A New Information Share Measure

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In this study, we modify the information share (IS) originally proposed by Hasbrouck, J. (1995). The proposed modified information share (MIS) leads to a unique measure of price discovery instead of the upper and lower IS bounds. Performance of MIS is compared with the Hasbrouck IS measure and the Gonzalo–Granger permanent–transitory decomposition (PT/GG)-based measure using simulations with 1,000 replications applied to the same three examples considered by Hasbrouck, J. (2002). The MIS is found to outperform both Hasbrouck IS measure and PT/GG measure. The empirical application of the MIS to three major stock indices indicates that price discovery takes place mostly in the futures market. Hence, the evidence supports the transaction cost hypothesis as well as the model proposed by Garbade, K. D., and Silber, W. L. (1983). © 2009 Wiley Periodicals, Inc. Jrl Fut Mark 29:377–395, 2009

INTRODUCTION

Whether price reflects the fundamental value of a security is one of the fundamental questions in finance. This question is of great interest to academicians, policy makers, as well as practitioners. In well-functioning and efficient financial markets, the price of a security reflects its fundamental value. This is because, in these markets, any new information that affects the fundamental value of the

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security is rapidly captured in the price. However, the new information would not be instantaneously impounded into the price due to the existence of market imperfections like transaction costs, information asymmetry, regulations, etc. For example, an investor with private information may split its total trade into many smaller trades in order to maximize her profit. This will lead to a situation whereby the price will reflect the new information in a gradual fashion.[1]

The process in which the information gets reflected in the price becomes even more interesting if there are more than one market where the same security or very similar securities trade. For example, the same stock may be listed in multiple countries or in multiple markets in the same country. In this case, one would be interested to know which market reflects the new information first. This gives rise to the concept of *dominant* and *satellite* markets (Garbade & Silber, 1983). For example, in the case of multiple listing in multiple countries, one would like to know if the price discovery takes place in the domestic or foreign markets.

A similar situation is encountered when dealing with securities that have derivatives. One can take a long position in an underlying security by buying the underlying security itself or by taking positions in derivative securities like futures and options. In this case, one would like to know whether the price discovery takes place in the security (cash) market or the derivatives (futures) market. If the price discovery depends on transaction costs, as suggested by Fleming, Ostdiek, and Whaley (1996), then one can expect the price discovery to take place in the futures market instead of cash market due to low transaction cost associated with futures trading. Alternatively, price discovery may depend on the relative number of participants in each market as shown in the theoretical model suggested by Garbade and Silber (1983).[2]

This brings one to the question of how the price discovery is measured. In recent years, price discovery measures are defined based on a framework where the prices in different markets are non-stationary (unit-root) processes with the number of cointegrating vectors equal to the number of markets minus one. One of these measures is based on the permanent–transitory decomposition proposed by Gonzalo and Granger (1995) (PT/GG) where the permanent component, for identification, is assumed to be a linear function of the original series. As the permanent component is considered to reflect the efficient price driving the prices in all the markets, this method, referred to as PT/GG method, uses the normalized linear coefficients as the measures of price discovery

[1]See Kyle (1985), Admati and Pfleiderer (1988), Barclay and Warner (1993), and Chakravarty (2001) for discussions on this issue of stealth trading strategy followed by investors with private information.

[2]As the market with the lowest transaction cost is expected to be the most liquid market or the market with highest trading volume that in turn is expected to be the market with most participants, the theoretical model proposed by Garbade and Silber (1983) is consistent with transaction cost hypothesis. The authors thank the anonymous referee for pointing this out.

(Booth, So, & Tse, 1999; Booth, Lin, Martikainen, & Tse, 2002; Harris, McInish, & Wood, 2002). This method has some desirable properties. First, PT/GG method permits the hypothesis testing of a market's contribution to price discovery. Second, it provides a unique price discovery measure. However, as the linear coefficient vector can be shown to be orthogonal to the error-correction coefficient matrix, one of the limitations of this method is the fact that it ignores the innovation variances.

In a pioneering work, Hasbrouck (1995) suggested a different measure of price discovery commonly known as information share (IS). This is one of the most commonly used methods. One of the attractive features of this method has to do with the fact that it incorporates both the system dynamics as well as the innovation variances. This method is also consistent with the conventional argument that security prices are martingale.

However, as the IS measure depends on the ordering of the series, one ends up with upper and lower bounds for the IS measure instead of a unique measure. This may not be problematic in situations where the bounds are close to each other. However, a specific conclusion may be hard to reach if the bounds are far apart.[3] In this study, the authors suggest a modified version of the information share (MIS) based on a different factor structure that would lead to a unique measure of price discovery.[4] Simulations of the three examples considered by Hasbrouck (2002) indicate that MIS performs better than the IS. The same simulations also indicate that MIS performs better compared to the PT/GG method.

The proposed MIS method as well as the original IS method is applied to empirically estimate the price discovery for the three major stock indices, namely, the S&P 500, Tokyo Stock Price Index (TOPIX), and Financial Times Stock Exchange (FTSE) 100, using intra-day price data. The upper and lower bounds for the Hasbrouck IS are found to be far apart due to high correlations between the futures and spot returns. This results in a significant overlap between the IS bounds of futures and spot markets. Therefore, it is hard to make conclusions based on the IS measure. The MIS method, which provides a unique measure, indicates that price discovery mainly takes place in the futures markets for all the three stock indices considered.

