# Koopman Analysis of Weather Dynamics Using SCALE Simulation Data



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## Summary

- ❖ Different from sea surface temperature (SST) data, the SCALE simulation data provide generally non-steady-state data trajectories.
- \* Spectral analysis for weather data systems by Koopman mode decomposition (KMD) and dynamic mode decomposition (DMD) methods are active. For example, proper orthogonal decomposition (POD), sparsity-promoting DMD (SPDMD), extended DMD, kernel DMD, and residual DMD, etc.
- \* We are interested in transient stability and growth behavior for humidity ratio and temperature dataset to capture onset of weather control.

## Koopman Operator and Dynamic Mode Decompositions

 $\triangleright$  **Data Collection**: From observed dataset  $\{x_k, y_k\}_{k=1}^N$  to establish the data-driven weather dynamics

 $y_t = \sum_{j=1}^{\infty} \phi_j \lambda_j^t b_j$ 

 $x_{t+1}=F(x_t), t=0,1,\cdots,N-1, \qquad \qquad Y=AX$  which can be estimated by a linear time invariant (LTI) system  $y_t=Ax_t$ 

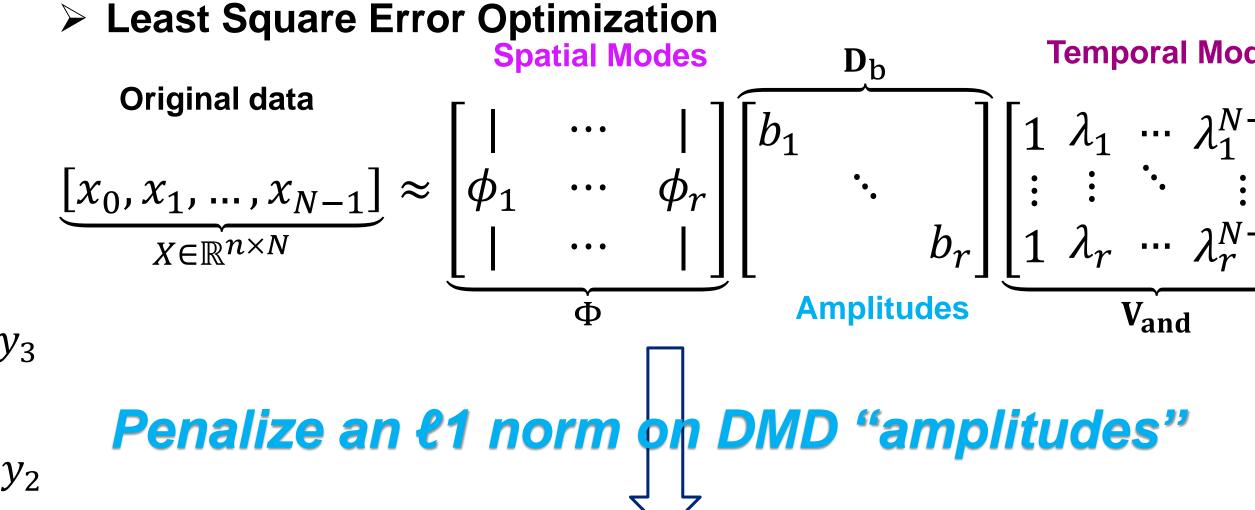
> Dynamic Mode Decomposition (DMD)

$$y_t = \sum_{j=1}^{N} \phi_j \lambda_j^t b_j$$
 Approximation
$$Truncated SVD$$

➤ Koopman Mode Decomposition (KMD): *Point* and *Continuous Spectrum* 

$$[\mathcal{K}^t g](x_0) = g \circ F(x) = \sum_j c_j \lambda_j^t \varphi_j(x) + \int_{[-\pi,\pi]_{per}} e^{i\omega t} \psi_{\omega,g}(x_0) d\omega$$

