Exploring SCALE Weather Data via Koopman Modes

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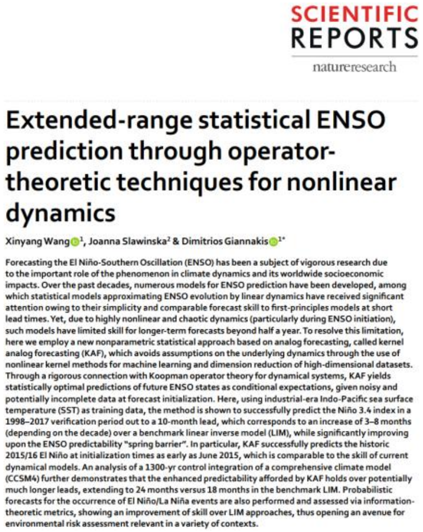
Purpose and Outline of Presentation

- Report a trial of Koopman mode decomposition (KMD) of SCALE simulation data
- Question 1: Can we capture a dominant (e.g., **transient**) Koopman mode (i.e., linear evolution of an observable) embedded in a **short-term** simulation data?
- Question 2: Which DMD (KMD algorithm) type is good to the simulation data?

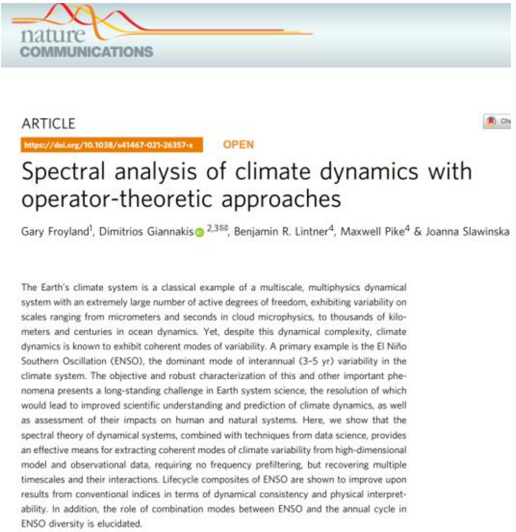
and so on…

1. Review of Koopman operator framework: KMD and DMD
2. Its application to vorticity magnitude (scalar-field) in SCALE simulation data
3. Discussion on humidity ratio field and low-dim. system representation .
4. Summary

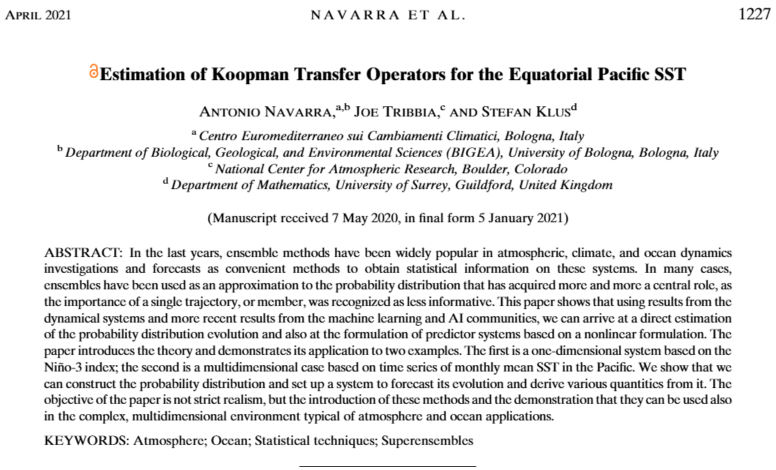
Previous Works on Weather and Climate Systems



