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Partial component consensus of leader-following multi-agent systems via intermittent pinning control ☆

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ABSTRACT

Partial component consensus means that some components of all state variables in a multi-agent system tend to be convergence as time tends to be infinite. It is a dynamics behavior that is weaker than identical consensus. In this paper, partial component consensus of nonlinear multi-agent systems via intermittent pinning control is investigated for the first time, and the intermittent signal can be aperiodic. With the help of permutation matrix method, the corresponding error system is reduced to a new error system. Then, partial component consensus in the multi-agent system is converted into the stability of the new error system with respect to partial variables. Based on matrix theory, graph theory and stability theory of partial variables, some sufficient conditions to guarantee exponential partial component consensus are derived. Finally, numerical simulations are shown to demonstrate correctness of the theoretical results.

1. Introduction

In recent years, with the rapid development of artificial intelligence technology, researches on multi-agent systems have received increasing attention from various fields, involving mathematics, control, communication, physics and many other disciplines. This is mainly due to broad applications of multi-agent systems in many areas such as intelligent transportation system control, distributed sensor networks, UAV formation [1] and persistent monitoring [2]. Currently, the primarily problems in multi-agent systems include flocking [3], tracking [4], swarming [5], controllability [6] and consensus [7,8].

Consensus problems, as a basic and hot research topic in distributed coordination control of multi-agent systems, have yielded fruitful results. For instance, cluster (group) consensus [9], finite-time consensus [10], output consensus [11], lag consensus [12,13]. Meanwhile, some scholars also considered other factors in consensus problems, such as leader-following architectures, dynamic models, time-domain characteristics and communication environment. Furthermore, synchronization for complex networks, which is closely related to consensus problems of multi-agent systems, has also been extensively explored [14].

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As we know, in the case where the multi-agent systems cannot reach consensus by itself, one of the critical problems is how to design appropriate external control algorithms such that a group of agents eventually reach agreement on a common value. Up to date, many substantial results in control problems of multi-agent systems consensus have been studied, such as feedback control [15], impulsive control [16], event-triggered control [17], intermittent control [18], and pinning control [19] etc. In order to reduce unnecessary waste of finite resources, some researchers gradually began to adopt discontinuous control protocols. More recently, a periodically intermittent control strategy is prevalent and its applications on synchronization and consensus are also investigated [20–25]. The exponential stability of nonlinear systems with periodically intermittent control was analyzed in [20]. In [23], consensus of leader-following multi-agent systems with time-delay via intermittent control has been explored. The periodically intermittent pinning control protocol has been employed in [24] to consider the exponential synchronization of complex networks with delayed nonlinear dynamics. Then, inspired by periodically intermittent strategy, some scholars put forward aperiodically intermittent control strategy idea to deal with the problems on synchronization and consensus [26–33]. For a class of complex dynamical networks under aperiodically intermittent pinning control, Liu and Chen obtained some sufficient conditions to reach synchronization [26]. In addition, they utilized the similar analysis method in [27] to further rationalize the synchronization of linearly coupled complex networks with time-delay. Ma and Cai developed several consensus criteria for Lagrangian systems with a directed graph and aperiodically intermittent pinning control [29]. In view of this, Zhou and Cai expanded intermittent control strategy's scope and obtained the less conservative synchronization condition by means of decentralized adaptive intermittent pinning control [31]. In [32], some sufficient conditions for successive lag synchronization via aperiodically and periodically intermittent control have been given.

It is worth noting that most of the above literatures focus on consensus for all state variables of multi-agent systems, while partial component consensus behaviors are neglected. As a matter of fact, there are partial component consensus behaviors in practice, including biological systems, chemical reaction, food webs and so on, relying on stability of partial variables. For example [34], during aircrafts flight performance, several aircrafts fly in side by side or in radial. As for the three-dimensional vector of displacement, the displacement component in the forward direction of flight will reach agreement, but in other directions is divergent. In other words, the components in some directions will reach consensus, while in other directions do not need to achieve consensus. Hence, some scholars start to investigate partial consensus problems [34–36]. Wu and Ma proposed the concept of partial component consensus in [34], and derived the sufficient conditions for first-order leader-following multi-agent systems, while the discussed control algorithms are continuous. To the best of our knowledge, there have been rare results addressing partial component consensus of multi-agent systems via intermittent pinning control. Motivated by the methods of above-mentioned results [20,26,29,34,36], the task of this paper is to explore partial component consensus of first-order multi-agent systems with nonlinear dynamics via intermittent pinning control. We design two kinds of intermittent pinning control protocols, in which the aperiodic and periodic, respectively. Compared with the existing results on consensus problems, the main contributions of this paper are threefold. First, partial component consensus problem via intermittent pinning control strategy is investigated for the first time and some sufficient criteria are derived based on the partial variables stability theory. Besides, there are two distinctive advantages in intermittent pinning control strategy. On one hand, intermittent control as a discontinuous control is more economic and effective than continuous control. On the other hand, pinning control is a powerful scheme because it can be easily realized by applying local feedback information to a small fraction of network nodes. Second, in this paper, by virtue of permutation matrix method, we transform the error system into a new error system and construct the appropriate Lyapunov function, then analyze the partial component exponential stability of the new error system instead of the asymptotic stability to discuss partial component consensus problems in [34,36]. Thirdly, the communication topology of agents is considered as directed graph, and then partial component consensus of leader-following multi-agents systems via aperiodically and periodically intermittent pinning control strategy are explored, respectively.

The structure of the paper is as follows. In Section 2, some preliminaries and problem descriptions are introduced. In Section 3, the models of dynamical systems with aperiodically and periodically intermittent pinning control strategies are introduced, and some sufficient but not necessary consensus conditions are derived to guarantee multi-agent system can reach partial component consensus. In Section 4, some numerical simulations demonstrate the theoretical results of the partial component consensus. Finally, this paper is concluded in Section 5.

Notations. Let $\mathbb{N} = \{0, 1, 2, \dots\}$ be the set of all natural numbers, $\mathbb{R}^{N\times N}$ represents a set of all $N \times N$ real matrices and $1_N = (1, \dots, 1)^T \in \mathbb{R}^N$. $I_N(0_N)$ is the N-dimensional identity (zero) matrix and $\|\cdot\|$ indicates the Euclidean norm. For $A \in \mathbb{R}^{N\times N}$, A^T stands for its transpose, A^{-1} is the inverse of matrix A and if all eigenvalues of a matrix A are all real, then $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denoted the maximum and minimum eigenvalue, respectively. $\lambda^{-1}_{\max}(A)$ is the reciprocal of the maximum eigenvalue of matrix A. The symmetric part of A is denoted by $A^s = (A^T + A)/2$. Denote matrix $A < 0$ ($A \leq 0$) if A is real symmetric and negative definite (semi-negative definite). The symbol $\otimes$ denotes the Kronecker product of matrices.

2. Preliminaries

In this section, some important preliminaries about graph theory, stability theory, definitions and model description for partial component consensus in multi-agent systems with nonlinear dynamics and intermittent pinning control are briefly introduced.

2.1. Graph theory

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ of order N, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the set of nodes, $\mathcal{E} \subseteq \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$ is the set of edges and $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}$ with element a_{ij}. A directed graph has a directed spanning tree if there exists at least one node called root which has a directed path to all the other nodes. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_j, v_i)$, if $e_{ij} \in \mathcal{E}$ then $a_{ij} > 0$, which means that the node v_i can receive information from node v_j, i.e., $e_{ij} \in \mathcal{E} \Leftrightarrow a_{ij} > 0$; Otherwise, $e_{ij} \notin \mathcal{E} \Leftrightarrow a_{ij} = 0$. The set of neighbors of node v_i is denoted by $N_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix $L = (l_{ij}) \in \mathbb{R}^{N\times N}$ corresponding to the graph $\mathcal{G}$ is defined as $l_{ij} = -a_{ij} (i \neq j)$, $l_{ii} = \sum_{i=1}^{N} a_{ij}$ and $\sum_{j=1}^{N} l_{ij} = 0$.