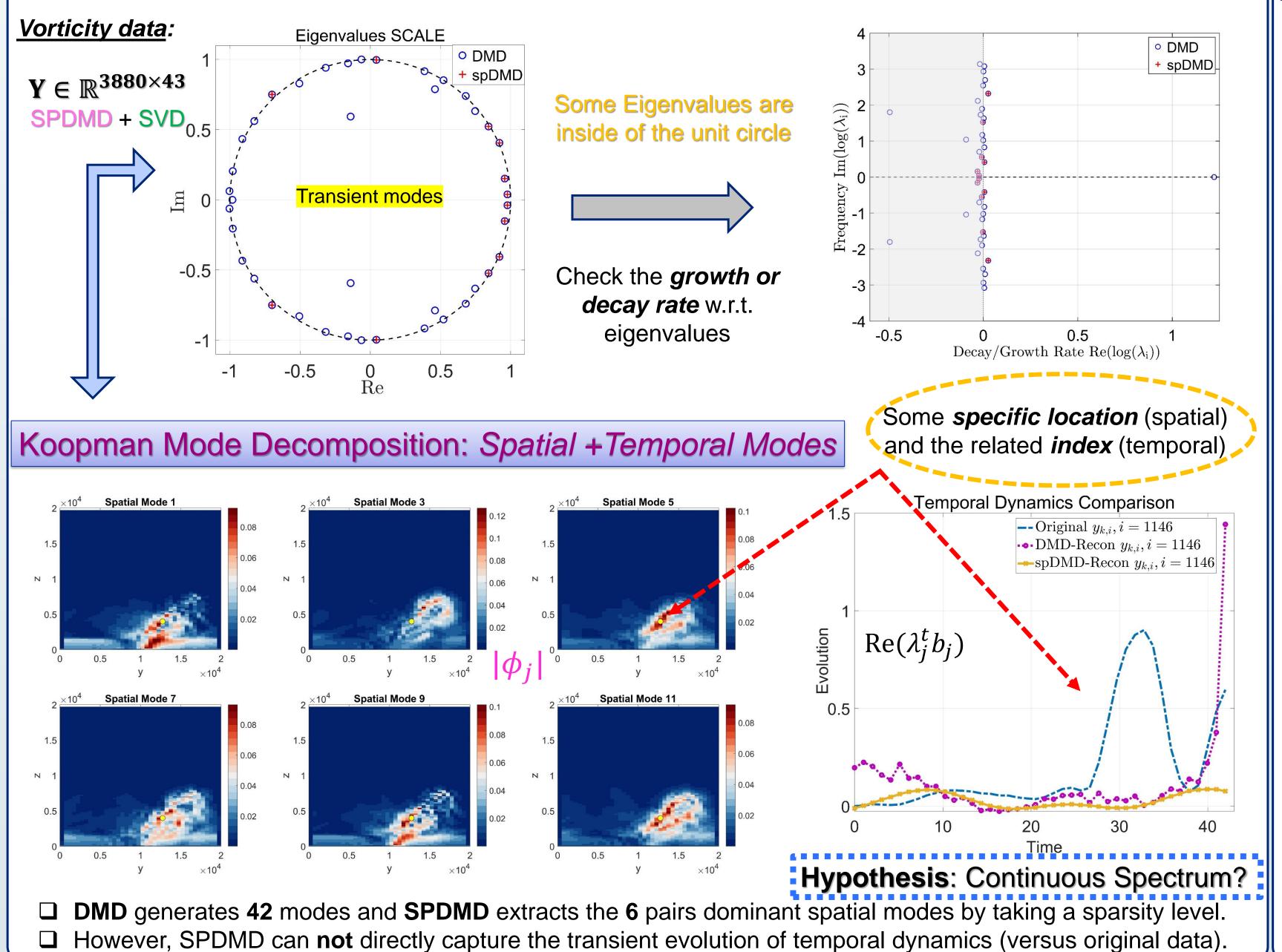
 $g \circ F^{k}$   $y_{3}$   $x_{2}$   $x_{1}$   $g \circ F^{2}$   $y_{2}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{7}$   $y_{1}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{1}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{1}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{7}$   $y_{1}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{7}$   $y_{1}$   $y_{2}$   $y_{3}$   $y_{4}$   $y_{5}$   $y_{6}$   $y_{7}$   $y_{8}$   $y_{1}$   $y_{1}$ 