The remainder of the article is divided into three sections. In the Measurement of Price Discovery section, the MIS, IS, and PT/GG methods are discussed. The simulation results are also presented in this section. The empirical

[3]In the case of two markets, it can be shown that the difference between the two bounds increases with the increase in correlation. The correlation in this case is expected to be high because the securities in question usually reflect the same information.

[4]The factor structure used by Hasbrouck (1995) involves a lower triangular matrix. Therefore, the ordering of the variable in vector autoregressive framework would matter. However, the factor structure used in the study is associated with the factorization of correlation matrix and involves full matrix instead of lower triangular matrix; therefore, the ordering of variables is irrelevant.

results are discussed in the Empirical Analysis section. The article is concluded in the Conclusion section.

MEASUREMENT OF PRICE DISCOVERY

In this section, the authors would discuss the two existing price discovery measures as well as the new measure, which is obtained by modifying one of the two existing measures, proposed in the study. The first measure, which would be referred to as PT/GG measure, is based on the permanent–transitory decomposition proposed by Gonzalo and Granger (1995). The second one is the IS measure proposed by Hasbrouck (1995). However, as the IS measure depends on the ordering of the series, different IS measures are obtained depending on the ordering of the series, which eventually leads to the upper and lower bounds for the IS measure. In this study, Hasbrouck's approach is modified in such a way that it leads to a unique IS measure that will be referred to as the MIS measure.[5]

First the basic framework on which both types of measure are based on would be presented. Let Y_t be an $n \times 1$ vector of unit-root series where it is assumed that there are $n - 1$ cointegrating vectors; i.e., the system consists of a single common stochastic trend (Stock & Watson, 1988). The series have the following vector error-correction representation (Engle & Granger, 1987):

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{k} A_i \Delta Y_{t-i} + \varepsilon_t,\ \Pi = \alpha\beta^T \tag{1}$$

where β and α are $n \times (n - 1)$ matrices of rank $(n - 1)$. The columns of β consist of the $(n - 1)$ cointegrating vectors and each column of α consists of adjustment coefficients. The matrix Π is decomposed in such a way that $\beta^T Y_t$ consists of $(n - 1)$ vectors of stationary series. The covariance matrix of the error term is given by $E[\varepsilon_t \varepsilon_t^T] = \Omega$.

Following Stock and Watson (1988), Equation (1) can be transformed into the following vector moving average (VMA) representation (Hasbrouck, 1995):

$$\Delta Y_t = \Psi(L)\varepsilon_t \tag{2}$$

or, alternatively,

$$Y_t = Y_0 + \Psi(1)\sum_{i=1}^{t} \varepsilon_i + \Psi^*(L)\varepsilon_t. \tag{3}$$

[5]Please see Baillie, Booth, Tse, and Zabotina (2002), De Jong (2002), and Lehmann (2002) for a detailed discussion on the PT/GG and IS measures and the relationship between these two measures.

As the series are cointegrated, the Engle–Granger representation theorem (Engle & Granger, 1987) implies the following (De Jong, 2002; Lehmann, 2002):

$$\beta^T \Psi(1) = 0 \quad \text{and} \quad \Psi(1)\alpha = 0. \tag{4}$$

Therefore, Equation (3) can be written as (see De Jong, 2002, Equation (4))

$$Y_t = Y_0 + \beta_\perp \alpha_\perp^T \sum_{i=1}^{t} \varepsilon_i + \Psi^*(L)\varepsilon_t \tag{5}$$

where $\alpha_\perp$ and $\beta_\perp$ are orthogonal vectors to α and β, respectively. Note that $\alpha_\perp^T \sum_{i=1}^{t} \varepsilon_i$ represents the common stochastic trend component, which follows a random walk process. Also note that $\Psi(1)\varepsilon_t$, which represents the long-run impact of innovations on prices, would be the main focus of the analysis.

Gonzalo–Granger Permanent–Temporary Measure (PT/GG)

Gonzalo and Granger (1995) suggested a way of decomposing the vector of non-stationary series Y_t into permanent (common factor) component f_t (which is non-stationary or $I(1)$ series) and transitory (stationary) component $\tilde{Y}_t$. The identification of these components is achieved by assuming that (i) the permanent component is a linear function of the original series and that (ii) the transitory component does not Granger cause the permanent component in the long run. Under these identification conditions, the series Y_t can be written as follows:

$$Y_t = Af_t + \tilde{Y}_t. \tag{6}$$

This method of decomposing the original series into permanent and transitory components will henceforth be referred to as PT/GG method. The permanent component f_t (under linearity condition) can be written as

$$f_t = \theta^T Y_t \tag{7}$$

where θ is the $n \times 1$ permanent component coefficient vector. The dimension of the permanent component (i.e., the number of permanent components) is equal to the number of common stochastic trends in the system, which, in this case, is equal to one. Therefore, in the case under consideration, f_t is a one-dimensional series obtained by taking the linear combination of the existing series as given by Equation (7).

The argument for using PT/GG method is based on the consideration that the permanent component represents the fundamental or efficient price.

