Risk-Aware Sparse Predictive Control

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Control Objectives

Seek a control sequence with "risk-aware" manner and "input-sparsity" sense that drives the state from initial state to a desired terminal set.

1. A discrete LTI system includes parametric uncertainties and additive disturbances
2. Soft state and hard input constraints
3. Data-driven sampling approach
4. Risk-aware solution vs. data-driven solution

Abstract

This abstract slide presents a risk-aware sparse predictive control (SPC) for a discrete linear time-invariant systems (LTI) subject to model parametric uncertainty and additive disturbances. We consider the case of probabilistic constraints on system state and hard constraints on control actions, which is equivalent to solving a chance-constrained sparse optimization problem. We leverage a data-driven sampling approach to seek a "randomized" solution by tackling a random convex program that approximates the risk-aware sparse solution with a high probability. We also give an explicit sample complexity to ensure the probabilistic robustness. Finally, numerical experiment illustrates the effectiveness of the proposed control strategy.

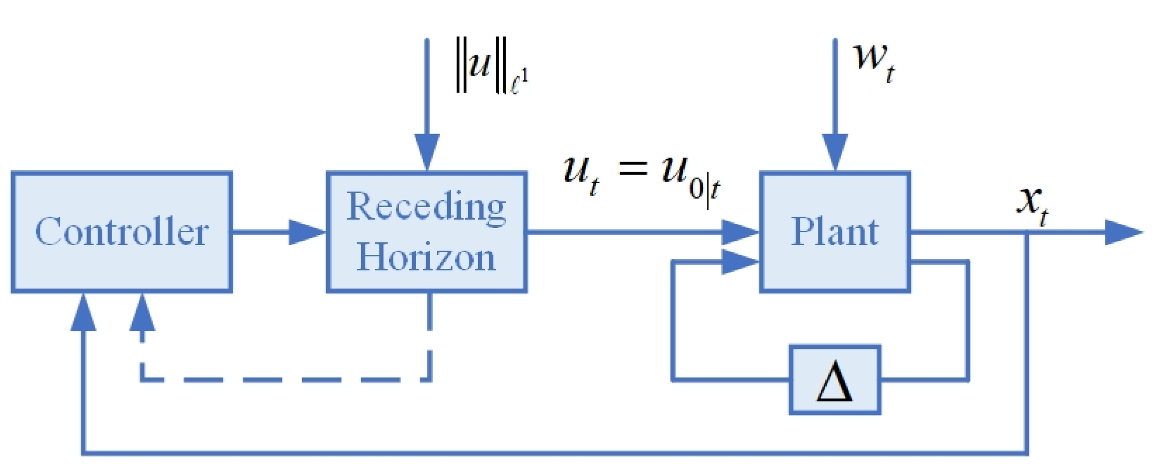


Figure: Sparse predictive control for uncertain systems

Consider an uncertain discrete time linear system

$$x_{t+1} = A(\delta)x_t + B(\delta)u_t + Ew_t, \quad x_0 \neq 0,$$

- state $x_t \in \mathbb{R}^n$, input $u_t \in \mathbb{R}^m$, $w_t \in \mathcal{W} \subseteq \mathbb{R}^{n_w}$
- $A(\delta)$, $B(\delta)$ w.r.t. model uncertainty $\delta \in \Delta$
- $(A(\delta), B(\delta))$ is stabilizable $\forall \delta \in \Delta \subseteq \mathbb{R}^{n_\delta}$
- uncertain instances $\delta \overset{i.i.d.}{\sim} \mathbb{P}_\delta$, $w_t \overset{i.i.d.}{\sim} \mathbb{P}_w$

