

MATH241 Chapter 11

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11.1 Cartesian Coordinates in Space

Given a point in 3 dimensional space P , then there are 3 planes that intersect P and are perpendicular to the x, y, and z axis.

So, P can be associated with an ordered triple of numbers (x, y, z) . This way of writing P is called the rectangular, or Cartesian, coordinates.

Distance

The distance between two points, P and Q , can be found using the equation:

$$|PQ| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

Some other notes:

$$\begin{aligned} |PQ| &= 0 \text{ iff } P = Q \\ |PQ| &= |QP| \\ |PQ| &\leq |PR| + |RQ| \text{ for any third point } R \end{aligned}$$

11.2 Vectors in Space

Definition: Vector

A vector is an ordered triple (a_1, a_2, a_3) of numbers. The numbers a_1, a_2 , and a_3 are called the components of the vector. The vector \vec{PQ} associated with the directed line segment with the initial point $P = (x_0, y_0, z_0)$ and the terminal point $Q = (x_1, y_1, z_1)$ is $(x_1 - x_0, y_1 - y_0, z_1 - z_0)$

Definition: Norm

The length(norm) of a vector $a = (a_1, a_2, a_3)$ is denoted as $\|a\|$ is defined as:

$$\|a\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

A **Unit Vector** is a vector with a norm of 1.

Some special unit vectors:

$$i = (1, 0, 0)$$

$$j = (0, 1, 0)$$

$$k = (0, 0, 1)$$

A vector $a = (a_1, a_2, a_3)$ can be written as:

$$a = a_1i + a_2j + a_3k$$

Vector Operations

Let $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$ and c be a scalar.

$$\begin{aligned}a + b &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\a - b &= (a_1 - b_1, a_2 - b_2, a_3 - b_3) \\ca &= (ca_1, ca_2, ca_3)\end{aligned}$$

There are four ways to describe a vector: 1. as (a_1, a_2, a_3) , an ordered triple of numbers

2. as (a_1, a_2, a_3) , a point in space

3. as a directed line segment with an initial point at (x_0, y_0, z_0) and a terminal point at $(x_0 + a_1, y_0 + a_2, z_0 + a_3)$

4. as $a_1i + a_2j + a_3k$

Parallel Vectors

Two nonzero vectors a and b are parallel iff there exists a scalar c such that $b = ca$.