2.2. Stability theory of partial variables

First, we consider nonautonomous ordinary differential equation of n dimensions

$$\frac{dx}{dt} = F(t, x), \tag{1}$$

where $F(t, x) \in \mathcal{C}[\mathbb{R}^+ \times \mathbb{R}^n, \mathbb{R}^n]$, $\mathbb{R}^+ = [0, +\infty)$, $t \in \mathbb{R}^+$, $F(t, 0) \equiv 0$, $x = (y^T, z^T)^T \in \mathbb{R}^n$, $y = (x_1, \dots, x_m)^T \in \mathbb{R}^m$, $z = (x_{m+1}, \dots, x_n)^T \in \mathbb{R}^P$, $m + p = n$, $\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$, $\|y\| = \left(\sum_{i=1}^{m} x_i^2\right)^{1/2}$, $\|z\| = \left(\sum_{i=m+1}^{n} x_i^2\right)^{1/2}$.

Definition 1 ([37]). The trivial solution of nonautonomous system (1) with respect to partial variables y is said to be exponentially stable if for every $\varepsilon > 0$, $\forall t_0 \in \mathbb{R}^+$, there exists $\lambda > 0$, $\exists \delta(t_0, \varepsilon) > 0$, such that $\|x_0\| < \delta(\varepsilon)$ and then the partial solution $\|y(t, t_0, x_0)\|$ of the solution $\|x(t, t_0, x_0)\|$ of system (1) satisfies

$$\|y(t, t_0, x_0)\| < \varepsilon e^{-\lambda(t-t_0)} (t \geq t_0).$$

Definition 2 ([37]). The trivial solution of nonautonomous system (1) with respect to partial variables y is said to be globally exponentially stable if for every $\varepsilon > 0$, $\forall t_0 \in \mathbb{R}^+$, there exists $\lambda > 0$, $\exists M(\delta) > 0$, such that $\|x_0\| < \delta$ and then the partial solution $\|y(t, t_0, x_0)\|$ of system (1) satisfies

$$\|y(t, t_0, x_0)\| < M(\delta) \|x_0\| e^{-\lambda(t-t_0)} (t \geq t_0).$$

2.3. Model description

In general, we consider the first-order leader-following multi-agent systems, which is composed of a leader agent and N follower agents. The leader agent x_0 is an isolated agent, can be defined as

$$\frac{dx_0(t)}{dt} = f(x_0(t)), \tag{2}$$

where $x_0 = (x_{01}, \dots, x_{0n})^T \in \mathbb{R}^n$ is the state vector of leader agent, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous mapping, $f(x_0) = (f_1(x_0), \dots, f_n(x_0))^T \in \mathbb{R}^n$ stands for its own dynamics of leader agent.

The dynamical behavior of the ith follower agent can be described by

$$\frac{dx_i(t)}{dt} = f(x_i(t)) + c \sum_{j\in N_i} a_{ij} \Gamma (x_j(t) - x_i(t)) + u_i(t), \tag{3}$$

where $x_i = (x_{i1}, \dots x_{in})^T \in \mathbb{R}^n$ is the state vector of ith follower agent, $i = 1, 2, \dots, N$, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous mapping, $f(x_i) = (f_1(x_i), \dots, f_n(x_i))^T \in \mathbb{R}^n$ represents the self-dynamics of the ith follower agent and c is coupling strength. The network communication topology is represented by the adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}$, $a_{ij} > 0$, if the ith agent can receive the information from the jth agent; otherwise, $a_{ij} = 0$. The semi-positive definite matrix $\Gamma = \mathrm{diag}(\gamma_{1,} \gamma_{2,} \dots \gamma_n)$ denotes the inner coupling matrix, $u_i(t)$ is the outer control of the ith agent.

For system (3), we consider the two kinds of intermittent pinning control protocols.

1. The aperiodically intermittent pinning control protocol is described by

$$u_i(t) = \begin{cases} -cb_i \Gamma (x_i(t) - x_0(t)), t \in [t_k, m_k], \\ 0, t \in (m_k, t_{k+1}), \end{cases} \tag{4}$$

where $k \in \mathbb{N}$, c is control gain, $b_i > 0$, if the ith agent can receive the information from the leader agent; otherwise, $b_i = 0$.

Remark 1. Taking time into consideration, we can know that for any time interval $[t_k, t_{k+1})$, $[t_k, m_k)$ is the control time (work time), and $m_k - t_k$ is called the kth work width (control duration); while (m_k, t_{k+1}) is rest time, and $t_{k+1} - m_k$ is the called kth rest width. Particularly, both the time of control and rest are aperiodic.

2. The Periodically intermittent pinning control is as following

$$u_i(t) = \begin{cases} -cb_i\Gamma(x_i(t) - x_0(t)), t \in [k\omega, k\omega + \theta], \\ 0, t \in (k\omega + \theta, (k+1)\omega), \end{cases} \tag{5}$$

where $k \in \mathbb{N}$, c is control gain, if the ith follower agent can measure leader's information, then $b_i > 0$; otherwise, $b_i = 0$.

Remark 2. Periodically intermittent pinning control means that both the control time and rest time are periodic. In any period of time, time is composed of "work time" and "rest time".

Remark 3. If the control protocol (4) satisfies $m_k - t_k \equiv \theta$ and $t_{k+1} - t_k \equiv \omega$, then aperiodically intermittent pinning control (5) equals to periodically intermittent pinning control (4). In fact, we can easily know that periodically intermittent control can be treated as a special case of aperiodically intermittent control. Therefore, in both application and theoretical analysis, it is necessary to investigate the consensus problem under these two control protocols.

Assumption 1. There exists a constant $\varepsilon_1 > 0$ such that the function $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the following inequality

$$(x-y)^T \Lambda(f(x) - f(y)) \le \varepsilon_1 (x-y)^T \Lambda(x-y), \tag{6}$$

where $\Lambda = \mathrm{diag}(\overbrace{1, \ldots, 1}^{l}, 0, \ldots, 0)$ and $x \in \mathbb{R}^n, y \in \mathbb{R}^n$.

Assumption 2. There exists a constant $\varepsilon_2 > 0$ such that the nonlinear function $f(\cdot) : \mathbb{R}^{nN} \to \mathbb{R}^{nN}$ satisfies the following inequality

$$(x-y)^T \Xi(f(x) - f(y)) \le \varepsilon_2 (x-y)^T \Xi(x-y), \tag{7}$$

where $\Xi = \mathrm{diag}(\overbrace{1, \ldots, 1}^{lN}, 0, \ldots, 0)$ and $x \in \mathbb{R}^{nN}, y \in \mathbb{R}^{nN}$.

Remark 4. Compared with Lipschitz condition, Assumptions 1 and 2 are the weaker conditions.

Assumption 3 (*[29]*)**.** For aperiodically intermittent control strategy, assume that there are two scalers θ and β to fit the following equalities

$$\begin{cases} \min\{m_k - t_k, k \in \mathbb{N}\} = \theta > 0, \\ \max\{t_{k+1} - m_k, k \in \mathbb{N}\} = \beta < +\infty, \end{cases} \tag{8}$$

where $\theta > 0$, $0 < \beta < +\infty$, $k \in \mathbb{N}$.

Remark 5. In this assumption, we can know that each control width is different, while each control width $[t_k, m_k]$ should be greater than or equal to θ. Similarly, each rest width (m_k, t_{k+1}) should be less than or equal to β, and the control width θ should be less than $[t_k, t_{k+1})$.

Lemma 1 (*[36]*)**.** *Suppose that $A = (a_{ij}) \in \mathbb{R}^{N\times N}$, $B = (b_{ij}) \in \mathbb{R}^{n\times n}$. Then there exists the nN order permutation matrix $P = P_s \cdots P_1$ (Each row and column has only one element of 1, while the remaining elements are 0), where P_i is the first class of elementary row transformation matrix (i.e., the matrix after permuting two rows of a unit matrix), such that the following equation holds*

$$P(A \otimes B)P^{-1} = B \otimes A, \tag{9}$$

where $i = 1, \ldots, s$, $\otimes$ is Kronecker product.

Lemma 2 (*[38]*)**.** *For matrices A, B, C and D with appropriate dimensions, the Kronecker product $\otimes$ which satisfies the following properties*

(i) $(\kappa A) \otimes B = A \otimes (\kappa B)$,
(ii) $(A + B) \otimes C = A \otimes C + B \otimes C$,
(iii) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$,
(iv) $(A \otimes B)^T = A^T \otimes B^T$,

where κ is a constant.

