

Zcash Protocol Specification

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1

Abstract. Zcash is an implementation of the *Decentralized Anonymous Payment* scheme **Zerocash**, with security fixes and improvements to performance and functionality. It bridges the existing transparent payment scheme used by **Bitcoin** with a *shielded* payment scheme secured by *zero-knowledge succinct non-interactive arguments of knowledge* (zk-SNARKs). It attempted to address the problem of mining centralization by use of the *Equihash* memory-hard *proof-of-work* algorithm.

This specification defines the **Zcash** consensus protocol at launch; after each of the upgrades codenamed **Overwinter**, **Sapling**, **Blossom**, **Heartwood**, **Canopy**, and **NU5**; and proposed changes for **NU6**. It is a work in progress. Protocol differences from **Zerocash** and **Bitcoin** are also explained.

Keywords: anonymity, applications, cryptographic protocols, electronic commerce and payment, financial privacy, proof of work, zero knowledge.

Contents	1
1 Introduction	7
1.1 Caution	7
1.2 High-level Overview	8
2 Notation	9

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¹ Jubbub bird image credit: Peter Newell 1902; Daira-Emma Hopwood 2018.

3	Concepts	12
3.1	Payment Addresses and Keys	12
3.2	Notes	13
3.2.1	Note Plaintexts and Memo Fields	14
3.2.2	Note Commitments	15
3.2.3	Nullifiers	16
3.3	The Block Chain	16
3.4	Transactions and Treestates	17
3.5	JoinSplit Transfers and Descriptions	18
3.6	Spend Transfers, Output Transfers, and their Descriptions	19
3.7	Action Transfers and their Descriptions	19
3.8	Note Commitment Trees	20
3.9	Nullifier Sets	21
3.10	Block Subsidy, Funding Streams, and Founders' Reward	21
3.11	Coinbase Transactions	21
3.12	Mainnet and Testnet	21
4	Abstract Protocol	22
4.1	Abstract Cryptographic Schemes	23
4.1.1	Hash Functions	23
4.1.2	Pseudo Random Functions	24
4.1.3	Pseudo Random Permutations	25
4.1.4	Symmetric Encryption	25
4.1.5	Key Agreement	25
4.1.6	Key Derivation	26
4.1.7	Signature	26
4.1.7.1	Signature with Re-Randomizable Keys	28
4.1.7.2	Signature with Signing Key to Validating Key Monomorphism	29
4.1.8	Commitment	29
4.1.9	Represented Group	31
4.1.10	Coordinate Extractor	32
4.1.11	Group Hash	32
4.1.12	Represented Pairing	33
4.1.13	Zero-Knowledge Proving System	33
4.2	Key Components	35
4.2.1	Sprout Key Components	35
4.2.2	Sapling Key Components	35
4.2.3	Orchard Key Components	37
4.3	JoinSplit Descriptions	38
4.4	Spend Descriptions	39
4.5	Output Descriptions	40
4.6	Action Descriptions	41
4.7	Sending Notes	42
4.7.1	Sending Notes (Sprout)	42
4.7.2	Sending Notes (Sapling)	43

4.7.3	Sending Notes (Orchard)	44
4.8	Dummy Notes	45
4.8.1	Dummy Notes (Sprout)	45
4.8.2	Dummy Notes (Sapling)	46
4.8.3	Dummy Notes (Orchard)	47
4.9	Merkle Path Validity	47
4.10	SIGHASH Transaction Hashing	49
4.11	Non-malleability (Sprout)	50
4.12	Balance (Sprout)	50
4.13	Balance and Binding Signature (Sapling)	51
4.14	Balance and Binding Signature (Orchard)	53
4.15	Spend Authorization Signature (Sapling and Orchard)	55
4.16	Computing ρ values and Nullifiers	56
4.17	Chain Value Pool Balances	57
4.18	Zk-SNARK Statements	58
4.18.1	JoinSplit Statement (Sprout)	58
4.18.2	Spend Statement (Sapling)	59
4.18.3	Output Statement (Sapling)	60
4.18.4	Action Statement (Orchard)	61
4.19	In-band secret distribution (Sprout)	63
4.19.1	Encryption (Sprout)	63
4.19.2	Decryption (Sprout)	64
4.20	In-band secret distribution (Sapling and Orchard)	65
4.20.1	Encryption (Sapling and Orchard)	65
4.20.2	Decryption using an Incoming Viewing Key (Sapling and Orchard)	66
4.20.3	Decryption using a Full Viewing Key (Sapling and Orchard)	68
4.21	Block Chain Scanning (Sprout)	69
4.22	Block Chain Scanning (Sapling and Orchard)	70
5	Concrete Protocol	71
5.1	Integers, Bit Sequences, and Endianness	71
5.2	Bit layout diagrams	71
5.3	Constants	72
5.4	Concrete Cryptographic Schemes	73
5.4.1	Hash Functions	73
5.4.1.1	SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions	73
5.4.1.2	BLAKE2 Hash Functions	74
5.4.1.3	Merkle Tree Hash Function	74
5.4.1.4	h_{sig} Hash Function	75
5.4.1.5	CRH^{ivk} Hash Function	75
5.4.1.6	$\text{DiversifyHash}^{\text{Sapling}}$ and $\text{DiversifyHash}^{\text{Orchard}}$ Hash Functions	76
5.4.1.7	Pedersen Hash Function	77
5.4.1.8	Mixing Pedersen Hash Function	79
5.4.1.9	Sinsemilla Hash Function	79
5.4.1.10	PoseidonHash Function	82

5.4.1.11	Equihash Generator	83
5.4.2	Pseudo Random Functions	84
5.4.3	Symmetric Encryption	86
5.4.4	Pseudo Random Permutations	86
5.4.5	Key Agreement And Derivation	86
5.4.5.1	Sprout Key Agreement	86
5.4.5.2	Sprout Key Derivation	87
5.4.5.3	Sapling Key Agreement	87
5.4.5.4	Sapling Key Derivation	87
5.4.5.5	Orchard Key Agreement	88
5.4.5.6	Orchard Key Derivation	88
5.4.6	Ed25519	88
5.4.7	RedDSA, RedJubjub, and RedPallas	90
5.4.7.1	Spend Authorization Signature (Sapling and Orchard)	93
5.4.7.2	Binding Signature (Sapling and Orchard)	93
5.4.8	Commitment schemes	93
5.4.8.1	Sprout Note Commitments	93
5.4.8.2	Windowed Pedersen commitments	94
5.4.8.3	Homomorphic Pedersen commitments (Sapling and Orchard)	95
5.4.8.4	Sinsemilla commitments	96
5.4.9	Represented Groups and Pairings	97
5.4.9.1	BN-254	97
5.4.9.2	BLS12-381	99
5.4.9.3	Jubjub	100
5.4.9.4	Coordinate Extractor for Jubjub	102
5.4.9.5	Group Hash into Jubjub	102
5.4.9.6	Pallas and Vesta	103
5.4.9.7	Coordinate Extractor for Pallas	104
5.4.9.8	Group Hash into Pallas and Vesta	105
5.4.10	Zero-Knowledge Proving Systems	108
5.4.10.1	BCTV14	108
5.4.10.2	Groth16	109
5.4.10.3	Halo 2	110
5.5	Encodings of Note Plaintexts and Memo Fields	110
5.6	Encodings of Addresses and Keys	111
5.6.1	Transparent Encodings	111
5.6.1.1	Transparent Addresses	111
5.6.1.2	Transparent Private Keys	112
5.6.2	Sprout Encodings	112
5.6.2.1	Sprout Payment Addresses	112
5.6.2.2	Sprout Incoming Viewing Keys	112
5.6.2.3	Sprout Spending Keys	113
5.6.3	Sapling Encodings	113
5.6.3.1	Sapling Payment Addresses	113
5.6.3.2	Sapling Incoming Viewing Keys	114

5.6.3.3	Sapling Full Viewing Keys	114
5.6.3.4	Sapling Spending Keys	114
5.6.4	Unified and Orchard Encodings	115
5.6.4.1	Unified Payment Addresses and Viewing Keys	115
5.6.4.2	Orchard Raw Payment Addresses	115
5.6.4.3	Orchard Raw Incoming Viewing Keys	116
5.6.4.4	Orchard Raw Full Viewing Keys	116
5.6.4.5	Orchard Spending Keys	116
5.7	BCTV14 zk-SNARK Parameters	117
5.8	Groth16 zk-SNARK Parameters	117
5.9	Randomness Beacon	117
6	Network Upgrades	118
7	Consensus Changes from Bitcoin	119
7.1	Transaction Encoding and Consensus	119
7.1.1	Transaction Identifiers	121
7.1.2	Transaction Consensus Rules	121
7.2	JoinSplit Description Encoding and Consensus	125
7.3	Spend Description Encoding and Consensus	125
7.4	Output Description Encoding and Consensus	126
7.5	Action Description Encoding and Consensus	127
7.6	Block Header Encoding and Consensus	128
7.7	Proof of Work	130
7.7.1	Equihash	130
7.7.2	Difficulty filter	131
7.7.3	Difficulty adjustment	131
7.7.4	nBits conversion	133
7.7.5	Definition of Work	133
7.8	Calculation of Block Subsidy, Funding Streams, and Founders' Reward	133
7.9	Payment of Founders' Reward	134
7.10	Payment of Funding Streams	136
7.10.1	ZIP 214 Funding Streams	137
7.11	Changes to the Script System	138
7.12	Bitcoin Improvement Proposals	138
8	Differences from the Zerocash paper	139
8.1	Transaction Structure	139
8.2	Memo Fields	139
8.3	Unification of Mints and Pours	139
8.4	Faerie Gold attack and fix	140
8.5	Internal hash collision attack and fix	141
8.6	Changes to PRF inputs and truncation	142
8.7	In-band secret distribution	143
8.8	Omission in Zerocash security proof	145
8.9	Miscellaneous	145

9	Acknowledgements	146
10	Change History	147
11	References	180
	Appendices	195
A	Circuit Design	195
A.1	Quadratic Constraint Programs	195
A.2	Elliptic curve background	195
A.3	Circuit Components	196
A.3.1	Operations on individual bits	196
A.3.1.1	Boolean constraints	196
A.3.1.2	Conditional equality	197
A.3.1.3	Selection constraints	197
A.3.1.4	Nonzero constraints	197
A.3.1.5	Exclusive-or constraints	197
A.3.2	Operations on multiple bits	197
A.3.2.1	[Un]packing modulo r_s	197
A.3.2.2	Range check	198
A.3.3	Elliptic curve operations	200
A.3.3.1	Checking that Affine-ctEdwards coordinates are on the curve	200
A.3.3.2	ctEdwards [de]compression and validation	200
A.3.3.3	ctEdwards \leftrightarrow Montgomery conversion	200
A.3.3.4	Affine-Montgomery arithmetic	201
A.3.3.5	Affine-ctEdwards arithmetic	202
A.3.3.6	Affine-ctEdwards nonsmall-order check	203
A.3.3.7	Fixed-base Affine-ctEdwards scalar multiplication	203
A.3.3.8	Variable-base Affine-ctEdwards scalar multiplication	204
A.3.3.9	Pedersen hash	205
A.3.3.10	Mixing Pedersen hash	207
A.3.4	Merkle path check	208
A.3.5	Windowed Pedersen Commitment	208
A.3.6	Homomorphic Pedersen Commitment	208
A.3.7	BLAKE2s hashes	209
A.4	The Sapling Spend circuit	212
A.5	The Sapling Output circuit	214
B	Batching Optimizations	215
B.1	RedDSA batch validation	215
B.2	Groth16 batch verification	216
B.3	Ed25519 batch validation	218
	List of Theorems and Lemmata	219
	Index	219

1 Introduction

Zcash is an implementation of the *Decentralized Anonymous Payment scheme Zerocash* [BCGGMTV2014], with security fixes and improvements to performance and functionality. It bridges the existing transparent payment scheme used by **Bitcoin** [Nakamoto2008] with a *shielded* payment scheme secured by *zero-knowledge succinct non-interactive arguments of knowledge (zk-SNARKs)*.

In this document, technical terms for concepts that play an important rôle in **Zcash** are written in *slanted text*, which links to an index entry. *Italics* are used for emphasis and for references between sections of the document. The symbol § precedes section numbers in cross-references.

The key words **MUST**, **MUST NOT**, **SHOULD**, **SHOULD NOT**, **RECOMMENDED**, **MAY**, and **OPTIONAL** in this document are to be interpreted as described in [RFC-2119] when they appear in **ALL CAPS**. These words may also appear in this document in lower case as plain English words, absent their normative meanings.

The most significant changes from the original **Zerocash** are explained in § 8 ‘*Differences from the Zerocash paper*’ on p. 139.

Changes specific to the **Overwinter** upgrade are highlighted in [bright blue](#).

Changes specific to the **Sapling** upgrade following **Overwinter** are highlighted in [green](#).

Changes specific to the **Blossom** upgrade following **Sapling** are highlighted in [red](#).

Changes specific to the **Heartwood** upgrade following **Blossom** are highlighted in [orange](#).

Changes specific to the **Canopy** upgrade following **Heartwood** are highlighted in [purple](#).

Changes specific to the **NU5** upgrade following **Canopy** are highlighted in [slate blue](#).

Changes specific to the proposed **NU6** upgrade following **NU5** are highlighted in [pink](#).

All of these are also changes from **Zerocash**. The name **Sprout** is used for the **Zcash** protocol prior to **Sapling** (both before and after **Overwinter**), and in particular its shielded protocol.

This specification is structured as follows:

- Notation – definitions of notation used throughout the document;
- Concepts – the principal abstractions needed to understand the protocol;
- Abstract Protocol – a high-level description of the protocol in terms of ideal cryptographic components;
- Concrete Protocol – how the functions and encodings of the abstract protocol are instantiated;
- Network Upgrades – the strategy for upgrading the **Zcash** protocol.
- Consensus Changes from **Bitcoin** – how **Zcash** differs from **Bitcoin** at the consensus layer, including the Proof of Work;
- Differences from the **Zerocash** protocol – a summary of changes from the protocol in [BCGGMTV2014].
- Appendix: Circuit Design – details of how the **Sapling** circuits are defined as *quadratic constraint programs*.
- Appendix: Batching Optimizations – improvements to the efficiency of validating multiple signatures and verifying multiple proofs.

1.1 Caution

Zcash security depends on consensus. Should a program interacting with the **Zcash** network diverge from consensus, its security will be weakened or destroyed. The cause of the divergence doesn’t matter: it could be a bug in your program, it could be an error in this documentation which you implemented as described, or it could be that you do everything right but other software on the network behaves unexpectedly. The specific cause will not matter to the users of your software whose wealth is lost.

Having said that, a specification of *intended* behaviour is essential for security analysis, understanding of the protocol, and maintenance of **Zcash** and related software. If you find any mistake in this specification, please file an issue at <https://github.com/zcash/zips/issues> or contact <security@z.cash>.

1.2 High-level Overview

The following overview is intended to give a concise summary of the ideas behind the protocol, for an audience already familiar with *block chain*-based cryptocurrencies such as **Bitcoin**. It is imprecise in some aspects and is not part of the normative protocol specification. This overview applies to **Sprout**, **Sapling**, and **Orchard**, differences in the cryptographic constructions used notwithstanding.

All value in **Zcash** belongs to some *chain value pool*. There is a single *transparent chain value pool*, a *chain value pool* for each *shielded* protocol (**Sprout** or **Sapling** or **Orchard**), and a *deferred development fund chain value pool*. Transfers of *transparent* value work essentially as in **Bitcoin** and have the same privacy properties. Value in a *shielded chain value pool* is carried by *notes*², which specify an amount and (indirectly) a *shielded payment address*, which is a destination to which *notes* can be sent. As in **Bitcoin**, this is associated with a *private key* that can be used to spend *notes* sent to the address; in **Zcash** this is called a *spending key*.

To each *note* there is cryptographically associated a *note commitment*. Once the *transaction* creating a *note* has been mined, the *note* is associated with a fixed *note position* in a tree of *note commitments*, and with a *nullifier*² unique to that *note*. Computing the *nullifier* requires the associated private *spending key* (or the *nullifier deriving key* for **Sapling** or **Orchard** *notes*). It is infeasible to correlate the *note commitment* or *note position* with the corresponding *nullifier* without knowledge of at least this key. An unspent valid *note*, at a given point on the *block chain*, is one for which the *note commitment* has been publically revealed on the *block chain* prior to that point, but the *nullifier* has not.

A *transaction* can contain *transparent* inputs, outputs, and scripts, which all work as in **Bitcoin** [Bitcoin-Protocol]. It also can include *JoinSplit* descriptions, *Spend descriptions*, *Output descriptions* and *Action descriptions*. Together these describe *shielded transfers* which take in *shielded input notes*, and/or produce *shielded output notes*. (For **Sprout**, each *JoinSplit* description handles up to two *shielded inputs* and up to two *shielded outputs*. For **Sapling**, each *shielded input* or *shielded output* has its own description. For **Orchard**, each *Action description* handles up to one *shielded input* and up to one *shielded output*.) It is also possible for value to be transferred between *chain value pools*, either *transparent* or *shielded*; this always reveals the amount transferred.

In each *shielded transfer*, the *nullifiers* of the input *notes* are revealed (preventing them from being spent again) and the commitments of the output *notes* are revealed (allowing them to be spent in future). A *transaction* also includes computationally sound zk-SNARK proofs and signatures, which prove that all of the following hold except with insignificant probability:

For each *shielded input*,

- [Sapling onward] there is a revealed *value commitment* to the same value as the input *note*;³
- if the value is nonzero, some revealed *note commitment* exists for this *note*;
- the prover knew the *proof authorizing key* of the *note*;
- the *nullifier* and *note commitment* are computed correctly.

and for each *shielded output*,

- [Sapling onward] there is a revealed *value commitment* to the same value as the output *note*;³
- the *note commitment* is computed correctly;
- it is infeasible to cause the *nullifier* of the output *note* to collide with the *nullifier* of any other *note*.

² In **Zerocash** [BCGGMTV2014], *notes* were called “coins”, and *nullifiers* were called “serial numbers”.

³ For **Orchard**, each *Action* reveals a single *value commitment* to the net value spent by the *Action*, rather than one *value commitment* for the input *note* and one for the output *note*.

For **Sprout**, the *JoinSplit statement* also includes an explicit balance check. For **Sapling** and **Orchard**, the *value commitments* corresponding to the inputs and outputs are checked to balance (together with any net transparent input or output) outside the zk-SNARK.

In addition, various measures (differing between **Sprout** and **Sapling** or **Orchard**) are used to ensure that the *transaction* cannot be modified by a party not authorized to do so.

Outside the zk-SNARK, it is checked that the *nullifiers* for the input *notes* had not already been revealed (i.e. they had not already been spent).

A *shielded payment address* includes a *transmission key* for a “key-private” asymmetric encryption scheme. *Key-private* means that ciphertexts do not reveal information about which key they were encrypted to, except to a holder of the corresponding *private key*, which in this context is called the *receiving key*. This facility is used to communicate encrypted output *notes* on the *block chain* to their intended recipient, who can use the *receiving key* to scan the *block chain* for *notes* addressed to them and then decrypt those *notes*.

In **Sapling** and **Orchard**, for each *spending key* there is a *full viewing key* that allows recognizing both incoming and outgoing *notes* without having *spending authority*. This is implemented by an additional ciphertext in each *Output description* or *Action description*.

The basis of the privacy properties of **Zcash** is that when a *note* is spent, the spender only proves that some commitment for it had been revealed, without revealing which one. This implies that a spent *note* cannot be linked to the *transaction* in which it was created. That is, from an adversary’s point of view the set of possibilities for a given *note* input to a *transaction* —its *note traceability set*— includes *all* previous notes that the adversary does not control or know to have been spent.⁴ This contrasts with other proposals for private payment systems, such as CoinJoin [Bitcoin-CoinJoin] or **CryptoNote** [vanSaberh2014], that are based on mixing of a limited number of transactions and that therefore have smaller *note traceability sets*.

The *nullifiers* are necessary to prevent *double-spending*: each *note* on the *block chain* only has one valid *nullifier*, and so attempting to spend a *note* twice would reveal the *nullifier* twice, which would cause the second *transaction* to be rejected.

2 Notation

\mathbb{B} means the type of bit values, i.e. $\{0, 1\}$. \mathbb{B}^8 means the type of byte values, i.e. $\{0 \dots 255\}$.

\mathbb{N} means the type of nonnegative integers. \mathbb{N}^+ means the type of positive integers. \mathbb{Z} means the type of integers. \mathbb{Q} means the type of rationals.

$x : T$ is used to specify that x has type T . A cartesian product type is denoted by $S \times T$, and a function type by $S \rightarrow T$. An argument to a function can determine other argument or result types.

The type of a randomized algorithm is denoted by $S \xrightarrow{R} T$. The domain of a randomized algorithm may be $()$, indicating that it requires no arguments. Given $f : S \xrightarrow{R} T$ and $s : S$, sampling a variable $x : T$ from the output of f applied to s is denoted by $x \xleftarrow{R} f(s)$.

Initial arguments to a function or randomized algorithm may be written as subscripts, e.g. if $x : X$, $y : Y$, and $f : X \times Y \rightarrow Z$, then an invocation of $f(x, y)$ can also be written $f_x(y)$.

$\{x : T \mid p_x\}$ means the subset of x from T for which p_x (a boolean expression depending on x) holds.

$T \subseteq U$ indicates that T is an inclusive subset or subtype of U .

$S \cup T$ means the set union of S and T .

$S \cap T$ means the set intersection of S and T , i.e. $\{x : S \mid x \in T\}$.

⁴ We make this claim only for *fully shielded transactions*. It does not exclude the possibility that an adversary may use data present in the cleartext of a *transaction* such as the number of inputs and outputs, or metadata-based heuristics such as timing, to make probabilistic inferences about *transaction* linkage. For consequences of this in the case of partially shielded *transactions*, see [Peterson2017], [Quesnelle2017], and [KYMM2018].

$S \setminus T$ means the set difference obtained by removing elements in T from S , i.e. $\{x : S \mid x \notin T\}$.

$x : T \mapsto e_x : U$ means the function of type $T \rightarrow U$ mapping formal parameter x to e_x (an expression depending on x). The types T and U are always explicit.

$x : T \mapsto_{\notin V} e_x : U$ means $x : T \mapsto e_x : U \cup V$ restricted to the domain $\{x : T \mid e_x \notin V\}$ and range U .

$\mathcal{P}(T)$ means the powerset of T .

\perp is a distinguished value used to indicate unavailable information, a failed decryption or validity check, or an exceptional case.

$T^{[\ell]}$, where T is a type and ℓ is an integer, means the type of sequences of length ℓ with elements in T . For example, $\mathbb{B}^{[\ell]}$ means the set of sequences of ℓ bits, and $\mathbb{B}^{[k]}$ means the set of sequences of k bytes.

$\mathbb{B}^{[N]}$ means the type of *byte sequences* of arbitrary length.

$\text{length}(S)$ means the length of (number of elements in) S .

$\text{truncate}_k(S)$ means the sequence formed from the first k elements of S .

0x followed by a string of monospace hexadecimal digits means the corresponding integer converted from hexadecimal. $[0x00]^\ell$ means the sequence of ℓ zero bytes.

“...” means the given string represented as a sequence of bytes in *US-ASCII*. For example, “abc” represents the *byte sequence* $[0x61, 0x62, 0x63]$.

$[0]^\ell$ means the sequence of ℓ zero bits. $[1]^\ell$ means the sequence of ℓ one bits.

$a..b$, used as a subscript, means the sequence of values with indices a through b inclusive. For example, $a_{pk,1..N}^{\text{new}}$ means the sequence $[a_{pk,1}^{\text{new}}, a_{pk,2}^{\text{new}}, \dots, a_{pk,N}^{\text{new}}]$. (For consistency with the notation in [BCGGMTV2014] and in [BK2016], this specification uses 1-based indexing and inclusive ranges, notwithstanding the compelling arguments to the contrary made in [EWD-831].)

$\{a..b\}$ means the set or type of integers from a through b inclusive.

$[f(x) \text{ for } x \text{ from } a \text{ up to } b]$ means the sequence formed by evaluating f on each integer from a to b inclusive, in ascending order. Similarly, $[f(x) \text{ for } x \text{ from } a \text{ down to } b]$ means the sequence formed by evaluating f on each integer from a to b inclusive, in descending order.

$a || b$ means the concatenation of sequences a then b .

$\text{concat}_{\mathbb{B}}(S)$ means the sequence of bits obtained by concatenating the elements of S as *bit sequences*.

$\text{sorted}(S)$ means the sequence formed by sorting the elements of S .

\mathbb{F}_n means the finite field with n elements, and \mathbb{F}_n^* means its group under multiplication (which excludes 0).

Where there is a need to make the distinction, we denote the unique representative of $a : \mathbb{F}_n$ in the range $\{0..n-1\}$ (or the unique representative of $a : \mathbb{F}_n^*$ in the range $\{1..n-1\}$) as $a \bmod n$. Conversely, we denote the element of \mathbb{F}_n corresponding to an integer $k : \mathbb{Z}$ as $k \pmod n$. We also use the latter notation in the context of an equality $k = k' \pmod n$ as shorthand for $k \bmod n = k' \bmod n$, and similarly $k \neq k' \pmod n$ as shorthand for $k \bmod n \neq k' \bmod n$. (When referring to constants such as 0 and 1 it is usually not necessary to make the distinction between field elements and their representatives, since the meaning is normally clear from context.)

$\mathbb{F}_n[z]$ means the ring of polynomials over z with coefficients in \mathbb{F}_n .

$a + b$ means the sum of a and b . This may refer to addition of integers, rationals, finite field elements, or group elements (see §4.1.9 ‘*Represented Group*’ on p. 31) according to context.

$-a$ means the value of the appropriate integer, rational, finite field, or group type such that $(-a) + a = 0$ (or when a is an element of a group \mathbb{G} , $(-a) + a = \mathcal{O}_{\mathbb{G}}$), and $a - b$ means $a + (-b)$.

$a \cdot b$ means the product of multiplying a and b . This may refer to multiplication of integers, rationals, or finite field elements according to context (this notation is not used for group elements).

a/b , also written $\frac{a}{b}$, means the value of the appropriate integer, rational, or finite field type such that $(a/b) \cdot b = a$.

$a \bmod q$, for $a : \mathbb{N}$ and $q : \mathbb{N}^+$, means the remainder on dividing a by q . (This usage does not conflict with the notation above for the unique representative of a field element.)

$a \oplus b$ means the bitwise-exclusive-or of a and b , and $a \& b$ means the bitwise-and of a and b . These are defined on integers (which include bits and bytes), or elementwise on equal-length sequences of integers, according to context.

$\sum_{i=1}^N a_i$ means the sum of $a_{1..N}$. $\prod_{i=1}^N a_i$ means the product of $a_{1..N}$. $\bigoplus_{i=1}^N a_i$ means the bitwise-exclusive-or of $a_{1..N}$.

When $N = 0$ these yield the appropriate neutral element, i.e. $\sum_{i=1}^0 a_i = 0$, $\prod_{i=1}^0 a_i = 1$, and $\bigoplus_{i=1}^0 a_i = 0$ or the all-zero *bit sequence* of length given by the type of a .

$\sqrt[a]{a}$, where $a : \mathbb{F}_q$, means the positive square root of a in \mathbb{F}_q , i.e. in the range $\{0 .. \frac{q-1}{2}\}$. It is only used in cases where the square root must exist.

$\sqrt[a]{a}$, where $a : \mathbb{F}_q$, means an arbitrary square root of a in \mathbb{F}_q , or \perp if no such square root exists.

$b ? x : y$ means x when $b = 1$, or y when $b = 0$.

a^b , for a an integer or finite field element and $b : \mathbb{Z}$, means the result of raising a to the exponent b , i.e.

$$a^b := \begin{cases} \prod_{i=1}^b a, & \text{if } b \geq 0 \\ \prod_{i=1}^{-b} \frac{1}{a}, & \text{otherwise.} \end{cases}$$

The $[k]$ P notation for scalar multiplication in a group is defined in §4.1.9 ‘*Represented Group*’ on p. 31.

The convention of affixing \star to a variable name is used for variables that denote *bit sequence* representations of group elements.

The binary relations $<$, \leq , $=$, \geq , and $>$ have their conventional meanings on integers and rationals, and are defined lexicographically on sequences of integers.

$\text{floor}(x)$ means the largest integer $\leq x$. $\text{ceiling}(x)$ means the smallest integer $\geq x$.

$\text{bitlength}(x)$, for $x : \mathbb{N}$, means the smallest integer ℓ such that $2^\ell > x$.

The following integer constants will be instantiated in §5.3 ‘*Constants*’ on p. 72:

MerkleDepth^{Sprout}, MerkleDepth^{Sapling}, MerkleDepth^{Orchard}, $\ell_{\text{Merkle}}^{\text{Sprout}}$, $\ell_{\text{Merkle}}^{\text{Sapling}}$, $\ell_{\text{Merkle}}^{\text{Orchard}}$, N^{old} , N^{new} , ℓ_{value} , ℓ_{hSig} , $\ell_{\text{PRF}}^{\text{Sprout}}$, $\ell_{\text{PRFexpand}}$, $\ell_{\text{PRFnsapling}}$, $\ell_{\text{rcm}}^{\text{Sprout}}$, ℓ_{Seed} , ℓ_{a_k} , $\ell_{\varphi}^{\text{Sprout}}$, ℓ_{sk} , ℓ_{d} , ℓ_{dk} , $\ell_{\text{ivk}}^{\text{Sapling}}$, ℓ_{ovk} , $\ell_{\text{scalar}}^{\text{Sapling}}$, $\ell_{\text{scalar}}^{\text{Orchard}}$, $\ell_{\text{base}}^{\text{Orchard}}$, MAX_MONEY, BlossomActivationHeight, CanopyActivationHeight, ZIP212GracePeriod, NUFiveActivationHeight, SlowStartInterval, PreBlossomHalvingInterval, MaxBlockSubsidy, NumFounderAddresses, PoWLimit, PoWAveragingWindow, PoWMedianBlockSpan, PoWDampingFactor, PreBlossomPoWTargetSpacing, and PostBlossomPoWTargetSpacing.

The rational constants FoundersFraction, PoWMaxAdjustDown, and PoWMaxAdjustUp; the *bit-sequence* constants Uncommitted^{Sprout} : $\mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]}$ and Uncommitted^{Sapling} : $\mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]}$; and the constant Uncommitted^{Orchard} : $\{0 .. q_{\mathbb{P}} - 1\}$ will also be defined in that section.

We use the abbreviation “*ctEdwards*” to refer to *complete twisted Edwards elliptic curves* and coordinates (see §5.4.9.3 ‘Jubjub’ on p. 100).

3 Concepts

3.1 Payment Addresses and Keys

Users who wish to receive shielded payments in the **Zcash** protocol must have a *shielded payment address*, which is generated from a *spending key*.

The following diagrams depict the relations between key components in **Sprout** and **Sapling** and **Orchard**. Arrows point from a component to any other component(s) that can be derived from it. Double lines indicate that the same component is used in multiple abstractions.



[**Sprout**] The *receiving key* sk_{enc} , *incoming viewing key* $ivk = (a_{pk}, sk_{enc})$, and *shielded payment address* $addr_{pk} = (a_{pk}, pk_{enc})$ are derived from the *spending key* a_{sk} , as described in § 4.2.1 ‘**Sprout Key Components**’ on p. 35.

[**Sapling onward**] An *expanded spending key* is composed of a *Spend authorizing key* ask , a *nullifier private key* nsk , and an *outgoing viewing key* ovk . From these components we can derive a *proof authorizing key* (ak, nsk) , a *full viewing key* (ak, nk, ovk) , an *incoming viewing key* ivk , and a set of *diversified payment addresses* $addr_d = (d, pk_d)$, as described in § 4.2.2 ‘**Sapling Key Components**’ on p. 35.

The consensus protocol does not depend on how an *expanded spending key* is constructed. Two methods of doing so are defined:

1. Generate a *spending key* sk at random and derive the *expanded spending key* (ask, nsk, ovk) from it, as shown in the diagram above and described in § 4.2.2 ‘**Sapling Key Components**’ on p. 35.
2. Obtain an *extended spending key* as specified in [ZIP-32]; this includes a superset of the components of an *expanded spending key*. This method is used in the context of a *Hierarchical Deterministic Wallet*.

[**NU5 onward**] An **Orchard** *spending key* sk is used to derive a *Spend authorizing key* ask , and a *full viewing key* $(ak, nk, rivk)$. From the *full viewing key* we can also derive an *incoming viewing key* (composed of a *diversifier* key dk and a $KA^{Orchard}$ private key ivk), an *outgoing viewing key* ovk , and a set of *diversified payment addresses* $addr_d = (d, pk_d)$, as described in § 4.2.3 ‘**Orchard Key Components**’ on p. 37.

Non-normative note: In **zcashd**, all **Sapling** and **Orchard** keys and addresses are derived according to [ZIP-32].

The composition of *shielded payment addresses*, *incoming viewing keys*, *full viewing keys*, and *spending keys* is a cryptographic protocol detail that should not normally be exposed to users. However, user-visible operations should be provided to obtain a *shielded payment address*, *incoming viewing key*, or *full viewing key* from a *spending key* or *extended spending key*.

Users can accept payment from multiple parties with a single *shielded payment address* and the fact that these payments are destined to the same payee is not revealed on the *block chain*, even to the paying parties. *However* if two parties collude to compare a *shielded payment address* they can trivially determine they are the same. In the case that a payee wishes to prevent this they should create a distinct *shielded payment address* for each payer.

[Sapling onward] **Sapling and Orchard** provide a mechanism to allow the efficient creation of *diversified payment addresses* with the same *spending authority*. A group of such addresses shares the same *full viewing key* and *incoming viewing key*, and so creating as many unlinkable addresses as needed does not increase the cost of scanning the *block chain* for relevant *transactions*.

Note: It is conventional in cryptography to call the key used to encrypt a message in an asymmetric encryption scheme a “*public key*”. However, the *public key* used as the *transmission key* component of an address (pk_{enc} or pk_d) need not be publically distributed; it has the same distribution as the *shielded payment address* itself. As mentioned above, limiting the distribution of the *shielded payment address* is important for some use cases. This also helps to reduce reliance of the overall protocol on the security of the cryptosystem used for *note* encryption (see § 4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63 and § 4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65), since an adversary would have to know pk_{enc} or some pk_d in order to exploit a hypothetical weakness in that cryptosystem.

3.2 Notes

A *note* (denoted n) can be a **Sprout note** or a **Sapling note** or an **Orchard note**. In each case it represents that a value v is spendable by the recipient who holds the *spending key* corresponding to a given *shielded payment address*.

Let MAX_MONEY , ℓ_{PRF}^{Sprout} , $\ell_{PRF}^{Sapling}$, ℓ_d , and ℓ_{value} be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let $NoteCommit^{Sprout}$ be as defined in § 5.4.8.1 ‘*Sprout Note Commitments*’ on p. 93.

Let $NoteCommit^{Sapling}$ be as defined in § 5.4.8.2 ‘*Windowed Pedersen commitments*’ on p. 94.

Let $KA^{Sapling}$ be as defined in § 5.4.5.3 ‘*Sapling Key Agreement*’ on p. 87.

Let $DiversifyHash^{Sapling}$ be as defined in § 5.4.1.6 ‘*DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions*’ on p. 76.

Let $NoteCommit^{Orchard}$ be as defined in § 5.4.8.4 ‘*Sinsemilla commitments*’ on p. 96.

Let $KA^{Orchard}$ be as defined in § 5.4.5.5 ‘*Orchard Key Agreement*’ on p. 88.

Let $DiversifyHash^{Orchard}$ be as defined in § 5.4.1.6 ‘*DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions*’ on p. 76.

Let $PRF^{nfOrchard}$ be as defined in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84.

Let q_p be as defined in § 5.4.9.6 ‘*Pallas and Vesta*’ on p. 103.

A **Sprout note** is a tuple (a_{pk}, v, ρ, rcm) , where:

- $a_{pk} : \mathbb{B}^{[\ell_{PRF}^{Sprout}]}$ is the *paying key* of the recipient’s *shielded payment address*;
- $v : \{0 .. MAX_MONEY\}$ is an integer representing the value of the *note* in *zatoshi* (1 ZEC = 10^8 *zatoshi*);
- $\rho : \mathbb{B}^{[\ell_{PRF}^{Sprout}]}$ is used as input to $PRF_{a_{sk}}^{nfSprout}$ to derive the *nullifier* of the *note*;
- $rcm : NoteCommit^{Sprout}.Trapdoor$ is a random *commitment trapdoor* as defined in § 4.1.8 ‘*Commitment*’ on p. 29.

Let $Note^{Sprout}$ be the type of a **Sprout note**, i.e.

$$Note^{Sprout} := \mathbb{B}^{[\ell_{PRF}^{Sprout}]} \times \{0 .. MAX_MONEY\} \times \mathbb{B}^{[\ell_{PRF}^{Sprout}]} \times NoteCommit^{Sprout}.Trapdoor.$$

A **Sapling** note is a tuple (d, pk_d, v, rcm) , where:

- $d : \mathbb{B}^{[\ell_d]}$ is the *diversifier* of the recipient's *shielded payment address*;
- $pk_d : KA^{\text{Sapling}}.\text{PublicPrimeSubgroup}$ is the *diversified transmission key* of the recipient's *shielded payment address*;
- $v : \{0 \dots \text{MAX_MONEY}\}$ is an integer representing the value of the *note* in *zatoshi*;
- $rcm : \text{NoteCommit}^{\text{Sapling}}.\text{Trapdoor}$ is a random *commitment trapdoor* as defined in §4.1.8 ‘*Commitment*’ on p.29.

Let $\text{Note}^{\text{Sapling}}$ be the type of a **Sapling** note, i.e.

$$\text{Note}^{\text{Sapling}} := \mathbb{B}^{[\ell_d]} \times KA^{\text{Sapling}}.\text{PublicPrimeSubgroup} \times \{0 \dots \text{MAX_MONEY}\} \times \text{NoteCommit}^{\text{Sapling}}.\text{Trapdoor}.$$

An **Orchard** note is a tuple $(d, pk_d, v, \rho, \psi, rcm)$, where:

- $d : \mathbb{B}^{[\ell_d]}$ is the *diversifier* of the recipient's *shielded payment address*;
- $pk_d : KA^{\text{Orchard}}.\text{Public}$ is the *diversified transmission key* of the recipient's *shielded payment address*;
- $v : \{0 \dots 2^{\ell_{\text{value}}}-1\}$ is an integer representing the value of the *note* in *zatoshi*;
- $\rho : \mathbb{F}_{q_P}$ is used as input to $\text{PRF}_{\text{nk}}^{\text{nfOrchard}}$ as part of deriving the *nullifier* of the *note*;
- $\psi : \mathbb{F}_{q_P}$ is additional randomness used in deriving the *nullifier*;
- $rcm : \text{NoteCommit}^{\text{Orchard}}.\text{Trapdoor}$ is a random *commitment trapdoor* as defined in §4.1.8 ‘*Commitment*’ on p.29.

Let $\text{Note}^{\text{Orchard}}$ be the type of an **Orchard** note, i.e.

$$\text{Note}^{\text{Orchard}} := \mathbb{B}^{[\ell_d]} \times KA^{\text{Orchard}}.\text{Public} \times \{0 \dots 2^{\ell_{\text{value}}}-1\} \times \mathbb{F}_{q_P} \times \mathbb{F}_{q_P} \times \text{NoteCommit}^{\text{Orchard}}.\text{Trapdoor}.$$

Creation of new *notes* is described in §4.7 ‘*Sending Notes*’ on p.42.

3.2.1 Note Plaintexts and Memo Fields

Transmitted *notes* are stored on the *block chain* in encrypted form, together with a representation of the *note commitment* cm described in §3.2.2 ‘*Note Commitments*’ on p.15.

A *note plaintext* also includes a 512-byte *memo field* associated with this *note*. The usage of the *memo field* is by agreement between the sender and recipient of the *note*. **RECOMMENDED** non-consensus constraints on the *memo field* contents are specified in [ZIP-302].

For **Sprout**, the *note plaintexts* in each *JoinSplit description* are encrypted to the respective *transmission keys* $pk_{\text{enc},1 \dots N}^{\text{new}}$, as specified in §4.7.1 ‘*Sending Notes (Sprout)*’ on p.42.

Each **Sprout** *note plaintext* (denoted **np**) consists of

$$(\text{leadByte} : \mathbb{B}^Y, v : \{0 \dots 2^{\ell_{\text{value}}}-1\}, \rho : \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]}], rcm : \text{NoteCommit}^{\text{Sprout}}.\text{Trapdoor}, \text{memo} : \mathbb{B}^{Y[512]}).$$

The field leadByte is always 0x00 for **Sprout**. The fields v , ρ , and rcm are as defined in §3.2 ‘*Notes*’ on p.13.

[Sapling onward] For **Sapling** and **Orchard**, the *note plaintext* in each *Output description* or *Action description* is encrypted to the *diversified payment address* (d, pk_d) , as specified in §4.7.2 ‘*Sending Notes (Sapling)*’ on p.43 or §4.7.3 ‘*Sending Notes (Orchard)*’ on p.44.

Each **Sapling** or **Orchard** *note plaintext* (denoted **np**) consists of

$$(\text{leadByte} : \mathbb{B}^Y, d : \mathbb{B}^{[\ell_d]}, v : \{0 \dots 2^{\ell_{\text{value}}}-1\}, \text{rseed} : \mathbb{B}^{Y[32]}, \text{memo} : \mathbb{B}^{Y[512]})$$

The field **leadByte** indicates the version of the encoding of a **Sapling** or **Orchard** *note plaintext*. For **Sapling** it is 0x01 before activation of the **Canopy network upgrade** and 0x02 afterward, as specified in [ZIP-212]. For **Orchard** *note plaintexts* it is always 0x02.

The fields **d** and **v** are as defined in § 3.2 ‘Notes’ on p. 13.

The use of the field **rseed** is described in [ZIP-212].

Encodings are given in § 5.5 ‘Encodings of Note Plaintexts and Memo Fields’ on p. 110. The result of encryption forms part of a *transmitted note(s) ciphertext*. For further details, see § 4.19 ‘In-band secret distribution (**Sprout**)’ on p. 63 and § 4.20 ‘In-band secret distribution (**Sapling** and **Orchard**)’ on p. 65.

3.2.2 Note Commitments

When a *note* is created as an output of a *transaction*, only a commitment (see § 4.1.8 ‘Commitment’ on p. 29) to the *note* contents is disclosed publically in the associated *JoinSplit description* or *Output description* or *Action description*. If the *transaction* is entered into the *block chain*, each such *note commitment* is appended to the *note commitment tree* of the associated *treestate*. This allows the value and recipient to be kept private, while the commitment is used by the *zk-SNARK proof* when the *note* is spent, to check that it exists on the *block chain*.

Treestates are described in § 3.4 ‘Transactions and Treestates’ on p. 17, and *note commitment trees* are described in § 3.8 ‘Note Commitment Trees’ on p. 20.

A **Sprout** *note commitment* on a *note* **n** = (**a_{pk}**, **v**, **ρ**, **rcm**) is computed as

$$\text{NoteCommitment}^{\text{Sprout}}(\mathbf{n}) = \text{NoteCommit}_{\text{rcm}}^{\text{Sprout}}(\mathbf{a}_{\text{pk}}, \mathbf{v}, \rho),$$

where $\text{NoteCommit}^{\text{Sprout}}$ is instantiated in § 5.4.8.1 ‘**Sprout** Note Commitments’ on p. 93.

A **Sapling** *note commitment* on a *note* **n** = (**d**, **pk_d**, **v**, **rcm**) is computed as

$$\begin{aligned} g_d &:= \text{DiversifyHash}^{\text{Sapling}}(d) \\ \text{NoteCommitment}^{\text{Sapling}}(\mathbf{n}) &:= \begin{cases} \perp, & \text{if } g_d = \perp \\ \text{NoteCommit}_{\text{rcm}}^{\text{Sapling}}(\text{repr}_{\mathbb{J}}(g_d), \text{repr}_{\mathbb{J}}(\text{pk}_d), v), & \text{otherwise.} \end{cases} \end{aligned}$$

where $\text{NoteCommit}^{\text{Sapling}}$ is instantiated in § 5.4.8.2 ‘Windowed Pedersen commitments’ on p. 94.

Notice that the above definition of a **Sapling** *note* does not have a **ρ** component. There is in fact a **ρ** value associated with each **Sapling** *note*, but this can only be computed once its position in the *note commitment tree* (see § 3.4 ‘Transactions and Treestates’ on p. 17) is known. We refer to the combination of a *note* and its *note position* **pos**, as a *positioned note*.

For a *positioned note*, we can compute the value **ρ** as described in § 4.16 ‘Computing **ρ** values and Nullifiers’ on p. 56.

A **Sapling** *note commitment* is represented in an *Output description* by the *u*-coordinate of a Jubjub curve point, as specified in § 4.5 ‘Output Descriptions’ on p. 40.

An **Orchard** *note commitment* on a *note* **n** = (**d**, **pk_d**, **v**, **ρ**, **ψ**, **rcm**) is computed as

$$\begin{aligned} g_d &:= \text{DiversifyHash}^{\text{Orchard}}(d) \\ \text{NoteCommitment}^{\text{Orchard}}(\mathbf{n}) &:= \text{NoteCommit}_{\text{rcm}}^{\text{Orchard}}(\text{repr}_{\mathbb{P}}(g_d), \text{repr}_{\mathbb{P}}(\text{pk}_d), v, \rho, \psi) \end{aligned}$$

where $\text{NoteCommit}^{\text{Orchard}}$ is instantiated in § 5.4.8.4 ‘Sinsemilla commitments’ on p. 96.

If `NoteCommit`^{Orchard} returns \perp (which happens with insignificant probability), the *note* is invalid and should be recreated with a different rseed.

Unlike in **Sapling**, the definition of an **Orchard** *note* includes the ρ component; the *note*'s position in the *note commitment tree* does not need to be known in order to compute this value.

An **Orchard** *note commitment* is represented in an *Action description* by the x -coordinate of a Pallas curve point, as specified in § 4.6 ‘*Action Descriptions*’ on p. 41.

3.2.3 Nullifiers

The *nullifier* for a *note*, denoted nf , is a value unique to the *note* that is used to prevent *double-spends*. When a *transaction* that contains one or more *JoinSplit descriptions* or *Spend descriptions* or *Action descriptions* is entered into the *block chain*, all of the *nullifiers* for *notes* spent by that *transaction* are added to the *nullifier set* of the associated *treestate*. A *transaction* is not valid if it would have added a *nullifier* to the *nullifier set* that already exists in the set.

Treestates are described in § 3.4 ‘*Transactions and Treestates*’ on p. 17, and *nullifier sets* are described in § 3.9 ‘*Nullifier Sets*’ on p. 21.

In more detail, when a *note* is spent, the spender creates a *zero-knowledge proof* that it knows (ρ, a_{sk}) or (ρ, ak, nsk) or (ρ, ak, nk) , consistent with the publically disclosed *nullifier* and some previously committed *note commitment*.

Because each *note* can have only a single *nullifier*, and the same *nullifier* value cannot appear more than once in a *valid block chain*, *double-spending* is prevented.

The *nullifier* for a **Sprout** *note* is derived from the ρ value and the recipient's *spending key* a_{sk} .

The *nullifier* for a **Sapling** *note* is derived from the ρ value and the recipient's *nullifier deriving key* nk .

The *nullifier* for an **Orchard** *note* is derived from the ρ and ψ values, the recipient's *nullifier deriving key* nk , and the *note commitment*.

The *nullifier* computation uses a *Pseudo Random Function* (see § 4.1.2 ‘*Pseudo Random Functions*’ on p. 24), as described in § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56.

3.3 The Block Chain

At a given point in time, each *full validator* is aware of a set of candidate *blocks*. These form a tree rooted at the *genesis block*, where each node in the tree refers to its parent via the `hashPrevBlock` *block header* field (see § 7.6 ‘*Block Header Encoding and Consensus*’ on p. 128).

A path from the root toward the leaves of the tree consisting of a sequence of one or more *valid blocks* consistent with consensus rules, is called a *valid block chain*.

Each *block* in a *block chain* has a *block height*. The *block height* of the *genesis block* is 0, and the *block height* of each subsequent *block* in the *block chain* increments by 1. Implementations **MUST** support *block heights* up to and including $2^{31} - 1$. As of **NU5**, there is a consensus rule that all *coinbase transactions* (see § 3.11 ‘*Coinbase Transactions*’ on p. 21) **MUST** have the `nExpiryHeight` field set to the *block height*, and this limits the maximum *block height* to $2^{32} - 1$, absent future consensus changes.

In order to choose the *best valid block chain* in its view of the overall *block tree*, a node sums the work, as defined in § 7.7.5 ‘*Definition of Work*’ on p. 133, of all *blocks* in each *valid block chain*, and considers the *valid block chain* with greatest total work to be best. To break ties between leaf *blocks*, a node will prefer the *block* that it received first.

The consensus protocol is designed to ensure that for any given *block height*, the vast majority of well-connected nodes should eventually agree on their *best valid block chain* up to that height. A *full validator*⁵ **SHOULD** attempt to obtain candidate *blocks* from multiple sources in order to increase the likelihood that it will find a *valid block chain* that reflects a recent consensus state.

A *network upgrade* is *settled* on a given *network* when there is a social consensus that it has activated with a given *activation block* hash. A *full validator* that potentially risks *Mainnet* funds or displays *Mainnet* transaction information to a user **MUST** do so only for a *block chain* that includes the *activation block* of the most recent *settled network upgrade*, with the corresponding *activation block* hash. Currently, there is social consensus that **NU5** has activated on the **Zcash Mainnet** and *Testnet* with the *activation block* hashes given in § 3.12 ‘*Mainnet and Testnet*’ on p. 21.

A *full validator* **MAY** impose a limit on the number of *blocks* it will “roll back” when switching from one *best valid block chain* to another that is not a descendent. For *zcashd* and *zebra* this limit is 100 *blocks*.

3.4 Transactions and Treestates

Each *block* contains one or more *transactions*.

Each *transaction* has a *transaction ID*. *Transaction IDs* are used to refer to *transactions* in *tx_out* fields, in *leaf nodes* of a *block’s transaction tree* rooted at *hashMerkleRoot*, and in other parts of the ecosystem; for example they are shown in *block chain* explorers and can be used in higher-level protocols. *Version 5 transactions* also have a *wtxid*, which is used instead of the *transaction ID* when gossiping *transactions* in the *peer-to-peer protocol* [ZIP-239]. The computation of *transaction IDs* and *wtxids* is described in § 7.1.1 ‘*Transaction Identifiers*’ on p. 121. For more detail on the distinction between these two identifiers and when to use each of them, see [ZIP-239] and [ZIP-244].

Transparent inputs to a *transaction* insert value into a *transparent transaction value pool* associated with the *transaction*, and *transparent outputs* remove value from this pool. The effect of **Sprout** *JoinSplit transfers*, **Sapling** *Spend transfers and Output transfers*, and **Orchard** *Action transfers* on the *transparent transaction value pool* are specified in § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58, § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51, and § 4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53 respectively.

As in **Bitcoin**, the remaining value in the *transparent transaction value pool* of a non-coinbase *transaction* is available to miners as a fee. That is, the sum of those values for non-coinbase *transactions* in each *block* is treated as an implicit input to the *transparent transaction value pool* balance of the *block’s coinbase transaction* (in addition to the implicit input created by issuance).

The remaining value in the *transparent transaction value pool* of *coinbase transactions in blocks prior to NU6* is destroyed. From **NU6**, this remaining value is required to be zero; that is, all of the available balance **MUST** be consumed by outputs of the *coinbase transaction*.

Consensus rule: The remaining value in the *transparent transaction value pool* **MUST** be nonnegative.

To each *transaction* there are associated initial *treestates* for **Sprout** and for **Sapling** and for **Orchard**. Each *treestate* consists of:

- a *note commitment tree* (§ 3.8 ‘*Note Commitment Trees*’ on p. 20);
- a *nullifier set* (§ 3.9 ‘*Nullifier Sets*’ on p. 21).

Validation state associated with *transparent* inputs and outputs, such as the *UTXO (unspent transaction output) set*, is not described in this document; it is used in essentially the same way as in **Bitcoin**.

⁵ There is reason to follow the requirements in this section also for non-full validators, but those are outside the scope of this protocol specification.

An *anchor* is a Merkle tree root of a *note commitment tree* (either the **Sprout** tree or the **Sapling** tree or the **Orchard** tree). It uniquely identifies a *note commitment tree* state given the assumed security properties of the Merkle tree's *hash function*. Since the *nullifier set* is always updated together with the *note commitment tree*, this also identifies a particular state of the associated *nullifier set*.

In a given *block chain*, for each of **Sprout** and **Sapling** and **Orchard**, *treestates* are chained as follows:

- The input *treestate* of the first *block* is the empty *treestate*.
- The input *treestate* of the first *transaction* of a *block* is the final *treestate* of the immediately preceding *block*.
- The input *treestate* of each subsequent *transaction* in a *block* is the output *treestate* of the immediately preceding *transaction*.
- The final *treestate* of a *block* is the output *treestate* of its last *transaction*.

JoinSplit descriptions also have interstitial input and output *treestates* for **Sprout**, explained in the following section. There is no equivalent of interstitial *treestates* for **Sapling** or for **Orchard**.

3.5 JoinSplit Transfers and Descriptions

A *JoinSplit description* is data included in a *transaction* that describes a *JoinSplit transfer*, i.e. a *shielded* value transfer. In **Sprout**, this kind of value transfer was the primary **Zcash**-specific operation performed by *transactions*.

A *JoinSplit transfer* spends N^{old} notes $\mathbf{n}_{1..N^{\text{old}}}^{\text{old}}$ and transparent input $v_{\text{pub}}^{\text{old}}$, and creates N^{new} notes $\mathbf{n}_{1..N^{\text{new}}}^{\text{new}}$ and transparent output $v_{\text{pub}}^{\text{new}}$. It is associated with a *JoinSplit statement* instance (§4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58), for which it provides a *zk-SNARK proof*.

Each *transaction* has a sequence of *JoinSplit descriptions*.

The total $v_{\text{pub}}^{\text{new}}$ value adds to, and the total $v_{\text{pub}}^{\text{old}}$ value subtracts from the *transparent transaction value pool* of the containing *transaction*.

The *anchor* of each *JoinSplit description* in a *transaction* refers to a **Sprout** *treestate*.

For each of the N^{old} *shielded inputs*, a *nullifier* is revealed. This allows detection of *double-spends* as described in §3.9 ‘*Nullifier Sets*’ on p. 21.

For each *JoinSplit description* in a *transaction*, an interstitial output *treestate* is constructed which adds the *note commitments* and *nullifiers* specified in that *JoinSplit description* to the input *treestate* referred to by its *anchor*. This interstitial output *treestate* is available for use as the *anchor* of subsequent *JoinSplit descriptions* in the same *transaction*. In general, therefore, the set of interstitial *treestates* associated with a *transaction* forms a tree in which the parent of each node is determined by its *anchor*.

Interstitial *treestates* are necessary because when a *transaction* is constructed, it is not known where it will eventually appear in a mined *block*. Therefore the *anchors* that it uses must be independent of its eventual position.

The input and output values of each *JoinSplit transfer* **MUST** balance exactly. This is not a consensus rule since it cannot be checked directly; it is enforced by the **Balance** rule of the *JoinSplit statement*.

Consensus rules:

- For the first *JoinSplit description* of a *transaction*, the *anchor* **MUST** be the output **Sprout** *treestate* of a previous *block*.
- The *anchor* of each *JoinSplit description* in a *transaction* **MUST** refer to either some earlier *block*’s final **Sprout** *treestate*, or to the interstitial output *treestate* of any prior *JoinSplit description* in the same *transaction*.

As in **Sapling**, we use the homomorphic property of *Pedersen commitments* to enforce balance: we add all of the *value commitments* and prove by use of an *Orchard binding signature* that the result commits to a value consistent with the net *transparent* value change (as described in §4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53). This approach allows all of the *zk-SNARK statements* to be independent of each other, potentially increasing opportunities for precomputation.

The fields of an *Action description* are essentially a merger of the fields of a *Spend description* and an *Output description*, but with only a single *value commitment*. Also, the *zk-SNARK proof* is encoded outside the *Action description*, in order to more easily take advantage of space and performance optimizations in the Halo 2 proof system (§5.4.10.3 ‘Halo 2’ on p. 110) that apply when multiple proofs are aggregated. Each *Action description* does not encode a separate *anchor* field, because that is encoded once in the `anchorOrchard` field of the *transaction*.

Non-normative note: As with **Sapling**, interstitial *treestates* are not necessary for **Orchard**, because an *Action transfer* in a given *transaction* cannot spend any of the *shielded outputs* of the same *transaction*.

Consensus rules:

- The *Action transfers* of a *transaction* **MUST** be consistent with its $v^{\text{balanceOrchard}}$ value as specified in §4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53.
- The `anchorOrchard` field of the *transaction*, whenever it exists (i.e. when there are any *Action descriptions*), **MUST** refer to some earlier *block*’s final **Orchard** *treestate*.

3.8 Note Commitment Trees

Let $\ell_{\text{Merkle}}^{\text{Sprout}}$, $\text{MerkleDepth}^{\text{Sprout}}$, $\ell_{\text{Merkle}}^{\text{Sapling}}$, $\text{MerkleDepth}^{\text{Sapling}}$, $\ell_{\text{Merkle}}^{\text{Orchard}}$, and $\text{MerkleDepth}^{\text{Orchard}}$ be as defined in §5.3 ‘*Constants*’ on p. 72.



A *note commitment tree* is an *incremental Merkle tree* of fixed depth used to store *note commitments* that *JoinSplit transfers* or *Spend transfers* or *Action transfers* produce. Just as the *UTXO (unspent transaction output) set* used in **Bitcoin**, it is used to express the existence of value and the capability to spend it. However, unlike the *UTXO set*, it is *not* the job of this tree to protect against *double-spending*, as it is *append-only*.

A *root* of a *note commitment tree* is associated with each *treestate* (§3.4 ‘*Transactions and Treestates*’ on p. 17).

Each *node* in the *incremental Merkle tree* is associated with a *hash value* of size $\ell_{\text{Merkle}}^{\text{Sprout}}$ or $\ell_{\text{Merkle}}^{\text{Sapling}}$ or $\ell_{\text{Merkle}}^{\text{Orchard}}$ bits. The *layer* numbered h , counting from *layer* 0 at the *root*, has 2^h *nodes* with *indices* 0 to $2^h - 1$ inclusive. The *hash value* associated with the *node* at *index* i in *layer* h is denoted M_i^h .

The *index* of a *note*’s *commitment* at the *leafmost layer* ($\text{MerkleDepth}^{\text{Sprout}}$ or $\text{MerkleDepth}^{\text{Sapling}}$ or $\text{MerkleDepth}^{\text{Orchard}}$) is called its *note position*.

Consensus rules:

- A block **MUST NOT** add **Sprout** note commitments that would result in the **Sprout** note commitment tree exceeding its capacity of $2^{\text{MerkleDepth}^{\text{Sprout}}}$ leaf nodes.
- [Sapling onward] A block **MUST NOT** add **Sapling** note commitments that would result in the **Sapling** note commitment tree exceeding its capacity of $2^{\text{MerkleDepth}^{\text{Sapling}}}$ leaf nodes.
- [NU5 onward] A block **MUST NOT** add **Orchard** note commitments that would result in the **Orchard** note commitment tree exceeding its capacity of $2^{\text{MerkleDepth}^{\text{Orchard}}}$ leaf nodes.

3.9 Nullifier Sets

Each full validator maintains a nullifier set logically associated with each treestate. As valid transactions containing JoinSplit transfers or Spend transfers or Action transfers are processed, the nullifiers revealed in JoinSplit descriptions and Spend descriptions and Action descriptions are inserted into the nullifier set associated with the new treestate. Nullifiers are enforced to be unique within a valid block chain, in order to prevent double-spends.

Consensus rule: A nullifier **MUST NOT** repeat either within a transaction, or across transactions in a valid block chain. **Sprout** and **Sapling** and **Orchard** nullifiers are considered disjoint, even if they have the same bit pattern.

3.10 Block Subsidy, Funding Streams, and Founders' Reward

Like **Bitcoin**, **Zcash** creates currency when blocks are mined. The value created on mining a block is called the block subsidy.

[Pre-Canopy] The block subsidy is composed of a miner subsidy and a Founders' Reward.

[Canopy onward] The block subsidy is composed of a miner subsidy and a series of funding streams.

As in **Bitcoin**, the miner of a block also receives transaction fees.

The calculations of the block subsidy, miner subsidy, Founders' Reward, and funding streams depend on the block height, as defined in § 3.3 'The Block Chain' on p. 16.

The calculations are described in § 7.8 'Calculation of Block Subsidy, Funding Streams, and Founders' Reward' on p. 133.

3.11 Coinbase Transactions

A transaction that has a single transparent input with a null prevout field, is called a coinbase transaction. Every block has a single coinbase transaction as the first transaction in the block. The purpose of this coinbase transaction is to collect and spend any miner subsidy, and transaction fees paid by other transactions included in the block.

[Pre-Canopy] As described in § 7.9 'Payment of Founders' Reward' on p. 134, the coinbase transaction **MUST** also pay the Founders' Reward.

[Canopy onward] As described in § 7.10 'Payment of Funding Streams' on p. 136, the coinbase transaction **MUST** also pay the funding streams.

3.12 Mainnet and Testnet

The production **Zcash** network, which supports the **ZEC** token, is called *Mainnet*. Governance of its protocol is by agreement between the Electric Coin Company and the Zcash Foundation [ECCZF2019]. Subject to errors and omissions, each version of this document intends to describe some version (or planned version) of that agreed protocol.

All *block hashes* given in this section are in *RPC byte order* (that is, byte-reversed relative to the normal order for a SHA-256 hash).

Mainnet genesis block: 00040fe8ec8471911baa1db1266ea15dd06b4a8a5c453883c000b031973dce08

Mainnet NU5 activation block: 0000000000d723156d9b65ffcf4984da7a19675ed7e2f06d9e5d5188af087bf8

There is also a public test *network* called *Testnet*. It supports a **TAZ** token which is intended to have no monetary value. By convention, *Testnet* activates *network upgrades* (as described in §6 ‘*Network Upgrades*’ on p. 118) before *Mainnet*, in order to allow for errors or ambiguities in their specification and implementation to be discovered. The *Testnet block chain* is subject to being rolled back to a prior *block* at any time.

Testnet genesis block: 05a60a92d99d85997cce3b87616c089f6124d7342af37106edc76126334a2c38

Testnet NU5 activation block: 0006d75c60b3093d1b671ff7da11c99ea535df9927c02e6ed9eb898605eb7381

We call the smallest units of currency (on either *network*) *zatoshi*.

On *Mainnet*, 1 **ZEC** = 10^8 *zatoshi*. On *Testnet*, 1 **TAZ** = 10^8 *zatoshi*.

Other *networks* using variants of the **Zcash** protocol may exist, but are not described by this specification.

4 Abstract Protocol

*‘We all know that the only mental tool by means of which a very finite piece of reasoning can cover a myriad cases is called “abstraction”; as a result the effective exploitation of [their] powers of abstraction must be regarded as one of the most vital activities of a competent programmer. In this connection it might be worth-while to point out that the purpose of abstracting is **not** to be vague, but to create a new semantic level in which one can be absolutely precise.’*

— Edsger Dijkstra, “The Humble Programmer” [EWD-340]

Abstraction is an incredibly important idea in the design of any complex system. Without abstraction, we would not be able to design anything as ambitious as a computer, or a cryptographic protocol. Were we to attempt it, the computer would be hopelessly unreliable or the protocol would be insecure, if they could be completed at all.

The aim of abstraction is primarily to limit how much a human working on a piece of a system has to keep in mind at one time, in order to apprehend the connections of that piece to the remainder. The work could be to extend or maintain the system, to understand its security or other properties, or to explain it to others.

In this specification, we make use wherever possible of abstractions that have been developed by the cryptography community to model cryptographic primitives: *Pseudo Random Functions*, *commitment schemes*, *signature schemes*, etc. Each abstract primitive has associated syntax (its interface as used by the rest of the system) and security properties, as documented in this part. Their instantiations are documented in part §5 ‘*Concrete Protocol*’ on p. 71.

In some cases this syntax or these security requirements have been extended to meet the needs of the **Zcash** protocol. For example, some of the PRFs used in **Zcash** need to be *collision-resistant*, which is not part of the usual security requirement for a PRF; some *signature schemes* need to support additional functionality and security properties; and so on. Also, security requirements are sometimes intentionally stronger than what is known to be needed, because the stronger property is simpler or less error-prone to work with, and/or because it has been studied in the cryptographic literature in more depth.