As each of the original non-stationary series potentially contributes to the permanent component (Equation (7)), one can use θ_i (the ith component of the coefficient vector θ) to measure the contribution of market i to the price discovery process. This is the approach taken by Booth et al. (1999, 2002) and Harris et al. (2002). For example, if $\theta_1 = 0$, this implies that the first market has no contribution to the price discovery. Similarly, if $\theta_2 = 0$, then the second market has no contribution to the price discovery. Specifically, Harris et al. (2002) suggested the use of the elements of θ as measures of price discovery after the normalization so that the sum of the elements is equal to 1.

Gonzalo and Granger (1995) have shown that $\theta = \alpha_\perp$, where $\alpha_\perp$ is a column vector orthogonal to the adjustment coefficient matrix α (Equation (1)); i.e., $\alpha_\perp^T \alpha = 0$.[6] Therefore, the permanent component f_t can be written as follows:

$$f_t = \alpha_\perp^T Y_t. \tag{8}$$

In order to see the relationship between this representation and Stock–Watson common stochastic trend representation, Equation (5) can be substituted into Equation (8) to get the following (see De Jong, 2002):

$$f_t = \alpha_\perp^T Y_t = \alpha_\perp^T Y_0 + \alpha_\perp^T \beta_\perp \alpha_\perp^T \sum_{i=1}^{t} \varepsilon_i + \alpha_\perp^T \Psi^*(L)\varepsilon_t. \tag{9}$$

Or, alternatively,

$$f_t = \alpha_\perp^T Y_t = \alpha_\perp^T Y_0 + \delta \alpha_\perp^T \sum_{i=1}^{t} \varepsilon_i + \alpha_\perp^T \Psi^*(L)\varepsilon_t \tag{10}$$

where δ is given by $\delta = \alpha_\perp^T \beta_\perp$. Therefore, it is clear that Gonzalo–Granger permanent component (or common factor) consists of the Stock–Watson common stochastic trend plus a stationary series. Therefore, Gonzalo–Granger permanent component is a non-stationary ($I(1)$) process, but not necessarily a pure martingale or a pure random walk process (Hasbrouck, 2002).

Hasbrouck IS Measure

As mentioned earlier, in the situation under consideration, there are $(n-1)$ cointegrating vectors and this implies that the impact matrix $\Psi(1)$ has rank 1.

[6]When $n = 2$, the normalized θ must satisfy $\theta_1\alpha_1 + \theta_2\alpha_2 = 0$ (orthogonality condition) and $\theta_1 + \theta_2 = 1$ (weights sum to 1). This leads to the following PT/GG measures of price discovery:

$$\theta_1 = \frac{\alpha_2}{\alpha_2 - \alpha_1}, \quad \theta_2 = 1 - \theta_1.$$

Above measures are the same as the ones given by Equation (25) in Lehmann (2002). As the error-correction coefficient α_1 is expected to be negative and α_2 is expected to be positive, the PT/GG measure is expected to be between 0 and 1. However, in the actual estimation one of these conditions may be violated in which case one may end up with a price discovery measure being negative or greater than 1.

Furthermore, in the case considered by Hasbrouck (1995), the rows of $\Psi(1)$ are identical.[7] Let $\psi = (\psi_1, \psi_2, \ldots, \psi_n)$ represent the identical row of $\Psi(1)$. Note that $\psi\varepsilon_t$ constitutes the long-run impact of innovations on each of the prices. For the case where the covariance matrix Ω is diagonal (i.e., the innovations are independent), Hasbrouck (1995) defines the IS of market j as follows:

$$S_j = \frac{\psi_j^2 \Omega_{jj}}{\psi \Omega \psi^T} \tag{11}$$

where ψ_j is the jth element of ψ. The IS measure when the covariance matrix is not diagonal is given by (Hasbrouck, 1995)

$$S_j = \frac{([\psi F]_j)^2}{\psi \Omega \psi^T} \tag{12}$$

where F is the Cholesky factorization of Ω and $[\psi F]_j$ represents the jth element of the row vector ψF.[8] As the Cholesky factorization depends on ordering, the IS computed using Equation (12) will depend on the particular ordering. By considering all possible orderings, one can compute the upper and lower bounds on IS (for details, see Hasbrouck, 1995).

Modified Information Share (MIS)

Here the way the IS bounds can be eliminated will be discussed. One shall look at the Hasbrouck IS measure with independent innovations from a different perspective. Note that in this case of independent innovations, $\Omega = \mathrm{diag}(\Omega_{11}, \Omega_{22}, \ldots, \Omega_{nn})$. Consider the following factor structure:

$$\varepsilon_t = \hat{F} z_t,\ E[z_t] = 0,\ E[z_t z_t^T] = I \tag{13}$$

where $\hat{F}$ ($n \times n$ matrix) is chosen such that $\Omega = \hat{F}\hat{F}^T$; i.e., $\hat{F} = \mathrm{diag}(\sqrt{\Omega_{11}}, \sqrt{\Omega_{22}}, \ldots, \sqrt{\Omega_{nn}})$. Then, the variance of $\psi\varepsilon_t$ is given by

$$\psi\Omega\psi^T = E[\psi\varepsilon_t\varepsilon_t^T\psi^T] = E[\psi\hat{F}z_t z_t^T\hat{F}^T\psi^T] = \hat{\psi}\hat{\psi}^T = \sum_{i=1}^{n} \hat{\psi}_i^2 \tag{14}$$

[7]In Hasbrouck (1995), different series correspond to the prices of the same security being traded in multiple markets. Therefore, in equilibrium, all the prices must be equal. This would impose special restrictions on cointegrating matrix β. These restriction combined with the condition given by Equation (4) would imply that the rows of $\Psi(1)$ would be identical.

