Koopman Analysis of Weather Dynamics Using SCALE Simulation Data

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Summary

- Different from sea surface temperature (SST) data, the SCALE simulations provide generally **non-steady-state** data trajectories.
- **Spectral analysis** for weather data by **Koopman mode decomposition** (**KMD**) and **dynamic mode decomposition** (**DMD**) methods are active. For example, proper orthogonal decomposition (POD), sparsity-promoting DMD (SPDMD), extended DMD, kernel DMD, and residual DMD, etc.
- We are interested in **transient stability** and **growth behavior** for **humidity ratio** and **temperature** dataset to capture **onset of weather control**.

Koopman Operator and Dynamic Mode Decompositions

- **Data Collection**: From observed dataset $\{x_k, y_k\}_{k=1}^N$ to establish the data-driven weather dynamics

$$x_{t+1} = F(x_t), t = 0, 1, \cdots, N-1, \quad \Longleftarrow \quad Y = AX$$

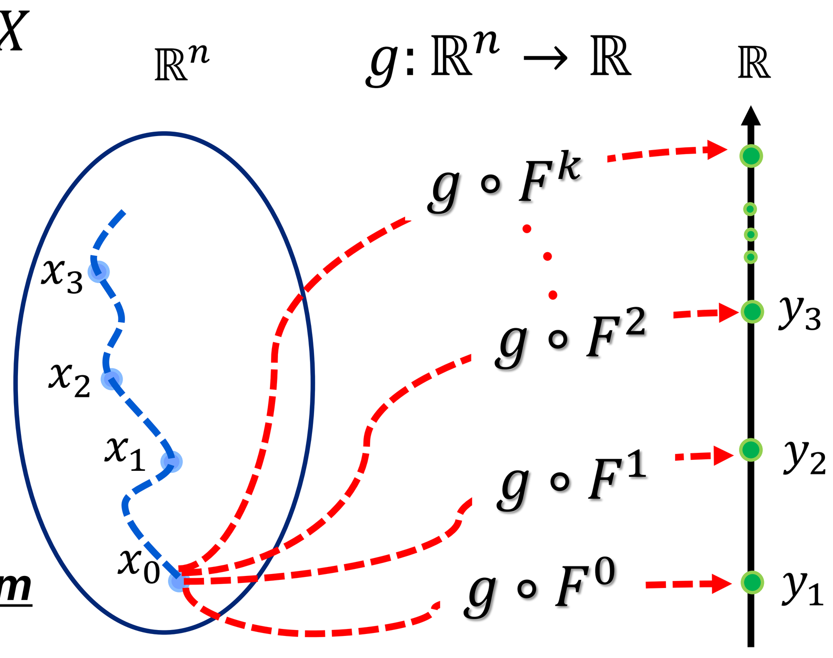
which can be estimated by a linear time invariant (LTI) system $y_t = Ax_t$

- **Dynamic Mode Decomposition (DMD)**

$$y_t = \sum_{j=1}^{N} \phi_j \lambda_j^t b_j \quad \xleftarrow[\text{Truncated SVD}]{\text{Approximation}} \quad y_t = \sum_{j=1}^{r} \phi_j \lambda_j^t b_j$$

- **Koopman Mode Decomposition (KMD):** ***Point*** **and** ***Continuous Spectrum***

$$[\mathcal{K}^t g](x_0) = g \circ F(x) = \sum_j c_j \lambda_j^t \varphi_j(x) + \int_{[-\pi,\pi]_{per}} e^{i\omega t} \psi_{\omega,g}(x_0) d\omega$$



Koopman Framework

- **Least Square Error Optimization**

$$\underbrace{[x_0, x_1, \ldots, x_{N-1}]}_{X \in \mathbb{R}^{n\times N}} \approx \underbrace{\begin{bmatrix} | & \cdots & | \\ \phi_1 & \cdots & \phi_r \\ | & \cdots & | \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_r \end{bmatrix}}_{\mathbf{D}_\mathrm{b}} \underbrace{\begin{bmatrix} 1 & \lambda_1 & \cdots & \lambda_1^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_r & \cdots & \lambda_r^{N-1} \end{bmatrix}}_{\mathbf{V}_\mathrm{and}}$$

Original data; Spatial Modes; Amplitudes; Temporal Modes

Penalize an ℓ1 norm on DMD "amplitudes"

