ATLAS: Interactive and Educational Linear Algebra System Containing Non-Standard Methods

Akhilesh Pai[1] James H. Davenport[1]

University of Bath, Bath BA2 7AY, United Kingdom
{abp34;J.H.Davenport}@bath.ac.uk

30 July 2021 at MathUI/CICM 2021

Plan of Talk

1. Introduction
2. Prior Work
3. Current State
4. Demonstration
5. Q&A

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools
- Most of these tools also do not provide step-by-step solutions for problems

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools
- Most of these tools also do not provide step-by-step solutions for problems
- This project aims to fill that gap, focusing specifically on methods like Strassen's fast matrix multiplication and other less common methods

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools
- Most of these tools also do not provide step-by-step solutions for problems
- This project aims to fill that gap, focusing specifically on methods like Strassen's fast matrix multiplication and other less common methods
- ATLAS aims to also provide the ability to compare the step-by-step solutions of methods simultaneously

Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools
- Most of these tools also do not provide step-by-step solutions for problems
- This project aims to fill that gap, focusing specifically on methods like Strassen's fast matrix multiplication and other less common methods
- ATLAS aims to also provide the ability to compare the step-by-step solutions of methods simultaneously
- By providing non-standard methods and step-by-step solutions for all methods, users have a greater choice and they can see exactly how problems can be solved using the methods

Prior Work I

Maple Maple has a feature called Tutor, allowing users to view the step-by-step solutions to a problem for linear algebra functions, such as calculation of eigenvalues [Map21]. Tutor may explain the popularity of Maple amongst students, as it can serves as a key tool in their education. However, this feature is also limited to only standard methods. Developing a Tutor-style feature in a linear algebra system, however, combined with non-standard methods could allow different methods to be compared against each other.

Prior Work II

MATLAB MATLAB does not contain a step-by-step solution feature. But users need to learn a new programming language to utilise it. It also does not offer alternative methods, such as Strassen's method. [Han09] conducted a study about whether MATLAB can act as a useful tool to supplement the learning of university students. Most students liked using MATLAB, with specific preference amongst lower achieving students as they liked getting an answer without working it out, which is not very helpful for their learning. Some students, however, did not like it because it did not show how it got to the answer and they wanted to see how the problem was solved step-by-step, otherwise it wouldn't help them. Some students also stated that they were "not good at computers", highlighting the need for simplicity in a linear algebra system for users who are unfamiliar with computers or programming.

Prior Work III

CoCalc CoCalc [Ste18] is web-based linear algebra system, which makes it more portable than Maple or MATLAB. CoCalc allows collaboration by multiple collaborators simultaneously [Sag21], in addition to allowing teachers to conduct their lessons entirely on CoCalc through an interactive platform. Whilst this is an interesting idea, it does little to directly help students to learn independently. Based on the features available, CoCalc acts more as an e-learning platform such as Moodle, used by many universities. CoCalc also suffers some of the flaws of MATLAB, such that users would need to learn new programming languages to utilise CoCalc.

Pedagogical Example: Strassen–Winograd

Pedagogical Example: Strassen–Winograd

- Very hard to motivate these.

Pedagogical Example: Strassen–Winograd

- Very hard to motivate these.
- But matrix multiplication is bilinear, and these expressions are also bilinear (verification easy).

Pedagogical Example: Strassen–Winograd

- Very hard to motivate these.
- But matrix multiplication is bilinear, and these expressions are also bilinear (verification easy).
- Hence it suffices to verify

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$$

(and three analogues)

Pedagogical Example: Strassen–Winograd

- Very hard to motivate these.
- But matrix multiplication is bilinear, and these expressions are also bilinear (verification easy).
- Hence it suffices to verify

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$$

  (and three analogues)
- Tedious by hand, but easy with this tool.

Current State (1)

Current State (1)

- ATLAS supports the calculation of determinants, inverses, eigenvectors and eigenvalues, in addition to matrix multiplication and solving systems of linear equations.

Current State (1)

- ATLAS supports the calculation of determinants, inverses, eigenvectors and eigenvalues, in addition to matrix multiplication and solving systems of linear equations.
- For calculating determinants, Laplace expansion, Sarrus' method and LU decomposition are supported, allowing users to compare all 3 methods step-by-step simultaneously.

Current State (1)

- ATLAS supports the calculation of determinants, inverses, eigenvectors and eigenvalues, in addition to matrix multiplication and solving systems of linear equations.
- For calculating determinants, Laplace expansion, Sarrus' method and LU decomposition are supported, allowing users to compare all 3 methods step-by-step simultaneously.
- This is also true of matrix multiplication, which supports the standard method, Strassen's method and the Laderman method, and calculation of inverses using both the Cramer's rule and the Cayley-Hamilton theorem.

Current State (1)

- ATLAS supports the calculation of determinants, inverses, eigenvectors and eigenvalues, in addition to matrix multiplication and solving systems of linear equations.
- For calculating determinants, Laplace expansion, Sarrus' method and LU decomposition are supported, allowing users to compare all 3 methods step-by-step simultaneously.
- This is also true of matrix multiplication, which supports the standard method, Strassen's method and the Laderman method, and calculation of inverses using both the Cramer's rule and the Cayley-Hamilton theorem.
- Systems of linear equations can be solved by Gaussian elimination, Cramer's rule and Cholesky decomposition, whilst being compared with each other simultaneously

Current State (2)

Current State (2)

- Unit testing completed.

Current State (2)

- Unit testing completed.
- Integration and user testing currently in progress.

Current State (2)

- Unit testing completed.
- Integration and user testing currently in progress.
- There is a desire to extend to comparisons of methods that are considered numerically good or bad to understand the effect of different methods on problems with floating point numbers.

Current State (2)

- Unit testing completed.
- Integration and user testing currently in progress.
- There is a desire to extend to comparisons of methods that are considered numerically good or bad to understand the effect of different methods on problems with floating point numbers.
- Also, there is an aspiration to improve the portability of ATLAS by creating a web-based equivalent, similar to CoCalc.

Demonstration

Demo Now we shall demonstrate some of the functionality of ATLAS

Q&A

? Any questions?

Bibliography I

Xiaoxu Han.
Teaching Elementary Linear Algebra Using MATLAB: An Initial Investigation.
https://www.researchgate.net/publication/27327767_Teaching_Elementary_Linear_Algebra_Using_Matlab_An_Initial_Investigation, 01 2009.

Maplesoft.
Student[LinearAlgebra][EigenvaluesTutor] — interactive matrix eigenvalues.
https://www.maplesoft.com/support/help/Maple/view.aspx\protect\penalty\z@?path=Student%2fLinearAlgebra%2fEigenvaluesTutor, 2021.

Bibliography II


SageMath.
About collaborators, 2021.
URL: https://doc.cocalc.com/project-settings.html#about-collaborators.


W.A. Stein.
COCALC: Making open source mathematical software easily available on the web (talk at ICMS 2018).
https://goo.gl/HWdEvc, 2018.