[8]When $n = 2$, it can be shown that

$$F = \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2(1 - \rho^2)^{1/2} \end{bmatrix}$$

and $\Omega = FF^T$.

where $\hat{\psi} = \psi\hat{F} = (\hat{\psi}_1, \hat{\psi}_2, \ldots, \hat{\psi}_n)$. Equation (14) shows a way of decomposing the variance $\psi\Omega\psi^T$ into separable additive parts. Then, the IS of market j can be defined as

$$S_j = \frac{\hat{\psi}_j^2}{\sum_{i=1}^{n}\hat{\psi}_i^2} = \frac{\hat{\psi}_j^2}{\hat{\psi}\Omega\hat{\psi}^T}. \tag{15}$$

The IS measure for market j given by Equation (15) is exactly the same as the one given by Equation (11) where it has been assumed that the innovations are independent.

However, for a more general case where the innovations are not independent (i.e., the matrix Ω is not diagonal), the factor structure that satisfies the condition given by Equation (13) is not unique. Hasbrouck (1995) chooses F to be the Cholesky factorization of Ω. Unfortunately, this leads to the IS that would depend on the ordering of the series. It would be more logical to choose the factor structure that leads to IS being independent of ordering. This is what the authors propose in the study. They would discuss the new factor structure next.

Here, the use of the factorization matrix that is based on the correlation matrix is suggested.[9] Specifically, let Φ represent the innovation correlation matrix. Let Λ represent the diagonal matrix with diagonal elements being the eigenvalues of the correlation matrix Φ, where the corresponding eigenvectors are given by the columns of matrix G. Finally, let V be a diagonal matrix containing the innovation standard deviations on the diagonal; i.e., $V = \mathrm{diag}(\sqrt{\Omega_{11}}, \sqrt{\Omega_{22}}, \ldots, \sqrt{\Omega_{nn}})$. Then, the following transformed innovation z_t^* can be shown to have zero mean and identity matrix as the covariance matrix; i.e., $E[z_t^*] = 0$ and $E[z_t^*(z_t^*)^T] = I$. Then, one has the following factor structure for innovations:

$$\varepsilon_t = F^* z_t^* \tag{16}$$

where $F^* = [G\Lambda^{-1/2}G^TV^{-1}]^{-1}$. Note that $\Omega = F^*(F^*)^T$. Under this factor structure, the MIS is given by

$$S_j^* = \frac{\psi_j^{*2}}{\sum_{i=1}^{n}\psi_i^{*2}} = \frac{\psi_j^{*2}}{\psi\Omega\psi^T} \tag{17}$$

where $\psi^* = \psi F^*$. It is important to note that under this new factor structure, the resulting ISs are independent of ordering (see the Appendix). Therefore, this leads to a measure of price discovery that is order invariant but not unique.

[9]See footnote 4 for the reason one expects the measure to be order independent and see the Appendix for the proof.

Due to the use of square-root matrix, one ends up with lack of uniqueness problem.[10] The new measure is referred to as MIS.

Note that when the innovations are not independent, it is necessary to choose a factor structure. As there are many possible factor structures one can choose from, the obvious question is which factor structure should be adopted. It is felt that one should adopt the factor structure that has some desirable properties such as uniqueness. Other desirable properties could be related to the performance of various factor structures under Monte Carlo simulation. As MIS leads to a unique price discovery measure and, as it is shown later, it also performs well under Monte Carlo simulation, it is felt that MIS is a better candidate to be used in the measurement of price discovery.[11]

If the number of series considered is 2, one has the following:

$$G = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} (1+\rho) & 0 \\ 0 & (1-\rho) \end{bmatrix} \quad \text{and}$$

$$F^* = \begin{bmatrix} 0.5(\sqrt{1+\rho} + \sqrt{1-\rho})\sigma_1 & 0.5(\sqrt{1+\rho} - \sqrt{1-\rho})\sigma_1 \\ 0.5(\sqrt{1+\rho} - \sqrt{1-\rho})\sigma_2 & 0.5(\sqrt{1+\rho} + \sqrt{1-\rho})\sigma_2 \end{bmatrix}.$$

Therefore, at least for the two-market case, MIS leads to some plausible conclusions. Firstly, the MIS of a market increases, ceteris paribus, with the increase in the corresponding element of ψ. This makes sense because the elements of the vector ψ represent the long-run impacts of one-unit innovation. Secondly, ceteris paribus, the MIS of a market would be higher for the market with a larger innovation variance. This also makes sense because the innovation variance is expected to represent the amount of information in the setup considered by Hasbrouck (1995). Thirdly, when the innovation correlation approaches zero, the MIS converges to the independent case given by Equation (11). Finally, when correlation approaches unity, the MIS would approach 50% for each market.