- **(Regularized) Least Square Error Optimization**

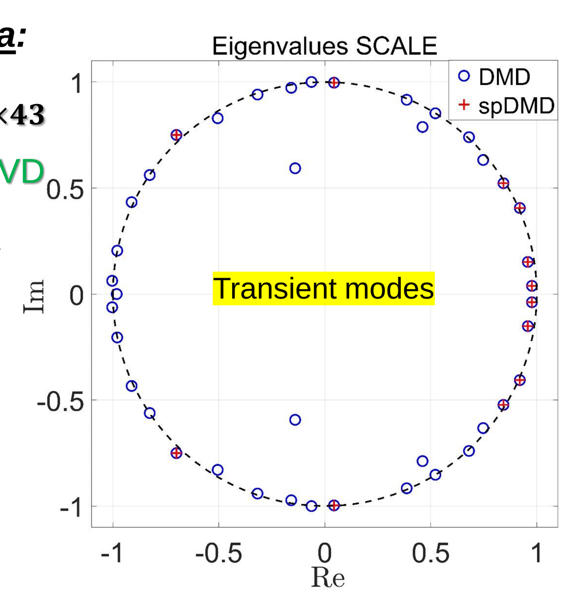
Sparsity-Promoting DMD

$$J_r(\boldsymbol{b}) = \min_{\boldsymbol{b}} ||X - \Phi \mathbf{D}_\mathrm{b} \mathbf{V}_\mathrm{and}||_2 + \gamma ||\boldsymbol{b}||_1$$

Experiment 1: Magnitude of Vortex Data via SPDMD

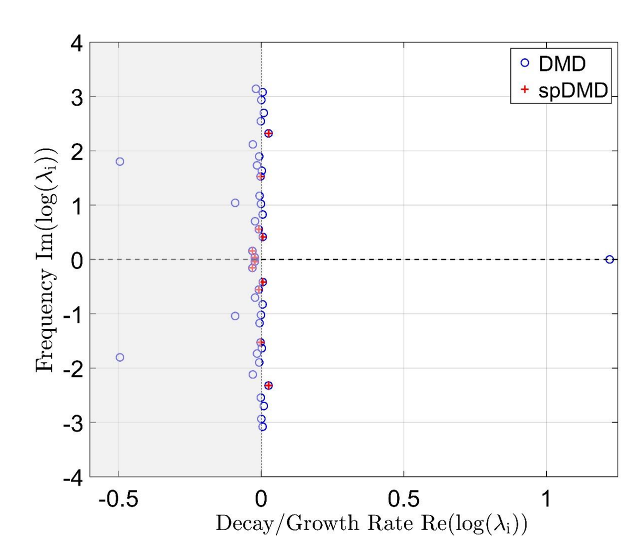
Vorticity data: $\mathbf{Y} \in \mathbb{R}^{3880\times 43}$