We explicitly *do not claim*, however, that all of these instantiations satisfying their documented syntax and security requirements would be sufficient for security or correctness of the overall **Zcash** protocol, or that it is always necessary. The claim is only that it helps to understand the protocol; that is, that analysis or extension is simplified by making use of the abstraction. In other words, *a good way to understand* the use of that primitive in the protocol is to model it as an instance of the given abstraction. And furthermore, if the instantiated primitive does not in fact satisfy the requirements of the abstraction, then this is an error that should be corrected –whether or not it leads to a vulnerability– since that would compromise the facility to understand its use in terms of the abstraction.

In this respect the abstractions play a similar rôle to that of a type system (which we also use): they add a form of redundancy to the specification that helps to express the intent.

Each property is a claim that may be incorrect (or that may be insufficiently precisely stated to determine whether it is correct). An example of an incorrect security claim occurs in the **Zerocash** protocol [BCGGMTV2014]: the instantiation of the *note commitment* scheme used in **Zerocash** failed to be *binding* at the intended security level (see §8.5 ‘*Internal hash collision attack and fix*’ on p.141).

Another hazard that we should be aware of is that abstractions can be “leaky”: an instantiation may impose conditions on its correct or secure use that are not captured by the abstraction’s interface and semantics. Ideally, the abstraction would be changed to explicitly document these conditions, or the protocol changed to rely only on the original abstraction.

An abstraction can also be incomplete (not quite the same thing as being leaky): it intentionally –usually for simplicity– does not model an aspect of behaviour that is important to security or correctness. An example would be resistance to *side-channel* attacks; this specification says little about *side-channel* defence, among many other implementation concerns.

4.1 Abstract Cryptographic Schemes

4.1.1 Hash Functions

Let $\text{MerkleDepth}^{\text{Sprout}}, \ell_{\text{Merkle}}^{\text{Sprout}}, \text{MerkleDepth}^{\text{Sapling}}, \ell_{\text{Merkle}}^{\text{Sapling}}, \text{MerkleDepth}^{\text{Orchard}}, \ell_{\text{Merkle}}^{\text{Orchard}}, \ell_{\text{ivk}}^{\text{Sapling}}, \ell_d, \ell_{\text{Seed}}, \ell_{\text{PRF}}^{\text{Sprout}}, \ell_{\text{hSig}}$, and N^{old} be as defined in §5.3 ‘*Constants*’ on p.72.

Let $\mathbb{J}, \mathbb{J}^{(n)}, \mathbb{J}^{(n)*}, r_{\mathbb{J}}$, and $\ell_{\mathbb{J}}$ be as defined in §5.4.9.3 ‘*Jubjub*’ on p.100.

Let \mathbb{P}^* be as defined in §5.4.9.6 ‘*Pallas and Vesta*’ on p.103.

The following *hash functions* are used in §4.9 ‘*Merkle Path Validity*’ on p.47:

$$\begin{aligned} \text{MerkleCRH}^{\text{Sprout}} &: \{0 \dots \text{MerkleDepth}^{\text{Sprout}} - 1\} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} \rightarrow \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} \\ \text{MerkleCRH}^{\text{Sapling}} &: \{0 \dots \text{MerkleDepth}^{\text{Sapling}} - 1\} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} \rightarrow \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} \\ \text{MerkleCRH}^{\text{Orchard}} &: \{0 \dots \text{MerkleDepth}^{\text{Orchard}} - 1\} \times \{0 \dots q_{\mathbb{P}} - 1\} \times \{0 \dots q_{\mathbb{P}} - 1\} \rightarrow \{0 \dots q_{\mathbb{P}} - 1\}. \end{aligned}$$

$\text{MerkleCRH}^{\text{Sprout}}$ is *collision-resistant* except on its first argument. $\text{MerkleCRH}^{\text{Sapling}}$ and $\text{MerkleCRH}^{\text{Orchard}}$ are *collision-resistant on all their arguments*.

These functions are instantiated in §5.4.1.3 ‘*Merkle Tree Hash Function*’ on p.74.

$\text{hSigCRH} : \mathbb{B}^{[\ell_{\text{Seed}}]} \times \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} [N^{\text{old}}] \times \text{JoinSplitSig.Public} \rightarrow \mathbb{B}^{[\ell_{\text{hSig}}]}$ is a *collision-resistant hash function* used in §4.3 ‘*JoinSplit Descriptions*’ on p.38. It is instantiated in §5.4.1.4 ‘*hSig Hash Function*’ on p.75.

$\text{EquiHashGen} : (n : \mathbb{N}^+) \times \mathbb{N}^+ \times \mathbb{B}^{\mathbb{Y}^{[N]}} \times \mathbb{N}^+ \rightarrow \mathbb{B}^{[n]}$ is another *hash function*, used in §7.7.1 ‘*EquiHash*’ on p.130 to generate input to the *EquiHash* solver. The first two arguments, representing the *EquiHash* parameters n and k , are written subscripted. It is instantiated in §5.4.1.11 ‘*EquiHash Generator*’ on p.83.

$\text{CRH}^{\text{ivk}} : \mathbb{B}^{[\ell_{\mathbb{J}}]} \times \mathbb{B}^{[\ell_{\mathbb{J}}]} \rightarrow \{0 \dots 2^{\ell_{\text{ivk}}^{\text{Sapling}}} - 1\}$ is a *collision-resistant hash function* used in §4.2.2 ‘*Sapling Key Components*’ on p.35 to derive an *incoming viewing key* for a **Sapling** shielded payment address. It is also used in the *Spend statement* (§4.18.2 ‘*Spend Statement (Sapling)*’ on p.59) to confirm use of the correct keys for the *note* being spent. It is instantiated in §5.4.1.5 ‘*CRH^{ivk} Hash Function*’ on p.75.

$\text{MixingPedersenHash} : \mathbb{J} \times \{0 \dots r_{\mathbb{J}} - 1\} \rightarrow \mathbb{J}$ is a *hash function* used in §4.16 ‘*Computing ρ values and Nullifiers*’ on p.56 to derive the unique ρ value for a **Sapling** note. It is also used in the *Spend statement* to confirm use of the correct ρ value as an input to *nullifier* derivation. It is instantiated in §5.4.1.8 ‘*Mixing Pedersen Hash Function*’ on p.79.

$\text{DiversifyHash}^{\text{Sapling}} : \mathbb{B}^{[\ell_d]} \rightarrow \mathbb{J}^{(r)*} \cup \{\perp\}$ and $\text{DiversifyHash}^{\text{Orchard}} : \mathbb{B}^{[\ell_d]} \rightarrow \mathbb{P}^*$ are *hash functions* instantiated in § 5.4.1.6 ‘*DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions*’ on p. 76, satisfying the Unlinkability security property described in that section. They are used to derive a *diversified base* from a *diversifier*, which is specified in § 4.2.2 ‘*Sapling Key Components*’ on p. 35 and in § 4.2.3 ‘*Orchard Key Components*’ on p. 37.

4.1.2 Pseudo Random Functions

PRF_x denotes a *Pseudo Random Function* keyed by x .

Let $\ell_{\text{ask}}, \ell_{\text{hSig}}, \ell_{\text{PRF}}^{\text{Sprout}}, \ell_{\phi}^{\text{Sprout}}, \ell_{\text{sk}}, \ell_{\text{ovk}}, \ell_{\text{PRFExpand}}, \ell_{\text{PRFnfSapling}}, N^{\text{old}}$, and N^{new} be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let Sym be as defined in § 5.4.3 ‘*Symmetric Encryption*’ on p. 86.

Let $\ell_{\mathbb{J}}$ and $\mathbb{J}_{\star}^{(r)}$ be as defined in § 5.4.9.3 ‘*Jubjub*’ on p. 100.

Let $\ell_{\mathbb{P}}$ and $q_{\mathbb{P}}$ be as defined in § 5.4.9.6 ‘*Pallas and Vesta*’ on p. 103.

For **Sprout**, four *independent* PRF_x are needed:

$$\begin{aligned} \text{PRF}^{\text{addr}} &: \mathbb{B}^{[\ell_{\text{ask}}]} \times \mathbb{B}^{\mathbb{Y}} \rightarrow \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \\ \text{PRF}^{\text{pk}} &: \mathbb{B}^{[\ell_{\text{ask}}]} \times \{1..N^{\text{old}}\} \times \mathbb{B}^{[\ell_{\text{hSig}}]} \rightarrow \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \\ \text{PRF}^{\rho} &: \mathbb{B}^{[\ell_{\phi}^{\text{Sprout}}]} \times \{1..N^{\text{new}}\} \times \mathbb{B}^{[\ell_{\text{hSig}}]} \rightarrow \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \\ \text{PRF}^{\text{nfSprout}} &: \mathbb{B}^{[\ell_{\text{ask}}]} \times \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \rightarrow \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \end{aligned}$$

These are used in § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58; PRF^{addr} is also used to derive a *shielded payment address* from a *spending key* in § 4.2.1 ‘*Sprout Key Components*’ on p. 35.

For **Sapling**, three additional PRF_x are needed:

$$\begin{aligned} \text{PRF}^{\text{expand}} &: \mathbb{B}^{[\ell_{\text{sk}}]} \times \mathbb{B}^{\mathbb{Y}^{[\mathbb{N}]}} \rightarrow \mathbb{B}^{\mathbb{Y}^{[\ell_{\text{PRFExpand}}/8]}} \\ \text{PRF}^{\text{ockSapling}} &: \mathbb{B}^{\mathbb{Y}^{[\ell_{\text{ovk}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{J}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{J}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{J}}/8]}} \rightarrow \text{Sym.K} \\ \text{PRF}^{\text{nfSapling}} &: \mathbb{J}_{\star}^{(r)} \times \mathbb{B}^{[\ell_{\mathbb{J}}]} \rightarrow \mathbb{B}^{\mathbb{Y}^{[\ell_{\text{PRFnfSapling}}/8]}} \end{aligned}$$

For **Orchard**, we need $\text{PRF}^{\text{expand}}$, and also:

$$\begin{aligned} \text{PRF}^{\text{ockOrchard}} &: \mathbb{B}^{\mathbb{Y}^{[\ell_{\text{ovk}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{P}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{P}}/8]}} \times \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{P}}/8]}} \rightarrow \text{Sym.K} \\ \text{PRF}^{\text{nfOrchard}} &: \mathbb{F}_{q_{\mathbb{P}}} \times \mathbb{F}_{q_{\mathbb{P}}} \rightarrow \mathbb{F}_{q_{\mathbb{P}}} \end{aligned}$$

$\text{PRF}^{\text{expand}}$ is used in the following places:

- § 4.2.2 ‘*Sapling Key Components*’ on p. 35, with inputs [0], [1], [2], and [3, $i : \mathbb{B}^{\mathbb{Y}}$];
- [NU5 onward] in § 4.2.3 ‘*Orchard Key Components*’ on p. 37, with inputs [6], [7], [8], and with first byte 0x82 (the last of these is also specified in [ZIP-32]);
- in the processes of sending (§ 4.7.2 ‘*Sending Notes (Sapling)*’ on p. 43 and § 4.7.3 ‘*Sending Notes (Orchard)*’ on p. 44) and of receiving (§ 4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65) *notes*, with inputs [4] and [5], and for **Orchard** $[t] \parallel \rho$ with $t \in \{5, 4, 9\}$;
- in [ZIP-32], with inputs [0], [1], [2] (intentionally matching § 4.2.2 on p. 35), [0x10], [0x13], [0x14], and with first byte in $\{0x11, 0x12, 0x15, 0x16, 0x17, 0x18, 0x80, 0x81, 0x82, 0x83\}$;
- in [ZIP-316], with first byte 0xD0.

$\text{PRF}^{\text{ockSapling}}$ and $\text{PRF}^{\text{ockOrchard}}$ are used in § 4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65.

$\text{PRF}^{\text{nfSapling}}$ is used in § 4.18.2 ‘*Spend Statement (Sapling)*’ on p. 59.

$\text{PRF}^{\text{nfOrchard}}$ is used in § 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61.

All of these *Pseudo Random Functions* are instantiated in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84.

Security requirements:

- Security definitions for *Pseudo Random Functions* are given in [BDJR2000, section 4].
- In addition to being *Pseudo Random Functions*, it is required that $\text{PRF}_x^{\text{addr}}$, PRF_x^{p} , $\text{PRF}_x^{\text{nfSprout}}$, $\text{PRF}_x^{\text{nfSapling}}$ and $\text{PRF}_x^{\text{nfOrchard}}$ be *collision-resistant* across all x — i.e. finding $(x, y) \neq (x', y')$ such that $\text{PRF}_x^{\text{addr}}(y) = \text{PRF}_{x'}^{\text{addr}}(y')$ should not be feasible, and similarly for PRF_x^{p} , $\text{PRF}_x^{\text{nfSprout}}$, $\text{PRF}_x^{\text{nfSapling}}$, and $\text{PRF}_x^{\text{nfOrchard}}$.
- See the note in §4.2.3 ‘*Orchard Key Components*’ on p. 37 for a security caveat about the use of $\text{PRF}^{\text{expand}}$.

Non-normative note: $\text{PRF}^{\text{nfSprout}}$ was called PRF^{sn} in *Zerocash* [BCGGMTV2014], and just PRF^{nf} in some previous versions of this specification.

4.1.3 Pseudo Random Permutations

PRP_x denotes a *Pseudo Random Permutation* keyed by x .

Let ℓ_{dk} and ℓ_{d} be as defined in §5.3 ‘*Constants*’ on p. 72.

One *Pseudo Random Permutation* is used for **Orchard**, to generate *diversifiers* from a *diversifier key* and index (an identical construction is also used for **Sapling** in [ZIP-32]):

$$\text{PRP}^{\text{d}} : \mathbb{B}^{\ell_{\text{dk}}/8} \times \mathbb{B}^{\ell_{\text{d}}} \rightarrow \mathbb{B}^{\ell_{\text{d}}}.$$

It is instantiated in §5.4.4 ‘*Pseudo Random Permutations*’ on p. 86.

Security requirement: PRP^{d} is a keyed *Pseudo Random Permutation* as defined in [BKR2001].

4.1.4 Symmetric Encryption

Let *Sym* be an *authenticated one-time symmetric encryption scheme* with keyspace *Sym.K*, encrypting plaintexts in *Sym.P* to produce ciphertexts in *Sym.C*.

$\text{Sym.Encrypt} : \text{Sym.K} \times \text{Sym.P} \rightarrow \text{Sym.C}$ is the encryption algorithm.

$\text{Sym.Decrypt} : \text{Sym.K} \times \text{Sym.C} \rightarrow \text{Sym.P} \cup \{\perp\}$ is the decryption algorithm, such that for any $K \in \text{Sym.K}$ and $P \in \text{Sym.P}$, $\text{Sym.Decrypt}_K(\text{Sym.Encrypt}_K(P)) = P$. \perp is used to represent the decryption of an invalid ciphertext.

Security requirement: *Sym* must be *one-time* (INT-CTXT \wedge IND-CPA)-secure [BN2007]. “*One-time*” here means that an honest protocol participant will almost surely encrypt only one message with a given key; however, the adversary may make many adaptive chosen ciphertext queries for a given key.

4.1.5 Key Agreement

A *key agreement scheme* is a cryptographic protocol in which two parties agree a shared secret, each using their *private key* and the other party’s *public key*.

A *key agreement scheme* *KA* defines a type of *public keys* *KA.Public*, a type of *private keys* *KA.Private*, and a type of *shared secrets* *KA.SharedSecret*. **Optionally, it also defines a type $\text{KA.PublicPrimeSubgroup} \subseteq \text{KA.Public}$.**

Optional: Let $\text{KA.FormatPrivate} : \mathbb{B}^{\ell_{\text{PRF}}^{\text{Sprout}}} \rightarrow \text{KA.Private}$ be a function to convert a bit string of length $\ell_{\text{PRF}}^{\text{Sprout}}$ to a *KA private key*.

Let $\text{KA.DerivePublic} : \text{KA.Private} \times \text{KA.Public} \rightarrow \text{KA.Public}$ be a function that derives the *KA public key* corresponding to a given *KA private key* and base point.

Let $\text{KA.Agree} : \text{KA.Private} \times \text{KA.Public} \rightarrow \text{KA.SharedSecret}$ be the agreement function.

Optional: Let $\text{KA.Base} : \text{KA.Public}$ be a public base point.

Note: The range of KA.DerivePublic may be a strict subset of *KA.Public*.

Security requirements:

- KA.FormatPrivate must preserve sufficient entropy from its input to be used as a secure KA *private key*.
- The key agreement and the KDF defined in the next section must together satisfy a suitable adaptive security assumption along the lines of [Bernstein2006, section 3] or [ABR1999, Definition 3].

More precise formalization of these requirements is beyond the scope of this specification.

4.1.6 Key Derivation

A *Key Derivation Function* is defined for a particular *key agreement scheme* and *authenticated one-time symmetric encryption scheme*; it takes the shared secret produced by the key agreement and additional arguments, and derives a key suitable for the encryption scheme.

The inputs to the *Key Derivation Function* differ between the **Sprout** and **Sapling** and **Orchard** KDFs:

$\text{KDF}^{\text{Sprout}}$ takes as input an output index in $\{1..N^{\text{new}}\}$, the value h_{Sig} , the shared Diffie–Hellman secret sharedSecret , the *ephemeral public key* epk , and the recipient’s public *transmission key* pk_{enc} . It is suitable for use with $\text{KA}^{\text{Sprout}}$ and derives keys for Sym.Encrypt .

$$\text{KDF}^{\text{Sprout}} : \{1..N^{\text{new}}\} \times \mathbb{B}^{[\ell_{h_{\text{Sig}}}] \times \text{KA}^{\text{Sprout}}.\text{SharedSecret} \times \text{KA}^{\text{Sprout}}.\text{Public} \times \text{KA}^{\text{Sprout}}.\text{Public} \rightarrow \text{Sym.K}$$

$\text{KDF}^{\text{Sapling}}$ takes as input the shared Diffie–Hellman secret sharedSecret and the *ephemeral public key* epk . (It does not have inputs taking the place of the output index, h_{Sig} , or pk_{enc} .) It is suitable for use with $\text{KA}^{\text{Sapling}}$ and derives keys for Sym.Encrypt .

$$\text{KDF}^{\text{Sapling}} : \text{KA}^{\text{Sapling}}.\text{SharedSecret} \times \mathbb{B}^{\mathbb{Y}[\ell_f/8]} \rightarrow \text{Sym.K}$$

As in **Sapling**, $\text{KDF}^{\text{Orchard}}$ takes as input the shared Diffie–Hellman secret sharedSecret and the *ephemeral public key* epk . It is suitable for use with $\text{KA}^{\text{Orchard}}$ and derives keys for Sym.Encrypt .

$$\text{KDF}^{\text{Orchard}} : \text{KA}^{\text{Orchard}}.\text{SharedSecret} \times \mathbb{B}^{\mathbb{Y}[\ell_f/8]} \rightarrow \text{Sym.K}$$

Security requirements:

- The asymmetric encryption scheme in §4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63, constructed from $\text{KA}^{\text{Sprout}}$, $\text{KDF}^{\text{Sprout}}$ and Sym , is required to be IND-CCA2-secure and *key-private*.
- The asymmetric encryption scheme in §4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65, constructed from $\text{KA}^{\text{Sapling}}$, $\text{KDF}^{\text{Sapling}}$ and Sym or from $\text{KA}^{\text{Orchard}}$, $\text{KDF}^{\text{Orchard}}$ and Sym , is required to be IND-CCA2-secure and *key-private*.

Key privacy is defined in [BBDP2001].

4.1.7 Signature

A *signature scheme* Sig defines:

- a type of *signing keys* Sig.Private ;
- a type of *validating keys* Sig.Public ;
- a type of *messages* Sig.Message ;
- a type of *signatures* Sig.Signature ;
- a randomized *signing key generation algorithm* $\text{Sig.GenPrivate} : () \xrightarrow{\mathcal{R}} \text{Sig.Private}$;
- an injective *validating key derivation algorithm* $\text{Sig.DerivePublic} : \text{Sig.Private} \rightarrow \text{Sig.Public}$;

- a randomized signing algorithm $\text{Sig.Sig} : \text{Sig.Private} \times \text{Sig.Message} \xrightarrow{R} \text{Sig.Signature}$;
- a validating algorithm $\text{Sig.Validate} : \text{Sig.Public} \times \text{Sig.Message} \times \text{Sig.Signature} \rightarrow \mathbb{B}$;

such that for any *signing key* $sk \xleftarrow{R} \text{Sig.GenPrivate}()$ and corresponding *validating key* $vk = \text{Sig.DerivePublic}(sk)$, and any $m : \text{Sig.Message}$ and $s : \text{Sig.Signature} \xleftarrow{R} \text{Sig.Sign}_{sk}(m)$, $\text{Sig.Validate}_{vk}(m, s) = 1$.

Zcash uses **four** signature schemes:

- one used for signatures that can be validated by script operations such as OP_CHECKSIG and OP_CHECKMULTISIG as in **Bitcoin**;
- one called JoinSplitSig which is used to sign *transactions* that contain at least one *JoinSplit description* (instantiated in § 5.4.6 ‘Ed25519’ on p. 88);
- [Sapling onward] one called SpendAuthSig which is used to sign authorizations of *Spend transfers* (instantiated in § 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93);
- [Sapling onward] one called BindingSig. A *Sapling binding signature* is used to enforce balance of *Spend transfers* and *Output transfers*, and to prevent their replay across *transactions*. Similarly, an *Orchard binding signature* is used to enforce balance of *Action transfers* and to prevent their replay. BindingSig is instantiated for both **Sapling** and **Orchard** in § 5.4.7.2 ‘*Binding Signature (Sapling and Orchard)*’ on p. 93.

The signature scheme used in script operations is instantiated by ECDSA on the secp256k1 curve. JoinSplitSig is instantiated by Ed25519. SpendAuthSig and BindingSig are instantiated by RedDSA; on the Jubjub curve in **Sapling**, and on the Pallas curve in **Orchard**.

The following security property is needed for JoinSplitSig and BindingSig. Security requirements for SpendAuthSig are defined in the next section, § 4.1.7.1 ‘*Signature with Re-Randomizable Keys*’ on p. 28. An additional requirement for BindingSig is defined in § 4.1.7.2 ‘*Signature with Signing Key to Validating Key Monomorphism*’ on p. 29.

Security requirement: JoinSplitSig and each instantiation of BindingSig must be Strongly Unforgeable under (non-adaptive) Chosen Message Attack (SU-CMA), as defined for example in [BDEHR2011, Definition 6].⁶ This allows an adversary to obtain signatures on chosen messages, and then requires it to be infeasible for the adversary to forge a previously unseen valid (message, signature) pair without access to the *signing key*.

Non-normative notes:

- We need separate *signing key* generation and *validating key* derivation algorithms, rather than the more conventional combined key pair generation algorithm $\text{Sig.Gen} : () \xrightarrow{R} \text{Sig.Private} \times \text{Sig.Public}$, to support the key derivation in § 4.2.2 ‘*Sapling Key Components*’ on p. 35 and in § 4.2.3 ‘*Orchard Key Components*’ on p. 37. The definitions of schemes with additional features in § 4.1.7.1 ‘*Signature with Re-Randomizable Keys*’ on p. 28 and in § 4.1.7.2 ‘*Signature with Signing Key to Validating Key Monomorphism*’ on p. 29 also become simpler.
- A fresh signature key pair is generated for each *transaction* containing a *JoinSplit description*. Since each key pair is only used for one signature (see § 4.11 ‘*Non-malleability (Sprout)*’ on p. 50), a *one-time signature scheme* would suffice for JoinSplitSig. This is also the reason why only security against *non-adaptive* chosen message attack is needed. In fact the instantiation of JoinSplitSig uses a scheme designed for security under adaptive attack even when multiple signatures are signed under the same key.
- [Sapling onward] The same remarks as above apply to BindingSig, except that the key is derived from the randomness of *value commitments*. This results in the same distribution as of freshly generated key pairs, for each *transaction* containing *Spend descriptions* or *Output descriptions* or *Action descriptions*.
- SU-CMA security requires it to be infeasible for the adversary, not knowing the *private key*, to forge a distinct signature on a previously seen message. That is, *JoinSplit signatures* and *Sapling binding signatures* and *Orchard binding signatures* are intended to be *nonmalleable* in the sense of [BIP-62].
- The terminology used in this specification is that we “validate” signatures, and “verify” zk-SNARK proofs.

⁶ The scheme defined in that paper was attacked in [LM2017], but this has no impact on the applicability of the definition.

Non-normative notes:

- The *randomizer* and key arguments to $\text{Sig.RandomizePrivate}$ and $\text{Sig.RandomizePublic}$ are swapped relative to [FKMSSS2016, section 3].
- The requirement for the identity *randomizer* $\mathcal{O}_{\text{Sig.Random}}$ simplifies the definition of SURK-CMA by removing the need for two oracles (because the oracle for original keys, called \mathcal{O}_1 in [FKMSSS2016], is a special case of the oracle for randomized keys).
- Since $\text{Sig.RandomizePrivate}(\alpha, \text{sk}) : \alpha \xleftarrow{\mathcal{R}} \text{Sig.Random}$ has an identical distribution to $\text{Sig.GenPrivate}()$, and since Sig.DerivePublic is a deterministic function, the combination of a *re-randomized validating key* and signature(s) under that key do not reveal the key from which it was *re-randomized*.
- Since $\text{Sig.RandomizePrivate}_\alpha$ is injective and easily invertible, knowledge of $\text{Sig.RandomizePrivate}(\alpha, \text{sk})$ *and* α implies knowledge of sk .

4.1.7.2 Signature with Signing Key to Validating Key Monomorphism

A *signature scheme with key monomorphism* Sig is a *signature scheme* that additionally defines:

- an abelian group on *signing keys*, with operation $\boxplus : \text{Sig.Private} \times \text{Sig.Private} \rightarrow \text{Sig.Private}$ and identity \mathcal{O}_{\boxplus} ;
- an abelian group on *validating keys*, with operation $\boxplus : \text{Sig.Public} \times \text{Sig.Public} \rightarrow \text{Sig.Public}$ and identity \mathcal{O}_{\boxplus} .

such that for any $\text{sk}_{1..2} : \text{Sig.Private}$, $\text{Sig.DerivePublic}(\text{sk}_1 \boxplus \text{sk}_2) = \text{Sig.DerivePublic}(\text{sk}_1) \boxplus \text{Sig.DerivePublic}(\text{sk}_2)$.

In other words, Sig.DerivePublic is a *monomorphism* (that is, an injective homomorphism) from the *signing key* group to the *validating key* group.

For $N : \mathbb{N}^+$,

- $\boxplus_{i=1}^N \text{sk}_i$ means $\text{sk}_1 \boxplus \text{sk}_2 \boxplus \dots \boxplus \text{sk}_N$;
- $\boxplus_{i=1}^N \text{vk}_i$ means $\text{vk}_1 \boxplus \text{vk}_2 \boxplus \dots \boxplus \text{vk}_N$.

When $N = 0$ these yield the appropriate group identity, i.e. $\boxplus_{i=1}^0 \text{sk}_i = \mathcal{O}_{\boxplus}$ and $\boxplus_{i=1}^0 \text{vk}_i = \mathcal{O}_{\boxplus}$.

$\boxplus \text{sk}$ means the *signing key* such that $(\boxplus \text{sk}) \boxplus \text{sk} = \mathcal{O}_{\boxplus}$, and $\text{sk}_1 \boxplus \text{sk}_2$ means $\text{sk}_1 \boxplus (\boxplus \text{sk}_2)$.

$\boxplus \text{vk}$ means the *validating key* such that $(\boxplus \text{vk}) \boxplus \text{vk} = \mathcal{O}_{\boxplus}$, and $\text{vk}_1 \boxplus \text{vk}_2$ means $\text{vk}_1 \boxplus (\boxplus \text{vk}_2)$.

With a change of notation from μ to Sig.DerivePublic , $+$ to \boxplus , and \cdot to \boxplus , this is similar to the definition of a “*Signature with Secret Key to Public Key Homomorphism*” in [DS2016, Definition 13], except for an additional requirement for the homomorphism to be injective.

Security requirement: For any $\text{sk}_1 : \text{Sig.Private}$, and an unknown $\text{sk}_2 \xleftarrow{\mathcal{R}} \text{Sig.GenPrivate}()$ chosen independently of sk_1 , the distribution of $\text{sk}_1 \boxplus \text{sk}_2$ is computationally indistinguishable from that of $\text{Sig.GenPrivate}()$. (Since \boxplus is an abelian group operation, this implies that for $n : \mathbb{N}^+$, $\boxplus_{i=1}^n \text{sk}_i$ is computationally indistinguishable from $\text{Sig.GenPrivate}()$ when at least one of $\text{sk}_{1..n}$ is unknown.)

4.1.8 Commitment

A *commitment scheme* is a function that, given a *commitment trapdoor* generated at random and an input, can be used to commit to the input in such a way that:

- no information is revealed about it without the *trapdoor* (“*hiding*”); and
- given the *trapdoor* and input, the commitment can be verified to “*open*” to that input and no other (“*binding*”).

A *commitment scheme* COMM defines a type of inputs COMM.Input , a type of commitments COMM.Output , a type of *commitment trapdoors* COMM.Trapdoor , and a *trapdoor generator* $\text{COMM.GenTrapdoor} : () \xrightarrow{\mathcal{R}} \text{COMM.Trapdoor}$.

Let $\text{COMM} : \text{COMM.Trapdoor} \times \text{COMM.Input} \rightarrow \text{COMM.Output}$ be a function satisfying the following security requirements.

Security requirements:

- **Computational *hiding*:** For all $x, x' : \text{COMM.Input}$, the distributions $\{ \text{COMM}_r(x) \mid r \xleftarrow{\mathbb{R}} \text{COMM.GenTrapdoor}() \}$ and $\{ \text{COMM}_r(x') \mid r \xleftarrow{\mathbb{R}} \text{COMM.GenTrapdoor}() \}$ are computationally indistinguishable.
- **Computational *binding*:** It is infeasible to find $x, x' : \text{COMM.Input}$ and $r, r' : \text{COMM.Trapdoor}$ such that $x \neq x'$ and $\text{COMM}_r(x) = \text{COMM}_{r'}(x')$.

Notes:

- COMM.GenTrapdoor need not produce the uniform distribution on COMM.Trapdoor . In that case, it is incorrect to choose a *trapdoor* from the latter distribution.
- If it were only feasible to find $x : \text{COMM.Input}$ and $r, r' : \text{COMM.Trapdoor}$ such that $r \neq r'$ and $\text{COMM}_r(x) = \text{COMM}_{r'}(x)$, this would not contradict the computational *binding* security requirement. (In fact, this is feasible for $\text{NoteCommit}^{\text{Sapling}}$ and $\text{ValueCommit}^{\text{Sapling}}$ because *trapdoors* are equivalent modulo $r_{\mathbb{J}}$, and the range of a *trapdoor* for those algorithms is $\{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}$ where $2^{\ell_{\text{scalar}}^{\text{Sapling}}} > r_{\mathbb{J}}$.)

Let $\ell_{\text{rcm}}^{\text{Sprout}}$, $\ell_{\text{Merkle}}^{\text{Sprout}}$, $\ell_{\text{PRF}}^{\text{Sprout}}$, and ℓ_{value} be as defined in § 5.3 ‘*Constants*’ on p. 72.

Define $\text{NoteCommit}^{\text{Sprout}}.\text{Trapdoor} := \mathbb{B}^{[\ell_{\text{rcm}}^{\text{Sprout}}]}$ and $\text{NoteCommit}^{\text{Sprout}}.\text{Output} := \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]}$.

Sprout uses a *note commitment scheme*

$$\begin{aligned} \text{NoteCommit}^{\text{Sprout}} : \text{NoteCommit}^{\text{Sprout}}.\text{Trapdoor} \times \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \times \{0 \dots 2^{\ell_{\text{value}}} - 1\} \times \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \\ \rightarrow \text{NoteCommit}^{\text{Sprout}}.\text{Output}, \end{aligned}$$

instantiated in § 5.4.8.1 ‘**Sprout** *Note Commitments*’ on p. 93.

Let $\ell_{\text{scalar}}^{\text{Sapling}}$ be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let $\mathbb{J}^{(r)}$, $\ell_{\mathbb{J}}$, and $r_{\mathbb{J}}$ be as defined in § 5.4.9.3 ‘*Jubjub*’ on p. 100.

Define:

$$\text{NoteCommit}^{\text{Sapling}}.\text{Trapdoor} := \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\} \text{ and } \text{NoteCommit}^{\text{Sapling}}.\text{Output} := \mathbb{J};$$

$$\text{ValueCommit}^{\text{Sapling}}.\text{Trapdoor} := \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\} \text{ and } \text{ValueCommit}^{\text{Sapling}}.\text{Output} := \mathbb{J}.$$

Sapling uses two additional *commitment schemes*:

$$\begin{aligned} \text{NoteCommit}^{\text{Sapling}} : \text{NoteCommit}^{\text{Sapling}}.\text{Trapdoor} \times \mathbb{B}^{[\ell_{\mathbb{J}}]} \times \mathbb{B}^{[\ell_{\mathbb{J}}]} \times \{0 \dots 2^{\ell_{\text{value}}} - 1\} &\rightarrow \text{NoteCommit}^{\text{Sapling}}.\text{Output} \\ \text{ValueCommit}^{\text{Sapling}} : \text{ValueCommit}^{\text{Sapling}}.\text{Trapdoor} \times \left\{ -\frac{r_{\mathbb{J}}-1}{2} \dots \frac{r_{\mathbb{J}}-1}{2} \right\} &\rightarrow \text{ValueCommit}^{\text{Sapling}}.\text{Output} \end{aligned}$$

$\text{NoteCommit}^{\text{Sapling}}$ is instantiated in § 5.4.8.2 ‘*Windowed Pedersen commitments*’ on p. 94, and $\text{ValueCommit}^{\text{Sapling}}$ is instantiated in § 5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p. 95.

Non-normative note: $\text{NoteCommit}^{\text{Sapling}}$ and $\text{ValueCommit}^{\text{Sapling}}$ always return points in the subgroup $\mathbb{J}^{(r)}$. However, we declare the type of these commitment outputs to be \mathbb{J} because they are not directly checked to be in the subgroup when $\text{ValueCommit}^{\text{Sapling}}$ outputs appear in *Spend descriptions* and *Output descriptions*, or when the *cmu* field derived from a $\text{NoteCommit}^{\text{Sapling}}$ appears in an *Output description*.

Let $\ell_{\text{scalar}}^{\text{Orchard}}$ be as defined in §5.3 ‘*Constants*’ on p.72.

Let \mathbb{P} , $\ell_{\mathbb{P}}$, $q_{\mathbb{P}}$, and $r_{\mathbb{P}}$ be as defined in §5.4.9.6 ‘*Pallas and Vesta*’ on p.103.

Define:

$\text{NoteCommit}^{\text{Orchard}}.\text{Trapdoor} := \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Orchard}}} - 1\}$ and $\text{NoteCommit}^{\text{Orchard}}.\text{Output} := \mathbb{P} \cup \{\perp\}$;

$\text{ValueCommit}^{\text{Orchard}}.\text{Trapdoor} := \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Orchard}}} - 1\}$ and $\text{ValueCommit}^{\text{Orchard}}.\text{Output} := \mathbb{P}$.

$\text{Commit}^{\text{ivk}}.\text{Trapdoor} := \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Orchard}}} - 1\}$ and $\text{Commit}^{\text{ivk}}.\text{Output} := \{0 \dots q_{\mathbb{P}} - 1\} \cup \{\perp\}$.

Orchard uses three additional *commitment schemes*:

$$\begin{aligned} \text{NoteCommit}^{\text{Orchard}} &: \text{NoteCommit}^{\text{Orchard}}.\text{Trapdoor} \times \mathbb{B}^{[\ell_{\mathbb{P}}]} \times \mathbb{B}^{[\ell_{\mathbb{P}}]} \times \{0 \dots 2^{\ell_{\text{value}}}-1\} \\ &\quad \times \mathbb{F}_{q_{\mathbb{P}}} \times \mathbb{F}_{q_{\mathbb{P}}} \rightarrow \text{NoteCommit}^{\text{Orchard}}.\text{Output} \\ \text{ValueCommit}^{\text{Orchard}} &: \text{ValueCommit}^{\text{Orchard}}.\text{Trapdoor} \times \left\{-\frac{r_{\mathbb{P}}-1}{2} \dots \frac{r_{\mathbb{P}}-1}{2}\right\} \rightarrow \text{ValueCommit}^{\text{Orchard}}.\text{Output} \\ \text{Commit}^{\text{ivk}} &: \text{Commit}^{\text{ivk}}.\text{Trapdoor} \times \{0 \dots q_{\mathbb{P}} - 1\} \times \mathbb{F}_{q_{\mathbb{P}}} \rightarrow \text{Commit}^{\text{ivk}}.\text{Output} \end{aligned}$$

Notes:

- $\text{NoteCommit}^{\text{Orchard}}$ and $\text{Commit}^{\text{ivk}}$ can return \perp (with insignificant probability).
- $\text{Commit}^{\text{ivk}}$ can return 0 (with insignificant probability) even though that is not a valid $\text{KA}^{\text{Orchard}}$ private key. The use of $\text{Commit}^{\text{ivk}}$ to obtain an **Orchard** incoming viewing key in §4.2.3 ‘**Orchard** Key Components’ on p.37 explicitly accounts for the 0 and \perp cases. Use of $\text{Commit}^{\text{ivk}}$ in the *Action circuit* does not require special handling of the 0 case.

$\text{NoteCommit}^{\text{Orchard}}$ and $\text{Commit}^{\text{ivk}}$ are instantiated in §5.4.8.4 ‘*Sinsemilla commitments*’ on p.96. $\text{ValueCommit}^{\text{Orchard}}$ is instantiated in §5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p.95.

4.1.9 Represented Group

A *represented group* \mathbb{G} consists of:

- a subgroup order parameter $r_{\mathbb{G}} : \mathbb{N}^+$, which must be prime;
- a cofactor parameter $h_{\mathbb{G}} : \mathbb{N}^+$;
- a group \mathbb{G} of order $h_{\mathbb{G}} \cdot r_{\mathbb{G}}$, written additively with operation $+$: $\mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}$, and additive identity $\mathcal{O}_{\mathbb{G}}$;
- a bit-length parameter $\ell_{\mathbb{G}} : \mathbb{N}$;
- a representation function $\text{repr}_{\mathbb{G}} : \mathbb{G} \rightarrow \mathbb{B}^{[\ell_{\mathbb{G}}]}$ and an abstraction function $\text{abst}_{\mathbb{G}} : \mathbb{B}^{[\ell_{\mathbb{G}}]} \rightarrow \mathbb{G} \cup \{\perp\}$, such that $\text{abst}_{\mathbb{G}}$ is a left inverse of $\text{repr}_{\mathbb{G}}$, i.e. for all $P \in \mathbb{G}$, $\text{abst}_{\mathbb{G}}(\text{repr}_{\mathbb{G}}(P)) = P$.

Note: Ideally, we would also have that for all S not in the image of $\text{repr}_{\mathbb{G}}$, $\text{abst}_{\mathbb{G}}(S) = \perp$. This may not be true in all cases, i.e. there can be *non-canonical* encodings $P\star$ such that $\text{repr}_{\mathbb{G}}(\text{abst}_{\mathbb{G}}(P\star)) \neq P\star$.

Define $\mathbb{G}^{(r)}$ as the order- $r_{\mathbb{G}}$ subgroup of \mathbb{G} , which is called a *represented subgroup*. Note that this includes $\mathcal{O}_{\mathbb{G}}$. For the set of points of order $r_{\mathbb{G}}$ (which excludes $\mathcal{O}_{\mathbb{G}}$), we write $\mathbb{G}^{(r)*}$.

Define $\mathbb{G}_{\star}^{(r)} := \{\text{repr}_{\mathbb{G}}(P) : \mathbb{B}^{[\ell_{\mathbb{G}}]} \mid P \in \mathbb{G}^{(r)}\}$. (This intentionally excludes *non-canonical* encodings if there are any.)

For $G : \mathbb{G}$ we write $-G$ for the negation of G , such that $(-G) + G = \mathcal{O}_{\mathbb{G}}$. We write $G - H$ for $G + (-H)$.

We also extend the \sum notation to addition on group elements.

For $G \in \mathbb{G}$ and $k \in \mathbb{Z}$ we write $[k]G$ for scalar multiplication on the group, i.e.

$$[k]G := \begin{cases} \sum_{i=1}^k G, & \text{if } k \geq 0 \\ \sum_{i=1}^{-k} (-G), & \text{otherwise.} \end{cases}$$

For $G \in \mathbb{G}$ and $a \in \mathbb{F}_{r_G}$, we may also write $[a]G$ meaning $[a \bmod r_G]G$ as defined above. (This variant is not defined for fields other than \mathbb{F}_{r_G} .)

4.1.10 Coordinate Extractor

A *coordinate extractor* for a *represented group* \mathbb{G} is a function $\text{Extract}_{\mathbb{G}^{(r)}} : \mathbb{G}^{(r)} \rightarrow T$ for some type T .

Note: Unlike the representation function $\text{repr}_{\mathbb{G}}$, $\text{Extract}_{\mathbb{G}^{(r)}}$ need not have an efficiently computable left inverse.

4.1.11 Group Hash

Given a *represented subgroup* $\mathbb{G}^{(r)}$, a *family of group hashes into the subgroup*, denoted $\text{GroupHash}^{\mathbb{G}^{(r)}}$, consists of:

- a type $\text{GroupHash}^{\mathbb{G}^{(r)}}.\text{URSType}$ of *Uniform Random Strings*;
- a type $\text{GroupHash}^{\mathbb{G}^{(r)}}.\text{Input}$ of inputs;
- a function $\text{GroupHash}^{\mathbb{G}^{(r)}} : \text{GroupHash}^{\mathbb{G}^{(r)}}.\text{URSType} \times \text{GroupHash}^{\mathbb{G}^{(r)}}.\text{Input} \rightarrow \mathbb{G}^{(r)}$.

In §5.4.9.5 ‘*Group Hash into Jubjub*’ on p. 102, we instantiate a family of *group hashes* into the Jubjub curve defined by §5.4.9.3 ‘*Jubjub*’ on p. 100.

Security requirement: For a randomly selected $\text{URS} \in \text{GroupHash}^{\mathbb{G}^{(r)}}.\text{URSType}$, it must be reasonable to model $\text{GroupHash}_{\text{URS}}^{\mathbb{G}^{(r)}}$ (restricted to inputs for which it does not return \perp) as a *random oracle*.

In §5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105, we instantiate *group hashes* into the Pallas and Vesta curves. These are not strictly speaking *families of group hashes*, because they have a trivial URS, and so the above security definition does not apply. Nevertheless, they can be heuristically modelled as *random oracles*.

Non-normative notes:

- $\text{GroupHash}^{\mathbb{J}^{(r)*}}$ is used to obtain generators of the Jubjub curve for various purposes: the bases $\mathcal{G}^{\text{Sapling}}$ and $\mathcal{H}^{\text{Sapling}}$ used in **Sapling** key generation, the *Pedersen hash* defined in §5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77, and the *commitment schemes* defined in §5.4.8.2 ‘*Windowed Pedersen commitments*’ on p. 94 and in §5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p. 95.

The security property needed for these uses can alternatively be defined in the standard model as follows:

Discrete Logarithm Independence: For a randomly selected member $\text{GroupHash}_{\text{URS}}^{\mathbb{G}^{(r)}}$ of the family, it is infeasible to find a sequence of *distinct* inputs $m_{1..n} \in \text{GroupHash}^{\mathbb{G}^{(r)}}.\text{Input}^{[n]}$ and a sequence of nonzero $x_{1..n} \in \mathbb{F}_{r_G}^{*[n]}$ such that $\sum_{i=1}^n ([x_i] \text{GroupHash}_{\text{URS}}^{\mathbb{G}^{(r)}}(m_i)) = \mathcal{O}_{\mathbb{G}}$.

- Under the *Discrete Logarithm* assumption on $\mathbb{G}^{(r)}$, a *random oracle* almost surely satisfies Discrete Logarithm Independence. Discrete Logarithm Independence implies *collision resistance*, since a collision (m_1, m_2) for $\text{GroupHash}_{\text{URS}}^{\mathbb{G}^{(r)}}$ trivially gives a discrete logarithm relation with $x_1 = 1$ and $x_2 = -1$.

- $\text{GroupHash}^{\mathbb{J}^{(r)*}}$ is used in §5.4.1.6 ‘DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions’ on p. 76 to instantiate DiversifyHash^{Sapling}. We do not know how to prove the Unlinkability property defined in that section in the standard model, but in a model where $\text{GroupHash}^{\mathbb{J}^{(r)*}}$ (restricted to inputs for which it does not return \perp) is taken as a *random oracle*, it is implied by the *Decisional Diffie–Hellman* assumption on $\mathbb{J}^{(r)}$, and similarly for $\text{GroupHash}^{\mathbb{P}}$.
- URS is a *Uniform Random String*; we chose it verifiably at random (see §5.9 ‘Randomness Beacon’ on p. 117), after fixing the concrete group hash algorithm to be used. This mitigates the possibility that the group hash algorithm could have been backdoored. For Orchard, we considered a URS to be unnecessary, because we follow [ID–hashtocurve] which does not use one.

4.1.12 Represented Pairing

A *represented pairing* \mathbb{PAIR} consists of:

- a group order parameter $r_{\mathbb{PAIR}} : \mathbb{N}^+$ which must be prime;
- two *represented subgroups* $\mathbb{PAIR}_{1,2}^{(r)}$, both of order $r_{\mathbb{PAIR}}$;
- a group $\mathbb{PAIR}_T^{(r)}$ of order $r_{\mathbb{PAIR}}$, written multiplicatively with operation $\cdot : \mathbb{PAIR}_T^{(r)} \times \mathbb{PAIR}_T^{(r)} \rightarrow \mathbb{PAIR}_T^{(r)}$ and group identity $\mathbf{1}_{\mathbb{PAIR}}$;
- three generators $\mathcal{P}_{\mathbb{PAIR}_{1,2,T}^{(r)}}$ of $\mathbb{PAIR}_{1,2,T}^{(r)}$ respectively;
- a pairing function $\hat{e}_{\mathbb{PAIR}} : \mathbb{PAIR}_1^{(r)} \times \mathbb{PAIR}_2^{(r)} \rightarrow \mathbb{PAIR}_T^{(r)}$ satisfying:
 - (Bilinearity) for all $a, b : \mathbb{F}_r^*$, $P : \mathbb{PAIR}_1^{(r)}$, and $Q : \mathbb{PAIR}_2^{(r)}$, $\hat{e}_{\mathbb{PAIR}}([a] P, [b] Q) = \hat{e}_{\mathbb{PAIR}}(P, Q)^{a \cdot b}$; and
 - (Nondegeneracy) there does not exist $P : \mathbb{PAIR}_1^{(r)*}$ such that for all $Q : \mathbb{PAIR}_2^{(r)}$, $\hat{e}_{\mathbb{PAIR}}(P, Q) = \mathbf{1}_{\mathbb{PAIR}}$.

4.1.13 Zero-Knowledge Proving System

A *zero-knowledge proving system* is a cryptographic protocol that allows proving a particular *statement*, dependent on *primary* and *auxiliary inputs*, in zero knowledge — that is, without revealing information about the *auxiliary inputs* other than that implied by the *statement*. The type of *zero-knowledge proving system* needed by Zcash is a *preprocessing zk-SNARK* [BCCGLRT2014].

A *preprocessing zk-SNARK* instance ZK defines:

- a type of *zero-knowledge proving keys*, ZK.ProvingKey ;
- a type of *zero-knowledge verifying keys*, ZK.VerifyingKey ;
- a type of *primary inputs* ZK.PrimaryInput ;
- a type of *auxiliary inputs* ZK.AuxiliaryInput ;
- a type of *zk-SNARK proofs* ZK.Proof ;
- a type $\text{ZK.SatisfyingInputs} \subseteq \text{ZK.PrimaryInput} \times \text{ZK.AuxiliaryInput}$ of inputs satisfying the *statement*;
- a randomized key pair generation algorithm $\text{ZK.Gen} : () \xrightarrow{\mathbb{R}} \text{ZK.ProvingKey} \times \text{ZK.VerifyingKey}$;
- a proving algorithm $\text{ZK.Prove} : \text{ZK.ProvingKey} \times \text{ZK.SatisfyingInputs} \rightarrow \text{ZK.Proof}$;
- a verifying algorithm $\text{ZK.Verify} : \text{ZK.VerifyingKey} \times \text{ZK.PrimaryInput} \times \text{ZK.Proof} \rightarrow \mathbb{B}$;

The security requirements below are supposed to hold with overwhelming probability for $(\text{pk}, \text{vk}) \xleftarrow{\mathbb{R}} \text{ZK.Gen}()$.

Security requirements:

- **Completeness:** An honestly generated proof will convince a verifier: for any $(x, w) \in \text{ZK.SatisfyingInputs}$, if $\text{ZK.Prove}_{\text{pk}}(x, w)$ outputs π , then $\text{ZK.Verify}_{\text{vk}}(x, \pi) = 1$.
- **Knowledge Soundness:** For any adversary \mathcal{A} able to find an $x : \text{ZK.PrimaryInput}$ and proof $\pi : \text{ZK.Proof}$ such that $\text{ZK.Verify}_{\text{vk}}(x, \pi) = 1$, there is an efficient extractor $\mathcal{E}_{\mathcal{A}}$ such that if $\mathcal{E}_{\mathcal{A}}(\text{vk}, \text{pk})$ returns w , then the probability that $(x, w) \notin \text{ZK.SatisfyingInputs}$ is insignificant.
- **Statistical Zero Knowledge:** An honestly generated proof is statistical zero knowledge. That is, there is a feasible stateful simulator \mathcal{S} such that, for all stateful distinguishers \mathcal{D} , the following two probabilities are not significantly different:

$$\Pr \left[\begin{array}{c} (x, w) \in \text{ZK.SatisfyingInputs} \\ \mathcal{D}(\pi) = 1 \end{array} \middle| \begin{array}{c} (\text{pk}, \text{vk}) \xleftarrow{\mathcal{R}} \text{ZK.Gen}() \\ (x, w) \xleftarrow{\mathcal{R}} \mathcal{D}(\text{pk}, \text{vk}) \\ \pi \xleftarrow{\mathcal{R}} \text{ZK.Prove}_{\text{pk}}(x, w) \end{array} \right] \text{ and } \Pr \left[\begin{array}{c} (x, w) \in \text{ZK.SatisfyingInputs} \\ \mathcal{D}(\pi) = 1 \end{array} \middle| \begin{array}{c} (\text{pk}, \text{vk}) \xleftarrow{\mathcal{R}} \mathcal{S}() \\ (x, w) \xleftarrow{\mathcal{R}} \mathcal{D}(\text{pk}, \text{vk}) \\ \pi \xleftarrow{\mathcal{R}} \mathcal{S}(x) \end{array} \right]$$

These definitions are derived from those in [BCTV2014b, Appendix C], adapted to state concrete security for a fixed circuit, rather than asymptotic security for arbitrary circuits. (ZK.Prove corresponds to P , ZK.Verify corresponds to V , and $\text{ZK.SatisfyingInputs}$ corresponds to \mathcal{R}_C in the notation of that appendix.)

The Knowledge Soundness definition is a way to formalize the property that it is infeasible to find a new proof π where $\text{ZK.Verify}_{\text{vk}}(x, \pi) = 1$ without *knowing* an *auxiliary input* w such that $(x, w) \in \text{ZK.SatisfyingInputs}$. Note that Knowledge Soundness implies Soundness – i.e. the property that it is infeasible to find a new proof π where $\text{ZK.Verify}_{\text{vk}}(x, \pi) = 1$ without *there existing* an *auxiliary input* w such that $(x, w) \in \text{ZK.SatisfyingInputs}$.

Non-normative notes:

- The above properties do not include *nonmalleability* [DSDCOPS2001], and the design of the protocol using the *zero-knowledge proving system* must take this into account.
- The terminology used in this specification is that we “validate” signatures, and “verify” *zk-SNARK proofs*.

Zcash uses three proving systems:

- BCTV14 (§5.4.10.1 ‘BCTV14’ on p.108) is used with the BN-254 pairing (§5.4.9.1 ‘BN-254’ on p.97), to prove and verify the **Sprout JoinSplit statement** (§4.18.1 ‘JoinSplit Statement (**Sprout**)’ on p.58) before **Sapling** activation.
- Groth16 (§5.4.10.2 ‘Groth16’ on p.109) is used with the BLS12-381 pairing (§5.4.9.2 ‘BLS12-381’ on p.99), to prove and verify the **Sapling Spend statement** (§4.18.2 ‘Spend Statement (**Sapling**)’ on p.59) and **Output statement** (§4.18.3 ‘Output Statement (**Sapling**)’ on p.60). It is also used to prove and verify the **JoinSplit statement** after **Sapling** activation.
- [NU5 onward] Halo 2 (§5.4.10.3 ‘Halo 2’ on p.110) is used with the Vesta curve (§5.4.9.6 ‘Pallas and Vesta’ on p.103) to prove and verify the **Orchard Action statement** (§4.18.4 ‘Action Statement (**Orchard**)’ on p.61).

These specializations are:

- ZKJoinSplit for the **Sprout JoinSplit statement** (with BCTV14 and BN-254, or Groth16 and BLS12-381);
- ZKSpending for the **Sapling Spend statement** and ZKOutput for the **Sapling Output statement**;
- [NU5 onward] ZKAction for the **Orchard Action statement**.

We omit key subscripts on ZKJoinSplit.Prove and ZKJoinSplit.Verify, taking them to be either the BCTV14 *proving key* and *verifying key* defined in §5.7 ‘BCTV14 zk-SNARK Parameters’ on p.117, or the sprout-groth16.params Groth16 *proving key* and *verifying key* defined in §5.8 ‘Groth16 zk-SNARK Parameters’ on p.117, according to whether the proof appears in a *block* before or after **Sapling** activation.

We omit subscripts on ZKSpending.Prove, ZKSpending.Verify, ZKOutput.Prove, and ZKOutput.Verify, taking them to be the relevant Groth16 *proving keys* and *verifying keys* defined in §5.8 ‘Groth16 zk-SNARK Parameters’ on p.117.

We also omit subscripts on ZKAction.Prove and ZKAction.Verify. For Halo 2, parameters for a given circuit implementation are generated on the fly by the halo2 library, and do not require parameter files.

4.2 Key Components

4.2.1 Sprout Key Components

Let $\ell_{a_{sk}}$ be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let PRF^{addr} be a *Pseudo Random Function*, instantiated in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84.

Let $\text{KA}^{\text{Sprout}}$ be a *key agreement scheme*, instantiated in § 5.4.5.1 ‘*Sprout Key Agreement*’ on p. 86.

A new **Sprout** *spending key* a_{sk} is generated by choosing a *bit sequence* uniformly at random from $\mathbb{B}^{[\ell_{a_{sk}}]}$.

a_{pk} , sk_{enc} and pk_{enc} are derived from a_{sk} as follows:

$$\begin{aligned} a_{pk} &:= \text{PRF}_{a_{sk}}^{\text{addr}}(0) \\ sk_{\text{enc}} &:= \text{KA}^{\text{Sprout}}.\text{FormatPrivate}(\text{PRF}_{a_{sk}}^{\text{addr}}(1)) \\ pk_{\text{enc}} &:= \text{KA}^{\text{Sprout}}.\text{DerivePublic}(sk_{\text{enc}}, \text{KA}^{\text{Sprout}}.\text{Base}). \end{aligned}$$

4.2.2 Sapling Key Components

Let $\ell_{\text{PRFexpand}}$, ℓ_{sk} , $\ell_{ivk}^{\text{Sapling}}$, ℓ_{ovk} , and ℓ_d be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let $\mathbb{J}^{(r)}$, $\mathbb{J}^{(r)*}$, $\mathbb{J}_{\star}^{(r)}$, $\text{repr}_{\mathbb{J}}$, and $r_{\mathbb{J}}$ be as defined in § 5.4.9.3 ‘*Jubjub*’ on p. 100, and let $\text{FindGroupHash}^{\mathbb{J}^{(r)*}}$ be as defined in § 5.4.9.5 ‘*Group Hash into Jubjub*’ on p. 102.

Let $\text{PRF}^{\text{expand}}$ and $\text{PRF}^{\text{ockSapling}}$, instantiated in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84, be *Pseudo Random Functions*.

Let $\text{KA}^{\text{Sapling}}$, instantiated in § 5.4.5.3 ‘*Sapling Key Agreement*’ on p. 87, be a *key agreement scheme*.

Let CRH^{ivk} , instantiated in § 5.4.1.5 ‘*CRH^{ivk} Hash Function*’ on p. 75, be a *hash function*.

Let $\text{DiversifyHash}^{\text{Sapling}}$, instantiated in § 5.4.1.6 ‘*DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions*’ on p. 76, be a *hash function*.

Let $\text{SpendAuthSig}^{\text{Sapling}}$, instantiated in § 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93, be a *signature scheme with re-randomizable keys*.

Let $\text{LEBS2OSP} : (\ell : \mathbb{N}) \times \mathbb{B}^{[\ell]} \rightarrow \mathbb{B}^{[\text{ceiling}(\ell/8)]}$ and $\text{LEOS2IP} : (\ell : \mathbb{N} \mid \ell \bmod 8 = 0) \times \mathbb{B}^{[\ell/8]} \rightarrow \{0 \dots 2^\ell - 1\}$ be as defined in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

Define $\mathcal{H}^{\text{Sapling}} := \text{FindGroupHash}^{\mathbb{J}^{(r)*}}(\text{"Zcash_H_"}, \text{""})$.

Define $\text{ToScalar}^{\text{Sapling}}(x : \mathbb{B}^{[\ell_{\text{PRFexpand}}/8]}) := \text{LEOS2IP}_{\ell_{\text{PRFexpand}}}(x) \pmod{r_{\mathbb{J}}}$.

A new **Sapling** *spending key* sk is generated by choosing a *bit sequence* uniformly at random from $\mathbb{B}^{[\ell_{sk}]}$.

From this *spending key*, the *Spend authorizing key* $ask : \mathbb{F}_{r_{\mathbb{J}}}^*$, the *proof authorizing key* $nsk : \mathbb{F}_{r_{\mathbb{J}}}$, and the *outgoing viewing key* $ovk : \mathbb{B}^{[\ell_{ovk}/8]}$ are derived as follows:

$$\begin{aligned} ask &:= \text{ToScalar}^{\text{Sapling}}(\text{PRF}_{sk}^{\text{expand}}([0])) \\ nsk &:= \text{ToScalar}^{\text{Sapling}}(\text{PRF}_{sk}^{\text{expand}}([1])) \\ ovk &:= \text{truncate}_{(\ell_{ovk}/8)}(\text{PRF}_{sk}^{\text{expand}}([2])) \end{aligned}$$

If $ask = 0$, discard this key and repeat with a new sk .

$ak : \mathbb{J}^{(r)*}$, $nk : \mathbb{J}^{(r)}$, and the *incoming viewing key* $ivk : \{0 \dots 2^{\ell_{ivk}^{\text{Sapling}}} - 1\}$ are then derived as:

$$\begin{aligned} ak &:= \text{SpendAuthSig}^{\text{Sapling}}.\text{DerivePublic}(ask) \\ nk &:= [nsk] \mathcal{H}^{\text{Sapling}} \\ ivk &:= \text{CRH}^{\text{ivk}}(\text{repr}_{\mathbb{J}}(ak), \text{repr}_{\mathbb{J}}(nk)). \end{aligned}$$

If $ivk = 0$, discard this key and repeat with a new sk .

Note: $ak_{\text{internal}} = ak$ and $nk_{\text{internal}} = nk$.

As explained in §3.1 ‘*Payment Addresses and Keys*’ on p.12, **Orchard** allows the efficient creation of multiple *diversified payment addresses* with the same *spending authority*. A group of such addresses shares the same *full viewing key*, *incoming viewing key*, and *outgoing viewing key*.

To create a new *diversified payment address* given an *incoming viewing key* (dk, ivk), pick a *diversifier index* $index$ uniquely from $\mathbb{B}^{[\ell_d]}$. Then calculate the *diversifier* d and the *diversified transmission key* pk_d :

$$\begin{aligned} d &:= \text{PRP}_{dk}^d(index) \\ g_d &:= \text{DiversifyHash}^{\text{Orchard}}(d) \\ pk_d &:= \text{KA}^{\text{Orchard}}.\text{DerivePublic}(ivk, g_d). \end{aligned}$$

The resulting *diversified payment address* is $(d : \mathbb{B}^{[\ell_d]}, pk_d : \text{KA}^{\text{Orchard}}.\text{Public})$.

The *diversified payment address* with *diversifier index* 0 is called the *default diversified payment address*.

Notes:

- *Diversifier indices* **SHOULD NOT** be chosen at random. [ZIP-32] specifies their usage in the context of hierarchical deterministic wallets.
- Address generators **MAY** encode information in the *diversifier index* that can be recovered by the recipient of a payment, given the *diversifier key*.
- ivk is used both as a randomizer for Commit^{ivk} , and as a key for $\text{PRF}^{\text{expand}}$ to derive dk and ovk . If dk and ovk are known to an adversary, then this reuse prevents proving that the use of Commit^{ivk} in this context is perfectly *hiding*. It is also not sufficient to model $\text{PRF}^{\text{expand}}$ only as a PRF. In practice, we believe it would be extremely surprising if there were an exploitable interaction between scalar multiplication used in Commit^{ivk} , and BLAKE2b used to instantiate $\text{PRF}^{\text{expand}}$. It is possible, albeit somewhat inelegantly, to model this usage by a joint assumption on Pallas scalar multiplication and $\text{PRF}^{\text{expand}}$.

Non-normative notes:

- The uses of $\text{ToScalar}^{\text{Orchard}}$ and $\text{ToBase}^{\text{Orchard}}$ produce output that is uniform on \mathbb{F}_{r_p} and \mathbb{F}_{q_p} respectively when applied to random input, by a similar argument to that used in §4.2.2 ‘*Sapling Key Components*’ on p.35.
- The output of Commit^{ivk} is the *affine-short-Weierstrass* x -coordinate of a Pallas curve point, which we then use as a $\text{KA}^{\text{Orchard}}$ *private key* ivk for *note* encryption. The fact that ivk is non-uniform on \mathbb{F}_{r_p} (since it can only take on roughly half of the possible values) is not expected to cause any security issue.

4.3 JoinSplit Descriptions

A *JoinSplit transfer*, as specified in §3.5 ‘*JoinSplit Transfers and Descriptions*’ on p.18, is encoded in *transactions* as a *JoinSplit description*.

Each *transaction* includes a sequence of zero or more *JoinSplit descriptions*. When this sequence is non-empty, the *transaction* also includes encodings of a *JoinSplitSig* public *validating key* and signature.

Let $\ell_{\text{Merkle}}^{\text{Sprout}}$, $\ell_{\text{PRF}}^{\text{Sprout}}$, ℓ_{Seed} , N^{old} , N^{new} , and MAX_MONEY be as defined in §5.3 ‘*Constants*’ on p.72.

Let hSigCRH be as defined in §4.1.1 ‘*Hash Functions*’ on p.23.

Let $\text{NoteCommit}^{\text{Sprout}}$ be as defined in §4.1.8 ‘*Commitment*’ on p.29.

Let $\text{KA}^{\text{Sprout}}$ be as defined in §4.1.5 ‘*Key Agreement*’ on p.25.

Let Sym be as defined in §4.1.4 ‘*Symmetric Encryption*’ on p.25.

Let ZKJoinSplit be as defined in §4.1.13 ‘*Zero-Knowledge Proving System*’ on p.33.

