Determination of dynamic security region in power systems using the dynamic mode decomposition

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Stability Constrained OPF

Power System
Operation Calculation

mathematical
essence

Stability Constrained OPF

$$\min \quad f(\boldsymbol{p})$$
$$\text{s.t.} \quad \boldsymbol{g}(\boldsymbol{p}) = 0$$
$$\boldsymbol{h}(\boldsymbol{p}) \leq 0$$

Stability constraint

$$\boldsymbol{h}_s(\boldsymbol{p}) \leq 0$$

?

Definition of Dynamic Security Region

Stability Constrained OPF

$$\min \quad f(\boldsymbol{p})$$
$$\text{s.t.} \quad \boldsymbol{g}(\boldsymbol{p}) = 0$$
$$\boldsymbol{h}(\boldsymbol{p}) \le 0$$

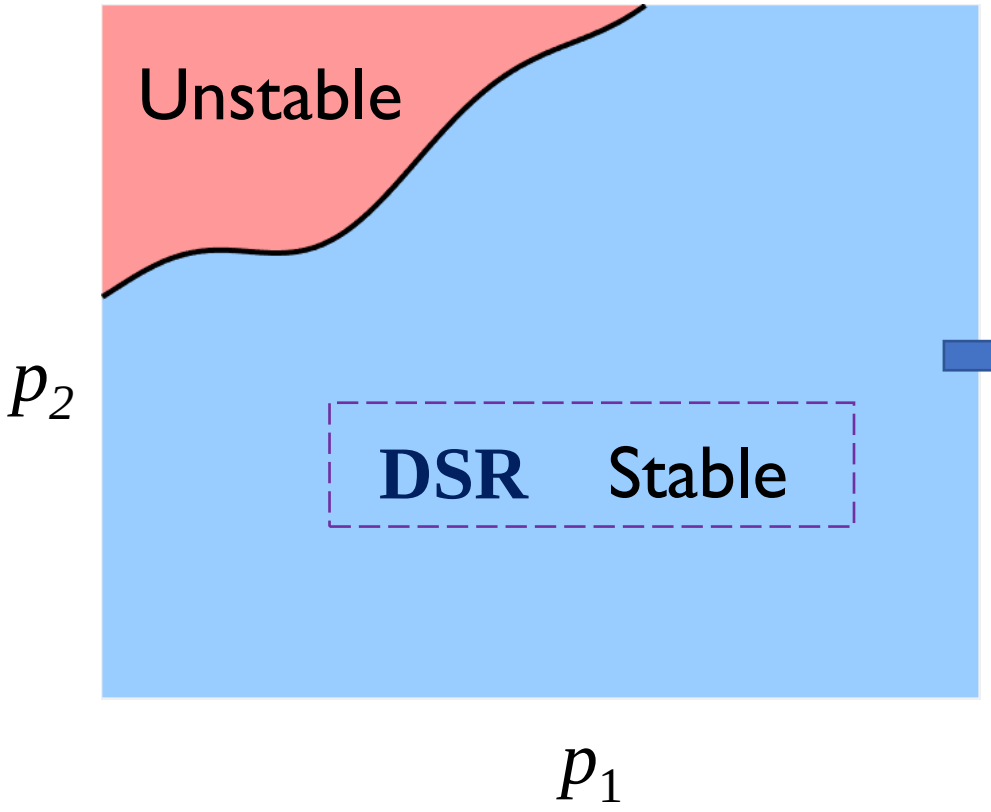
Stability constraint

$$\boldsymbol{h}_s(\boldsymbol{p}) \le 0$$

Large-signal stability

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, p)$$

Parameter space

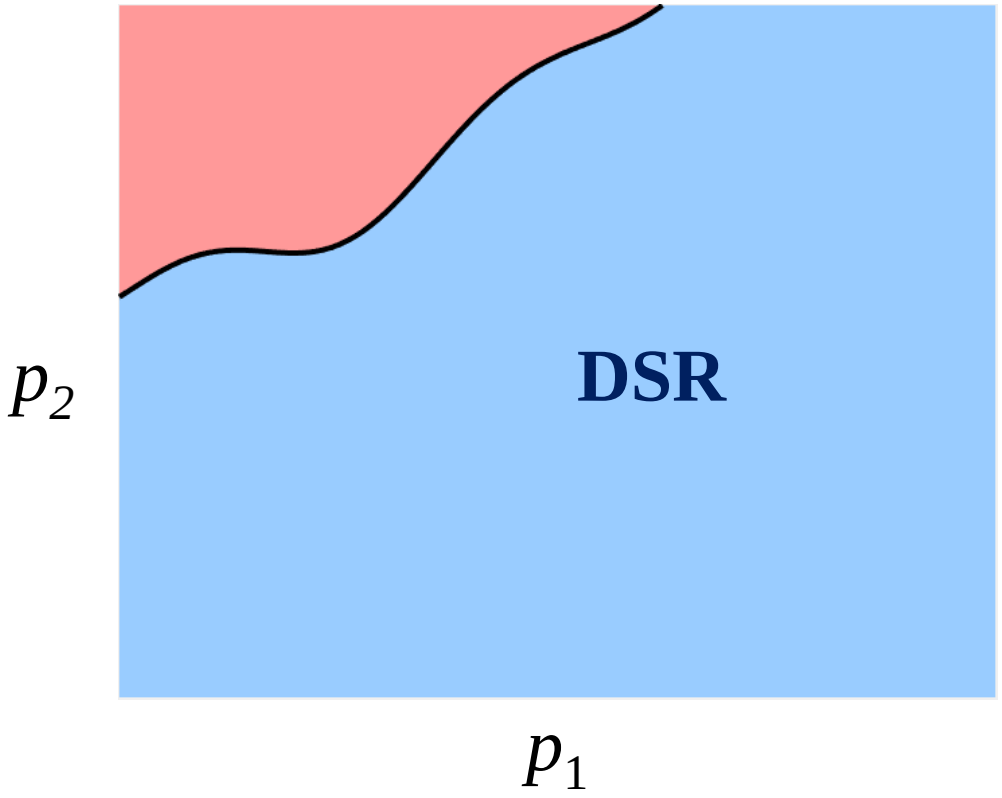


$$\boldsymbol{h}_s(\boldsymbol{p}) \le 0$$

Dynamic Security Region (DSR)