SPDMD + SVD

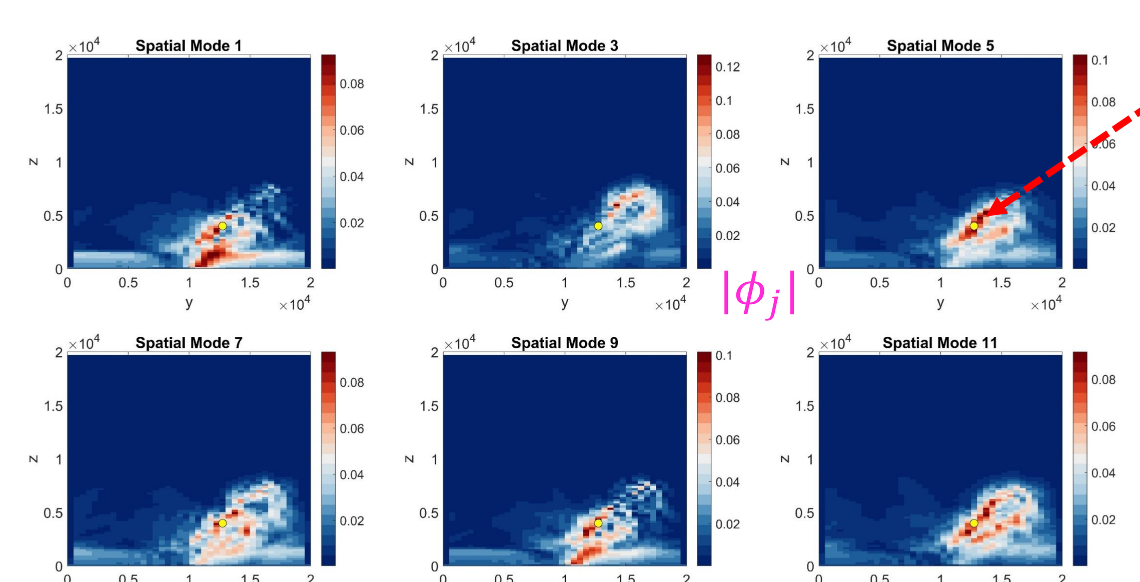


Some Eigenvalues are inside of the unit circle

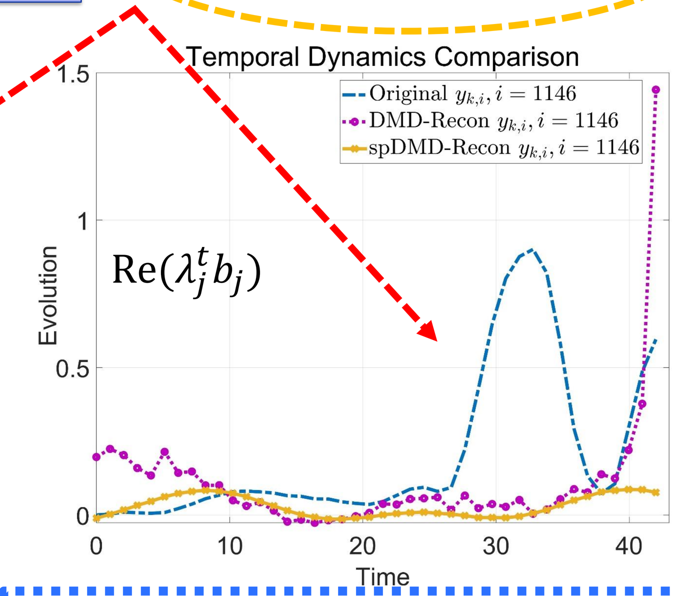
Check the ***growth or decay rate*** w.r.t. eigenvalues



Koopman Mode Decomposition: *Spatial +Temporal Modes*



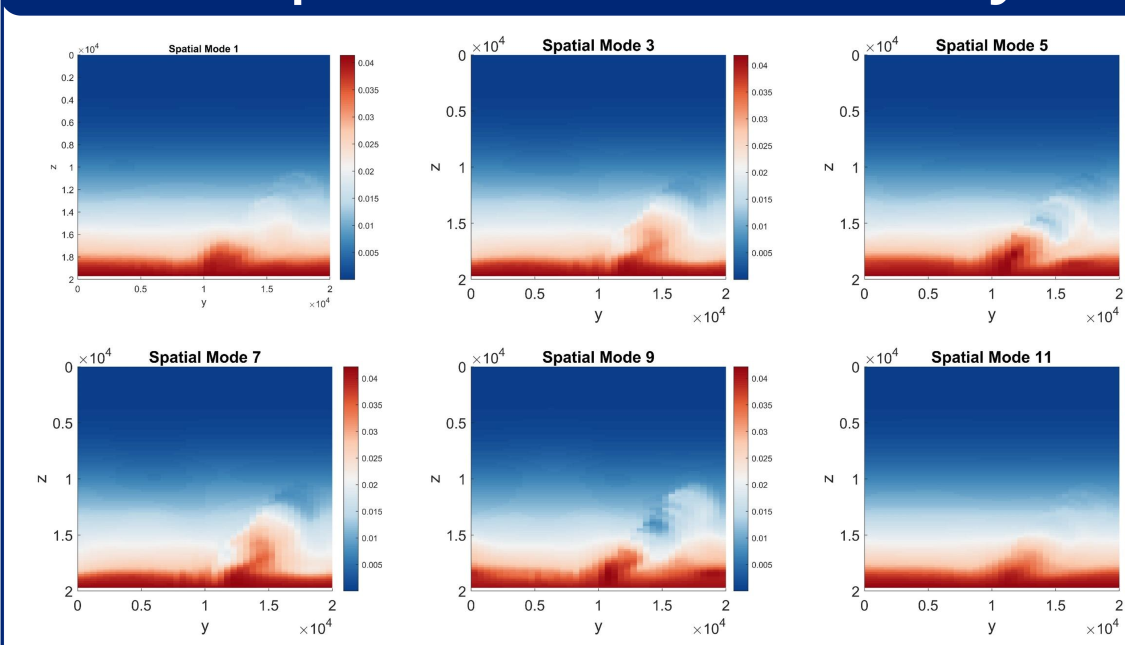
Some ***specific location*** (spatial) and the related ***index*** (temporal)