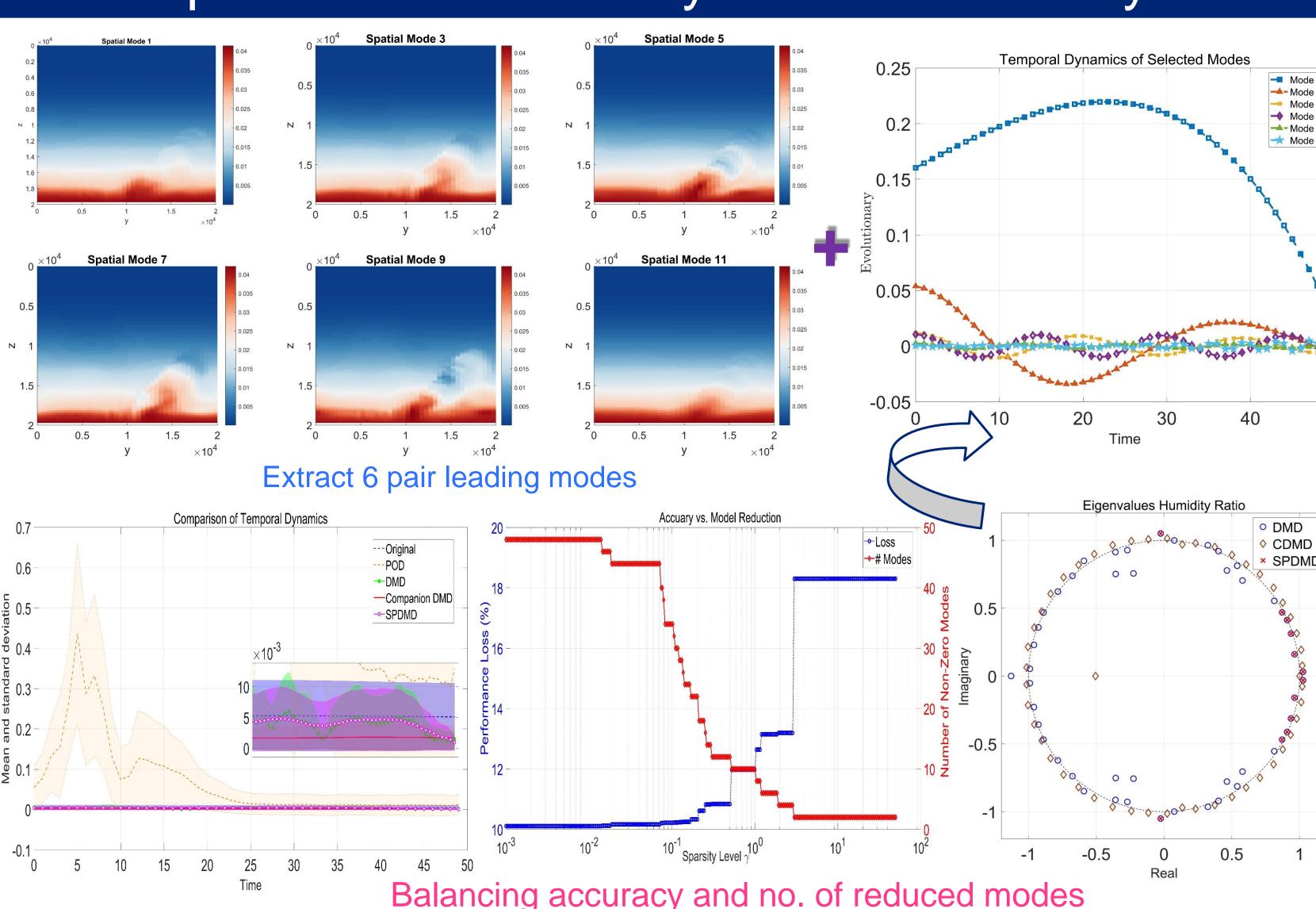
> (Regularized) Least Square Error Optimization Sparsity-Promoting DMD

$$J_r(\boldsymbol{b}) = \min_{\boldsymbol{b}} ||X - \Phi \mathbf{D}_{\mathbf{b}} \mathbf{V}_{\text{and}}||_2 + \gamma ||\boldsymbol{b}||_1$$

# Experiment 1: Magnitude of Vortex Data via SPDMD



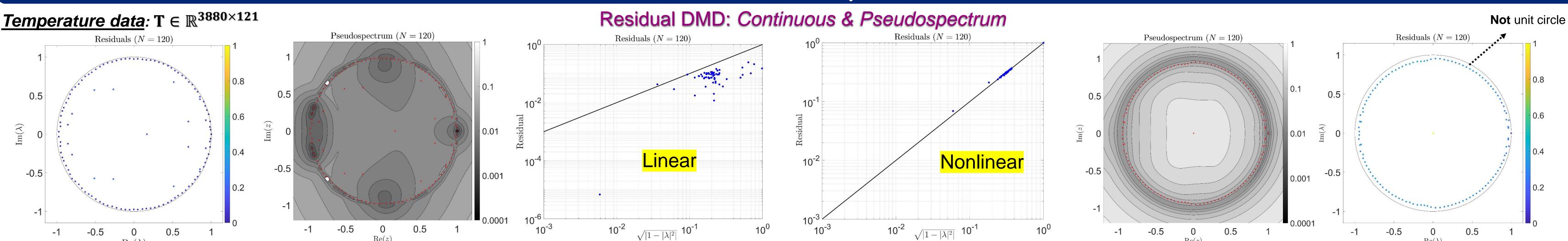
### Experiment 2: Humidity Ratio Data Analysis



- ❖ Humidity ratio data H ∈ R<sup>3880×51</sup> has the 40\*97=3880 grids with 51 snapshots and it directly shows the evolution of SCALE simulations, which is useful to explore the Koopman modes.
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  ❖ Various DMD algorithms were applied to the humidity data field, and the mean and standard deviation indicate that DMD performs well.

### Another Residual Trial: Temperature Data Field



Residual DMD tests the temperature data with linear (left) and nonlinear Laplacian (right) dictionary of observables, where the related residuals possibly lie far from the curve  $\sqrt{1-|\lambda|^2}$  and the pseudospectrum show the presences of the non-normality and transient behavior.

#### Conclusions & Discussions

- ✓ Koopman analysis for SCLAE simulation data by using various DMD algorithms to identify the
  dominant spatial and temporal modes, and further realize the model reduction.
- ✓ Vorticity magnitude data, humidity ratio data, and temperature data fields of SCALE simulations (i.e., different observable function spaces) are explored, and analyze the point spectrum (resp., continuous spectrum) of the linearized dynamics and find the corresponding leading modes.

#### Discussions

• The pressure data field plays a crucial role in studying the SCALE weather systems. However, changes in pressure data are not always immediately apparent or easy to interpret.

#### References

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