3. Main results

3.1. Partial component consensus via aperiodically intermittent pinning control

In this subsection, the ith follower agent system (3) with aperiodically intermittent pinning control protocol (5) can be described as

$$\begin{cases} \dfrac{dx_i(t)}{dt} = f(x_i(t)) + c\sum_{j\in N_i} a_{ij}\Gamma(x_j(t)-x_i(t)) - cb_i\Gamma(x_i(t)-x_0(t)), \\ \qquad t\in[t_k, m_k], \\ \dfrac{dx_i(t)}{dt} = f(x_i(t)) + c\sum_{j\in N_i} a_{ij}\Gamma(x_j(t)-x_i(t)), \\ \qquad t\in(m_k, t_{k+1}), \end{cases} \tag{10}$$

where $i = 1, 2, \ldots, N$, $k \in \mathbb{N}$ and $x_i(t) \in \mathbb{R}^n$ stands for the n-dimensional state vector of the ith agent.

Remark 6. It is obvious that the system (10) can be divided into two parts. The pinning control protocol is activated during the control duration $[t_k, m_k] > 0$ and off during the rest interval $(m_k, t_{k+1}) > 0$. This control strategy is referred to as aperiodically intermittent pinning control strategy.

Taking the state error as $e_i(t) = x_i(t) - x_0(t)$, $i = 1, 2, \ldots, N$, the error system can be written as

$$\begin{cases} \dfrac{de_i(t)}{dt} = f(x_i(t)) - f(x_0(t)) - c\sum_{j\in N_i} l_{ij}\Gamma(x_j(t)-x_0(t)) + cb_i\Gamma(x_0(t)-x_i(t)), \\ \qquad i = 1, 2, \ldots, N,\ t\in[t_k, m_k],\ k\in\mathbb{N}, \\ \dfrac{de_i(t)}{dt} = f(x_i(t)) - f(x_0(t)) - c\sum_{j\in N_i} l_{ij}\Gamma(x_j(t)-x_0(t)), \\ \qquad i = 1, 2, \ldots, N,\ t\in(m_k, t_{k+1}),\ k\in\mathbb{N}. \end{cases} \tag{11}$$

It is simple to show that the error system in the vector form as

$$\begin{cases} \dfrac{de(t)}{dt} = F(e(t)) - (c\tilde{L}\otimes\Gamma)e(t),\ t\in[t_k, m_k], \\ \dfrac{de(t)}{dt} = F(e(t)) - (cL\otimes\Gamma)e(t),\ t\in(m_k, t_{k+1}), \end{cases} \tag{12}$$

where $e(t) = (e_1(t)^T, \ldots, e_N(t)^T)^T \in \mathbb{R}^{Nn}$, $e_i(t) = (e_{i1}(t), \ldots, e_{in}(t))^T = (x_{i1}-x_{01}, \ldots, x_{in}-x_{0n})^T \in \mathbb{R}^n$, $i = 1, 2, \ldots, N$, x_{ij} denotes the jth component of ith agent, $\tilde{L} = L + \mathrm{diag}(b_1, \ldots, b_N)$ and $F(e(t)) = ((f(x_1(t)) - f(x_0(t)))^T, \ldots, (f(x_N(t)) - f(x_0(t)))^T)^T$.

In light of Lemma 1 and permutation matrix method, we will rearrange the components such that the ones that need to be studied can rank ahead in the error system (12). Therefore, the order of Nn error system turns to the new error system of nN dimensions. The crucial technique for proving partial component consensus is the following new error system

$$\begin{cases} \dfrac{d\tilde{e}(t)}{dt} = \tilde{F}(\tilde{e}(t)) - [c\Gamma\otimes\tilde{L}]\tilde{e}(t),\ t\in[t_k, m_k], \\ \dfrac{d\tilde{e}(t)}{dt} = \tilde{F}(\tilde{e}(t)) - [c\Gamma\otimes L]\tilde{e}(t),\ t\in(m_k, t_{k+1}), \end{cases} \tag{13}$$

where $\tilde{e}(t) = (\tilde{e}_1(t)^T, \ldots, \tilde{e}_n(t)^T)^T \in \mathbb{R}^{nN}$, $\tilde{e}_q(t) = (\tilde{e}_{1q}(t), \ldots, \tilde{e}_{Nq}(t))^T = (x_{1q}-x_{0q}, \ldots, x_{Nq}-x_{0q})^T \in \mathbb{R}^N$, $q = 1, 2, \ldots, n$, $\tilde{F}(\tilde{e}(t)) = (\tilde{f}_1^T, \tilde{f}_2^T, \ldots, \tilde{f}_n^T)^T$ and $\tilde{f}_q^T = (f_q(x_1(t)), f_q(x_2(t)), \ldots, f_q(x_N(t)))^T - f_q(x_0(t))1_N$.

Remark 7. Noting that most of consensus problems directly analyze the stability of the error system defined by the state error equation, while partial component consensus problem needs to transform the original state error system into a new error system by means of permutation matrix method to analyze its stability.

Definition 3 (*[34]*)**.** The multi-agent system with (2) and (3) with respect to the first l components can reach partial component consensus, if there exists $1 \le l \le n$, for any initial condition, such that the solutions of the system with (2) and (3) satisfy

$$\lim_{t\to+\infty}\sum_{q=1}^{l}\left\|\tilde{e}_q(t)\right\| = 0, \tag{14}$$

where $q = 1, 2, \ldots, n$, $\tilde{e}_q(t) = (\tilde{e}_{1q}, \ldots, \tilde{e}_{Nq})^T = (x_{1q}-x_{0q}, \ldots, x_{Nq}-x_{0q})^T \in \mathbb{R}^N$.

Remark 8. From Definition 3, if $l = n$, then partial component consensus is called identical consensus. It means that partial component consensus is more general dynamics behavior than identical consensus.

Remark 9. Note that if the output variable of system (1) in [11] is regarded as a partial variable of this system, then the leader-following output consensus is slightly stronger than partial component consensus in this paper. In fact, in [11] the output consensus is one more bounded condition than the partial component consensus.

Theorem 1. *Suppose that the nonlinear function $f(\cdot)$ satisfies Assumption 1 with constant ε_1. Let*

$$\Phi = \mathrm{diag}(\phi_1, \phi_2, \dots, \phi_N) > 0, (\Phi\tilde{L})^s = \frac{(\Phi\tilde{L})^T + \Phi\tilde{L}}{2}, (\Phi L)^s = \frac{(\Phi L)^T + \Phi L}{2}.$$

Then, partial component consensus in system (3) with aperiodically intermittent pinning control (4) is achieved if the following conditions hold

$$\text{(i) } c > \frac{\lambda_{\max}(\Phi)\varepsilon_1}{\min\limits_{1\le q\le l} \gamma_q \lambda_{\min}((\Phi\tilde{L})^s)}, \tag{15}$$

$$\text{(ii) } \lim_{p\to+\infty}\left[\mu_1 \sum_{k=0}^{p}(m_k - t_k) + \mu_2 \sum_{k=0}^{p}(t_{k+1} - m_k)\right] = -\infty, \tag{16}$$

where $\mu_1 = 2\Big(\varepsilon_1 - c\,\min_{1\le q\le l}\gamma_q\lambda_{\min}((\Phi\tilde{L})^s)\lambda_{\max}^{-1}(\Phi)\Big)$, $\mu_2 = 2\Big(\varepsilon_1 - c\,\max_{1\le q\le l}\gamma_q\lambda_{\min}((\Phi L)^s)\lambda_{\max}^{-1}(\Phi)\Big)$.

Proof. Denote $\Lambda = \mathrm{diag}(\overbrace{1, \dots, 1}^{l}, 0, \dots, 0)$ and positive diagonal matrix $\Phi = \mathrm{diag}(\phi_1, \phi_2, \dots, \phi_N)$. Take the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\sum_{q=1}^{n}\tilde{e}_q(t)(\Lambda_q \otimes \Phi)\tilde{e}_q(t) = \frac{1}{2}\tilde{e}(t)^T(\Lambda \otimes \Phi)\tilde{e}(t). \tag{17}$$

In this proof, we only need to prove that the first l components of Lyapunov function $V(t)$ is positive definite and obtain that

$$\begin{aligned} V(t) &= \frac{1}{2}\tilde{e}(t)^T(\Lambda \otimes \Phi)\tilde{e}(t) \\ &\ge \frac{1}{2}\lambda_{\min}(\Phi)\tilde{e}(t)^T(\Lambda \otimes I_N)\tilde{e}(t), \end{aligned} \tag{18}$$

where $\lambda_{\min}(\Phi) > 0$.

