ATLAS: Interactive and Educational Linear Algebra System Containing Non-Standard Methods

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30 July 2021 at MathUI/CICM 2021

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- This project aims to fill that gap, focusing specifically on methods like Strassen's fast matrix multiplication and other less common methods
- ATLAS aims to also provide the ability to compare the step-by-step solutions of methods simultaneously
- By providing non-standard methods and step-by-step solutions for all methods, users have a greater choice and they can see exactly how problems can be solved using the methods

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CoCalc CoCalc [Ste18] is web-based linear algebra system, which makes it more portable than Maple or MATLAB. Cocal allows teachers to conduct their lessons entirely on CoCalc through an interactive platform. CoCalc acts more as an e-learning platform such as Moodle.

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- Tedious by hand, but easy with this tool.

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- For calculating determinants, Laplace expansion, Sarrus' method and LU decomposition are supported, allowing users to compare all 3 methods step-by-step simultaneously.
- This is also true of matrix multiplication, which supports the standard method, Strassen's method and the Laderman method, and calculation of inverses using both the Cramer's rule and the Cayley-Hamilton theorem.
- Systems of linear equations can be solved by Gaussian elimination, Cramer's rule and Cholesky decomposition, whilst being compared with each other simultaneously

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- The next stage of development is the implementation of symbolic inputs, as ATLAS is currently limited to only numerical inputs.
- There is a desire to extend to comparisons of methods that are considered numerically good or bad to understand the effect of different methods on problems with floating point numbers.
- Also, there is an aspiration to improve the portability of ATLAS by creating a web-based equivalent, similar to CoCalc.

Demonstration

Demo Now we shall demonstrate some of the functionality of ATLAS

Q&A

? Any questions?

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