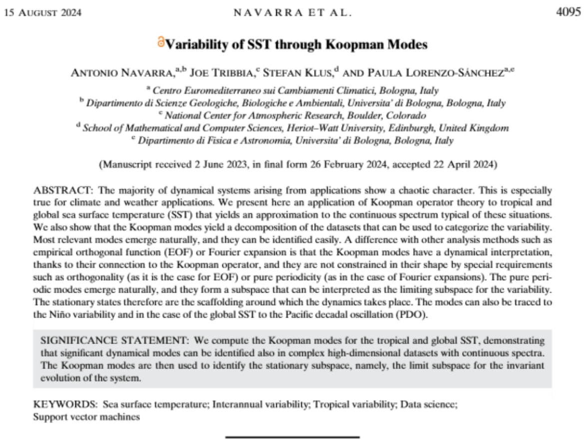
Kernel approach on ENSO



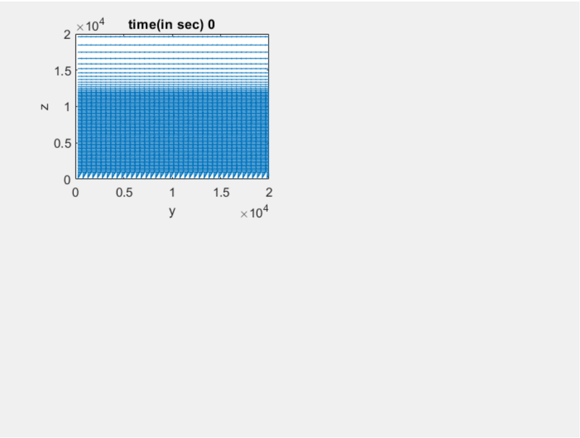
Spectral analysis of dynamics



Koopman/transfer operator for SST

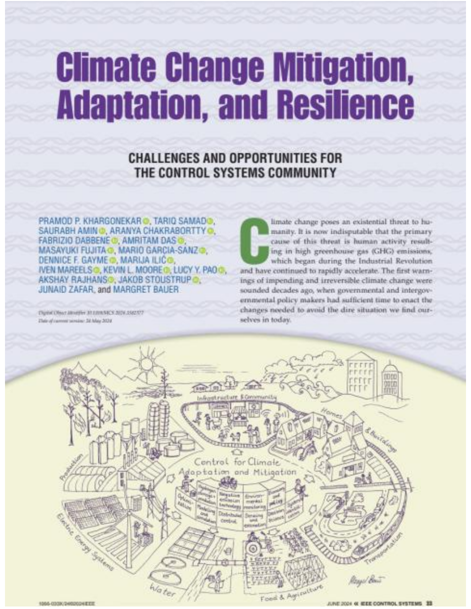


Continuous spectrum for SST



- In this presentation, we will focus on:
  1) **SCALE** weather simulation **data** (rather than SST)
  2) **non-steady-state** (or non-stationary) behavior
  3) **short-time** dynamics (**transient** pattern)
  4) Koopman mode decomposition (KMD) and model-reduction

     (a low-dim. linear system representation of original high-dim. nonlinear systems)



The controllability of climate or weather systems is gaining increasing attention.

Weather Dynamics & Observables

❖ Fact: true weather dynamical systems often go beyond the mathematical toy model, and can be unknown

➢ **Nonlinear CT/DT dynamical system:**

$$x^+ = \boldsymbol{F}(x_t), \qquad x \in \mathbb{X} \subseteq \mathbb{R}^n \qquad t = 0,1,2,\ldots$$

State equation State/Phase space

$$\boldsymbol{F}: \mathbb{R}^n \to \mathbb{R}^n$$

❑ Nonlinear map: describes the time evolution of the state, such as the nonlinear weather or climate systems

EX: Lorenz 96 model, chaotic dynamics

➢ Output equation Observable space

$$y_t = g(x_t), \qquad \forall g \in \mathcal{O}$$

❖ Hilbert / Banach space (complete inner product / normed space)

$$g: \mathbb{R}^n \to \mathbb{R} \text{ or } \mathbb{C}$$

❑ Observable: the function of the state that can be *measured* directly *from data*

❑ E.g., scalar, vortex, pressure, temperature, humidity ratio fields in SCALE Simulation

❑ **Importance**: The choice of dictionaries of observable (linear/nonlinear) Open issue!

Q: How to realize system representation?