By virtue of diagonal matrix Φ and Λ, we can easily get $V(t) \ge 0$ and $V_q(t) > 0$, $q = 1, \dots, l$.

Similarly,

$$\begin{aligned} V(t) &= \frac{1}{2}\tilde{e}(t)^T(\Lambda \otimes \Phi)\tilde{e}(t) \\ &\le \frac{1}{2}\lambda_{\max}(\Phi)\tilde{e}(t)^T(\Lambda \otimes I_N)\tilde{e}(t), \end{aligned} \tag{19}$$

where $\lambda_{\max}(\Phi) > 0$.

When $t \in [t_k, m_k]$, $k \in \mathbb{N}$, by Assumption 1, we can calculate the time derivative of function $V(t)$ along the trajectories of (13) as follows

$$\begin{aligned} \left.\frac{dV(t)}{dt}\right|_{(13)} &= \tilde{e}(t)^T(\Lambda \otimes \Phi)\frac{d\tilde{e}(t)}{dt} \\ &= \tilde{e}(t)^T(\Lambda \otimes \Phi)\tilde{F}(\tilde{e}(t)) - c\tilde{e}(t)^T(\Lambda \otimes \Phi)(\Gamma \otimes \tilde{L})\tilde{e}(t) \\ &= \tilde{e}(t)^T(\Lambda \otimes \Phi)\tilde{F}(\tilde{e}(t)) - c\tilde{e}(t)^T(\Lambda\Gamma \otimes (\Phi\tilde{L}))\tilde{e}(t) \end{aligned}$$

$$
\begin{aligned}
&= \tilde{e}(t)^T \begin{pmatrix} \Phi & & & & \\ & \ddots & & & \\ & & \Phi & & \\ & & & 0_N & \\ & & & & \ddots \\ & & & & & 0_N \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \vdots \\ \tilde{f}_l \\ \tilde{f}_{l+1} \\ \vdots \\ \tilde{f}_n \end{pmatrix} - c\tilde{e}(t)^T \begin{pmatrix} \gamma_1 \Phi \tilde{L} & & & & \\ & \ddots & & & \\ & & \gamma_l \Phi \tilde{L} & & \\ & & & 0_N & \\ & & & & \ddots \\ & & & & & 0_N \end{pmatrix} \tilde{e}(t) \\
&= \sum_{q=1}^{l} \tilde{e}_q(t)^T \Lambda_q \Phi \tilde{f}_q - c \sum_{q=1}^{l} \tilde{e}_q(t)^T \gamma_q \Lambda_q \Phi \tilde{L} \tilde{e}_q(t) \\
&\le \sum_{q=1}^{l} \tilde{e}_q(t)^T [\varepsilon_1 \Phi - c\gamma_q \Lambda_q \Phi \tilde{L}] \tilde{e}_q(t) \\
&= \sum_{q=1}^{l} \tilde{e}_q(t)^T [\varepsilon_1 \Phi - c\gamma_q \Lambda_q (\Phi \tilde{L})^s] \tilde{e}_q(t) \\
&= \varepsilon_1 \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c\tilde{e}(t)^T (\Lambda \Gamma \otimes (\Phi \tilde{L})^s) \tilde{e}(t) \\
&\le \varepsilon_1 \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c\lambda_{\min}((\Phi \tilde{L})^s) \tilde{e}(t)^T (\Lambda \Gamma \otimes I_N) \tilde{e}(t) \\
&\le 2\varepsilon_1 V(t) - c \min_{1 \le q \le l} \gamma_q \lambda_{\min}((\Phi \tilde{L})^s) \tilde{e}(t)^T (\Lambda \otimes I_N) \tilde{e}(t) \\
&\le 2\left(\varepsilon_1 - c \min_{1 \le q \le l} \gamma_q \lambda_{\min}((\Phi \tilde{L})^s) \lambda_{\max}^{-1}(\Phi)\right) V(t) \\
&\triangleq \mu_1 V(t).
\end{aligned} \tag{20}
$$

Instead, when $t \in (m_k, t_{k+1})$, $k \in \mathbb{N}$, in an analogous way, one has

$$
\begin{aligned}
\left.\frac{dV(t)}{dt}\right|_{(13)} &= \tilde{e}^T(t)(\Lambda \otimes \Phi) \frac{d\tilde{e}(t)}{dt} \\
&= \tilde{e}(t)^T (\Lambda \otimes \Phi)(\tilde{F}(\tilde{e}(t)) - (c\Gamma \otimes L)\tilde{e}(t)) \\
&= \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c\tilde{e}(t)^T (\Lambda \Gamma \otimes \Phi L) \tilde{e}(t) \\
&= \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c\tilde{e}(t)^T (\Lambda \Gamma \otimes (\Phi L)^s) \tilde{e}(t) \\
&\le \varepsilon_1 \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c\tilde{e}(t)^T (\Lambda \Gamma \otimes (\Phi L)^s) \tilde{e}(t) \\
&\le \varepsilon_1 \tilde{e}(t)^T (\Lambda \otimes \Phi) \tilde{e}(t) - c \max_{1 \le q \le l} \gamma_q \lambda_{\min}((\Phi L)^s) \tilde{e}(t)^T (\Lambda \otimes I_N) \tilde{e}(t) \\
&\le 2\left(\varepsilon_1 - c \max_{1 \le q \le l} \gamma_q \lambda_{\min}((\Phi L)^s) \lambda_{\max}^{-1}(\Phi)\right) V(t) \\
&\triangleq \mu_2 V(t).
\end{aligned} \tag{21}
$$

Combining with inequalities (20) and (21), one obtains

$$
\begin{cases}
\dfrac{dV(t)}{dt} \le \mu_1 V(t), t \in [t_k, m_k], k \in \mathbb{N}. \\
\dfrac{dV(t)}{dt} \le \mu_2 V(t), t \in (m_k, t_{k+1}), k \in \mathbb{N}.
\end{cases} \tag{22}
$$

Next we will discuss two cases. The following proof is analogous to that in [29,30].

Case 1. $\mu_1 < 0, \mu_2 < 0$.

In this case, we denote $\varsigma = \max\{\mu_1, \mu_2\} < 0$. From the inequality (22), we can obtain

$$
\frac{dV(t)}{dt} < \varsigma V(t). \tag{23}
$$

It is evident that $\lim_{t\to+\infty} V(t) = 0$, which means that $\lim_{t\to+\infty} \tilde{e}_1(t) = \lim_{t\to+\infty} \tilde{e}_2(t) = \cdots = \lim_{t\to+\infty} \tilde{e}_l(t) = 0$. That is, partial component consensus is realized.

Case 2. $\mu_1 < 0, \mu_2 \ge 0$.

In this case, we will consider two subcases according to t in different time intervals $t \in [t_k, m_k]$ and $t \in (m_k, t_{k+1})$, respectively. Suppose that $[t_i, m_i] \subset [t_k, m_k]$ and $(m_i, t_{i+1}) \subset (m_k, t_{k+1})$, where $i \in \mathbb{N}$.

Subcase 1. For $t \in [t_k, m_k]$, $t \to +\infty$, $k \in \mathbb{N}$, one has

$$V(t) \leq V(0)\exp\left\{\sum_{i=0}^{i=k-1}\int_{t_i}^{m_i}\mu_1 dt + \int_{t_k}^{t}\mu_1 dt + \sum_{i=0}^{i=k-1}\int_{m_i}^{t_{i+1}}\mu_2 dt\right\}. \tag{24}$$

If $t - t_k \to +\infty$, owing to $\mu_1 < 0$, then we have

$$\left\{\sum_{i=0}^{i=k-1}\int_{t_i}^{m_i}\mu_1 dt + \int_{t_k}^{t}\mu_1 dt + \sum_{i=0}^{i=k-1}\int_{m_i}^{t_{i+1}}\mu_2 dt\right\} \to -\infty.$$

It is easy to obtain $\lim_{t\to+\infty}V(t) = 0$. In other words, $\lim_{t\to+\infty}\tilde{e}_1(t) = \lim_{t\to+\infty}\tilde{e}_2(t) = \cdots = \lim_{t\to+\infty}\tilde{e}_l(t) = 0$.