The region for large-signal stability in **parameter space**

Formulation of Dynamic Security Region



Definition of the stability criteria

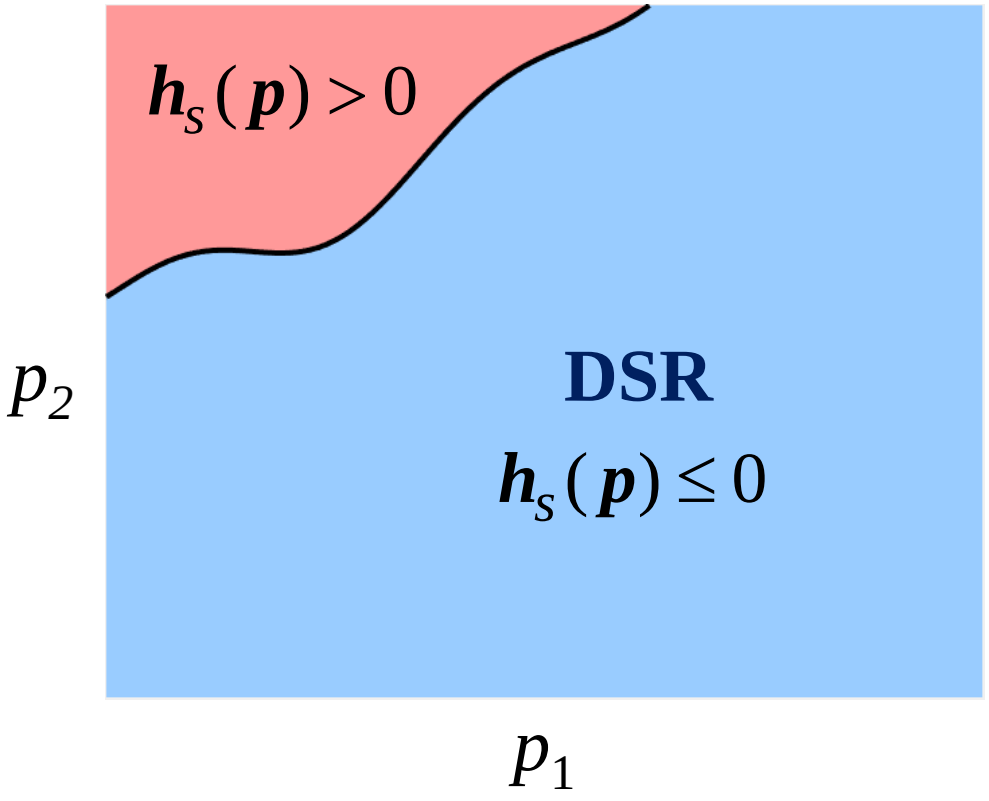
Nonlinear dynamical systems theory

- Lyapunov Stability Criterion
- Energy Function Method
- Direct Method…

Difficult to be expressed by a explicit function

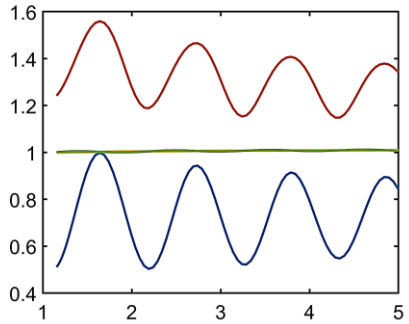
Ref.) Chiang & Alberto, Stability of regions of nonlinear dynamical systems, Cambridge Univ. Press, 2015

Formulation of Dynamic Security Region



Definition of the stability criteria

Specific variables constrained by thresholds



- Thresholds are typically determined by empirical wisdom
- Each variable requires a separate threshold

$$-\delta_{\max} \leq \delta_i^k(t|u) - \delta_{COI}^k(t|u) \leq \delta_{\max}$$