A *JoinSplit description* comprises $(v_{\text{pub}}^{\text{old}}, v_{\text{pub}}^{\text{new}}, \text{rt}^{\text{Sprout}}, \text{nf}_{1..N}^{\text{old}}, \text{cm}_{1..N}^{\text{new}}, \text{epk}, \text{randomSeed}, h_{1..N}^{\text{old}}, \pi_{\text{ZKJoinSplit}}, C_{1..N}^{\text{enc}})$ where

- $v_{\text{pub}}^{\text{old}} : \{0 \dots \text{MAX_MONEY}\}$ is the value that the *JoinSplit transfer* removes from the *transparent transaction value pool*;
- $v_{\text{pub}}^{\text{new}} : \{0 \dots \text{MAX_MONEY}\}$ is the value that the *JoinSplit transfer* inserts into the *transparent transaction value pool*;
- $\text{rt}^{\text{Sprout}} : \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]}$ is an *anchor*, as defined in § 3.4 ‘*Transactions and Treestates*’ on p. 17, for the output *treestate* of either a previous *block*, or a previous *JoinSplit transfer* in this *transaction*.
- $\text{nf}_{1..N}^{\text{old}} : \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}][N^{\text{old}}]}$ is the sequence of *nullifiers* for the input *notes*;
- $\text{cm}_{1..N}^{\text{new}} : \text{NoteCommit}^{\text{Sprout}}.\text{Output}^{[N^{\text{new}}]}$ is the sequence of *note commitments* for the output *notes*;
- $\text{epk} : \text{KA}^{\text{Sprout}}.\text{Public}$ is a key agreement *public key*, used to derive the key for encryption of the *transmitted notes ciphertext* (§ 4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63);
- $\text{randomSeed} : \mathbb{B}^{[\ell_{\text{Seed}}]}$ is a seed that must be chosen independently at random for each *JoinSplit description*;
- $h_{1..N}^{\text{old}} : \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}][N^{\text{old}}]}$ is a sequence of tags that bind h_{Sig} to each a_{sk} of the input *notes*;
- $\pi_{\text{ZKJoinSplit}} : \text{ZKJoinSplit}.\text{Proof}$ is a *zk proof* with *primary input* $(\text{rt}^{\text{Sprout}}, \text{nf}_{1..N}^{\text{old}}, \text{cm}_{1..N}^{\text{new}}, v_{\text{pub}}^{\text{old}}, v_{\text{pub}}^{\text{new}}, h_{\text{Sig}}, h_{1..N}^{\text{old}})$ for the *JoinSplit statement* defined in § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58 (this is a BCTV14 proof before Sapling activation, and a Groth16 proof after Sapling activation);
- $C_{1..N}^{\text{enc}} : \text{Sym.C}^{[N^{\text{new}}]}$ is a sequence of ciphertext components for the encrypted output *notes*.

The *ephemeralKey* and *encCiphertexts* fields together form the *transmitted notes ciphertext*.

The value h_{Sig} is also computed from randomSeed , $\text{nf}_{1..N}^{\text{old}}$, and the *joinSplitPubKey* of the containing *transaction*:

$$h_{\text{Sig}} := h_{\text{SigCRH}}(\text{randomSeed}, \text{nf}_{1..N}^{\text{old}}, \text{joinSplitPubKey}).$$

Consensus rules:

- Elements of a *JoinSplit description* **MUST** have the types given above (for example: $0 \leq v_{\text{pub}}^{\text{old}} \leq \text{MAX_MONEY}$ and $0 \leq v_{\text{pub}}^{\text{new}} \leq \text{MAX_MONEY}$).
- The proof $\pi_{\text{ZKJoinSplit}}$ **MUST** be valid given a *primary input* formed from the relevant other fields and h_{Sig} — i.e. $\text{ZKJoinSplit}.\text{Verify}((\text{rt}^{\text{Sprout}}, \text{nf}_{1..N}^{\text{old}}, \text{cm}_{1..N}^{\text{new}}, v_{\text{pub}}^{\text{old}}, v_{\text{pub}}^{\text{new}}, h_{\text{Sig}}, h_{1..N}^{\text{old}}), \pi_{\text{ZKJoinSplit}}) = 1$.
- Either $v_{\text{pub}}^{\text{old}}$ or $v_{\text{pub}}^{\text{new}}$ **MUST** be zero.
- [Canopy onward] $v_{\text{pub}}^{\text{old}}$ **MUST** be zero.

4.4 Spend Descriptions

A *Spend transfer*, as specified in § 3.6 ‘*Spend Transfers, Output Transfers, and their Descriptions*’ on p. 19, is encoded in *transactions* as a *Spend description*.

Each *transaction* includes a sequence of zero or more *Spend descriptions*.

Each *Spend description* is authorized by a signature, called the *spend authorization signature*.

Let $\ell_{\text{Merkle}}^{\text{Sapling}}$ and $\ell_{\text{PRF}}^{\text{Sapling}}$ be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let \mathcal{O}_{J} , abst_{J} , repr_{J} , and h_{J} be as defined in § 5.4.9.3 ‘*Jubjub*’ on p. 100.

Let $\text{ValueCommit}^{\text{Sapling}}.\text{Output}$ be as defined in § 4.1.8 ‘*Commitment*’ on p. 29.

Let $\text{SpendAuthSig}^{\text{Sapling}}$ be as defined in § 4.15 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 55.

Let ZKSpend be as defined in § 4.1.13 ‘*Zero-Knowledge Proving System*’ on p. 33.

An *Action description* comprises $(cv^{\text{net}}, rt^{\text{Orchard}}, nf, rk, \text{spendAuthSig}, cm_x, epk, C^{\text{enc}}, C^{\text{out}}, \text{enableSpends}, \text{enableOutputs}, \pi)$ where

- $cv^{\text{net}} : \text{ValueCommit}^{\text{Orchard}}.$ Output is the *value commitment* to the value of the input *note* minus the value of the output *note*;
- $rt^{\text{Orchard}} : \{0 \dots q_{\mathbb{P}} - 1\}$ is an *anchor*, as defined in § 3.4 ‘*Transactions and Treestates*’ on p.17, for the output *treestate* of a previous *block*;
- $nf : \{0 \dots q_{\mathbb{P}} - 1\}$ is the *nullifier* for the input *note*;
- $rk : \text{SpendAuthSig}^{\text{Orchard}}.$ Public is a randomized *validating key* that should be used to validate *spendAuthSig*;
- $\text{spendAuthSig} : \text{SpendAuthSig}^{\text{Orchard}}.$ Signature is a *spend authorization signature*, validated as specified in § 4.15 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 55;
- $cm_x : \{0 \dots q_{\mathbb{P}} - 1\}$ is the result of applying $\text{Extract}_{\mathbb{P}}$ to the *note commitment* for the output *note*;
- $epk : \text{KA}^{\text{Orchard}}.$ Public is a key agreement *public key*, used to derive the key for encryption of the *transmitted note ciphertext* (§ 4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65);
- $C^{\text{enc}} : \text{Sym.C}$ is a ciphertext component for the encrypted output *note*;
- $C^{\text{out}} : \text{Sym.C}$ is a ciphertext component that allows the holder of the *outgoing cipher key* (which can be derived from a *full viewing key*) to recover the recipient *diversified transmission key* pk_d and the *ephemeral private key* esk , hence the entire *note plaintext*;
- $\text{enableSpends} : \mathbb{B}$ is a flag that is set in order to enable non-zero-valued spends in this Action;
- $\text{enableOutputs} : \mathbb{B}$ is a flag that is set in order to enable non-zero-valued outputs in this Action;
- $\pi : \text{ZKAction.Proof}$ is a *zk-SNARK proof* with *primary input* $(cv, rt^{\text{Orchard}}, nf, rk, cm_x, \text{enableSpends}, \text{enableOutputs})$ for the *Action statement* defined in § 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61.

Note: The rt^{Orchard} , enableSpends , and enableOutputs components are the same for all *Action transfers* in a *transaction*. They are encoded once in the *transaction body* (see § 7.1 ‘*Transaction Encoding and Consensus*’ on p.119), not in the *ActionDescription* structure. π is aggregated with other Action proofs and encoded in the *proofsOrchard* field of a *transaction*.

Consensus rules:

- Elements of an *Action description* **MUST** be canonical encodings of the types given above.
- Let SigHash be the *SIGHASH transaction hash* of this *transaction*, not associated with an input, as defined in § 4.10 ‘*SIGHASH Transaction Hashing*’ on p. 49 using SIGHASH_ALL .
The *spend authorization signature* **MUST** be a valid $\text{SpendAuthSig}^{\text{Orchard}}$ signature over SigHash using rk as the *validating key*— i.e. $\text{SpendAuthSig}^{\text{Orchard}}.\text{Validate}_{rk}(\text{SigHash}, \text{spendAuthSig}) = 1$. As specified in § 5.4.7 ‘*RedDSA, RedJubjub, and RedPallas*’ on p. 90, validation of the \underline{R} component of the signature prohibits *non-canonical* encodings.
- The proof π **MUST** be valid given a *primary input* $(cv, rt^{\text{Orchard}}, nf, rk, cm_x, \text{enableSpends}, \text{enableOutputs})$ — i.e. $\text{ZKAction.Verify}((cv, rt^{\text{Orchard}}, nf, rk, cm_x, \text{enableSpends}, \text{enableOutputs}), \pi) = 1$.

Non-normative notes:

- cv and rk can be the zero point $\mathcal{O}_{\mathbb{P}}$. epk cannot be $\mathcal{O}_{\mathbb{P}}$.
- nf and cm_x are *not* checked to be valid *affine-short-Weierstrass x-coordinates* on the Pallas curve; they are only checked to encode integers in $\{0 \dots q_{\mathbb{P}} - 1\}$.

4.7 Sending Notes

4.7.1 Sending Notes (Sprout)

In order to send *Sprout shielded value*, the sender constructs a *transaction* containing one or more *JoinSplit descriptions*.

Let JoinSplitSig be as specified in § 4.1.7 ‘*Signature*’ on p. 26.

Let $\text{NoteCommit}^{\text{Sprout}}$ be as specified in § 4.1.8 ‘*Commitment*’ on p. 29.

Let ℓ_{Seed} and $\ell_{\varphi}^{\text{Sprout}}$ be as specified in § 5.3 ‘*Constants*’ on p. 72.

Sending a *transaction* containing *JoinSplit descriptions* involves first generating a new JoinSplitSig key pair:

```
joinSplitPrivKey  $\xleftarrow{R}$  JoinSplitSig.GenPrivate()
joinSplitPubKey := JoinSplitSig.DerivePublic(joinSplitPrivKey).
```

For each *JoinSplit description*, the sender chooses randomSeed uniformly at random on $\mathbb{B}^{[\ell_{\text{Seed}}]}$, and selects the input *notes*. At this point there is sufficient information to compute h_{Sig} , as described in the previous section. The sender also chooses φ uniformly at random on $\mathbb{B}^{[\ell_{\varphi}^{\text{Sprout}}]}$. Then it creates each output *note* with index $i : \{1..N^{\text{new}}\}$:

- Choose uniformly random $\text{rcm}_i \xleftarrow{R} \text{NoteCommit}^{\text{Sprout}}.\text{GenTrapdoor}()$.
- Compute $\rho_i = \text{PRF}_{\varphi}^{\rho}(i, h_{\text{Sig}})$.
- Compute $\text{cm}_i = \text{NoteCommit}^{\text{Sprout}}_{\text{rcm}_i}(a_{\text{pk},i}, v_i, \rho_i)$.
- Let $\text{np}_i = (0x00, v_i, \rho_i, \text{rcm}_i, \text{memo}_i)$.

$\text{np}_{1..N^{\text{new}}}$ are then encrypted to the recipient *transmission keys* $\text{pk}_{\text{enc},1..N^{\text{new}}}$, giving the *transmitted notes ciphertext* $(\text{epk}, \text{C}_{1..N^{\text{new}}}^{\text{enc}})$, as described in § 4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63.

In order to minimize information leakage, the sender **SHOULD** randomize the order of the input *notes* and of the output *notes*. Other considerations relating to information leakage from the structure of *transactions* are beyond the scope of this specification.

After generating all of the *JoinSplit descriptions*, the sender obtains $\text{dataToBeSigned} : \mathbb{B}^{[N]}$ as described in § 4.11 ‘*Non-malleability (Sprout)*’ on p. 50, and signs it with the private *JoinSplit signing key*:

```
joinSplitSig  $\xleftarrow{R}$  JoinSplitSig.Sign_{joinSplitPrivKey}(\text{dataToBeSigned})
```

Then the encoded *transaction* including joinSplitSig is submitted to the *peer-to-peer network*.

[Canopy onward] **Note:** [ZIP-211] specifies that nodes and wallets **MUST** disable any facilities to send to **Sprout** addresses. This **SHOULD** be made clear in user interfaces and API documentation.

The facility to send to **Sprout** addresses is in any case **OPTIONAL** for a particular node or wallet implementation.

4.7.2 Sending Notes (Sapling)

In order to send **Sapling shielded** value, the sender constructs a *transaction* with one or more *Output descriptions*.

Let $\text{ValueCommit}^{\text{Sapling}}$ and $\text{NoteCommit}^{\text{Sapling}}$ be as specified in § 4.1.8 ‘*Commitment*’ on p. 29.

Let $\text{KA}^{\text{Sapling}}$ be as specified in § 4.1.5 ‘*Key Agreement*’ on p. 25.

Let $\text{DiversifyHash}^{\text{Sapling}}$ be as specified in § 4.1.1 ‘*Hash Functions*’ on p. 23.

Let $\text{ToScalar}^{\text{Sapling}}$ be as specified in § 4.2.2 ‘*Sapling Key Components*’ on p. 35.

Let repr_J and r_J be as defined in § 5.4.9.3 ‘*Jubjub*’ on p. 100.

Let ovk be a **Sapling outgoing viewing key** that is intended to be able to decrypt this payment. This may be one of:

- the *outgoing viewing key* for the address (or one of the addresses) from which the payment was sent;
- the *outgoing viewing key* for all payments associated with an “account”, to be defined in [ZIP-32];
- \perp , if the sender should not be able to decrypt the payment once it has deleted its own copy.

Let ovk be an **Orchard** *outgoing viewing key* that is intended to be able to decrypt this payment. The considerations for choosing *outgoing viewing keys* are as described for **Sapling** in § 4.7.2 ‘*Sending Notes (Sapling)*’ on p. 43.

Let $leadByte$ be the *note plaintext lead byte*, which **MUST** be $0x02$.

For each *Action description*, the sender selects a value $v : \{0 \dots MAX_MONEY\}$ and a destination **Orchard** *shielded payment address* (d, pk_d) , and performs the following steps:

Check that pk_d is of type $KA^{Orchard}.Public$.

Calculate $g_d = \text{DiversifyHash}^{Orchard}(d)$.

Choose a uniformly random *commitment trapdoor* $rcv \xleftarrow{R} \text{ValueCommit}^{Orchard}.GenTrapdoor()$.

Choose uniformly random $rseed \xleftarrow{R} \mathbb{BY}^{[32]}$.

Let $\rho = nf^{old}$ from the same *Action description*, and let $\underline{\rho} = I2LEOSP_{256}(\rho)$.

Derive $esk = \text{ToScalar}^{Orchard}(\text{PRF}_{rseed}^{expand}([4] || \underline{\rho}))$.

If $esk = 0 \pmod{r_{\mathbb{P}}}$, repeat the above steps using a different $rseed$.

Derive $rcm = \text{ToScalar}^{Orchard}(\text{PRF}_{rseed}^{expand}([5] || \underline{\rho}))$.

Derive $\psi = \text{ToBase}^{Orchard}(\text{PRF}_{rseed}^{expand}([9] || \underline{\rho}))$.

Let cv^{net} be the *value commitment* to the value of the input *note* minus the value v of the output *note* for this *Action transfer*, using rcv , as described in § 4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53.

Let $cm_x = \text{Extract}_{\mathbb{P}}^{\perp}(\text{NoteCommit}_{rcm}^{Orchard}(\text{repr}_{\mathbb{P}}(g_d), \text{repr}_{\mathbb{P}}(pk_d), v, \rho, \psi))$.

If $cm_x = \perp$, repeat the above steps using a different $rseed$.

Let $np = (leadByte, d, v, rseed, memo)$.

Encrypt np to the recipient *diversified transmission key* pk_d with *diversified base* g_d , and to the *outgoing viewing key* ovk , giving the *transmitted note ciphertext* (epk, C^{enc}, C^{out}) . This procedure is described in § 4.20.1 ‘*Encryption (Sapling and Orchard)*’ on p. 65; it uses cv^{net} and cm_x to derive ock , and takes esk as input.

Fill in the spending side of the *Action transfer* (§ 4.15 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 55), and generate a proof π for the *Action statement* in § 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61.

Return $(cv, cm_x, epk, C^{enc}, C^{out}, \pi)$.

If no real **Orchard** *note* is being spent in the same *Action transfer*, the sender **SHOULD** create a *dummy note* to spend as described in § 4.8.3 ‘*Dummy Notes (Orchard)*’ on p. 47, and use that *dummy note*’s nullifier as the ρ value.

In order to minimize information leakage, the sender **SHOULD** randomize the order of *Action descriptions* in a *transaction*. Other considerations relating to information leakage from the structure of *transactions* are beyond the scope of this specification. The encoded *transaction* is submitted to the *peer-to-peer network*.

Note: The domain separators $[4]$ and $[5]$ used in the input to $\text{PRF}_{rseed}^{expand}$ are swapped for **Orchard** relative to **Sapling**. This was due to an oversight and there is no good reason for it.

4.8 Dummy Notes

4.8.1 Dummy Notes (Sprout)

The fields in a *JoinSplit description* allow for N^{old} input *notes*, and N^{new} output *notes*. In practice, we may wish to encode a *JoinSplit transfer* with fewer input or output *notes*. This is achieved using *dummy notes*.

Let ℓ_{ask} and ℓ_{PRF}^{Sprout} be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let $\text{PRF}^{\text{nfSprout}}$ be as defined in §4.1.2 ‘*Pseudo Random Functions*’ on p. 24.

Let $\text{NoteCommit}^{\text{Sprout}}$ be as defined in §4.1.8 ‘*Commitment*’ on p. 29.

A *dummy Sprout* input note, with index i in the *JoinSplit* description, is constructed as follows:

- Generate a new uniformly random *spending* key $a_{\text{sk},i}^{\text{old}} \xleftarrow{\mathbb{R}} \mathbb{B}^{[\ell_{\text{sk}}]}$ and derive its *paying* key $a_{\text{pk},i}^{\text{old}}$.
- Set $v_i^{\text{old}} = 0$.
- Choose uniformly random $\rho_i^{\text{old}} \xleftarrow{\mathbb{R}} \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]}$ and $\text{rcm}_i^{\text{old}} \xleftarrow{\mathbb{R}} \text{NoteCommit}^{\text{Sprout}}.\text{GenTrapdoor}()$.
- Compute $\text{nf}_i^{\text{old}} = \text{PRF}_{a_{\text{sk},i}^{\text{old}}}^{\text{nfSprout}}(\rho_i^{\text{old}})$.
- Let path_i be a *dummy Merkle path* for the *auxiliary input* to the *JoinSplit* statement (this will not be checked).
- When generating the *JoinSplit* proof, set $\text{enforceMerklePath}_i$ to 0.

A *dummy Sprout* output note is constructed as normal but with zero value, and sent to a random *shielded payment address*.

4.8.2 Dummy Notes (Sapling)

In *Sapling* there is no need to use *dummy notes* simply in order to fill otherwise unused inputs as in the case of a *JoinSplit* description; nevertheless it may be useful for privacy to obscure the number of real *shielded inputs* from *Sapling* notes.

Let ℓ_{sk} be as defined in §5.3 ‘*Constants*’ on p. 72.

Let $\text{ValueCommit}^{\text{Sapling}}$ and $\text{NoteCommit}^{\text{Sapling}}$ be as defined in §4.1.8 ‘*Commitment*’ on p. 29.

Let $\text{DiversifyHash}^{\text{Sapling}}$ be as specified in §4.1.1 ‘*Hash Functions*’ on p. 23.

Let $\text{ToScalar}^{\text{Sapling}}$ be as specified in §4.2.2 ‘*Sapling Key Components*’ on p. 35.

Let $\text{repr}_{\mathbb{J}}$ and $r_{\mathbb{J}}$ be as defined in §5.4.9.3 ‘*Jubjub*’ on p. 100.

Let $\text{PRF}^{\text{nfSapling}}$ be as defined in §4.1.2 ‘*Pseudo Random Functions*’ on p. 24.

Let $\text{NoteCommit}^{\text{Sapling}}$ be as defined in §4.1.8 ‘*Commitment*’ on p. 29.

A *Spend* description for a *dummy Sapling* input note is constructed as follows:

- Choose uniformly random $\text{sk} \xleftarrow{\mathbb{R}} \mathbb{B}^{[\ell_{\text{sk}}]}$.
- Generate the ak and nk components of a *full viewing key* and a *diversified payment address* (d, pk_d) for sk , as described in §4.2.2 ‘*Sapling Key Components*’ on p. 35.
- Let $v = 0$ and $\text{pos} = 0$.
- Choose uniformly random $\text{rcv} \xleftarrow{\mathbb{R}} \text{ValueCommit}^{\text{Sapling}}.\text{GenTrapdoor}()$.
- Choose uniformly random $\text{rseed} \xleftarrow{\mathbb{R}} \mathbb{B}^{\text{Y}[32]}$.
- Derive $\text{rcm} = \text{ToScalar}^{\text{Sapling}}(\text{PRF}_{\text{rseed}}^{\text{expand}}([d]))$.
- Let $\text{cv} = \text{ValueCommit}_{\text{rcv}}^{\text{Sapling}}(v)$.
- Let $\text{cm} = \text{NoteCommit}_{\text{rcm}}^{\text{Sapling}}(\text{repr}_{\mathbb{J}}(g_d), \text{repr}_{\mathbb{J}}(\text{pk}_d), v)$.
- Let $\rho_{\star} = \text{repr}_{\mathbb{J}}(\text{MixingPedersenHash}(\text{cm}, \text{pos}))$.
- Let $\text{nk}_{\star} = \text{repr}_{\mathbb{J}}(\text{nk})$.
- Let $\text{nf} = \text{PRF}_{\text{nk}_{\star}}^{\text{nfSapling}}(\rho_{\star})$.
- Construct a *dummy Merkle path* path for use in the *auxiliary input* to the *Spend* statement (this will not be checked, because $v = 0$).

As in *Sprout*, a *dummy Sapling* output note is constructed as normal but with zero value, and sent to a random *shielded payment address*.

4.8.3 Dummy Notes (Orchard)

As for **Sapling**, it may be useful for privacy to obscure the number of real *shielded inputs* from **Orchard** notes.

Let ℓ_{sk} be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let $\text{ValueCommit}^{\text{Orchard}}$ and $\text{NoteCommit}^{\text{Orchard}}$ be as defined in § 4.1.8 ‘*Commitment*’ on p. 29.

Let $\text{DiversifyHash}^{\text{Orchard}}$ be as specified in § 4.1.1 ‘*Hash Functions*’ on p. 23.

Let $\text{ToScalar}^{\text{Orchard}}$ and $\text{ToBase}^{\text{Orchard}}$ be as specified in § 4.2.3 ‘*Orchard Key Components*’ on p. 37.

Let $\text{repr}_{\mathbb{P}}$ and $r_{\mathbb{P}}$ be as defined in § 5.4.9.6 ‘*Pallas and Vesta*’ on p. 103.

Let DeriveNullifier be as defined in § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56.

Let $\text{NoteCommit}^{\text{Orchard}}$ be as defined in § 4.1.8 ‘*Commitment*’ on p. 29.

Let I2LEOSP be as defined in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

The spend-related fields of an *Action description* for a dummy **Orchard** input *note* are constructed as follows:

- Choose uniformly random $sk \xleftarrow{\mathbb{R}} \mathbb{B}^{[\ell_{sk}]}$.
- Generate a *full viewing key* ($ak, nk, rivk$) and a *diversified payment address* (d, pk_d) for sk as described in § 4.2.3 ‘*Orchard Key Components*’ on p. 37.
- Let $v = 0$.
- Choose uniformly random $rseed \xleftarrow{\mathbb{R}} \mathbb{B}^{[32]}$.
- Choose uniformly random $\rho \xleftarrow{\mathbb{R}} \mathbb{P}$.
- Let $\rho = \text{Extract}_{\mathbb{P}}(\rho^{\mathbb{P}})$ and $\underline{\rho} = \text{I2LEOSP}_{256}(\rho)$.
- Derive $rcm = \text{ToScalar}^{\text{Orchard}}(\text{PRF}_{rseed}^{\text{expand}}([5] || \underline{\rho}))$.
- Derive $\psi = \text{ToBase}^{\text{Orchard}}(\text{PRF}_{rseed}^{\text{expand}}([9] || \underline{\rho}))$.
- Let $cm = \text{NoteCommit}_{rcm}^{\text{Orchard}}(\text{repr}_{\mathbb{P}}(g_d), \text{repr}_{\mathbb{P}}(pk_d), v, \rho, \psi)$.
- If $cm = \perp$, repeat the above steps using a different $rseed$.
- Let $nf = \text{DeriveNullifier}_{nk}(\rho, \psi, cm)$.
- Construct a *dummy Merkle path* $path$ for use in the *auxiliary input* to the *Action statement* (this will not be checked, because $v = 0$).

As in **Sprout** and **Sapling**, a *dummy Orchard* output *note* is constructed as normal but with zero value, and sent to a random *shielded payment address*.

Note: The domain separators [4] and [5] used in the input to $\text{PRF}_{rseed}^{\text{expand}}$ are swapped for **Orchard** relative to **Sapling**. This was due to an oversight and there is no good reason for it.

4.9 Merkle Path Validity

Let MerkleDepth be $\text{MerkleDepth}^{\text{Sprout}}$ for the **Sprout** note commitment tree, or $\text{MerkleDepth}^{\text{Sapling}}$ for the **Sapling** note commitment tree, or $\text{MerkleDepth}^{\text{Orchard}}$ for the **Orchard** note commitment tree. These constants are defined in § 5.3 ‘*Constants*’ on p. 72.

Similarly, let MerkleCRH be $\text{MerkleCRH}^{\text{Sprout}}$ for **Sprout**, or $\text{MerkleCRH}^{\text{Sapling}}$ for **Sapling**, or $\text{MerkleCRH}^{\text{Orchard}}$ for **Orchard**.

The following discussion applies independently to the **Sprout and Sapling and Orchard** note commitment trees.

Each *node* in the *incremental Merkle tree* is associated with a *hash value*, which is a *bit sequence*.

The *layer* numbered h , counting from *layer* 0 at the *root*, has 2^h nodes with *indices* 0 to $2^h - 1$ inclusive.

Let M_i^h be the *hash value* associated with the *node* at *index* i in *layer* h .

The *nodes* at *layer* MerkleDepth are called *leaf nodes*. When a *note commitment* is added to the tree, it occupies the *leaf node hash value* $M_i^{\text{MerkleDepth}}$ for the next available i .

As-yet unused *leaf nodes* are associated with a distinguished *hash value* $\text{Uncommitted}^{\text{Sprout}}$ or $\text{Uncommitted}^{\text{Sapling}}$ or $\text{Uncommitted}^{\text{Orchard}}$. It is assumed to be infeasible to find a preimage *note* \mathbf{n} such that $\text{NoteCommitment}^{\text{Sprout}}(\mathbf{n}) = \text{Uncommitted}^{\text{Sprout}}$. (No similar assumption is needed for **Sapling** or **Orchard** because we use a representation for $\text{Uncommitted}^{\text{Sapling}}$ that cannot occur as an output of $\text{NoteCommitment}^{\text{Sapling}}$, and similarly for **Orchard**.)

The *nodes* at *layers* 0 to $\text{MerkleDepth} - 1$ inclusive are called *internal nodes*, and are associated with MerkleCRH outputs. *Internal nodes* are computed from their children in the next *layer* as follows: for $0 \leq h < \text{MerkleDepth}$ and $0 \leq i < 2^h$,

$$M_i^h := \text{MerkleCRH}(h, M_{2i}^{h+1}, M_{2i+1}^{h+1}).$$

A *Merkle path* from *leaf node* $M_i^{\text{MerkleDepth}}$ in the *incremental Merkle tree* is the sequence

$$[M_{\text{sibling}(h,i)}^h \text{ for } h \text{ from } \text{MerkleDepth} \text{ down to } 1],$$

where

$$\text{sibling}(h, i) := \text{floor}\left(\frac{i}{2^{\text{MerkleDepth}-h}}\right) \oplus 1$$

Given such a *Merkle path*, it is possible to verify that *leaf node* $M_i^{\text{MerkleDepth}}$ is in a tree with a given *root* $\text{rt} = M_0^0$.

Notes:

- For **Sapling**, *Merkle hash values* are specified to be encoded as *bit sequences*, but the *root* $\text{rt}^{\text{Sapling}}$ is encoded for the *primary input* of a *Spend proof* as an element of \mathbb{F}_{q_p} , as specified in § A.4 ‘*The Sapling Spend circuit*’ on p. 212. The *Spend circuit* allows inputs to $\text{MerkleCRH}^{\text{Sapling}}$ at each *node* to be *non-canonically* encoded, as specified in § A.3.4 ‘*Merkle path check*’ on p. 208.
- For **Orchard**, *Merkle hash values* have type $\{0 \dots q_{\mathbb{P}} - 1\}$ as defined in § 5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p. 104. Similarly to **Sapling**, the *Action circuit* allows inputs to $\text{MerkleCRH}^{\text{Orchard}}$ at each *node* to be *non-canonically* encoded.
- The *Action circuit* is permitted to be implemented in such a way that the *Merkle path* validity check can pass if any *hash value* on the path, including the *root*, is 0. This can only happen if SinsemillaHash returned \perp for that hash, because 0 is not the *affine-short-Weierstrass x-coordinate* of any point on the Pallas curve (as shown in a note at § 5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p. 104), and $\text{SinsemillaHashToPoint}$ cannot return $\mathcal{O}_{\mathbb{P}}$. Allowing the validity check to pass in that case models the fact that incomplete addition is used to implement Sinsemilla in the circuit. As proven in Theorem 5.4.4 on p. 82, a \perp output from SinsemillaHash yields a nontrivial discrete logarithm relation. Since we assume finding such a relation to be infeasible, we can argue that it is safe to allow an adversary to create a proof that passes the *Merkle* validity check in such a case.

4.10 SIGHASH Transaction Hashing

Bitcoin and **Zcash** use signatures and/or non-interactive proofs associated with *transaction* inputs to authorize spending. Because these signatures or proofs could otherwise be replayed in a different *transaction*, it is necessary to “bind” them to the *transaction* for which they are intended. This is done by hashing information about the *transaction* and (where applicable) the specific input, to give a *SIGHASH transaction hash* which is then used for the Spend authorization. The means of authorization differs between *transparent inputs*, inputs to **Sprout** *JoinSplit* transfers, and **Sapling** *Spend* transfers or **Orchard** *Action* transfers, but for a given *transaction* version the same *SIGHASH transaction hash* algorithm is used.

In the case of **Zcash**, the BCTV14 and **Groth16** and **Halo 2** proving systems used are *malleable*, meaning that there is the potential for an adversary who does not know all of the *auxiliary inputs* to a proof, to malleate it in order to create a new proof involving related *auxiliary inputs* [DSDCOPS2001]. This can be understood as similar to a malleability attack on an encryption scheme, in which an adversary can malleate a ciphertext in order to create an encryption of a related plaintext, without knowing the original plaintext. **Zcash** has been designed to mitigate malleability attacks, as described in § 4.11 ‘*Non-malleability (Sprout)*’ on p. 50, § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51, and § 4.15 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 55.

To provide additional flexibility when combining spend authorizations from different sources, **Bitcoin** defines several *SIGHASH* types that cover various parts of a transaction [Bitcoin-SigHash]. One of these types is *SIGHASH_ALL*, which is used for **Zcash**-specific signatures, i.e. *JoinSplit* signatures, *spend authorization signatures*, *Sapling binding signatures*, and *Orchard binding signatures*. In these cases the *SIGHASH transaction hash* is not associated with a *transparent input*, and so the input to hashing excludes *all* of the *scriptSig* fields in the non-**Zcash**-specific parts of the *transaction*.

In **Zcash**, all *SIGHASH* types are extended to cover the **Zcash**-specific fields *nJoinSplit*, *vJoinSplit*, and if present *joinSplitPubKey*. These fields are described in § 7.1 ‘*Transaction Encoding and Consensus*’ on p. 119. The hash *does not* cover the field *joinSplitSig*. After **Overwinter** activation, all *SIGHASH* types are also extended to cover *transaction* fields introduced in that upgrade, and similarly after **Sapling** activation and after **NU5** activation.

The original *SIGHASH* algorithm defined by **Bitcoin** suffered from some deficiencies as described in [ZIP-143]; in **Zcash** these were addressed by changing this algorithm as part of the **Overwinter** upgrade.

Orchard and the **NU5** network upgrade introduce *transaction* version 5, which **MUST** be used if any *Action* transfers are present. This version also provides nonmalleable *transaction* identifiers, and **MAY** be used for that reason whether or not *Action* transfers are present.

Consensus rules:

- [NU5 onward] Any *SIGHASH* type encoding used in a version 5 *transaction* **MUST** be the canonical encoding of one of the defined *SIGHASH* types, i.e. one of 0x01, 0x02, 0x03, 0x81, 0x82, or 0x83. (Previously, undefined bits of a *SIGHASH* type encoding were ignored.)
- [Pre-Overwinter] The *SIGHASH* algorithm used prior to **Overwinter** activation, i.e. for version 1 and 2 *transactions*, will be defined in [ZIP-76] (to be written).
- [Overwinter only, pre-Sapling] The *SIGHASH* algorithm used after **Overwinter** activation and before **Sapling** activation, i.e. for version 3 *transactions*, is defined in [ZIP-143].
- [Overwinter only, pre-Sapling] All *transactions* **MUST** use the **Overwinter** consensus branch ID 0x5BA81B19 as defined in [ZIP-201].
- [Sapling onward] The *SIGHASH* algorithm used after **Sapling** activation, i.e. for version 4 *transactions*, is defined in [ZIP-243].
- [Sapling only, pre-Blossom] All *transactions* **MUST** use the **Sapling** consensus branch ID 0x76B809BB as defined in [ZIP-205].
- [Blossom only, pre-Heartwood] All *transactions* **MUST** use the **Blossom** consensus branch ID 0x2BB40E60 as defined in [ZIP-206].
- [Heartwood only, pre-Canopy] All *transactions* **MUST** use the **Heartwood** consensus branch ID 0xF5B9230B as defined in [ZIP-250].

- [Canopy only, pre-NU5] All *transactions* **MUST** use the **Canopy consensus branch ID** 0xE9FF75A6 as defined in [ZIP-251].
- [NU5 onward] The *SIGHASH algorithm* used for version 5 *transactions* introduced by the **NU5 network upgrade** is defined in [ZIP-244]. Version 4 *transactions* continue to use the *SIGHASH algorithm* defined in [ZIP-243].
- [NU5 only, pre-NU6] All *transactions* **MUST** use the **NU5 consensus branch ID** 0xF919A198 as defined in [ZIP-252].
- [NU6 only] All *transactions* **MUST** use the **NU6 consensus branch ID** 0xC8E71055 as defined in [ZIP-253].

4.11 Non-malleability (Sprout)

Let dataToBeSigned be the hash of the *transaction*, not associated with an input, using the `SIGHASH_ALL` *SIGHASH* type.

In order to ensure that a *JoinSplit* description is cryptographically bound to the *transparent* inputs and outputs corresponding to $v_{\text{pub}}^{\text{new}}$ and $v_{\text{pub}}^{\text{old}}$, and to the other *JoinSplit* descriptions in the same *transaction*, an ephemeral *JoinSplitSig* key pair is generated for each *transaction*, and the dataToBeSigned is signed with the private *signing key* of this key pair. The corresponding public *validating key* is included in the *transaction* encoding as `joinSplitPubKey`. *JoinSplitSig* is instantiated in § 5.4.6 ‘Ed25519’ on p. 88.

If `nJoinSplit` is zero, the `joinSplitPubKey` and `joinSplitSig` fields are omitted. Otherwise, a *transaction* has a correct *JoinSplit signature* if and only if $\text{JoinSplitSig.Validate}_{\text{joinSplitPubKey}}(\text{dataToBeSigned}, \text{joinSplitSig}) = 1$.

Let h_{sig} be computed as specified in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38.

Let PRF^{pk} be as defined in § 4.1.2 ‘*Pseudo Random Functions*’ on p. 24.

For each $i \in \{1..N^{\text{old}}\}$, the creator of a *JoinSplit* description calculates $h_i = \text{PRF}_{a_{\text{sk}, i}}^{\text{pk}}(i, h_{\text{sig}})$.

The correctness of $h_{1..N^{\text{old}}}$ is enforced by the *JoinSplit statement* given in § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58. This ensures that a holder of all of the $a_{\text{sk}, 1..N^{\text{old}}}^{\text{old}}$ for every *JoinSplit* description in the *transaction* has authorized the use of the private *signing key* corresponding to `joinSplitPubKey` to sign this *transaction*.

4.12 Balance (Sprout)

In **Bitcoin**, all inputs to and outputs from a *transaction* are *transparent*. The total value of *transparent outputs* must not exceed the total value of *transparent inputs*. The net value of *transparent inputs* minus *transparent outputs* is transferred to the miner of the *block* containing the *transaction*; it is added to the *miner subsidy* in the *coinbase transaction* of the *block*.

Zcash Sprout extends this by adding *JoinSplit transfers*. Each *JoinSplit transfer* can be seen, from the perspective of the *transparent transaction value pool*, as an input and an output simultaneously.

$v_{\text{pub}}^{\text{old}}$ takes value from the *transparent transaction value pool* and $v_{\text{pub}}^{\text{new}}$ adds value to the *transparent transaction value pool*. As a result, $v_{\text{pub}}^{\text{old}}$ is treated like an *output* value, whereas $v_{\text{pub}}^{\text{new}}$ is treated like an *input* value.

Unlike original **Zerocash** [BCGGMV2014], **Zcash** does not have a distinction between *Mint* and *Pour* operations. The addition of $v_{\text{pub}}^{\text{old}}$ to a *JoinSplit description* subsumes the functionality of both *Mint* and *Pour*.

Also, a difference in the number of real input *notes* does not by itself cause two *JoinSplit descriptions* to be distinguishable.

As stated in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38, either $v_{\text{pub}}^{\text{old}}$ or $v_{\text{pub}}^{\text{new}}$ **MUST** be zero. No generality is lost because, if a *transaction* in which both $v_{\text{pub}}^{\text{old}}$ and $v_{\text{pub}}^{\text{new}}$ were nonzero were allowed, it could be replaced by an equivalent one in which $\min(v_{\text{pub}}^{\text{old}}, v_{\text{pub}}^{\text{new}})$ is subtracted from both of these values. This restriction helps to avoid unnecessary distinctions between *transactions* according to client implementation.

For each *Spend description* or *Action description*, the signer chooses a fresh *spend authorization randomizer* α :

1. Choose $\alpha \xleftarrow{R} \text{SpendAuthSig.GenRandom}()$.
2. Let $\text{rsk} = \text{SpendAuthSig.RandomizePrivate}(\alpha, \text{ask})$.
3. Let $\text{rk} = \text{SpendAuthSig.DerivePublic}(\text{rsk})$.
4. Generate a proof π of the *Spend statement* (§ 4.18.2 ‘*Spend Statement (Sapling)*’ on p. 59) or *Action statement* (§ 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61), with α in the *auxiliary input* and rk in the *primary input*.
5. Let $\text{spendAuthSig} = \text{SpendAuthSig.Sign}_{\text{rsk}}(\text{SigHash})$.

The resulting spendAuthSig and π are included in the *Spend description*, or in the $\text{vSpendAuthSigsSapling}$ or $\text{vSpendAuthSigsOrchard}$ field of a version 5 *transaction*.

Note: If the spender is computationally or memory-limited, step 4 (and only step 4) **MAY** be delegated to a different party that is capable of performing the *zk-SNARK proof*. In this case privacy will be lost to that party since it needs ak and the *proof authorizing key* nsk ; this allows also deriving the nk component of the *full viewing key*. (In **Orchard**, that party needs the nk directly to make the *zk-SNARK proof*.) Together ak and nk are sufficient to recognize spent *notes* and to recognize and decrypt incoming *notes*. However, the other party will not obtain *spending authority* for other *transactions*, since it is not able to create a *spend authorization signature* by itself.

4.16 Computing ρ values and Nullifiers

In **Sprout** and **Orchard**, each *note* has a ρ component, defined as part of the *note*.

In **Sapling**, each *positioned note* (as defined in § 3.2.2 ‘*Note Commitments*’ on p. 15) has an associated ρ value, which is computed from its *note commitment* cm and *note position* pos as follows:

$$\rho := \text{MixingPedersenHash}(\text{cm}, \text{pos}).$$

$\text{MixingPedersenHash}$ is defined in § 5.4.1.8 ‘*Mixing Pedersen Hash Function*’ on p. 79.

Let $\text{PRF}^{\text{nfSprout}}$ and $\text{PRF}^{\text{nfSapling}}$ and $\text{PRF}^{\text{nfOrchard}}$ be as instantiated in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84.

For a **Sprout** *note*, the *nullifier* (see § 3.2.3 ‘*Nullifiers*’ on p. 16) is derived as $\text{PRF}_{\text{ask}}^{\text{nfSprout}}(\rho)$, where ask is the *spending key* associated with the *note*.

For a **Sapling** *note*, the *nullifier* is derived as $\text{PRF}_{\text{nk}^*}^{\text{nfSapling}}(\rho^*)$, where nk^* is a representation of the *nullifier deriving key* associated with the *note* and $\rho^* = \text{repr}_{\mathbb{J}}(\rho)$.

The derivation of *nullifiers* for **Orchard** *notes* is a little more complicated.

Let \mathbb{P} and $q_{\mathbb{P}}$ be as defined in § 5.4.9.6 ‘*Pallas and Vesta*’ on p. 103.

Let $\text{Extract}_{\mathbb{P}}$ be as defined in § 5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p. 104.

Let $\text{GroupHash}^{\mathbb{P}}$ be as defined in § 5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105.

Define $\mathcal{K}^{\text{Orchard}} := \text{GroupHash}^{\mathbb{P}}(\text{"z.cash:Orchard"}, \text{"K"})$.

To avoid repetition, we define a function $\text{DeriveNullifier} : \mathbb{F}_{q_{\mathbb{P}}} \times \mathbb{F}_{q_{\mathbb{P}}} \times \mathbb{F}_{q_{\mathbb{P}}} \times \mathbb{P} \rightarrow \mathbb{F}_{q_{\mathbb{P}}}$ as follows:

$$\text{DeriveNullifier}_{\text{nk}}(\rho, \psi, \text{cm}) = \text{Extract}_{\mathbb{P}}\left(\left[(\text{PRF}_{\text{nk}}^{\text{nfOrchard}}(\rho) + \psi) \bmod q_{\mathbb{P}}\right] \mathcal{K}^{\text{Orchard}} + \text{cm}\right).$$

where nk is the *nullifier deriving key* associated with the *note*; ρ and ψ are part of the *note*; and cm is the *note commitment*.

Note: The addition of $\text{PRF}_{\text{nk}}^{\text{nfOrchard}}(\rho)$ and ψ is intentionally done modulo $q_{\mathbb{P}}$, even though the scalar multiplication is on the Pallas curve which has scalar field $\mathbb{F}_{r_{\mathbb{P}}}$.

Security requirement: For each shielded protocol, the requirements on *nullifier* derivation are as follows:

- The derived *nullifier* must be determined completely by the fields of the *note*, **and possibly its position**, in a way that can be checked in the corresponding statement that controls spends (i.e. the *JoinSplit statement*, *Spend statement*, or *Action statement*).
- Under the assumption that p values are unique, it must not be possible to generate two *notes* with distinct *note commitments* but the same *nullifier*. (See §8.4 ‘*Faerie Gold attack and fix*’ on p.140 for further discussion.)
- Given a set of *nullifiers* of *a priori* unknown *notes*, they must not be linkable to those *notes* with probability greater than expected by chance, even to an adversary with the corresponding *incoming viewing keys* (but not *full viewing keys*), and even if the adversary may have created the *notes*.

4.17 Chain Value Pool Balances

The *transparent chain value pool balance* for a given *block chain* is the sum of the values of all *UTXOs* in the *UTXO* (*unspent transaction output*) set for that chain. It is denoted by $\text{ChainValuePoolBalance}^{\text{Transparent}}(\text{height})$.

As defined in [ZIP-209], the ***Sprout*** *chain value pool balance* for a given *block chain* is the sum of all $v_{\text{pub}}^{\text{old}}$ field values for *transactions* in the *block chain*, minus the sum of all $v_{\text{pub}}^{\text{new}}$ field values for *transactions* in the *block chain*. It is denoted by $\text{ChainValuePoolBalance}^{\text{Sprout}}(\text{height})$.

Consensus rule: If the ***Sprout*** *chain value pool balance* would become negative in the *block chain* created as a result of accepting a *block*, then all nodes **MUST** reject the block as invalid.

As defined in [ZIP-209], the ***Sapling*** *chain value pool balance* for a given *block chain* is the negation of the sum of all $\text{valueBalanceSapling}$ values for *transactions* in the *block chain*. It is denoted by $\text{ChainValuePoolBalance}^{\text{Sapling}}(\text{height})$.

Consensus rule: If the ***Sapling*** *chain value pool balance* would become negative in the *block chain* created as a result of accepting a *block*, then all nodes **MUST** reject the block as invalid.

Similarly to the ***Sapling*** *chain value pool balance* defined in [ZIP-209], the ***Orchard*** *chain value pool balance* for a given *block chain* is the negation of the sum of all $\text{valueBalanceOrchard}$ field values for *transactions* in the *block chain*. It is denoted by $\text{ChainValuePoolBalance}^{\text{Orchard}}(\text{height})$.

Consensus rule: If the ***Orchard*** *chain value pool balance* would become negative in the *block chain* created as a result of accepting a *block*, then all nodes **MUST** reject the block as invalid.

Define $\text{totalDeferredOutput}$ as in §7.8 ‘*Calculation of Block Subsidy, Funding Streams, and Founders’ Reward*’ on p.133.

Then, consistent with [ZIP-207], the *deferred development fund chain value pool balance* for a *block chain* up to and including height height is given by $\text{ChainValuePoolBalance}^{\text{Deferred}}(\text{height}) := \sum_{h=0}^{\text{height}} \text{totalDeferredOutput}(h)$.

Non-normative notes:

- $\text{totalDeferredOutput}(h)$ is necessarily zero for heights h prior to **NU6** activation.
- Currently there is no way to withdraw from the *deferred development fund chain value pool*, so there is no possibility of it going negative. Therefore, no consensus rule to prevent that eventuality is needed at this time.

The *total issued supply* of a *block chain* at *block height* height is given by the function:

$$\begin{aligned} \text{IssuedSupply}(\text{height}) := & \text{ChainValuePoolBalance}^{\text{Transparent}}(\text{height}) \\ & + \text{ChainValuePoolBalance}^{\text{Sprout}}(\text{height}) \\ & + \text{ChainValuePoolBalance}^{\text{Sapling}}(\text{height}) \\ & + \text{ChainValuePoolBalance}^{\text{Orchard}}(\text{height}) \\ & + \text{ChainValuePoolBalance}^{\text{Deferred}}(\text{height}) \end{aligned}$$

4.18 Zk-SNARK Statements

4.18.1 JoinSplit Statement (Sprout)

Let $\ell_{\text{Merkle}}^{\text{Sprout}}, \ell_{\text{PRF}}^{\text{Sprout}}, \text{MerkleDepth}^{\text{Sprout}}, \ell_{\text{value}}, \ell_{\text{a}_{\text{sk}}}, \ell_{\varphi}^{\text{Sprout}}, \ell_{\text{hSig}}, N^{\text{old}}, N^{\text{new}}$ be as defined in §5.3 ‘Constants’ on p. 72.

Let $\text{PRF}^{\text{addr}}, \text{PRF}^{\text{nfSprout}}, \text{PRF}^{\text{pk}}$, and PRF^{φ} be as defined in §4.1.2 ‘Pseudo Random Functions’ on p. 24.

Let $\text{NoteCommit}^{\text{Sprout}}$ be as defined in §4.1.8 ‘Commitment’ on p. 29, and let $\text{Note}^{\text{Sprout}}$ and $\text{NoteCommitment}^{\text{Sprout}}$ be as defined in §3.2 ‘Notes’ on p. 13.

A valid instance of a *JoinSplit statement*, $\pi_{\text{ZKJoinSplit}}$, assures that given a *primary input*:

$$\begin{aligned} \text{rt}^{\text{Sprout}} & \circ \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]}, \\ \text{nf}_{1..N^{\text{old}}}^{\text{old}} & \circ \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}][N^{\text{old}}]}, \\ \text{cm}_{1..N^{\text{new}}}^{\text{new}} & \circ \text{NoteCommit}^{\text{Sprout}}.\text{Output}^{[N^{\text{new}}]}, \\ v_{\text{pub}}^{\text{old}} & \circ \{0 \dots 2^{\ell_{\text{value}}}-1\}, \\ v_{\text{pub}}^{\text{new}} & \circ \{0 \dots 2^{\ell_{\text{value}}}-1\}, \\ \text{h}_{\text{Sig}} & \circ \mathbb{B}^{[\ell_{\text{hSig}}]}, \\ \text{h}_{1..N^{\text{old}}} & \circ \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}][N^{\text{old}}]}), \end{aligned}$$

the prover knows an *auxiliary input*:

$$\begin{aligned} (\text{path}_{1..N^{\text{old}}} & \circ \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}][\text{MerkleDepth}^{\text{Sprout}}][N^{\text{old}}]}, \\ \text{pos}_{1..N^{\text{old}}} & \circ \{0 \dots 2^{\text{MerkleDepth}^{\text{Sprout}}}-1\}^{[N^{\text{old}}]}, \\ \text{n}_{1..N^{\text{old}}}^{\text{old}} & \circ \text{Note}^{\text{Sprout}}[N^{\text{old}}], \\ \text{a}_{\text{sk},1..N^{\text{old}}}^{\text{old}} & \circ \mathbb{B}^{[\ell_{\text{a}_{\text{sk}}}]^{[N^{\text{old}}]}}, \\ \text{n}_{1..N^{\text{new}}}^{\text{new}} & \circ \text{Note}^{\text{Sprout}}[N^{\text{new}}], \\ \varphi & \circ \mathbb{B}^{[\ell_{\varphi}^{\text{Sprout}}]}, \\ \text{enforceMerklePath}_{1..N^{\text{old}}} & \circ \mathbb{B}^{[N^{\text{old}}]}), \end{aligned}$$

where:

$$\begin{aligned} \text{for each } i \in \{1..N^{\text{old}}\}: \mathbf{n}_i^{\text{old}} &= (\mathbf{a}_{\text{pk},i}^{\text{old}}, v_i^{\text{old}}, \rho_i^{\text{old}}, \text{rcm}_i^{\text{old}}); \\ \text{for each } i \in \{1..N^{\text{new}}\}: \mathbf{n}_i^{\text{new}} &= (\mathbf{a}_{\text{pk},i}^{\text{new}}, v_i^{\text{new}}, \rho_i^{\text{new}}, \text{rcm}_i^{\text{new}}) \end{aligned}$$

such that the following conditions hold:

Merkle path validity for each $i \in \{1..N^{\text{old}}\}$ | $\text{enforceMerklePath}_i = 1$: $(\text{path}_i, \text{pos}_i)$ is a valid *Merkle path* (see §4.9 ‘Merkle Path Validity’ on p. 47) of depth $\text{MerkleDepth}^{\text{Sprout}}$ from $\text{NoteCommitment}^{\text{Sprout}}(\mathbf{n}_i^{\text{old}})$ to the anchor $\text{rt}^{\text{Sprout}}$.

Note: Merkle path validity covers conditions 1.(a) and 1.(d) of the NP *statement* in [BCGGMTV2014, section 4.2].

Merkle path enforcement for each $i \in \{1..N^{\text{old}}\}$, if $v_i^{\text{old}} \neq 0$ then $\text{enforceMerklePath}_i = 1$.

Balance $v_{\text{pub}}^{\text{old}} + \sum_{i=1}^{N^{\text{old}}} v_i^{\text{old}} = v_{\text{pub}}^{\text{new}} + \sum_{i=1}^{N^{\text{new}}} v_i^{\text{new}} \in \{0 \dots 2^{\ell_{\text{value}}}-1\}.$

Nullifier integrity for each $i \in \{1..N^{\text{old}}\}$: $\text{nf}_i^{\text{old}} = \text{PRF}_{\text{a}_{\text{sk},i}^{\text{old}}}^{\text{nfSprout}}(\rho_i^{\text{old}}).$

Spend authority for each $i \in \{1..N^{\text{old}}\}$: $\mathbf{a}_{\text{pk},i}^{\text{old}} = \text{PRF}_{\text{a}_{\text{sk},i}^{\text{old}}}^{\text{addr}}(0).$

Non-malleability for each $i \in \{1..N^{\text{old}}\}$: $\mathbf{h}_i = \text{PRF}_{\text{a}_{\text{sk},i}^{\text{old}}}^{\text{pk}}(i, \text{h}_{\text{Sig}}).$

Uniqueness of ρ_i^{new} for each $i \in \{1..N^{\text{new}}\}$: $\rho_i^{\text{new}} = \text{PRF}_{\varphi}^{\varphi}(i, \text{h}_{\text{Sig}}).$

Note commitment integrity for each $i \in \{1..N^{\text{new}}\}$: $\text{cm}_i^{\text{new}} = \text{NoteCommitment}^{\text{Sprout}}(\mathbf{n}_i^{\text{new}}).$

For details of the form and encoding of proofs, see §5.4.10.1 ‘BCTV14’ on p. 108.

$\text{SpendAuthSig}^{\text{Orchard}}.\text{RandomizePublic}$. In particular, the representation of $(\text{PRF}_{\text{nk}}^{\text{nfOrchard}}(\rho) + \psi) \bmod q_{\mathbb{P}}$ that is used for the scalar multiplication in DeriveNullifier **MUST** be checked to be canonical in order to avoid a potential *double-spend* vulnerability, and similarly for the representation of ivk in $[\text{ivk}]g_d^{\text{old}}$.

Non-normative notes:

- The procedure in §4.2.3 ‘**Orchard Key Components**’ on p. 37 will always produce a *spend authorization address key* that effectively has the compressed y -coordinate, \tilde{y} , set to 0. The *Action statement*, on the other hand, allows the prover to witness $\text{ak}^{\mathbb{P}}$ with \tilde{y} set to 0 or 1. This is harmless because if the prover and signer(s) of the *spend authorization signature* collectively know rsk and α , we can conclude that they collectively know ask up to sign, which is sufficient for spend authorization.
- There is intentionally no equivalent to the **Ephemeral public key integrity** check from the **Sapling Output statement**. It is unnecessary for the sender of an **Orchard note** to prove knowledge of esk , because the potential attack this originally addressed for **Sapling** is prevented by checks added at **Canopy** activation in [ZIP-212]. These checks are required after the end of the ZIP 212 grace period, which precedes **NU5** activation.
- If $\text{NoteCommit}^{\text{Orchard}}$ returns \perp for the old or new *note*, then the corresponding **note commitment integrity** check is satisfied. Similarly, if $\text{Commit}^{\text{ivk}}$ returns \perp , then the **diversified address integrity** check is satisfied. This models the fact that the implemented circuit uses incomplete point addition to compute $\text{SinsemillaHashToPoint}$. If an exceptional case were to occur, the prover could arbitrarily choose the intermediate λ value in an addition, which must be assumed to allow them to control the output. (The formal output of $\text{SinsemillaHashToPoint}$ is \perp in such a case, while the output computed by the circuit would be nondeterministic.) But as proven in Theorem 5.4.4 on p. 82, these exceptional cases allow immediately finding a nontrivial discrete logarithm relation. If the *Discrete Logarithm Problem* is hard on the Pallas curve, then finding such a case is infeasible.

4.19 In-band secret distribution (Sprout)

In **Sprout**, the secrets that need to be transmitted to a recipient of funds in order for them to later spend, are v , ρ , and rcm . A *memo field* (§3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14) is also transmitted.

To transmit these secrets securely to a recipient *without* requiring an *out-of-band* communication channel, the *transmission key* pk_{enc} is used to encrypt them. The recipient’s possession of the associated *incoming viewing key* ivk is used to reconstruct the original *note* and *memo field*.

A single *ephemeral public key* is shared between encryptions of the N^{new} *shielded outputs* in a *JoinSplit description*. All of the resulting ciphertexts are combined to form a *transmitted notes ciphertext*.

For both encryption and decryption,

- let Sym be the scheme instantiated in §5.4.3 ‘*Symmetric Encryption*’ on p. 86;
- let $\text{KDF}^{\text{Sprout}}$ be the *Key Derivation Function* instantiated in §5.4.5.2 ‘**Sprout Key Derivation**’ on p. 87;
- let $\text{KA}^{\text{Sprout}}$ be the *key agreement scheme* instantiated in §5.4.5.1 ‘**Sprout Key Agreement**’ on p. 86;
- let h_{sig} be the value computed for this *JoinSplit description* in §4.3 ‘*JoinSplit Descriptions*’ on p. 38.

4.19.1 Encryption (Sprout)

Let $\text{KA}^{\text{Sprout}}$ be the *key agreement scheme* instantiated in §5.4.5.1 ‘**Sprout Key Agreement**’ on p. 86.

Let $\text{pk}_{\text{enc}, 1..N^{\text{new}}}$ be the *transmission keys* for the intended recipient addresses of each new *note*.

Let $\text{np}_{1..N^{\text{new}}}$ be **Sprout note plaintexts** defined in §3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14.

Then to encrypt:

- Generate a new $\text{KA}^{\text{Sprout}}$ (public, private) key pair (epk, esk).
- For $i \in \{1..N^{\text{new}}\}$,
 - Let P_i^{enc} be the *raw encoding* of np_i .
 - Let $\text{sharedSecret}_i = \text{KA}^{\text{Sprout}}.\text{Agree}(\text{esk}, \text{pk}_{\text{enc},i})$.
 - Let $K_i^{\text{enc}} = \text{KDF}^{\text{Sprout}}(i, h_{\text{Sig}}, \text{sharedSecret}_i, \text{epk}, \text{pk}_{\text{enc},i})$.
 - Let $C_i^{\text{enc}} = \text{Sym.Encrypt}_{K_i^{\text{enc}}}(P_i^{\text{enc}})$.

The resulting *transmitted notes ciphertext* is (epk, $C_{1..N^{\text{new}}}^{\text{enc}}$).

Note: It is technically possible to replace C_i^{enc} for a given *note* with a random (and undecryptable) dummy ciphertext, relying instead on *out-of-band* transmission of the *note* to the recipient. In this case the ephemeral key **MUST** still be generated as a random *public key* (rather than a random *bit sequence*) to ensure indistinguishability from other *JoinSplit descriptions*. This mode of operation raises further security considerations, for example of how to validate a *Sprout note* received *out-of-band*, which are not addressed in this document.

4.19.2 Decryption (Sprout)

Let $\text{ivk} = (\text{a}_{\text{pk}}, \text{sk}_{\text{enc}})$ be the recipient's *incoming viewing key*, and let pk_{enc} be the corresponding *transmission key* derived from sk_{enc} as specified in § 4.2.1 '*Sprout Key Components*' on p. 35.

Let $\text{cm}_{1..N^{\text{new}}}$ be the *note commitments* of each output coin.

Then for each $i \in \{1..N^{\text{new}}\}$, the recipient will attempt to decrypt that ciphertext component (epk, C_i^{enc}) as follows:

```

let sharedSecreti = KASprout.Agree(skenc, epk)
let Kienc = KDFSprout(i, hSig, sharedSecreti, epk, pkenc)
return DecryptNoteSprout(Kienc, Cienc, cmi, apk).

```

$\text{DecryptNoteSprout}(K_i^{\text{enc}}, C_i^{\text{enc}}, \text{cm}_i, \text{a}_{\text{pk}})$ is defined as follows:

```

let Pienc = Sym.DecryptKienc(Cienc)
if Pienc = ⊥, return ⊥
extract npi = (leadBytei : BY, vi : {0..2ℓvalue}-1}, ρi : B[ℓPRF]Sprout], rcmi : NoteCommitSprout.Trapdoor, memoi : BY[512])
from Pienc
let ni = (apk, vi, ρi, rcmi)
if leadBytei ≠ 0x00 or NoteCommitSprout(ni) ≠ cmi, return ⊥
return (ni, memoi).

```

Notes:

- The decryption algorithm corresponds to step 3 (b) i. and ii. (first bullet point) of the Receive algorithm shown in [BCGGMTV2014, Figure 2].
- To test whether a *note* is unspent in a particular *block chain* also requires the *spending key* a_{sk} ; the coin is unspent if and only if $\text{nf} = \text{PRF}_{\text{a}_{\text{sk}}}^{\text{nfSprout}}(\rho)$ is not in the *nullifier set* for that *block chain*.
- A *note* can change from being unspent to spent as a node's view of the *best valid block chain* is extended by new *transactions*. Also, *block chain reorganizations* can cause a node to switch to a different *best valid block chain* that does not contain the *transaction* in which a *note* was output.

See § 8.7 '*In-band secret distribution*' on p. 143 for further discussion of the security and engineering rationale behind this encryption scheme.

Notes:

- g_d has already been computed when applying NoteCommitment, and need not be computed again.
- A previous version of this specification did not have the requirement for the decoded point pk_d of a **Sapling** note to be in the set of *prime-order* points $\mathbb{J}^{(r)*}$ (i.e. "if ... $pk_d \notin \mathbb{J}^{(r)*}$, return \perp "). That did not match the implementation in zcashd. In fact the history is a little more complicated. The current specification matches the implementation in librustzcash as of [librustzcash-109], which has been used in zcashd since zcashd v2.1.2. However, there was another implementation of **Sapling** note decryption used in zcashd for consensus checks, specifically the check that a *shielded* coinbase output decrypts successfully with the zero ovk. This was corrected to enforce the same restriction on the decrypted pk_d in zcashd v5.5.0, originally set to activate in a soft fork at *block height* 2121200 on both *Mainnet* and *Testnet* [zcashd-6459]. (On *Testnet* this height was in the past as of the zcashd v5.5.0 release, and so the change would have been immediately enforced on upgrade.) Since the soft fork was observed to be retrospectively valid after that height, the implementation was simplified in [zcashd-6725] to use the librustzcash implementation in all cases, which reflects the specification above. zebra always used the librustzcash implementation.
- As explained in the note in § 5.4.9.3 ‘Jubjub’ on p. 100, $abst_j$ accepts *non-canonical* compressed encodings of Jubjub curve points. Therefore, an implementation **MUST** use the original ephemeralKey field as encoded in the *transaction* as input to PRF^{ock} and $KDF^{Sapling}$, and in the comparison against $repr_G(KA^{Sapling}.DerivePublic(esk, g_d))$. For consistency this is also what is specified for **Orchard**.
- For **Sapling** outgoing ciphertexts, pk_{\star_d} could also be *non-canonical*. After **NU5** activation, the above algorithm explicitly returns \perp if $repr_P(pk_d) \neq pk_{\star_d}$. However, this is technically redundant with the later check that returns \perp if $pk_d \notin \mathbb{J}^{(r)*}$, because only *small-order* Jubjub curve points have *non-canonical* encodings. This check is enforced retrospectively for consensus by current zcashd and zebra versions, and for wallet rescanning by current zcashd. Versions of zcashd prior to [zcashd-6725] could however have accepted notes for which the *outgoing ciphertext* contains either a canonical or a *non-canonical* encoding of \mathcal{O}_J for pk_d .
- [NU5 onward] For **Orchard** outgoing ciphertexts, it is not possible for pk_{\star_d} to be *non-canonical*.
- [NU5 onward] The domain separators [4] and [5] used in the input to PRF_{rseed}^{expand} are swapped for **Orchard** relative to **Sapling**. This was due to an oversight and there is no good reason for it.
- The comments in § 4.20.2 ‘Decryption using an Incoming Viewing Key (**Sapling** and **Orchard**)’ on p. 66 concerning calculation of ρ , detection of spent notes, and decryption of *transmitted note ciphertexts* for *transactions* in the *mempool* also apply to notes decrypted by this procedure.

Non-normative note: Implementors should pay close attention to similarities and differences between this procedure and § 4.20.2 ‘Decryption using an Incoming Viewing Key (**Sapling** and **Orchard**)’ on p. 66, especially that:

- in this procedure, the ephemeral *private key* esk' derived from $rseed$ is checked to be identical to that obtained from **op** (when $leadByte \neq 0x01$);
- in this procedure, pk_d is obtained from **op** rather than being derived as $KA^{Sapling}.DerivePublic(ivk, g_d)$;
- in this procedure, the check that $KA^{Sapling}.DerivePublic(esk, g_d) = epk$ is unconditional rather than being dependent on $leadByte \neq 0x01$, and it uses the esk obtained from **op**;
- [NU5 onward] for the same reason as in § 4.20.2 on p. 66, epk cannot be \mathcal{O}_P .

4.21 Block Chain Scanning (Sprout)

Let ℓ_{PRF}^{Sprout} be as defined in § 5.3 ‘Constants’ on p. 72.

Let $Note^{Sprout}$ be as defined in § 3.2 ‘Notes’ on p. 13.

Let KA^{Sprout} be as defined in § 5.4.5.1 ‘**Sprout** Key Agreement’ on p. 86.

Let $ivk = (a_{pk} : \mathbb{B}^{[\ell_{PRF}^{Sprout}]}, sk_{enc} : KA^{Sprout}.Private)$ be the *incoming viewing key* corresponding to a_{sk} , and let pk_{enc} be the associated *transmission key*, as specified in § 4.2.1 ‘**Sprout** Key Components’ on p. 35.