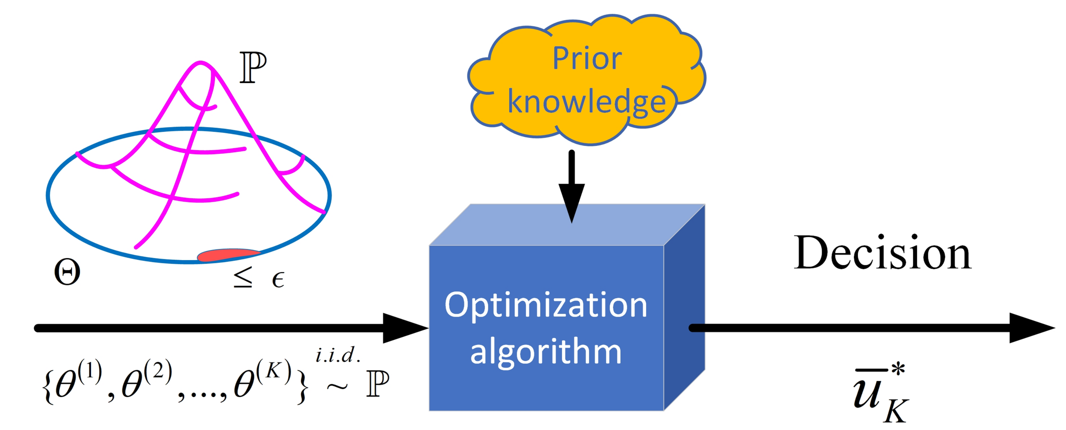
Risk-aware SPC

Solving a risk-aware sparse predictive control for uncertain discrete LTI system

$$\begin{aligned}
\min_{x,u} \quad & \|\bar{u}\|_1 \\
\text{s.t.} \quad & x_{j+1|t} = A(\delta)x_{j|t} + B(\delta)u_{j|t} + Ew_{j|t}, \\
& x_{0|t} = x_t, \quad j = 0, 1, \ldots, N-1, \\
& \mathbb{P}\{C_f x_{N|t} \leq c_f,\ Cx_{j+1|t} \leq c, \\
& \qquad\qquad j = 0, 1, \ldots, N-1\} \geq 1 - \epsilon, \\
& Du_{j|t} \leq d, \quad j = 0, 1, \ldots, N-1,
\end{aligned}$$

- Chance constrained sparse optimization problem.
- Risk-aware sparse solution $\bar{u}^*_\epsilon$ is hard to calculate.

Data-Driven Sampling Approach



Receding Horizon Strategy:

$$u_t \doteq u_{0|t} = \underbrace{\begin{bmatrix} I_m & 0_{m\times(N-1)} \end{bmatrix}}_{F} \overbrace{\begin{bmatrix} u_{0|t} \\ \vdots \\ u_{N-1|t} \end{bmatrix}}^{\bar{u}}$$

Data-Driven SPC

Using data-driven sampling, risk-aware SPC reduces to data-driven SPC as follows

$$\begin{aligned}
\min_{x,u} \quad & \|\bar{u}\|_1 \\
\text{s.t.} \quad & x^{(i)}_{j+1|t} = A(\delta^{(i)})x^{(i)}_{j|t} + B(\delta^{(i)})u_{j|t} + Ew^{(i)}_{j|t}, \\
& x^{(i)}_{0|t} = x_t,\ j = 0, 1, \ldots, N-1,\ i = 1, 2, \ldots, K, \\
& C_f x^{(i)}_{N|t} \leq c_f,\ Cx^{(i)}_{j|t} \leq c,\ j = 0, 1, \ldots, N-1, \\
& \qquad\qquad i = 1, 2, \ldots, K, \\
& Du_{j|t} \leq d, \quad j = 0, 1, \ldots, N-1,
\end{aligned}$$

- Random convex program.
- Data-driven randomized solution $\bar{u}^*_K$ is tractable.

Theorem (Probabilistic Robustness Guarantee)

Given a convex function $h(\bar{u}, \theta)$ to quantify joint chance constraints. Let $\Theta^K \doteq \{\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(K)}\}$ be a multi-sample of θ collected from probability $\mathbb{P}$, where K is a priori knowledge, satisfying

$$\sum_{i=0}^{mN-1} \binom{K}{i} \epsilon^i (1-\epsilon)^{K-i} \leq \beta, \quad \overset{i.e.}{\Longrightarrow} \quad K \geq \frac{mN - 1 + \ln(1/\beta) + \sqrt{2(mN-1)\ln(1/\beta)}}{\epsilon}$$

for specified risk $\epsilon \in (0, 1)$ and confidence $\beta \in (0, 1)$. Solving RCP, we obtain the optimal input

$$\bar{u}^*_K = \arg\min_{\bar{u}} \quad \|\bar{u}\|_1 \quad \text{s.t.} \quad h(\bar{u}, \theta^{(i)}) \leq 0, \quad i = 1, 2, \ldots, K, \quad Du_{j|t} \leq d, \quad j = 0, 1, \ldots, N-1.$$