Ref). Rong & Geng & Jiang, Data-driven transient stability boundary generation for online security monitoring, IEEE TPS'21

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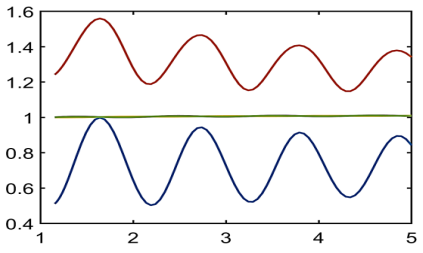
Dynamic Mode Decomposition

- Collect historical data from power system



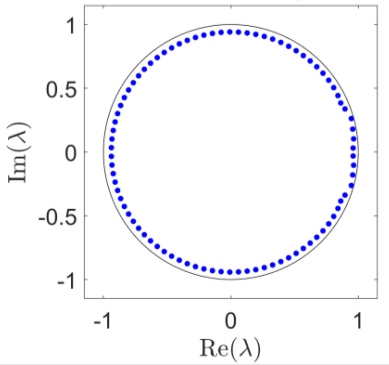
$$\{x_1, x_2, \dots, x_N, x_{N+1}\}$$

$$\underbrace{\begin{bmatrix} | & | & \cdots & | \\ x_1 & x_2 & \cdots & x_N \\ | & | & \cdots & | \end{bmatrix}}_{X \in \mathbb{R}^{m \times N}}$$

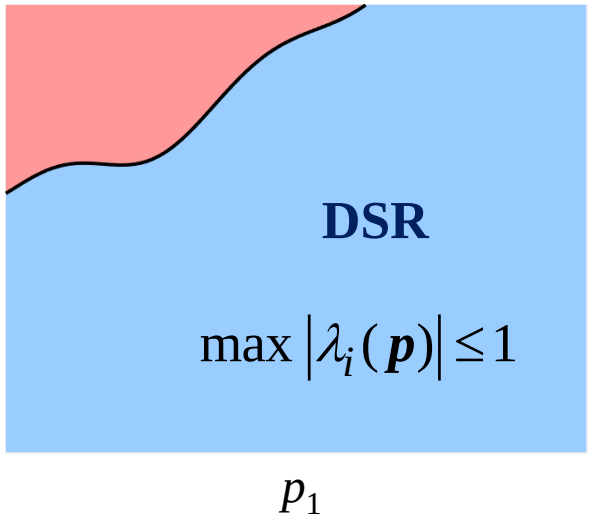
$$X' \approx AX$$

$$X = [x_1, x_2, \dots, x_{N-1}]$$

$$X' = [x_2, x_3, \dots, x_N]$$



Eigenvalues



Stability criteria

$$\boldsymbol{h_s(p)} \le 0$$

$$\max |\lambda_i(\boldsymbol{p})| \le 1$$

A unified and theoretically grounded stability criterion applicable across all variables in power system.