The following algorithm can be used, given the *block chain* and a **Sprout** *spending key* a_{sk} , to obtain each *note* sent to the corresponding *shielded payment address*, its *memo field*, and its final status (spent or unspent).

```

let mutable ReceivedSet :  $\mathcal{P}(\text{Note}^{\text{Sprout}} \times \mathbb{B}^{\mathbb{Y}[512]}) \leftarrow \{\}$ 
let mutable SpentSet :  $\mathcal{P}(\text{Note}^{\text{Sprout}}) \leftarrow \{\}$ 
let mutable NullifierMap :  $\mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]} \rightarrow \text{Note}^{\text{Sprout}} \leftarrow$  the empty mapping

for each transaction tx:
  for each JoinSplit description in tx:
    let  $(\text{epk}, C_{1..N}^{\text{enc}})$  be the transmitted notes ciphertext of the JoinSplit description
    for  $i$  in  $1..N^{\text{new}}$ :
      Attempt to decrypt the transmitted notes ciphertext component  $(\text{epk}, C_i^{\text{enc}})$  using  $\text{ivk}$  with the
      algorithm in § 4.19.2 ‘Decryption (Sprout)’ on p. 64. If this succeeds with  $(\mathbf{n}, \text{memo})$ :
        Add  $(\mathbf{n}, \text{memo})$  to ReceivedSet.
        Calculate the nullifier  $\text{nf}$  of  $\mathbf{n}$  using  $a_{sk}$  as in § 4.16 ‘Computing  $\rho$  values and Nullifiers’ on p. 56.
        Add the mapping  $\text{nf} \rightarrow \mathbf{n}$  to NullifierMap.
    let  $\text{nf}_{1..N^{\text{old}}}$  be the nullifiers of the JoinSplit description
    for  $i$  in  $1..N^{\text{old}}$ : if  $\text{nf}_i$  is present in NullifierMap, add NullifierMap( $\text{nf}_i$ ) to SpentSet

return (ReceivedSet, SpentSet).

```

4.22 Block Chain Scanning (Sapling and Orchard)

In **Sapling** and **Orchard**, *block chain* scanning requires only the nk and ivk key components, rather than a *spending key* as in **Sprout**. Typically, these components are derived from a *full viewing key* (§ 4.2.2 ‘**Sapling Key Components**’ on p. 35 or § 4.2.3 ‘**Orchard Key Components**’ on p. 37).

Let $\ell_{\text{PRF}\text{FntSapling}}$ be as defined in § 5.3 ‘**Constants**’ on p. 72.

Let Note be $\text{Note}^{\text{Sapling}}$ or $\text{Note}^{\text{Orchard}}$ as defined in § 3.2 ‘**Notes**’ on p. 13.

Let KA be either $\text{KA}^{\text{Sapling}}$ as defined in § 5.4.5.3 on p. 87, or $\text{KA}^{\text{Orchard}}$ as defined in § 5.4.5.5 on p. 88.

Let NullifierType be $\mathbb{B}^{\mathbb{Y}[\ell_{\text{PRF}\text{FntSapling}}/8]}$ for **Sapling**, or \mathbb{F}_{q_F} for **Orchard**.

The following algorithm can be used, given the *block chain* and (nk, ivk) , to obtain each *note* sent to the corresponding *shielded payment address*, its *memo field*, and its final status (spent or unspent).

```

let mutable ReceivedSet :  $\mathcal{P}(\text{Note} \times \mathbb{B}^{\mathbb{Y}[512]}) \leftarrow \{\}$ 
let mutable SpentSet :  $\mathcal{P}(\text{Note}) \leftarrow \{\}$ 
let mutable NullifierMap :  $(\text{NullifierType} \rightarrow \text{Note}) \leftarrow$  the empty mapping

for each transaction tx:
  for each Output description or Action description in tx:
    Attempt to decrypt the transmitted note ciphertext components  $\text{epk}$  and  $C^{\text{enc}}$  using  $\text{ivk}$  with the algorithm
    § 4.20.2 ‘Decryption using an Incoming Viewing Key (Sapling and Orchard)’ on p. 66.
    If this succeeds with  $(\mathbf{n}, \text{memo})$ :
      Add  $(\mathbf{n}, \text{memo})$  to ReceivedSet.
      Calculate the nullifier  $\text{nf}$  of  $\mathbf{n}$  using  $\text{nk}$  as in § 4.16 ‘Computing  $\rho$  values and Nullifiers’ on p. 56.
      (This also requires  $\text{pos}$  from the Output description for Sapling notes.)
      Add the mapping  $\text{nf} \rightarrow \mathbf{n}$  to NullifierMap.
  for each nullifier  $\text{nf}$  of a Spend description or Action description in tx:
    if  $\text{nf}$  is present in NullifierMap, add NullifierMap( $\text{nf}$ ) to SpentSet

return (ReceivedSet, SpentSet).

```

Non-normative notes:

- The above algorithm does not use the `ovk` key component, or the C^{out} *transmitted note ciphertext* component. When scanning the whole *block chain*, these are indeed not necessary. The advantage of supporting decryption using `ovk` as described in §4.20.3 ‘*Decryption using a Full Viewing Key (Sapling and Orchard)*’ on p. 68, is that it allows recovering information about the *note plaintexts* sent in a *transaction* from that *transaction* alone.
- When scanning only part of a *block chain*, it may be useful to augment the above algorithm with decryption of C^{out} components for each *transaction*, in order to obtain information about *notes* that were spent in the scanned period but received outside it.
- The above algorithm does not detect *notes* that were sent “out-of-band” or with incorrect *transmitted note ciphertexts*. It is possible to detect whether such *notes* were spent only if their *nullifiers* are known.

5 Concrete Protocol

5.1 Integers, Bit Sequences, and Endianness

All integers in **Zcash**-specific encodings are unsigned, have a fixed bit length, and are encoded in little-endian byte order *unless otherwise specified*.

The following functions convert between sequences of bits, sequences of bytes, and integers:

- $\text{I2LEBSP} : (\ell : \mathbb{N}) \times \{0 \dots 2^\ell - 1\} \rightarrow \mathbb{B}^{[\ell]}$, such that $\text{I2LEBSP}_\ell(x)$ is the sequence of ℓ bits representing x in little-endian order;
- $\text{I2LEOSP} : (\ell : \mathbb{N}) \times \{0 \dots 2^\ell - 1\} \rightarrow \mathbb{B}^{\lceil \ell/8 \rceil}$, such that $\text{I2LEOSP}_\ell(x)$ is the sequence of $\lceil \ell/8 \rceil$ bytes representing x in little-endian order;
- $\text{I2BEBSP} : (\ell : \mathbb{N}) \times \{0 \dots 2^\ell - 1\} \rightarrow \mathbb{B}^{[\ell]}$ such that $\text{I2BEBSP}_\ell(x)$ is the sequence of ℓ bits representing x in big-endian order.
- $\text{LEBS2IP} : (\ell : \mathbb{N}) \times \mathbb{B}^{[\ell]} \rightarrow \{0 \dots 2^\ell - 1\}$ such that $\text{LEBS2IP}_\ell(S)$ is the integer represented in little-endian order by the *bit sequence* S of length ℓ .
- $\text{LEOS2IP} : (\ell : \mathbb{N} \mid \ell \bmod 8 = 0) \times \mathbb{B}^{\lceil \ell/8 \rceil} \rightarrow \{0 \dots 2^\ell - 1\}$ such that $\text{LEOS2IP}_\ell(S)$ is the integer represented in little-endian order by the *byte sequence* S of length $\ell/8$.
- $\text{BEOS2IP} : (\ell : \mathbb{N} \mid \ell \bmod 8 = 0) \times \mathbb{B}^{\lceil \ell/8 \rceil} \rightarrow \{0 \dots 2^\ell - 1\}$ such that $\text{BEOS2IP}_\ell(S)$ is the integer represented in big-endian order by the *byte sequence* S of length $\ell/8$.
- $\text{LEBS2OSP} : (\ell : \mathbb{N}) \times \mathbb{B}^{[\ell]} \rightarrow \mathbb{B}^{\lceil \ell/8 \rceil}$ defined as follows: pad the input on the right with $8 \cdot \lceil \ell/8 \rceil - \ell$ zero bits so that its length is a multiple of 8 bits. Then convert each group of 8 bits to a byte value with the *least* significant bit first, and concatenate the resulting bytes in the same order as the groups.
- $\text{LEOS2BSP} : (\ell : \mathbb{N} \mid \ell \bmod 8 = 0) \times \mathbb{B}^{\lceil \ell/8 \rceil} \rightarrow \mathbb{B}^{[\ell]}$ defined as follows: convert each byte to a group of 8 bits with the *least* significant bit first, and concatenate the resulting groups in the same order as the bytes.

5.2 Bit layout diagrams

We sometimes use bit layout diagrams, in which each box of the diagram represents a sequence of bits. Diagrams are read from left to right, with lines read from top to bottom; the breaking of boxes across lines has no significance. The bit length ℓ is given explicitly in each box, except when it is obvious (e.g. for a single bit, or for the notation $[0]^\ell$ representing the sequence of ℓ zero bits, or for the output of LEBS2OSP_ℓ).

The entire diagram represents the sequence of *bytes* formed by first concatenating these *bit sequences*, and then treating each subsequence of 8 bits as a byte with the bits ordered from *most significant* to *least significant*. Thus the *most significant* bit in each byte is toward the left of a diagram. (This convention is used only in descriptions of the **Sprout** design; in the **Sapling** and **Orchard** additions, *bit sequence/byte sequence conversions are always specified explicitly*.) Where bit fields are used, the text will clarify their position in each case.

5.3 Constants

Define:

$$\begin{aligned}
 \text{MerkleDepth}^{\text{Sprout}} &: \mathbb{N} := 29 \\
 \text{MerkleDepth}^{\text{Sapling}} &: \mathbb{N} := 32 \\
 \text{MerkleDepth}^{\text{Orchard}} &: \mathbb{N} := 32 \\
 \ell_{\text{Merkle}}^{\text{Sprout}} &: \mathbb{N} := 256 \\
 \ell_{\text{Merkle}}^{\text{Sapling}} &: \mathbb{N} := 255 \\
 \ell_{\text{Merkle}}^{\text{Orchard}} &: \mathbb{N} := 255 \\
 N^{\text{old}} &: \mathbb{N} := 2 \\
 N^{\text{new}} &: \mathbb{N} := 2 \\
 \ell_{\text{value}} &: \mathbb{N} := 64 \\
 \ell_{\text{hSig}} &: \mathbb{N} := 256 \\
 \ell_{\text{PRF}}^{\text{Sprout}} &: \mathbb{N} := 256 \\
 \ell_{\text{PRFexpand}} &: \mathbb{N} := 512 \\
 \ell_{\text{PRFntSapling}} &: \mathbb{N} := 256 \\
 \ell_{\text{rcm}}^{\text{Sprout}} &: \mathbb{N} := 256 \\
 \ell_{\text{Seed}} &: \mathbb{N} := 256 \\
 \ell_{\text{a}_{\text{sk}}} &: \mathbb{N} := 252 \\
 \ell_{\varphi}^{\text{Sprout}} &: \mathbb{N} := 252 \\
 \ell_{\text{sk}} &: \mathbb{N} := 256 \\
 \ell_{\text{d}} &: \mathbb{N} := 88 \\
 \ell_{\text{dk}} &: \mathbb{N} := 256 \\
 \ell_{\text{ivk}}^{\text{Sapling}} &: \mathbb{N} := 251 \\
 \ell_{\text{ovk}} &: \mathbb{N} := 256 \\
 \ell_{\text{scalar}}^{\text{Sapling}} &: \mathbb{N} := 252 \\
 \ell_{\text{scalar}}^{\text{Orchard}} &: \mathbb{N} := 255 \\
 \ell_{\text{base}}^{\text{Orchard}} &: \mathbb{N} := 255 \\
 \text{Uncommitted}^{\text{Sprout}} &: \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} := [0]^{\ell_{\text{Merkle}}^{\text{Sprout}}} \\
 \text{Uncommitted}^{\text{Sapling}} &: \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} := \text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}(1) \\
 \text{Uncommitted}^{\text{Orchard}} &: \{0 \dots q_{\mathbb{P}} - 1\} := 2 \\
 \text{MAX_MONEY} &: \mathbb{N} := 2.1 \cdot 10^{15} \text{ (zatoshi)} \\
 \text{BlossomActivationHeight} &: \mathbb{N} := \begin{cases} 653600, & \text{for Mainnet} \\ 584000, & \text{for Testnet} \end{cases} \\
 \text{CanopyActivationHeight} &: \mathbb{N} := \begin{cases} 1046400, & \text{for Mainnet} \\ 1028500, & \text{for Testnet} \end{cases} \\
 \text{ZIP212GracePeriod} &: \mathbb{N} := 32256 \\
 \text{NUFiveActivationHeight} &: \mathbb{N} := \begin{cases} 1687104, & \text{for Mainnet} \\ 1842420, & \text{for Testnet} \end{cases}
 \end{aligned}$$

$\text{SlowStartInterval} : \mathbb{N} := 20000$
 $\text{PreBlossomHalvingInterval} : \mathbb{N} := 840000$
 $\text{MaxBlockSubsidy} : \mathbb{N} := 1.25 \cdot 10^9$ (*zatoshi*)
 $\text{NumFounderAddresses} : \mathbb{N} := 48$
 $\text{FoundersFraction} : \mathbb{Q} := \frac{1}{5}$
 $\text{PoWLimit} : \mathbb{N} := \begin{cases} 2^{243} - 1, & \text{for Mainnet} \\ 2^{251} - 1, & \text{for Testnet} \end{cases}$
 $\text{PoWAveragingWindow} : \mathbb{N} := 17$
 $\text{PoWMedianBlockSpan} : \mathbb{N} := 11$
 $\text{PoWMaxAdjustDown} : \mathbb{Q} := \frac{32}{100}$
 $\text{PoWMaxAdjustUp} : \mathbb{Q} := \frac{16}{100}$
 $\text{PoWDampingFactor} : \mathbb{N} := 4$
 $\text{PreBlossomPoWTargetSpacing} : \mathbb{N} := 150$ (seconds).
 $\text{PostBlossomPoWTargetSpacing} : \mathbb{N} := 75$ (seconds).

5.4 Concrete Cryptographic Schemes

5.4.1 Hash Functions

5.4.1.1 SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions

SHA-256 and SHA-512 are defined by [NIST2015].

Zcash uses the full SHA-256 *hash function* to instantiate $\text{NoteCommitment}^{\text{Sprout}}$.

$$\text{SHA-256} : \mathbb{B}^{\mathbb{N}} \rightarrow \mathbb{B}^{[32]}$$

[NIST2015] strictly speaking only specifies the application of SHA-256 to messages that are *bit sequences*, producing outputs (“message digests”) that are also *bit sequences*. In practice, SHA-256 is universally implemented with a byte-sequence interface for messages and outputs, such that the *most significant* bit of each byte corresponds to the first bit of the associated *bit sequence*. (In the NIST specification “first” is conflated with “leftmost”)

SHA-256d, defined as a double application of SHA-256, is used to hash *block headers*:

$$\text{SHA-256d} : \mathbb{B}^{\mathbb{N}} \rightarrow \mathbb{B}^{[32]}$$

Zcash also uses the SHA-256 compression function, SHA256Compress . This operates on a single 512-bit block and *excludes* the padding step specified in [NIST2015, section 5.1].

That is, the input to SHA256Compress is what [NIST2015, section 5.2] refers to as “the message and its padding”. The Initial Hash Value is the same as for full SHA-256.

SHA256Compress is used to instantiate several *Pseudo Random Functions* and $\text{MerkleCRH}^{\text{Sprout}}$.

$$\text{SHA256Compress} : \mathbb{B}^{[512]} \rightarrow \mathbb{B}^{[256]}$$

The ordering of bits within words in the interface to SHA256Compress is consistent with [NIST2015, section 3.1], i.e. big-endian.

Ed25519 uses SHA-512:

$$\text{SHA-512} : \mathbb{B}^{\mathbb{N}} \rightarrow \mathbb{B}^{[64]}$$

The comment above concerning bit vs byte-sequence interfaces also applies to SHA-512.

5.4.1.2 BLAKE2 Hash Functions

BLAKE2 is defined by [ANWW2013]. **Zcash** uses both the BLAKE2b and BLAKE2s variants.

BLAKE2b- $\ell(p, x)$ refers to unkeyed BLAKE2b- ℓ in sequential mode, with an output digest length of $\ell/8$ bytes, 16-byte personalization string p , and input x .

BLAKE2b is used to instantiate hSigCRH, EquihashGen, and KDF^{Sprout}. From **Overwinter** onward, it is used to compute *SIGHASH transaction hashes* as specified in [ZIP-143], or as in [ZIP-243] after **Sapling** activation, or as in [ZIP-244] for version 5 transactions. For **Sapling**, it is also used to instantiate PRF^{expand}, PRF^{ockSapling}, KDF^{Sapling}, and in the *RedJubjub signature scheme* which instantiates SpendAuthSig^{Sapling} and BindingSig^{Sapling}.

$$\text{BLAKE2b-}\ell : \mathbb{BY}^{[16]} \times \mathbb{BY}^{[N]} \rightarrow \mathbb{BY}^{[\ell/8]}$$

Note: BLAKE2b- ℓ is not the same as BLAKE2b-512 truncated to ℓ bits, because the digest length is encoded in the parameter block.

BLAKE2s- $\ell(p, x)$ refers to unkeyed BLAKE2s- ℓ in sequential mode, with an output digest length of $\ell/8$ bytes, 8-byte personalization string p , and input x .

BLAKE2s is used to instantiate PRF^{nfSapling}, CRH^{ivk}, and GroupHash ^{$\mathbb{J}^{(r)*}$} .

$$\text{BLAKE2s-}\ell : \mathbb{BY}^{[8]} \times \mathbb{BY}^{[N]} \rightarrow \mathbb{BY}^{[\ell/8]}$$

5.4.1.3 Merkle Tree Hash Function

MerkleCRH^{Sprout} and MerkleCRH^{Sapling} and MerkleCRH^{Orchard} are used to hash *incremental Merkle tree hash values* for **Sprout** and **Sapling** and **Orchard** respectively.

MerkleCRH^{Sprout} Hash Function

MerkleCRH^{Sprout} : $\{0 \dots \text{MerkleDepth}^{\text{Sprout}} - 1\} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]} \rightarrow \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sprout}}]}$ is defined as follows:

$$\text{MerkleCRH}^{\text{Sprout}}(\text{layer}, \text{left}\star, \text{right}\star) := \text{SHA256Compress} \left(\begin{array}{|c|c|} \hline 256\text{-bit left}\star & 256\text{-bit right}\star \\ \hline \end{array} \right).$$

SHA256Compress is defined in § 5.4.1.1 ‘SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions’ on p. 73.

Security requirement: SHA256Compress must be *collision-resistant*, and it must be infeasible to find a preimage x such that $\text{SHA256Compress}(x) = [0]^{256}$.

Notes:

- The layer argument does not affect the output.
- SHA256Compress is not the same as the SHA-256 function, which hashes arbitrary-length *byte sequences*.

MerkleCRH^{Sapling} Hash Function

Let PedersenHash be as specified in § 5.4.1.7 ‘Pedersen Hash Function’ on p. 77.

MerkleCRH^{Sapling} : $\{0 \dots \text{MerkleDepth}^{\text{Sapling}} - 1\} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} \times \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]} \rightarrow \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]}$ is defined as follows:

$$\text{MerkleCRH}^{\text{Sapling}}(\text{layer}, \text{left}\star, \text{right}\star) := \text{PedersenHash}(\text{"Zcash_PH"}, l\star || \text{left}\star || \text{right}\star)$$

$$\text{where } l\star = \text{I2LEBSP}_6(\text{MerkleDepth}^{\text{Sapling}} - 1 - \text{layer}).$$

Security requirement: PedersenHash must be *collision-resistant*.

Note: The prefix $l\star$ provides domain separation between inputs at different layers of the *note commitment tree*. $\text{NoteCommit}^{\text{Sapling}}$, like PedersenHash , is defined in terms of $\text{PedersenHashToPoint}$, but using a prefix that cannot collide with a layer prefix, as noted in § 5.4.8.2 ‘*Windowed Pedersen commitments*’ on p. 94.

MerkleCRH^{Orchard} Hash Function

Let SinsemillaHash be as specified in § 5.4.1.9 ‘*Sinsemilla Hash Function*’ on p. 79.

$\text{MerkleCRH}^{\text{Orchard}} : \{0 \dots \text{MerkleDepth}^{\text{Orchard}} - 1\} \times \{0 \dots q_{\mathbb{P}} - 1\} \times \{0 \dots q_{\mathbb{P}} - 1\} \rightarrow \{0 \dots q_{\mathbb{P}} - 1\}$ is defined as follows:

$$\text{MerkleCRH}^{\text{Orchard}}(\text{layer}, \text{left}, \text{right}) := \begin{cases} 0, & \text{if hash} = \perp \\ \text{hash}, & \text{otherwise} \end{cases}$$

where $\text{hash} = \text{SinsemillaHash}(\text{"z.cash:Orchard-MerkleCRH"}, l\star \parallel \text{left}\star \parallel \text{right}\star)$

$$l\star = \text{I2LEBSP}_{10}(\text{MerkleDepth}^{\text{Orchard}} - 1 - \text{layer})$$

$$\text{left}\star = \text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Orchard}}}(\text{left})$$

$$\text{right}\star = \text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Orchard}}}(\text{right}).$$

Security requirements:

- SinsemillaHash must be *collision-resistant*.
- It must be infeasible to find a input of length $10 + 2 \cdot \ell_{\text{Merkle}}^{\text{Orchard}}$ bits to SinsemillaHash that yields output \perp .

Note: The prefix $l\star$ provides domain separation between inputs at different layers of the *note commitment tree*.

5.4.1.4 h_{Sig} Hash Function

$h_{\text{Sig}}\text{CRH}$ is used to compute the value h_{Sig} in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38.

$$h_{\text{Sig}}\text{CRH}(\text{randomSeed}, \text{nf}_{1 \dots N^{\text{old}}}^{\text{old}}, \text{joinSplitPubKey}) := \text{BLAKE2b-256}(\text{"ZcashComputehSig"}, h_{\text{Sig}}\text{Input})$$

where

$$h_{\text{Sig}}\text{Input} := \begin{array}{|c|c|c|c|} \hline 256\text{-bit randomSeed} & 256\text{-bit nf}_1^{\text{old}} & \dots & 256\text{-bit nf}_{N^{\text{old}}}^{\text{old}} & 256\text{-bit joinSplitPubKey} \\ \hline \end{array}.$$

$\text{BLAKE2b-256}(p, x)$ is defined in § 5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74.

Security requirement: $\text{BLAKE2b-256}(\text{"ZcashComputehSig"}, x)$ must be *collision-resistant* on x .

5.4.1.5 CRH^{ivk} Hash Function

CRH^{ivk} is used to derive the *incoming viewing key* ivk for a **Sapling** shielded payment address. For its use when generating an address see § 4.2.2 ‘*Sapling Key Components*’ on p. 35, and for its use in the *Spend statement* see § 4.18.2 ‘*Spend Statement (Sapling)*’ on p. 59.

It is defined as follows:

$$\text{CRH}^{\text{ivk}}(\text{ak}\star, \text{nk}\star) := \text{LEOS2IP}_{256}(\text{BLAKE2s-256}(\text{"Zcashivk"}, \text{crhInput})) \bmod 2^{\ell_{\text{ivk}}^{\text{Sapling}}}$$

where

$$\text{crhInput} := \begin{array}{|c|c|} \hline \text{LEBS2OSP}_{256}(\text{ak}\star) & \text{LEBS2OSP}_{256}(\text{nk}\star) \\ \hline \end{array}$$

$\text{BLAKE2s-256}(p, x)$ is defined in § 5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74.

Non-normative notes:

- The choice of MDS matrix and the number of rounds take into account cryptanalytic results in [KR2020] and [BCD+2020]. A detailed analysis of related matrix properties is given in [GRS2020].
- [BCD+2020] says that “... finite fields \mathbb{F}_q with a limited number of multiplicative subgroups might be preferable, i.e. one might want to avoid $q-1$ being smooth. This implies that the fields which are suitable for implementing FFT may be more vulnerable to integral attacks.” \mathbb{F}_{q_p} is such a field; the factorization of $q_p - 1$ is $2^{32} \cdot 3 \cdot 463 \cdot 539204044132271846773 \cdot 8999194758858563409123804352480028797519453$.

Furthermore, previous cryptanalysis of Poseidon has focussed mainly on the case of S-box $x \mapsto x^3$. That variant cannot be used in \mathbb{F}_{q_p} because $x \mapsto x^3$ would not be a permutation. $\alpha = 5$ is the smallest integer for which $x \mapsto x^\alpha$ is a permutation in \mathbb{F}_{q_p} .

On the other hand, the number of rounds chosen includes a significant security margin, even taking into account these considerations. For small t , such as $t = 3$ as used here, the results of [KR2020] are positive for security since they indicate that the number of active S-boxes through the middle rounds is larger than originally estimated by the Poseidon designers (and the number of rounds is based on this original conservative estimate).

Also note that the use of Poseidon in **Orchard** is very conservative. First, the sponge mode limits an adversary to only being able to influence part of the Poseidon permutation input, and we use it only to construct a PRF ($\text{PRF}^{\text{nfOrchard}}$ as described in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84). Half of the sponge input is a random key nk , known only to holders of a *full viewing key*, and the remaining half p comes from a previous *nullifier* which is effectively a random *affine-short-Weierstrass* x -coordinate on the Pallas curve. Then the PRF is used to enhance the security of a discrete-logarithm-based nullifier construction (described in [Zcash-Orchard, Section 3.5 Nullifiers]) against a potential discrete-log-breaking adversary. Given the weak assumption that the PoseidonHash sponge produces output that preserves sufficient entropy from the inputs nk and p , this nullifier construction would still be secure under a *Decisional Diffie-Hellman* assumption on the Pallas curve, even if the Poseidon-based PRF were distinguishable from an ideal PRF.

- The constant 2^{65} comes from [GKRRS2019, section 4.2]: “Constant-Input-Length Hashing. The capacity value is $\text{length} \cdot (2^{64}) + (o - 1)$ where o is the output length.” In this case the input length (*length*) is 2 field elements, and the output length is 1 field element.

5.4.1.11 Equihash Generator

$\text{EquihashGen}_{n,k}$ is a specialized *hash function* that maps an input and an index to an output of length n bits. It is used in § 7.7.1 ‘*Equihash*’ on p. 130.

Let $\text{powtag} := \boxed{\text{64-bit “ZcashPoW”}} \boxed{\text{32-bit } n} \boxed{\text{32-bit } k}$.

Let $\text{powcount}(g) := \boxed{\text{32-bit } g}$.

Let $\text{EquihashGen}_{n,k}(S, i) := T_{h+1 \dots h+n}$, where

$$m = \text{floor}\left(\frac{512}{n}\right);$$

$$h = (i - 1 \bmod m) \cdot n;$$

$$T = \text{BLAKE2b}-(n \cdot m)(\text{powtag}, S \parallel \text{powcount}(\text{floor}\left(\frac{i-1}{m}\right))).$$

Indices of bits in T are 1-based.

$\text{BLAKE2b-}\ell(p, x)$ is defined in § 5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74.

Security requirement: $\text{BLAKE2b-}\ell(\text{powtag}, x)$ must generate output that is sufficiently unpredictable to avoid short-cuts to the *Equihash* solution process. It would suffice to model it as a *random oracle*.

Note: When *EquihashGen* is evaluated for sequential indices, as in the *Equihash* solving process (§7.7.1 ‘*Equihash*’ on p. 130), the number of calls to *BLAKE2b* can be reduced by a factor of $\text{floor}(\frac{512}{n})$ in the best case (which is a factor of 2 for $n = 200$).

5.4.2 Pseudo Random Functions

Let *SHA256Compress* be as given in §5.4.1.1 ‘*SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions*’ on p. 73.

The *Pseudo Random Functions* PRF^{addr} , $\text{PRF}^{\text{nfSprout}}$, PRF^{pk} , and PRF^{p} from §4.1.2 ‘*Pseudo Random Functions*’ on p. 24, are all instantiated using *SHA256Compress*:

$$\begin{aligned} \text{PRF}_x^{\text{addr}}(t) &:= \text{SHA256Compress} \left(\begin{array}{|c|c|c|c|} \hline 1 & 1 & 0 & 0 \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 252\text{-bit } x \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 8\text{-bit } t \\ \hline \end{array} \parallel \begin{array}{|c|} \hline [0]^{248} \\ \hline \end{array} \right) \\ \text{PRF}_{a_{\text{sk}}}^{\text{nfSprout}}(\rho) &:= \text{SHA256Compress} \left(\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 0 \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 252\text{-bit } a_{\text{sk}} \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 256\text{-bit } \rho \\ \hline \end{array} \right) \\ \text{PRF}_{a_{\text{sk}}}^{\text{pk}}(i, h_{\text{sig}}) &:= \text{SHA256Compress} \left(\begin{array}{|c|c|c|c|} \hline 0 & i-1 & 0 & 0 \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 252\text{-bit } a_{\text{sk}} \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 256\text{-bit } h_{\text{sig}} \\ \hline \end{array} \right) \\ \text{PRF}_{\varphi}^{\text{p}}(i, h_{\text{sig}}) &:= \text{SHA256Compress} \left(\begin{array}{|c|c|c|c|} \hline 0 & i-1 & 1 & 0 \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 252\text{-bit } \varphi \\ \hline \end{array} \parallel \begin{array}{|c|} \hline 256\text{-bit } h_{\text{sig}} \\ \hline \end{array} \right) \end{aligned}$$

Security requirements:

- *SHA256Compress* must be *collision-resistant*.
- *SHA256Compress* must be a *PRF* when keyed by the bits corresponding to x , a_{sk} or φ in the above diagrams, with input in the remaining bits.

Note: The first four bits –i.e. the most significant four bits of the first byte– are used to separate distinct uses of *SHA256Compress*, ensuring that the functions are independent. As well as the inputs shown here, bits 1011 in this position are used to distinguish uses of the full *SHA-256* hash function; see §5.4.8.1 ‘*Sprout Note Commitments*’ on p. 93.

(The specific bit patterns chosen here were motivated by the possibility of future extensions that might have increased N^{old} and/or N^{new} to 3, or added an additional bit to a_{sk} to encode a new key type, or that would have required an additional *PRF*. In fact since **Sapling switches to non-SHA256Compress-based cryptographic primitives, these extensions are unlikely to be necessary.**)

$\text{PRF}_{\text{sk}}^{\text{expand}}$ is used in §4.2.2 ‘**Sapling Key Components**’ on p. 35 to derive the *Spend authorizing key* a_{sk} and the *proof authorizing key* n_{sk} .

It is instantiated using the *BLAKE2b hash function* defined in §5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74:

$$\text{PRF}_{\text{sk}}^{\text{expand}}(t) := \text{BLAKE2b-512}(\text{"Zcash_ExpandSeed"}, \text{LEBS2OSP}_{256}(\text{sk}) \parallel t)$$

Security requirement: $\text{BLAKE2b-512}(\text{"Zcash_ExpandSeed"}, \text{LEBS2OSP}_{256}(\text{sk}) \parallel t)$ must be a *PRF* for output range $\mathbb{Y}^{[\ell_{\text{PRF}^{\text{expand}}}/8]}$ when keyed by the bits corresponding to sk , with input in the bits corresponding to t .

$\text{PRF}_{\text{ovk}}^{\text{ockSapling}}$ is used in §4.20.1 ‘*Encryption (Sapling and Orchard)*’ on p. 65 to derive the *outgoing cipher key* ock used to encrypt an *outgoing ciphertext*.

It is instantiated using the *BLAKE2b hash function* defined in §5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74:

$$\text{PRF}_{\text{ovk}}^{\text{ockSapling}}(\text{cv}, \text{cmu}, \text{ephemeralKey}) := \text{BLAKE2b-256}(\text{"Zcash_Derive_ock"}, \text{ockInput})$$

$$\text{where } \text{ockInput} = \begin{array}{|c|c|c|c|} \hline \text{LEBS2OSP}_{256}(\text{ovk}) & 32\text{-byte } \text{cv} & 32\text{-byte } \text{cmu} & 32\text{-byte } \text{ephemeralKey} \\ \hline \end{array}.$$

5.4.3 Symmetric Encryption

Let $\text{Sym.K} := \mathbb{B}^{[256]}$, $\text{Sym.P} := \mathbb{B}^{\mathbb{Y}^{[N]}}$, and $\text{Sym.C} := \mathbb{B}^{\mathbb{Y}^{[N]}}$.

Let the *authenticated one-time symmetric encryption scheme* $\text{Sym.Encrypt}_K(P)$ be authenticated encryption using AEAD_CHACHA20_POLY1305 [RFC-7539] encryption of plaintext $P \in \text{Sym.P}$, with empty “associated data”, all-zero nonce $[0]^{96}$, and 256-bit key $K \in \text{Sym.K}$.

Similarly, let $\text{Sym.Decrypt}_K(C)$ be AEAD_CHACHA20_POLY1305 decryption of ciphertext $C \in \text{Sym.C}$, with empty “associated data”, all-zero nonce $[0]^{96}$, and 256-bit key $K \in \text{Sym.K}$. The result is either the plaintext *byte sequence*, or \perp indicating failure to decrypt.

Note: The “IETF” definition of AEAD_CHACHA20_POLY1305 from [RFC-7539] is used; this has a 32-bit block count and a 96-bit nonce, rather than a 64-bit block count and 64-bit nonce as in the original definition of ChaCha20.

5.4.4 Pseudo Random Permutations

Let ℓ_{dk} and ℓ_d be as defined in § 5.3 ‘Constants’ on p. 72.

$\text{PRP}^d : \mathbb{B}^{\mathbb{Y}^{[\ell_{dk}/8]}} \times \mathbb{B}^{[\ell_d]} \rightarrow \mathbb{B}^{[\ell_d]}$ is a *Pseudo Random Permutation* specified in § 4.1.3 ‘Pseudo Random Permutations’ on p. 25. In this specification, it is used to generate *diversifiers* for **Orchard** shielded payment addresses in § 4.2.3 ‘Orchard Key Components’ on p. 37. ([ZIP-32] uses an identical construction to generate *diversifiers* for **Sapling** shielded payment addresses.)

Let $\text{FF1-AES256}_K(\text{tweak}, x)$ be the FF1 format-preserving encryption algorithm [NIST2016] using AES with a 256-bit key K , and parameters $\text{radix} = 2$, $\text{minlen} = 88$, $\text{maxlen} = 88$. It will be used only with the empty string “” as the *tweak*. x is a sequence of 88 bits, as is the output.

Define $\text{PRP}_K^d(d) := \text{FF1-AES256}_K(\text{“”}, d)$.

Security requirement: FF1-AES256 with tweak fixed to “” must be a secure *Pseudo Random Permutation*.

Non-normative note: [DKLS2020] describes attacks against FF1 that are practical for some parameterizations. However, for an 88-bit domain, and 10 rounds as specified in [NIST2016], even the distinguishing attack is no better than a brute force search for the 256-bit key. Specifically we have $r = 5$ (half the number of rounds) and $n = 44$ (half the domain size in bits), so according to [DKLS2020, section 4.2] the data complexity is $2^{2n((r-1)-\frac{1}{2})-n} = 2^{264}$, and the time complexity is $2^{2n((r-1)-\frac{1}{2})} = 2^{308}$.

5.4.5 Key Agreement And Derivation

5.4.5.1 Sprout Key Agreement

$\text{KA}^{\text{Sprout}}$ is a *key agreement scheme* as specified in § 4.1.5 ‘Key Agreement’ on p. 25.

It is instantiated as Curve25519 key agreement, described in [Bernstein2006], as follows.

Let $\text{KA}^{\text{Sprout}}.\text{Public}$ and $\text{KA}^{\text{Sprout}}.\text{SharedSecret}$ be the type of Curve25519 *public keys* (i.e. $\mathbb{B}^{\mathbb{Y}^{[32]}}$), and let $\text{KA}^{\text{Sprout}}.\text{Private}$ be the type of Curve25519 *secret keys*.

Let $\text{Curve25519}(\underline{n}, \underline{q})$ be the result of point multiplication of the Curve25519 *public key* represented by the byte sequence \underline{q} by the Curve25519 *secret key* represented by the *byte sequence* \underline{n} , as defined in [Bernstein2006, section 2].

Let $\text{KA}^{\text{Sprout}}.\text{Base} := \underline{9}$ be the public *byte sequence* representing the Curve25519 base point.

Let $\text{clamp}_{\text{Curve25519}}(x)$ take a 32-byte sequence x as input and return a *byte sequence* representing a Curve25519 *private key*, with bits “clamped” as described in [Bernstein2006, section 3]: “clear bits 0, 1, 2 of the first byte, clear bit 7 of the last byte, and set bit 6 of the last byte.” Here the bits of a byte are numbered such that bit b has numeric weight 2^b .

Define $\text{KA}^{\text{Sprout}}.\text{FormatPrivate}(x) := \text{clamp}_{\text{Curve25519}}(x)$.

Define $\text{KA}^{\text{Sprout}}.\text{DerivePublic}(n, q) := \text{Curve25519}(n, q)$.

Define $\text{KA}^{\text{Sprout}}.\text{Agree}(n, q) := \text{Curve25519}(n, q)$.

5.4.5.2 Sprout Key Derivation

$\text{KDF}^{\text{Sprout}}$ is a *Key Derivation Function* as specified in §4.1.6 ‘*Key Derivation*’ on p. 26.

It is instantiated using BLAKE2b-256 as follows:

$$\text{KDF}^{\text{Sprout}}(i, h_{\text{Sig}}, \text{sharedSecret}_i, \text{epk}, \text{pk}_{\text{enc}, i}^{\text{new}}) := \text{BLAKE2b-256}(\text{kdf_tag}, \text{kdf_input})$$

where:

$$\text{kdf_tag} := \begin{array}{|c|c|c|} \hline 64\text{-bit “ZcashKDF”} & 8\text{-bit } i-1 & [0]^{56} \\ \hline \end{array}$$

$$\text{kdf_input} := \begin{array}{|c|c|c|c|} \hline 256\text{-bit } h_{\text{Sig}} & 256\text{-bit } \text{sharedSecret}_i & 256\text{-bit } \text{epk} & 256\text{-bit } \text{pk}_{\text{enc}, i}^{\text{new}} \\ \hline \end{array}.$$

$\text{BLAKE2b-256}(p, x)$ is defined in §5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74.

5.4.5.3 Sapling Key Agreement

$\text{KA}^{\text{Sapling}}$ is a *key agreement scheme* as specified in §4.1.5 ‘*Key Agreement*’ on p. 25.

It is instantiated as Diffie–Hellman with cofactor multiplication on Jubjub as follows:

Let \mathbb{J} , $\mathbb{J}^{(r)}$, $\mathbb{J}^{(r)*}$, and the cofactor $h_{\mathbb{J}}$ be as defined in §5.4.9.3 ‘*Jubjub*’ on p. 100.

Define $\text{KA}^{\text{Sapling}}.\text{Public} := \mathbb{J}$.

Define $\text{KA}^{\text{Sapling}}.\text{PublicPrimeSubgroup} := \mathbb{J}^{(r)}$.

Define $\text{KA}^{\text{Sapling}}.\text{SharedSecret} := \mathbb{J}^{(r)}$.

Define $\text{KA}^{\text{Sapling}}.\text{Private} := \mathbb{F}_{r_{\mathbb{J}}}$.

Define $\text{KA}^{\text{Sapling}}.\text{DerivePublic}(\text{sk}, B) := [\text{sk}] B$.

Define $\text{KA}^{\text{Sapling}}.\text{Agree}(\text{sk}, P) := [h_{\mathbb{J}} \cdot \text{sk}] P$.

5.4.5.4 Sapling Key Derivation

$\text{KDF}^{\text{Sapling}}$ is a *Key Derivation Function* as specified in §4.1.6 ‘*Key Derivation*’ on p. 26.

It is instantiated using BLAKE2b-256 as follows:

$$\text{KDF}^{\text{Sapling}}(\text{sharedSecret}, \text{ephemeralKey}) := \text{BLAKE2b-256}(\text{“Zcash_SaplingKDF”}, \text{kdf_input}).$$

where:

$$\text{kdf_input} := \begin{array}{|c|c|} \hline \text{LEBS2OSP}_{256}(\text{repr}_{\mathbb{J}}(\text{sharedSecret})) & \text{ephemeralKey} \\ \hline \end{array}.$$

$\text{BLAKE2b-256}(p, x)$ is defined in §5.4.1.2 ‘*BLAKE2 Hash Functions*’ on p. 74.

5.4.5.5 Orchard Key Agreement

Define $\text{KA}^{\text{Orchard}}.\text{Public} := \mathbb{P}^*$.

Define $\text{KA}^{\text{Orchard}}.\text{SharedSecret} := \mathbb{P}^*$.

Define $\text{KA}^{\text{Orchard}}.\text{Private} := \mathbb{F}_{r_{\mathbb{P}}}^*$.

Define $\text{KA}^{\text{Orchard}}.\text{DerivePublic}(\text{sk}, B) := [\text{sk}] B$.

Define $\text{KA}^{\text{Orchard}}.\text{Agree}(\text{sk}, P) := [\text{sk}] P$.

5.4.5.6 Orchard Key Derivation

It is instantiated using BLAKE2b-256 as follows:

where:

5.4.6 Ed25519

Ed25519 is a *signature scheme* as specified in §4.1.7 ‘*Signature*’ on p. 26. It is used to instantiate JoinSplitSig as described in §4.11 ‘*Non-malleability (Sprout)*’ on p. 50.

Let $\text{PreCanopyExcludedPointEncodings} : \mathcal{P}(\mathbb{BY}^{[32]}) = \{$

[illegible]

}

Let $p = 2^{255} - 19$.

Let $a = -1$.

Let $d = -121665/121666 \pmod{p}$.

Let $\ell = 2^{252} + 27742317777372353535851937790883648493$ (the order of the Ed25519 curve's *prime-order subgroup*).

Let B be the base point given in [BDLSY2012].

Notes:

- It is *not* required that the integer encoding of the y -coordinate⁹ of the points represented by \underline{R} or \underline{A} are less than $2^{255} - 19$.
- It is *not* required that $\underline{A} \notin \text{PreCanopyExcludedPointEncodings}$.
- [Canopy onward] Appendix § B.3 ‘Ed25519 *batch validation*’ on p. 218 describes an optimization that **MAY** be used to speed up validation of batches of Ed25519 signatures.

Non-normative note: The exclusion, before Canopy activation, of PreCanopyExcludedPointEncodings from \underline{R} is due to a quirk of version 1.0.15 of the libsodium library [libsodium] which was initially used to implement Ed25519 signature validation in zcashd. (The ED25519_COMPAT compile-time option was not set.) The intent was to exclude points of order less than ℓ ; however, not all such points were covered.

[Canopy onward] **Non-normative note:** Because the post-Canopy rules for Ed25519 signatures are a relaxation of the pre-Canopy rules, a *full validator* implementation that checkpoints on the Canopy activation block **MAY** validate using the post-Canopy rules for the whole chain (and zcashd does so since zcashd v4.2.0). We retain the pre-Canopy rules in the specification in order to accurately document the history of consensus changes.

5.4.7 RedDSA, RedJubjub, and RedPallas

RedDSA is a Schnorr-based *signature scheme*, optionally supporting key *re-randomization* as described in § 4.1.7.1 ‘*Signature with Re-Randomizable Keys*’ on p. 28. It also supports a Secret Key to Public Key Monomorphism as described in § 4.1.7.2 ‘*Signature with Signing Key to Validating Key Monomorphism*’ on p. 29. It is based on a scheme from [FKMSSS2016, section 3], with some ideas from EdDSA [BJLSY2015].

RedJubjub is a specialization of RedDSA to the Jubjub curve (§ 5.4.9.3 ‘Jubjub’ on p. 100), using the BLAKE2b-512 hash function.

The *spend authorization signature scheme* $\text{SpendAuthSig}^{\text{Sapling}}$ is instantiated by RedJubjub, using parameters defined in § 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93.

The *binding signature scheme* $\text{BindingSig}^{\text{Sapling}}$ is instantiated by RedJubjub without key *re-randomization*, using parameters defined in § 5.4.7.2 ‘*Binding Signature (Sapling and Orchard)*’ on p. 93.

RedPallas is a specialization of RedDSA to the Pallas curve defined in § 5.4.9.6 ‘Pallas *and* Vesta’ on p. 103, using the BLAKE2b-512 hash function.

The *spend authorization signature scheme* $\text{SpendAuthSig}^{\text{Orchard}}$ is instantiated by RedPallas, using parameters defined in § 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93.

The *binding signature scheme* $\text{BindingSig}^{\text{Orchard}}$ is instantiated by RedPallas without key *re-randomization*, using parameters defined in § 5.4.7.2 ‘*Binding Signature (Sapling and Orchard)*’ on p. 93.

Let I2LEBSP, I2LEOSP, LEOS2IP, and LEBS2OSP be as defined in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

We first describe the scheme RedDSA over a general *represented group*. Its parameters are:

- a *represented group* \mathbb{G} , which also defines a subgroup $\mathbb{G}^{(r)}$ of order $r_{\mathbb{G}}$, a cofactor $h_{\mathbb{G}}$, a group operation $+$, an additive identity $\mathcal{O}_{\mathbb{G}}$, a bit-length $\ell_{\mathbb{G}}$, a representation function $\text{repr}_{\mathbb{G}}$, and an abstraction function $\text{abst}_{\mathbb{G}}$, as specified in § 4.1.9 ‘*Represented Group*’ on p. 31;
- $\mathcal{P}_{\mathbb{G}}$, a generator of $\mathbb{G}^{(r)}$;
- a bit-length $\ell_{\mathbb{H}} : \mathbb{N}$ such that $2^{\ell_{\mathbb{H}}-128} \geq r_{\mathbb{G}}$ and $\ell_{\mathbb{H}} \bmod 8 = 0$;
- a cryptographic *hash function* $\mathbb{H} : \mathbb{B}^{\mathbb{N}} \rightarrow \mathbb{B}^{\ell_{\mathbb{H}}/8}$.

5.4.7.1 Spend Authorization Signature (Sapling and Orchard)

Let RedJubjub be as defined in § 5.4.7 ‘RedDSA, RedJubjub, *and* RedPallas’ on p. 90.

Define $\mathcal{G}^{\text{Sapling}} := \text{FindGroupHash}^{\mathbb{J}^{(*)}}(\text{"Zcash_G_"}, \text{""})$.

The *spend authorization signature scheme* $\text{SpendAuthSig}^{\text{Sapling}}$ is instantiated as RedJubjub with key *re-randomization* and with generator $\mathcal{P}_G = \mathcal{G}^{\text{Sapling}}$.

Let RedPallas be as defined in § 5.4.7 ‘RedDSA, RedJubjub, *and* RedPallas’ on p. 90.

Define $\mathcal{G}^{\text{Orchard}} := \text{GroupHash}^{\mathbb{P}}(\text{"z.cash:Orchard"}, \text{"G"})$.

The *spend authorization signature scheme* $\text{SpendAuthSig}^{\text{Orchard}}$ is instantiated as RedPallas with key *re-randomization* and with generator $\mathcal{P}_G = \mathcal{G}^{\text{Orchard}}$.

See § 4.15 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 55 for details on the use of this signature scheme.

Security requirement: Each instantiation of SpendAuthSig must be a SURK-CMA-secure signature scheme with *re-randomizable keys* as defined in § 4.1.7.1 ‘*Signature with Re-Randomizable Keys*’ on p. 28.

5.4.7.2 Binding Signature (Sapling and Orchard)

Let RedJubjub and RedPallas be as defined in § 5.4.7 ‘RedDSA, RedJubjub, *and* RedPallas’ on p. 90.

The **Sapling** binding signature scheme, $\text{BindingSig}^{\text{Sapling}}$, is instantiated as RedJubjub without key *re-randomization*, using generator $\mathcal{P}_G = \mathcal{R}^{\text{Sapling}}$ defined in § 5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p. 95. See § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51 for details on the use of this signature scheme.

The **Orchard** binding signature scheme, $\text{BindingSig}^{\text{Orchard}}$, is instantiated as RedPallas without key *re-randomization*, using generator $\mathcal{P}_G = \mathcal{R}^{\text{Orchard}}$ defined in § 5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p. 95. See § 4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53 for details on the use of this signature scheme.

Security requirement: Each instantiation of BindingSig must be a SUF-CMA-secure signature scheme with key *monomorphism* as defined in § 4.1.7.2 ‘*Signature with Signing Key to Validating Key Monomorphism*’ on p. 29. A signature must prove knowledge of the discrete logarithm of the *validating key* with respect to the base $\mathcal{R}^{\text{Sapling}}$ or $\mathcal{R}^{\text{Orchard}}$.

5.4.8 Commitment schemes

5.4.8.1 Sprout Note Commitments

The *note commitment scheme* $\text{NoteCommit}^{\text{Sprout}}$ specified in § 4.1.8 ‘*Commitment*’ on p. 29 is instantiated using SHA-256 as follows:

$$\text{NoteCommit}^{\text{Sprout}}_{\text{rcm}}(a_{\text{pk}}, v, \rho) := \text{SHA-256} \left(\begin{array}{|c|c|c|c|c|c|} \hline 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array} \parallel \begin{array}{|c|} \hline \text{256-bit } a_{\text{pk}} \\ \hline \end{array} \parallel \begin{array}{|c|} \hline \text{64-bit } v \\ \hline \end{array} \parallel \begin{array}{|c|} \hline \text{256-bit } \rho \\ \hline \end{array} \parallel \begin{array}{|c|} \hline \text{256-bit rcm} \\ \hline \end{array} \right)$$

$\text{NoteCommit}^{\text{Sprout}}.\text{GenTrapdoor}()$ generates the uniform distribution on $\text{NoteCommit}^{\text{Sprout}}.\text{Trapdoor}$.

Note: The leading byte of the SHA-256 input is 0xB0.

Security requirements:

- SHA256Compress must be *collision-resistant*.
- SHA256Compress must be a *PRF* when keyed by the bits corresponding to the position of rcm in the second block of SHA-256 input, with input to the *PRF* in the remaining bits of the block and the chaining variable.

5.4.8.2 Windowed Pedersen commitments

§ 5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77 defines a *Pedersen hash* construction. We construct “*windowed Pedersen commitments*” by reusing that construction, and adding a randomized point on the Jubjub curve (see § 5.4.9.3 ‘Jubjub’ on p. 100):

$$\text{WindowedPedersenCommit}_r(s) := \text{PedersenHashToPoint}(\text{"Zcash_PH"}, s) + [r] \text{FindGroupHash}^{\mathbb{J}(r)*}(\text{"Zcash_PH"}, \text{"r"})$$

See § A.3.5 ‘*Windowed Pedersen Commitment*’ on p. 208 for rationale and efficient circuit implementation of this function.

The *note commitment scheme* $\text{NoteCommit}^{\text{Sapling}}$ specified in § 4.1.8 ‘*Commitment*’ on p. 29 is instantiated as follows using $\text{WindowedPedersenCommit}$:

$$\text{NoteCommit}^{\text{Sapling}}_{\text{rcm}}(g_{\star_d}, \text{pk}_{\star_d}, v) := \text{WindowedPedersenCommit}_{\text{rcm}}([1]^6 \parallel \text{I2LEBSP}_{64}(v) \parallel g_{\star_d} \parallel \text{pk}_{\star_d})$$

$\text{NoteCommit}^{\text{Sapling}}.\text{GenTrapdoor}()$ generates the uniform distribution on $\mathbb{F}_{r_{\mathbb{J}}}$.

Security requirements:

- $\text{WindowedPedersenCommit}$, and hence $\text{NoteCommit}^{\text{Sapling}}$, must be computationally *binding* and at least computationally *hiding commitment schemes*.

(They are in fact unconditionally *hiding commitment schemes*.)

Notes:

- $\text{MerkleCRH}^{\text{Sapling}}$ is also defined in terms of $\text{PedersenHashToPoint}$ (see § 5.4.1.3 ‘*MerkleCRH^{Sapling} Hash Function*’ on p. 74). The prefix $[1]^6$ distinguishes the use of $\text{WindowedPedersenCommit}$ in $\text{NoteCommit}^{\text{Sapling}}$ from the layer prefix used in $\text{MerkleCRH}^{\text{Sapling}}$. That layer prefix is a 6-bit *little-endian* encoding of an integer in the range $\{0 \dots \text{MerkleDepth}^{\text{Sapling}} - 1\}$; because $\text{MerkleDepth}^{\text{Sapling}} < 64$, it cannot collide with $[1]^6$.
- The arguments to $\text{NoteCommit}^{\text{Sapling}}$ are in a different order to their encodings in $\text{WindowedPedersenCommit}$. There is no particularly good reason for this.

Theorem 5.4.5. $\text{Uncommitted}^{\text{Sapling}}$ is not in the range of $\text{NoteCommit}^{\text{Sapling}}$.

Proof. $\text{Uncommitted}^{\text{Sapling}}$ is defined as $\text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}(1)$. By injectivity of $\text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}$ and definitions of $\text{Extract}_{\mathbb{J}(r)}$, $\text{WindowedPedersenCommit}$, and $\text{NoteCommit}^{\text{Sapling}}$, $\text{I2LEBSP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}(1)$ can be in the range of $\text{NoteCommit}^{\text{Sapling}}$ only if there exist $\text{rcm} : \text{NoteCommit}^{\text{Sapling}}.\text{Trapdoor}$, $D : \mathbb{B}^{\mathbb{V}[8]}$, and $M : \mathbb{B}^{[\mathbb{N}^+]}$ such that $\mathcal{U}(\text{WindowedPedersenCommit}_{\text{rcm}}(D, M)) = 1$. The latter can only be the *affine-ctEdwards* u -coordinate of a point in \mathbb{J} . We show that there are no points in \mathbb{J} with *affine-ctEdwards* u -coordinate 1. Suppose for a contradiction that $(u, v) \in \mathbb{J}$ for $u = 1$ and some $v : \mathbb{F}_{r_{\mathbb{S}}}$. By writing the curve equation as $v^2 = (1 - a_{\mathbb{J}} \cdot u^2) / (1 - d_{\mathbb{J}} \cdot u^2)$, and noting that $1 - d_{\mathbb{J}} \cdot u^2 \neq 0$ because $d_{\mathbb{J}}$ is nonsquare, we have $v^2 = (1 - a_{\mathbb{J}}) / (1 - d_{\mathbb{J}})$. The right-hand-side is a nonsquare in $\mathbb{F}_{r_{\mathbb{S}}}$ (for the Jubjub curve parameters), so there are no solutions for v (contradiction). \square

Security requirements:

- SinsemillaCommit and SinsemillaShortCommit, and hence NoteCommit^{Orchard} and Commit^{ivk}, must be computationally *binding* and at least computationally *hiding commitment schemes*. They are in fact unconditionally *hiding commitment schemes* provided that no \perp output is observed.

Theorem 5.4.6. Uncommitted^{Orchard} is not in the range of NoteCommit^{Orchard}.

Proof. Uncommitted^{Orchard} is defined as 2. By the definitions of Extract $_{\mathbb{P}}^{\perp}$, SinsemillaShortCommit, and NoteCommit^{Orchard}, 2 can be in the range of NoteCommit^{Orchard} only if there exist $\text{rcm} : \text{NoteCommit}^{\text{Orchard}}.\text{Trapdoor}$, $D : \mathbb{B}^{\mathbb{Y}[\mathbb{N}]}$, and $M : \mathbb{B}^{[\mathbb{N}^+]}$ such that $\text{Extract}_{\mathbb{P}}^{\perp}(\text{SinsemillaCommit}_{\text{rcm}}(D, M)) = 2$. $\text{Extract}_{\mathbb{P}}^{\perp}(\text{SinsemillaCommit}_{\text{rcm}}(D, M))$ can only be \perp or 0 or the *affine-short-Weierstrass x-coordinate* of a point in \mathbb{P} . But $0 \neq 2 \pmod{q_{\mathbb{P}}}$, and there are no points in \mathbb{P} with *affine-short-Weierstrass x-coordinate* $2 \pmod{q_{\mathbb{P}}}$, since $2^3 + b_{\mathbb{P}} = 13$ is not square in $\mathbb{F}_{q_{\mathbb{P}}}$. \square

Non-normative notes:

- Although the given theorem is correct for the definition of NoteCommit^{Orchard} in this specification, the implementation in the *Action circuit* constrains the result to an unspecified set of values when an input results in an exceptional case for any incomplete addition. If this occurs then it yields a nontrivial discrete logarithm relation for the Pallas curve, as proven in Theorem 5.4.4 on p. 82. We can therefore assume that it is infeasible to find such inputs with nonnegligible probability.
- There are also no points in \mathbb{P} with *affine-short-Weierstrass x-coordinate* $0 \pmod{q_{\mathbb{P}}}$, as shown in a note at §5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p.104. We do not choose Uncommitted^{Orchard} = 0 because MerkleCRH^{Orchard} returns 0 in exceptional cases. Although the *hash values* of *leaf nodes* are separated from the *hash values* at other *layers* by the layer input to MerkleCRH^{Orchard}, it would arguably be confusing to rely on that.

5.4.9 Represented Groups and Pairings

5.4.9.1 BN-254

The *represented pairing* BN-254 is defined in this section.

Let $q_{\mathbb{G}} := 2188824287183927522246405745257275088696311157297823662689037894645226208583$.

Let $r_{\mathbb{G}} := 2188824287183927522246405745257275088548364400416034343698204186575808495617$.

Let $b_{\mathbb{G}} := 3$.

($q_{\mathbb{G}}$ and $r_{\mathbb{G}}$ are prime.)

Let $\mathbb{G}_1^{(r)}$ be the group (of order $r_{\mathbb{G}}$) of rational points on a Barreto–Naehrig ([BN2005]) curve $E_{\mathbb{G}_1}$ over $\mathbb{F}_{q_{\mathbb{G}}}$ with equation $y^2 = x^3 + b_{\mathbb{G}}$. This curve has embedding degree 12 with respect to $r_{\mathbb{G}}$.

Let $\mathbb{G}_2^{(r)}$ be the subgroup of order $r_{\mathbb{G}}$ in the sextic twist $E_{\mathbb{G}_2}$ of $E_{\mathbb{G}_1}$ over $\mathbb{F}_{q_{\mathbb{G}}^2}$ with equation $y^2 = x^3 + \frac{b_{\mathbb{G}}}{\xi}$, where $\xi : \mathbb{F}_{q_{\mathbb{G}}^2}$.

We represent elements of $\mathbb{F}_{q_{\mathbb{G}}^2}$ as polynomials $a_1 \cdot t + a_0 : \mathbb{F}_{q_{\mathbb{G}}}[t]$, modulo the irreducible polynomial $t^2 + 1$; in this representation, ξ is given by $t + 9$.

Let $\mathbb{G}_T^{(r)}$ be the subgroup of $r_{\mathbb{G}}^{\text{th}}$ roots of unity in $\mathbb{F}_{q_{\mathbb{G}}^2}^*$, with multiplicative identity $1_{\mathbb{G}}$.

Let $\hat{e}_{\mathbb{G}}$ be the optimal ate pairing (see [Vercauter2009] and [AKLGL2010, section 2]) of type $\mathbb{G}_1^{(r)} \times \mathbb{G}_2^{(r)} \rightarrow \mathbb{G}_T^{(r)}$.

For $i : \{1 \dots 2\}$, let $\mathcal{O}_{\mathbb{G}_i}$ be the point at infinity (which is the additive identity) in $\mathbb{G}_i^{(r)}$, and let $\mathbb{G}_i^{(r)*} := \mathbb{G}_i^{(r)} \setminus \{\mathcal{O}_{\mathbb{G}_i}\}$.

Let $\mathcal{P}_{\mathbb{G}_1} : \mathbb{G}_1^{(r)*} := (1, 2)$.

Let $\mathcal{P}_{\mathbb{G}_2} : \mathbb{G}_2^{(r)*} := (11559732032986387107991004021392285783925812861821192530917403151452391805634 \cdot t + 10857046999023057135944570762232829481370756359578518086990519993285655852781, 4082367875863433681332203403145435568316851327593401208105741076214120093531 \cdot t + 8495653923123431417604973247489272438418190587263600148770280649306958101930)$.

$\mathcal{P}_{\mathbb{G}_1}$ and $\mathcal{P}_{\mathbb{G}_2}$ are generators of $\mathbb{G}_1^{(r)}$ and $\mathbb{G}_2^{(r)}$ respectively.

Define $\text{l2BEBSP} : (\ell : \mathbb{N}) \times \{0 \dots 2^\ell - 1\} \rightarrow \mathbb{B}^{[\ell]}$ as in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

For a point $P : \mathbb{G}_1^{(r)*} = (x_P, y_P)$:

- The field elements x_P and $y_P : \mathbb{F}_q$ are represented as integers x and $y : \{0 \dots q-1\}$.
- Let $\tilde{y} = y \bmod 2$.
- P is encoded as

0	0	0	0	0	0	1
---	---	---	---	---	---	---

 1-bit \tilde{y} 256-bit $\text{l2BEBSP}_{256}(x)$.

For a point $P : \mathbb{G}_2^{(r)*} = (x_P, y_P)$:

- Define $\text{FE2IP} : \mathbb{F}_{q_{\mathbb{G}}}[t]/(t^2 + 1) \rightarrow \{0 \dots q_{\mathbb{G}}^2 - 1\}$ such that $\text{FE2IP}(a_{w,1} \cdot t + a_{w,0}) = a_{w,1} \cdot q + a_{w,0}$.
- Let $x = \text{FE2IP}(x_P)$, $y = \text{FE2IP}(y_P)$, and $y' = \text{FE2IP}(-y_P)$.
- Let $\tilde{y} = \begin{cases} 1, & \text{if } y > y' \\ 0, & \text{otherwise.} \end{cases}$
- P is encoded as

0	0	0	0	1	0	1
---	---	---	---	---	---	---

 1-bit \tilde{y} 512-bit $\text{l2BEBSP}_{512}(x)$.

Non-normative notes:

- Only the $r_{\mathbb{G}}$ -order subgroups $\mathbb{G}_{2,T}^{(r)}$ are used in the protocol, not their containing groups $\mathbb{G}_{2,T}$. Points in $\mathbb{G}_2^{(r)*}$ are *always* checked to be of order $r_{\mathbb{G}}$ when decoding from external representation. (The group of rational points \mathbb{G}_1 on $E_{\mathbb{G}_1}/\mathbb{F}_{q_{\mathbb{G}}}$ is of order $r_{\mathbb{G}}$ so no subgroup checks are needed in that case, and elements of $\mathbb{G}_T^{(r)}$ are never represented externally.) The (r) superscripts on $\mathbb{G}_{1,2,T}^{(r)}$ are used for consistency with notation elsewhere in this specification.
- The points at infinity $\mathcal{O}_{\mathbb{G}_{1,2}}$ never occur in proofs and have no defined encodings in this protocol.
- A rational point $P \neq \mathcal{O}_{\mathbb{G}_2}$ on the curve $E_{\mathbb{G}_2}$ can be verified to be of order $r_{\mathbb{G}}$, and therefore in $\mathbb{G}_2^{(r)*}$, by checking that $r_{\mathbb{G}} \cdot P = \mathcal{O}_{\mathbb{G}_2}$.
- The use of *big-endian* order by l2BEBSP is different from the encoding of most other integers in this protocol. The encodings for $\mathbb{G}_{1,2}^{(r)*}$ are consistent with the definition of EC2OSP for compressed curve points in [IEEE2004, section 5.5.6.2]. The LSB compressed form (i.e. EC2OSP-XL) is used for points in $\mathbb{G}_1^{(r)*}$, and the SORT compressed form (i.e. EC2OSP-XS) for points in $\mathbb{G}_2^{(r)*}$.
- Testing $y > y'$ for the compression of $\mathbb{G}_2^{(r)*}$ points is equivalent to testing whether $(a_{y,1}, a_{y,0}) > (a_{-y,1}, a_{-y,0})$ in lexicographic order.
- Algorithms for decompressing points from the above encodings are given in [IEEE2000, Appendix A.12.8] for $\mathbb{G}_1^{(r)*}$, and [IEEE2004, Appendix A.12.11] for $\mathbb{G}_2^{(r)*}$.

When computing square roots in $\mathbb{F}_{q_{\mathbb{G}}}$ or $\mathbb{F}_{q_{\mathbb{G}}^2}$ in order to decompress a point encoding, the implementation **MUST NOT** assume that the square root exists, or that the encoding represents a point on the curve.