Koopman Operator: A (Bounded) Linear Operator

Koopman Framework

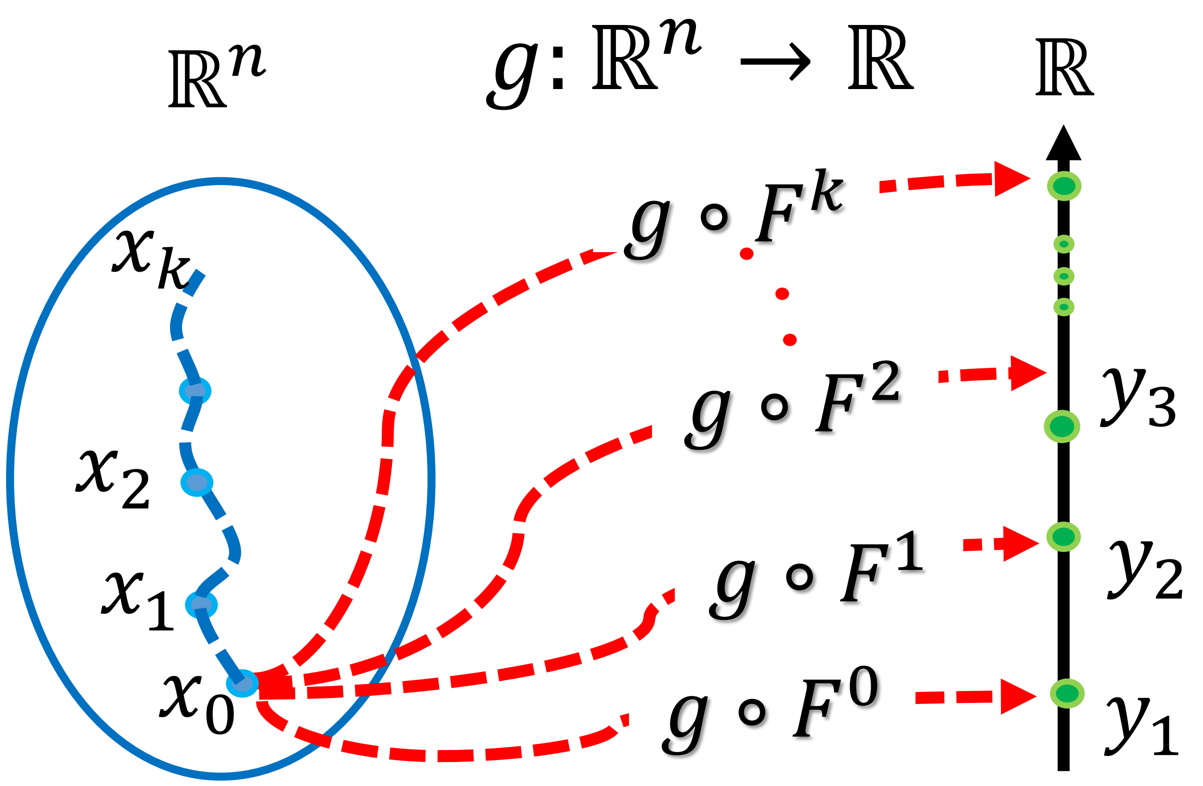
$$x_{t+1} = \boldsymbol{F}(x_t)$$

REALIZATION

LIFTING

$$\mathcal{K}^t g = g \circ \boldsymbol{F}$$

$$g_{t+1} = \mathcal{K}^t g_t = g \circ \boldsymbol{F} = g(x_{t+1})$$



State space (*Finite dim. Nonlinear* dynamics) —LIFTING→ Observable space (*Infinite dim. linear* evolution)

- Lifts the **nonlinear evolution of state** into a *linear evolution of functional observable*
- Linearity: $\mathcal{K}^t(\alpha_1 g_1 + \alpha_2 g_2) = \alpha_1 \mathcal{K}^t g_1 + \alpha_2 \mathcal{K}^t g_2$
  $\forall \alpha_1, \alpha_2 \in \mathbb{R},\ g_1, g_2 \in \mathcal{O}$

- **Linear operator** → (**Spectral Analysis**)

Refs.) Koopman, PNAS'1931; Koopman & von Neumann. PNAS'1932; Mezic, Nonlinear Dynamics'05

Spectral Analysis: Koopman & Dynamic Mode Decomposition

Dynamic mode decomposition (DMD):

$$y_t = \sum_{j=1}^{N} \phi_j \lambda_j^t b_j \quad \xrightarrow{\text{SVD}} \quad y_t = \sum_{j=1}^{r} \phi_j \lambda_j^t b_j$$

$A = Y(Y^{+})^{\dagger}$ Approx.

Refs.) Kutz et. al, SIAM'16; Mauroy, Mezic & Susuki, Springer'20

Koopman mode decomposition (KMD):

Point spectra

$$[\mathcal{K}^t g](x_0) = g \circ \boldsymbol{F}(x) = \sum_{j} \lambda_j \varphi_j(x) \mathrm{v}_j$$

Eigenpairs: $\{\lambda_j, \varphi_j\}_j^{\infty}$ $\quad\quad \{g_1, g_2, \dots\}$

Finite dim. approx:

$$x_i \in \mathrm{span}\{g_1, g_2, \dots, g_M, \dots, g_N\}$$

state realizations ≈ a linear combination of observables

➢ **Least Square Error Optimization (DMD)**

Data, Spatial Modes, Temporal Modes

spatial / temporal

$$\underbrace{\begin{bmatrix} | & \cdots & | \\ y_1 & \cdots & y_N \\ | & \cdots & | \end{bmatrix}}_{Y \in \mathbb{R}^{n \times N}} \approx \underbrace{\begin{bmatrix} | & \cdots & | \\ \phi_1 & \cdots & \phi_r \\ | & \cdots & | \end{bmatrix}}_{\Phi} \overbrace{\begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_r \end{bmatrix}}^{\mathbf{D}_{\mathrm{b}}} \underbrace{\begin{bmatrix} 1 & \lambda_1 & \cdots & \lambda_1^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_r & \cdots & \lambda_r^{N-1} \end{bmatrix}}_{\mathbf{V}_{\mathrm{and}}}$$

Amplitudes

Penalize an $\ell 1$ norm on the amplitudes

➢ **(Regularized) Least Square Error Optimization**

Sparsity-Promoting DMD

$$J_r(\boldsymbol{b}) = \min_{\boldsymbol{b}} ||\mathrm{Y} - \Phi \mathbf{D}_{\mathrm{b}} \mathbf{V}_{\mathrm{and}}||_2 + \gamma ||\boldsymbol{b}||_1$$