[10]This method involves the use of the matrix square root $\Lambda^{-1/2}$, which is a diagonal matrix with diagonal elements being the reciprocal of the square root of the eigenvalues (say, λ_is) of correlation matrix Φ. Being the eigenvalues of correlation matrix, all the eigenvalues are non-negative. It would be natural to use the positive square root $(+\sqrt{\lambda_i})$ instead of the negative square root $(-\sqrt{\lambda_i})$ even though either could be used. In this study, the positive square root is used. Note that a diagonal matrix has non-diagonal square-root matrices. However, it is more natural to consider diagonal square-root matrices. The authors thank the anonymous referee for pointing out the lack of uniqueness.

[11]Baillie et al. (2002) suggested the use of the average of the upper and lower IS bounds in order to solve the uniqueness problem. Even though this may work well for two-market case, this method is associated with some weaknesses. Firstly, the average cannot be shown to be the result of any particular factor structure. Therefore, the solution seems to be arbitrary. Secondly, when dealing with more than two markets, the average IS (average of upper and lower IS bounds) for all markets would not necessarily add up to 100%.

Relationship Between PT/GG Measure and IS-Based Measures

As both PT/GG method and IS-based methods are derived from the same framework (n non-stationary series with $(n - 1)$ cointegrating vectors), they are related. For example, the condition represented by Equation (4) (i.e., $\Psi(1)\alpha = 0$) implies that the identical row of $\Psi(1)$ is orthogonal to the error-correction matrix; i.e., $\psi = \alpha_{\perp}$. As the coefficient vector θ, used by PT/GG, is also orthogonal to the error-correction vector, θ and ψ differ by a scalar multiple; i.e., $\theta = s\psi$, where s is a scalar. Therefore, it is clear that PT/GG method uses only information on ψ, whereas IS and MIS use information on both ψ and innovation covariance matrix Ω. This fact would help characterize the price discovery measures based on PT/GG method and IS method.

From the analysis presented thus far, one can summarize the characteristics of PT/GG-based measure and the measure based on IS. PT/GG measure has some attractive features. Firstly, PT/GG method leads to a unique measure of price discovery. Secondly, PT/GG method permits the hypothesis testing on the individual elements of θ. Therefore, a hypothesis test on a particular market's contribution to price discovery can be performed. IS-based method does not permit the hypothesis testing unless the innovations are independent.[12] However, PT/GG method uses only the error-correction coefficients and completely ignores the innovation covariances. As to the IS-based method, the modification presented in the study solves the uniqueness problem. Furthermore, IS and MIS methods incorporate both the error-correction coefficients as well as the innovation covariances. The IS-based methods also have possible theoretical appeal due to the fact that the underlying permanent component follows pure martingale process. However, it has one drawback in that it does not allow hypothesis testing. Therefore, it is felt that both methods have roles to play in an empirical analysis.

Performance Comparisons Using Simulated Market Data

In order to compare the performance of alternative methods, these methods are applied to simulated market data for the three examples considered by Hasbrouck (2002). In the simulation, a sample size of 100,000 observations is used as in Hasbrouck (2002).[13] However, rather than one run, the estimation is repeated 1,000 times with the same sample size. This allows one to create the

[12]If innovations are independent, the null hypothesis that the market j does not contribute to price discovery can be tested by testing the null hypothesis: $\psi_j = 0$.

[13]The random number generator used is based on Matsumoto and Nishimura's (1998) Mersenne Twister that has a super astronomical period of $2^{19,937} - 1$.

confidence intervals for comparison. The simulation is performed for the same three examples considered by Hasbrouck (2002), which are briefly presented below.

Let m_t denote the unobservable efficient price. The trade direction for market $i = 1, 2$ is denoted by q_{it}, which takes values of either 1 or -1 with equal probability. It is assumed that q_{1t} is independent of q_{2t}. Finally, p_{it} denotes the price in market i. The three examples are given below:

Example 1 (A two-market Roll model):

$$m_t = m_{t-1} + u_t,\ u_t \sim N(0, 1) \tag{18}$$

$$p_{it} = m_t + q_{it}. \tag{19}$$

Example 2 (Two markets with private information):

$$m_t = m_{t-1} + q_{1t} \tag{20}$$

$$p_{1t} = m_t + q_{1t} \tag{19}$$

$$p_{2t} = m_{t-1} + q_{2t}. \tag{21}$$

Example 3 (Two markets with public and private information):

$$m_t = m_{t-1} + q_{1t} + u_t,\ u_t \sim N(0, 1) \tag{22}$$

$$p_{1t} = m_t + q_{1t} \tag{19}$$

$$p_{2t} = m_{t-1} + q_{2t}. \tag{21}$$

The result of the simulation is reported in Table I. For the first example (example 1), the average MIS value is very close to the theoretical value of 50. However, the average lower and upper IS bounds are quite far apart (78.11 and 21.95) from the theoretical value. Furthermore, the 90% confidence interval for the MIS is also close to the theoretical value. Similarly, the average PT/GG measure is also close to the theoretical value. However, the 90% confidence interval for PT/GG measure (47.61–52.18) is wider compared to the interval for MIS (48.63–51.33). Therefore, in example 1, MIS seems to perform the best among the three measures considered.