Non-normative notes:

- The length of D is in practice limited to $233 - \text{length}(\text{curveName}_{\mathbb{G}})$ bytes due to the restriction of DST to at most 255 bytes. This limit is not exceeded by any use of $\text{GroupHash}^{\mathbb{P}}$ or $\text{GroupHash}^{\mathbb{V}}$ in this specification.
- $\text{GroupHash}^{\mathbb{P}}$ and $\text{GroupHash}^{\mathbb{V}}$ are intended to be instantiations of hash_to_curve using “Simplified SWU for $AB = 0$ ” described in [ID-hashtocurve, section 6.6.3]. In the event of any discrepancy or change to the Internet Draft, the definition here takes precedence.
- It is not necessary to use the clear_cofactor function specified in the Internet Draft, because Pallas and Vesta (and therefore iso-Pallas and iso-Vesta) are *prime-order curves*.
- The above description incorporates optimizations from [WB2019] that avoid inversions and unnecessary square tests in the computation of $\text{map_to_curve_simple_swu}^{\text{iso-}\mathbb{G}}$. In order to fully avoid inversions, the output of $\text{map_to_curve_simple_swu}^{\text{iso-}\mathbb{G}}$ can be expressed in Jacobian coordinates, as can the input and output of $\text{iso_map}^{\mathbb{G}}$. It is outside the scope of this document to describe Jacobian coordinates, but for example, the $E_{\text{iso-}\mathbb{G}}$ point with *affine-short-Weierstrass* coordinates $(x_{\text{num}}/x_{\text{div}}, y)$, has Jacobian coordinates $(x_{\text{num}} \cdot x_{\text{div}} : y \cdot x_{\text{div}}^3 : x_{\text{div}})$.

Note: The uses of $\text{GroupHash}^{\mathbb{P}}$ for $\text{DiversifyHash}^{\text{Orchard}}$, and of both $\text{GroupHash}^{\mathbb{P}}$ and $\text{GroupHash}^{\mathbb{V}}$ to generate independent bases, need a *random oracle*. The hash_to_curve algorithm in [ID-hashtocurve] is designed to be indifferentiable from a *random oracle* (in the framework of [MRH2003]), given that XMD:BLAKE2b satisfies the requirements of [ID-hashtocurve, section 5.5.4]. The security of the Brier et al. construction on which this algorithm is based is analysed in [FFSTV2013] and [KT2015], with a verified proof in [BGHOZ2013].

5.4.10 Zero-Knowledge Proving Systems

5.4.10.1 BCTV14

Before Sapling activation, **Zcash** uses *zk-SNARKs* generated by a fork of *libsnark* [Zcash-libsnark] with the BCTV14 *proving system* described in [BCTV2014a], which is a modification of the systems in [PHGR2013] and [BCGTV2013].

A BCTV14 proof comprises $(\pi_A : \mathbb{G}_1^{(r)*}, \pi'_A : \mathbb{G}_1^{(r)*}, \pi_B : \mathbb{G}_2^{(r)*}, \pi'_B : \mathbb{G}_1^{(r)*}, \pi_C : \mathbb{G}_1^{(r)*}, \pi'_C : \mathbb{G}_1^{(r)*}, \pi_K : \mathbb{G}_1^{(r)*}, \pi_H : \mathbb{G}_1^{(r)*})$. It is computed as described in [BCTV2014a, Appendix B], using the pairing parameters specified in § 5.4.9.1 ‘BN-254’ on p. 97.

Note: Many details of the *proving system* are beyond the scope of this protocol document. For example, the *quadratic constraint program* verifying the *JoinSplit statement*, or its translation to a *Quadratic Arithmetic Program* [BCTV2014a, section 2.3], are not specified in this document. In 2015, Bryan Parno found a bug in this translation, which is corrected by the *libsnark* implementation¹² [WCBTV2015] [Parno2015] [BCTV2014a, Remark 2.5]. In practice it will be necessary to use the specific proving and verifying keys that were generated for the **Zcash** production *block chain*, given in § 5.7 ‘BCTV14 *zk-SNARK Parameters*’ on p. 117, together with a *proving system* implementation that is interoperable with the **Zcash** fork of *libsnark*, to ensure compatibility.

Vulnerability disclosure: BCTV14 is subject to a security vulnerability, separate from [Parno2015], that could allow violation of Knowledge Soundness (and Soundness) [CVE-2019-7167] [SWB2019] [Gabizon2019]. The consequence for **Zcash** is that balance violation could have occurred before activation of the **Sapling network upgrade**, although there is no evidence of this having happened. Use of the vulnerability to produce false proofs is believed to have been fully mitigated by activation of **Sapling**. The use of BCTV14 in **Zcash** is now limited to verifying proofs that were made prior to the **Sapling network upgrade**.

Due to this issue, new forks of **Zcash** **MUST NOT** use BCTV14, and any other users of the **Zcash** protocol **SHOULD** discontinue use of BCTV14 as soon as possible.

¹²Confusingly, the bug found by Bryan Parno was fixed in *libsnark* in 2015, but that fix was incompletely described in the May 2015 update [BCTV2014a-old, Theorem 2.4]. It is described completely in [BCTV2014a, Theorem 2.4] and in [Gabizon2019].

The vulnerability does not affect the Zero Knowledge property of the scheme (as described in any version of [BCTV2014a] or as implemented in any version of *libsark* that has been used in **Zcash**), even under subversion of the parameter generation [BGG2017, Theorem 4.10].

[Sapling onward] An implementation of **Zcash** that checkpoints on a *block* after **Sapling** **MAY** choose to skip verification of BCTV14 proofs. Note that in §3.3 ‘*The Block Chain*’ on p.16, there is a requirement that a *full validator* that potentially risks *Mainnet* funds or displays *Mainnet transaction* information to a user **MUST** do so only for a *block chain* that includes the *activation block* of the most recent *settled network upgrade*, with its known *block hash* as specified in §3.12 ‘*Mainnet and Testnet*’ on p.21. Since the most recent *settled network upgrade* is after the **Sapling network upgrade**, this mitigates the potential risks due to skipping BCTV14 proof verification.

Encoding of BCTV14 Proofs

A BCTV14 proof is encoded by concatenating the encodings of its elements; for the BN-254 pairing this is:

264-bit π_A	264-bit π'_A	520-bit π_B	264-bit π'_B	264-bit π_C	264-bit π'_C	264-bit π_K	264-bit π_H
-----------------	------------------	-----------------	------------------	-----------------	------------------	-----------------	-----------------

The resulting proof size is 296 bytes.

In addition to the steps to verify a proof given in [BCTV2014a, Appendix B], the verifier **MUST** check, for the encoding of each element, that:

- the lead byte is of the required form;
- the remaining bytes encode a *big-endian* representation of an integer in $\{0 \dots q_S - 1\}$ or (for π_B) $\{0 \dots q_S^2 - 1\}$;
- the encoding represents a point in $\mathbb{G}_1^{(r)*}$ or (for π_B) $\mathbb{G}_2^{(r)*}$, including checking that it is of order r_G in the latter case.

5.4.10.2 Groth16

After **Sapling** activation, **Zcash** uses *zk-SNARKs* with the **Groth16 proving system** described in [BGM2017], which is a modification of the system in [Groth2016]. An independent security proof of this system and its setup is given in [Maller2018].

Groth16 *zk-SNARK proofs* are used in *transaction version 4* and later (§7.1 ‘*Transaction Encoding and Consensus*’ on p.119), both in **Sprout JoinSplit** descriptions and in **Sapling Spend** descriptions and *Output descriptions*. They are generated by the *bellman* library [Bowe-bellman].

A Groth16 proof comprises $(\pi_A : \mathbb{S}_1^{(r)*}, \pi_B : \mathbb{S}_2^{(r)*}, \pi_C : \mathbb{S}_1^{(r)*})$. It is computed as described in [Groth2016, section 3.2], using the pairing parameters specified in §5.4.9.2 ‘BLS12-381’ on p.99. The proof elements are in a different order to the presentation in [Groth2016].

Note: The *quadratic constraint programs* verifying the *Spend statement* and *Output statement* are described in Appendix §A ‘*Circuit Design*’ on p.195. However, many other details of the *proving system* are beyond the scope of this protocol document. For example, certain details of the translations of the *Spend statement* and *Output statement* to *Quadratic Arithmetic Programs* are not specified in this document. In practice it will be necessary to use the specific proving and verifying keys generated for the **Zcash** production *block chain* (see §5.8 ‘Groth16 *zk-SNARK Parameters*’ on p.117), and a *proving system* implementation that is interoperable with the *bellman* library used by **Zcash**, to ensure compatibility.

Encoding of Groth16 Proofs

A Groth16 proof is encoded by concatenating the encodings of its elements; for the BLS12-381 pairing this is:

384-bit π_A	768-bit π_B	384-bit π_C
-----------------	-----------------	-----------------

The resulting proof size is 192 bytes.

In addition to the steps to verify a proof given in [Groth2016], the verifier **MUST** check, for the encoding of each element, that:

- the leading bitfield is of the required form;
- the remaining bits encode a *big-endian* representation of an integer in $\{0 \dots q_S - 1\}$ or (in the case of π_B) two integers in that range;
- the encoding represents a point in $S_1^{(r)*}$ or (in the case of π_B) $S_2^{(r)*}$, including checking that it is of order r_S in each case.

5.4.10.3 Halo 2

For **Orchard** *Action descriptions* in version 5 transactions, **Zcash** uses *zk-SNARKs* with the Halo 2 proving system described in [Zcash-halo2].

Encoding of Halo 2 Proofs

Halo 2 proofs are defined as *byte sequences*, and so the encoding is the proof itself.

5.5 Encodings of Note Plaintexts and Memo Fields

As explained in § 3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14, transmitted *notes* are stored on the *block chain* in encrypted form. The components and usage of *note plaintexts*, and which keys they are encrypted to, are defined in that section.

The encoding of a **Sprout** *note plaintext* consists of:

8-bit leadByte	64-bit v	256-bit p	256-bit rcm	memo (512 bytes)
----------------	----------	-----------	-------------	------------------

- A byte, 0x00, indicating this version of the encoding of a **Sprout** *note plaintext*.
- 8 bytes specifying v.
- 32 bytes specifying p.
- 32 bytes specifying rcm.
- 512 bytes specifying memo.

The encoding of a **Sapling** or **Orchard** *note plaintext* consists of:

8-bit leadByte	88-bit d	64-bit v	256-bit rseed	memo (512 bytes)
----------------	----------	----------	---------------	------------------

- A byte, 0x01 or 0x02 as specified in § 3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14, indicating this version of the encoding of a **Sapling** or **Orchard** *note plaintext*.
- 11 bytes specifying d.
- 8 bytes specifying v.
- 32 bytes specifying rseed.
- 512 bytes specifying memo.

5.6 Encodings of Addresses and Keys

This section describes how **Zcash** encodes *shielded payment addresses*, *incoming viewing keys*, and *spending keys*. Addresses and keys can be encoded as a *byte sequence*; this is called the *raw encoding*. For **Sprout** *shielded payment addresses*, this *byte sequence* can then be further encoded using *Base58Check*. The *Base58Check* layer is the same as for upstream **Bitcoin** addresses [Bitcoin-Base58].

For **Sapling**-specific key and address formats, *Bech32* [ZIP-173] is used instead of *Base58Check*.

Non-normative note: ZIP 173 is similar to **Bitcoin**'s BIP 173, except for dropping the limit of 90 characters on an encoded *Bech32* string (which does not hold for **Sapling** viewing keys, for example), and requirements specific to **Bitcoin**'s Segwit addresses.

Orchard introduces a new address format called a *unified payment address*. This can encode an **Orchard** address, but also a **Sapling** address, a *transparent address*, and potentially future address formats, all in the same *unified payment address*. It is **RECOMMENDED** to use *unified payment addresses* for all new applications, unless compatibility with software that only accepts previous address formats is required.

Unified payment addresses and **Orchard** *spending keys* are encoded with *Bech32m* [BIP-350] rather than *Bech32*.

Payment addresses **MAY** be encoded as QR codes; in this case, the **RECOMMENDED** format for a **Sapling** *payment address* is the *Bech32* form converted to uppercase, using the *Alphanumeric mode* [ISO2015, sections 7.3.4 and 7.4.4]. Similarly, the **RECOMMENDED** format for a *unified payment address* is the *Bech32m* form converted to uppercase, using the *Alphanumeric mode*.

5.6.1 Transparent Encodings

5.6.1.1 Transparent Addresses

Transparent addresses are either *P2SH* (Pay to Script Hash) addresses [BIP-13] or *P2PKH* (Pay to Public Key Hash) addresses [Bitcoin-P2PKH].

The *raw encoding* of a *P2SH* address consists of:

8-bit 0x1C	8-bit 0xBD	160-bit script hash
------------	------------	---------------------

- Two bytes [0x1C, 0xBD], indicating this version of the *raw encoding* of a *P2SH* address on *Mainnet*. (Addresses on *Testnet* use [0x1C, 0xBA] instead.)
- 20 bytes specifying a script hash [Bitcoin-P2SH].

The *raw encoding* of a *P2PKH* address consists of:

8-bit 0x1C	8-bit 0xB8	160-bit <i>validating</i> key hash
------------	------------	------------------------------------

- Two bytes [0x1C, 0xB8], indicating this version of the *raw encoding* of a *P2PKH* address on *Mainnet*. (Addresses on *Testnet* use [0x1D, 0x25] instead.)
- 20 bytes specifying a *validating* key hash, which is a RIPEMD-160 hash [RIPEMD160] of a SHA-256 hash [NIST2015] of a compressed ECDSA key encoding.

Notes:

- In **Bitcoin** a single byte is used for the version field identifying the address type. In **Zcash** two bytes are used. For addresses on *Mainnet*, this and the encoded length cause the first two characters of the *Base58Check* encoding to be fixed as “t3” for *P2SH* addresses, and as “t1” for *P2PKH* addresses. (This does *not* imply that a *transparent Zcash* address can be parsed identically to a **Bitcoin** address just by removing the “t”.)
- **Zcash** does not yet support Hierarchical Deterministic Wallet addresses [BIP-32].

5.6.1.2 Transparent Private Keys

These are encoded in the same way as in **Bitcoin** [Bitcoin-Base58], for both *Mainnet* and *Testnet*.

5.6.2 Sprout Encodings

5.6.2.1 Sprout Payment Addresses

Let KA^{Sprout} be as defined in § 5.4.5.1 ‘**Sprout Key Agreement**’ on p. 86.

A **Sprout shielded payment address** consists of $a_{pk} : \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]}$ and $pk_{\text{enc}} : KA^{\text{Sprout}}.\text{Public}$.

a_{pk} is a SHA256Compress output. pk_{enc} is a $KA^{\text{Sprout}}.\text{Public}$ key, for use with the encryption scheme defined in § 4.19 ‘**In-band secret distribution (Sprout)**’ on p. 63. These components are derived from a *spending key* as described in § 4.2.1 ‘**Sprout Key Components**’ on p. 35.

The *raw encoding* of a **Sprout shielded payment address** consists of:

8-bit 0x16	8-bit 0x9A	256-bit a_{pk}	256-bit pk_{enc}
------------	------------	------------------	---------------------------

- Two bytes [0x16, 0x9A], indicating this version of the *raw encoding* of a **Sprout shielded payment address** on *Mainnet*. (Addresses on *Testnet* use [0x16, 0xB6] instead.)
- 32 bytes specifying a_{pk} .
- 32 bytes specifying pk_{enc} , using the normal encoding of a Curve25519 *public key* [Bernstein2006].

Note: For addresses on *Mainnet*, the lead bytes and encoded length cause the first two characters of the *Base58Check* encoding to be fixed as “zc”. For *Testnet*, the first two characters are fixed as “zt”.

5.6.2.2 Sprout Incoming Viewing Keys

Let KA^{Sprout} be as defined in § 5.4.5.1 ‘**Sprout Key Agreement**’ on p. 86.

A **Sprout incoming viewing key** consists of $a_{pk} : \mathbb{B}^{[\ell_{\text{PRF}}^{\text{Sprout}}]}$ and $sk_{\text{enc}} : KA^{\text{Sprout}}.\text{Private}$.

a_{pk} is a SHA256Compress output. sk_{enc} is a $KA^{\text{Sprout}}.\text{Private}$ key, for use with the encryption scheme defined in § 4.19 ‘**In-band secret distribution (Sprout)**’ on p. 63. These components are derived from a *spending key* as described in § 4.2.1 ‘**Sprout Key Components**’ on p. 35.

The *raw encoding* of a **Sprout incoming viewing key** consists of:

8-bit 0xA8	8-bit 0xAB	8-bit 0xD3	256-bit a_{pk}	256-bit sk_{enc}
------------	------------	------------	------------------	---------------------------

- Three bytes [0xA8, 0xAB, 0xD3], indicating this version of the *raw encoding* of a **Zcash incoming viewing key** on *Mainnet*. (Addresses on *Testnet* use [0xA8, 0xAC, 0x0C] instead.)
- 32 bytes specifying a_{pk} .
- 32 bytes specifying sk_{enc} , using the normal encoding of a Curve25519 *private key* [Bernstein2006].

sk_{enc} **MUST** be “clamped” using $KA^{\text{Sprout}}.\text{FormatPrivate}$ as specified in § 4.2.1 ‘**Sprout Key Components**’ on p. 35. That is, a decoded *incoming viewing key* **MUST** be considered invalid if $sk_{\text{enc}} \neq KA^{\text{Sprout}}.\text{FormatPrivate}(sk_{\text{enc}})$.

$KA^{\text{Sprout}}.\text{FormatPrivate}$ is defined in § 5.4.5.1 ‘**Sprout Key Agreement**’ on p. 86.

Note: For addresses on *Mainnet*, the lead bytes and encoded length cause the first four characters of the *Base58Check* encoding to be fixed as “ZiVK”. For *Testnet*, the first four characters are fixed as “ZiVt”.

5.6.2.3 Sprout Spending Keys

A **Sprout** spending key consists of a_{sk} , which is a sequence of 252 bits (see § 4.2.1 ‘**Sprout** Key Components’ on p. 35).

The raw encoding of a **Sprout** spending key consists of:

8-bit 0xAB	8-bit 0x36	$[0]^4$	252-bit a_{sk}
------------	------------	---------	------------------

- Two bytes [0xAB, 0x36], indicating this version of the raw encoding of a **Zcash** spending key on *Mainnet*. (Addresses on *Testnet* use [0xAC, 0x08] instead.)
- 32 bytes: 4 zero padding bits and 252 bits specifying a_{sk} .

The zero padding occupies the most significant 4 bits of the third byte.

Notes:

- If an implementation represents a_{sk} internally as a sequence of 32 bytes with the 4 bits of zero padding intact, it will be in the correct form for use as an input to PRF^{addr} , $\text{PRF}^{\text{nfSprout}}$, and PRF^{pk} without need for bit-shifting.
- For addresses on *Mainnet*, the lead bytes and encoded length cause the first two characters of the *Base58Check* encoding to be fixed as “SK”. For *Testnet*, the first two characters are fixed as “ST”.

5.6.3 Sapling Encodings

5.6.3.1 Sapling Payment Addresses

Let $\text{KA}^{\text{Sapling}}$ be as defined in § 5.4.5.3 ‘**Sapling** Key Agreement’ on p. 87.

Let ℓ_d be as defined in § 5.3 ‘**Constants**’ on p. 72.

Let $\mathbb{J}^{(r)}$, $\text{abst}_{\mathbb{J}}$, and $\text{repr}_{\mathbb{J}}$ be as defined in § 5.4.9.3 ‘**Jubjub**’ on p. 100.

Let $\text{LEBS2OSP} : (\ell : \mathbb{N}) \times \mathbb{B}^{[\ell]} \rightarrow \mathbb{B}^{\lceil \ell/8 \rceil}$ be as defined in § 5.1 ‘**Integers, Bit Sequences, and Endianness**’ on p. 71.

A **Sapling shielded payment address** consists of $d : \mathbb{B}^{[\ell_d]}$ and $\text{pk}_d : \text{KA}^{\text{Sapling}}.\text{PublicPrimeSubgroup}$.

pk_d is an encoding of a $\text{KA}^{\text{Sapling}}$ public key of type $\text{KA}^{\text{Sapling}}.\text{PublicPrimeSubgroup}$, for use with the encryption scheme defined in § 4.20 ‘**In-band secret distribution (Sapling and Orchard)**’ on p. 65. d is a *diversifier*. These components are derived as described in § 4.2.2 ‘**Sapling** Key Components’ on p. 35.

The raw encoding of a **Sapling shielded payment address** consists of:

$\text{LEBS2OSP}_{88}(d)$	$\text{LEBS2OSP}_{256}(\text{repr}_{\mathbb{J}}(\text{pk}_d))$
---------------------------	--

- 11 bytes specifying d .
- 32 bytes specifying the *ctEdwards compressed encoding* of pk_d (see § 5.4.9.3 ‘**Jubjub**’ on p. 100).

When decoding the representation of pk_d , the address **MUST** be considered invalid if $\text{abst}_{\mathbb{J}}$ returns \perp .

[ZIP-216] specifies that the address **MUST** also be considered invalid if the resulting pk_d is not in the *prime-order subgroup* $\mathbb{J}^{(r)}$, or if it is a *non-canonical* encoding as defined in § 4.1.9 ‘**Represented Group**’ on p. 31. This **MAY** be enforced in advance of activation of **NU5**.

For addresses on *Mainnet*, the *Human-Readable Part* (as defined in [ZIP-173]) is “zs”. For addresses on *Testnet*, the *Human-Readable Part* is “ztestsapling”.

5.6.4 Unified and Orchard Encodings

5.6.4.1 Unified Payment Addresses and Viewing Keys

Rather than defining a *Bech32* string encoding of **Orchard** *shielded payment addresses*, we instead define, in [ZIP-316], a *unified payment address* format that is able to encode a set of *payment addresses* of different types. This enables the consumer of an address to choose the best address type it supports, providing a better user experience as new formats are added in the future.

Similarly, *unified incoming viewing keys* and *unified full viewing keys* are defined to encode sets of *incoming viewing keys* and *full viewing keys* respectively.

Since [ZIP-316] includes a full specification of encoding, decoding, and other processing of *unified payment addresses*, *unified incoming viewing keys*, and *unified full viewing keys*, we give only a summary here.

A *unified payment address* includes zero or one address of each type in the following Priority List:

- typecode 0x03 – § 5.6.4.2 ‘**Orchard** Raw Payment Addresses’ on p. 115;
- typecode 0x02 – § 5.6.3.1 ‘**Sapling** Payment Addresses’ on p. 113;
- typecode 0x01 – *transparent P2SH address*, or typecode 0x00 – *transparent P2PKH address*.

with the restrictions that there **MUST** be at least one *shielded payment address* (typecodes $\geq 0x02$), and that both *P2SH* and *P2PKH* cannot be present.

When sending a payment, the consumer of a *unified payment address* **MUST** use the most preferred address type that it supports from the set, i.e. the first in the above list. See [ZIP-316] for additional requirements, and for discussion of *unified incoming viewing keys* and *unified full viewing keys*.

Note that there is intentionally no typecode defined for a **Sprout** *shielded payment address* (or **Sprout** viewing keys). Since it is no longer possible (since activation of [ZIP-211] in the **Canopy network upgrade**) to send funds into the **Sprout chain value pool**, this would not be generally useful.

The format uses *Bech32m* [BIP-350] (ignoring any length restrictions) for the checksum algorithm and string encoding. This is chosen over *Bech32* in order for the checksum to better handle variable-length inputs.

A “jumbling” algorithm is used in order to mitigate address replacement attacks given that a user might only check part of the address. See [ZIP-316] for full details.

5.6.4.2 Orchard Raw Payment Addresses

Let $\text{KA}^{\text{Orchard}}$ be as defined in § 5.4.5.5 ‘**Orchard Key Agreement**’ on p. 88.

An **Orchard** *shielded payment address* consists of $d : \mathbb{B}^{[\ell_d]}$ and $\text{pk}_d : \text{KA}^{\text{Orchard}}.\text{Public}$.

pk_d is an encoding of a $\text{KA}^{\text{Orchard}}$ public key of type $\text{KA}^{\text{Orchard}}.\text{Public}$, for use with the encryption scheme defined in § 4.20 ‘*In-band secret distribution (**Sapling** and **Orchard**)*’ on p. 65. d is a sequence of 11 bytes. These components are derived as described in § 4.2.3 ‘**Orchard Key Components**’ on p. 37.

The *raw encoding* of an **Orchard** *shielded payment address* consists of:

$\text{LEBS2OSP}_{88}(d)$	$\text{LEBS2OSP}_{256}(\text{repr}_{\mathbb{P}}(\text{pk}_d))$
---------------------------	--

- 11 bytes specifying d .
- 32 bytes specifying the *short Weierstrass compressed encoding* of pk_d (see § 5.4.9.6 ‘**Pallas and Vesta**’ on p. 103).

When decoding the representation of pk_d , the address **MUST** be considered invalid if $\text{abst}_{\mathbb{P}}$ returns \perp or $\mathcal{O}_{\mathbb{P}}$.

There is no *Bech32[m]* encoding defined for an individual **Orchard** *shielded payment address*; instead use a *unified payment address* as defined in [ZIP-316].

The *raw encoding* of an **Orchard** *spending key* consists of:

$\text{LEBS2OSP}_{256}(\text{sk})$

- 32 bytes specifying sk.

Orchard *spending keys* are encoded using *Bech32m* (not *Bech32*).

For *spending keys* on *Mainnet*, the *Human-Readable Part* is “**secret-orchard-sk-main**”. For *spending keys* on *Testnet*, the *Human-Readable Part* is “**secret-orchard-sk-test**”.

5.7 BCTV14 zk-SNARK Parameters

The SHA-256 hashes of the *proving key* and *verifying key* for the **Sprout** *JoinSplit circuit*, encoded in *libsnaek* format, are:

```
8bc20a7f013b2b58970cddd2e7ea028975c88ae7ceb9259a5344a16bc2c0eef7 sprout-proving.key
4bd498dae0aacfd8e98dc306338d017d9c08dd0918ead18172bd0aec2fc5df82 sprout-verifying.key
```

These parameters were obtained by a *multi-party computation* described in [BGG-mpc] and [BGG2017]. **They are used only before Sapling activation.** Due to the security vulnerability described in § 5.4.10.1 ‘BCTV14’ on p. 108, it is not recommended to use these parameters in new protocols, and it is recommended to stop using them in protocols other than **Zcash** where they are currently used.

5.8 Groth16 zk-SNARK Parameters

bellman [Bowe-bellman] encodes the *proving key* and *verifying key* for a *zk-SNARK circuit* in a single parameters file. The BLAKE2b-512 hashes of this file for the **Sapling** *Spend circuit* and *Output circuit*, and for the implementation of the **Sprout** *JoinSplit circuit* used after **Sapling** activation, are respectively:

```
8270785a1a0d0bc77196f000ee6d221c9c9894f55307bd9357c3f0105d31ca63
991ab91324160d8f53e2bbd3c2633a6eb8bdf5205d822e7f3f73edac51b2b70c sapling-spend.params
657e3d38dbb5cb5e7dd2970e8b03d69b4787dd907285b5a7f0790dcc8072f60b
f593b32cc2d1c030e00ff5ae64bf84c5c3beb84ddc841d48264b4a171744d028 sapling-output.params
e9b238411bd6c0ec4791e9d04245ec350c9c5744f5610dfcce4365d5ca49dfef
d5054e371842b3f88fa1b9d7e8e075249b3ebabd167fa8b0f3161292d36c180a sprout-groth16.params
```

These parameters were obtained by a *multi-party computation* described in [BGM2017].

5.9 Randomness Beacon

Let $\text{URS} := \text{“096b36a5804bfacef1691e173c366a47ff5ba84a44f26ddd7e8d9f79d5b42df0”}$.

This value is used in the definition of $\text{GroupHash}^{\mathbb{J}^{(r)*}}$ in § 5.4.9.5 ‘*Group Hash into Jubjub*’ on p. 102, and in the *multi-party computation* to obtain the **Sapling** parameters given in § 5.8 ‘*Groth16 zk-SNARK Parameters*’ on p. 117.

It is derived as described in [Bowe2018]:

- Take the hash of the **Bitcoin** *block* at height 514200 in *RPC byte order*, i.e. the *big-endian* 32-byte representation of 0x00.
- Apply SHA-256 2^{42} times.
- Convert to a *US-ASCII* lowercase hexadecimal string.

Note: URS is a 64-byte *US-ASCII* string, i.e. the first byte is 0x30, not 0x09.

6 Network Upgrades

Zcash launched with a protocol revision that we call **Sprout**.

A first upgrade, called **Overwinter**, activated on *Mainnet* on 26 June, 2018 at *block height* 347500 [Swihart2018]. Its specifications are described in this document, [ZIP-201], [ZIP-202], [ZIP-203], and [ZIP-143].

A second upgrade, called **Sapling**, activated on *Mainnet* on 28 October, 2018 at *block height* 419200 [Hamdon2018]. Its specifications are described in this document, [ZIP-205], and [ZIP-243].

A third upgrade, called **Blossom**, activated on *Mainnet* on 11 December, 2019 at *block height* 653600 [Zcash-Blossom]. Its specifications are described in this document, [ZIP-206], and [ZIP-208].

A fourth upgrade, called **Heartwood**, activated on *Mainnet* on 16 July, 2020 at *block height* 903000 [Zcash-Heartwd]. Its specifications are described in this document, [ZIP-250], [ZIP-213], and [ZIP-221].

A fifth upgrade, called **Canopy**, activated on *Mainnet* on 18 November, 2020 at *block height* 1046400 (coinciding with the first *block subsidy halving*) [Zcash-Canopy]. Its specifications are described in this document, [ZIP-251], [ZIP-207], [ZIP-211], [ZIP-212], [ZIP-214], and [ZIP-215]. Additional information and rationale is given in [ZIP-1014].

A sixth upgrade, called **NU5**, activated on *Mainnet* on 31 May, 2022 at *block height* 1687104 [Zcash-Nu5]. Its specifications are described in this document, [ZIP-252], [ZIP-216], [ZIP-221], [ZIP-224], [ZIP-225], [ZIP-239], [ZIP-244], and [ZIP-316], with updates to [ZIP-32], [ZIP-203], [ZIP-209], [ZIP-212], [ZIP-213], and [ZIP-221]. Additional information and rationale is given in [Zcash-Orchard] and [Zcash-halo2].

This draft specification describes the set of changes proposed for the **NU6 network upgrade**, for which the *Mainnet* activation *block height* has been set at 2726400. Its specifications are described in this document, [ZIP-253], [ZIP-236], and [ZIP-2001], with updates to [ZIP-207] and [ZIP-214]. Additional information and rationale is given in [ZIP-1015].

This section summarizes the strategy for upgrading from **Sprout** to subsequent versions of the protocol (**Overwinter**, **Sapling**, **Blossom**, **Heartwood**, **Canopy**, **NU5**, and **NU6**), and for future upgrades.

The *network upgrade* mechanism is described in [ZIP-200].

Each *network upgrade* is introduced as a “*bilateral consensus rule change*”. In this kind of upgrade,

- there is an *activation block height* at which the *consensus rule change* takes effect;
- *blocks* and *transactions* that are valid according to the post-upgrade rules are not valid before the upgrade *block height*;
- *blocks* and *transactions* that are valid according to the pre-upgrade rules are no longer valid at or after the *activation block height*.

Full support for each *network upgrade* is indicated by a minimum version of the *peer-to-peer protocol*. At the planned *activation block height*, nodes that support a given upgrade will disconnect from (and will not reconnect to) nodes with a protocol version lower than this minimum. See [ZIP-201] for how this applies to the **Overwinter** upgrade, for example.

This ensures that upgrade-supporting nodes transition cleanly from the old protocol to the new protocol. Nodes that do not support the upgrade will find themselves on a network that uses the old protocol and is fully partitioned from the upgrade-supporting network. This allows us to specify arbitrary protocol changes that take effect at a given *block height*.

Note, however, that a *block chain reorganization* across the upgrade *activation block height* is possible. In the case of such a reorganization, *blocks* at a height before the *activation block height* will still be created and validated according to the pre-upgrade rules, and upgrade-supporting nodes **MUST** allow for this.

7 Consensus Changes from Bitcoin

7.1 Transaction Encoding and Consensus

The **Zcash** *transaction* format up to and including *transaction version* 4 is as follows (this should be read in the context of consensus rules later in the section):

Version*	Bytes	Name	Data Type	Description
1..4	4	header	uint32	Contains: · <code>f0verwintered</code> flag (bit 31) · <code>version</code> (bits 30..0) – <i>transaction version</i> .
3..4	4	<code>nVersionGroupId</code>	uint32	Version group ID (nonzero).
1..4	Varies	<code>tx_in_count</code>	compactSize	Number of <i>transparent inputs</i> .
1..4	Varies	<code>tx_in</code>	<code>tx_in</code>	<i>Transparent</i> inputs, encoded as in Bitcoin .
1..4	Varies	<code>tx_out_count</code>	compactSize	Number of <i>transparent outputs</i> .
1..4	Varies	<code>tx_out</code>	<code>tx_out</code>	<i>Transparent</i> outputs, encoded as in Bitcoin .
1..4	4	<code>lock_time</code>	uint32	Unix-epoch UTC time or <i>block height</i> , encoded as in Bitcoin .
3..4	4	<code>nExpiryHeight</code>	uint32	A <i>block height</i> after which the <i>transaction</i> will expire, or 0 to disable expiry. [ZIP-203]
4	8	<code>valueBalanceSapling</code>	int64	The net value of Sapling spends minus outputs.
4	Varies	<code>nSpendsSapling</code>	compactSize	The number of <i>Spend descriptions</i> in <code>vSpendsSapling</code> .
4	384· <code>nSpendsSapling</code>	<code>vSpendsSapling</code>	<code>SpendDescriptionV4</code> [<code>nSpendsSapling</code>]	A sequence of <i>Spend descriptions</i> , encoded per §7.3 ‘ <i>Spend Description Encoding and Consensus</i> ’ on p.125.
4	Varies	<code>nOutputsSapling</code>	compactSize	The number of <i>Output descriptions</i> in <code>vOutputsSapling</code> .
4	948· <code>nOutputsSapling</code>	<code>vOutputsSapling</code>	<code>OutputDescriptionV4</code> [<code>nOutputsSapling</code>]	A sequence of <i>Output descriptions</i> , encoded per §7.4 ‘ <i>Output Description Encoding and Consensus</i> ’ on p.126.
2..4	Varies	<code>nJoinSplit</code>	compactSize	The number of <i>JoinSplit descriptions</i> in <code>vJoinSplit</code> .
2..3	1802· <code>nJoinSplit</code>	<code>vJoinSplit</code>	<code>JSDescriptionBCTV14</code> [<code>nJoinSplit</code>]	A sequence of <i>JoinSplit descriptions</i> using BCTV14 proofs, encoded per §7.2 ‘ <i>JoinSplit Description Encoding and Consensus</i> ’ on p.125.
4	1698· <code>nJoinSplit</code>	<code>vJoinSplit</code>	<code>JSDescriptionGroth16</code> [<code>nJoinSplit</code>]	A sequence of <i>JoinSplit descriptions</i> using Groth16 proofs, encoded per §7.2 ‘ <i>JoinSplit Description Encoding and Consensus</i> ’ on p.125.
2..4 †	32	<code>joinSplitPubKey</code>	byte[32]	An encoding of a <code>JoinSplitSig</code> public validating key.
2..4 †	64	<code>joinSplitSig</code>	byte[64]	A signature on a prefix of the <i>transaction</i> encoding, validated using <code>joinSplitPubKey</code> as specified in §4.11 ‘ <i>Non-malleability (Sprout)</i> ’ on p.50.
4 ‡	64	<code>bindingSigSapling</code>	byte[64]	A Sapling binding signature on the SIGHASH transaction hash, validated as specified in §5.4.7.2 ‘ <i>Binding Signature (Sapling and Orchard)</i> ’ on p.93.

* Version constraints apply to the `effectiveVersion`, which is equal to $\min(2, \text{version})$ when `f0verwintered` = 0 and to `version` otherwise. If `effectiveVersion` ≥ 5 once `header` has been parsed, the remainder of the *transaction* encoding **MUST** be parsed according to the v5 format described in the next table. The consensus rules later in this section specify constraints on `nVersionGroupId` depending on `effectiveVersion`.

† The `joinSplitPubKey` and `joinSplitSig` fields are present if and only if `effectiveVersion` ≥ 2 and `nJoinSplit` > 0 .

‡ `bindingSigSapling` is present if and only if `effectiveVersion` = 4 and `nSpendsSapling` + `nOutputsSapling` > 0 .

Note that the `valueBalanceSapling` field is always present for these *transaction versions*.

Several **Sapling** fields have been renamed from previous versions of this specification:

`valueBalance` → `valueBalanceSapling`; `nShieldedSpend` → `nSpendsSapling`; `vShieldedSpend` → `vSpendsSapling`; `nShieldedOutput` → `nOutputsSapling`; `vShieldedOutput` → `vOutputsSapling`; `bindingSig` → `bindingSigSapling`.

7.1.1 Transaction Identifiers

The *transaction ID* of a **version 4** or earlier *transaction* is the SHA-256d hash of the *transaction* encoding in the pre-v5 format described above.

The *transaction ID* of a version 5 *transaction* is as defined in [ZIP-244]. A v5 *transaction* also has a *wtxid* (used for example in the *peer-to-peer protocol*) as defined in [ZIP-239].

7.1.2 Transaction Consensus Rules

Consensus rules:

- The *transaction version number* **MUST** be greater than or equal to 1.
- [Pre-Overwinter] The `f0verwintered` flag **MUST NOT** be set.
- [Overwinter onward] The `f0verwintered` flag **MUST** be set.
- [Overwinter onward] The *version group ID* **MUST** be recognized.
- [Overwinter only, pre-Sapling] The *transaction version number* **MUST** be 3, and the *version group ID* **MUST** be 0x03C48270.
- [Sapling to Canopy inclusive, pre-NU5] The *transaction version number* **MUST** be 4, and the *version group ID* **MUST** be 0x892F2085.
- [NU5 onward] The *transaction version number* **MUST** be 4 or 5. If the *transaction version number* is 4 then the *version group ID* **MUST** be 0x892F2085. If the *transaction version number* is 5 then the *version group ID* **MUST** be 0x26A7270A.
- [NU5 onward] If `effectiveVersion` \geq 5, the `nConsensusBranchId` field **MUST** match the *consensus branch ID* used for *SIGHASH transaction hashes*, as specified in [ZIP-244].
- [Pre-Sapling] The encoded size of the *transaction* **MUST** be less than or equal to 100000 bytes.
- [NU5 onward] `nSpendsSapling`, `nOutputsSapling`, and `nActionsOrchard` **MUST** all be less than 2^{16} .
- [Pre-Sapling] If `effectiveVersion` = 1 or `nJoinSplit` = 0, then both `tx_in_count` and `tx_out_count` **MUST** be nonzero.
- [Sapling onward] If `effectiveVersion` < 5, then at least one of `tx_in_count`, `nSpendsSapling`, and `nJoinSplit` **MUST** be nonzero.
- [Sapling onward] If `effectiveVersion` < 5, then at least one of `tx_out_count`, `nOutputsSapling`, and `nJoinSplit` **MUST** be nonzero.
- [NU5 onward] If `effectiveVersion` \geq 5 then this condition **MUST** hold: `tx_in_count` > 0 or `nSpendsSapling` > 0 or (`nActionsOrchard` > 0 and `enableSpendsOrchard` = 1).
- [NU5 onward] If `effectiveVersion` \geq 5 then this condition **MUST** hold: `tx_out_count` > 0 or `nOutputsSapling` > 0 or (`nActionsOrchard` > 0 and `enableOutputsOrchard` = 1).
- [NU5 onward] If `effectiveVersion` \geq 5 and `nActionsOrchard` > 0, then at least one of `enableSpendsOrchard` and `enableOutputsOrchard` **MUST** be 1.
- A *transaction* with one or more *transparent inputs* from *coinbase transactions* **MUST** have no *transparent outputs* (i.e. `tx_out_count` **MUST** be 0). Inputs from *coinbase transactions* include *Founders' Reward outputs* and *funding stream outputs*.
- If `effectiveVersion` \geq 2 and `nJoinSplit` > 0, then:
 - `joinSplitPubKey` **MUST** be a valid encoding (see §5.4.6 ‘Ed25519’ on p. 88) of an Ed25519 *validating key*.
 - `joinSplitSig` **MUST** represent a valid signature under `joinSplitPubKey` of `dataToBeSigned`, as defined in §4.11 ‘Non-malleability (*Sprout*)’ on p. 50.

- [Sapling onward] If $\text{effectiveVersion} \geq 4$ and $\text{nSpendSapling} + \text{nOutputsSapling} > 0$, then:
 - let $\text{bvK}^{\text{Sapling}}$ and SigHash be as defined in § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51;
 - bindingSigSapling **MUST** represent a valid signature under the *transaction binding validating key* $\text{bvK}^{\text{Sapling}}$ of SigHash — i.e. $\text{BindingSig}^{\text{Sapling}}.\text{Validate}_{\text{bvK}^{\text{Sapling}}}(\text{SigHash}, \text{bindingSigSapling}) = 1$. [NU5 onward] As specified in § 5.4.7 ‘RedDSA, RedJubjub, and RedPallas’ on p. 90, the validation of the \underline{R} component of the signature changes to prohibit *non-canonical* encodings.
- [Sapling onward] If $\text{effectiveVersion} = 4$ and there are no *Spend descriptions* or *Output descriptions*, then $\text{valueBalanceSapling}$ **MUST** be 0.
- [NU5 onward] If $\text{effectiveVersion} \geq 5$ and $\text{nActionsOrchard} > 0$, then:
 - let $\text{bvK}^{\text{Orchard}}$ and SigHash be as defined in § 4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53;
 - bindingSigOrchard **MUST** represent a valid signature under the *transaction binding validating key* $\text{bvK}^{\text{Orchard}}$ of SigHash — i.e. $\text{BindingSig}^{\text{Orchard}}.\text{Validate}_{\text{bvK}^{\text{Orchard}}}(\text{SigHash}, \text{bindingSigOrchard}) = 1$. As specified in § 5.4.7 ‘RedDSA, RedJubjub, and RedPallas’ on p. 90, validation of the \underline{R} component of the signature prohibits *non-canonical* encodings.
- Let $\text{totalDeferredOutput}$ be as defined in § 7.8 on p. 133. For the *block* at *block height* height :
 - define the *total output value* of its *coinbase transaction* to be the total value in *zatoshi* of its *transparent outputs*, minus $v^{\text{balanceSapling}}$, minus $v^{\text{balanceOrchard}}$, plus $\text{totalDeferredOutput}(\text{height})$;
 - define the *total input value* of its *coinbase transaction* to be the value in *zatoshi* of the *block subsidy*, plus the *transaction fees* paid by *transactions* in the *block*.
- [Pre-NU6] The *total output value* of a *coinbase transaction* **MUST NOT** be greater than its *total input value*.
- [NU6 onward] The *total output value* of a *coinbase transaction* **MUST** be equal to its *total input value*.
- A *coinbase transaction* **MUST NOT** have any *JoinSplit descriptions*.
- A *coinbase transaction* **MUST NOT** have any *Spend descriptions*.
- [Pre-Heartwood] A *coinbase transaction* **MUST NOT** have any *Output descriptions*.
- [NU5 onward] In a version 5 *coinbase transaction*, the $\text{enableSpendOrchard}$ flag **MUST** be 0.
- [NU5 onward] In a version 5 *transaction*, the reserved bits 2 .. 7 of the flagsOrchard field **MUST** be zero.
- A *coinbase transaction* for a *block* at *block height* greater than 0 **MUST** have a *script* that, as its first item, encodes the *block height* height as follows. For height in the range $\{1 \dots 16\}$, the encoding is a single byte of value $0x50 + \text{height}$. Otherwise, let heightBytes be the signed *little-endian* representation of height , using the minimum nonzero number of bytes such that the most significant byte is $< 0x80$. The length of heightBytes **MUST** be in the range $\{1 \dots 5\}$. Then the encoding is the length of heightBytes encoded as one byte, followed by heightBytes itself. This matches the encoding used by **Bitcoin** in the implementation of [BIP-34] (but the description here is to be considered normative).
- A *coinbase transaction script* **MUST** have length in $\{2 \dots 100\}$ bytes.
- A *transparent input* in a non-*coinbase transaction* **MUST NOT** have a null *prevout*.
- Every non-null *prevout* **MUST** point to a unique *UTXO* in either a preceding *block*, or a *previous transaction* in the same *block*.
- A *transaction* **MUST NOT** spend a *transparent output* of a *coinbase transaction* from a *block* less than 100 *blocks* prior to the spend. Note that *transparent outputs* of *coinbase transactions* include *Founders’ Reward outputs* and *transparent funding stream outputs*.
- A *transaction* **MUST NOT** spend an output of the *genesis block coinbase transaction*. (There is one such zero-valued output, on each of *Testnet* and *Mainnet*.)
- [Overwinter to Canopy inclusive, pre-NU5] nExpiryHeight **MUST** be less than or equal to 499999999.
- [NU5 onward] nExpiryHeight **MUST** be less than or equal to 499999999 for non-*coinbase transactions*.

- [Overwinter onward] If a *transaction* is not a *coinbase transaction* and its `nExpiryHeight` field is nonzero, then it **MUST NOT** be mined at a *block height* greater than its `nExpiryHeight`.
- [NU5 onward] The `nExpiryHeight` field of a *coinbase transaction* **MUST** be equal to its *block height*.
- [Sapling onward] `valueBalanceSapling` **MUST** be in the range $\{-\text{MAX_MONEY} .. \text{MAX_MONEY}\}$.
- [NU5 onward] `valueBalanceOrchard` **MUST** be in the range $\{-\text{MAX_MONEY} .. \text{MAX_MONEY}\}$ for version 5 transactions.
- [Heartwood onward] All **Sapling** and **Orchard** outputs in *coinbase transactions* **MUST** decrypt to a *note plaintext*, i.e. the procedure in §4.20.3 ‘*Decryption using a Full Viewing Key (Sapling and Orchard)*’ on p. 68 does not return \perp , using a sequence of 32 zero bytes as the *outgoing viewing key*. (This implies that before **Canopy** activation, **Sapling** outputs of a *coinbase transaction* **MUST** have *note plaintext lead byte* equal to 0x01.)
- [Canopy onward] Any **Sapling** or **Orchard** output of a *coinbase transaction* decrypted to a *note plaintext* according to the preceding rule **MUST** have *note plaintext lead byte* equal to 0x02. (This applies even during the “grace period” specified in [ZIP-212].)
- TODO: Other rules inherited from **Bitcoin**.

The types specified in §7.1 ‘*Transaction Encoding and Consensus*’ on p. 119 are part of the consensus rules.

Consensus rules associated with each *JoinSplit* description (§7.2 ‘*JoinSplit Description Encoding and Consensus*’ on p. 125), each *Spend* description (§7.3 ‘*Spend Description Encoding and Consensus*’ on p. 125), each *Output* description (§7.4 ‘*Output Description Encoding and Consensus*’ on p. 126), and each *Action* description (§7.5 ‘*Action Description Encoding and Consensus*’ on p. 127) **MUST** also be followed.

Notes:

- Previous versions of this specification defined what is now the header field as a signed int32 field which was required to be positive. The consensus rule that the `fOverwintered` flag **MUST NOT** be set before **Overwinter** has activated, has the same effect.
- The semantics of *transactions* with *version number* not equal to 1, 2, 3, 4, or 5 is not currently defined.
- The exclusion of *transactions* with *transaction version number greater than* 2 is not a consensus rule before **Overwinter** activation. Such *transactions* may exist in the *block chain* and **MUST** be treated identically to version 2 *transactions*.
- [Overwinter onward] Once **Overwinter** has activated, limits on the maximum *transaction version number* are consensus rules.
- The *transaction version number* 0x7FFFFFFF, and the *version group ID* 0xFFFFFFFF, are reserved for use in experimental extensions to *transaction* format or semantics on private testnets. They **MUST NOT** be used on the **Zcash Mainnet** or *Testnet*.
- Note that a future upgrade might use *any transaction version number or version group ID*. It is likely that an upgrade that changes the *transaction version number or version group ID* will also change the *transaction* format, and software that parses *transactions* **SHOULD** take this into account.
- [Overwinter onward] The purpose of *version group ID* is to allow unambiguous parsing of “loose” *transactions*, independent of the context of a *block chain*. Code that parses *transactions* is likely to be reused between *consensus branches* as defined in [ZIP-200], and in that case the `fOverwintered` and *version* fields alone may be insufficient to determine the format to be used for parsing.
- A *transaction version number* of 2 does not have the same meaning as in **Bitcoin**, where it is associated with support for OP_CHECKSEQUENCEVERIFY as specified in [BIP-68]. **Zcash** was forked from Bitcoin Core v0.11.2 and does not currently support BIP 68.
- [Sapling onward] Because *coinbase transactions* have no *Spend descriptions*, the `valueBalanceSapling` field of a *coinbase transaction* must have a negative or zero value. The negative case can only occur after **Heartwood** activation, for *transactions* with [ZIP-213] *shielded outputs*.

- Prior to the **Heartwood** network upgrade, it was not possible for *coinbase transactions* to have *shielded* outputs, and therefore the “coinbase maturity” rule and the requirement to spend coinbase outputs only in *transactions with no transparent outputs*, applied to *all* coinbase outputs.
- [Canopy onward] The rule that **Sapling** outputs in *coinbase transactions* **MUST** decrypt to a *note plaintext* with lead byte 0x02, also applies to *funding stream* outputs that specify **Sapling** *shielded payment addresses*, if there are any.
- [NU5 onward] The flags in `flagsOrchard` allow a version 5 *transaction* to declare that no funds are spent from **Orchard** *notes* (by setting `enableSpendsOrchard` to 0), or that no new **Orchard** *notes* with nonzero values are created (by setting `enableOutputsOrchard` to 0). This has two primary purposes. First, the `enableSpendsOrchard` flag is set to 0 in version 5 *coinbase transactions* to ensure that they cannot spend from existing **Orchard** outputs. This maintains a restriction present in *coinbase transactions* for *transparent*, **Sprout**, or **Sapling** funds, which would not otherwise be enforceable in the combined *Action transfer* design. Second, if a security vulnerability were found that affected only the input side, or only the output side of the *Action circuit*, it would be possible to use these flags in a soft fork (i.e. a strictly contracting consensus change) to effectively “switch off” non-zero-valued transfers only on the relevant side. Setting either of these flags to 0 does not affect the presence or validation of *spend authorization signatures*, or other consensus rules associated with *Action descriptions*. These *note* spending and creation consensus rules are specified as part of the **Orchard** *Action statement* (§ 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61).
- [NU5 onward] Because `enableSpendsOrchard` is set to 0 in version 5 *coinbase transactions* –which disables non-zero-valued **Orchard** spends– the `valueBalanceOrchard` field of a *coinbase transaction* must have a negative or zero value. The negative case can only occur for *transactions* with [ZIP-213] *shielded outputs*.
- [NU5 onward] The rule that `nSpendsSapling`, `nOutputsSapling`, and `nActionsOrchard` **MUST** all be less than 2^{16} , is technically redundant because a *transaction* that could violate this rule would not fit within the 2 MB block size limit. It is included in order to simplify the security argument for balance preservation.
- [NU5 onward] The rule that from NU5 activation, the `nExpiryHeight` field of a *coinbase transaction* **MUST** be equal to the *block height*, is needed to maintain the property that all *transactions* have unique *transaction IDs*. All non-coinbase *transactions* necessarily have some effecting data that is unique across all *transactions* in a *valid block chain*: either a `tx_in` referring to a previous unique `tx_out`, or a *Spend description* or *Action description* referring to a unique *nullifier*. However, *coinbase transactions* do not necessarily have any such unique *effecting* data; the *block height* encoded in the coinbase script is unique in a *valid block chain*, but for v5 *transactions*, it is not included in the *transaction ID* hash specified by [ZIP-244]. Requiring `nExpiryHeight` to be set to the *block height* ensures that the effecting data that contributes to the *transaction ID* is unique, even for v5 *coinbase transactions*. In order to avoid the *block height* being limited to 499999999, we also remove that bound on `nExpiryHeight` for *coinbase transactions*. For consistency, these changes apply to *all* *coinbase transactions*, not just v5 *coinbase transactions*.

The changes relative to **Bitcoin** version 1 *transactions* as described in [Bitcoin-Format] are:

- *Transaction version* 0 is not supported.
- A version 1 *transaction* is equivalent to a version 2 *transaction* with `nJoinSplit` = 0.
- The fields `nJoinSplit`, `vJoinSplit`, `joinSplitPubKey`, and `joinSplitSig` have been added.
- [Overwinter onward] The field `nVersionGroupId` has been added.
- [Sapling onward] The following fields have been added: `nSpendsSapling`, `vSpendsSapling`, `nOutputsSapling`, `vOutputsSapling`, and `bindingSigSapling`.
- [NU5 onward] In version 5 *transactions*, these fields have been added: `nConsensusBranchId`, `nActionsOrchard`, `vActionsOrchard`, `flagsOrchard`, `valueBalanceOrchard`, `anchorOrchard`, `sizeProofsOrchard`, `proofsOrchard`, `bindingSigOrchard`, and `vSpendAuthSigsOrchard`.
- In **Zcash** it is permitted for a *transaction* to have no *transparent inputs*, provided at least one of `nJoinSplit`, `nSpendsSapling`, `nOutputsSapling`, and `nActionsOrchard` are nonzero.
- A consensus rule limiting *transaction* size has been added. In **Bitcoin** there is a corresponding standard rule but no consensus rule.

7.2 JoinSplit Description Encoding and Consensus

An abstract *JoinSplit description*, as described in § 3.5 ‘*JoinSplit Transfers and Descriptions*’ on p. 18, is encoded in a *transaction* as an instance of a `JoinSplitDescription` type:

Bytes	Name	Data Type	Description
8	<code>vpub_old</code>	<code>uint64</code>	A value $v_{\text{pub}}^{\text{old}}$ that the <i>JoinSplit</i> transfer removes from the <i>transparent transaction value pool</i> .
8	<code>vpub_new</code>	<code>uint64</code>	A value $v_{\text{pub}}^{\text{new}}$ that the <i>JoinSplit</i> transfer inserts into the <i>transparent transaction value pool</i> .
32	<code>anchor</code>	<code>byte[32]</code>	A root rt^{Sprout} of the Sprout note commitment tree at some <i>block height</i> in the past, or the root produced by a previous <i>JoinSplit</i> transfer in this <i>transaction</i> .
64	<code>nullifiers</code>	<code>byte[32] [N^{old}]</code>	A sequence of <i>nullifiers</i> of the input notes $nf_{1..N}^{\text{old}}$.
64	<code>commitments</code>	<code>byte[32] [N^{new}]</code>	A sequence of <i>note commitments</i> for the output notes $cm_{1..N}^{\text{new}}$.
32	<code>ephemeralKey</code>	<code>byte[32]</code>	A Curve25519 <i>public key</i> <code>epk</code> .
32	<code>randomSeed</code>	<code>byte[32]</code>	A 256-bit seed that must be chosen independently at random for each <i>JoinSplit description</i> .
64	<code>vmacs</code>	<code>byte[32] [N^{old}]</code>	A sequence of message authentication tags $h_{1..N}^{\text{old}}$ binding h_{sig} to each a_{sk} of the <i>JoinSplit description</i> , computed as described in § 4.11 ‘ <i>Non-malleability (Sprout)</i> ’ on p. 50.
296 †	<code>zkproof</code>	<code>byte[296]</code>	An encoding of the <i>zk-SNARK proof</i> $\pi_{\text{ZKJoinSplit}}$ (see § 5.4.10.1 ‘BCTV14’ on p. 108).
192 ‡	<code>zkproof</code>	<code>byte[192]</code>	An encoding of the <i>zk-SNARK proof</i> $\pi_{\text{ZKJoinSplit}}$ (see § 5.4.10.2 ‘Groth16’ on p. 109).
1202	<code>encCiphertexts</code>	<code>byte[601] [N^{new}]</code>	A sequence of ciphertext components for the encrypted output notes, $C_{1..N}^{\text{enc}}$.

† BCTV14 proofs are used when the *transaction version* is 2 or 3, i.e. before **Sapling** activation.

‡ Groth16 proofs are used when the *transaction version* is ≥ 4 , i.e. after **Sapling** activation.

The `ephemeralKey` and `encCiphertexts` fields together form the *transmitted notes ciphertext*, which is computed as described in § 4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63.

Consensus rules applying to a *JoinSplit description* are given in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38.

7.3 Spend Description Encoding and Consensus

Let `LEBS2OSP` be as defined in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

Let `reprJ` and `qJ` be as defined in § 5.4.9.3 ‘Jubjub’ on p. 100.

Let `spendAuthSig` be the *spend authorization signature* for this *Spend transfer*, and let π_{ZKSpnd} be the *zk-SNARK proof* of the corresponding *Spend statement*. In a version 4 *transaction* these are encoded in the `spendAuthSig` field and `zkproof` field respectively of the *Spend description*. In a version 5 *transaction*, *spend authorization signatures* in `vSpendAuthSigsSapling` and *proofs* in `vSpendProofsSapling` are in one-to-one correspondence with *Spend descriptions* in `vSpendsSapling`.

7.6 Block Header Encoding and Consensus

The **Zcash** *block header* format is as follows (this should be read in the context of consensus rules later in the section):

Bytes	Name	Data Type	Description
4	nVersion	int32	The <i>block version number</i> indicates which set of <i>block</i> validation rules to follow. The current and only defined <i>block version number</i> for Zcash is 4.
32	hashPrevBlock	byte[32]	A SHA-256d hash in internal byte order of the previous <i>block's header</i> . This ensures no previous <i>block</i> can be changed without also changing this <i>block's header</i> .
32	hashMerkleRoot	byte[32]	A SHA-256d hash in internal byte order. The merkle root is derived from the hashes of all <i>transactions</i> included in this <i>block</i> , ensuring that none of those <i>transactions</i> can be modified without modifying the <i>header</i> .
32	hashReserved / hashFinalSaplingRoot / hashLightClientRoot / hashBlockCommitments	byte[32]	[Pre-Sapling] A reserved field, to be ignored. [Sapling and Blossom only, pre-Heartwood] The root $\text{LEBS2OSP}_{256}(\text{rt}^{\text{Sapling}})$ of the Sapling <i>note commitment tree</i> corresponding to the final Sapling <i>treestate</i> of this <i>block</i> . [Heartwood and Canopy only, pre-NU5] The <i>hashChainHistoryRoot</i> of this <i>block</i> [ZIP-221]. [NU5 onward] The <i>hashBlockCommitments</i> of this <i>block</i> [ZIP-244].
4	nTime	uint32	The <i>block timestamp</i> is a Unix epoch time (UTC) when the miner started hashing the <i>header</i> (according to the miner).
4	nBits	uint32	An encoded version of the <i>target threshold</i> this <i>block's header</i> hash must be less than or equal to, in the same nBits format used by Bitcoin . [Bitcoin-nBits]
32	nNonce	byte[32]	An arbitrary field that miners can change to modify the <i>header</i> hash in order to produce a hash less than or equal to the <i>target threshold</i> .
3	solutionSize	compactSize	The size of an <i>Equihash</i> solution in bytes (always 1344).
1344	solution	byte[1344]	The <i>Equihash</i> solution.

A *block* consists of a *block header* and a sequence of *transactions*. How *transactions* are encoded in a *block* is part of the Zcash *peer-to-peer protocol* but not part of the consensus protocol.

Let *ThresholdBits* be as defined in § 7.7.3 ‘*Difficulty adjustment*’ on p. 131, and let *PoWMedianBlockSpan* be the constant defined in § 5.3 ‘*Constants*’ on p. 72.

Define the *median-time-past* of a *block* to be the median (as defined in § 7.7.3 ‘*Difficulty adjustment*’ on p. 131) of the *nTime* fields of the *preceding* *PoWMedianBlockSpan* *blocks* (or all preceding *blocks* if there are fewer than *PoWMedianBlockSpan*). The *median-time-past* of a *genesis block* is not defined.

Consensus rules:

- The *block version number* **MUST** be greater than or equal to 4.
- For a *block* at *block height* *height*, *nBits* **MUST** be equal to `ThresholdBits(height)`.
- The *block* **MUST** pass the difficulty filter defined in §7.7.2 ‘*Difficulty filter*’ on p. 131.
- *solution* **MUST** represent a *valid Equihash solution* as defined in §7.7.1 ‘*Equihash*’ on p. 130.
- For each *block* other than the *genesis block*, *nTime* **MUST** be strictly greater than the *median-time-past* of that *block*.
- For each *block* at *block height* 2 or greater on *Mainnet*, or *block height* 653606 or greater on *Testnet*, *nTime* **MUST** be less than or equal to the *median-time-past* of that *block* plus $90 \cdot 60$ seconds.
- The size of a *block* **MUST** be less than or equal to 2000000 bytes.
- [Sapling and Blossom only, pre-Heartwood] *hashLightClientRoot* **MUST** be $\text{LEBS2OSP}_{256}(\text{rt}^{\text{Sapling}})$ where $\text{rt}^{\text{Sapling}}$ is the root of the Sapling note commitment tree for the final Sapling treestate of this *block*.
- [Heartwood and Canopy only, pre-NU5] *hashLightClientRoot* **MUST** be set to the *hashChainHistoryRoot* for this *block*, as specified in [ZIP-221].
- [NU5 onward] *hashBlockCommitments* **MUST** be set to the value of *hashBlockCommitments* for this *block*, as specified in [ZIP-244].
- A *block* **MUST** have at least one *transaction*.
- The first *transaction* in a *block* **MUST** be a *coinbase transaction*, and subsequent *transactions* **MUST NOT** be *coinbase transactions*.
- TODO: Other rules inherited from Bitcoin.

In addition, a *full validator* **MUST NOT** accept *blocks* with *nTime* more than two hours in the future according to its clock. This is not strictly a consensus rule because it is nondeterministic, and clock time varies between nodes. Also note that a *block* that is rejected by this rule at a given point in time may later be accepted.

Notes:

- The semantics of *blocks* with *block version number* not equal to 4 is not currently defined. Miners **MUST NOT** create such *blocks*.
- The exclusion of *blocks* with *block version number* *greater than* 4 is not a consensus rule; such *blocks* may exist in the *block chain* and **MUST** be treated identically to version 4 *blocks* by *full validators*. Note that a future upgrade might use *block version number* either greater than or less than 4. It is likely that such an upgrade will change the *block* header and/or *transaction* format, and software that parses *blocks* **SHOULD** take this into account.
- The *nVersion* field is a signed integer. (It was specified as unsigned in a previous version of this specification.) A future upgrade might use negative values for this field, or otherwise change its interpretation.
- There is no relation between the values of the *version* field of a *transaction*, and the *nVersion* field of a *block header*.
- Like other serialized fields of type *compactSize*, the *solutionSize* field **MUST** be encoded with the minimum number of bytes (3 in this case), and other encodings **MUST** be rejected. This is necessary to avoid a potential attack in which a miner could test several distinct encodings of each *Equihash* solution against the difficulty filter, rather than only the single intended encoding.
- As in **Bitcoin**, the *nTime* field **MUST** represent a time *strictly greater than* the median of the timestamps of the past `PoWMedianBlockSpan` *blocks*. The Bitcoin Developer Reference [Bitcoin-Block] was previously in error on this point, but has now been corrected.
- The rule limiting *nTime* to be no later than $90 \cdot 60$ seconds after the *median-time-past* is a retrospective consensus change, applied as a soft fork in `zcashd` v2.1.1-1. It had not been violated by any *block* from the given *block heights* in the consensus *block chains* of either *Mainnet* or *Testnet*.
- There are no changes to the *block version number* or format for **Overwinter**.

- Although the *block version number* does not change for **Sapling**, the previously reserved (and ignored) field `hashReserved` has been repurposed for `hashFinalSaplingRoot`. There are no other format changes.
- There are no changes to the *block version number* or format for **Blossom**.
- For **Heartwood**, the `hashFinalSaplingRoot` field is renamed to `hashLightClientRoot`. Once **Heartwood** activates, the meaning of this field changes according to [ZIP-221].
- There are no changes to the *block version number* or format for **Canopy**.
- For **NU5**, the `hashLightClientRoot` field is renamed to `hashBlockCommitments`. Once **NU5** activates, the meaning of this field changes according to [ZIP-244].
- There are no changes to the *block version number* or format for **NU6**.

The changes relative to **Bitcoin** version 4 blocks as described in [Bitcoin-Block] are:

- *Block versions* less than 4 are not supported.
- The `hashReserved` / `hashFinalSaplingRoot` / `hashLightClientRoot` / `hashBlockCommitments`, `solutionSize`, and `solution` fields have been added.
- The type of the `nNonce` field has changed from `uint32` to `byte[32]`.
- The maximum *block* size has been doubled to 2000000 bytes.