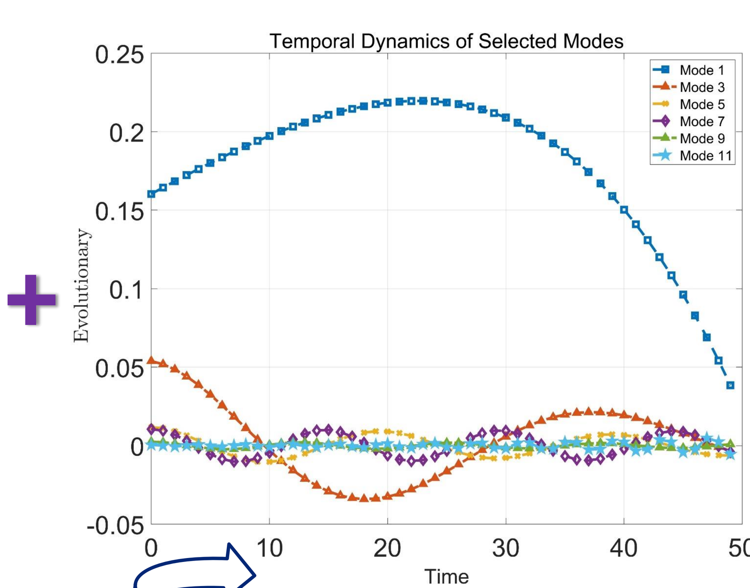


Hypothesis: Continuous Spectrum?

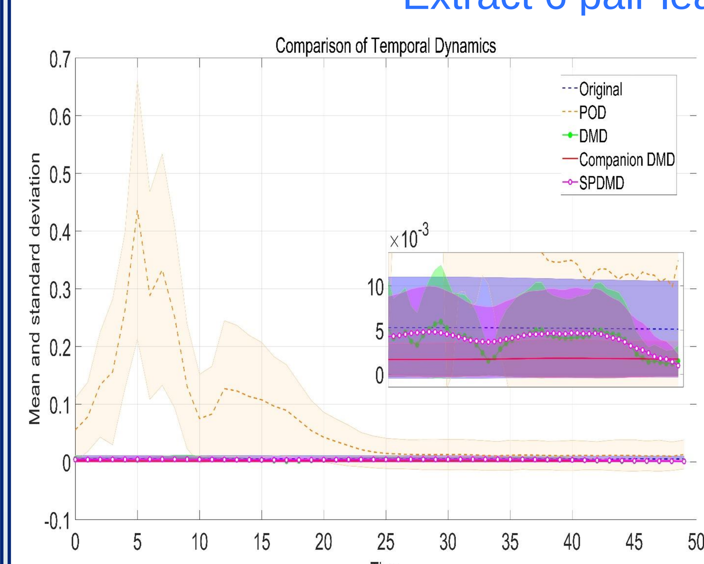
- **DMD** generates **42** modes and **SPDMD** extracts the **6** pairs dominant spatial modes by taking a sparsity level.
- However, SPDMD can **not** directly capture the transient evolution of temporal dynamics (versus original data).

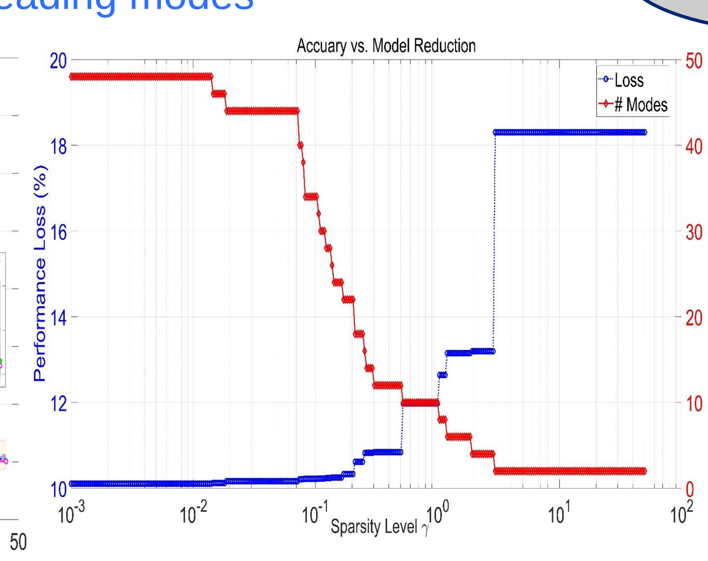
Experiment 2: Humidity Ratio Data Analysis

