第 3、4 章作业

3-3 (1) $E[Y(t)]=0$

$E[Y^2(t)]=\sigma^2$

(2) $f(y)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-y^2}{2\sigma^2}\right)$

(3) $R(t_1,t_2)=\sigma^2\cos\omega_0(t_1-t_2)$

$B(t_1,t_2)=R(t_1,t_2)$

3-12 (1) 是

(2) $P_y(w)=2w^2(1+\cos wT)P_x(w)$

4-1 44.7 km

4-8 80/s

4-9 56.79W

3-3 解:

(1) $E[Y(t)] = E[X_1\cos\omega_0 t - X_2\sin\omega_0 t]$

$= \cos\omega_0 t\, E[X_1] - \sin\omega_0 t\, E[X_2] = 0$

$E[Y^2(t)] = E[(X_1\cos\omega_0 t - X_2\sin\omega_0 t)^2]$

$= E[X_1^2\cos^2\omega_0 t - 2X_1X_2\cos\omega_0 t\sin\omega_0 t + X_2^2\sin^2\omega_0 t]$

$= \cos^2\omega_0 t\, E[X_1^2] - 2\sin 2\omega_0 t\, E[X_1X_2] + \sin^2\omega_0 t\, E[X_2^2]$

$\because E[X_1] = E[X_2] = 0 \quad D[X_1] = D[X_2] = \sigma^2$

$\therefore E[X_1^2] = E[X_2^2] = \sigma^2$

$\because X_1, X_2$ 相互独立

$\therefore E[X_1X_2] = E[X_1]\cdot E[X_2] = 0$

$\therefore E[Y^2(t)] = \cos^2\omega_0 t\cdot\sigma^2 + \sin^2\omega_0 t\cdot\sigma^2 = \sigma^2$

(2) $\because X_1, X_2$ 服从高斯分布 $\quad Y(t) = X_1\cos\omega_0 t - X_2\sin\omega_0 t$

$\therefore Y(t)$ 也服从高斯分布

$\therefore f(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right)$

(3) $R(t_1,t_2) = E[Y(t_1)Y(t_2)] = E[(X_1\cos\omega_0 t_1 - X_2\sin\omega_0 t_1)(X_1\cos\omega_0 t_2 - X_2\sin\omega_0 t_2)]$

$= E[X_1^2\cos\omega_0 t_1\cos\omega_0 t_2 - X_1X_2\cos\omega_0 t_1\sin\omega_0 t_2 - X_1X_2\sin\omega_0 t_1\cos\omega_0 t_2 + X_2^2\sin\omega_0 t_1\sin\omega_0 t_2)]$

$= (\cos\omega_0 t_1\cos\omega_0 t_2 + \sin\omega_0 t_1\sin\omega_0 t_2)\sigma^2$

$= \sigma^2\cos\omega_0(t_1 - t_2)$

$B(t_1,t_2) = R(t_1,t_2) - E[Y(t_1)]E[Y(t_2)] = R(t_1,t_2) = \sigma^2\cos\omega_0(t_1 - t_2)$

3-12 解:

(1) 由图知 $Y(t) = \frac{d}{dt}[X(t) + X(t-T)]$ 为线性系统

$X(t)$ 是平稳过程，故 $Y(t)$ 也是平稳过程

(2) $\frac{d}{dt}[X(t) + X(t-T)] = Y(t) \quad \therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = j\omega(1 + e^{-j\omega T})$

$\rightarrow j\omega X(\omega) + j\omega e^{-j\omega T}X(\omega) = Y(\omega) \quad P_Y(\omega) = |H(\omega)|^2 P_X(\omega) = 2\omega^2(1 + \cos\omega T)P_X(\omega)$

4-1 解：设D为两天线间的距离，h为天线高度。

$D^2 = 8rh \approx 50h = 50 \times 40 = 2000$

最远通信距离 $D = \sqrt{2000} \approx 44.7$ km

4-8 解：信道容量：$C_t = B\log_2(1+\frac{S}{N}) = 3000\log_2(1+100) \approx 19974$ (b/s)

一幅相片的信息量：$I = 4\times10^6 \log_2 16 = 1.6\times10^7$ bit

传输时间：$t = \frac{I}{C_t} = \frac{1.6\times10^7}{19974} \approx 801$ s

4-9 解：电磁波波长 $\lambda = \frac{c}{f} = \frac{3\times10^8}{1.8\times10^8} = \frac{1}{6}$ m

由 $L_{fr} = \frac{P_T}{P_R} = \frac{16\pi^2 d^2}{\lambda^2 G_T G_R}$ 得

$P_T = \frac{16\pi^2 d^2 P_R}{\lambda^2 G_T G_R} = \frac{16\pi^2 (50\times10^3)^2 \, 4000\times10^{-12}}{(\frac{1}{6})^2 \times 100\times10} \approx 56.79$ W