Ref.) Jovanovic et. al, Physics of Fluids'14

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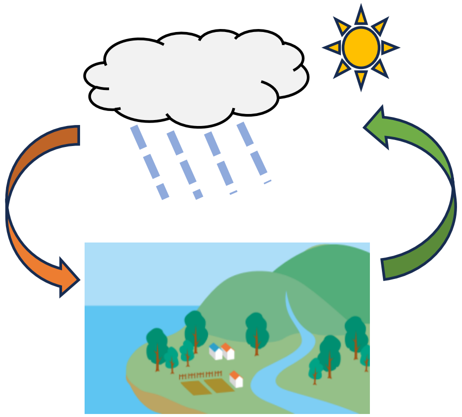
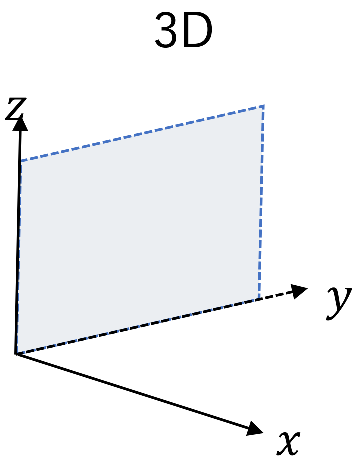
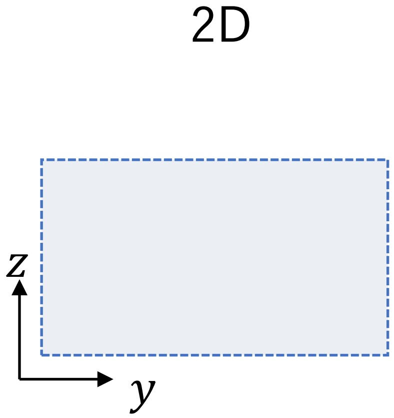
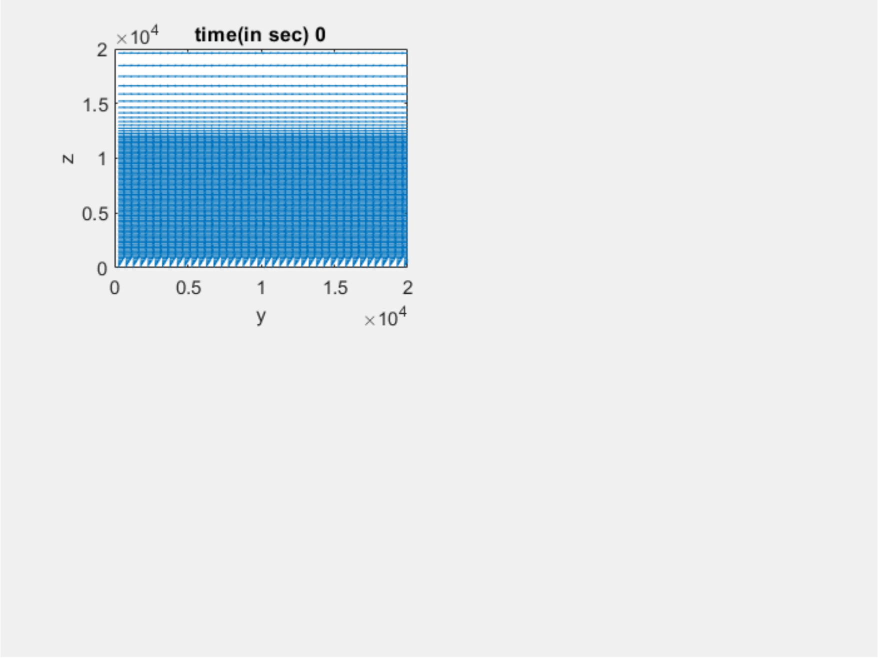
Model Configuration: SCALE Weather Simulation







- horizontal velocity component in the y-direction (**data.V**)
- vertical velocity component in the z-direction (**data.W**)



SCALE Weather dynamics Simulation

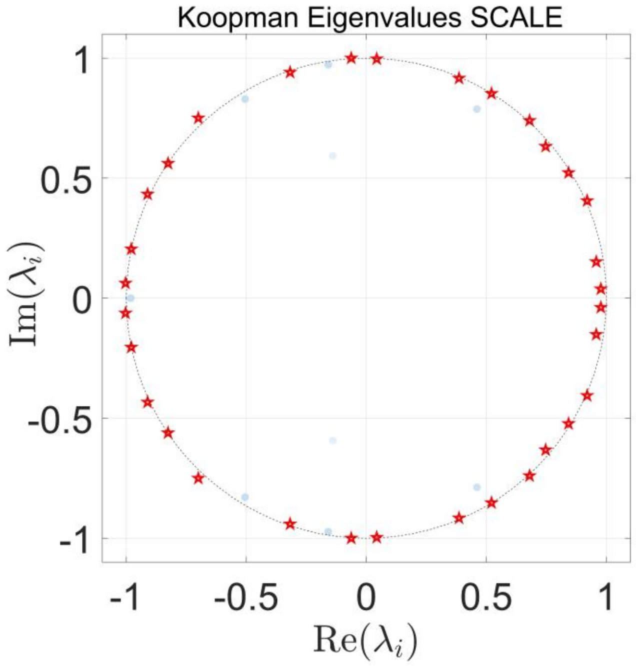
- Time unit**:** 30 seconds per interval/iteration
- Original data **:**
  121 snapshots & **40 * 97 = 3380 grids**