Ref.) Kutz et al., Dynamic mode decomposition: Data-driven modeling of complex system, SIAM, 2016

Filter: Proper Orthogonal Decomposition (POD)

Ref.) Susuki & Mezic & Hikihara, Coherent swing instability of power grids, J.Nonlin. Sci., 2011

- DMD exhibits sensitivity to the characteristics of the input data
- Only a limited subset of factors significantly influences the system's behavior

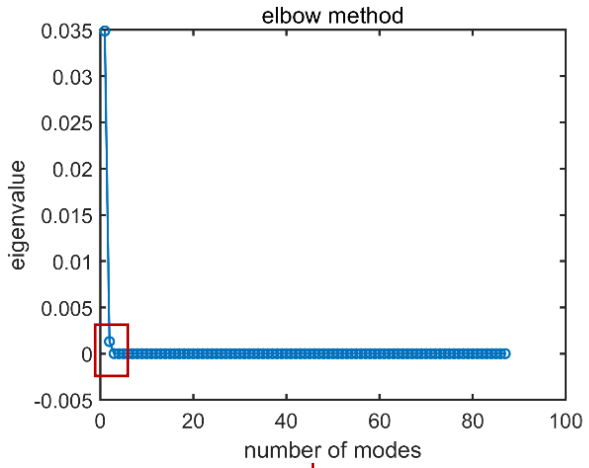
The unimportant factor may interfere the estimation result

$$\underbrace{\begin{bmatrix} | & | & \cdots & | \\ x_1 & x_2 & \cdots & x_N \\ | & | & \cdots & | \end{bmatrix}}_{X \in \mathbb{R}^{m \times N}}$$

POD

$$\underbrace{\begin{bmatrix} | & | & \cdots & | \\ x_1 & x_2 & \cdots & x_N \\ | & | & \cdots & | \end{bmatrix}}_{X \in \mathbb{R}^{r \times N}}$$

Elbow Method



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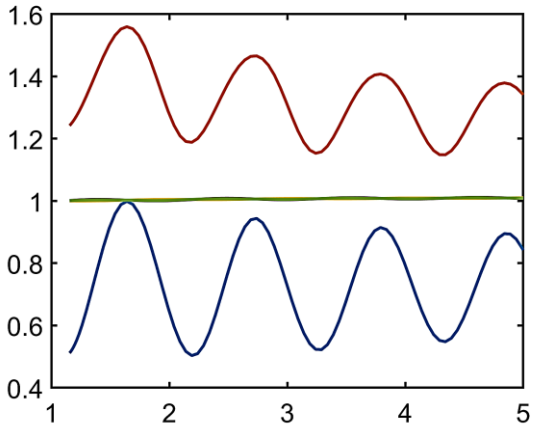
Companion-based DMD



Dynamic trajectory of power system

Flat data matrix: dimension much smaller than the number of temporal snapshots

$X' \approx AX$

CDMD

Ref.) Rowley et al, Spectral analysis of nonlinear flows, J. Fluid Mech., 2008

Companion-based DMD

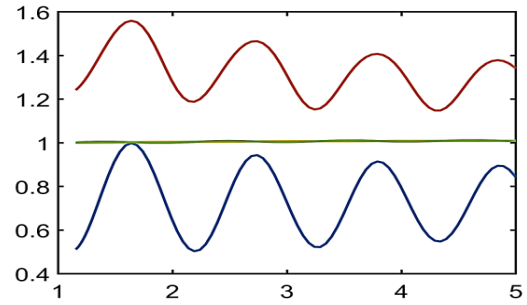
Companion matrix

$$\tilde{C} = \begin{bmatrix} 0 & 0 & \dots & 0 & \tilde{c}_0 \\ 1 & 0 & \dots & 0 & \tilde{c}_1 \\ 0 & 1 & \dots & 0 & \tilde{c}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & c_{m-1} \end{bmatrix}$$

Eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_{N-1}$

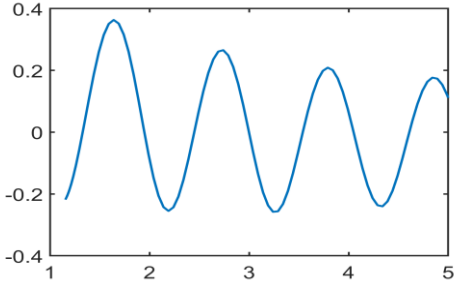
Stability criteria $\max \left|\lambda_i(\boldsymbol{p})\right| \leq 1$

The Procedure of Determining DSR by DMD



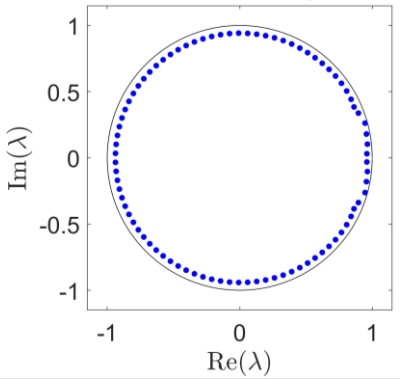
Dynamic trajectory of power system

POD



Critical data

Companion-based DMD

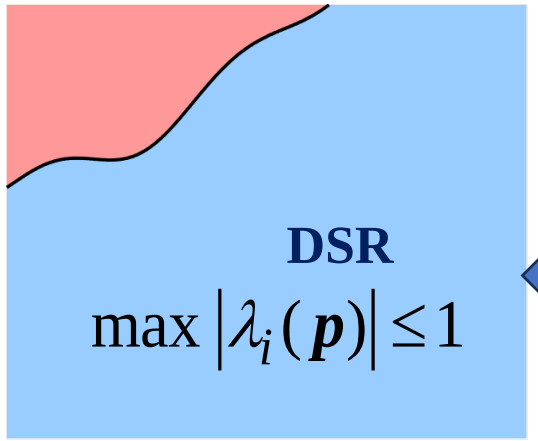


Eigenvalue

Stability criteria

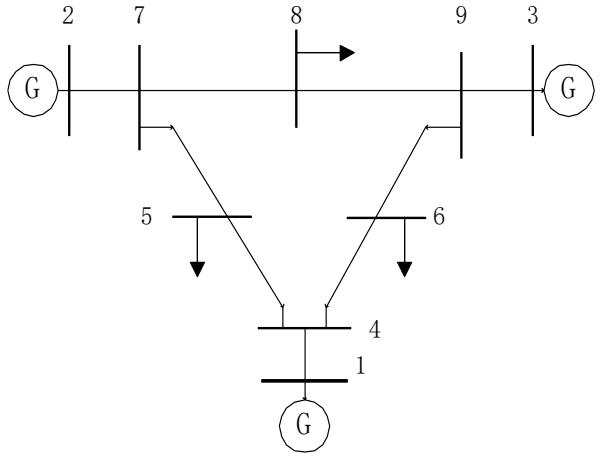
$$\max\left|\lambda_i(\boldsymbol{p})\right| \le 1$$

