\alpha_1, ..., \alpha_n$ and $\beta_1, ..., \beta_n$ for which the function

$$\alpha_1 x_1^2 + \cdots + \alpha_n x_n^2 + \cos\left(\beta_1 x_1^3 + \cdots + \beta_n x_n^3\right) \tag{3}$$

is coercive.

ii) Given a constant a, prove that the function $x_1^2 + x_2^2 + a x_1 x_2$ is coercive if $|a| < 2$ and is not coercive if $|a| \geq 2$.