Other Atmospheric factors:

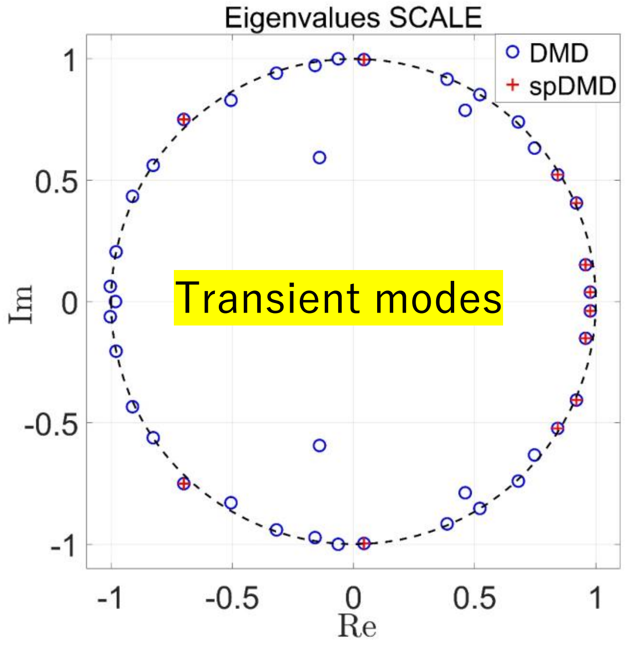
- pressure (**data.PRES**) and temperature (**data.T**)
- relative humidity (**data.QV**) and precipitation (**data.PREC**)

Ref.) **SCALE**: Scalable Computing for Advanced Library and Environment, RIKEN, url: https://scale.riken.jp

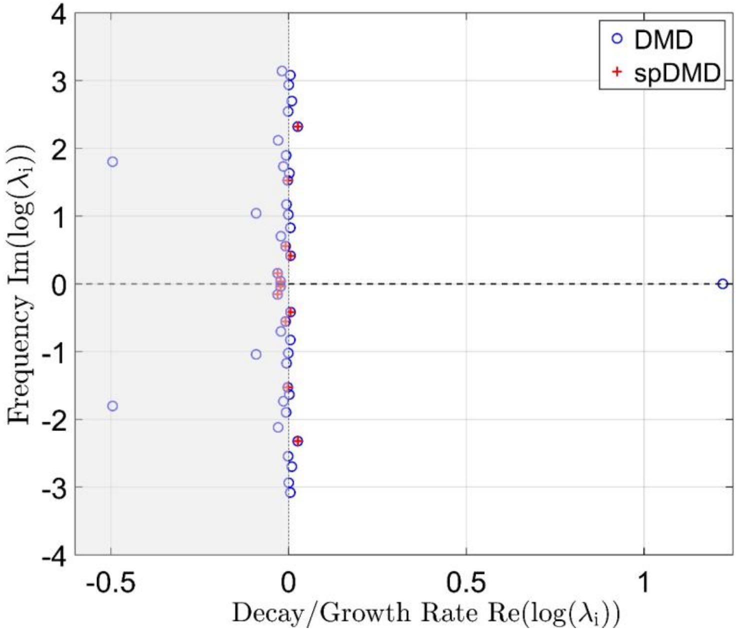
Experiment I: Magnitude of Vorticity Field