Sampling



Simulation: IEEE 9-bus case

Test system setup

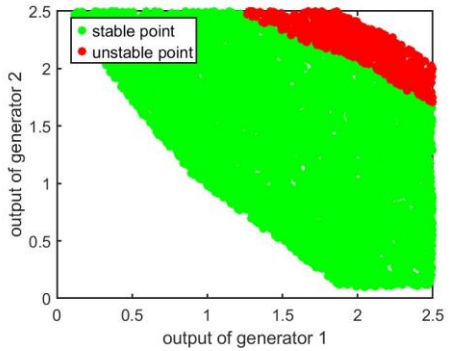


The IEEE-9 bus system

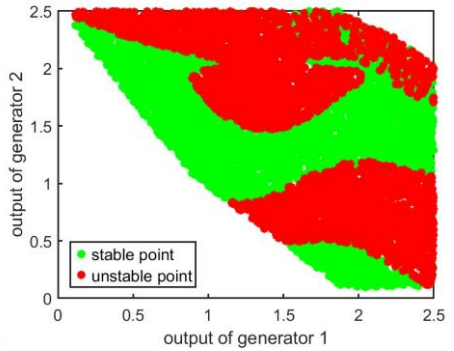
- Parameter space: generator outputs
- Sampling scale: **10000** operating points
- Time scale of simulation: first 5 seconds of the trajectory
- Clearing time: 0.16s

Table 1. Stability detection error

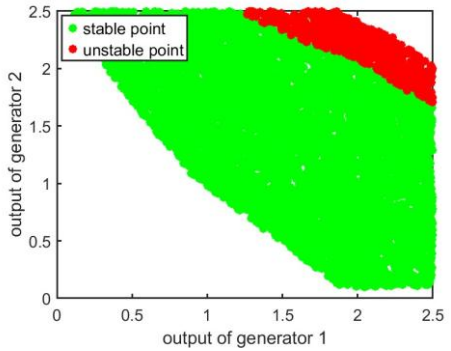
Method	Error
Companion-matrix DMD	49.89%
POD+Companion-matrix DMD	0.0874%



Benchmark



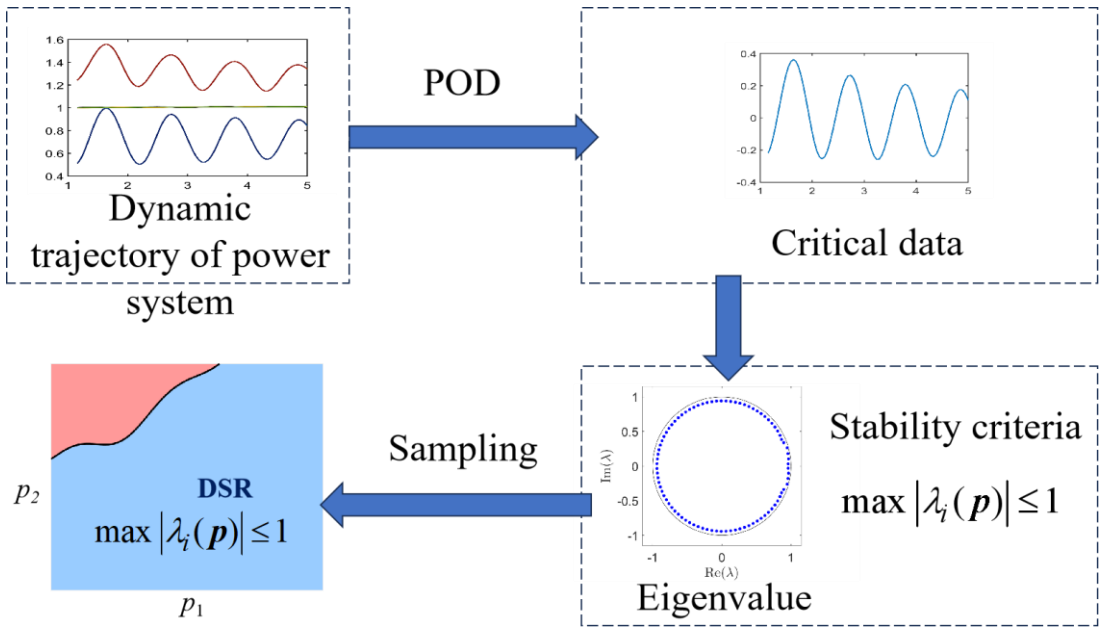
Companion-matrix DMD



Companion-matrix DMD+POD

A 20-second simulation

Conclusion and Future Work



- ✓DMD provides an accurate and scalable stability criterion for determining the dynamic security region based on eigenvalues.
- ✓POD can enhance the accuracy of DMD estimations.

Next…

- Formulate stability constraints based on the dynamic security region using a data-driven method
- Integrate stability constraints into Optimal Power Flow (OPF)