If $t - t_k < +\infty$, then $t \to +\infty$, which implies that $k \to +\infty$. By inequality (22), we can get

$$\begin{aligned} V(t) &\leq V(0)\exp\left\{\sum_{i=0}^{i=k-1}\int_{t_i}^{m_i}\mu_1 dt + \sum_{i=0}^{i=k-1}\int_{m_i}^{t_{i+1}}\mu_2 dt\right\} \\ &= V(0)\exp\left\{\sum_{i=0}^{i=k-1}\mu_1(m_i - t_i) + \sum_{i=0}^{i=k-1}\mu_2(t_{i+1} - m_i)\right\} \\ &\leq V(0)\exp\left\{\sum_{k=0}^{p}\mu_1(m_k - t_k) + \sum_{k=0}^{p}\mu_2(t_{k+1} - m_k)\right\}. \end{aligned} \tag{25}$$

According to the condition (16), it follows that

$$\left\{\sum_{k=0}^{p}\mu_1(m_k - t_k) + \sum_{k=0}^{p}\mu_2(t_{k+1} - m_k)\right\} \to -\infty.$$

Therefore, $p \to +\infty$ means $t \to +\infty$. It is easy to show that $\lim_{t\to+\infty}V(t) = 0$. That is, $\lim_{t\to+\infty}\tilde{e}_1(t) = \lim_{t\to+\infty}\tilde{e}_2(t) = \cdots = \lim_{t\to+\infty}\tilde{e}_l(t) = 0$.

Subcase 2. For $t \in (m_k, t_{k+1})$, $k \in \mathbb{N}$. In this case, we have

$$\begin{aligned} V(t) &\leq V(0)\exp\left\{\sum_{i=0}^{i=k}\int_{t_i}^{m_i}\mu_1 dt + \sum_{i=0}^{i=k-1}\int_{m_i}^{t_{i+1}}\mu_2 dt + \int_{m_k}^{t}\mu_1 dt\right\} \\ &\leq V(0)\exp\left\{\sum_{i=0}^{i=k}\int_{t_i}^{m_i}\mu_1 dt + \sum_{i=0}^{i=k-1}\int_{m_i}^{t_{i+1}}\mu_2 dt\right\} \\ &= V(0)\exp\left\{\sum_{i=0}^{i=k}\mu_1(m_i - t_i) + \sum_{i=0}^{i=k}\mu_2(t_{i+1} - m_i)\right\} \\ &\leq V(0)\exp\left\{\sum_{k=0}^{p}\mu_1(m_k - t_k) + \sum_{k=0}^{p}\mu_2(t_{k+1} - m_k)\right\}. \end{aligned} \tag{26}$$

In summary, adding up equations (23), (25) and (26), we can obtain

$$\lim_{p\to+\infty}\left[\mu_1\sum_{k=0}^{p}(m_k - t_k) + \mu_2\sum_{k=0}^{p}(t_{k+1} - m_k)\right] = -\infty.$$

Thus, $t \to +\infty$ means $p \to +\infty$. It is clearly that $\lim_{t\to+\infty}V(t) = 0$, which implies that $\tilde{e}_q(t) \to 0$, as $t \to +\infty$, $q = 1, \ldots, l$ (i.e., $\lim_{t\to+\infty}\tilde{e}_1(t) = \lim_{t\to+\infty}\tilde{e}_2(t) = \cdots = \lim_{t\to+\infty}\tilde{e}_l(t) = 0$). It indicates that error systems (13) with respect to the first l components convergence exponentially. Thereby, the multi-agent systems (10) can achieve partial component consensus under aperiodically intermittent pinning control. The complete the proof of Theorem 1. □

Remark 10. Compared with the use of the continuous control strategy to reach partial component consensus in literatures [34,36], this paper utilizes the aperiodically intermittent pinning control algorithm, which is more reasonable and effective in practical applications. Note that if for any $k \in \mathbb{N}$, $t_{k+1} - m_k \equiv 0$, that is to say, the control is continuous, then the results in [34,36] are special case in our results. Moreover, in this paper the control protocol (4) is also pinning control protocol, which means that only parts of agents need to be pinned such that the multi-agent systems can reach consensus.

Remark 11. When $l = n$, our result is similar to the Theorem 1 of literature [26]. Besides, In [8], it proved that for first-order multi-agent systems if and only if the network topology contains a spanning tree, the systems can achieve consensus. In [34], the authors pointed that for a weighted directed graph, if the node with weak ability to receive neighbor information, then it should also be controlled by the leader agent to ensure that system can reach partial component consensus. Hence, we should select some nodes, such as root nodes of connected branches, weak weight nodes, isolated nodes as pinning nodes in networks to investigate partial component consensus.

Remark 12. In [11], the output consensus criteria of multi-agent systems under event-triggered control are derived by means of Barbalat's lemma and Hurwitz matrix properties. In this paper, based on exponential stability of partial variables, the sufficient conditions for the partial component consensus of multi-agent systems via aperiodically intermittent pinning control are obtained.

If there exists an infinite time sequence with uniformly bounded and no-overlapping time $[t_k, t_{k+1})$, $k \in \mathbb{N}$ such that each time interval $[t_k, t_{k+1})$ satisfies the work width $\theta = \min\{m_k - t_k, k \in \mathbb{N}\}$ and rest width $\beta = \max\{t_{k+1} - m_k, k \in \mathbb{N}\}$ in Theorem 1, then one can easily get the following corollary.

Corollary 1. *Suppose that Assumptions 1 and 3 hold. The designed controller $u_i(t)$ is aperiodically intermittent pinning control (4). If the condition (i) of Theorem 1 is satisfied, and there exist two constants θ and β, such that*

$$\mu_1\theta + \mu_2\beta < 0, \tag{27}$$

the corresponding parameters are defined uniformly by Theorem 1. Then, the multi-agent system with (2) and (3) can achieve partial component consensus for any initial value.

Remark 13. In [26–30], the intermittent control assumption of control durations and rest durations in time intermittent intervals is similar to Assumption 3, and the results are conservative. In order to accord with actual requirements, the authors in the Remark 6 of [31] reassumed that for all $k \in \mathbb{N}^+ = \{1, 2, \ldots\}$, $\mathrm{T}_k = t_{k+1} - t_k$, $\mathrm{m}_k = t_k + \theta_k$, $\sup_{k\in\mathbb{N}^+}\{t_{k+1} - t_k\} = \mathrm{T}_{\sup}$ and $\inf_{k\in\mathbb{N}^+}\{t_{k+1} - t_k\} = \mathrm{T}_{\inf}$. Denote $\mathrm{T}_0 = \hat{\mathrm{T}}_0 = t_1$, $\hat{\mathrm{T}}_k = \sum_{j=0}^{k}\mathrm{T}_j$, $\delta_k = \theta_k/\mathrm{T}_k$, where δ_k is the ratio of the kth control width θ_k to the kth time width T_k. Then, one gets $t_k = \hat{\mathrm{T}}_{k-1}$ and $\theta_k = \delta_k\mathrm{T}_k$, $k \in \mathbb{N}^+$. As a result, Corollary 2 can be obtained.

Corollary 2. *Suppose that $\inf_{k\in\mathbb{N}^+}\delta_k = \inf_{k\in\mathbb{N}^+}\left\{\frac{\theta_k}{\mathrm{T}_k}\right\} = \delta_{\inf} > 0$. Under Assumption 1 and the aperiodically intermittent pinning controller (4), if the condition (i) of Theorem 1 is satisfied, and*

$$(\mu_1 - \mu_2)\,\delta_{\inf} + \mu_2 < 0, \tag{28}$$

the μ_1 and μ_2 are defined identically by Theorem 1. Then, the multi-agent system with (2) and (3) can reach partial component consensus for any initial value.

Remark 14. According to Assumption 3 and Corollary 2, it follows that $\beta_k < \beta$, $(t_{k+1} - t_k) - \theta_k < \mathrm{T}_{\sup} - \theta_{\inf}$. Clearly, inequality (27) can be written as $\mu_1\theta_{\inf} + \mu_2\left(\mathrm{T}_{\sup} - \theta_{\inf}\right) < 0$. Using the fact that $\mathrm{T}_{\sup} > 0$, then dividing both sides by $\mathrm{T}_{\sup}$, one has

$$(\mu_1 - \mu_2)\frac{\theta_{\inf}}{\mathrm{T}_{\sup}} + \mu_2 < 0. \tag{29}$$

Notice that for all $k \in \mathbb{N}^+$, $\theta_k > \inf_{k\in\mathbb{N}^+}\theta_k = \theta_{\inf}$, $\mathrm{T}_k < \mathrm{T}_{\sup}$, and therefore

$$\delta_{\inf} = \inf_{k\in\mathbb{N}^+}\left\{\frac{\theta_k}{\mathrm{T}_k}\right\} \geq \frac{\theta_{\inf}}{\mathrm{T}_{\sup}}.$$

It means that the limitation condition of intermittent control strategy of inequality (28) derived by using $\delta_{\inf}$ is easier to be satisfied and less conservative than inequality (29) obtained by using $\theta_{\inf}/\mathrm{T}_{\sup}$.