7.7 Proof of Work

Zcash uses *Equihash* [BK2016] as its *proof-of-work*. The original motivations for changing the *proof-of-work* from SHA-256d used by **Bitcoin** were described in [WG2016].

A *block* satisfies the *proof-of-work* if and only if:

- The `solution` field encodes a *valid Equihash solution* according to § 7.7.1 ‘*Equihash*’ on p. 130.
- The *block header* satisfies the difficulty check according to § 7.7.2 ‘*Difficulty filter*’ on p. 131.

7.7.1 Equihash

An instance of the *Equihash* algorithm is parameterized by positive integers n and k , such that n is a multiple of $k + 1$. We assume $k \geq 3$.

The *Equihash* parameters for *Mainnet* and *Testnet* are $n = 200, k = 9$.

Equihash is based on a variation of the Generalized Birthday Problem [AR2017]: given a sequence $X_{1..N}$ of n -bit strings, find 2^k distinct X_{i_j} such that $\bigoplus_{j=1}^{2^k} X_{i_j} = 0$.

In *Equihash*, $N = 2^{\frac{n}{k+1}+1}$, and the sequence $X_{1..N}$ is derived from the *block header* and a nonce.

Let `powheader` :=

32-bit <code>nVersion</code>	256-bit <code>hashPrevBlock</code>	256-bit <code>hashMerkleRoot</code>		
256-bit <code>hashReserved</code>	32-bit <code>nTime</code>	32-bit <code>nBits</code>	256-bit <code>nNonce</code>	

For $i \in \{1..N\}$, let $X_i = \text{EquihashGen}_{n,k}(\text{powheader}, i)$.

EquihashGen is instantiated in § 5.4.1.11 ‘*Equihash Generator*’ on p. 83.

Define $\text{l2BEBSP} : (\ell : \mathbb{N}) \times \{0..2^\ell - 1\} \rightarrow \mathbb{B}^{[\ell]}$ as in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

Generalized Birthday condition $\bigoplus_{j=1}^{2^k} X_{i_j} = 0$.

Algorithm Binding conditions

- For all $r \in \{1 \dots k-1\}$, for all $w \in \{0 \dots 2^{k-r}-1\}$: $\bigoplus_{j=1}^{2^r} X_{i_{w \cdot 2^r + j}}$ has $\frac{n \cdot r}{k+1}$ leading zeros; and
- For all $r \in \{1 \dots k\}$, for all $w \in \{0 \dots 2^{k-r}-1\}$: $i_{w \cdot 2^r + 1, w \cdot 2^r + 2^{r-1}} < i_{w \cdot 2^r + 2^{r-1} + 1, w \cdot 2^r + 2^r}$ lexicographically.

Notes:

- This does not include a difficulty condition, because here we are defining validity of an *Equi*hash solution independent of difficulty.
- Previous versions of this specification incorrectly specified the range of r to be $\{1 \dots k-1\}$ for both parts of the algorithm binding condition. The implementation in `zcashd` was as intended.

An *Equihash* solution with $n = 200$ and $k = 9$ is encoded in the solution field of a *block header* as follows:

$\text{I2BEBSP}_{21}(i_1 - 1)$	$\text{I2BEBSP}_{21}(i_2 - 1)$	\dots	$\text{I2BEBSP}_{21}(i_{512} - 1)$
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Recall from § 5.2 ‘*Bit layout diagrams*’ on p. 71 that the bits in the above diagram are ordered from most to least significant in each byte. For example, if the first 3 elements of i are $[69, 42, 2^{21}]$, then the corresponding bit array is:

[illegible]

and so the first 7 bytes of `solution` would be `[0, 2, 32, 0, 10, 127, 255]`.

Note: I2BEBSP is *big-endian*, while integer field encodings in powheader and in the instantiation of EquihashGen are *little-endian*. The rationale for this is that *little-endian* serialization of *block headers* is consistent with **Bitcoin**, but *little-endian* ordering of bits in the solution encoding would require bit-reversal (as opposed to only shifting).

7.7.2 Difficulty filter

Let ToTarget be as defined in §7.7.4 ‘*nBits conversion*’ on p.133.

Difficulty is defined in terms of a *target threshold*, which is adjusted for each *block* according to the algorithm defined in §7.7.3 ‘*Difficulty adjustment*’ on p. 131.

The difficulty filter is unchanged from **Bitcoin**, and is calculated using SHA-256d on the whole *block header* (including `solutionSize` and `solution`). The result is interpreted as a 256-bit integer represented in *little-endian* byte order, which **MUST** be less than or equal to the *target threshold* given by `ToTarget(nBits)`.

7.7.3 Difficulty adjustment

The desired time between *blocks* is called the *block target spacing*. **Zcash** uses a difficulty adjustment algorithm based on DigiShield v3/v4 [DigiByte-PoW], with simplifications and altered parameters, to adjust difficulty to target the desired *block target spacing*. Unlike **Bitcoin**, the difficulty adjustment occurs after every *block*.

The constants `PoWLimit`, `PreBlossomHalvingInterval`, `PoWAveragingWindow`, `PoWMaxAdjustDown`, `PoWMaxAdjustUp`, `PoWDampingFactor`, `PreBlossomPoWTargetSpacing`, and `PostBlossomPoWTargetSpacing` are specified in section §5.3 ‘*Constants*’ on p.72.

Let ToCompact and ToTarget be as defined in §7.7.4 ‘*nBits conversion*’ on p. 133.

Let nTime(height) be the value of the nTime field in the *header* of the *block* at *block height* height.

Let nBits(height) be the value of the nBits field in the *header* of the *block* at *block height* height.

Block header fields are specified in §7.6 ‘*Block Header Encoding and Consensus*’ on p. 128.

Define:

$$\text{mean}(S) := \frac{\sum_{i=1}^{\text{length}(S)} S_i}{\text{length}(S)}$$

$$\text{median}(S) := \text{sorted}(S)_{\text{ceiling}((\text{length}(S)+1)/2)}$$

$$\text{bound}_{\text{lower}}^{\text{upper}}(x) := \max(\text{lower}, \min(\text{upper}, x))$$

$$\text{trunc}(x) := \begin{cases} \text{floor}(x), & \text{if } x \geq 0 \\ -\text{floor}(-x), & \text{otherwise} \end{cases}$$

$$\text{IsBlossomActivated}(\text{height} : \mathbb{N}) := (\text{height} \geq \text{BlossomActivationHeight})$$

$$\text{BlossomPoWTargetSpacingRatio} := \frac{\text{PreBlossomPoWTargetSpacing}}{\text{PostBlossomPoWTargetSpacing}}$$

$$\text{PostBlossomHalvingInterval} := \text{floor}(\text{PreBlossomHalvingInterval} \cdot \text{BlossomPoWTargetSpacingRatio})$$

$$\text{PoWTargetSpacing}(\text{height} : \mathbb{N}) := \begin{cases} \text{PreBlossomPoWTargetSpacing}, & \text{if not IsBlossomActivated}(\text{height}) \\ \text{PostBlossomPoWTargetSpacing}, & \text{otherwise} \end{cases}$$

$$\text{AveragingWindowTimespan}(\text{height} : \mathbb{N}) := \text{PoWAveragingWindow} \cdot \text{PoWTargetSpacing}(\text{height})$$

$$\text{MinActualTimespan}(\text{height} : \mathbb{N}) := \text{floor}(\text{AveragingWindowTimespan}(\text{height}) \cdot (1 - \text{PoWMaxAdjustUp}))$$

$$\text{MaxActualTimespan}(\text{height} : \mathbb{N}) := \text{floor}(\text{AveragingWindowTimespan}(\text{height}) \cdot (1 + \text{PoWMaxAdjustDown}))$$

$$\text{MedianTime}(\text{height} : \mathbb{N}) := \text{median}([\text{nTime}(i) \text{ for } i \text{ from } \max(0, \text{height} - \text{PoWMedianBlockSpan}) \text{ up to } \text{height} - 1])$$

$$\text{ActualTimespan}(\text{height} : \mathbb{N}) := \text{MedianTime}(\text{height}) - \text{MedianTime}(\text{height} - \text{PoWAveragingWindow})$$

$$\text{ActualTimespanDamped}(\text{height} : \mathbb{N}) :=$$

$$\text{AveragingWindowTimespan}(\text{height}) + \text{trunc}\left(\frac{\text{ActualTimespan}(\text{height}) - \text{AveragingWindowTimespan}(\text{height})}{\text{PoWDampingFactor}}\right)$$

$$\text{ActualTimespanBounded}(\text{height} : \mathbb{N}) := \text{bound}_{\text{MinActualTimespan}(\text{height})}^{\text{MaxActualTimespan}(\text{height})}(\text{ActualTimespanDamped}(\text{height}))$$

$$\text{MeanTarget}(\text{height} : \mathbb{N}) := \begin{cases} \text{PoWLimit}, & \text{if } \text{height} \leq \text{PoWAveragingWindow} \\ \text{mean}([\text{ToTarget}(\text{nBits}(i)) \text{ for } i \text{ from } \text{height} - \text{PoWAveragingWindow} \text{ up to } \text{height} - 1]), & \text{otherwise.} \end{cases}$$

The *target threshold* for a given *block height* height is then calculated as:

$$\text{Threshold}(\text{height} : \mathbb{N}) := \begin{cases} \text{PoWLimit}, & \text{if } \text{height} = 0 \\ \min(\text{PoWLimit}, \text{floor}\left(\frac{\text{MeanTarget}(\text{height})}{\text{AveragingWindowTimespan}}\right) \cdot \text{ActualTimespanBounded}(\text{height})), & \text{otherwise} \end{cases}$$

$$\text{ThresholdBits}(\text{height} : \mathbb{N}) := \text{ToCompact}(\text{Threshold}(\text{height})).$$

Notes:

- The convention used for the height parameters to the functions MedianTime, MeanTarget, ActualTimespan, ActualTimespanDamped, ActualTimespanBounded, Threshold, and ThresholdBits is that these functions use only information from *blocks preceding* the given *block height*.

- When the median function is applied to a sequence of even length (which only happens in the definition of MedianTime during the first PoWAveragingWindow – 1 blocks of the block chain), the element that begins the second half of the sequence is taken. This corresponds to the zcashd implementation, but was not specified correctly in versions of this specification prior to v2019.0.0.

On *Testnet* from *block height* 299188 onward, the difficulty adjustment algorithm is changed to allow minimum-difficulty blocks, as described in [ZIP-205]. The **Blossom network upgrade changes the minimum-difficulty time threshold to 6 times the block target spacing**, as described in [ZIP-208]. These changes do not apply to *Mainnet*.

7.7.4 nBits conversion

Deterministic conversions between a *target threshold* and a “compact” nBits value are not fully defined in the Bitcoin documentation [Bitcoin-nBits], and so we define them here:

$$\begin{aligned} \text{size}(x) &:= \text{ceiling}\left(\frac{\text{bitlength}(x)}{8}\right) \\ \text{mantissa}(x) &:= \text{floor}\left(x \cdot 256^{3-\text{size}(x)}\right) \\ \text{ToCompact}(x) &:= \begin{cases} \text{mantissa}(x) + 2^{24} \cdot \text{size}(x), & \text{if } \text{mantissa}(x) < 2^{23} \\ \text{floor}\left(\frac{\text{mantissa}(x)}{256}\right) + 2^{24} \cdot (\text{size}(x) + 1), & \text{otherwise} \end{cases} \\ \text{ToTarget}(x) &:= \begin{cases} 0, & \text{if } x \& 2^{23} = 2^{23} \\ (x \& (2^{23} - 1)) \cdot 256^{\text{floor}(x/2^{24})-3}, & \text{otherwise.} \end{cases} \end{aligned}$$

7.7.5 Definition of Work

As explained in § 3.3 ‘*The Block Chain*’ on p.16, a node chooses the “best” *block chain* visible to it by finding the chain of valid *blocks* with the greatest total work.

Let ToTarget be as defined in § 7.7.4 ‘*nBits conversion*’ on p.133.

The work of a *block* with value nBits for the nBits field in its *block header* is defined as $\text{floor}\left(\frac{2^{256}}{\text{ToTarget}(\text{nBits}) + 1}\right)$.

7.8 Calculation of Block Subsidy, Funding Streams, and Founders’ Reward

In § 3.10 ‘*Block Subsidy, Funding Streams, and Founders’ Reward*’ on p.21 the *block subsidy*, *miner subsidy*, *Founders’ Reward*, and *funding streams* are defined. Their amounts in *zatoshi* are calculated from the *block height* using the formulae below.

Let the constants SlowStartInterval, PreBlossomHalvingInterval, PostBlossomHalvingInterval, BlossomActivationHeight, MaxBlockSubsidy, and FoundersFraction be as defined in § 5.3 ‘*Constants*’ on p.72.

Let FundingStreams be as specified in § 7.10.1 ‘*ZIP 214 Funding Streams*’ on p.137.

$$\begin{aligned} \text{SlowStartShift} : \mathbb{N} &:= \frac{\text{SlowStartInterval}}{2} \\ \text{SlowStartRate} : \mathbb{N} &:= \frac{\text{MaxBlockSubsidy}}{\text{SlowStartInterval}} \\ \text{Halving}(\text{height} : \mathbb{N}) &:= \begin{cases} 0, & \text{if } \text{height} < \text{SlowStartShift} \\ \text{floor}\left(\frac{\text{height} - \text{SlowStartShift}}{\text{PreBlossomHalvingInterval}}\right), & \text{if not IsBlossomActivated}(\text{height}) \\ \text{floor}\left(\frac{\text{BlossomActivationHeight} - \text{SlowStartShift}}{\text{PreBlossomHalvingInterval}} + \frac{\text{height} - \text{BlossomActivationHeight}}{\text{PostBlossomHalvingInterval}}\right), & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{BlockSubsidy}(\text{height} : \mathbb{N}) := \begin{cases} \text{SlowStartRate} \cdot \text{height}, & \text{if } \text{height} < \text{SlowStartShift} \\ \text{SlowStartRate} \cdot (\text{height} + 1), & \text{if } \text{SlowStartShift} \leq \text{height} \text{ and } \text{height} < \text{SlowStartInterval} \\ \text{floor}\left(\frac{\text{MaxBlockSubsidy}}{2^{\text{Halving}(\text{height})}}\right), & \text{if } \text{SlowStartInterval} \leq \text{height} \text{ and not } \text{IsBlossomActivated}(\text{height}) \\ \text{floor}\left(\frac{\text{MaxBlockSubsidy}}{\text{BlossomPoWTargetSpacingRatio} \cdot 2^{\text{Halving}(\text{height})}}\right), & \text{otherwise} \end{cases}$$

$$\text{FoundersReward}(\text{height} : \mathbb{N}) := \begin{cases} \text{BlockSubsidy}(\text{height}) \cdot \text{FoundersFraction}, & \text{if } \text{Halving}(\text{height}) < 1 \\ 0, & \text{otherwise} \end{cases}$$

for $\text{fs} \in \text{FundingStreams}$, $\text{fs.Value}(\text{height}) :=$

$$\begin{cases} 0, & \text{if } \text{height} < \text{CanopyActivationHeight} \\ \text{floor}\left(\text{BlockSubsidy}(\text{height}) \cdot \frac{\text{fs.Numerator}}{\text{fs.Denominator}}\right), & \text{if } \text{fs.StartHeight} \leq \text{height} \text{ and } \text{height} < \text{fs.EndHeight} \\ 0, & \text{otherwise} \end{cases}$$

$$\text{totalDeferredOutput}(\text{height}) := \sum_{\text{fs} \in \text{FundingStreams} : \text{fs.Recipient}(\text{height}) = \text{DEFERRED_POOL}} \text{fs.Value}(\text{height})$$

$$\text{MinerSubsidy}(\text{height}) := \text{BlockSubsidy}(\text{height}) - \text{FoundersReward}(\text{height}) - \sum_{\text{fs} \in \text{FundingStreams}} \text{fs.Value}(\text{height}).$$

7.9 Payment of Founders' Reward

The *Founders' Reward* is paid by a *transparent output* in the *coinbase transaction*, to one of `NumFounderAddresses` *transparent addresses*, depending on the *block height*.

For *Mainnet*, `FounderAddressList`_{1..NumFounderAddresses} is:

```
[ "t3Vz22vK5z2LcKEdg16Yv4FFneEL1zg9oJd", "t3cL9AucCajm3HXDhb5jBnJK2vapVoXsop3",
  "t3fqvkzrrNaMcamlQMwAyHRjfdM2xQvDTR", "t3TgZ9ZT2CTSK44AnUPi6qeNaHa2eC7pUyF",
  "t3SpkcPQPfuRYHsP5vz3Pv86PgKo5m9KVmx", "t3Xt4oQMRPagwbpQqkgAViQgtST4VoSWR6S",
  "t3ayBkZ4w6kKXynwoHZFUSSgXRktogTXngb", "t3adJBQuaa21u7NxbR8YMzp3km3TbSZ4MGB",
  "t3K4aLYagSSBySdrfAGGeUd5H9z5Qvz88t2", "t3RYnsc5nhEvKiva3ZPhfRSk7eyh1CrA6Rk",
  "t3Ut4KUq2ZSMTPNE67pBU5LqYCi2q36KpXQ", "t3ZnCNAvgu6CSyHm1vWtrx3aiN98dSAGpnD",
  "t3fB9cB3eSYim64BS9xfwAHQUKLGqQroBDG", "t3cwZfKNNj2vXMAHBQeewm6pXhkFdhk18kD",
  "t3YcoujXfspNy7rbNUSGKxFEWZqNstGpeG4", "t3bLvCLigc6rbNrUTS5NwkgYVrZcZumTra4",
  "t3VvHwA7r3oy67YtU4LZKGCWa2J6eGHvShi", "t3eF9X6X2dSo7MCvTjffZEzWwVzquxRLNeY",
  "t3esCNwmmcyc8i9qQfyTbYhTqmYXZ9AwK3X", "t3M4jN7hYE2e27yLsuQPPjuVek81WV3VbBj",
  "t3gGWxdC67CYNoBbPjNvrrWLAwxPqZLxrVY", "t3LTweoxeWPbmdkUD3NWBquk4WkazhFBmvU",
  "t3P5KKX97gXYFSaSJPIruQEX84yF5z3Tjq", "t3f3T3nCWsEpzmD35VK62JgQfFig74dV8C9",
  "t3Rqonuzz7afkF7156ZA4vi4iimRSEn41hj", "t3fJZ5jYsyxDtvNrWBeoMbvJaQCj4JJgbgX",
  "t3Pnbg7XjP7FGPBUuz75H65aczphHgkpoJW", "t3WeKQDxCijL5X7rwFem1MTL9ZwVJkUFhpF",
  "t3Y9FNi26J7UtAUC4moaETLbMo8KS1Be6ME", "t3aNRLlsL2y8xcjPheZZwFy3Pcv7CsTwBec",
  "t3gQDEavk5VzAAHK8TrQu2BwDLxEiF1unBm", "t3Rbykhx1TUFrgXrmBYrAJe2STxRKFL7G9r",
  "t3aaW4aTdP7a8d1VTE1Bod2yhbeggHgMajR", "t3YEiAa6uEjXwFL2v5ztU1fn3yKgZMQqNyo",
  "t3g1yUWt2PbmDvMDevTCPWUcbDatL2iQGP", "t3dPwneP6YqGPuY1CecgbeZrY9iUwH8Yd4z",
  "t3QRZXHDP2hwU46iQs2776kRuuWfwFp4dV", "t3enhACRxi1ZD7e8ePomVGKn7wp7N9ffJ3r",
  "t3PkLgT71TnF112nSwBToXsD77yNbx2gJJY", "t3LQtHUDoe7ZhhvddRv4vnaoNAhCr2f4oFN",
  "t3fNcdBUbycvbCtsD2n9q3LuxG7jVPvFB8L", "t3dKojUU2EMjs28nHV84TvKVEUDu1M1FaEx",
  "t3aKH6NiWN1ofGd8c19rZiqgYpkJ3n679ME", "t3MEXDF9Wsi63KwpPuQdD6by32Mw2bNTbEa",
  "t3WDhPfik343yNmpTqtKZAoQZeqA83K7Y3f", "t3PSn5TbMMAEw7Eu36DYctFezRzpX1hzf3M",
  "t3R3Y5vnBLrEn8L6wFjPjBLnxSUQsKnmFpv", "t3Pcm737EsVkgTbhsu2NekKtJeG92mvYyoN" ]
```


For *Testnet*, `FounderAddressList`_{1..NumFounderAddresses} is:

```
[ "t2UNzUUx8mWBCRYPrezvA363EYXyEpHokyI", "t2N9PH9Wk9xjqYg9iin1Ua3aekJqfAtE543",
  "t2NGQjYMQhFndDHgUvUw4wZdNdsssa6K7x2", "t2ENG7hHVqqs9JwU5cgjvSbxnT2a9USNfhy",
  "t2BkYdVCHzvTJJUTx4yZB8qeegD8QsPx8bo", "t2J8q1xH1EuigJ52MfExyyjYtN3VgvsKdF",
  "t2Crq9mydTM37kZokC68HzT6yez3t2FBnFj", "t2EaMPUiQ1kthqcP5UEkF42CAFKJqXCkXC9",
  "t2F9dtQc63JDDyrhnfpzvVYTJcr57MkqA12", "t2LPirmnfYSZc481GgZBa6xUGcoovfytBnC",
  "t26xfxoSw2UV9Pe5o3C8V4YybQD4SESfxtP", "t2D3k4fNdErd66YxtvXEdft9xuLoKD7CcVo",
  "t2DWYBkxKNivdmsMiiVnJzutaQGqmoRjRnL", "t2C3kFF9iQRxfc4B9zgbWo4dQLLqzqjpuGQ",
  "t2MnT5tzU9HSKcppRyUNwoTp8MUueuSGNaB", "t2AREsWdoW1F8EQYsScsJkgqobmgrkKeUkK",
  "t2Vf4wKcJ3ZFtLj4jezUUKwYR92BLHn5UT", "t2K3fdViH6R5tRuXLphKyoYXyZhyWGghDNY",
  "t2VEn3KiKyHSGydz3nDw6ESWtaCQHwuv9WC", "t2F8XouqdNMq6zzEvxQXHV1TjwZRHwRg8gC",
  "t2BS7Mrbaef3fA4xrmkvDisFVXvRbNz6QJ", "t2FuSwoLCdBVPwdZuYoHrEzxAb9qy4qjbnL",
  "t2SX3U8NtrT6gz5Db1AtQCSGjrpptR8JC6h", "t2V51gZNSoJ5kRL74bf9YTtbZuv8Fcqx2FH",
  "t2FyTsLjjdm4jeVwir4xzj7FAkUidbr1b4R", "t2EYbGLEkmpqHyn8UBF6kqpahrYm7D6N1Le",
  "t2NQTrStZhtJECNFT3dUBLYA9AerxPCmkka", "t2GSWZZJzoesYxfPTWXkFn5UaxjiYxGBU2a",
  "t2RpfkzyLRevGM3w9aWdqMX6bd8uuAK3vn", "t2JzjoQqnuXtTGSN7k7yk5keURBGvYofh1d",
  "t2AEefc72ieTnsXKmgK2bZNckiWvZe3oPNL", "t2Nns3ZGZFsNj2wvmVd8BSwSfvETgiLRd8J",
  "t2ECCQPvcxUCSSQopdNquguEPE14HsVfcUn", "t2JabDUkG8TaqVKYfqDJ3rqkVdHKp6hwXvG",
  "t2FGzW5Zdc8Cy98ZKmRygsVGi6oKcmYir9n", "t2DUD8a21FtEFn42oVlp5NGbogY13uyjy9t",
  "t2UjVsd3zheHPgAkux8WQW2CiC9xHQ8EvWp", "t2TBUAhELyHUn8i6SXYsXz5Lmy7kDzA1uT5",
  "t2Tz3uCyhP6eizUWdc3bGH7XUC9GQsEyQnc", "t2NysJSZtLwMLWEJ6MH3BsXRh6h27mNcsSy",
  "t2KXJVVyyrjVxxSeazbY9ksGyft4qsXUNm9", "t2J9YYtH31cveiLZzjaE4AcuwVho6qjTNzp",
  "t2QgVw4sP9zaGpPMH1GRzy7cpydmuRfB4AZ", "t2NDTJP9MosKpyFPHJmfjc5pGCvAU58XGa4",
  "t29pHDBWq7qN4EjwSEHg8wEqYe9pkmVrtRP", "t2Ez9KM8VJLuArcxuEkNRakhNvidKkzXcjJ",
  "t2D5y7J5fpXajLbGrMBQkFg2mFN8fo3n8cX", "t2UV2wr1PTaUiypbkV3FdSdGxUJeZdZztyt" ]
```

Note: For *Testnet* only, the addresses from index 4 onward have been changed from what was implemented at launch. This reflects an upgrade on *Testnet*, starting from *block height* 53127. [Zcash-Issue2113]

Each address representation in `FounderAddressList` denotes a *transparent P2SH multisig address*.

Let `SlowStartShift` and `Halving` be defined as in the previous section.

Define:

$$\text{FounderAddressChangeInterval} := \text{ceiling} \left(\frac{\text{SlowStartShift} + \text{PreBlossomHalvingInterval}}{\text{NumFounderAddresses}} \right)$$

$$\text{FounderAddressAdjustedHeight}(\text{height} : \mathbb{N}) :=$$

$$\begin{cases} \text{height}, & \text{if not IsBlossomActivated}(\text{height}), \\ \text{BlossomActivationHeight} + \text{floor} \left(\frac{\text{height} - \text{BlossomActivationHeight}}{\text{BlossomPoWTargetSpacingRatio}} \right), & \text{otherwise} \end{cases}$$

$$\text{FounderAddressIndex}(\text{height} : \mathbb{N}) := 1 + \text{floor} \left(\frac{\text{FounderAddressAdjustedHeight}(\text{height})}{\text{FounderAddressChangeInterval}} \right)$$

$$\text{FoundersRewardLastBlockHeight} := \max(\{\text{height} : \mathbb{N} \mid \text{Halving}(\text{height}) < 1\}).$$

Let `FounderRedeemScriptHash`(*height* : \mathbb{N}) be the *standard redeem script* hash, as specified in [Bitcoin-Multisig], for the *P2SH multisig address* with *Base58Check* form given by `FounderAddressList`_{`FounderAddressIndex`(*height*)}.

Consensus rule: [Pre-Canopy] A coinbase transaction at *height* $\in \{1 \dots \text{FoundersRewardLastBlockHeight}\}$ **MUST** include at least one output that pays exactly `FoundersReward`(*height*) *zatoshi* with a *standard P2SH script* of the form `OP_HASH160 FounderRedeemScriptHash`(*height*) `OP_EQUAL` as its *scriptPubKey*.

Notes:

- No *Founders' Reward* is required to be paid for height $>$ FoundersRewardLastBlockHeight (i.e. after the first halving), or for height $= 0$ (i.e. the genesis block), or after **Canopy** activation.
- The *Founders' Reward* addresses are not treated specially in any other way, and there can be other outputs to them, in *coinbase transactions* or otherwise. In particular, it is valid for a *coinbase transaction* with height $\in \{1 \dots \text{FoundersRewardLastBlockHeight}\}$ to have other outputs, possibly to the same address, that do not meet the criterion in the above consensus rule, as long as at least one output meets it.
- The assertion $\text{FounderAddressIndex}(\text{FoundersRewardLastBlockHeight}) \leq \text{NumFounderAddresses}$ holds, ensuring that the *Founders' Reward* address index remains in range for the whole period in which the *Founders' Reward* is paid.

Non-normative notes:

- **[Blossom onward]** FoundersRewardLastBlockHeight = 1046399.
- **Blossom** is not intended to change the total *Founders' Reward* or the effective period over which it is paid.

7.10 Payment of Funding Streams

Let PostBlossomHalvingInterval be as defined in § 5.3 ‘*Constants*’ on p. 72.

Let Halving be as defined in § 7.8 ‘*Calculation of Block Subsidy, Funding Streams, and Founders' Reward*’ on p. 133.

The *funding streams* are paid to one of a pre-defined set of recipients, depending on the *block height*. Each recipient identifier is either the string encoding of an address to be paid by an output in the *coinbase transaction*, or the identifier DEFERRED_POOL. The latter indicates that the value is to be paid to a reserve to be used for development funding, the distribution of which is to be determined via a future ZIP.

A *funding stream* fs is defined by a *block subsidy* fraction (represented as a numerator and denominator), a start *block height* (inclusive), an end *block height* (exclusive), and a sequence of recipients:

$fs.\text{Numerator} : \mathbb{N}^+$
 $fs.\text{Denominator} : \mathbb{N}^+$
 $fs.\text{StartHeight} : \mathbb{N}$
 $fs.\text{EndHeight} : \mathbb{N}$
 $fs.\text{Recipients} : (\mathbb{B}^{\mathbb{Y}[\mathbb{N}]} \cup \{\text{DEFERRED_POOL}\})^{[\mathbb{N}^+]}$.

Define:

$\text{HeightForHalving}(\text{halving} : \mathbb{N}^+) := \min(\{\text{height} : \mathbb{N} \mid \text{Halving}(\text{height}) = \text{halving}\})$
 $\text{FSRecipientChangeInterval} := \text{PostBlossomHalvingInterval} / 48$
 $\text{FSRecipientPeriod}(\text{height}) := \text{floor}\left(\frac{\text{height} - (\text{HeightForHalving}(1) - \text{PostBlossomHalvingInterval})}{\text{FSRecipientChangeInterval}}\right)$.

For each *funding stream* fs , define:

$fs.\text{RecipientIndex}(\text{height}) := 1 + \text{FSRecipientPeriod}(\text{height}) - \text{FSRecipientPeriod}(fs.\text{StartHeight})$
 $fs.\text{Recipient}(\text{height}) := fs.\text{Recipients}_{fs.\text{RecipientIndex}(\text{height})}$
 $fs.\text{NumRecipients} := fs.\text{RecipientIndex}(fs.\text{EndHeight} - 1)$.

$fs.\text{Recipients}$ **MUST** be of length $fs.\text{NumRecipients}$. Each element of $fs.\text{Recipients}$ **MUST** represent either a *transparent P2SH address* as specified in § 5.6.1.1 ‘*Transparent Addresses*’ on p. 111, or a **Sapling shielded payment address** as specified in § 5.6.3.1 ‘**Sapling Payment Addresses**’ on p. 113, or the identifier DEFERRED_POOL.

Recall from § 7.8 ‘*Calculation of Block Subsidy, Funding Streams, and Founders' Reward*’ on p. 133 the definition of $fs.\text{Value}$. A *funding stream* fs is “active” at *block height* height when $fs.\text{Value}(\text{height}) > 0$.

[**NU6** onward] [ZIP-214] Revision 1 defines these *funding streams* for *Mainnet*:

Stream	Numerator	Denominator	Start height	End height
FS_FPF_ZCG	8	100	2726400	3146400
FS_DEFERRED	12	100	2726400	3146400

It also defines these *funding streams* for *Testnet*:

Stream	Numerator	Denominator	Start height	End height
FS_FPF_ZCG	8	100	2976000	3396000
FS_DEFERRED	12	100	2976000	3396000

Note: The new funding streams begin at the second halving for Mainnet, but the second halving on Testnet occurred prior to the introduction of the new funding streams. For both new funding streams on each network, the associated duration corresponds to approximately one year’s worth of blocks.

7.11 Changes to the Script System

The OP_CODESEPARATOR opcode has been disabled. This opcode also no longer affects the calculation of *SIGHASH transaction hashes*.

7.12 Bitcoin Improvement Proposals

In general, Bitcoin Improvement Proposals (BIPs) do not apply to **Zcash** unless otherwise specified in this section.

All of the BIPs referenced below should be interpreted by replacing “BTC”, or “bitcoin” used as a currency unit, with “ZEC”; and “satoshi” with “zatoshi”.

The following BIPs apply, otherwise unchanged, to **Zcash**: [BIP-11], [BIP-14], [BIP-31], [BIP-35], [BIP-37], [BIP-61].

The following BIPs apply starting from the **Zcash** *genesis block*, i.e. any activation rules or exceptions for particular *blocks* in the **Bitcoin** *block chain* are to be ignored: [BIP-16], [BIP-30], [BIP-65], [BIP-66].

The effect of [BIP-34] has been incorporated into the consensus rules (§ 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121). This excludes the *Mainnet* and *Testnet* *genesis blocks*, for which the “height in coinbase” was inadvertently omitted.

[BIP-13] applies with the changes to address version bytes described in § 5.6.1.1 ‘*Transparent Addresses*’ on p. 111.

[BIP-111] applies from *peer-to-peer protocol* version 170004 onward; that is:

- references to protocol version 70002 are to be replaced by 170003;
- references to protocol version 70011 are to be replaced by 170004;
- the reference to protocol version 70000 is to be ignored (**Zcash** nodes have supported Bloom-filtered connections since launch).

8 Differences from the Zerocash paper

8.1 Transaction Structure

Zerocash introduces two new operations, which are described in the paper as new transaction types, in addition to the original transaction type of the cryptocurrency on which it is based (e.g. **Bitcoin**).

In **Zcash**, there is only the original **Bitcoin** transaction type, which is extended to contain a sequence of zero or more **Zcash**-specific operations.

This allows for the possibility of chaining transfers of *shielded* value in a single **Zcash** transaction, e.g. to spend a *shielded note* that has just been created. (In **Zcash**, we refer to value stored in *UTXOs* as *transparent*, and value stored in output *notes* of *JoinSplit* transfers or *Output transfers* as *shielded*.) This was not possible in the **Zerocash** design without using multiple transactions. It also allows *transparent* and *shielded* transfers to happen atomically – possibly under the control of nontrivial script conditions, at some cost in distinguishability.

Computation of *SIGHASH* transaction hashes, as described in § 4.10 ‘*SIGHASH Transaction Hashing*’ on p. 49, was changed to clean up handling of an error case for `SIGHASH_SINGLE`, to remove the special treatment of `OP_CODESEPARATOR`, and to include **Zcash**-specific fields in the hash [ZIP-76].

8.2 Memo Fields

Zcash adds a *memo field* sent from the creator of a *JoinSplit* description to the recipient of each output *note*. This feature is described in more detail in § 3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14.

8.3 Unification of Mints and Pours

In the original **Zerocash** protocol, there were two kinds of transaction relating to *shielded notes*:

- a “Mint” transaction takes value from *UTXOs* (*unspent transaction outputs*) as input and produces a new *shielded note* as output.
- a “Pour” transaction takes up to N^{old} *shielded notes* as input, and produces up to N^{new} *shielded notes* and a *UTXO* as output.

Only “Pour” transactions included a *zk-SNARK* proof.

[Pre-Sapling] In **Zcash**, the sequence of operations added to a transaction (see § 8.1 ‘*Transaction Structure*’ on p. 139) consists only of *JoinSplit* transfers. A *JoinSplit* transfer is a Pour operation generalized to take a *UTXO* as input, allowing *JoinSplit* transfers to subsume the functionality of Mints. An advantage of this is that a **Zcash** transaction that takes input from a *UTXO* can produce up to N^{new} output *notes*, improving the indistinguishability properties of the protocol. A related change conceals the input arity of the *JoinSplit* transfer: an unused (zero-valued) input is indistinguishable from an input that takes value from a *note*.

This unification also simplifies the fix to the Faerie Gold attack described below, since no special case is needed for Mints.

[Sapling onward] In **Sapling**, there are still no “Mint” transactions. Instead of *JoinSplit* transfers, there are *Spend transfers* and *Output transfers*. These make use of *Pedersen value commitments* to represent the shielded values that are transferred. Because these commitments are additively homomorphic, it is possible to check that all *Spend transfers* and *Output transfers* balance; see § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51 for detail. This reduces the granularity of the circuit, allowing a substantial performance improvement (orthogonal to other **Sapling** circuit improvements) when the numbers of *shielded* inputs and outputs are significantly different. This comes at the cost of revealing the exact number of *shielded* inputs and outputs, but *dummy* (zero-valued) outputs are still possible.

8.4 Faerie Gold attack and fix

When a *shielded note* is created in **Zerocash**, the creator is supposed to choose a new ρ value at random. The *nullifier* of the *note* is derived from its *spending key* (a_{sk}) and ρ . The *note commitment* is derived from the recipient address component a_{pk} , the value v , and the *commitment trapdoor* rcm , as well as ρ . However nothing prevents creating multiple *notes* with different v and rcm (hence different *note commitments*) but the same ρ .

An adversary can use this to mislead a *note* recipient, by sending two *notes* both of which are verified as valid by Receive (as defined in [BCGGMTV2014, Figure 2]), but only one of which can be spent.

We call this a “Faerie Gold” attack — referring to various Celtic legends in which faeries pay mortals in what appears to be gold, but which soon after reveals itself to be leaves, gorse blossoms, gingerbread cakes, or other less valuable things [LG2004].

This attack does not violate the security definitions given in [BCGGMTV2014]. The issue could be framed as a problem either with the definition of Completeness, or the definition of Balance:

- The Completeness property asserts that a validly received *note* can be spent provided that its *nullifier* does not appear on the ledger. This does not take into account the possibility that distinct *notes*, which are validly received, could have the same *nullifier*. That is, the security definition depends on a protocol detail – *nullifiers* – that is not part of the intended abstract security property, and that could be implemented incorrectly.
- The Balance property only asserts that an adversary cannot obtain *more* funds than they have minted or received via payments. It does not prevent an adversary from causing others’ funds to decrease. In a Faerie Gold attack, an adversary can cause spending of a *note* to reduce (to zero) the effective value of another *note* for which the adversary does not know the *spending key*, which violates an intuitive conception of global balance.

These problems with the security definitions need to be repaired; how to do so is discussed in [Hopwood2022], but that is outside the scope of this specification. Here we only describe how **Zcash** addresses the immediate attack.

It would be possible to address the attack by requiring that a recipient remember all of the ρ values for all *notes* they have ever received, and reject duplicates (as proposed in [GGM2016]). However, this requirement would interfere with the intended **Zcash** feature that a holder of a *spending key* can recover access to (and be sure that they are able to spend) all of their funds, even if they have forgotten everything but the *spending key*.

[Sprout] Instead, **Zcash** enforces that an adversary must choose distinct values for each ρ , by making use of the fact that all of the *nullifiers* in *JoinSplit descriptions* that appear in a *valid block chain* must be distinct. This is true regardless of whether the *nullifiers* corresponded to real or *dummy notes* (see § 4.8.1 ‘*Dummy Notes (Sprout)*’ on p. 45). The *nullifiers* are used as input to $hSigCRH$ to derive a public value h_{Sig} which uniquely identifies the transaction, as described in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38. (h_{Sig} was already used in **Zerocash** in a way that requires it to be unique in order to maintain indistinguishability of *JoinSplit descriptions*; adding the *nullifiers* to the input of the hash used to calculate it has the effect of making this uniqueness property robust even if the *transaction* creator is an adversary.)

[Sprout] The ρ value for each output *note* is then derived from a random private seed φ and h_{Sig} using PRF^{ρ}_{φ} . The correct construction of ρ for each output *note* is enforced by § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58 in the *JoinSplit statement*.

[Sprout] Now even if the creator of a *JoinSplit description* does not choose φ randomly, uniqueness of *nullifiers* and *collision resistance* of both $hSigCRH$ and PRF^{ρ} will ensure that the derived ρ values are unique, at least for any two *JoinSplit descriptions* that get into a *valid block chain*. This is sufficient to prevent the Faerie Gold attack.

A variation on the attack attempts to cause the *nullifier* of a sent *note* to be repeated, without repeating ρ . However, since the *nullifier* is computed as $PRF^{\rho}_{a_{sk}}(\rho)$ or $PRF^{\rho}_{nk}(\rho)$ (for **Orchard**, see below); this is only possible if the adversary finds a collision across both inputs on $PRF^{\rho}_{a_{sk}}$ or PRF^{ρ}_{nk} , which is assumed to be infeasible — see § 4.1.2 ‘*Pseudo Random Functions*’ on p. 24.

[Sprout] Crucially, “*nullifier integrity*” is enforced whether or not the `enforceMerklePathi` flag is set for an input *note* (§ 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58). If this were not the case then an adversary could perform the attack by creating a zero-valued *note* with a repeated *nullifier*, since the *nullifier* would not depend on the value.

[**Sprout**] *Nullifier* integrity also prevents a “roadblock attack” in which the adversary sees a victim’s *transaction*, and is able to publish another *transaction* that is mined first and blocks the victim’s *transaction*. This attack would be possible if the public value(s) used to enforce uniqueness of ρ could be chosen arbitrarily by the *transaction* creator: the victim’s *transaction*, rather than the adversary’s, would be considered to be repeating these values. In the chosen solution that uses *nullifiers* for these public values, they are enforced to be dependent on *spending* keys controlled by the original *transaction* creator (whether or not each input *note* is a *dummy*), and so a roadblock attack cannot be performed by another party who does not know these keys.

[**Sapling** onward] In **Sapling**, uniqueness of ρ is ensured by making it dependent on the position of the *note commitment* in the **Sapling** *note commitment tree*. Specifically, $\rho = \text{cm} + [\text{pos}] \mathcal{J}^{\text{Sapling}}$, where $\mathcal{J}^{\text{Sapling}}$ is a generator independent of the generators used in $\text{NoteCommit}^{\text{Sapling}}$. Therefore, ρ commits uniquely to the *note* and its position, and this commitment is *collision-resistant* by the same argument used to prove *collision resistance* of Pedersen hashes. Note that it is possible for two distinct **Sapling** *positioned notes* (having different ρ values and *nullifiers*, but different *note positions*) to have the same *note commitment*, but this causes no security problem. Roadblock attacks are not possible because a given *note position* does not repeat for outputs of different *transactions* in the same *block chain*. Note that this depends on the fact that the value is bound by the *note commitment*: it could be the case that the adversary uses a *dummy note* that is not required to have a *note commitment* in the *note commitment tree* when it is spent. If this happens and the victim’s *note* is not a *dummy*, the *note commitments* will differ and so will the *nullifiers*. If both *notes* are dummies, the adversary cannot know the inputs to the *note commitment* since they are generated at random for the victim’s spend, regardless of the adversary’s potential knowledge of viewing keys.

[**NU5** onward] In **Orchard**, the *nullifier* is computed using a construction that combines elliptic curve cryptography and the Poseidon-based $\text{PRF}^{\text{nfOrchard}}$ in a way that, for privacy, aims to provide defence in depth against potential weaknesses in either (see [Zcash-Orchard, Section 3.5 Nullifiers] and § 4.16 ‘Computing ρ values and Nullifiers’ on p. 56).

Resistance to Faerie Gold attacks, on the other hand, depends entirely on hardness of the *Discrete Logarithm Problem*. The ρ value of a *note* created in a given *Action transfer* is obtained from the *nullifier* of the *note* spent in that *Action transfer*; this ensures (without any cryptographic assumption) that all ρ values of *notes* added to the *note commitment tree* are unique. Then, the *nullifier* derivation can be considered as computing a vector Pedersen commitment on input that includes ρ , so that the *binding* property of that *commitment scheme* ensures that **Orchard** *nullifiers* will be unique. (Specifically, this is a Sinsemilla commitment with an additional term having base $\mathcal{K}^{\text{Orchard}}$, truncated to its x -coordinate. The x -coordinate truncation cannot harm *collision resistance* because, assuming hardness of the *Discrete Logarithm Problem* on the Pallas curve, § 5.4.1.9 ‘Security argument’ on p. 80 covers the case where the additional term is added.) Roadblock attacks are not possible because ρ does not repeat for *notes* in the *note commitment tree*, and by a corresponding argument to **Sapling** for *dummy notes*.

8.5 Internal hash collision attack and fix

The **Zerocash** security proof requires that the composition of COMM_{rcm} and COMM_s is a computationally *binding* commitment to its inputs a_{pk} , v , and ρ . However, the instantiation of COMM_{rcm} and COMM_s in section 5.1 of the paper did not meet the definition of a *binding* commitment at a 128-bit security level. Specifically, the internal hash of a_{pk} and ρ is truncated to 128 bits (motivated by providing statistical *hiding* security). This allows an attacker, with a work factor on the order of 2^{64} , to find distinct pairs (a_{pk}, ρ) and (a'_{pk}, ρ') with colliding outputs of the truncated hash, and therefore the same *note commitment*. This would have allowed such an attacker to break the Balance property by *double-spending notes*, potentially creating arbitrary amounts of currency for themselves [HW2016].

Zcash uses a simpler construction with a single hash evaluation for the commitment: SHA-256 for **Sprout** *notes*, **PedersenHashToPoint** for **Sapling** *notes*, and **SinsemillaHashToPoint** for **Orchard** *notes*. The motivation for the nested construction in **Zerocash** was to allow Mint transactions to be publically verified without requiring *zk-SNARK proofs* ([BCGGMTV2014, section 1.3, under step 3]). Since **Zcash** combines “Mint” and “Pour” transactions into generalized *JoinSplit transfers* (for **Sprout**), or *Spend transfers and Output transfers* (for **Sapling**), or *Action transfers* (for **Orchard**), and each transfer always uses a *zk-SNARK proof*, **Zcash** does not require the nesting. A side benefit is that this reduces the cost of computing the *note commitments*: for **Sprout** it reduces the number of SHA256Compress evaluations needed to compute each *note commitment* from three to two, saving a total of four SHA256Compress evaluations in the *JoinSplit statement*.

[**Sprout**] **Note:** The full SHA-256 algorithm is used for $\text{NoteCommit}^{\text{Sprout}}$, with randomness appended after the commitment input. The commitment input can be split into two blocks, call them x of length 64 bytes, and y of the remaining length (9 bytes). Let $\text{COMM}'_r(z : \mathbb{B}^{[41]})$ be the *commitment scheme* that applies SHA256Compress with the first 32 bytes of z in the IV, and the rest of z (9 bytes), the randomness r (32 bytes), and padding up to 64 bytes in the SHA256Compress input block. Then we have $\text{NoteCommit}^{\text{Sprout}}_r(x \parallel y) = \text{COMM}'_r(\text{SHA256Compress}(x) \parallel y)$. Suppose we make the reasonable assumption that COMM' is a computationally *binding* and *hiding* commitment scheme. If SHA256Compress is *collision-resistant* with the standard IV¹³, then $\text{NoteCommit}^{\text{Sprout}}$ is as secure for *binding* as COMM' . Also $\text{NoteCommit}^{\text{Sprout}}$ is as secure for *hiding* as COMM' (without any assumption on SHA256Compress). This effectively rules out potential concerns about the Merkle–Damgård structure [Damgård1989] of SHA-256 causing any security problem for $\text{NoteCommit}^{\text{Sprout}}$.

[**Sprout**] **Note:** **Sprout** note commitments are not statistically *hiding*, so for **Sprout** notes, **Zcash** does not support the “everlasting anonymity” property described in [BCGGMTV2014, section 8.1], even when used as described in that section. While it is possible to define a statistically *hiding*, computationally *binding* commitment scheme for this use at a 128-bit security level, the overhead of doing so within the *JoinSplit* statement was not considered to justify the benefits.

[**Sapling onward**] In **Sapling**, **Pedersen** or **Sinsemilla** commitments are used instead of SHA256Compress . These commitments are statistically *hiding*, and so “everlasting anonymity” is supported for **Sapling** and **Orchard** notes under the same conditions as in **Zerocash** (by the protocol, not necessarily by **zcashd**). Note that *diversified payment addresses* can be linked if the *Decisional Diffie–Hellman Problem* on the Jubjub curve or the Pallas curve can be broken.

8.6 Changes to PRF inputs and truncation

The format of inputs to the *PRFs* instantiated in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84 has changed relative to **Zerocash**. There is also a requirement for another *PRF*, PRF^p , which must be domain-separated from the others.

In the **Zerocash** protocol, ρ_i^{old} is truncated from 256 to 254 bits in the input to PRF^{sn} (which corresponds to $\text{PRF}^{\text{nfSprout}}$ in **Zcash**). Also, h_{sig} is truncated from 256 to 253 bits in the input to PRF^{pk} . These truncations are not taken into account in the security proofs.

Both truncations affect the validity of the proof sketch for Lemma D.2 in the proof of Ledger Indistinguishability in [BCGGMTV2014, Appendix D].

In more detail:

- In the argument relating **H** and \mathcal{D}_2 , it is stated that in \mathcal{D}_2 , “for each $i \in \{1, 2\}$, $\text{sn}_i := \text{PRF}^{\text{sn}}_{a_{\text{sk}}}(\rho)$ for a random (and not previously used) ρ ”. It is also argued that “the calls to $\text{PRF}^{\text{sn}}_{a_{\text{sk}}}$ are each by definition unique”. The latter assertion depends on the fact that ρ is “not previously used”. However, the argument is incorrect because the truncated input to $\text{PRF}^{\text{sn}}_{a_{\text{sk}}}$, i.e. $[\rho]_{254}$, may repeat even if ρ does not.
- In the same argument, it is stated that “with overwhelming probability, h_{sig} is unique”. In fact what is required to be unique is the truncated input to PRF^{pk} , i.e. $[h_{\text{sig}}]_{253} = [\text{CRH}(\text{pk}_{\text{sig}})]_{253}$. In practice this value will be unique under a plausible assumption on CRH provided that pk_{sig} is chosen randomly, but no formal argument for this is presented.

Note that ρ is truncated in the input to PRF^{sn} but not in the input to COMM_{rcm} , which further complicates the analysis.

As further evidence that it is essential for the proofs to explicitly take any such truncations into account, consider a slightly modified protocol in which ρ is truncated in the input to COMM_{rcm} but not in the input to PRF^{sn} . In that case, it would be possible to violate balance by creating two *notes* for which ρ differs only in the truncated bits. These *notes* would have the same *note commitment* but different *nullifiers*, so it would be possible to spend the same value twice.

¹³If SHA256Compress is not *collision-resistant* with the standard IV, then SHA-256 is not *collision-resistant* for a 2-block input.

[**Sprout**] For resistance to Faerie Gold attacks as described in § 8.4 ‘*Faerie Gold attack and fix*’ on p. 140, **Zcash** depends on *collision resistance* of hSigCRH and PRF^ρ (instantiated using BLAKE2b-256 and SHA256Compress respectively). *Collision resistance* of a truncated hash does not follow from *collision resistance* of the original hash, even if the truncation is only by one bit. This motivated avoiding truncation along any path from the inputs to the computation of h_{Sig} to the uses of ρ .

[**Sprout**] Since the *PRFs* are instantiated using SHA256Compress which has an input block size of 512 bits (of which 256 bits are used for the *PRF* input and 4 bits are used for domain separation), it was necessary to reduce the size of the *PRF* key to 252 bits. The key is set to a_{sk} in the case of PRF^{addr} , $\text{PRF}^{\text{nfSprout}}$, and PRF^{pk} , and to \varnothing (which does not exist in **Zerocash**) for PRF^ρ , and so those values have been reduced to 252 bits. This is preferable to requiring reasoning about truncation, and 252 bits is quite sufficient for security of these cryptovalues.

Sapling uses *Pedersen hashes* and BLAKE2s where **Sprout** used SHA256Compress. *Pedersen hashes* can be efficiently instantiated for arbitrary input lengths. BLAKE2s has an input block size of 512 bits, and uses a finalization flag rather than padding of the last input block; it also supports domain separation via a personalization parameter distinct from the input. Therefore, there is no need for truncation in the inputs to any of these hashes. Note however that the *output* of CRH^{ivk} is truncated, requiring a security assumption on BLAKE2s truncated to 251 bits (see § 5.4.1.5 ‘ CRH^{ivk} Hash Function’ on p. 75).

Orchard replaces *Pedersen hashes* by *Sinsemilla hashes* which can also be efficiently instantiated for arbitrary input lengths. It replaces uses of BLAKE2s in the circuit by the *commitment scheme* $\text{Commit}^{\text{ivk}}$, and by a construction for *nullifier* derivation that uses the Poseidon-based $\text{PRF}^{\text{nfOrchard}}$ (along with scalar multiplication on the Pallas curve). Again, there is no need for truncation in the inputs to any of these functions, and the need for truncation in the derivation of *ivk* is removed.

8.7 In-band secret distribution

Zerocash specified ECIES (referencing Certicom’s SEC 1 standard) as the encryption scheme used for the *in-band* secret distribution. This has been changed to a *key agreement scheme* based on Curve25519 (for **Sprout**) or **Jubjub** (for **Sapling**) and the authenticated encryption algorithm AEAD_CHACHA20_POLY1305. This scheme is still loosely based on ECIES, and on the `crypto_box_seal` scheme defined in `libsodium` [`libsodium-Seal`].

The motivations for this change were as follows:

- The **Zerocash** paper did not specify the curve to be used. We believe that Curve25519 has significant *side-channel* resistance, performance, implementation complexity, and robustness advantages over most other available curve choices, as explained in [Bernstein2006]. For **Sapling**, the Jubjub curve was designed according to a similar design process following the “Safe curves” criteria [BL-SafeCurves] [Hopwood2018]. This retains Curve25519’s advantages while keeping *shielded payment address* sizes short, because the same *public key* material supports both encryption and spend authentication. For **Orchard**, we define a *prime-order curve* Pallas [Hopwood2020], with similar advantages to Jubjub.
- ECIES permits many options, which were not specified. There are at least –counting conservatively– 576 possible combinations of options and algorithms over the four standards (ANSI X9.63, IEEE Std 1363a-2004, ISO/IEC 18033-2, and SEC 1) that define ECIES variants [MÁEÁ2010].
- Although the **Zerocash** paper states that ECIES satisfies *key privacy* (as defined in [BBDP2001]), it is not clear that this holds for all curve parameters and key distributions. For example, if a group of non-prime order is used, the distribution of ciphertexts could be distinguishable depending on the order of the points representing the ephemeral and recipient *public keys*. Public key validity is also a concern. Curve25519 (and Jubjub) key agreement is defined in a way that avoids these concerns due to the curve structure and the “clamping” of *private keys* (or explicit cofactor multiplication and point validation for **Sapling**). The Pallas curve is prime-order, but we still validate points, and use a similar *key agreement scheme* to **Sapling** for consistency and ease of analysis.
- Unlike the DHAES/DHIES proposal on which it is based [ABR1999], ECIES does not require a representation of the sender’s *ephemeral public key* to be included in the input to the KDF, which may impair the security properties of the scheme. (The Std 1363a-2004 version of ECIES [IEEE2004] has a “DHAES mode” that allows

this, but the representation of the key input is underspecified, leading to incompatible implementations.) The scheme we use for **Sprout** has both the ephemeral and recipient *public key* encodings –which are unambiguous for Curve25519– and also h_{Sig} and a nonce as described below, as input to the KDF. For **Sapling** and **Orchard**, it is only possible to include the ephemeral public key encoding, but this is sufficient to retain the original security properties of DHAES. Note that being able to break the Elliptic Curve Diffie–Hellman Problem on Curve25519 or Jubjub or Pallas (without breaking AEAD_CHACHA20_POLY1305 as an authenticated encryption scheme or BLAKE2b-256 as a KDF) would not help to decrypt the *transmitted note(s) ciphertext* unless pk_{enc} or pk_d is known or guessed.

- [**Sprout**] The KDF also takes a public seed h_{Sig} as input. This can be modeled as using a different “randomness extractor” for each *JoinSplit transfer*, which limits degradation of security with the number of *JoinSplit transfers*. This facilitates security analysis as explained in [DGKM2011] – see section 7 of that paper for a security proof that can be applied to this construction under the assumption that single-block BLAKE2b-256 is a “weak PRF”. Note that h_{Sig} is authenticated, by the zk-SNARK proof, as having been chosen with knowledge of $a_{\text{sk},1..N}^{\text{old}}$, so an adversary cannot modify it in a ciphertext from someone else’s transaction for use in a chosen-ciphertext attack without detection. (In **Sapling** and **Orchard**, there is no equivalent to h_{Sig} , but the binding signature and spend authorization signatures prevent such modifications.)
- [**Sprout**] The scheme used by **Sprout** includes an optimization that reuses the same ephemeral key (with different nonces) for the two ciphertexts encrypted in each *JoinSplit description*.

The security proofs of [ABR1999] can be adapted straightforwardly to the resulting scheme. Although DHAES as defined in that paper does not pass the recipient *public key* or a public seed to the *hash function* H , this does not impair the proof because we can consider H to be the specialization of our KDF to a given recipient key and seed. (Passing the recipient *public key* to the KDF could in principle compromise *key privacy*, but not confidentiality of encryption.) [**Sprout**] It is necessary to adapt the “HDH independence” assumptions and the proof slightly to take into account that the ephemeral key is reused for two encryptions.

Note that the 256-bit key for AEAD_CHACHA20_POLY1305 maintains a high concrete security level even under attacks using parallel hardware [Bernstein2005] in the multi-user setting [Zaverucha2012]. This is especially necessary because the privacy of **Zcash** transactions may need to be maintained far into the future, and upgrading the encryption algorithm would not prevent a future adversary from attempting to decrypt ciphertexts encrypted before the upgrade. Other cryptovalues that could be attacked to break the privacy of transactions are also sufficiently long to resist parallel brute force in the multi-user setting: for **Sprout**, a_{sk} is 252 bits, and sk_{enc} is no shorter than a_{sk} .

In **Sapling**, ivk is an output of CRH^{ivk} , which is a 251-bit value. In **Orchard**, ivk is an x -coordinate on the Pallas curve. This degree of divergence from a uniform distribution on the scalar field is not expected to cause any weakness in note encryption.

For all shielded protocols, the checking of *note commitments* makes *partitioning oracle attacks* [LGR2021] against the *transmitted note ciphertext* infeasible, at least in the absence of *side-channel attacks*. The following argument applies to **Sapling** and **Orchard**, but can be adapted to **Sprout** by replacing ivk with sk_{enc} , pk_d with pk_{enc} , and using a fixed base. The decryption procedure for *transmitted note ciphertexts* in **Sapling** and **Orchard** is specified in §4.20.2 ‘*Decryption using an Incoming Viewing Key (Sapling and Orchard)*’ on p. 66; it ensures that a successful decryption cannot occur unless the decrypted *note plaintext* encodes a *note* consistent with the *note commitment* (encoded as the cm_u field of the *Output description* or the cm_x field of the *Action description*). Suppose that it were feasible to find a pair of *transmitted note ciphertext* and *note commitment* that decrypts successfully for two different *incoming viewing keys* ivk_1 and ivk_2 . Assuming that the *note commitment scheme* is *binding* and that *note commitment* opens to a *note* with pk_d and g_d , we must have $pk_d = \text{KA.Agree}(ivk_1, g_d) = \text{KA.Agree}(ivk_2, g_d)$. But this is impossible given that g_d has order greater than the maximum value of ivk that can be an output of CRH^{ivk} or Commit^{ivk} .

There is also a decryption procedure that makes use of *outgoing ciphertexts* in **Sapling** and **Orchard**, as specified in §4.20.3 ‘*Decryption using a Full Viewing Key (Sapling and Orchard)*’ on p. 68. It checks (via KA.DerivePublic , and also via $\text{PRF}_{\text{rseed}}^{\text{expand}}$ in the case of post-[ZIP-212] ciphertexts with *note plaintext lead byte* $\neq 0x01$) that the decrypted esk value is consistent with the *transmitted note ciphertext*, which is protected from *partitioning oracle attacks* as described above. It also checks that the pk_d value is consistent with the *note commitment*. Since these

are the only fields in an *outgoing ciphertext*, even if a *partitioning oracle attack* occurred against an *outgoing ciphertext*, it could not result in any equivocation of the decrypted data. Because ovk and ock are each 256 bits, *partitioning oracle attacks* that speed up a search for these keys (analogous to the attacks against Password-based AEAD in [LGR2021]) are infeasible, even given knowledge of ivk .

8.8 Omission in Zerocash security proof

The abstract **Zerocash** protocol requires PRF^{addr} only to be a *PRF*; it is not specified to be *collision-resistant*. This reveals a flaw in the proof of the Balance property.

Suppose that an adversary finds a collision on PRF^{addr} such that a_{sk}^1 and a_{sk}^2 are distinct *spending keys* for the same a_{pk} . Because the *note commitment* is to a_{pk} , but the *nullifier* is computed from a_{sk} (and ρ), the adversary is able to *double-spend* the note, once with each a_{sk} . This is not detected because each Spend reveals a different *nullifier*. The *JoinSplit statements* are still valid because they can only check that the a_{sk} in the witness is *some* preimage of the a_{pk} used in the *note commitment*.

The error is in the proof of Balance in [BCGGMTV2014, Appendix D.3]. For the “ \mathcal{A} violates Condition I” case, the proof says:

“(i) If $cm_1^{old} = cm_2^{old}$, then the fact that $sn_1^{old} \neq sn_2^{old}$ implies that the witness a contains two distinct openings of cm_1^{old} (the first opening contains $(a_{sk,1}^{old}, \rho_1^{old})$, while the second opening contains $(a_{sk,2}^{old}, \rho_2^{old})$). This violates the *binding* property of the *commitment scheme* COMM.”

In fact the openings do not contain $a_{sk,i}^{old}$; they contain $a_{pk,i}^{old}$. (In **Sprout** cm_i^{old} opens directly to $(a_{pk,i}^{old}, v_i^{old}, \rho_i^{old})$, and in **Zerocash** it opens to $(v_i^{old}, COMM_s(a_{pk,i}^{old}, \rho_i^{old}))$.)

A similar error occurs in the argument for the “ \mathcal{A} violates Condition II” case.

The flaw is not exploitable for the actual instantiations of PRF^{addr} in **Zerocash** and **Sprout**, which *are* *collision-resistant* assuming that SHA256Compress is.

The proof can be straightforwardly repaired. The intuition is that we can rely on *collision resistance* of PRF^{addr} (on both its arguments) to argue that distinctness of $a_{sk,1}^{old}$ and $a_{sk,2}^{old}$, together with constraint 1(b) of the *JoinSplit statement* (see §4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58), implies distinctness of $a_{pk,1}^{old}$ and $a_{pk,2}^{old}$, therefore distinct openings of the *note commitment* when Condition I or II is violated.

8.9 Miscellaneous

- The paper defines a *note* as $((a_{pk}, pk_{enc}), v, \rho, rcm, s, cm)$, whereas this specification defines a **Sprout** *note* as (a_{pk}, v, ρ, rcm) . The instantiation of $COMM_s$ in section 5.1 of the paper did not actually use s , and neither does the new instantiation of $NoteCommit^{Sprout}$ in **Sprout**. pk_{enc} is also not needed as part of a *note*: it is not an input to $NoteCommit^{Sprout}$ nor is it constrained by the **Zerocash** *POUR statement* or the **Zcash** *JoinSplit statement*. cm can be computed from the other fields. (The definition of *notes* for **Sapling** is different again.)
- The length of proof encodings given in the paper is 288 bytes. [**Sprout**] This differs from the 296 bytes specified in §5.4.10.1 ‘BCTV14’ on p. 108, because both the x -coordinate and compressed y -coordinate of each point need to be represented. Although it is possible to encode a proof in 288 bytes by making use of the fact that elements of \mathbb{F}_q can be represented in 254 bits, we prefer to use the standard formats for points defined in [IEEE2004]. The fork of *libsnark* used by **Zcash** uses this standard encoding rather than the less efficient (uncompressed) one used by upstream *libsnark*. In **Sapling**, a customized encoding is used for BLS12-381 points in Groth16 proofs to minimize length, and similarly for Pallas and Vesta points in Orchard.
- The range of monetary values differs. In **Zcash** this range is $\{0 \dots MAX_MONEY\}$, while in **Zerocash** it is $\{0 \dots 2^{\ell_{value}} - 1\}$. (The *JoinSplit statement* still only directly enforces that the sum of amounts in a given *JoinSplit transfer* is in the latter range; this enforcement is technically redundant given that the Balance property holds.)

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The error in the **Zerocash** proof of Balance relating to *collision resistance* of PRF^{addr} was found by Daira-Emma Hopwood.

The errors in the proof of Ledger Indistinguishability mentioned in § 8.6 ‘*Changes to PRF inputs and truncation*’ on p. 142 were also found by Daira-Emma Hopwood.

The 2015 Soundness vulnerability in BCTV14 [Parno2015] was found by Bryan Parno. An additional condition needed to resist this attack was documented by Ariel Gabizon [Gabizon2019, section 3]. The 2019 Soundness vulnerability in BCTV14 [Gabizon2019] was found by Ariel Gabizon.

The design of **Sapling** is primarily due to Matthew Green, Ian Miers, Daira-Emma Hopwood, Sean Bowe, Jack Grigg, and Jack Gavigan. A potential attack linking *diversified payment addresses*, avoided in the adopted design, was found by Brian Warner.

The design of **Orchard** is primarily due to Daira-Emma Hopwood, Sean Bowe, Jack Grigg, Kris Nuttycombe, Ying Tong Lai, and Steven Smith.

The observation in § 5.4.1.6 ‘*DiversifyHash^{Sapling} and DiversifyHash^{Orchard} Hash Functions*’ on p. 76 that *diversified payment address* unlinkability can be proven in the same way as *key privacy* for ElGamal, is due to Mary Maller.