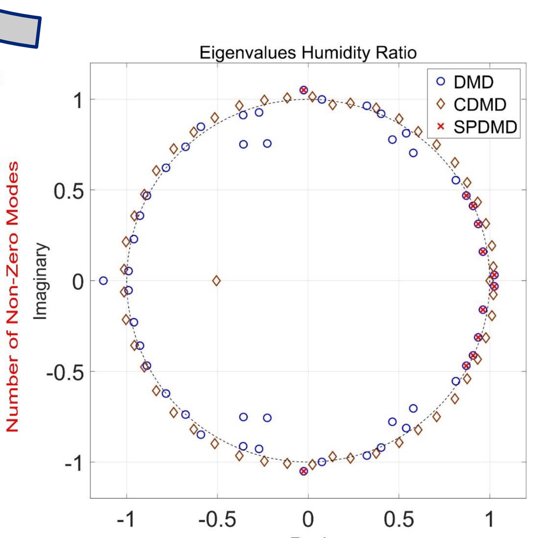




Extract 6 pair leading modes







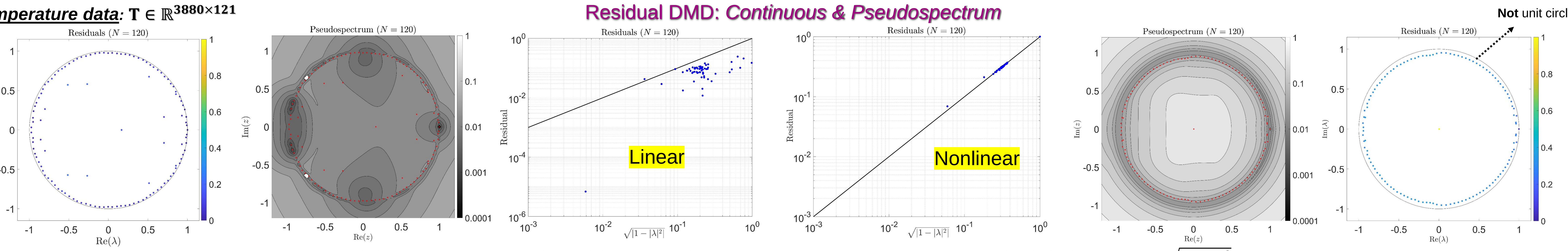
Balancing accuracy and no. of reduced modes

- **Humidity ratio data** $\mathrm{H} \in \mathbb{R}^{3880\times 51}$ has the **40*97=3880** grids with **51** snapshots and it directly shows the evolution of SCALE simulations, which is useful to explore the Koopman modes.
- Various DMD algorithms were applied to the humidity data field, and the mean and standard deviation indicate that DMD performs well.

Another Residual Trial: Temperature Data Field

Temperature data: $\mathbf{T} \in \mathbb{R}^{3880\times 121}$

Residual DMD: *Continuous & Pseudospectrum*



- **Residual DMD** tests the temperature data with **linear** (left) and **nonlinear Laplacian** (right) dictionary of observables, where the related residuals possibly lie far from the curve $\sqrt{1-|\lambda|^2}$ and the pseudospectrum show the presences of the ***non-normality*** and ***transient behavior***.

Conclusions & Discussions

- ✓ Koopman analysis for SCLAE simulation data by using various DMD algorithms to identify the dominant spatial and temporal modes, and further realize the model reduction.
- ✓ Vorticity magnitude data, humidity ratio data, and temperature data fields of SCALE simulations (i.e., different observable function spaces) are explored, and analyze the point spectrum (resp., continuous spectrum) of the linearized dynamics and find the corresponding leading modes.

Discussions

- The pressure data field plays a crucial role in studying the SCALE weather systems. However, changes in pressure data are not always immediately apparent or easy to interpret.

References

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