3.2. Partial component consensus with periodically intermittent pinning control

In this subsection, by using a similar heuristic analysis, we will discuss the partial component consensus problem of leader-following multi-agent system (3) via periodically intermittent pinning control (5), and then obtain the relevant sufficient conditions.

The ith follower agent system (3) with periodically intermittent pinning control is given by

$$\begin{cases} \dfrac{dx_i(t)}{dt} = f(x_i(t)) + c\displaystyle\sum_{j\in N_i} a_{ij}\Gamma(x_j(t) - x_i(t)) + cb_i\Gamma(x_0(t) - x_i(t)), \\ \qquad t \in [k\omega, k\omega + \theta], \\ \dfrac{dx_i(t)}{dt} = f(x_i(t)) + c\displaystyle\sum_{j\in N_i} a_{ij}\Gamma(x_j(t) - x_i(t)), \\ \qquad t \in (k\omega + \theta, (k+1)\omega), \end{cases} \tag{30}$$

where $i = 1, 2, \dots, N$, $k \in \mathbb{N}$. $x_i(t) \in \mathbb{R}^n$ stands for the n-dimensional state vector of the ith agent. For any period of time $\omega > 0$, time intervals θ and $\omega - \theta$ represent the "work time" and "rest time", respectively.

According to the state error $e_i(t) = x_i(t) - x_0(t)$ and permutation matrix method, a new error system under periodically intermittent control can be obtained by the same method as employed in the last section

$$\begin{cases} \dfrac{d\tilde{e}(t)}{dt} = \tilde{F}(\tilde{e}(t)) - [c\Gamma \otimes \tilde{L}]\tilde{e}(t), t \in [k\omega, k\omega + \theta], \\ \dfrac{d\tilde{e}(t)}{dt} = \tilde{F}(\tilde{e}(t)) - [c\Gamma \otimes L]\,\tilde{e}(t), t \in (k\omega + \theta, (k+1)\omega), \end{cases} \tag{31}$$

where $\tilde{L} = L + \text{diag}(b_1, \dots, b_N)$, $\tilde{e}(t) = (\tilde{e}_1(t)^T, \dots, \tilde{e}_n(t)^T)^T \in \mathbb{R}^{nN}$, $\tilde{e}_q(t) = (\tilde{e}_{1q}(t), \dots, \tilde{e}_{Nq}(t))^T \in \mathbb{R}^N$, $q = 1, 2, \dots, n$, $\tilde{F}(\tilde{e}(t)) = (\tilde{f}_1^T, \tilde{f}_2^T, \dots, \tilde{f}_n^T)^T$ and $\tilde{f}_q^T = (f_q(x_1(t)), f_q(x_2(t)), \dots, f_q(x_N(t)))^T - 1_N \otimes f_q(x_0(t))$.

Theorem 2. *Suppose that the nonlinear function $f(\cdot)$ satisfies Assumption 2 with constant ε_2. The system with (2) and (3) can reach partial component consensus by using periodically intermittent pinning control (5). If there exist two positive constants $\bar{\mu}_1$ and $\bar{\mu}_2$, such that the following conditions hold*

$$\text{(i) } (\varepsilon_2 + \frac{1}{2}\bar{\mu}_1)\Xi - \Xi(c\Gamma \otimes \tilde{L}^s) \le 0, \tag{32}$$

$$\text{(ii) } (\varepsilon_2 - \frac{1}{2}\bar{\mu}_2)\Xi - \Xi(c\Gamma \otimes L^s) \le 0, \tag{33}$$

$$\text{(iii) } -\bar{\mu}_1\theta + \bar{\mu}_2(\omega - \theta) < 0. \tag{34}$$

Proof. Denote $\Xi = \text{diag}(\overbrace{1, \dots, 1}^{lN}, 0, \dots, 0)$. Construct the following Lyapunov function candidate

$$W(t) = \frac{1}{2}\tilde{e}(t)\Xi\tilde{e}(t). \tag{35}$$

It will be shown that $W(t)$ is a valid Lyapunov function for analyzing the stability of new error dynamical system described by system (31).

When $t \in [k\omega, k\omega + \theta]$, $k \in \mathbb{N}$, the derivative of $W(t)$ along with the trajectories of (31) is

$$\begin{aligned} \left.\frac{dW(t)}{dt}\right|_{(31)} &= \tilde{e}(t)^T\Xi\frac{d\tilde{e}(t)}{dt} \\ &= \tilde{e}(t)^T\Xi(\tilde{F}(\tilde{e}(t)) - (c\Gamma \otimes \tilde{L})\tilde{e}(t)) \\ &= \tilde{e}(t)^T\Xi\tilde{F}(\tilde{e}(t)) - c\tilde{e}(t)^T\Xi(\Gamma \otimes \tilde{L})\tilde{e}(t) \\ &\le \varepsilon_2\tilde{e}(t)^T\Xi\tilde{e}(t) - c\tilde{e}(t)^T\Xi(\Gamma \otimes \tilde{L}^s)\tilde{e}(t) \\ &= \tilde{e}(t)^T[\varepsilon_2\Xi - \Xi(c\Gamma \otimes \tilde{L}^s) + \frac{1}{2}\bar{\mu}_1\Xi]\tilde{e}(t) - \frac{1}{2}\bar{\mu}_1\tilde{e}(t)^T\Xi\tilde{e}(t) \\ &\le -\bar{\mu}_1 W(t). \end{aligned} \tag{36}$$

Therefore, it is clearly that the inequality $\frac{dW(t)}{dt} \le -\bar{\mu}_1 W(t)$. By integral the both sides of the inequality

$$\int_{k\omega}^{k\omega+\theta} \frac{dW(t)}{W(t)} \le \int_{k\omega}^{k\omega+\theta} -\bar{\mu}_1 dt,$$

we can obtain

$$W(t) \le W(k\omega)e^{-\bar{\mu}_1(t-k\omega)}. \tag{37}$$

Similarly, when $t \in (k\omega + \theta, (k+1)\omega)$, $k \in \mathbb{N}$, one has

$$\begin{aligned} \left.\frac{dW(t)}{dt}\right|_{(31)} &= \tilde{e}(t)^T\Xi\frac{d\tilde{e}(t)}{dt} \\ &= \tilde{e}(t)^T\Xi(\tilde{F}(\tilde{e}(t)) - (c\Gamma \otimes L)\tilde{e}(t)) \\ &= \tilde{e}(t)^T\Xi\tilde{F}(\tilde{e}(t)) - c\tilde{e}(t)^T\Xi(\Gamma \otimes L^s)\tilde{e}(t) \\ &\le \varepsilon_2\tilde{e}(t)^T\Xi\tilde{e}(t) - c\tilde{e}(t)^T\Xi(\Gamma \otimes L^s)\tilde{e}(t) \\ &= \tilde{e}(t)^T[\varepsilon_2\Xi - \Xi(c\Gamma \otimes L^s) - \frac{1}{2}\bar{\mu}_2\Xi]\tilde{e}(t) + \frac{1}{2}\bar{\mu}_2\tilde{e}(t)^T\Xi\tilde{e}(t) \\ &\le \bar{\mu}_2 W(t). \end{aligned} \tag{38}$$

Hence, according to $\frac{dW(t)}{dt} \le \bar{\mu}_2 W(t)$, it follows that

$$W(t) \le W(k\omega + \theta)e^{\bar{\mu}_2(t-k\omega-\theta)}. \tag{39}$$

The following statement can be proved by the similar method in [22]. Combining with inequalities (37) and (39), we get

$$\begin{cases} W(t) \le W(k\omega)e^{-\bar{\mu}_1(t-k\omega)}, \\ W(t) \le W(k\omega + \theta)e^{\bar{\mu}_2(t-k\omega-\theta)}. \end{cases} \tag{40}$$