Extract 32 modes

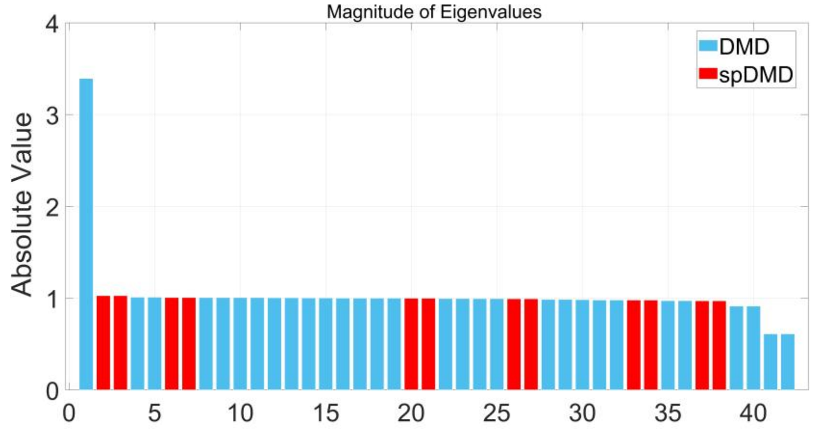


Extract 12 modes

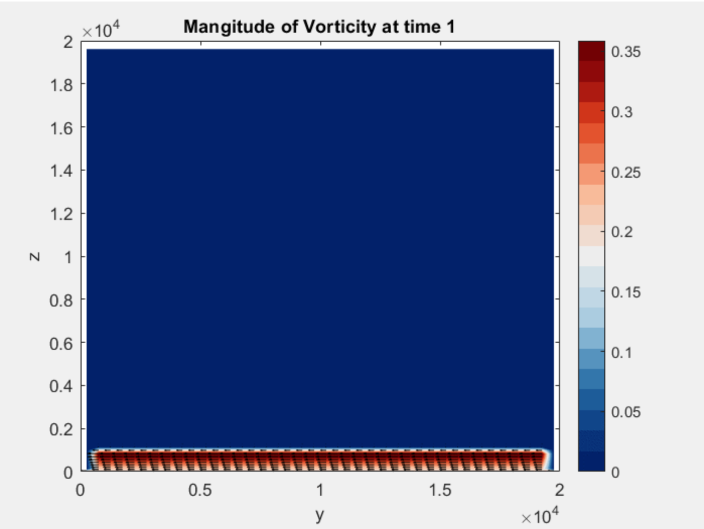


Check ***growth/decay rate***
w.r.t. eigenvalues

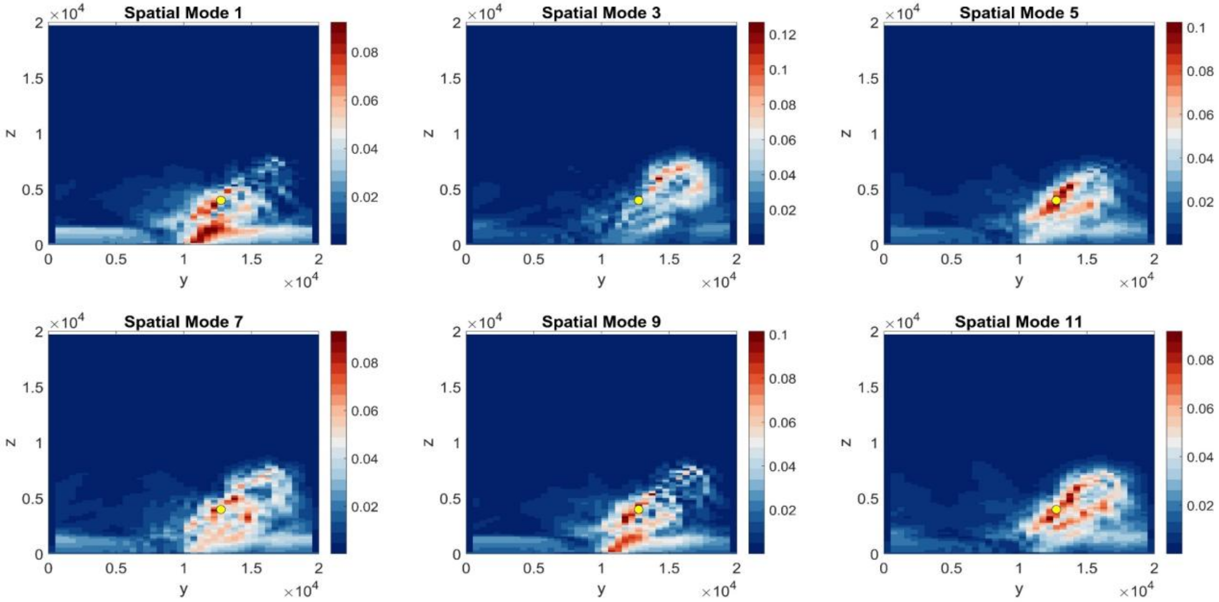
- **Vorticity Data:** $\mathbf{Y} \in \mathbb{R}^{3880\times 43}$
- Only takes the <u>first **43 snapshots** </u>and has <u>**40 * 97 = 3380 grids**</u>
- Generate **a tall &** short **matrix**
- Rich spatial knowledge while less temporal information



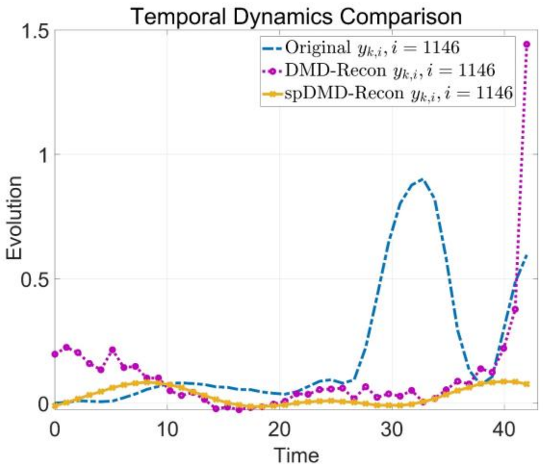
Experiment I: Magnitude of Vorticity Field