Then, with confidence $1 - \beta$, a probabilistic robustness guarantee

$$\mathbb{P}^K\{h(\bar{u}^*_K, \theta) \leq 0\} \geq 1 - \epsilon \quad \text{holds.}$$

Constrained Prediction Systems

Given the state x_t observed at time t, the predicted state is modeled as

$$\begin{aligned}
x_{j+1|t} &= A(\delta)x_{j|t} + B(\delta)u_{j|t} + Ew_{j|t}, \\
x_{0|t} &\doteq x_t, \quad j = 0, 1, \ldots, N-1
\end{aligned}$$

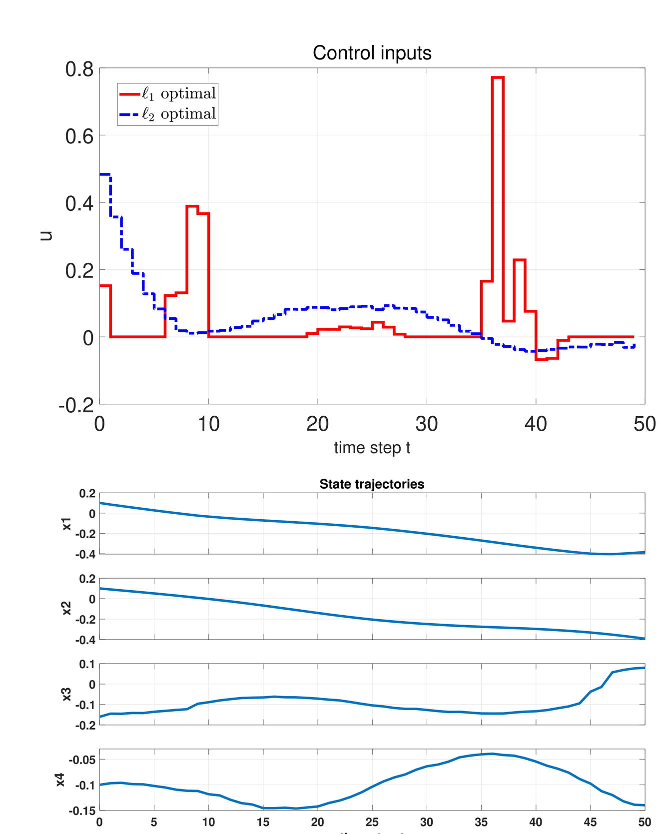
- e.g., $x_{j|t}$ denotes the jth step forward prediction of the state at sampling time t (resp. u, w)

"Soft State + Hard Input" Constraints

$$\mathbb{P}\{\theta \in \Theta : Cx_{j+1|t} \leq c,\ j = 0, 1, \ldots, N-1\} \geq 1 - \epsilon,$$
$$Du_{j|t} \leq d, \quad j = 0, 1, \ldots, N-1$$

- a desired risk $\epsilon \in (0, 1)$
- $\theta \doteq (\delta, \bar{w})$, $\theta \in \Theta \doteq \Delta \times \mathcal{W}^N$, $\Theta \overset{i.i.d.}{\sim} \mathbb{P} \doteq \mathbb{P}_\delta \times \mathbb{P}^N_w$

Simulations



- Data-driven SPC enjoys robustness and sparsity

Conclusion

A data-driven sampling approach for risk-aware SPC for uncertain discrete LTI system.

- Risk-aware $\bar{u}^*_\epsilon$ is approximated by data-driven $\bar{u}^*_K$ with a high probability.
- Provide a probabilistic robustness, sparsity, and finite sample complexity guarantees for SPC.

References

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[2] Cmapi, M.C. & Garatti. S. (2018), Introduction to the scenario approach. *SIAM*

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