Consequently, we infer that

$$\begin{aligned} W((k+1)\omega) &\le W(k\omega + \theta)e^{\bar{\mu}_2(\omega-\theta)} \\ &\le W(k\omega)\exp\{-\bar{\mu}_1\theta + \bar{\mu}_2(\omega-\theta)\} \\ &\le \cdots \le W(0)\exp\{(-\bar{\mu}_1\theta + \bar{\mu}_2(\omega-\theta))(k+1)\}. \end{aligned} \tag{41}$$

Thus, we have derived that $\lim_{t\to+\infty} W(t) = 0$, which implies that $\tilde{e}_q(t) \to 0$, as $t \to +\infty$, $q = 1, \ldots, l$. i.e., $\lim_{t\to+\infty}\tilde{e}_1(t) = \lim_{t\to+\infty}\tilde{e}_2(t) = \cdots = \lim_{t\to+\infty}\tilde{e}_l(t) = 0$. Therefore, the error system (31) is exponential stability, namely, the multi-agent system (30) can reach partial component consensus via periodically intermittent pinning control. The complete the proof of Theorem 2. □

Remark 15. Differing from [21,23], the periodically intermittent control law was used to deal with partial component consensus problem in this paper. Besides, if there are no time-delay in [21,23], then the results can be easily derived by Theorem 2. It means that identical consensus is a special case for partial component consensus.

Remark 16. Compared with the use of two fixed values to limit periodically intermittent control law in [21–23], this paper chooses two optional values to analyze the periodically intermittent constraint conditions, which is more flexible and controllable in practical applications.

Remark 17. Note that Theorem 2 cannot be derived directly from Theorem 1, since the constants μ_1 and μ_2 in Theorem 1 are explicit rather than the implicit values $\bar{\mu}_1$ and $\bar{\mu}_2$ in Theorem 2. That is to say, μ_1 and μ_2 in Theorem 1 are fixed values, while $\bar{\mu}_1$ and $\bar{\mu}_2$ in Theorem 2 are optional values. Moreover, if we only consider the constraint condition of intermittent control and the relevant parameters are same, then the condition (iii) in Theorem 2 can be regarded as a special case of condition (ii) in Theorem 1. In particular, with the help of Corollary 1, one can easily obtain the condition (iii) of Theorem 2.

4. Numerical simulations

In this section, we give some numerical simulations to illustrate the effectiveness of the theoretical results.

Without loss generality, we will consider the multi-agent system with (10) and (30) which consists of a leader agent and four follower agents, where the network communication topology structures are shown in Fig. 1. The nonlinear function $f(\cdot)$ is described by Chua's circuit [34]

$$f(x_i(t)) = \begin{pmatrix} ax_{i2} - bx_{i1} + \zeta(x_{i1}), \\ -x_{i1} + x_{i2} - x_{i2}x_{i3}^2, \\ -\epsilon x_{i2} \end{pmatrix}, \tag{42}$$

where $\zeta(x_{i1}) = \varpi(|x_{i1} + 1| - |x_{i1} - 1|)$, $a = 1$, $b = 3$, $\varpi = 2$, $\epsilon = 2$. In view of Assumptions 1 and 2, by computations, one obtains $\varepsilon_1 = \varepsilon_2 = 1$.

For simplicity, In this simulation we may choose $\Gamma = \mathrm{diag}(1, 1, 0)$, $\Phi = I_3$ and let $N = 4$, $n = 3$, then we consider that the multi-agent system (3) via intermittent pinning control (4) and (5) can reach consensus with respect to the first two components (that is, $l = 2$). It is seen from communication network topology graphs Fig. 1 that the networks contain a directed spanning tree and the Laplacian matrix L_a and L_b are as follows

$$L_a = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, L_b = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{pmatrix}.$$

Example 1. At first, we discuss the multi-agent system (3) with periodically intermittent pinning control protocol (4). Motivated by the pinned-node selection scheme [19,34], we can pick the agents such as root nodes, weak weight nodes as pinned agent in the networks. By virtue of the theory of algebra graph, it is observed from network topology graph Fig. 1(a) and Laplacian matrix L_a that the fourth node is a root node, so we can take $B = \mathrm{diag}(0, 0, 0, 1)$. Therefore, it is easy to see that $\lambda_{\max}(\Phi) = 1$, $\lambda_{\min}((\Phi\tilde{L})^s) = 0.1910$ and $\lambda_{\min}((\Phi L)^s) = -0.3873$. We set time interval as $[0, 3]$, which

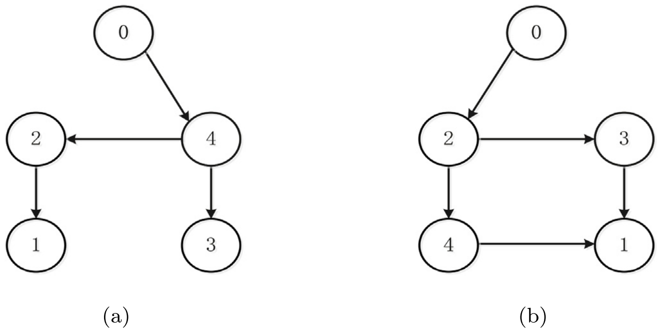


Fig. 1. The network communication topology graphs of agents, where 0 and $1-4$ stand for the leader agent and follower agents, respectively.

can guarantee the convergence of consensus error. Let the intermittent control exist on the following time interval in the next simulations

$$[0, 0.3] \cup [0.5, 0.7] \cup [0.8, 1.2] \cup [1.5, 2] \cup [2.1, 2.2] \cup [2.6, 2.85] \cup \cdots.$$

According to the control time span, we can select coupling strength or control strength $c = 6$. It is not difficult to verify that $c = 6 > 5.236$, $\mu_1 = -0.2920$ and $\mu_2 = 6.6476$, which implies that conditions (i) and (ii) in Theorem 1 are hold.

In addition, the initial values of multi-agent system (3) are taken arbitrarily as $x_1(0) = [-1.5, 3.6, -3.5]^T$, $x_2(0) = [-3.7, -3.6, -1]^T$, $x_3(0) = [-0.08, 2, -0.3]^T$, $x_4(0) = [4, 0.1, 1.5]^T$. Through numerical simulation, Fig. 2 displays the consensus errors $e_i(t)$ relative to the desired orbit for three components of four agents under the aperiodically intermittent pinning control protocol (4), where each agent contains three components. Fig. 2(a) represents the first component error trajectory of four agents, Fig. 2(b) stands for the second component error orbit of four agents and Fig. 2(c) is the third component error state of four agents. It is apparent from Figs. 2(a) and 2(b) that the first two components of the multi-agent system with (2) and (3) can reach consensus for all agents. However, Fig. 2(c) indicates that the third component of all agents is divergent. Particularly, whatever how many agents we pinned (that is, each follower agent can receive the information from the leader agent, i.e., taking $B = \mathrm{diag}(b_1, b_2, b_3, b_4) > 0$) or how bigger enough coupling strength c we take, the third component of all agents still cannot achieve consensus. It follows that the partial state variable of this system is stable. This consensus is referred to as partial component consensus.

Example 2. In this simulation, we consider the multi-agent system (3) with periodically intermittent pinning control protocol (5). According to the topology structure Fig. 1(b) and Laplacian matrix L_b, the second agent is root-node. Consequently, we can select $B = \mathrm{diag}(0, 2, 0, 0)$ and get the eigenvalue $\lambda_{\min}(\tilde{L}^s) = 0.2929$, $\lambda_{\min}(L^s) = -0.3660$. According to the inequality $(\varepsilon_2 + \frac{1}{2}\bar{\mu}_1)\Xi - \Xi(c\Gamma \otimes \tilde{L}^s) < 0$, we choose $c = 7$, $\bar{\mu}_1 = 2.1006 > 0$ and $\bar{\mu}_2 = 7.1240 > 0$ to ensure that the inequalities (32) and (33) in Theorem 2 are satisfied. In the following, we fix the intermittent control time $\theta = 0.4$ and take $\omega = 0.5$, then set time interval as $[0, 3]$. We perform intermittent control on control interval