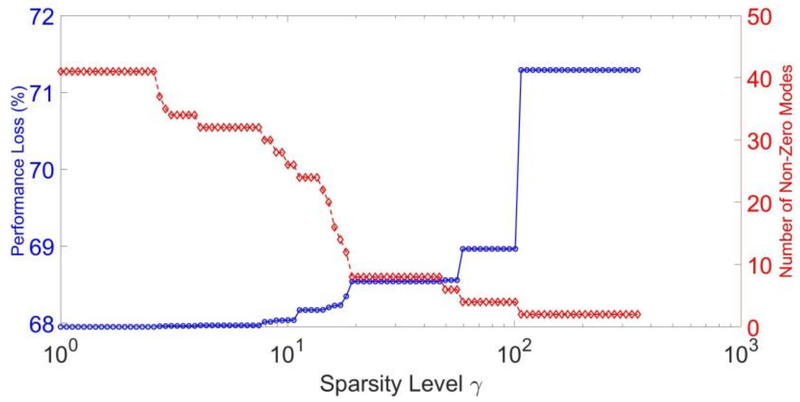


$|\phi_j|$



$\Re e(\lambda_j^t b_j)$

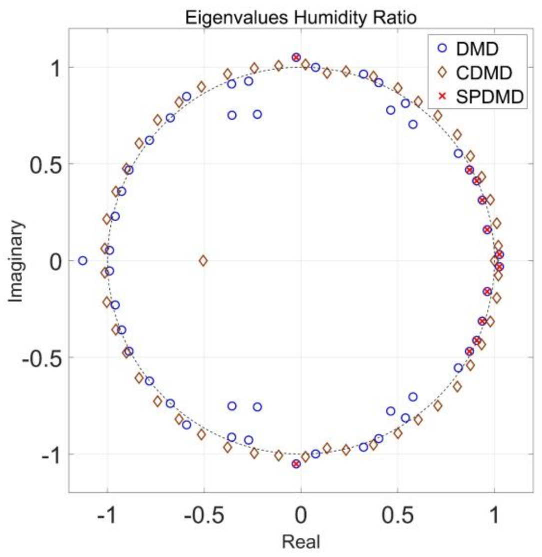


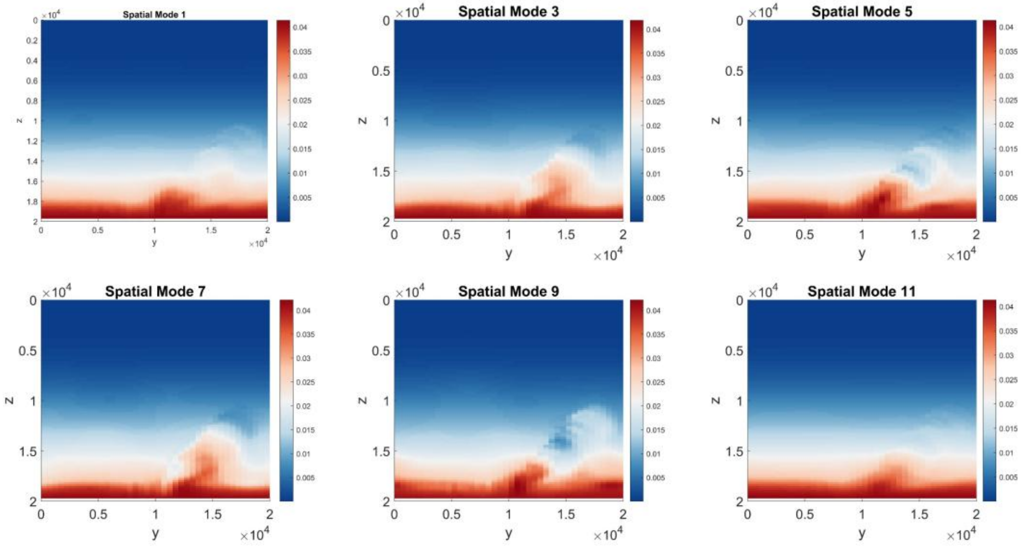


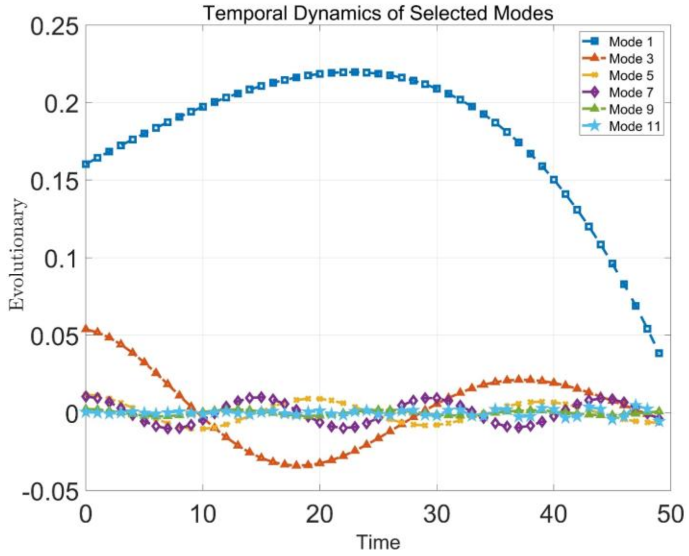
- ❑ **DMD** generates **42** modes and **SPDMD** extracts the **6** pairs dominant spatial modes by taking a sparsity level
- ❑ However, SPDMD can **not** directly capture the **transient** evolution of ***temporal dynamics*** (vs. original data)

Experiment II: Humidity Ratio (Relative Humidity) Field

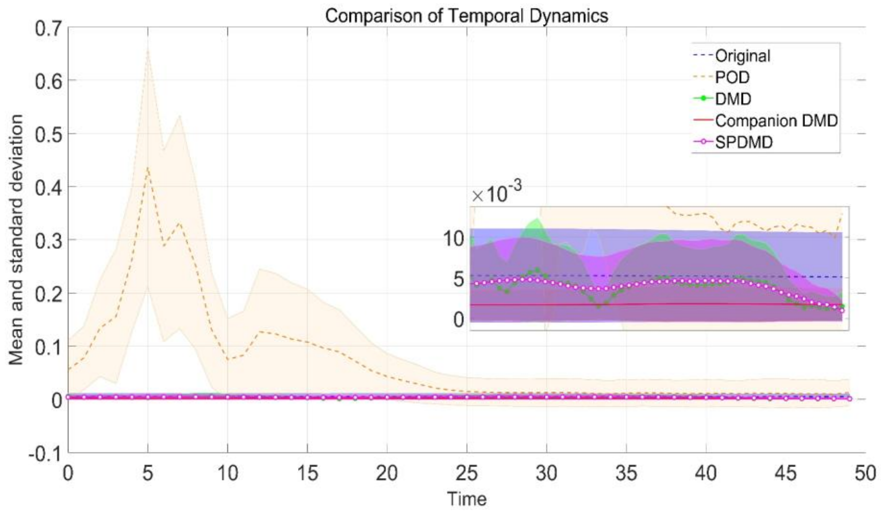
Goal: Extract bubble-like behavior in dynamics
(i.e., transient modes)

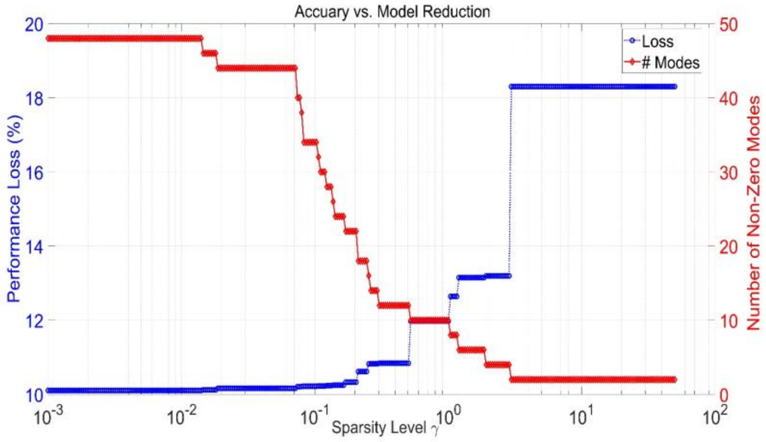






- **Humidity ratio:** $Y \in \mathbb{R}^{3880\times51}$
- Various DMD algorithms (Companion DMD, SPDMD)
- Extract 6 pairs leading modes





➢ Balancing accuracy and no. of reduced modes

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Conclusion:

- ✓ Koopman analysis for SCLAE simulation data by using DMD algorithms to identify the dominant (spatial & temporal) modes and realize the model reduction.
- ✓ Vorticity magnitude data, humidity ratio data fields of SCALE simulations (i.e., ***different observable function spaces***) are explored, and analyze the **point spectrum** of the linearized dynamics and find the corresponding leading modes.
- ✓ Use the key modes to represent a low dim. system.

Next…

- ❑ Generate more realistic SACLE simulation data (3D) and explore Koopman modes.
- ❑ Ensemble initial conditions: Koopman operator on **multiple trajectories**.
- ❑ Design a controller to predict the *precipitation* with a low dim. System.

Thank you for your listening !
Q&A