In example 2, the upper and lower IS bounds are quite close to each other and close to the theoretical value of 100. The average MIS is also close to the theoretical value. The average PT/GG measure is also close to the theoretical value. However, the 90% confidence interval is significantly wider for PT/GG measure compared to MIS.[14] Therefore, in example 2, MIS seem to perform

[14]The 95th percentile is greater than 100 for PT/GG method. This is due to the reason explained in footnote 6. For example, even though the theoretical value of α_1 is zero, for more than 5% of the runs, the estimated value of α_1 came out to be positive.

TABLE I
Market 1's Price Discovery Share

	Market 1's Price Discovery Share (%)	*Variance of Efficient Price Change Estimates*	*First-Order Autocorrelation of Efficient Price Change*
Panel A: Example 1			
Structural model	50	1	0
PT (average)	49.97	2.000	−0.250
5%	47.61	1.990	−0.255
95%	52.18	2.010	−0.245
Standard deviation	1.402	0.009	0.003
IS (average)	78.11–21.95	1.000	0
5%	76.93–20.77	0.956	0
95%	79.18–23.06	1.045	0
Standard deviation	0.69–0.69	0.028	0
MIS (average)	50.03	1.000	0
5%	48.63	0.956	0
95%	51.33	1.045	0
Standard deviation	0.83	0.028	0
Panel B: Example 2			
Structural model	100	1	0
PT (average)	100.16	5.026	−0.400
5%	95.78	4.514	−0.412
95%	104.80	5.599	−0.388
Standard deviation	2.77	0.335	0.007
IS (average)	99.72–99.71	0.997	0
5%	99.56–99.56	0.954	0
95%	99.85–99.85	1.044	0
Standard deviation	0.08–0.09	0.028	0
MIS (average)	99.72	0.997	0
5%	99.57	0.954	0
95%	99.85	1.044	0
Standard deviation	0.09	0.028	0
Panel C: Example 3			
Structural model	100	2	0
PT (average)	60.05	2.011	0.001
5%	55.62	1.753	−0.067
95%	64.81	2.312	0.070
Standard deviation	2.80	0.169	0.042
IS (average)	98.25–90.12	2.000	0
5%	97.91–89.28	1.902	0
95%	98.58–90.97	2.099	0
Standard deviation	0.21–0.51	0.059	0
MIS (average)	94.98	2.000	0
5%	94.39	1.902	0
95%	95.54	2.099	0
Standard deviation	0.36	0.059	0

Note. Average indicates the arithmetic average of 1,000 estimates resulting from 1,000 simulation runs. Similarly, "Standard deviation" denotes the standard deviation of the 1,000 estimates. Finally, the values associated with structural model represent the theoretical values. PT, permanent–transitory; IS, information share; MIS, modified information share.

the best among the three measures considered even though performance of IS is close to that of MIS.

In example 3, both MIS and IS measures are downward biased. The upper and lower IS bounds are farther apart compared to example 2. However, PT/GG measure performs badly. The performance of PT/GG method is consistent with the one obtained by Hasbrouck (2002) (see Table 3, p. 338). This may be explained by the fact that PT/GG method ignores the characteristics of innovations. Therefore, in example 3, MIS seems to perform the best among the three measures.

The simulation results indicate that MIS performs the best among the three measures considered except in example 2 where IS measure performs close to MIS. It is important to note that MIS clearly performs the best in example 3 where both public and private information is present, a situation considered to be normal. Therefore, in normal situations, where both public and private information is present or where neither market is the sole source of price discovery, it is hoped that MIS would significantly improve the empirical analysis by providing a unique and efficient measure of price discovery.

EMPIRICAL ANALYSIS

In this study, the authors empirically test and estimate the price discovery mechanism in the spot and futures markets using the MIS approach as discussed in the previous section. For comparison purposes, they also report the results obtained by using the Hasbrouck IS method. Intra-day data with a five-minute interval for S&P 500, TOPIX, and FTSE 100 indices are used. In each case, June and September 2006 contracts are used and the data cover the period from March to September 2006. The data are obtained from Reuters. Only the data where the trading hours between the futures and spot markets overlap are used. This results in sample size of more than 10,000, 7,000, and 13,000 for S&P 500, TOPIX, and FTSE 100, respectively. The logarithm of spot and futures prices is used.

Before one proceeds to compute the ISs, one needs to establish the fact that each of the series has single unit root and the spot and futures prices are cointegrated with single cointegrating vector. Table II summarizes the unit-root tests based on the Augmented Dickey–Fuller (Dickey & Fuller, 1979) and Phillips–Perron (Phillips & Perron, 1988) tests. From Table II, it is clear that all the series considered have single unit root. The Johansen (1991) cointegration test results are reported in Table III. When performing the cointegration tests, a rule where one starts out with most restrictive model to less restrictive model and with zero cointegrating vector to higher number of cointegrating vectors until one accepts the null hypothesis is followed. One also needs to