We thank Ariel Gabizon for teaching us the techniques of [BFIJSV2010] used in § B.2 ‘*Groth16 batch verification*’ on p. 216, by applying them to BCTV14.

The arithmetization used by Halo 2 is based on that used by PLONK [GWC2019], which was designed by Ariel Gabizon, Zachary Williamson, and Oana Ciobotaru.

Numerous people have contributed to the science of *zero-knowledge proving systems*, but we would particularly like to acknowledge the work of Shafi Goldwasser, Silvio Micali, Oded Goldreich, Mihir Bellare, Charles Rackoff, Rosario Gennaro, Bryan Parno, Jon Howell, Craig Gentry, Mariana Raykova, Jens Groth, Rafail Ostrovsky, and Amit Sahai.

We thank the organizers of the ZKProof standardization effort and workshops; and also Anna Rose, Fredrik Harrysson, Terun Chitra, James Prestwich, Josh Cincinnati, Tanya Karsou, Henrik Jose, Chris Ward, and others for their work on the Zero Knowledge Podcast, ZK Summits, and ZK Study Club. These efforts have enriched the zero knowledge community immeasurably.

Many of the ideas used in **Zcash**—including the use of *zero-knowledge proofs* to resolve the tension between privacy and auditability, Merkle trees over note commitments (using Pedersen hashes as in **Sapling**), and the use of “serial numbers” or *nullifiers* to detect or prevent *double-spends*—were first applied to privacy-preserving digital currencies by Tomas Sander and Amnon Ta-Shma. To a large extent **Zcash** is a refinement of their “Auditable, Anonymous Electronic Cash” proposal in [ST1999].

We thank Alexandra Elbakyan for her tireless work in dismantling barriers to scientific research.

This document is set in the beautiful Quattrocento font designed by Pablo Impallari. The **New Century Schoolbook** font by URW Type Foundry, based on Century Schoolbook designed by Morris Fuller, is used for *italics*.

Finally, we would like to thank the Internet Archive for their scan of Peter Newell’s illustration of the Jubjub bird, from [Carroll1902].

10 Change History

Unreleased

- Clarify § 3.4 ‘*Transactions and Treestates*’ on p. 17, taking into account the **NU6** consensus changes from [ZIP-236].
- Added dark mode rendering (<https://zips.z.cash/protocol/protocol-dark.pdf>).

2024.5.1 2024-09-26

- Apply **NU6** consensus changes from [ZIP-2001] and [ZIP-236].
- Collect the definitions of balances for each *chain value pool* into § 4.17 ‘*Chain Value Pool Balances*’ on p. 57.
- Add a definition of *total issued supply*.
- Add acknowledgements to Aditya Bharadwaj, Andrew Arnott, Arya, Andrea Kobrlova, Lukas Korba, and Honza Rychnovský for discussions on the Zcash protocol.

2024.5.0 2024-08-28

- Add the hyphen in Daira-Emma Hopwood.
- Correct some author lists in the References.
- Prevent incorrect line-breaking on hyphens.
- In § 5.4.1.9 ‘*Sinsemilla Hash Function*’ on p. 79, declare use of LEBS2IP instead of LEOS2IP.
- Add an acknowledgement to Conrado Gouvêa for discussions on the Zcash protocol.
- Add boilerplate support for **NU6**.

2023.4.0 2023-12-19

- The domain separators [4] and [5] used in the input to $\text{PRF}_{\text{rseed}}^{\text{expand}}$ for **Sapling** were accidentally swapped in the protocol specification relative to [ZIP-212]. The implementation in zcashd followed [ZIP-212], using [4] to derive rcm and [5] to derive esk. This has been corrected in the protocol specification.

For **Orchard**, the implementation in the orchard crate (and therefore in zcashd) followed the protocol specification, using $[5] \parallel \text{I2LEOSP}_{256}(\rho)$ to derive rcm and $[4] \parallel \text{I2LEOSP}_{256}(\rho)$ to derive esk. This cannot now be changed, and so [ZIP-212] has been updated to follow this implementation. Notes have been added pointing out the discrepancy.

- Document that the attacks in [DKLS2020] are no better than brute force key search against FF1-AES256 as specified in § 5.4.4 ‘*Pseudo Random Permutations*’ on p. 86.
- In the table of § 7.6 ‘*Block Header Encoding and Consensus*’ on p. 128, clarify that `hashLightClientRoot` is used in **Heartwood** and **Canopy**, but not in **NU5** or later.
- The return type of $\text{GroupHash}^{\mathbb{J}^{(r)*}}$ in § 5.4.9.5 ‘*Group Hash into Jubjub*’ on p. 102 was incorrectly given as $\mathbb{J}^{(r)*}$, rather than the correct $\mathbb{J}^{(r)*} \cup \{\perp\}$.
- In the discussion of partitioning oracle attacks on *note* encryption in § 8.7 ‘*In-band secret distribution*’ on p. 143, we now use the fact that g_d has order greater than the maximum value of *ivk*, rather than assuming that g_d is a non-zero point in the *prime-order subgroup*. (In the case of **Sapling**, the circuits only enforce that g_d is not a *small-order* point, not that it is in the *prime-order subgroup*. It is true that honestly generated addresses have *prime-order* g_d which would have been sufficient for the security argument against this class of attacks, but the chosen fix is more direct.)
- Delete a confusing claim in § 4.4 ‘*Spend Descriptions*’ on p. 39 that “The check that *rk* is not of *small order* is technically redundant with a check in the *Spend circuit*...”. The *small-order* check excludes the zero point $\mathcal{O}_{\mathbb{J}}$, which the **Spend authority** check that this claim was intending to reference does not.
- An implementation of $\text{HomomorphicPedersenCommit}^{\text{Sapling}}$ **MAY** resample the *commitment trapdoor* until the resulting commitment is not $\mathcal{O}_{\mathbb{J}}$, in order to avoid it being rejected as the *cv* field of a *Spend description* or *Output description*. Add notes in § 4.4 ‘*Spend Descriptions*’ on p. 39, § 4.5 ‘*Output Descriptions*’ on p. 40, and § 5.4.8.3 ‘*Homomorphic Pedersen commitments (Sapling and Orchard)*’ on p. 95 to that effect.
- Rename the section “Note Commitments and Nullifiers” to § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56, to more accurately reflect its contents.
- Split some of the content of the section “Notes” into subsections § 3.2.2 ‘*Note Commitments*’ on p. 15 and § 3.2.3 ‘*Nullifiers*’ on p. 16. Make the descriptions of how *note commitments* and *nullifiers* are used more precise and explicit, and add forward references where helpful.
- Remove redundancy in the definition of *note plaintexts* between § 3.2.1 ‘*Note Plaintexts and Memo Fields*’ on p. 14 and § 5.5 ‘*Encodings of Note Plaintexts and Memo Fields*’ on p. 110.
- The abstract no longer describes the **NU5** version of the specification as a draft.
- Acknowledge Greg Pfeil as a co-designer of the **Zcash** protocol.
- Acknowledge Daira-Emma Hopwood for the fix to the Faerie Gold attack in **Sprout**, and add a reference to her *Explaining the Security of Zcash* talk at Zcon3 [Hopwood2022] for repairs to the **Zerocash** security definitions.
- Acknowledge the font designers Pablo Impallari and Morris Fuller.
- Change Daira-Emma Hopwood’s name.

2022.3.8 2022-09-15

- Correct Jurgen Bos’ name.

2022.3.7 2022-09-10

- Remove a now-unused sampling of `rcv` in § 4.8.3 ‘*Dummy Notes (Orchard)*’ on p. 47.
- Specify in § 3.3 ‘*The Block Chain*’ on p. 16 that **NU5** is the most recent settled *network upgrade*.

2022.3.6 2022-09-01

- Correct Kexin Hu’s name.
- Correct cross-references for the definition of an *anchor*.
- Remove a calculation of `cv` in § 4.8.3 ‘*Dummy Notes (Orchard)*’ on p. 47 that is not applicable to **Orchard** (since `cv` for an *Action description* depends on both the spent and output *notes*).
- Clarify that the recommended format for a QR code starts with a *Bech32* encoding for a **Sapling** payment address and with a *Bech32m* encoding for a *unified payment address*.
- Replace ResearchGate links for [CDvdG1987] and [BDPA2007] with alternatives that do not cause false-positive link checker errors.
- In `protocol/README.rst`: update the build dependency documentation for Debian Bullseye, mention the “`make linkcheck`” target, and correct the description of “`make all`”.
- Update the `Makefile` to build correctly with newer versions of `latexmk`.

2022.3.5 2022-08-02

- ZIP 244 is not modified by ZIP 225.
- § 5.4.1.5 ‘**CRH**^{ivk} *Hash Function*’ on p. 75 incorrectly cross-referenced BLAKE2b-256 rather than BLAKE2s-256. The actual specification was correct.

2022.3.4 2022-06-22

- Document in § 5.4.6 ‘Ed25519’ on p. 88 that a *full validator* implementation that checkpoints on the **Canopy** activation block **MAY** validate Ed25519 signatures using the post-**Canopy** rules for the whole chain.
- Update references for [ECCZF2019] and [ZIP-302] and [ZIP-252].

2022.3.3 2022-06-21

- In § 3.12 ‘*Mainnet and Testnet*’ on p. 21, update the settled activation *block hashes* to be those for **NU5** on *Mainnet* and *Testnet*.
- Correct the history entry for v2022.3.2 to include the entry about the calculation for `sizeProofsOrchard`.
- Rename `ExcludedPointEncodings` to `PreCanopyExcludedPointEncodings`.
- In § 5.6.2.3 ‘**Sprout** *Spending Keys*’ on p. 113, remove the statement that future key representations might use the padding bits of **Sprout** spending keys.
- Give a full-text URL for [Nakamoto2008].

2022.3.2 2022-06-06

- Set `NUFiveActivationHeight` for *Testnet* and *Mainnet*.
- An [**NU5** onward] consensus rule requiring the `nConsensusBranchId` field to match the *consensus branch ID* used for *SIGHASH transaction hashes*, should apply only when `effectiveVersion` ≥ 5 (since v4 *transactions* did not explicitly encode the `nConsensusBranchId` field).
- Correction in § 5.3 ‘*Constants*’ on p. 72: $\text{Uncommitted}^{\text{Orchard}} : \{0 \dots q_{\mathbb{P}} - 1\}$ is not a *bit sequence*.

- In §7.1 ‘*Transaction Encoding and Consensus*’ on p. 119, add the calculation for `sizeProofsOrchard` to the v5 *transaction* format table.
- Make §1.2 ‘*High-level Overview*’ on p. 8 more precise about *chain value pools*.

2022.3.1 2022-04-28

- In §4.2.3 ‘*Orchard Key Components*’ on p. 37, do not allow construction of **Orchard** *spending keys* such that the corresponding internal *incoming viewing key* is 0 or \perp . (This was already specified for the external *incoming viewing key*.) Similarly in §5.6.4.4 ‘*Orchard Raw Full Viewing Keys*’ on p. 116, do not consider a decoded key valid if either its external or internal *incoming viewing key* would be 0 or \perp .
- Clarify how to determine which table in §7.1 ‘*Transaction Encoding and Consensus*’ on p. 119 to use for *transaction* parsing, depending on the `effectiveVersion` as determined by the header field.
- Correct “block chain branch” to “*consensus branch*” to match [ZIP-200].
- Add an acknowledgement to Mary Maller for reviewing the Halo 2 security proofs.
- Add an acknowledgement to Josh Cincinnati for discussions on the Zcash protocol.
- Add acknowledgements to more people associated with the ZK Podcast.

2022.3.0 2022-03-18

- Correct a type error in the usage of $\text{Commit}^{\text{ivk}}$: the output type $\text{Commit}^{\text{ivk}}.\text{Output}$ includes 0, but the type of *incoming viewing keys* should not include 0 because $\text{KA}^{\text{Orchard}}.\text{Private}$ does not. This is now handled by explicitly rejecting 0 as output from $\text{Commit}^{\text{ivk}}$ when generating *ivk* in §4.2.3 ‘*Orchard Key Components*’ on p. 37. An encoding of *ivk* as 0 is also rejected in §5.6.4.3 ‘*Orchard Raw Incoming Viewing Keys*’ on p. 116 when parsing an *incoming viewing key*. The *Action circuit* needed no changes because pk_d already could not be $\mathcal{O}_{\mathbb{P}}$, and therefore the **Diversified address integrity** condition fails when *ivk* = 0.
- In §3.3 ‘*The Block Chain*’ on p. 16, define what a *settled network upgrade* is, specify requirements for checkpointing, and allow nodes to impose a limitation on rollback depth.
- In §5.4.10.1 ‘*BCTV14*’ on p. 108, note that the above checkpointing requirement mitigates the risks of not performing BCTV14 zk proof verification.
- Document the consensus rule that coinbase script length **MUST** be $\{2 \dots 100\}$ bytes.
- §3.11 ‘*Coinbase Transactions*’ on p. 21 effectively defined a *coinbase transaction* as the first *transaction* in a *block*. This wording was copied from the Bitcoin Developer Reference [Bitcoin-CbInput], but it does not match the implementation in `zcashd` that was inherited from Bitcoin Core. Instead, a *coinbase transaction* should be, and now is, defined as a *transaction* with a single null *prevout*. The specifications of consensus rules have been clarified and adjusted (without any actual consensus change) to take this into account, as follows:
 - a *block* **MUST** have at least one *transaction*;
 - the first *transaction* in a *block* **MUST** be a *coinbase transaction*, and subsequent *transactions* **MUST NOT** be *coinbase transactions*;
 - a *transparent input* in a non-coinbase *transaction* **MUST NOT** have a null *prevout*;
 - every non-null *prevout* **MUST** point to a unique *UTXO* in either a preceding *block*, or a *previous transaction* in the same *block* (this rule was previously not given explicitly because it was assumed to be inherited from **Bitcoin**);
 - the rule that “A *coinbase transaction* **MUST NOT** have any *transparent inputs* with non-null *prevout* fields” is removed as an explicit consensus rule because it is implied by the corrected definition of *coinbase transaction*.

2022.2.19 2022-01-19

- In § 4.10 ‘*SIGHASH Transaction Hashing*’ on p. 49, add a consensus rule that *SIGHASH* type encodings **MUST** be canonical for v5 transactions.
- In § 3.5 ‘*JoinSplit Transfers and Descriptions*’ on p. 18, clarify that balance for *JoinSplit* transfers is enforced by the *JoinSplit* statement, and that there is no consensus rule to check it directly.
- In § 8.5 ‘*Internal hash collision attack and fix*’ on p. 141, add a security argument for why the SHA-256-based commitment scheme $\text{NoteCommit}^{\text{Sprout}}$ is *binding* and *hiding*, under reasonable assumptions about SHA256Compress.

2022.2.18 2022-01-03

- Change the types of cm_x , $\text{Uncommitted}^{\text{Orchard}}$, and ak in **Orchard** to $\{0 \dots q_{\mathbb{P}} - 1\}$, avoiding type errors and reflecting the implementation in zcashd. This eliminates all uses of \mathbb{P}_x (except that ak in an **Orchard** full viewing key is still required to be a valid Pallas affine-short-Weierstrass x -coordinate).
- Refine the security argument about *partitioning oracle attacks* in § 8.7 ‘*In-band secret distribution*’ on p. 143:
 - The argument for decryption with an *incoming viewing key* does not need to depend on the *Decisional Diffie-Hellman Problem*, since g_d is committed to by the *note commitment* as well as pk_d .
 - It is necessary to say that the *note commitment* is always checked for a successful decryption.
 - Pedantically, it was not correct to conclude from the given security argument that *partitioning oracle attacks* against an *outgoing ciphertext* are necessarily prevented, according to the definition in [LGR2021]. Instead, the correct conclusions are that such attacks could not feasibly result in any equivocation of the decrypted data, or in recovery of ovk or ock .
- Correct the note about domain separators for $\text{PRF}^{\text{expand}}$ in § 4.1.2 ‘*Pseudo Random Functions*’ on p. 24, and ensure that new domain separators for deriving internal keys from [ZIP-32] and [ZIP-316] are included.

2021.2.17 2021-12-01

- Add notes in § B.1 ‘*RedDSA batch validation*’ on p. 215, § B.2 ‘*Groth16 batch verification*’ on p. 216 and § B.3 ‘*Ed25519 batch validation*’ on p. 218 that z_j may be sampled from $\{0 \dots 2^{128} - 1\}$ instead of $\{1 \dots 2^{128} - 1\}$.
- Add note in § 8.7 ‘*In-band secret distribution*’ on p. 143 about resistance of *note encryption* to *partitioning oracle attacks* [LGR2021].
- Add acknowledgement to Mihir Bellare for contributions to the science of *zero-knowledge proofs*.
- Add acknowledgement to Sasha Meyer.

2021.2.16 2021-09-30

- Use complete addition in SinsemillaCommit .
- Correct the proof of Theorem 5.4.6 on p. 97.
- Change the type of cm^{old} in **Orchard** to \mathbb{P} rather than \mathbb{P}^* , i.e. allow the identity point.
- Change the type of $\text{rt}^{\text{Orchard}}$ from \mathbb{P}_x (i.e. a Pallas x -coordinate or 0) to $\{0 \dots q_{\mathbb{P}} - 1\}$. This reflects the existing zcashd implementation; also checking $\text{rt}^{\text{Orchard}} \in \mathbb{P}_x$ would require a square root and is unnecessary.
- Witness g_d^{new} and pk_d^{new} in the **Orchard** Action circuit as \mathbb{P}^* , i.e. non-identity Pallas points, rather than witnessing their representations as *bit sequences*. This reflects the existing zcashd implementation.
- Note that $\text{ak}^{\mathbb{P}}$ in **Orchard** cannot be the identity.
- Correct the consensus rule about the maximum value of outputs in a *coinbase transaction*: it should reference the *block subsidy* rather than the *miner subsidy*.

2021.2.15 2021-09-01

- Correct a minor error in the proof of Theorem 5.4.3 on p. 81: the condition $\text{SinsemillaHashToPoint}(D, M) \neq \perp$ is required in the proof. (The case $\text{SinsemillaHashToPoint}(D, M) = \perp$ is covered by Theorem 5.4.4 on p. 82.) The proof had not been updated correctly when the statement was revised in v2021.2.0. Also add a missing D argument to $\text{SinsemillaHashToPoint}$ in that proof.
- Fix a reference to nonexistent version 2019.0-beta-40 of this specification (in §7.7.3 ‘*Difficulty adjustment*’ on p. 131) that should be v2019.0.0.
- Fix URL links to [BBDP2001] and [BDJR2000].
- Improve `protocol/links_and_dests.py` to eliminate false positives when checking DOI links.

2021.2.14 2021-08-12

- Fix the URL for [ZIP-239] in the References.
- **Reword the reference to a *Sapling* full viewing key in §4.8.2 ‘*Dummy Notes (Sapling)*’ on p. 46 (the full viewing key would include *ovk*, although it is not used in that section).**

2021.2.13 2021-07-29

- Add consensus rules in §3.8 ‘*Note Commitment Trees*’ on p. 20 that, for each *note commitment tree*, a block **MUST NOT** add *note commitments* that exceed the capacity of that tree.

2021.2.12 2021-07-29

- Change the number of partial rounds, R_P , for Poseidon from 58 to 56. This matches the number calculated by `calc_round_numbers.py` (for 128-bit security “with margin”) in Version 1.1 of the Poseidon reference implementation [Poseidon-1.1] [Poseidon-Zc1.1].

2021.2.11 2021-07-20

- Change the definition of inputs to the *Action circuit* to split `enableSpend`s and `enableOutput`s into two field elements.

2021.2.10 2021-07-13

- Clarify that decomposition of scalars for scalar multiplication in the *Action circuit* **MUST** be canonical, unless a non-canonical decomposition can be proven to result in an equivalent statement – and clarify for which multiplications the latter case applies.
- The encoding of the *block height* in the `scriptSig` of a *coinbase transaction* is now at most 5 bytes (rather than 9 bytes), because *block height* **MUST** also be encoded in the 32-bit `nExpiryHeight` field of *coinbase transactions* after **NU5** activation.
- Clarify in §3.4 ‘*Transactions and Treestates*’ on p. 17 that the remaining value in a *transparent transaction value pool* is only available to miners as a fee in the case of non-coinbase *transactions*, and that the remaining value in the *transparent transaction value pool* of a *coinbase transaction* is destroyed.
- Remove a spurious reference to `rseed` in §4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63. There were no changes for **Sprout** in [ZIP-212].

2021.2.9 2021-07-01

- Add a consensus rule for version 5 or later *transactions*, that if $\text{nActionsOrchard} > 0$ then at least one of $\text{enableSpendsOrchard}$ and $\text{enableOutputsOrchard}$ **MUST** be 1.
- Delete the consensus rule in § 3.4 ‘*Transactions and Treestates*’ on p. 17 that required checking that each intermediate *root* of the *note commitment tree* is not \perp . Checking this rule would have imposed a significant performance penalty, since intermediate roots do not otherwise need to be computed.
- Change the type of $\text{MerkleCRH}^{\text{Orchard}}$ to have $\{0 \dots q_{\mathbb{P}} - 1\}$ in place of $\{0 \dots q_{\mathbb{P}} - 1\} \cup \{\perp\}$ for the inputs and output, and map a \perp output from SinsemillaHash to 0. (We retain the original definitions of SinsemillaHash and $\text{SinsemillaHashToPoint}$ both because it would be disruptive to change them at this point in the Network Upgrade Process, and because it is necessary to track \perp outputs in order to correctly model non-determinism in the *Action circuit*.)
- Allow the Merkle path validity check in the *Action circuit* to pass if any output of $\text{MerkleCRH}^{\text{Orchard}}$ is 0, and add a note in § 4.9 ‘*Merkle Path Validity*’ on p. 47 arguing that this is safe.
- Fix a typo in the Security Requirements for § 5.4.1.3 ‘ $\text{MerkleCRH}^{\text{Orchard}}$ *Hash Function*’ on p. 75: the length of the input to SinsemillaHash is $10 + 2 \cdot \ell_{\text{Merkle}}^{\text{Orchard}}$ bits, not $6 + 2 \cdot \ell_{\text{Merkle}}^{\text{Orchard}}$ bits.
- Replace “must” with “**MUST**” in two consensus rules specified in § 7.1 ‘*Transaction Encoding and Consensus*’ on p. 119.
- Add a clarification in § 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121 that after **Heartwood** and before **Canopy** activation, **Sapling** outputs of a *coinbase transaction* **MUST** have *note plaintext lead byte* equal to 0x01. This was implied by the existing rule that such outputs **MUST** decrypt successfully with an all-zero *outgoing viewing key*.
- Correct *l* to *l** in two places in § 5.4.1.3 ‘ $\text{MerkleCRH}^{\text{Sapling}}$ *Hash Function*’ on p. 74.
- Correct an erroneous statement in § 3.4 ‘*Transactions and Treestates*’ on p. 17 that claimed *transaction IDs* are not part of the consensus protocol.

2021.2.8 2021-06-29

- Change one of the [**Sapling onward**] consensus rules in § 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121 to have the correct applicability: [**Sapling to Canopy inclusive**, pre-**NU5**].
- Describe *transaction IDs* and *wtxids* in § 3.4 ‘*Transactions and Treestates*’ on p. 17.
- Add a section § 7.1.1 ‘*Transaction Identifiers*’ on p. 121 on how to compute *transaction IDs* and *wtxids*.
- Split the *transaction*-related consensus rules into their own subsection § 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121, for more precise cross-referencing.

2021.2.7 2021-06-28

- Correct the type of $\text{Uncommitted}^{\text{Orchard}}$, which should be \mathbb{P}_x rather than a *bit sequence*.
- Explicitly say that padding in § 5.4.1.9 ‘*Sinsemilla Hash Function*’ on p. 79 is by appending zero bits.
- Add a step to the algorithm for generating an **Orchard** note in § 4.7.3 ‘*Sending Notes (Orchard)*’ on p. 44, to restart if $\text{esk} = 0$.

2021.2.6 2021-06-26

- Require that from **NU5** activation, the nExpiryHeight field of a *coinbase transaction* is set to the *block height*. This is needed to maintain the property that all *transactions* have unique *transaction IDs*, as explained in a note in § 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121. In order to avoid the *block height* being limited to 499999999, we also remove that bound on nExpiryHeight for *coinbase transactions*.
- Remove the recommendation to support 63-bit *block heights* in § 3.3 ‘*The Block Chain*’ on p. 16 (since it is incompatible with the above consensus rule for *coinbase nExpiryHeight*).

- Ensure that the *layer* number is passed to MerkleCRH in § 4.9 ‘*Merkle Path Validity*’ on p. 47.
- Refine the key components diagram in § 3.1 ‘*Payment Addresses and Keys*’ on p. 12 to show that **Orchard incoming viewing keys** include both *dk* and *ivk*.
- Clarify that the $\{-\text{MAX_MONEY} \dots \text{MAX_MONEY}\}$ range restriction applies to both *valueBalanceSapling* and *valueBalanceOrchard*.
- Update § 5.5 ‘*Encodings of Note Plaintexts and Memo Fields*’ on p. 110 for **Orchard**.
- Add [ZIP-203], [ZIP-212], and [ZIP-213] to the list of ZIPs updated for **NU5**.
- Give cross-references to § 2 ‘*Notation*’ on p. 9 where $\sqrt[3]{\cdot}$ and $\sqrt[4]{\cdot}$ are used.

2021.2.5 2021-06-19

- Change the consensus rule that requires at least one input to, and at least one output from a v5 or later *transaction*, to take into account the *enableSpendsOrchard* and *enableOutputsOrchard* flags.
- Correct the type of $\text{Extract}_{\mathbb{P}}^{\perp}$ imported in § 5.4.1.9 ‘*Sinsemilla Hash Function*’ on p. 79 (from $\mathbb{P} \rightarrow \mathbb{P}_x$ to $\mathbb{P} \cup \{\perp\} \rightarrow \mathbb{P}_x \cup \{\perp\}$).
- Add [ZIP-209] to the list of ZIPs updated for **NU5**.

2021.2.4 2021-06-08

- Add an explicit consensus rule in § 7.1.2 ‘*Transaction Consensus Rules*’ on p. 121 that the reserved bits of the *flagsOrchard* field **MUST** be zero.
- Correct a cut-and-paste error in the algorithm for § 4.8.3 ‘*Dummy Notes (Orchard)*’ on p. 47, which should refer to the *Action statement* rather than the *Spend statement*.

2021.2.3 2021-06-06

- Specify (as a note in § 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61) the encoding of *primary inputs* to the *Action circuit*. This uses new helper functions \mathcal{X} and \mathcal{Y} defined in § 5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p. 104. The specification of $\text{Extract}_{\mathbb{P}}$ has also been refactored to use \mathcal{X} (this does not change the **Orchard** protocol).
- In § 5.4.1.10 ‘*PoseidonHash Function*’ on p. 82, say that the round constants as well as the MDS matrices are generated according to Version 1.1 of the reference implementation.
- Clarify that *epk* encoded in an *Action description* cannot be $\mathcal{O}_{\mathbb{P}}$.
- Specify that **Orchard spending keys** are encoded using *Bech32m*.
- Add [ZIP-239] to the list of ZIPs included in **NU5**.
- Move the section on abstraction (previously section 5.1) to § 4 ‘*Abstract Protocol*’ on p. 22. Section 5.2 has been split into two (§ 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71 and § 5.2 ‘*Bit layout diagrams*’ on p. 71) to avoid renumbering later subsections.
- Correct an error in the encoding of height-in-coinbase for *blocks* at heights 1 .. 16.
- Clarify, in § 3.3 ‘*The Block Chain*’ on p. 16, requirements on the range of *block heights* that should be supported.
- Delete the sentence “All conversions between Ed25519 points, *byte sequences*, and integers used in this section are as specified in [BDLSY2012].” from § 5.4.6 ‘*Ed25519*’ on p. 88. This sentence was misleading given that the conversions in [BDLSY2012] are not sufficiently well-specified for a consensus protocol; it should have been deleted earlier when explicit definitions for $\text{reprBytes}_{\text{Ed25519}}$ and $\text{abstBytes}_{\text{Ed25519}}$ were added.
- Make the **NU5** specification the default.

2021.1.24 2021-04-23

- Add the `nConsensusBranchId` field to *v5 transactions*, matching the *consensus branch ID* used for *SIGHASH transaction hashes*.
- Include the *diversifier* key in an encoded **Orchard** Incoming Viewing Key.
- Remove an unused precomputation in § 5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105.
- Clarify that only an *outgoing cipher key* is strictly needed to decrypt an *outgoing ciphertext*.
- Explicitly say that *coinbase transactions* **MUST NOT** have *transparent inputs* (this is a consensus rule inherited from **Bitcoin** which has been present since launch).

2021.1.23 2021-04-19

- Correct errors in the definitions of $\text{Extract}_{\mathbb{P}}$ and $\text{Extract}_{\mathbb{P}}^{\perp}$ in § 5.4.9.7 ‘*Coordinate Extractor for Pallas*’ on p. 104: $\text{Extract}_{\mathbb{P}}(\mathcal{O}_{\mathbb{P}})$ should be 0, and $\text{Extract}_{\mathbb{P}}^{\perp}(\perp)$ should be \perp .
- Change the type of $\text{KA}^{\text{Orchard}}$ public keys and shared secrets to \mathbb{P}^* (i.e. exclude $\mathcal{O}_{\mathbb{P}}$), and the type of $\text{KA}^{\text{Orchard}}$ private keys to $\mathbb{F}_{r_{\mathbb{P}}}^*$ (i.e. exclude 0).
- Change the type of an **Orchard** *ivk* to $\{1 \dots q_{\mathbb{P}} - 1\}$ (i.e. exclude 0).
- Change the types of pk_d^{old} , cm^{old} and $\text{ak}^{\mathbb{P}}$ to \mathbb{P}^* in the *auxiliary inputs* to the *Action statement*.
- When creating **Orchard** *notes*, repeat with another *rseed* if *cm* is \perp .
- Add a note in § 4.2.3 ‘**Orchard** Key Components’ on p. 37 about non-uniformity of *ivk*.
- Fix a typo: “Decryption” to “Description”.
- Add *Action descriptions* to the introduction of § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56.
- Use a different footnote symbol for each **Sapling** field cardinality rule in *v5 transactions*.
- Fix some URLs in references.

2021.1.22 2021-04-05

- Specify that a *unified payment address* **MUST** contain at least one *shielded payment address*.
- Further clarifications to Theorem 5.4.3 on p. 81.
- **Correct ZKSpend.Verify to ZKOutput.Verify in § 4.5 ‘Output Descriptions’ on p. 40.**
- Make sure that Change History entries are URL destinations.

2021.1.21 2021-04-01

- Correct and clarify Theorem 5.4.3 on p. 81 and Theorem 5.4.4 on p. 82.
- Clarify that a *dummy note* should be created if no real **Orchard** *note* is being spent in an *Action transfer*.
- Add a caveat in § 4.2.3 ‘**Orchard** Key Components’ on p. 37 about reuse of *rivk* between $\text{PRF}^{\text{expand}}$ and $\text{Commit}^{\text{ivk}}$.
- Expand the set of ZIPs associated with **NU5** in § 6 ‘*Network Upgrades*’ on p. 118, and reference [Zcash-Orchard] and [Zcash-halo2] there.
- Section § 5.4.5.6 ‘**Orchard** Key Derivation’ on p. 88 should be in slate blue.
- Explicitly note that the end of the [ZIP-212] grace period precedes **NU5** activation.
- Change the condition for presence of *anchorSapling* in a version 5 *transaction* to $\text{vSpendSapling} > 0$.
- **Fix type error in *kdfinput* for $\text{KDF}^{\text{Sapling}}$ and $\text{KDF}^{\text{Orchard}}$ (*ephemeralKey* is already a *byte sequence*).**
- **Make a note in § 8.7 ‘In-band secret distribution’ on p. 143 of the divergence of *ivk* for **Sapling** and **Orchard** from a uniform scalar.**

- Correct the set of inputs to $\text{PRF}^{\text{expand}}$ used for [ZIP-32] and **Orchard** in § 4.1.2 ‘*Pseudo Random Functions*’ on p. 24.
- Write the caution about linkage between the abstract and concrete protocols in § 4 ‘*Abstract Protocol*’ on p. 22.
- Update the **Sprout** key component diagram in § 3.1 ‘*Payment Addresses and Keys*’ on p. 12 to remove magenta highlighting.

2021.1.20 2021-03-25

- Credit Eirik Ogilvie-Wigley as a designer of the Zcash protocol. Add Andre Serrano, Brad Miller, Charlie O’Keefe, David Campbell, Elena Giral, Francisco Gindre, Joseph Van Geffen, Josh Swihart, Kevin Gorham, Larry Ruane, Marshall Gaucher, and Ryan Taylor to the acknowledgements.
- Add proof of *collision resistance* for Sinsemilla.
- Correct some interim findings of the NCC specification audit:
 - Fix typos.
 - Correct the definition of c in § 5.4.1.9 ‘*Sinsemilla Hash Function*’ on p. 79.
 - Propagate \perp intermediate results to the output of Sinsemilla primitives.
 - Change the output types of $\text{NoteCommit}^{\text{Orchard}}$ and $\text{Commit}^{\text{ivk}}$ to reflect that these can return \perp , and change the *Action statement* to be satisfied if they do.
 - Propagate \perp from the inputs of $\text{MerkleCRH}^{\text{Orchard}}$ to its output, and add an explicit consensus rule that $\text{rt}^{\text{Orchard}}$ computed from appending a *note commitment* is not \perp .
 - Correct the definition of $\text{PRF}^{\text{nfOrchard}}$ in § 5.4.2 ‘*Pseudo Random Functions*’ on p. 84 by changing Poseidon to PoseidonHash.
 - Restrict the definition of a *short Weierstrass elliptic curve* in § 5.4.9.6 ‘*Pallas and Vesta*’ on p. 103 to base fields of characteristic greater than 3.
 - Define \mathbb{G} in § 5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105.
 - Fix type confusion between integers and field elements (including additional cases not found in the audit, involving *nullifiers* and cm_x).
 - Fix a discrepancy between § 5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105 and [ID-hashtocurve]: the zero padding in `expand_message_xmd` should be 128 bytes (matching the input block size of BLAKE2b), rather than 64 bytes.
 - Document that the use of $k = 256$ when extracting field elements in `hash_to_field` is intentional, despite the Pallas curve only having 126-bit conjectured security against generic attacks.
 - Correct the output type of $\text{sqrt_ratio}_{\mathbb{F}_{q_{\mathbb{G}}}}$.
 - Document that the choice of nonsquare for $\lambda_{\mathbb{G}}$ in § 5.4.9.8 ‘*Group Hash into Pallas and Vesta*’ on p. 105 makes no difference to the output of `map_to_curve_simple_swu`^{iso- \mathbb{G}} .
 - Document the limitation on the domain separation string for the *group hash* into Pallas and Vesta.
 - Correct the sizes of `SpendDescriptionV5` and `OutputDescriptionV5` in the version 5 *transaction* format.
 - Make the description of when fields are included in v5 *transactions* consistent between the protocol specification and [ZIP-225].
 - Make the naming of `enableSpend`s and `enableOutput`s consistent.
- Change the specifications of *note* decryption in § 4.19 ‘*In-band secret distribution (Sprout)*’ on p. 63 and § 4.20 ‘*In-band secret distribution (Sapling and Orchard)*’ on p. 65 to return the *note* and *memo field*, rather than a *note plaintext*.
- Generalize the *block chain* scanning algorithm in § 4.22 ‘*Block Chain Scanning (Sapling and Orchard)*’ on p. 70 to support **Orchard**.
- Update the `hashFinalSaplingRoot/hashLightClientRoot/hashBlockCommitments` field for **NU5**.

- Update specification of Poseidon.
- Fix errors in **Orchard** due to cut-and-paste from **Sapling**.
- Add references to [Zcash-halo2].
- Correct the description of length in § 5.6.4.1 ‘*Unified Payment Addresses and Viewing Keys*’ on p. 115.
- Correct the type signature of $\text{DiversifyHash}^{\text{Orchard}}$ in § 4.1.1 ‘*Hash Functions*’ on p. 23.
- Various rationale updates for **NU5**.
- Other fixes to the **Orchard** specification, including generation of *dummy notes* and *output notes*.
- Describe the recommended way to encode a **Sapling** or unified *payment address* as a QR code.
- Move the definition of \perp to before its first use.
- Delete a confusing part of the definition of $\text{concat}_{\mathbb{B}}$ that we don’t rely on.
- Add a definition for the $\$$ symbol in § 1 ‘*Introduction*’ on p. 7, before its first use.
- Remove specification of *memo field* contents, which will be in [ZIP-302].
- Remove support for building the **Sprout**-only specification (*sprout.pdf*).
- Remove magenta highlighting of differences from **Zerocash**.

2021.1.19 2021-03-17

- Correct the range of input to $\text{ValueCommit}^{\text{Orchard}}$ in the *Action statement*, and the corresponding security argument in § 4.14 ‘*Balance and Binding Signature (Orchard)*’ on p. 53.
- Update the consensus rules that prevent trivial transactions (with no inputs or outputs) to take into account *Action transfers* in the v5 *transaction* format.
- Make $\text{DiversifyHash}^{\text{Orchard}}$ total, by replacing an output of $\mathcal{O}_{\mathbb{P}}$ with another base.
- Fix a type error in the non-normative note at the end of § 5.4.8.4 ‘*Sinsemilla commitments*’ on p. 96.

2021.1.18 2021-03-17

- Define *unified payment addresses* in place of the *Bech32* form of **Orchard** *shielded payment addresses*.
- Remove **Sprout**-specific fields from the v5 *transaction* format.
- The ρ value for an **Orchard** output *note* was incorrectly described as being derived from *rseed*, instead of being set to the nullifier from the same *Action description* as intended.
- The ψ value is now derived using the $\text{PRF}^{\text{expand}}$ input [9], instead of [10].
- Correct a note about the range of the Merkle hash inputs in § 4.18.4 ‘*Action Statement (Orchard)*’ on p. 61.
- Correct the validity condition for ak in § 5.6.4.4 ‘*Orchard Raw Full Viewing Keys*’ on p. 116.
- Add a definition for $\mathcal{K}^{\text{Orchard}}$ in § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56.
- Correct the number of full and partial rounds for Poseidon.
- Add a note explaining the origin of the 2^{65} constant in the definition of PoseidonHash .
- The subgroup check added to § 4.20.3 ‘*Decryption using a Full Viewing Key (Sapling and Orchard)*’ on p. 68 for **Sapling** in v2021.1.17 was applied to the wrong variable (g_d , when it should have been pk_d), despite being described correctly in the Change History entry below.

2021.1.17 2021-03-15

- Draft **NU5** specification.
- In the consensus rule that a *transaction* with one or more *transparent* inputs from *coinbase transactions* **MUST** have no *transparent* outputs, explicitly say that inputs from *coinbase transactions* include *funding stream* outputs.
- The definition of an abstraction function in §4.1.9 ‘*Represented Group*’ on p. 31 incorrectly required canonicity, i.e. that $\text{abst}_{\mathbb{G}}$ does not accept inputs outside the range of $\text{repr}_{\mathbb{G}}$. While this was originally intended, it is not true of $\text{abst}_{\mathbb{J}}$. (It is also not true of $\text{abstBytes}_{\text{Ed25519}}$, but Ed25519 is not strictly defined as a *represented group* in this specification.)
- Correct Theorem 5.4.5 on p. 94, which was proving the wrong thing. It needs to prove that $\text{NoteCommit}^{\text{Sapling}}$ does not return $\text{Uncommitted}^{\text{Sapling}}$, but was previously proving that PedersenHash does not return that value.
- The note about *non-canonical* encodings in §5.4.9.3 ‘*Jubjub*’ on p. 100 gave incorrect values for the encodings of the point of order 2, by omitting a $q_{\mathbb{J}}$ term.
- The specification of decryption in §4.20.3 ‘*Decryption using a Full Viewing Key (Sapling and Orchard)*’ on p. 68 differed from its implementation in *zcashd*, in two respects:
 - The specification had a type error in that it failed to check whether $\text{abst}_{\mathbb{J}}(\text{pk} \star_d) = \perp$, which is needed in order for its use as input to $\text{KA}^{\text{Sapling}}$. Agree to be well-typed.
 - The specification did not require pk_d to be in the subgroup $\mathbb{J}^{(r)}$, while the implementation in *zcashd* did. This check is not needed for security; however, since Jubjub public keys are normally of type $\text{KA}^{\text{Sapling}}.\text{PublicPrimeSubgroup}$, we change the specification to match *zcashd*.
- Correct the procedure for generating *dummy Sapling notes* in §4.8.2 ‘*Dummy Notes (Sapling)*’ on p. 46.
- Add a note in §5.4.10.1 ‘*BCTV14*’ on p. 108 describing conditions under which an implementation that check-points on **Sapling** can omit verifying BCTV14 proofs.
- Rename “hash extractor” to *coordinate extractor*. This is a more accurate name since it is also used on commitments.
- Rename *char* to *byte* in field type declarations.

2021.1.16 2021-01-11

- Add macros and *Makefile* support for building the **NU5** draft specification.
- Clarify the encoding of *block heights* for the “height in coinbase” rule. The description of this rule has also moved from §7.6 on p. 128 to §7.1.2 ‘*Transaction Consensus Rules*’ on p. 121.
- Include the activation dates of **Heartwood** and **Canopy** in §6 ‘*Network Upgrades*’ on p. 118.
- Section links in the **Heartwood** and **Canopy** versions of the specification now go to the correct document URL.
- Attempt to improve search and cut-and-paste behaviour for ligatures in some PDF readers.

2020.1.15 2020-11-06

- Add a missing consensus rule that has always been implemented in *zcashd*: there must be at least one *transparent output*, *Output description*, or *JoinSplit description* in a *transaction*.
- Add a consensus rule that the (zero-valued) *coinbase transaction* output of the *genesis block* cannot be spent.
- Define **Sprout** chain value pool balance and **Sapling** chain value pool balance, and include consensus rules from [ZIP-209].
- Correct the **Sapling** note decryption algorithms:
 - *ephemeralKey* is kept as a *byte sequence* rather than immediately converted to a curve point; this matters because of *non-canonical* encoding.

- The representation of pk_d in a *note plaintext* may also be *non-canonical* and need not be in the *prime-order subgroup*.
- Move checking of cm_u in decryption with ivk to the end of the algorithm, to more closely match the implementation.
- The note about decryption of outputs in *mempool transactions* should have been normative.
- Reserve *transaction version number* $0x7FFFFFFF$ and *version group ID* $0xFFFFFFFF$ for experimental use.
- Remove a statement that the language consisting of key and address encoding possibilities is prefix-free. (The human-readable forms are prefix-free but the raw encodings are not; for example, the *raw encoding* of a **Sapling** *spending key* can be a prefix of several of the other encodings.)
- Use “let mutable” to introduce mutable variables in algorithms.
- Include a reference to [BFI]SV2010] for batch pairing verification techniques.
- Acknowledge Jack Gavigan as a co-designer of **Sapling** and of the **Zcash** protocol.
- Acknowledge Izaak Meckler, Zac Williamson, Vitalik Buterin, and Jakub Zalewski.
- Acknowledge Alexandra Elbakyan.

2020.1.14 2020-08-19

- The consensus rule that a *coinbase transaction* must not spend more than is available from the *block subsidy* and *transaction fees*, was not explicitly stated. (This rule was correctly implemented in *zcashd*.)
- Fix a type error in the output of $PRF^{nfSapling}$; a **Sapling nullifier** is a sequence of 32 bytes, not a *bit sequence*.
- Correct an off-by-one in an expression used in the definition of c in § 5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77 (this does not change the value of c).

2020.1.13 2020-08-11

- Rename the type of **Sapling transmission keys** from $KA^{Sapling}.PublicPrimeOrder$ to $KA^{Sapling}.PublicPrimeSubgroup$. This type is defined as $\mathbb{J}^{(n)}$, which reflects the implementation in *zcashd* (subject to the next point below); it was never enforced that a *transmission key* (pk_d) cannot be \mathcal{O}_j .
- Add a non-normative note saying that *zcashd* does not fully conform to the requirement to treat *transmission keys* not in $KA^{Sapling}.PublicPrimeSubgroup$ as invalid when importing *shielded payment addresses*.
- Set *CanopyActivationHeight* for *Testnet*.
- Modify the tables and notes in § 7.10.1 ‘*ZIP 214 Funding Streams*’ on p. 137 to reflect changes in [ZIP-214].
- Updates to reflect [ZIP-211]: add a consensus rule on v_{pub}^{old} in § 4.3 ‘*JoinSplit Descriptions*’ on p. 38, and a rule about node and wallet support for sending to **Sprout** addresses in § 4.7.1 ‘*Sending Notes (Sprout)*’ on p. 42.
- Refine the domain of *HeightForHalving* from \mathbb{N} to \mathbb{N}^+ .
- Make *Halving(height)* return 0 (rather than -1) for $height < SlowStartShift$. This has no effect on consensus since the *Halving* function is not used in that case, but it makes the definition match the intuitive meaning of the function.
- Rename sections under § 7 ‘*Consensus Changes from Bitcoin*’ on p. 119 to clarify that these sections do not only concern encoding, but also consensus rules.
- Make the **Canopy** specification the default.

2020.1.12 2020-08-03

- Include SHA-512 in § 5.4.1.1 ‘SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions’ on p. 73.
- Add a reference to [BCCGLRT2014] in § 4.1.13 ‘Zero-Knowledge Proving System’ on p. 33.
- Use $\text{abstBytes}_{\text{Ed25519}}$ and $\text{reprBytes}_{\text{Ed25519}}$ for conversions in § B.3 ‘Ed25519 batch validation’ on p. 218, and fix a missing requirement that $S_j < \ell$ for all signatures.

2020.1.11 2020-07-13

- Change instances of “the production network” to “Mainnet”, and “the test network” to *Testnet*. This follows the terminology used in ZIPs.
- Update stale references to **Bitcoin** documentation.
- Add changes for [ZIP-207] and [ZIP-214].

2020.1.10 2020-07-05

- Corrections to a note in § 5.4.6 ‘Ed25519’ on p. 88.

2020.1.9 2020-07-05

- Add § 3.12 ‘Mainnet and Testnet’ on p. 21.
- Acknowledge Jane Lusby and teor.
- Precisely specify the encoding and decoding of Ed25519 points.
- Correct an error introduced in v2020.1.8; “ $-O_j$ ” was incorrectly used when the point $(0, -1)$ on Jubjub was meant.
- Precisely specify the conversion from a *bit sequence* in abst_j .

2020.1.8 2020-07-04

- Add Ying Tong Lai and Kris Nuttycombe as **Zcash** protocol designers.
- Change the specification of abst_j in § 5.4.9.3 ‘Jubjub’ on p. 100 to match the implementation.
- Repair the argument for $\text{GroupHash}_{\text{URS}}^{(*)}$ being usable as a *random oracle*, which previously depended on abst_j being injective.
- In RedDSA verification, clarify that \underline{R} used as part of the input to H° **MUST** be exactly as encoded in the signature.
- Specify that *shielded outputs* of coinbase transactions **MUST** use v2 *note plaintexts* after **Canopy** activation.
- Correct a bug in § 4.20.3 ‘Decryption using a Full Viewing Key (**Sapling** and **Orchard**)’ on p. 68: esk is only to be checked against $\text{ToScalar}(\text{PRF}_{\text{rseed}}^{\text{expand}}([4]))$ when $\text{leadByte} \neq 0x01$. [Later edit: this should have been $\text{ToScalar}(\text{PRF}_{\text{rseed}}^{\text{expand}}([5]))$.]

2020.1.7 2020-06-26

- Delete some ‘new’ superscripts that only added notational clutter.
- Add an explicit lead byte field to **Sprout** *note plaintexts*, and clearly specify the error handling when it is invalid.
- Define a **Sapling** *note plaintext lead byte* as having type \mathbb{B}^Y (so that decoding to a *note plaintext* always succeeds, and error handling is more explicit).
- Fix a sign error in the fixed-base term of the batch validation equation in § B.1 ‘RedDSA batch validation’ on p. 215.

- Fix a sign error in the fixed-base term of the batch validation equation in § B.3 ‘Ed25519 *batch validation*’ on p. 218.

2020.1.6 2020-06-17

- Incorporate changes to **Sapling** *note* encryption from [ZIP-212].
- Correct an error in the specification of Ed25519 *validating* keys: they should not have been specified to be checked against PreCanopyExcludedPointEncodings, since libsodium v1.0.15 does not do so.
- Incorporate Ed25519 changes for **Canopy** from [ZIP-215].
- Add Appendix § B.3 ‘Ed25519 *batch validation*’ on p. 218.
- Consistently use “validating” for signatures and “verifying” for proofs.
- Use the symbol $\sqrt[+]{}$ for positive square root.

2020.1.5 2020-06-02

- Reference [ZIP-173] instead of BIP 173.
- Mark more index entries as definitions.

2020.1.4 2020-05-27

- Reference [BIP-32] and [ZIP-32] when describing keys and their encodings.
- Network Upgrade 4 has been given the name **Canopy**.
- Reference [ZIP-211], [ZIP-212], and [ZIP-215] for the **Canopy** upgrade.
- Improve LaTeX portability of this specification.

2020.1.3 2020-04-22

- Correct a wording error transposing *transparent inputs* and *transparent outputs* in § 4.12 ‘*Balance (Sprout)*’ on p. 50.
- Minor wording clarifications.
- Reference [ZIP-251], [ZIP-207], and [ZIP-214] for the **Canopy** upgrade.

2020.1.2 2020-03-20

- The implementation of **Sprout** Ed25519 signature validation in zcashd differed from what was specified in § 5.4.6 ‘Ed25519’ on p. 88. The specification has been changed to match the implementation.
- Add consensus rules for **Heartwood**.
- Remove “pvc” Makefile targets.
- Make the **Heartwood** specification the default.
- Add macros and Makefile support for building the **Canopy** specification.

2020.1.1 2020-02-13

- Resolve conflicts in the specification of *memo fields* by deferring to [ZIP-302].

2020.1.0 2020-02-06

- Specify a retrospective soft fork implemented in zcashd v2.1.1-1 that limits the `nTime` field of a *block* relative to its *median-time-past*.
- Correct the definition of *median-time-past* for the first PoWMedianBlockSpan *blocks* in a *block chain*.
- Add acknowledgements to Henry de Valence, Deirdre Connolly, Chelsea Komlo, and Zancas Wilcox.
- Add an acknowledgement to Trail of Bits for their security audit.
- Change indices in the *incremental Merkle tree* diagram to be zero-based.
- Use the term “*monomorphism*” for an injective homomorphism, in the context of a *signature scheme with key monomorphism*.

2019.0.9 2019-12-27

- No changes to **Sprout** or **Sapling**.
- Specify the height at which **Blossom** activated.
- Add **Blossom** to § 6 ‘*Network Upgrades*’ on p. 118.
- Add a non-normative note giving the explicit value of FoundersRewardLastBlockHeight.
- Clarify the effect of **Blossom** on *SIGHASH transaction hashes*.
- Makefile updates for **Heartwood**.

2019.0.8 2019-09-24

- Fix a typo in the generator \mathcal{P}_{S_1} in § 5.4.9.2 ‘*BLS12-381*’ on p. 99 found by magrady.
- Clarify the type of v^{new} in § 4.7.2 ‘*Sending Notes (Sapling)*’ on p. 43.

2019.0.7 2019-09-24

- Fix a discrepancy in the number of constraints for BLAKE2s found by QED-it.
- Fix an error in the expression for Δ in § A.3.3.9 ‘*Pedersen hash*’ on p. 205, and add acknowledgement to Kobi Gurkan.
- Fix a typo in § 4.9 ‘*Merkle Path Validity*’ on p. 47 and add acknowledgement to Weikeng Chen.
- Update references to ZIPs and to the Electric Coin Company blog.
- Makefile improvements to suppress unneeded output.

2019.0.6 2019-08-23

- No changes to **Sprout** or **Sapling**.
- Replace dummy **Blossom** activation block height with the *Testnet* height, and a reference to [ZIP-206].

2019.0.5 2019-08-23

- Note the change to the minimum-difficulty threshold time on *Testnet* for **Blossom**.
- Correct the packing of nf^{old} into input elements in § A.4 ‘*The Sapling Spend circuit*’ on p. 212.
- Add an epigraph from [Carroll1876] to the start of § 5.4.9.3 ‘Jubjub’ on p. 100.
- Clarify how the constant c in § 5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77 is obtained.
- Add a footnote that *zcashd* uses [ZIP-32] *extended spending keys* instead of the derivation from sk in § 3.1 ‘*Payment Addresses and Keys*’ on p. 12.
- Remove “optimized” *Makefile* targets (which actually produced a larger PDF, with TeXLive 2019).
- Remove “html” *Makefile* targets.
- Make the **Blossom** spec the default.

2019.0.4 2019-07-23

- Clicking on a section heading now shows section labels.
- Add a **List of Theorems and Lemmata**.
- Changes needed to support TeXLive 2019.

2019.0.3 2019-07-08

- Experimental support for building using LuaTeX and XeTeX.
- Add an **Index**.

2019.0.2 2019-06-18

- Correct a misstatement in the security argument in § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51: *binding* for a *commitment scheme* does not imply that the commitment determines its randomness. The rest of the security argument did not depend on this; it is simpler to rely on knowledge soundness of the Spend and Output proofs.
- Give a definition for *complete twisted Edwards elliptic curves* in § 5.4.9.3 ‘Jubjub’ on p. 100.
- Clarify that Theorem 5.4.5 on p. 94 depends on the parameters of the Jubjub curve.
- Ensure that this document builds correctly and without missing characters on recent versions of TeXLive.
- Update the *Makefile* to use Ghostscript for PDF optimization.
- Ensure that hyperlinks are preserved, and available as “Destination names” in URL fragments and links from other PDF documents.

2019.0.1 2019-05-20

- No changes to **Sprout** or **Sapling**.
- Minor fix to the list of integer constants in § 2 ‘*Notation*’ on p. 9.
- Use `IsBlossomActivated` in the definition of `FounderAddressAdjustedHeight` for consistency.

2019.0.0 2019-05-01

- Fix a specification error in the *Founders' Reward* calculation during the slow start period.
- Correct an inconsistency in difficulty adjustment between the spec and zcashd implementation for the first PoWAveragingWindow – 1 *blocks* of the *block chain*. This inconsistency was pointed out by NCC Group in their **Blossom** specification audit.
- Revert changes for *funding streams* from Withdrawn ZIP 207.

2019.0-beta-39 2019-04-18

- Change author affiliations from “Zerocoin Electric Coin Company” to “Electric Coin Company”.
- Add acknowledgement to Mary Maller for the observation that *diversified payment address* unlinkability can be proven in the same way as *key privacy* for ElGamal.

2019.0-beta-38 2019-04-18

- Update the following sections to match the current draft of [ZIP-208]:
 - § 7.7.3 ‘*Difficulty adjustment*’ on p.131
 - § 7.8 ‘*Calculation of Block Subsidy, Funding Streams, and Founders' Reward*’ on p.133
- Specify *funding streams*, along with the draft *funding streams* defined in the current draft of ZIP 207.
- Update the following sections to match the current draft of ZIP 207:
 - § 3.10 ‘*Block Subsidy, Funding Streams, and Founders' Reward*’ on p. 21
 - § 3.11 ‘*Coinbase Transactions*’ on p. 21
 - § 7.8 ‘*Calculation of Block Subsidy, Funding Streams, and Founders' Reward*’ on p.133
 - § 7.9 ‘*Payment of Founders' Reward*’ on p.134
- Correct the generators \mathcal{P}_{S_1} and \mathcal{P}_{S_2} for BLS12-381.
- Update README.rst to include Makefile targets for **Blossom**.
- Makefile updates:
 - Fix a typo for the pvcblossom target.
 - Update the pinned git hashes for sam2p and pdfsizeopt.

2019.0-beta-37 2019-02-22

- The rule that miners **SHOULD NOT** mine *blocks* that chain to other *blocks* with a *block version number* greater than 4, has been removed. This is because such *blocks* (mined nonconformantly) exist in the current *Mainnet* consensus *block chain*.
- Clarify that *Equihash* is based on a *variation* of the Generalized Birthday Problem, and cite [AR2017].
- Update reference [BGG2017] (previously [BGG2016]).
- Clarify which transaction fields are added by **Overwinter** and **Sapling**.
- Correct the rule about when a *transaction* is permitted to have no *transparent* inputs.
- Explain the differences between the system in [Groth2016] and what we refer to as Groth16.
- Reference Mary Maller’s security proof for Groth16 [Maller2018].
- Correct [BGM2018] to [BGM2017].
- Fix a typo in § B.2 ‘*Groth16 batch verification*’ on p. 216 and clarify the costs of Groth16 batch verification.
- Add macros and Makefile support for building the **Blossom** specification.

2019.0-beta-36 2019-02-09

- Correct isis agora lovecruft's name.

2019.0-beta-35 2019-02-08

- Cite [Gabizon2019] and acknowledge Ariel Gabizon.
- Correct [SBB2019] to [SWB2019].
- The [Gabizon2019] vulnerability affected Soundness of BCTV14 as well as Knowledge Soundness.
- Clarify the history of the [Parno2015] vulnerability and acknowledge Bryan Parno.
- Specify the difficulty adjustment change that occurred on *Testnet* at *block height* 299188.
- Add Eirik Ogilvie-Wigley and Benjamin Winston to acknowledgements.
- Rename *zk-SNARK Parameters* sections to be named according to the proving system (BCTV14 or Groth16), not the shielded protocol construction (**Sprout** or **Sapling**).
- In § 6 '*Network Upgrades*' on p. 118, say when **Sapling** activated.

2019.0-beta-34 2019-02-05

- Disclose a security vulnerability in BCTV14 that affected **Sprout** before activation of the **Sapling network upgrade** (see § 5.4.10.1 'BCTV14' on p. 108).
- Rename PHGR13 to BCTV2014.
- Rename reference [BCTV2015] to [BCTV2014a], and [BCTV2014] to [BCTV2014b].

2018.0-beta-33 2018-11-14

- Complete § A.4 '*The **Sapling** Spend circuit*' on p. 212.
- Add § A.5 '*The **Sapling** Output circuit*' on p. 214.
- Change the description of window lookup in § A.3.3.7 '*Fixed-base Affine-ctEdwards scalar multiplication*' on p. 203 to match sapling-crypto.
- Describe 2-bit window lookup with conditional negation in § A.3.3.9 '*Pedersen hash*' on p. 205.
- Fix or complete various calculations of constraint costs.
- Adjust the notation used for scalar multiplication in Appendix A to allow *bit sequences* as scalars.

2018.0-beta-32 2018-10-24

- Correct the input to H° used to derive the nonce r in RedDSA.Sign , from $T \parallel M$ to $T \parallel \underline{vk} \parallel M$. This matches the sapling-crypto implementation; the specification of this input was unintentionally changed in v2018.0-beta-20.
- Clarify the description of the Merkle path check in § A.3.4 '*Merkle path check*' on p. 208.

2018.0-beta-28 2018-08-14

- Finish § A.3.7 *'BLAKE2s hashes'* on p. 209.
- Minor corrections to § A.3.3.8 *'Variable-base Affine-ctEdwards scalar multiplication'* on p. 204.

2018.0-beta-27 2018-08-12

- Notational changes:
 - Use a superscript $^{(r)}$ to mark the subgroup order, instead of a subscript.
 - Use $\mathbb{G}^{(r)*}$ for the set of $r_{\mathbb{G}}$ -order points in \mathbb{G} .
 - Mark the subgroup order in pairing groups, e.g. use $\mathbb{G}_1^{(r)}$ instead of \mathbb{G}_1 .
 - Make the bit-representation indicator \star an affix instead of a superscript.
- Clarify that when validating a Groth16 proof, it is necessary to perform a subgroup check for π_A and π_C as well as for π_B .
- Correct the description of Groth16 batch verification to explicitly take account of how verification depends on *primary inputs*.
- Add Charles Rackoff, Rafail Ostrovsky, and Amit Sahai to the acknowledgements section for their work on *zero-knowledge proofs*.

2018.0-beta-26 2018-08-05

- Add § B.2 *'Groth16 batch verification'* on p. 216.

2018.0-beta-25 2018-08-05

- Add the hashes of parameter files for **Sapling**.
- Add cross references for parameters and functions used in RedDSA batch validation.
- Makefile changes: name the PDF file for the **Sprout** version of the specification as `sprout.pdf`, and make `protocol.pdf` link to the **Sapling** version.

2018.0-beta-24 2018-07-31

- Add a missing consensus rule for version 4 *transactions*: if there are no **Sapling** Spends or Outputs, then `valueBalanceSapling` **MUST** be 0.

2018.0-beta-23 2018-07-27

- Update RedDSA validation to use cofactor multiplication. This is necessary in order for the output of batch validation to match that of unbatched validation in all cases.
- Add § B.1 *'RedDSA batch validation'* on p. 215.

2018.0-beta-22 2018-07-18

- Update § 6 ‘*Network Upgrades*’ on p. 118 to take account that **Overwinter** has activated.
- The recommendation for *transactions* without *JoinSplit* descriptions to be version 1 applies only before **Overwinter**, not before **Sapling**.
- Complete the proof of Theorem A.3.5 on p. 206.
- Add a note about redundancy in the nonsmall-order checking of rk .
- Clarify the use of cv^{new} and cm^{new} , and the selection of *outgoing viewing key*, in sending **Sapling** notes.
- Delete the description of optimizations for the affine twisted Edwards nonsmall-order check, since the **Sapling** circuit does not use them. Also clarify that some other optimizations are not used.

2018.0-beta-21 2018-06-22

- Remove the consensus rule “If $nJoinSplit > 0$, the *transaction* **MUST NOT** use *SIGHASH* types other than *SIGHASH_ALL*,” which was never implemented.
- Add section on signature hashing.
- Briefly describe the changes to computation of *SIGHASH* transaction hashes in **Sprout**.
- Clarify that interstitial *treestates* form a tree for each *transaction* containing *JoinSplit* descriptions.
- Correct the description of *P2PKH* addresses in § 5.6.1.1 ‘*Transparent Addresses*’ on p. 111 — they use a hash of a compressed, not an uncompressed ECDSA key representation.
- Clarify the wording of the caveat⁴ about the claimed security of shielded *transactions*.
- Correct the definition of set difference $(S \setminus T)$.
- Add a note concerning malleability of *zk-SNARK* proofs.
- Clarify attribution of the **Zcash** protocol design.
- Acknowledge Alex Biryukov and Dmitry Khovratovich as the designers of *Equihash*.
- Acknowledge Shafi Goldwasser, Silvio Micali, Oded Goldreich, Rosario Gennaro, Bryan Parno, Jon Howell, Craig Gentry, Mariana Raykova, and Jens Groth for their work on *zero-knowledge proving systems*.
- Acknowledge Tomas Sander and Amnon Ta-Shma for [ST1999].
- Acknowledge Kudelski Security’s audit.
- Use the more precise subgroup types $\mathbb{G}^{(r)}$ and $\mathbb{J}^{(r)}$ in preference to \mathbb{G} and \mathbb{J} where applicable.
- Change the types of *auxiliary inputs* to the *Spend statement* and *Output statement*, to be more faithful to the implementation.
- Rename the cm field of an *Output description* to cm_u , reflecting the fact that it is a Jubjub curve u -coordinate.
- Add explicit consensus rules that the *anchorSapling* field of a *Spend description* and the cm_u field of an *Output description* must be canonical encodings.
- Enforce that esk in *outCiphertext* is a canonical encoding.
- Add consensus rules that cv in a *Spend description*, and cv and epk in an *Output description*, are not of *small order*. Exclude 0 from the range of esk when encrypting **Sapling** notes.
- Add a consensus rule that $valueBalanceSapling$ is in the range $\{-MAX_MONEY .. MAX_MONEY\}$.
- Enforce stronger constraints on the types of key components pk_d , ak , and nk .
- Correct the conformance rule for $fOverwintered$ (it must not be set before **Overwinter** has activated, not before **Sapling** has activated).
- Correct the argument that v^* is in range in § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51.

