Chapter 1

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1.1 System of Linear Equations

A linear equation in the variables $x_1, x_2, ..., x_n$ is an equation that can be written as:

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b$$

where a_1, a_2, a_n, b are real or complex numbers.

Example 1:

$$2x_1 - 5x_2 + 2 = x_1$$
$$\mathrm{x}_1 - 5x_2 + 2 = 0$$
$$\mathrm{x}_1 - 5x_2 = -2$$

A solution of the system is a list $(s_1, ..., s_n)$ of numbers that makes each equation true when $(s_1, ..., s_n)$ are plugged into $(x_1, ..., x_n)$.

A system of linear equations has:
1. No solutions
2. One solution
3. Many solutions

A system of linear equations is said to be consistent if it has one or many solutions, an inconsistent system has no solutions.

Matrix Notation

The essential information of a linear system can be recorded in a matrix.

Example

Given:

$$x_1 - x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

the coefficents can be aligned in columns into:

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}$$