TABLE II
Results of the Unit-Root Tests on Futures and Spot Prices

	Logarithm of Spot Price (S)				*Logarithm of Futures Price (F)*			
	Level		*First Difference*		*Level*		*First Difference*	
	ADF	*PP*	*ADF*	*PP*	*ADF*	*PP*	*ADF*	*PP*
FTSE 100	−1.871	−1.928	−114.77***	−114.80***	−1.882	−1.933	−119.29***	−119.25***
TOPIX	−1.301	−1.346	−53.83***	−81.62***	−1.354	−1.372	−87.35***	−87.33***
S&P 500	−1.804	−1.804	−100.72***	−100.68***	−2.051	−1.964	−103.61***	−103.64***

Note. This table lists the results of the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) Unit-Root Tests on the logarithm of spot and futures prices. The critical values are −2.57, −2.87, and −3.43 at 10, 5, and 1% level of significance respectively.
***, **, and * indicate the test statistic to be significant at 1, 5, and 10%, respectively.
FTSE, Financial Times Stock Exchange; TOPIX, Tokyo Stock Price Index.

TABLE III
Johansen Tests on Number of Cointegrating Vectors

	Cointegrating Vector		*Zero Cointegrating Vectors (r = 0)*			*One Cointegrating Vector (r = 1)*		
	Coeff. of S	*Coeff. of F*	*Eigenvalue*	λ_{max}	*Trace*		λ_{max}	*Trace*
FTSE 100	1	−0.9999	0.00184	24.67***	24.68***	3.570E−07	0.0048	0.0048
TOPIX	1	−1.0001	0.00269	19.57***	19.57***	1.710E−07	0.0012	0.0012
S&P 500	1	−0.9995	0.00123	13.25**	13.26**	5.536E−07	0.0059	0.0059

Note. The critical values for λ_{max} statistic for $r = 0$ are 9.52, 11.44, and 15.69 at 10, 5, and 1%, respectively, and for $r = 1$ are 2.68, 3.84, and 6.51 at 10, 5, and 1%, respectively. The critical values for Trace statistic for $r = 0$ are 10.47, 12.53, and 16.31 at 10, 5, and 1%, respectively, and for $r = 1$ they are the same as for λ_{max} statistic. The critical values are taken from Osterwald-Lenum (1992) (Table 0).
***, **, and * indicate the test statistic to be significant at 1, 5, and 10%, respectively.
FTSE, Financial Times Stock Exchange; TOPIX, Tokyo Stock Price Index.

consider the fact that for the IS and MIS methods to be applicable the cointegrating vectors should be (1, −1). Then, one ends up using the model without intercept and trend in cointegrating vector and no intercept and trend in test Vector Autoregressive (VAR).[15] Both the Trace and λ-Max statistics indicate that the spot and futures prices are cointegrated.

The estimates of IS as well as MIS are summarized in Table IV. For all three indices, the IS upper and lower bounds are far apart. As mentioned earlier

[15]This refers to case 0 in Osterwald-Lenum (1992). The authors also experimented with tests where the intercept or the trend term is included. In each case, it is found that one cannot reject the existence of a single cointegrating vector. In some cases, the Johansen cointegration-based tests on S&P 500 index reject the cointegrating vector being (1, −1), which is required by the Hasbrouck IS measure as well as the authors' MIS measure. However, unit-root tests cannot reject the hypothesis that the cointegrating variable constructed by the (1, –1) vector is stationary. Because this variable bears important practical implications (i.e., it represents the basis), it is adopted to proceed with further analysis.

TABLE IV
Estimation of Information Share

		Information Share of the Spot Market			*Information Share of the Futures Market*			*Innovation Standard Deviation*	
	ρ	*Modified IS*	*Upper Bound*	*Lower Bound*	*Modified IS*	*Upper Bound*	*Lower Bound*	*Spot (%)*	*Futures (%)*
FTSE 100	74.961	**0.232**	0.639	0.006	**0.768**	0.994	0.361	0.067	0.086
TOPIX	91.184	**0.389**	0.899	0.010	**0.611**	0.990	0.101	0.131	0.160
S&P 500	85.669	**0.286**	0.776	0.002	**0.714**	0.998	0.224	0.068	0.077

Note. ρ denotes the sample correlation between the spot and futures innovations.
IS, information share; FTSE, Financial Times Stock Exchange; TOPIX, Tokyo Stock Price Index.

large innovation correlation coefficients would lead to such results. Indeed, large sample innovation correlations, reported in Table II, can explain such results.

It is interesting to note that the innovation standard deviation for the futures market is larger than the innovation standard deviation for the spot market for each index being considered. However, the difference is not that large. Among the indices, TOPIX seems to have the largest innovation standard deviation for both spot and futures markets.

Based on MIS, it is found that price discovery takes place mostly in the futures market instead of the spot market for all the three indices. The evidence supports the transaction cost hypothesis, which states that price discovery is expected to take place in the market with the lower transaction cost (Fleming et al., 1996). This evidence is consistent with the evidence reported by Garbade and Silber (1983), Oellermann, Brorsen, and Farris (1989), Schroeder and Goodwin (1991), Yang, Bessler, and Leatham (2001), and So and Tse (2004). Finally, it is important to point out that the finding is also consistent with the highly liquid futures market and the theoretical work of Garbade and Silver (1983) where it is shown that price discovery is determined by the relative number of participants in each market, which could eventually be related to trading volume or liquidity.[16]

CONCLUSION

In a pioneering work, Hasbrouck (1995) suggested a measure of price discovery commonly known as the IS. The attractive features of IS have to do with the fact that it incorporates both the system dynamics as well as innovation

[16]See footnote 2.

covariances. In addition, it is consistent with the conventional argument that security prices are martingales. However, Hasbrouck's method leads to upper and lower IS bounds instead of a unique measure. In this study, the authors propose an MIS that results in a unique measure of price discovery by using a different factor structure that is based on the correlation matrix.