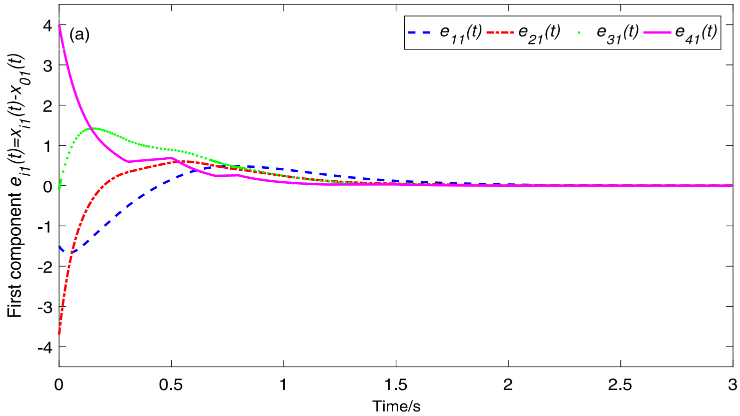
$$[0, 0.4] \cup [0.5, 0.9] \cup [1, 1.4] \cup [1.5, 1.9] \cup [2, 2.4] \cup [2.5, 2.9] \cup \cdots.$$

It is clearly that $-2.1006\theta + 7.1240(\omega - \theta) < 0$, which implies that the condition (iii) in Theorem 2 is also hold. In this simulation, we select the same initial value in Example 1, and then the validity of Theorem 2 can be demonstrated by Fig. 3. It shows the error trajectories of the four agents by pinning the second agent via periodically intermittent pinning control protocol (5). One can obtain from the Figs. 3(a) and 3(b) that the first two component of all agents can reach consensus, while the third component of all agents is divergent Fig. 3(c).

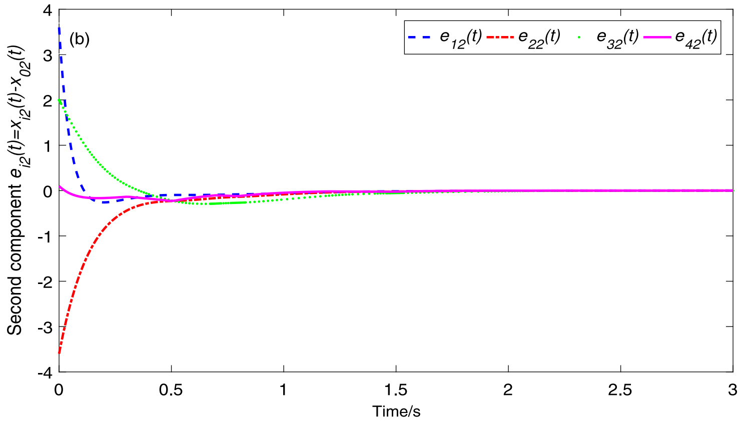
As is known to us, a routine consensus definition indicates that with time evolution of all agents will converge to a desired trajectory, while the partial component consensus is asked to with respect to the first l components of all agents reach agreement, the remainder $n - l$ components of all agents do not need to realize consensus. According to these two simulations and theoretical results, it follows that partial component consensus is a weaker group dynamics behavior than identical consensus.

5. Conclusion

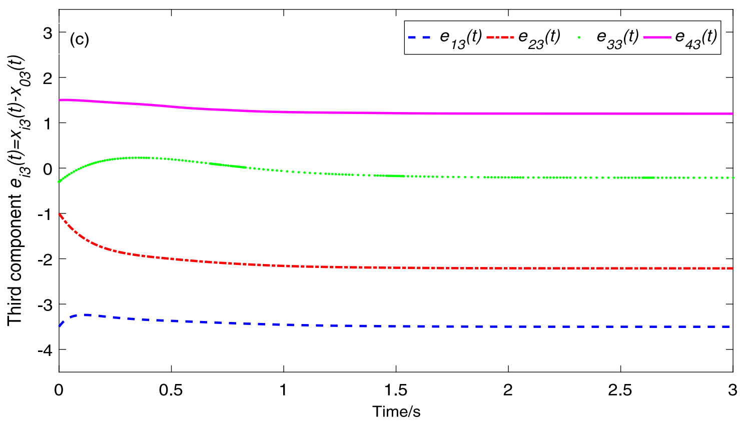
This paper has further studied the partial component consensus problems of first-order leader-following multi-agent systems with nonlinear dynamics via intermittent pinning control. Detailed analysis has been performed for two cases. For the multi-agent systems with aperiodically intermittent pinning control, its control strategy is discontinuous control input and based on pinning control protocol, only a fraction of agents need to be pinned in each work time to ensure that the systems reach partial component consensus instead of pinning many agents. Furthermore, compared with the previous



(a)



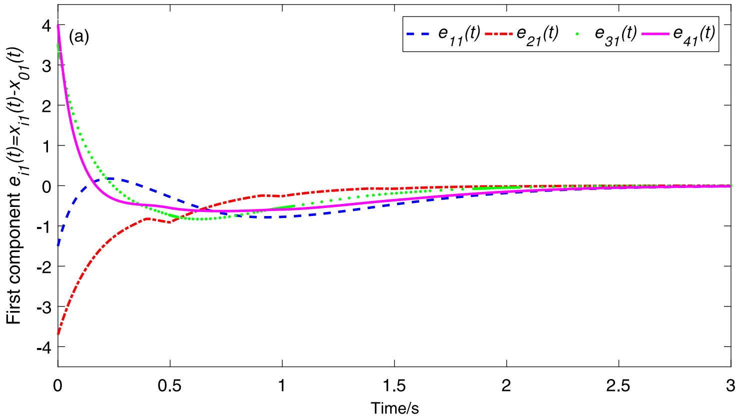
(b)



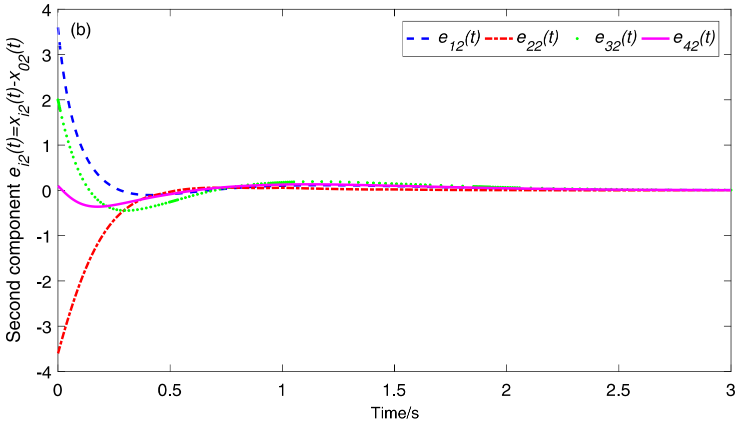
(c)

Fig. 2. The time evolution of error trajectories between the leader and each follower agent under the aperiodically intermittent pinning control (4), where $i = 1, 2, 3, 4$.

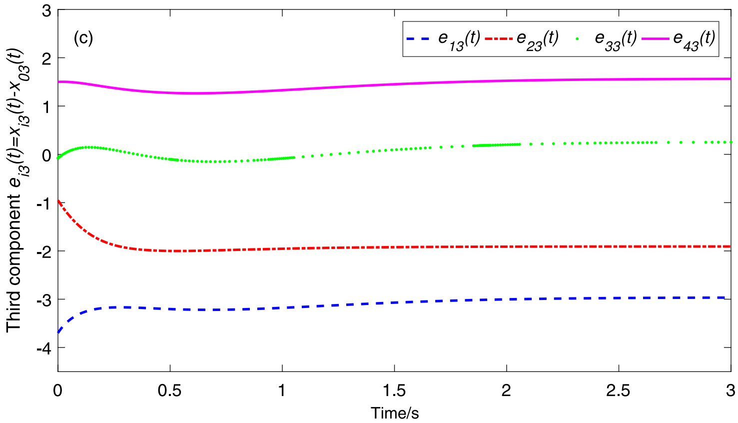
results, this paper explores partial component consensus of multi-agent systems under aperiodically and periodically intermittent pinning control, and then obtains some sufficient but not necessary consensus criteria. In addition, we substitute the new error systems for the traditional error systems to analyze the exponential stabilization of multi-agent



(a)



(b)



(c)

Fig. 3. The time evolution of error trajectories between the leader and each follower agent under the periodically intermittent pinning control (5), where $i = 1, 2, 3, 4$.

systems. Finally, two numerical examples have been provided to verify the proposed theoretical results. In future work, the communication delay factors of partial component consensus will be considered. The results obtained in this paper can be easily applied in various fields, which can promote the development of partial component consensus in the future.

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