In this study, using Monte Carlo simulations, the performances of the MIS and the IS measures are compared. In the simulation, 1,000 replications are used (with individual series of 100,000 observations) for the same three examples considered by Hasbrouck (2002). It is found that MIS performs best compared to Hasbrouck IS as well as the measure based on permanent–transitory decomposition proposed by Gonzalo and Granger (1995).

As an empirical application, the authors estimate the MIS for the spot and futures markets for S&P 500, FTSE 100, and TOPIX stock indices using intraday data. The estimates are used to test the transaction cost hypothesis, which states that the price discovery should mostly take place in the futures market due to the lower transaction cost associated with futures trading as compared to spot trading. The upper and lower IS bounds are found to be far apart making it difficult to make judgment about the price discovery. The large difference in the bounds can be explained by the high correlation between the innovations in futures and spot markets. However, the proposed MIS provides a unique measure. Based on MIS, it is found that indeed price discovery takes place mostly in the futures market for all three indices. The evidence clearly supports the transaction cost hypothesis. This is also consistent with the theoretical model proposed by Garbade and Silber (1983) where the price discovery is determined by the relative participants in each market.

Finally, the application of MIS on the price discovery in spot and futures market constitutes only one of the potential applications of MIS. It is hoped that the method proposed would be used by other researchers in many other areas where the price discovery mechanism plays an important role.

APPENDIX A

Uniqueness of MIS Measure

In this section, the authors show that the MIS measure is independent of the ordering of series. Let E_{ij} denote the elementary matrix obtained by interchanging the ith and jth rows of an $n \times n$ identity matrix. In the proof of the uniqueness of MIS measure, the following properties are used:

Property 1: An elementary matrix is symmetric and is its own inverse; i.e., $E_{ij}E_{ij} = I$ and $E_{ij}^{-1} = E_{ij}$.

Property 2: If v_1 is an eigenvector of a square matrix A with the corresponding eigenvalue λ_l, then $E_{ij}v_l$ is the eigenvector of $E_{ij}A$ corresponding to the same eigenvalue λ_l.

Now, let us consider the VMA representation given by Equation (2) with ith and jth series being interchanged. Then the resulting VMA can be written as

$$E_{ij}\Delta Y_t = E_{ij}\Psi(L)\varepsilon_t.$$

According to Property 1, one has

$$E_{ij}\Delta Y_t = E_{ij}\Psi(L)E_{ij}E_{ij}\varepsilon_t.$$

Or,

$$\Delta\tilde{Y}_t = \tilde{\Psi}(L)\tilde{\varepsilon}_t \tag{A1}$$

where $\Delta\tilde{Y}_t = E_{ij}\Delta Y_t$, $\tilde{\Psi}(L) = E_{ij}\Psi(L)E_{ij}$, and $\tilde{\varepsilon}_t = E_{ij}\varepsilon_t$. The correlation matrix of the reordered innovation vector $\tilde{\varepsilon}_t$ is given by $\tilde{\Phi} = E_{ij}\Phi E_{ij}$. From Property 2, it is clear that $\tilde{\Lambda} = \Lambda$ and the orthonormal matrix with columns as the eigenvectors $\tilde{G}$ is given by $\tilde{G} = E_{ij}G$. One also has the following:

$$\tilde{V} = E_{ij}VE_{ij},\ \tilde{\psi} = \psi E_{ij}$$

and

$$\tilde{\psi}\tilde{\Omega}\tilde{\psi}^T = \psi E_{ij}E_{ij}\Omega E_{ij}E_{ij}^T\Psi^T = \psi\Omega\Psi^T$$

where the symbols with "$\sim$" on top for the reordered system (Equation (A1)) correspond to the same symbols without "$\sim$" for the original system. Also note the following:

$$\begin{aligned}
\tilde{F}^* &= F^* = [\tilde{G}\tilde{\Lambda}^{-1/2}\tilde{G}^T\tilde{V}^{-1}]^{-1} \\
&= [E_{ij}G\Lambda^{-1/2}G^TE_{ij}E_{ij}V^{-1}E_{ij}]^{-1} = [E_{ij}G\Lambda^{-1/2}G^TV^{-1}E_{ij}]^{-1} \\
\tilde{F}^* &= E_{ij}^{-1}[G\Lambda^{-1/2}G^TV^{-1}]^{-1}E_{ij}^{-1} = E_{ij}F^*E_{ij}
\end{aligned}$$

and

$$\tilde{\psi}\tilde{F}^* = \psi E_{ij}E_{ij}F^*E_{ij} = \psi F^*E_{ij}.$$

Therefore, it is clear that the MIS measure is independent of the reordering.

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