- Correct an error in the algorithm for `RedDSA.Validate`: the *validating key* `vk` is given directly to this algorithm and should not be computed from the unknown *signing key* `sk`.
- Correct or improve the types of `GroupHashJ(r)*`, `FindGroupHashJ(r)*`, `ExtractJ(r)`, `PRFexpand`, `PRFockSapling`, and `CRHivk`.
- Instantiate `PRFockSapling` using BLAKE2b-256.
- Change the syntax of a *commitment scheme* to add `COMM.GenTrapdoor`. This is necessary because the intended distribution of *commitment trapdoors* may not be uniform on all values that are acceptable *trapdoor* inputs.
- Add notes on the purpose of *outgoing viewing keys*.
- Correct the encoding of a *full viewing key* (`ovk` was missing).
- Ensure that **Sprout** functions and values are given **Sprout**-specific types where appropriate.
- Improve cross-referencing.
- Clarify the use of BCTV14 vs Groth16 proofs in *JoinSplit statements*.
- Clarify that the \sqrt{a} notation refers to the positive square root. (This matters for the conversion in § A.3.3.3 ‘*ctEdwards* \leftrightarrow *Montgomery conversion*’ on p.200.)
- Model the group hash as a *random oracle*. This appears to be unavoidable in order to allow proving unlinkability of `DiversifyHashSapling`. Explain how this relates to the Discrete Logarithm Independence assumption used previously, and justify this modelling by showing that it follows from treating BLAKE2s-256 as a *random oracle* in the instantiation of `GroupHashJ(r)*`.
- Rename CRS (Common Random String) to URS (*Uniform Random String*), to match the terminology adopted at the first ZKProof workshop held in Boston, Massachusetts on May 10–11, 2018.
- Generalize `PRFexpand` to accept an arbitrary-length input. (This specification does not use that generalization, but [ZIP-32] does.)
- Change the notation for a multiplication constraint in Appendix § A ‘*Circuit Design*’ on p. 195 to avoid potential confusion with cartesian product.
- Clarify the wording of the abstract.
- Correct statements about which algorithms are instantiated by BLAKE2s and BLAKE2b.
- Add a note explaining which conformance requirements of BIP 173 (defining *Bech32*) apply.
- Add the Jubjub bird image to the title page. This image has been edited from a scan of Peter Newell’s original illustration (as it appeared in [Carroll1902]) to remove the background and Bandersnatch, and to restore the bird’s clipped right wing.
- Change the light yellow background to white (indicating that this **Overwinter** and **Sapling** specification is no longer a draft).

2018.0-beta-20 2018-05-22

- Add Michael Dixon and Andrew Poelstra to acknowledgements.
- Minor improvements to cross-references.
- Correct the order of arguments to `RedDSA.RandomizePrivate` and `RedDSA.RandomizePublic`.
- Correct a reference to `RedDSA.RandomizePrivate` that was intended to be `RedDSA.RandomizePublic`.
- Fix the description of the *Sapling balancing value* in § 4.13 ‘*Balance and Binding Signature (Sapling)*’ on p. 51.
- Correct a type error in § 5.4.9.5 ‘*Group Hash into Jubjub*’ on p. 102.
- Correct a type error in `RedDSA.Sign` in § 5.4.7 ‘*RedDSA, RedJubjub, and RedPallas*’ on p. 90.

- Ensure $\mathcal{G}^{\text{Sapling}}$ is defined in § 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93.
- Make the *validating key* prefix part of the input to the *hash function* in RedDSA, not part of the message.
- Correct the statement about $\text{FindGroupHash}^{\mathbb{J}^{(r)*}}$ never returning \perp .
- Correct an error in the computation of generators for *Pedersen hashes*.
- Change the order in which $\text{NoteCommit}^{\text{Sapling}}$ commits to its inputs, to match the sapling-crypto implementation.
- Fail **Sapling** key generation if $\text{ivk} = 0$. (This has negligible probability.)
- Change the notation H^* to H° in § 5.4.7 ‘RedDSA, RedJubjub, *and* RedPallas’ on p. 90, to avoid confusion with the $*$ convention for representations of group elements.
- cmu encodes only the u -coordinate of the *note commitment*, not the full curve point.
- rk is checked to be not of *small order* outside the *Spend statement*, not in the *Spend statement*.
- Change terminology describing constraint systems.

2018.0-beta-19 2018-04-23

- Minor clarifications.

2018.0-beta-18 2018-04-23

- Clarify the security argument for balance in **Sapling**.
- Correct a subtle problem with the type of the value input to $\text{ValueCommit}^{\text{Sapling}}$, although it is only directly used to commit to values in $\{0 \dots 2^{\ell_{\text{value}}}-1\}$, the security argument depends on a sum of commitments being *binding* on $\{-\frac{r_2-1}{2} \dots \frac{r_2-1}{2}\}$.
- Fix the loss of tightness in the use of $\text{PRF}^{\text{nfSapling}}$ by specifying the keyspace more precisely.
- Correct type ambiguities for ρ .
- Specify the representation of i in group \mathbb{G}_2 of BLS12-381.

2018.0-beta-17 2018-04-21

- Correct an error in the definition of **DefaultDiversifier**.

2018.0-beta-16 2018-04-21

- Explicitly note that outputs from *coinbase transactions* include *Founders’ Reward* outputs.
- The point represented by \underline{R} in an Ed25519 signature is checked to not be of *small order*; this is not the same as checking that it is of *prime order* ℓ .
- Specify support for [BIP-111] (the `NODE_BLOOM` service bit) in *peer-to-peer protocol* version 170004.
- Give references [Vercauter2009] and [AKLGL2010] for the optimal ate pairing.
- Give references for BLS [BLS2002] and BN [BN2005] curves.
- Define $\text{KA}^{\text{Sprout}}.\text{DerivePublic}$ for Curve25519.
- Caveat the claim about *note traceability set* in § 1.2 ‘*High-level Overview*’ on p. 8 and link to [Peterson2017] and [Quesnelle2017].
- Do not require a generator as part of the specification of a *represented group*; instead, define it in the *represented pairing* or scheme using the group.

- Refactor the abstract definition of a *signature scheme* to allow derivation of *validating keys* independent of key pair generation.
- Correct the explanation in §1.2 ‘*High-level Overview*’ on p. 8 to apply to **Sapling**.
- Add the definition of a *signing key* to *validating key* homomorphism for *signature schemes*.
- Remove the output index as an input to $\text{KDF}^{\text{Sapling}}$.
- Allow dummy **Sapling** input *notes*.
- Specify RedDSA and RedJubjub.
- Specify *Sapling binding signatures* and *spend authorization signatures*.
- Specify the randomness beacon.
- Add *outgoing ciphertexts* and *ock*.
- Define DefaultDiversifier.
- Change the *Spend circuit* and *Output circuit* specifications to remove unintended differences from sapling-crypto.
- Use h_J to refer to the Jubjub curve cofactor, rather than 8.
- Correct an error in the y -coordinate formula for addition in §A.3.3.4 ‘*Affine-Montgomery arithmetic*’ on p. 201 (the constraints were correct).
- Add acknowledgements for Brian Warner, Mary Maller, and the Least Authority audit.
- Makefile improvements.

2018.0-beta-15 2018-03-19

- Clarify the bit ordering of SHA-256.
- Drop `_t` from the names of representation types.
- Remove functions from the **Sprout** specification that it does not use.
- Updates to transaction format and consensus rules for Overwinter and Sapling.
- Add specification of the *Output statement*.
- Change $\text{MerkleDepth}^{\text{Sapling}}$ from 29 to 32.
- Updates to **Sapling** construction, changing how the *nullifier* is computed and separating it from the *randomized Spend validating key* (rk).
- Clarify conversions between *bit sequences* and *byte sequences* for sk , $\text{repr}_J(ak)$, and $\text{repr}_J(nk)$.
- Change the Makefile to avoid multiple reloads in PDF readers while rebuilding the PDF.
- Spacing and pagination improvements.

2018.0-beta-14 2018-03-11

- Only cosmetic changes to **Sprout**.
- Simplify $\text{FindGroupHash}^{J^{(r)*}}$ to use a single-byte index.
- Changes to diversification for *Pedersen hashes* and *Pedersen commitments*.
- Improve security definitions for signatures.

2018.0-beta-13 2018-03-11

- Only cosmetic changes to **Sprout**.
- Change how (ask, nsk) are derived from the *spending key* sk to ensure they are on the full range of \mathbb{F}_{r_j} .
- Change PRF^{nr} to produce output computationally indistinguishable from uniform on \mathbb{F}_{r_j} .
- Change $Uncommitted^{Sapling}_t$ to be a u -coordinate for which there is no point on the curve.
- Appendix A updates:
 - categorize components into larger sections
 - fill in the [de]compression and validation algorithm
 - more precisely state the assumptions for inputs and outputs
 - delete not-all-one component which is no longer needed
 - factor out xor into its own component
 - specify [un]packing more precisely; separate it from boolean constraints
 - optimize checking for non-*small order*
 - notation in variable-base multiplication algorithm.

2018.0-beta-12 2018-03-06

- Add references to **Overwinter** ZIPs and update the section on **Overwinter/Sapling** transitions.
- Add a section on re-randomizable signatures.
- Add definition of PRF^{nr} .
- Work-in-progress on **Sapling statements**.
- Rename “raw” to “homomorphic” *Pedersen commitments*.
- Add packing modulo the field size and range checks to Appendix A.
- Update the algorithm for variable-base scalar multiplication to what is implemented by **sapling-crypto**.

2018.0-beta-11 2018-02-26

- Add sections on *Spend descriptions* and *Output descriptions*.
- Swap order of cv and rt in a *Spend description* for consistency.
- Fix off-by-one error in the range of ivk .

2018.0-beta-10 2018-02-26

- Split the descriptions of SHA-256 and SHA256Compress and of **BLAKE2**, into their own sections. Specify SHA256Compress more precisely.
- Add Kexin Hu to acknowledgements (for the idea of explicitly encoding the root of the **Sapling note commitment tree** in *block headers*).
- Move bit/byte/integer conversion primitives into §5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.
- Refer to **Overwinter** and **Sapling** just as “upgrades” in the abstract, not as the next “minor version” and “major version”.
- PRF^{nr} must be *collision-resistant*.
- Correct an error in the *Pedersen hash* specification.
- Use a named variable, c , for chunks per segment in the *Pedersen hash* specification, and change its value from 61 to 63. Add a proof justifying this value of c .

- Specify *Pedersen commitments*.
- Notation changes.
- Generalize the *distinct- x criterion* (Theorem A.3.4 on p. 201) to allow negative indices.

2018.0-beta-9 2018-02-10

- Specify the coinbase maturity rule, and the rule that *coinbase transactions* cannot contain *JoinSplit descriptions*, *Spend descriptions*, or *Output descriptions*.
- Delay lifting the 100000-byte *transaction* size limit from **Overwinter** to **Sapling**.
- Improve presentation of the proof of injectivity for $\text{Extract}_{\mathbb{J}^{(r)}}$.
- Specify $\text{GroupHash}^{\mathbb{J}^{(r)*}}$.
- Specify *Pedersen hashes*.

2018.0-beta-8 2018-02-08

- Add instantiation of CRH^{ivk} .
- Add instantiation of a hash extractor (later renamed to *coordinate extractor*) for Jubjub.
- Make the background lighter and the **Sapling** green darker, for contrast.

2018.0-beta-7 2018-02-07

- Specify the 100000-byte limit on *transaction* size. (The implementation in *zcashd* was as intended.)
- Specify that 0xF6 followed by 511 zero bytes encodes an empty *memo field*.
- Reference security definitions for *Pseudo Random Functions* and *Pseudo Random Generators*.
- Rename *clamp* to *bound* and *ActualTimespanClamped* to *ActualTimespanBounded* in the difficulty adjustment algorithm, to avoid a name collision with Curve25519 scalar “clamping”.
- Change uses of the term *full node* to *full validator*. A *full node* by definition participates in the *peer-to-peer network*, whereas a *full validator* just needs a copy of the *block chain* from somewhere. The latter is what was meant.
- Add an explanation of how **Sapling** prevents Faerie Gold and roadblock attacks.
- **Sapling** work in progress.

2018.0-beta-6 2018-01-31

- **Sapling** work in progress, mainly on Appendix § A ‘*Circuit Design*’ on p. 195.

2018.0-beta-5 2018-01-30

- Specify more precisely the requirements on Ed25519 *validating keys* and signatures.
- **Sapling** work in progress.

2018.0-beta-4 2018-01-25

- Update key components diagram for **Sapling**.

2018.0-beta-3 2018-01-22

- Explain how the chosen fix to Faerie Gold avoids a potential “roadblock” attack.
- Update some explanations of changes from **Zerocash** for **Sapling**.
- Add a description of the Jubjub curve.
- Add an acknowledgement to George Tankersley.
- Add an appendix on the design of the **Sapling** circuits at the *quadratic constraint program* level.

2017.0-beta-2.9 2017-12-17

- Refer to sk_{enc} as a *receiving key* rather than as a viewing key.
- Updates for *incoming viewing key* support.
- Refer to Network Upgrade 0 as **Overwinter**.

2017.0-beta-2.8 2017-12-02

- Correct the non-normative note describing how to check the order of π_B .
- Initial version of draft **Sapling** protocol specification.

2017.0-beta-2.7 2017-07-10

- Fix an off-by-one error in the specification of the *Equihash* algorithm binding condition. (The implementation in *zcashd* was as intended.)
- Correct the types and consensus rules for *transaction version numbers* and *block version numbers*. (Again, the implementation in *zcashd* was as intended.)
- Clarify the computation of h_i in a *JoinSplit* statement.

2017.0-beta-2.6 2017-05-09

- Be more precise when talking about curve points and pairing groups.

2017.0-beta-2.5 2017-03-07

- Clarify the consensus rule preventing *double-spends*.
- Clarify what a *note commitment* opens to in § 8.8 ‘*Omission in **Zerocash** security proof*’ on p. 145.
- Correct the order of arguments to COMM in § 5.4.8.1 ‘***Sprout** Note Commitments*’ on p. 93.
- Correct a statement about indistinguishability of *JoinSplit* descriptions.
- Change the *Founders’ Reward* addresses, for *Testnet* only, to reflect the hard-fork upgrade described in [Zcash-Issue2113].

2017.0-beta-2.4 2017-02-25

- Explain a variation on the Faerie Gold attack and why it is prevented.
- Generalize the description of the InternalH attack to include finding collisions on (a_{pk}, ρ) rather than just on ρ .
- Rename $enforce_i$ to $enforceMerklePath_i$.

2017.0-beta-2.3 2017-02-12

- Specify the security requirements on the SHA256Compress function, in order for the scheme in §5.4.8.1 ‘**Sprout Note Commitments**’ on p. 93 to be a secure commitment.
- Specify \mathbb{G}_2 more precisely.
- Explain the use of interstitial *treestates* in chained *JoinSplit transfers*.

2017.0-beta-2.2 2017-02-11

- Give definitions of computational *binding* and computational *hiding* for *commitment schemes*.
- Give a definition of statistical zero knowledge.
- Reference the white paper on MPC parameter generation [BGG2017].

2017.0-beta-2.1 2017-02-06

- ℓ_{Merkle} is a bit length, not a byte length.
- Specify the maximum *block* size.

2017.0-beta-2 2017-02-04

- Add abstract and keywords.
- Fix a typo in the definition of *nullifier* integrity.
- Make the description of *block chains* more consistent with upstream **Bitcoin** documentation (referring to “best” chains rather than using the concept of a *block chain view*).
- Define how nodes select a *best valid block chain*.

2016.0-beta-1.13 2017-01-20

- Specify the difficulty adjustment algorithm.
- Clarify some definitions of fields in a *block header*.
- Define PRF^{addr} in §4.2.1 ‘**Sprout Key Components**’ on p. 35.

2016.0-beta-1.12 2017-01-09

- Update the hashes of proving and verifying keys for the final Sprout parameters.
- Add cross references from *shielded payment address* and *spending key* encoding sections to where the key components are specified.
- Add acknowledgements for Filippo Valsorda and Zaki Manian.

2016.0-beta-1.11 2016-12-19

- Specify a check on the order of π_B in a *zk-SNARK proof*.
- Note that due to an oversight, the **Zcash** *genesis block* does not follow [BIP-34].

2016.0-beta-1.10 2016-10-30

- Update reference to the *Equihash* paper [BK2016]. (The newer version has no algorithmic changes, but the section discussing potential ASIC implementations is substantially expanded.)
- Clarify the discussion of proof size in “Differences from the **Zerocash** paper”.

2016.0-beta-1.9 2016-10-28

- Add *Founders’ Reward* addresses for *Mainnet*.
- Change “*protected*” terminology to “*shielded*”.

2016.0-beta-1.8 2016-10-04

- Revise the lead bytes for *transparent P2SH* and *P2PKH* addresses, and reencode the *Testnet Founders’ Reward* addresses.
- Add a section on which BIPs apply to **Zcash**.
- Specify that OP_CODESEPARATOR has been disabled, and no longer affects *SIGHASH transaction hashes*.
- Change the representation type of *vpub_old* and *vpub_new* to uint64. (This is not a consensus change because the type of v_{pub}^{old} and v_{pub}^{new} was already specified to be $\{0..MAX_MONEY\}$; it just better reflects the implementation.)
- Correct the representation type of the *block nVersion* field to uint32.

2016.0-beta-1.7 2016-10-02

- Clarify the consensus rule for payment of the *Founders’ Reward*, in response to an issue raised by the NCC audit.

2016.0-beta-1.6 2016-09-26

- Fix an error in the definition of the sortedness condition for *Equihash*: it is the sequences of indices that are sorted, not the sequences of hashes.
- Correct the number of bytes in the encoding of *solutionSize*.
- Update the section on encoding of *transparent addresses*. (The precise prefixes are not decided yet.)
- Clarify why BLAKE2b- ℓ is different from truncated BLAKE2b-512.
- Clarify a note about SU-CMA security for signatures.
- Add a note about $PRF^{nfSprout}$ corresponding to PRF^{sn} in **Zerocash**.
- Add a paragraph about key length in §8.7 ‘*In-band secret distribution*’ on p.143.
- Add acknowledgements for John Tromp, Paige Peterson, Maureen Walsh, Jay Graber, and Jack Gavigan.

2016.0-beta-1.5 2016-09-22

- Update the *Founders’ Reward* address list.
- Add some clarifications based on Eli Ben-Sasson’s review.

2016.0-beta-1.4 2016-09-19

- Specify the *block subsidy*, *miner subsidy*, and the *Founders' Reward*.
- Specify *coinbase transaction* outputs to *Founders' Reward* addresses.
- Improve notation (for example “.” for multiplication and “ $T^{[\ell]}$ ” for sequence types) to avoid ambiguity.

2016.0-beta-1.3 2016-09-16

- Correct the omission of `solutionSize` from the *block header* format.
- Document that `compactSize` encodings must be canonical.
- Add a note about conformance language in the introduction.
- Add acknowledgements for Solar Designer, Ling Ren and Alison Stevenson, and for the NCC Group and Coinspect security audits.

2016.0-beta-1.2 2016-09-11

- Remove GeneralCRH in favour of specifying `hSigCRH` and `EquihashGen` directly in terms of `BLAKE2b- ℓ` .
- Correct the security requirement for `EquihashGen`.

2016.0-beta-1.1 2016-09-05

- Add a specification of abstract signatures.
- Clarify what is signed in the “Sending Notes” section.
- Specify ZK parameter generation as a randomized algorithm, rather than as a distribution of parameters.

2016.0-beta-1 2016-09-04

- Major reorganization to separate the abstract cryptographic protocol from the algorithm instantiations.
- Add type declarations.
- Add a “High-level Overview” section.
- Add a section specifying the *zero-knowledge proving system* and the encoding of proofs. Change the encoding of points in proofs to follow IEEE Std 1363[a].
- Add a section on consensus changes from **Bitcoin**, and the specification of *Equihash*.
- Complete the “Differences from the **Zerocash** paper” section.
- Correct the Merkle tree depth to 29.
- Change the length of *memo fields* to 512 bytes.
- Switch the *JoinSplit signature* scheme to Ed25519, with consequent changes to the computation of `hSig`.
- Fix the lead bytes in *shielded payment address* and *spending key* encodings to match the implemented protocol.
- Add a consensus rule about the ranges of $v_{\text{pub}}^{\text{old}}$ and $v_{\text{pub}}^{\text{new}}$.
- Clarify cryptographic security requirements and added definitions relating to the *in-band* secret distribution.
- Add various citations: the “Fixing Vulnerabilities in the Zcash Protocol” and “Why Equihash?” blog posts, several crypto papers for security definitions, the **Bitcoin** whitepaper, the **CryptoNote** whitepaper, and several references to **Bitcoin** documentation.
- Reference the extended version of the **Zerocash** paper rather than the Oakland proceedings version.

- Add *JoinSplit transfers* to the Concepts section.
- Add a section on Coinbase Transactions.
- Add acknowledgements for Jack Grigg, Simon Liu, Ariel Gabizon, jl777, Ben Blaxill, Alex Balducci, and Jake Tarren.
- Fix a *Makefile* compatibility problem with the escaping behaviour of *echo*.
- Switch to *biber* for the bibliography generation, and add backreferences.
- Make the date format in references more consistent.
- Add visited dates to all URLs in references.
- Terminology changes.

2016.0-alpha-3.1 2016-05-20

- Change main font to Quattrocento.

2016.0-alpha-3 2016-05-09

- Change version numbering convention (no other changes).

2.0-alpha-3 2016-05-06

- Allow anchoring to any previous output *treestate* in the same *transaction*, rather than just the immediately preceding output *treestate*.
- Add change history.

2.0-alpha-2 2016-04-21

- Change from truncated BLAKE2b-512 to BLAKE2b-256.
- Clarify endianness, and that uses of BLAKE2b are unkeyed.
- Minor correction to what *SIGHASH types* cover.
- Add “as intended for the **Zcash** release of summer 2016” to title page.
- Require PRF^{addr} to be *collision-resistant* (see § 8.8 ‘*Omission in **Zerocash** security proof*’ on p.145).
- Add specification of path computation for the *incremental Merkle tree*.
- Add a note in § 4.18.1 ‘*JoinSplit Statement (**Sprout**)*’ on p. 58 about how this condition corresponds to conditions in the **Zerocash** paper.
- Changes to terminology around keys.

2.0-alpha-1 2016-03-30

- First version intended for public review.

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Appendices

A Circuit Design

A.1 Quadratic Constraint Programs

Sapling defines two circuits, Spend and Output, each implementing an abstract *statement* described in § 4.18.2 ‘*Spend Statement (Sapling)*’ on p. 59 and § 4.18.3 ‘*Output Statement (Sapling)*’ on p. 60 respectively. It also adds a Groth16 circuit for the *JoinSplit statement* described in § 4.18.1 ‘*JoinSplit Statement (Sprout)*’ on p. 58.

At the next lower level, each circuit is defined in terms of a *quadratic constraint program* (specifying a *Rank 1 Constraint System*), as detailed in this section. In the BCTV14 or Groth16 proving systems, this program is translated to a *Quadratic Arithmetic Program* [BCTV2014a, section 2.3] [WCBTV2015]. The circuit descriptions given here are necessary to compute witness elements for each circuit, as well as the proving and verifying keys.

Let \mathbb{F}_{r_s} be the finite field over which Jubjub is defined, as given in § 5.4.9.3 ‘Jubjub’ on p. 100.

A *quadratic constraint program* consists of a set of constraints over variables in \mathbb{F}_{r_s} , each of the form:

$$(A) \times (B) = (C)$$

where (A) , (B) , and (C) are *linear combinations* of variables and constants in \mathbb{F}_{r_s} .

Here \times and \cdot both represent multiplication in the field \mathbb{F}_{r_s} , but we use \times for multiplications corresponding to gates of the circuit, and \cdot for multiplications by constants in the terms of a *linear combination*. \times should not be confused with \times which is defined as cartesian product in § 2 ‘*Notation*’ on p. 9.

A.2 Elliptic curve background

The **Sapling** circuits make use of a *complete twisted Edwards elliptic curve* (“*ctEdwards curve*”) Jubjub, defined in § 5.4.9.3 ‘Jubjub’ on p. 100, and also a *Montgomery elliptic curve* \mathbb{M} that is birationally equivalent to Jubjub. Following the notation in [BL2017] we use (u, v) for affine coordinates on the *ctEdwards curve*, and (x, y) for affine coordinates on the *Montgomery curve*.

A point P is normally represented by two \mathbb{F}_{r_s} variables, which we name as (P^u, P^v) for an *affine-ctEdwards* point, for instance.

The implementations of scalar multiplication require the scalar to be represented as a *bit sequence*. We therefore allow the notation $[k\star] P$ meaning $[\text{LEBS2IP}_{\text{length}(k\star)}(k\star)] P$. There will be no ambiguity because variables representing *bit sequences* are named with a \star suffix.

The *Montgomery curve* \mathbb{M} has parameters $A_{\mathbb{M}} = 40962$ and $B_{\mathbb{M}} = 1$. We use an affine representation of this curve with the formula:

$$B_{\mathbb{M}} \cdot y^2 = x^3 + A_{\mathbb{M}} \cdot x^2 + x$$

Usually, elliptic curve arithmetic over prime fields is implemented using some form of projective coordinates, in order to reduce the number of expensive inversions required. In the circuit, it turns out that a division can be implemented at the same cost as a multiplication, i.e. one constraint. Therefore it is beneficial to use affine coordinates for both curves.

We define the following types representing *affine-ctEdwards* and *affine-Montgomery* coordinates respectively:

$$\begin{aligned} \text{AffineCtEdwardsJubjub} &:= (u : \mathbb{F}_{r_s}) \times (v : \mathbb{F}_{r_s}) : a_{\mathbb{J}} \cdot u^2 + v^2 = 1 + d_{\mathbb{J}} \cdot u^2 \cdot v^2 \\ \text{AffineMontJubjub} &:= (x : \mathbb{F}_{r_s}) \times (y : \mathbb{F}_{r_s}) : B_{\mathbb{M}} \cdot y^2 = x^3 + A_{\mathbb{M}} \cdot x^2 + x \end{aligned}$$

We also define a type representing compressed, *not necessarily valid*, ctEdwards coordinates:

$$\text{CompressedCtEdwardsJubjub} := (\tilde{u} : \mathbb{B}) \times (v : \mathbb{F}_{r_s})$$

See §5.4.9.3 ‘Jubjub’ on p. 100 for how this type is represented as a *byte sequence* in external encodings.

We use *affine-Montgomery* arithmetic in parts of the circuit because it is more efficient, in terms of the number of constraints, than *affine-ctEdwards* arithmetic.

An important consideration when using Montgomery arithmetic is that the addition formula is not complete, that is, there are cases where it produces the wrong answer. We must ensure that these cases do not arise.

We will need the theorem below about y -coordinates of points on *Montgomery curves*.

Fact: $A_{\mathbb{M}}^2 - 4$ is a nonsquare in \mathbb{F}_{r_s} .

Theorem A.2.1. $(0, 0)$ is the only point with $y = 0$ on certain Montgomery curves.

Let $P = (x, y)$ be a point other than $(0, 0)$ on a Montgomery curve $E_{\text{Mont}(A, B)}$ over \mathbb{F}_r , such that $A^2 - 4$ is a nonsquare in \mathbb{F}_r . Then $y \neq 0$.

Proof. Substituting $y = 0$ into the *Montgomery curve* equation gives $0 = x^3 + A \cdot x^2 + x = x \cdot (x^2 + A \cdot x + 1)$. So either $x = 0$ or $x^2 + A \cdot x + 1 = 0$. Since $P \neq (0, 0)$, the case $x = 0$ is excluded. In the other case, complete the square for $x^2 + A \cdot x + 1 = 0$ to give the equivalent $(2 \cdot x + A)^2 = A^2 - 4$. The left-hand side is a square, so if the right-hand side is a nonsquare, then there are no solutions for x . \square

A.3 Circuit Components

Each of the following sections describes how to implement a particular component of the circuit, and counts the number of constraints required. Some components make use of others; the order of presentation is “bottom-up”.

It is important for security to ensure that variables intended to be of boolean type are boolean-constrained; and for efficiency that they are boolean-constrained only once. We explicitly state for the boolean inputs and outputs of each component whether they are boolean-constrained by the component, or are assumed to have been boolean-constrained separately.

Affine coordinates for elliptic curve points are assumed to represent points on the relevant curve, unless otherwise specified.

In this section, variables have type \mathbb{F}_{r_s} unless otherwise specified. In contrast to most of this document, we use zero-based indexing in order to more closely match the implementation.

A.3.1 Operations on individual bits

A.3.1.1 Boolean constraints

A boolean constraint $b \in \mathbb{B}$ can be implemented as:

$$(1 - b) \times (b) = (0)$$

A.3.1.2 Conditional equality

The constraint “either $a = 0$ or $b = c$ ” can be implemented as:

$$(a) \times (b - c) = (0)$$

A.3.1.3 Selection constraints

A selection constraint $(b ? x : y) = z$, where $b : \mathbb{B}$ has been boolean-constrained, can be implemented as:

$$(b) \times (y - x) = (y - z)$$

A.3.1.4 Nonzero constraints

Since only nonzero elements of \mathbb{F}_{r_S} have a multiplicative inverse, the assertion $a \neq 0$ can be implemented by witnessing the inverse, $a_{\text{inv}} = a^{-1} \pmod{r_S}$:

$$(a_{\text{inv}}) \times (a) = (1)$$

This technique comes from [SVPBABW2012, Appendix D.1].

Non-normative note: A global optimization allows to use a single inverse computation outside the circuit for any number of nonzero constraints. Suppose that we have n variables (or *linear combinations*) that are supposed to be nonzero: $a_0 \dots a_{n-1}$. Multiply these together (using $n-1$ constraints) to give $a^* = \prod_{i=0}^{n-1} a_i$; then, constrain a^* to be nonzero. This works because the product a^* is nonzero if and only if all of $a_0 \dots a_{n-1}$ are nonzero. However, the **Sapling** circuit does not use this optimization.

A.3.1.5 Exclusive-or constraints

An exclusive-or operation $a \oplus b = c$, where $a, b : \mathbb{B}$ are already boolean-constrained, can be implemented in one constraint as:

$$(2 \cdot a) \times (b) = (a + b - c)$$

This automatically boolean-constrains c . Its correctness can be seen by checking the truth table of (a, b) .

A.3.2 Operations on multiple bits

A.3.2.1 [Un]packing modulo r_S

Let $n : \mathbb{N}^+$ be a constant. The operation of converting a field element, $a : \mathbb{F}_{r_S}$, to a sequence of boolean variables $b_0 \dots b_{n-1} : \mathbb{B}^{[n]}$ such that $a = \sum_{i=0}^{n-1} b_i \cdot 2^i \pmod{r_S}$, is called “*unpacking*”. The inverse operation is called “*packing*”.

In the *quadratic constraint program* these are the same operation (but see the note about canonical representation below). We assume that the variables $b_0 \dots b_{n-1}$ are boolean-constrained separately.

We have $a \bmod r_S = \left(\sum_{i=0}^{n-1} b_i \cdot 2^i \right) \bmod r_S = \left(\sum_{i=0}^{n-1} b_i \cdot (2^i \bmod r_S) \right) \bmod r_S$.

This can be implemented in one constraint:

$$\left(\sum_{i=0}^{n-1} b_i \cdot (2^i \bmod r_{\mathbb{S}}) \right) \times (1) = (a)$$

Notes:

- The bit length n is not limited by the field element size.
- Since the constraint has only a trivial multiplication, it is possible to eliminate it by merging it into the boolean constraint of one of the output bits, expressing that bit as a linear combination of the others and a . However, this optimization requires substitutions that would interfere with the modularity of the circuit implementation (for a saving of only one constraint per unpacking operation), and so we do not use it for the **Sapling** circuit.
- In the case $n = 255$, for $a < 2^{255} - r_{\mathbb{S}}$ there are two possible representations of $a : \mathbb{F}_{r_{\mathbb{S}}}$ as a sequence of 255 bits, corresponding to $\text{I2LEBSP}_{255}(a)$ and $\text{I2LEBSP}_{255}(a + r_{\mathbb{S}})$. This is a potential hazard, but it may or may not be necessary to force use of the canonical representation $\text{I2LEBSP}_{255}(a)$, depending on the context in which the [un]packing operation is used. We therefore do not consider this to be part of the [un]packing operation itself.

A.3.2.2 Range check

Let $n : \mathbb{N}^+$ be a constant, and let $a = \sum_{i=0}^{n-1} a_i \cdot 2^i : \mathbb{N}$. Suppose we want to constrain $a \leq c$ for some *constant* $c = \sum_{i=0}^{n-1} c_i \cdot 2^i : \mathbb{N}$.

Without loss of generality we can assume that $c_{n-1} = 1$, because if it were not then we would decrease n accordingly.

Note that since a and c are provided in binary representation, their bit length n is not limited by the field element size. We *do not* assume that the bits $a_0 \dots a_{n-1}$ are already boolean-constrained.

Define $\Pi_m = \prod_{i=m}^{n-1} (c_i = 0 \vee a_i = 1)$ for $m \in \{0 \dots n-1\}$. Notice that for any $m < n-1$ such that $c_m = 0$, we have $\Pi_m = \Pi_{m+1}$, and so it is only necessary to allocate separate variables for the Π_m such that $m < n-1$ and $c_m = 1$. Furthermore if $c_{n-2} \dots c_0$ has $t > 0$ trailing 1 bits, then we do not need to allocate variables for $\Pi_0 \dots \Pi_{t-1}$ because those variables will not be used below.

More explicitly:

Let $\Pi_{n-1} = a_{n-1}$.

For i from $n-2$ down to t ,

- if $c_i = 0$, then let $\Pi_i = \Pi_{i+1}$;
- if $c_i = 1$, then constrain $(\Pi_{i+1}) \times (a_i) = (\Pi_i)$.

Then we constrain the a_i as follows:

For i from $n-1$ down to 0 ,

- if $c_i = 0$, constrain $(1 - \Pi_{i+1} - a_i) \times (a_i) = (0)$;
- if $c_i = 1$, boolean-constrain a_i as in § A.3.1.1 ‘*Boolean constraints*’ on p. 196.

Note that the constraints corresponding to zero bits of c are *in place of* boolean constraints on bits of a_i .

This costs $n + k$ constraints, where k is the number of non-trailing 1 bits in $c_{n-2} \dots c_0$.

Theorem A.3.1. *Correctness of a constraint system for range checks.*

Assume $c_{0..n-1} : \mathbb{B}^{[n]}$ and $c_{n-1} = 1$. Define $A_m := \sum_{i=m}^{n-1} a_i \cdot 2^i$ and $C_m := \sum_{i=m}^{n-1} c_i \cdot 2^i$. For any $m \in \{0..n-1\}$, $A_m \leq C_m$ if and only if the restriction of the above constraint system to $i \in \{m..n-1\}$ is satisfied. Furthermore the system at least boolean-constrains $a_{0..n-1}$.

Proof. For $i \in \{0..n-1\}$ such that $c_i = 1$, the corresponding a_i are unconditionally boolean-constrained. This implies that the system constrains $\Pi_i \in \mathbb{B}$ for all $i \in \{0..n-1\}$. For $i \in \{0..n-1\}$ such that $c_i = 0$, the constraint $(1 - \Pi_{i+1} - a_i) \times (a_i) = (0)$ constrains a_i to be 0 if $\Pi_{i+1} = 1$, otherwise it constrains $a_i \in \mathbb{B}$. So all of $a_{0..n-1}$ are at least boolean-constrained.

To prove the rest of the theorem we proceed by induction on decreasing m , i.e. taking successively longer prefixes of the *big-endian* binary representations of a and c .

Base case $m = n - 1$: since $c_{n-1} = 1$, the constraint system has just one boolean constraint on a_{n-1} , which fulfils the theorem since $A_{n-1} \leq C_{n-1}$ is always satisfied.

Inductive case $m < n - 1$:

- If $A_{m+1} > C_{m+1}$, then by the inductive hypothesis the constraint system must fail, which fulfils the theorem regardless of the value of a_m .
- If $A_{m+1} \leq C_{m+1}$, then by the inductive hypothesis the constraint system restricted to $i \in \{m+1..n-1\}$ succeeds. We have $\Pi_{m+1} = \prod_{i=m+1}^{n-1} (c_i = 0 \vee a_i = 1) = \prod_{i=m+1}^{n-1} (a_i \geq c_i)$.
 - If $A_{m+1} = C_{m+1}$, then $a_i = c_i$ for all $i \in \{m+1..n-1\}$ and so $\Pi_{m+1} = 1$. Also $A_m \leq C_m$ if and only if $a_m \leq c_m$.
 When $c_m = 1$, only a boolean constraint is added for a_m which fulfils the theorem.
 When $c_m = 0$, a_m is constrained to be 0 which fulfils the theorem.
 - If $A_{m+1} < C_{m+1}$, then it cannot be the case that $a_i \geq c_i$ for all $i \in \{m+1..n-1\}$, so $\Pi_{m+1} = 0$.
 This implies that the constraint on a_m is always equivalent to a boolean constraint, which fulfils the theorem because $A_m \leq C_m$ must be true regardless of the value of a_m .

This covers all cases. □

Correctness of the full constraint system follows by taking $m = 0$ in the above theorem.

The algorithm in § A.3.3.2 ‘*ctEdwards [de]compression and validation*’ on p. 200 uses range checks with $c = r_{\mathbb{S}} - 1$ to validate *ctEdwards compressed encodings*. In that case $n = 255$ and $k = 132$, so the cost of each such range check is 387 constraints.

Non-normative note: It is possible to optimize the computation of $\Pi_{t..n-2}$ further. Notice that Π_m is only used when m is the index of the last bit of a run of 1 bits in c . So for each such run of 1 bits $c_m..m+N-2$ of length $N - 1$, it is sufficient to compute an N -ary AND of $a_m..m+N-2$ and Π_{m+N-1} : $R = \prod_{i=0}^{N-1} X_i$. This can be computed in 3 constraints for any N ; boolean-constrain the output R , and then add constraints

$$\left(N - \sum_{i=0}^{N-1} X_i\right) \times (\text{inv}) = (1 - R) \quad \text{to enforce that } \sum_{i=0}^{N-1} X_i \neq N \text{ when } R = 0;$$

$$\left(N - \sum_{i=0}^{N-1} X_i\right) \times (R) = (0) \quad \text{to enforce that } \sum_{i=0}^{N-1} X_i = N \text{ when } R = 1.$$

where inv is witnessed as $\left(N - \sum_{i=0}^{N-1} X_i\right)^{-1}$ if $R = 0$ or is unconstrained otherwise. (Since $N < r_{\mathbb{S}}$, the sums cannot overflow.)

In fact the last constraint is not needed in this context because it is sufficient to compute an upper bound on each Π_m (i.e. it does not benefit a malicious prover to witness $R = 1$ when the result of the AND should be 0). So the cost of computing Π variables for an arbitrarily long run of 1 bits can be reduced to 2 constraints. For example, for $c = r_{\mathbb{S}} - 1$ the overall cost would be reduced to $255 + 68 = 323$ constraints.

These optimizations are not used in **Sapling**.

A.3.3 Elliptic curve operations

A.3.3.1 Checking that Affine-ctEdwards coordinates are on the curve

To check that (u, v) is a point on the *ctEdwards curve*, the **Sapling** circuit uses 4 constraints:

$$\begin{aligned} (u) \times (u) &= (uu) \\ (v) \times (v) &= (vv) \\ (uu) \times (vv) &= (uuvv) \\ (a_{\mathbb{J}} \cdot uu + vv) \times (1) &= (1 + d_{\mathbb{J}} \cdot uuvv) \end{aligned}$$

Non-normative note: The last two constraints can be combined into $(d_{\mathbb{J}} \cdot uu) \times (vv) = (a_{\mathbb{J}} \cdot uu + vv - 1)$. The **Sapling** circuit does not use this optimization.

A.3.3.2 ctEdwards [de]compression and validation

Define $\text{DecompressValidate} : \text{CompressedCtEdwardsJubjub} \rightarrow \text{AffineCtEdwardsJubjub}$ as follows:

```
DecompressValidate( $\tilde{u}, v$ ) :
  // Prover supplies the  $u$ -coordinate.
  Let  $u : \mathbb{F}_{r_{\mathbb{S}}}$ .

  // §A.3.3.1 ‘Checking that Affine-ctEdwards coordinates are on the curve’ on p.200.
  Check that  $(u, v)$  is a point on the ctEdwards curve.

  // §A.3.2.1 ‘[Un]packing modulo  $r_{\mathbb{S}}$ ’ on p.197.
  Unpack  $u$  to  $\sum_{i=0}^{254} u_i \cdot 2^i$ , equating  $\tilde{u}$  with  $u_0$ .

  // §A.3.2.2 ‘Range check’ on p.198.
  Check that  $\sum_{i=0}^{254} u_i \cdot 2^i \leq r_{\mathbb{S}} - 1$ .

  Return  $(u, v)$ .
```

This costs 4 constraints for the curve equation check, 1 constraint for the unpacking, and 387 constraints for the range check (as computed in §A.3.2.2 ‘Range check’ on p.198) for a total of 392 constraints. The cost of the range check includes boolean-constraining $u_0 \dots u_{254}$.

The same *quadratic constraint program* is used for compression and decompression.

Non-normative note: The point-on-curve check could be omitted if (u, v) were already known to be on the curve. However, the **Sapling** circuit never omits it; this provides a consistency check on the elliptic curve arithmetic.

A.3.3.3 ctEdwards \leftrightarrow Montgomery conversion

Define the notation $\sqrt[+]{\cdot}$ as in §2 ‘Notation’ on p.9.

Define $\text{CtEdwardsToMont} : \text{AffineCtEdwardsJubjub} \rightarrow \text{AffineMontJubjub}$ as follows:

$$\text{CtEdwardsToMont}(u, v) = \left(\frac{1+v}{1-v}, \sqrt[+]{-40964} \cdot \frac{1+v}{(1-v) \cdot u} \right) \quad [1-v \neq 0 \text{ and } u \neq 0]$$

Define $\text{MontToCtEdwards} : \text{AffineMontJubjub} \rightarrow \text{AffineCtEdwardsJubjub}$ as follows:

$$\text{MontToCtEdwards}(x, y) = \left(\sqrt[+]{-40964} \cdot \frac{x}{y}, \frac{x-1}{x+1} \right) \quad [x+1 \neq 0 \text{ and } y \neq 0]$$

Either of these conversions can be implemented by the same *quadratic constraint program*:

$$\begin{aligned}(y) \times (u) &= \left(\sqrt[4]{-40964} \cdot x \right) \\ (x+1) \times (v) &= (x-1)\end{aligned}$$

The above conversions should only be used if the input is guaranteed to be a point on the relevant curve. If that is the case, the theorems below enumerate all exceptional inputs that may violate the side-conditions.

Theorem A.3.2. *Exceptional points (ctEdwards \rightarrow Montgomery).*

Let (u, v) be an affine point on a ctEdwards curve $E_{\text{ctEdwards}(a,d)}$. Then the only points with $u = 0$ or $1 - v = 0$ are $(0, 1) = \mathcal{O}_{\mathbb{J}}$, and $(0, -1)$ of order 2.

Proof. The curve equation is $a \cdot u^2 + v^2 = 1 + d \cdot u^2 \cdot v^2$ with $a \neq d$ (see [BBJLP2008, Definition 2.1]). By substituting $u = 0$ we obtain $v = \pm 1$, and by substituting $v = 1$ and using $a \neq d$ we obtain $u = 0$. \square

Theorem A.3.3. *Exceptional points (Montgomery \rightarrow ctEdwards).*

Let (x, y) be an affine point on a Montgomery curve $E_{\text{Mont}(A,B)}$ over \mathbb{F}_r , with parameters A and B such that $A^2 - 4$ is a nonsquare in \mathbb{F}_r , that is birationally equivalent to a ctEdwards curve. Then $x + 1 \neq 0$, and the only point (x, y) with $y = 0$ is $(0, 0)$ of order 2.

Proof. That the only point with $y = 0$ is $(0, 0)$ is proven by Theorem A.2.1 on p. 196.

If $x + 1 = 0$, then substituting $x = -1$ into the Montgomery curve equation gives $B \cdot y^2 = x^3 + A \cdot x^2 + x = A - 2$. So in that case $y^2 = (A - 2)/B$. The right-hand-side is equal to the parameter d of a particular ctEdwards curve birationally equivalent to the Montgomery curve (see [BL2017, section 4.3.5]). For all ctEdwards curves, d is nonsquare, so this equation has no solutions for y , hence $x + 1 \neq 0$. \square

(When the theorem is applied with $E_{\text{Mont}(A,B)} = \mathbb{M}$ defined in §A.2 ‘*Elliptic curve background*’ on p.195, the ctEdwards curve referred to in the proof is an isomorphic rescaling of the Jubjub curve.)

A.3.3.4 Affine-Montgomery arithmetic

The incomplete *affine-Montgomery* addition formulae given in [BL2017, section 4.3.2] are:

$$\begin{aligned}x_3 &= B_{\mathbb{M}} \cdot \lambda^2 - A_{\mathbb{M}} - x_1 - x_2 \\ y_3 &= (x_1 - x_3) \cdot \lambda - y_1 \\ \text{where } \lambda &= \begin{cases} \frac{3 \cdot x_1^2 + 2 \cdot A_{\mathbb{M}} \cdot x_1 + 1}{2 \cdot B_{\mathbb{M}} \cdot y_1}, & \text{if } x_1 = x_2 \\ \frac{y_2 - y_1}{x_2 - x_1}, & \text{otherwise.} \end{cases}\end{aligned}$$

The following theorem helps to determine when these incomplete addition formulae can be safely used:

Theorem A.3.4. *Distinct- x theorem.*

Let Q be a point of odd-prime order s on a Montgomery curve $\mathbb{M} = E_{\text{Mont}(A_{\mathbb{M}}, B_{\mathbb{M}})}$ over \mathbb{F}_{r_s} . Let $k_1 \dots k_2$ be integers in $\{-\frac{s-1}{2} \dots \frac{s-1}{2}\} \setminus \{0\}$. Let $P_i = [k_i]Q = (x_i, y_i)$ for $i \in \{1 \dots 2\}$, with $k_2 \neq \pm k_1$. Then the non-unified addition constraints

$$\begin{aligned}(x_2 - x_1) \times (\lambda) &= (y_2 - y_1) \\ (B_{\mathbb{M}} \cdot \lambda) \times (\lambda) &= (A_{\mathbb{M}} + x_1 + x_2 + x_3) \\ (x_1 - x_3) \times (\lambda) &= (y_3 + y_1)\end{aligned}$$

implement the affine-Montgomery addition $P_1 + P_2 = (x_3, y_3)$ for all such $P_1 \dots P_2$.

Proof. The given constraints are equivalent to the Montgomery addition formulae under the side condition that $x_1 \neq x_2$. (Note that neither P_i can be the zero point since $k_{1..2} \not\equiv 0 \pmod{s}$.) Assume for a contradiction that $x_1 = x_2$. For any $P_1 = [k_1] Q$, there can be only one other point $-P_1$ with the same x -coordinate. (This follows from the fact that the curve equation determines $\pm y$ as a function of x .) But $-P_1 = [-1] [k_1] Q = [-k_1] Q$. Since $k : \{-\frac{s-1}{2} .. \frac{s-1}{2}\} \mapsto [k] Q : \mathbb{M}$ is injective and $k_{1..2}$ are in $\{-\frac{s-1}{2} .. \frac{s-1}{2}\}$, then $k_2 = \pm k_1$ (contradiction). \square

The conditions of this theorem are called the *distinct- x criterion*.

In particular, if $k_{1..2}$ are integers in $\{1 .. \frac{s-1}{2}\}$ then it is sufficient to require $k_2 \neq k_1$, since that implies $k_2 \neq \pm k_1$.

Affine-Montgomery doubling can be implemented as:

$$\begin{aligned} (x) \times (x) &= (xx) \\ (2 \cdot B_{\mathbb{M}} \cdot y) \times (\lambda) &= (3 \cdot xx + 2 \cdot A_{\mathbb{M}} \cdot x + 1) \\ (B_{\mathbb{M}} \cdot \lambda) \times (\lambda) &= (A_{\mathbb{M}} + 2 \cdot x + x_3) \\ (x - x_3) \times (\lambda) &= (y_3 + y) \end{aligned}$$

This doubling formula is valid when $y \neq 0$, which is the case when (x, y) is not the point $(0, 0)$ (the only point of order 2), as proven in Theorem A.2.1 on p. 196.

A.3.3.5 Affine-ctEdwards arithmetic

Formulae for *affine-ctEdwards* addition are given in [BBJLP2008, section 6]. With a change of variable names to match our convention, the formulae for $(u_1, v_1) + (u_2, v_2) = (u_3, v_3)$ are:

$$\begin{aligned} u_3 &= \frac{u_1 \cdot v_2 + v_1 \cdot u_2}{1 + d_{\mathbb{J}} \cdot u_1 \cdot u_2 \cdot v_1 \cdot v_2} \\ v_3 &= \frac{v_1 \cdot v_2 - a_{\mathbb{J}} \cdot u_1 \cdot u_2}{1 - d_{\mathbb{J}} \cdot u_1 \cdot u_2 \cdot v_1 \cdot v_2} \end{aligned}$$

We use an optimized implementation found by Daira-Emma Hopwood making use of an observation by Bernstein and Lange in [BL2017, last paragraph of section 4.5.2]:

$$\begin{aligned} (u_1 + v_1) \times (v_2 - a_{\mathbb{J}} \cdot u_2) &= (T) \\ (u_1) \times (v_2) &= (A) \\ (v_1) \times (u_2) &= (B) \\ (d_{\mathbb{J}} \cdot A) \times (B) &= (C) \\ (1 + C) \times (u_3) &= (A + B) \\ (1 - C) \times (v_3) &= (T - A + a_{\mathbb{J}} \cdot B) \end{aligned}$$

The correctness of this implementation can be seen by expanding $T - A + a_{\mathbb{J}} \cdot B$:

$$\begin{aligned} T - A + a_{\mathbb{J}} \cdot B &= (u_1 + v_1) \cdot (v_2 - a_{\mathbb{J}} \cdot u_2) - u_1 \cdot v_2 + a_{\mathbb{J}} \cdot v_1 \cdot u_2 \\ &= v_1 \cdot v_2 - a_{\mathbb{J}} \cdot u_1 \cdot u_2 + u_1 \cdot v_2 - a_{\mathbb{J}} \cdot v_1 \cdot u_2 - u_1 \cdot v_2 + a_{\mathbb{J}} \cdot v_1 \cdot u_2 \\ &= v_1 \cdot v_2 - a_{\mathbb{J}} \cdot u_1 \cdot u_2 \end{aligned}$$

The above addition formulae are “unified”, that is, they can also be used for doubling. *Affine-ctEdwards* doubling [2] $(u, v) = (u_3, v_3)$ can also be implemented slightly more efficiently as:

$$\begin{aligned}(u + v) \times (v - a_{\mathbb{J}} \cdot u) &= (T) \\ (u) \times (v) &= (A) \\ (d_{\mathbb{J}} \cdot A) \times (A) &= (C) \\ (1 + C) \times (u_3) &= (2 \cdot A) \\ (1 - C) \times (v_3) &= (T + (a_{\mathbb{J}} - 1) \cdot A)\end{aligned}$$

This implementation is obtained by specializing the addition formulae to $(u, v) = (u_1, v_1) = (u_2, v_2)$ and observing that $u \cdot v = A = B$.

A.3.3.6 Affine-ctEdwards nonsmall-order check

In order to avoid small-subgroup attacks, we check that certain points used in the circuit are not of *small order*. In practice the **Sapling** circuit uses this in combination with a check that the coordinates are on the curve (§ A.3.3.1 ‘*Checking that Affine-ctEdwards coordinates are on the curve*’ on p. 200), so we combine the two operations.

The Jubjub curve has a large *prime-order subgroup* with a cofactor of 8. To check for a point P of order 8 or less, the **Sapling** circuit doubles three times (as in § A.3.3.5 ‘*Affine-ctEdwards arithmetic*’ on p. 202) and checks that the resulting u -coordinate is not 0 (as in § A.3.1.4 ‘*Nonzero constraints*’ on p. 197).

On a *ctEdwards* curve, only the zero point $\mathcal{O}_{\mathbb{J}}$, and the unique point of order 2 at $(0, -1)$ have zero u -coordinate. The point of order 2 cannot occur as the result of three doublings. So this u -coordinate check rejects only $\mathcal{O}_{\mathbb{J}}$.

The total cost, including the curve check, is $4 + 3 \cdot 5 + 1 = 20$ constraints.

Note: This *does not* ensure that the point is in the *prime-order subgroup*.

Non-normative notes:

- It would have been sufficient to do two doublings rather than three, because the check that the u -coordinate is nonzero would reject both $\mathcal{O}_{\mathbb{J}}$ and the point of order 2.
- It is possible to reduce the cost to 8 constraints by eliminating the redundant constraint in the curve point check (as mentioned in § A.3.3.1 ‘*Checking that Affine-ctEdwards coordinates are on the curve*’ on p. 200); merging the first doubling with the curve point check; and then optimizing the second doubling based on the fact that we only need to check whether the resulting u -coordinate is zero. The **Sapling** circuit does not use these optimizations.

A.3.3.7 Fixed-base Affine-ctEdwards scalar multiplication

If the base point B is fixed for a given scalar multiplication $[k] B$, we can fully precompute window tables for each window position.

It is most efficient to use 3-bit fixed windows. Since the length of $r_{\mathbb{J}}$ is 252 bits, we need 84 windows.

Express k in base 8, i.e. $k = \sum_{i=0}^{83} k_i \cdot 8^i$.

Then $[k] B = \sum_{i=0}^{83} w_{(B, i, k_i)}$, where $w_{(B, i, k_i)} = [k_i \cdot 8^i] B$.

We precompute all of $w_{(B, i, s)}$ for $i \in \{0 \dots 83\}$, $s \in \{0 \dots 7\}$.

To look up a given window entry $w_{(B,i,s)} = (u_s, v_s)$, where $s = 4 \cdot s_2 + 2 \cdot s_1 + s_0$, we use:

$$\begin{aligned}
(s_1) \times (s_2) &= (s_{\otimes}) \\
(s_0) \times (-u_0 \cdot s_{\otimes} + u_0 \cdot s_2 + u_0 \cdot s_1 - u_0 + u_2 \cdot s_{\otimes} - u_2 \cdot s_1 + u_4 \cdot s_{\otimes} - u_4 \cdot s_2 - u_6 \cdot s_{\otimes} \\
&\quad + u_1 \cdot s_{\otimes} - u_1 \cdot s_2 - u_1 \cdot s_1 + u_1 - u_3 \cdot s_{\otimes} + u_3 \cdot s_1 - u_5 \cdot s_{\otimes} + u_5 \cdot s_2 + u_7 \cdot s_{\otimes}) = \\
&\quad (u_s - u_0 \cdot s_{\otimes} + u_0 \cdot s_2 + u_0 \cdot s_1 - u_0 + u_2 \cdot s_{\otimes} - u_2 \cdot s_1 + u_4 \cdot s_{\otimes} - u_4 \cdot s_2 - u_6 \cdot s_{\otimes}) \\
(s_0) \times (-v_0 \cdot s_{\otimes} + v_0 \cdot s_2 + v_0 \cdot s_1 - v_0 + v_2 \cdot s_{\otimes} - v_2 \cdot s_1 + v_4 \cdot s_{\otimes} - v_4 \cdot s_2 - v_6 \cdot s_{\otimes} \\
&\quad + v_1 \cdot s_{\otimes} - v_1 \cdot s_2 - v_1 \cdot s_1 + v_1 - v_3 \cdot s_{\otimes} + v_3 \cdot s_1 - v_5 \cdot s_{\otimes} + v_5 \cdot s_2 + v_7 \cdot s_{\otimes}) = \\
&\quad (v_s - v_0 \cdot s_{\otimes} + v_0 \cdot s_2 + v_0 \cdot s_1 - v_0 + v_2 \cdot s_{\otimes} - v_2 \cdot s_1 + v_4 \cdot s_{\otimes} - v_4 \cdot s_2 - v_6 \cdot s_{\otimes})
\end{aligned}$$

For a full-length (252-bit) scalar this costs 3 constraints for each of 84 window lookups, plus 6 constraints for each of 83 *ctEdwards* additions (as in §A.3.3.5 ‘*Affine-ctEdwards arithmetic*’ on p.202), for a total of 750 constraints.

Fixed-base scalar multiplication is also used in two places with shorter scalars:

- §A.3.6 ‘*Homomorphic Pedersen Commitment*’ on p.208 uses 64 bits for the v input to `ValueCommit`^{Sapling}, requiring 22 windows at a cost of $3 \cdot 22 - 1 + 6 \cdot 21 = 191$ constraints;
- §A.3.3.10 ‘*Mixing Pedersen hash*’ on p.207 uses 32 bits for the pos input to `MixingPedersenHash`, requiring 11 windows at a cost of $3 \cdot 11 - 1 + 6 \cdot 10 = 92$ constraints.

None of these costs include the cost of boolean-constraining the scalar.

Non-normative notes:

- It would be more efficient to use arithmetic on the *Montgomery curve*, as in §A.3.3.9 ‘*Pedersen hash*’ on p.205. However since there are only three instances of fixed-base scalar multiplication in the *Spend circuit* and two in the *Output circuit*¹⁴, the additional complexity was not considered justified for **Sapling**.
- For the multiplications with 64-bit and 32-bit scalars, the scalar is padded to a multiple of 3 bits with zeros. This causes the computation of s_{\otimes} in the lookup for the most significant window to be optimized out, which is where the “ -1 ” comes from in the above cost calculations. No further optimization is done for this lookup.

A.3.3.8 Variable-base Affine-ctEdwards scalar multiplication

When the base point B is not fixed, the method in the preceding section cannot be used. Instead we use a naïve double-and-add method.

Given $k = \sum_{i=0}^{250} k_i \cdot 2^i$, we calculate $R = [k] B$ using:

```

// Basei = [2i] B
let Base0 = B
let Acc0u = k0 ? Base0u : 0
let Acc0v = k0 ? Base0v : 1

for i from 1 up to 250:
    let Basei = [2] Basei-1
    // select Basei or  $\mathcal{O}_{\mathbb{J}}$  depending on the bit  $k_i$ 
    let Addendiu = ki ? Baseiu : 0
    let Addendiv = ki ? Baseiv : 1
    let Acci = Acci-1 + Addendi

let R = Acc250.

```

This costs 5 constraints for each of 250 *ctEdwards* doublings, 6 constraints for each of 250 *ctEdwards* additions, and 2 constraints for each of 251 point selections, for a total of 3252 constraints.

¹⁴ A *Pedersen commitment* uses fixed-base scalar multiplication as a subcomponent.

Non-normative note: It would be more efficient to use 2-bit fixed windows, and/or to use arithmetic on the *Montgomery curve* in a similar way to § A.3.3.9 ‘*Pedersen hash*’ on p. 205. However since there are only two instances of variable-base scalar multiplication in the *Spend circuit* and one in the *Output circuit*, the additional complexity was not considered justified for **Sapling**.

A.3.3.9 Pedersen hash

The specification of the *Pedersen hashes* used in **Sapling** is given in § 5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77. It is based on the scheme from [CvHP1991, section 5.2] –for which a tighter security reduction to the *Discrete Logarithm Problem* was given in [BGG1995]– but tailored to allow several optimizations in the circuit implementation.

Pedersen hashes are the single most commonly used primitive in the **Sapling** circuits. $\text{MerkleDepth}^{\text{Sapling}}$ *Pedersen hash* instances are used in the *Spend circuit* to check a *Merkle path* to the *note commitment* of the *note* being spent. We also reuse the *Pedersen hash* implementation to construct the *note commitment scheme* $\text{NoteCommit}^{\text{Sapling}}$.

This motivates considerable attention to optimizing this circuit implementation of this primitive, even at the cost of complexity.

First, we use a windowed scalar multiplication algorithm with signed digits. Each 3-bit message chunk corresponds to a window; the chunk is encoded as an integer from the set $\text{Digits} = \{-4 \dots 4\} \setminus \{0\}$. This allows a more efficient lookup of the window entry for each chunk than if the set $\{1 \dots 8\}$ had been used, because a point can be conditionally negated using only a single constraint.

Next, we optimize the cost of point addition by allowing as many additions as possible to be performed on the *Montgomery curve*. An incomplete Montgomery addition costs 3 constraints, in comparison with a *ctEdwards* addition which costs 6 constraints.

However, we cannot do all additions on the *Montgomery curve* because the Montgomery addition is incomplete. In order to be able to prove that exceptional cases do not occur, we need to ensure that the *distinct- x criterion* from § A.3.3.4 ‘*Affine-Montgomery arithmetic*’ on p. 201 is met. This requires splitting the input into segments (each using an independent generator), calculating an intermediate result for each segment, and then converting to the *ctEdwards curve* and summing the intermediate results using *ctEdwards* addition.

Abstracting away the changes of curve, this calculation can be written as:

$$\text{PedersenHashToPoint}(D, M) = \sum_{j=1}^N [\langle M_j \rangle] \mathcal{I}(D, j)$$

where $\langle \cdot \rangle$ and $\mathcal{I}(D, j)$ are defined as in § 5.4.1.7 ‘*Pedersen Hash Function*’ on p. 77.

We have to prove that:

- the Montgomery-to-*ctEdwards* conversions can be implemented without exceptional cases;
- the *distinct- x criterion* is met for all Montgomery additions within a segment.

The proof of Theorem 5.4.1 on p. 78 showed that all indices of addition inputs are in the range $\{-\frac{r_{\mathbb{J}}-1}{2} \dots \frac{r_{\mathbb{J}}-1}{2}\} \setminus \{0\}$.

Because the $\mathcal{I}(D, j)$ (which are outputs of $\text{GroupHash}^{\mathbb{J}^{(*)}}$) are all of *prime order*, and $\langle M_j \rangle \neq 0 \pmod{r_{\mathbb{J}}}$, it is guaranteed that all of the terms $[\langle M_j \rangle] \mathcal{I}(D, j)$ to be converted to *ctEdwards* form are of *prime order*. From Theorem A.3.3 on p. 201, we can infer that the conversions will not encounter exceptional cases.

We also need to show that the indices of addition inputs are all distinct disregarding sign.

Theorem A.3.5. *Concerning addition inputs in the Pedersen circuit.*

For all disjoint nonempty subsets S and S' of $\{1 \dots c\}$, all $m \in \mathbb{B}^{[3][c]}$, and all $\Theta \in \{-1, 1\}$:

$$\sum_{j \in S} \text{enc}(m_j) \cdot 2^{4 \cdot (j-1)} \neq \Theta \cdot \sum_{j' \in S'} \text{enc}(m_{j'}) \cdot 2^{4 \cdot (j'-1)}.$$

Proof. Suppose for a contradiction that S, S', m, Θ is a counterexample. Taking the multiplication by Θ on the right hand side inside the summation, we have:

$$\sum_{j \in S} \text{enc}(m_j) \cdot 2^{4 \cdot (j-1)} = \sum_{j' \in S'} \Theta \cdot \text{enc}(m_{j'}) \cdot 2^{4 \cdot (j'-1)}.$$

Define $\text{enc}' : \{-1, 1\} \times \mathbb{B}^{[3]} \rightarrow \{0 \dots 8\} \setminus \{4\}$ as $\text{enc}'_{\theta}(m_i) := 4 + \theta \cdot \text{enc}(m_i)$.

Let $\Delta = 4 \cdot \sum_{i=1}^c 2^{4 \cdot (i-1)}$ as in the proof of Theorem 5.4.1 on p. 78. By adding Δ to both sides, we get

$$\sum_{j \in S} \text{enc}'_1(m_j) \cdot 2^{4 \cdot (j-1)} + \sum_{j \in \{1 \dots c\} \setminus S} 4 \cdot 2^{4 \cdot (j-1)} = \sum_{j' \in S'} \text{enc}'_{\Theta}(m_{j'}) \cdot 2^{4 \cdot (j'-1)} + \sum_{j' \in \{1 \dots c\} \setminus S'} 4 \cdot 2^{4 \cdot (j'-1)}$$

where all of the $\text{enc}'_1(m_j)$ and $\text{enc}'_{\Theta}(m_{j'})$ are in $\{0 \dots 8\} \setminus \{4\}$.

Each term on the left and on the right affects the single hex digit indexed by j and j' respectively. Since S and S' are disjoint subsets of $\{1 \dots c\}$ and S is nonempty, $S \cap (\{1 \dots c\} \setminus S')$ is nonempty. Therefore the left hand side has at least one hex digit not equal to 4 such that the corresponding right hand side digit is 4; contradiction. \square

This implies that the terms in the Montgomery addition –as well as any intermediate results formed from adding a distinct subset of terms– have distinct indices disregarding sign, hence distinct x -coordinates by Theorem A.3.4 on p. 201. (We make no assumption about the order of additions.)

We now describe the subcircuit used to process each chunk, which contributes most of the constraint cost of the hash. This subcircuit is used to perform a lookup of a Montgomery point in a 2-bit window table, conditionally negate the result, and add it to an accumulator holding another Montgomery point.

Suppose that the bits of the chunk, $[s_0, s_1, s_2]$, are already boolean-constrained.

We aim to compute $C = A + [(1 - 2 \cdot s_2) \cdot (1 + s_0 + 2 \cdot s_1)] P$ for some fixed base point P and accumulated sum A .

We first compute $s_{\&} = s_0 \& s_1$:

$$(s_0) \times (s_1) = (s_{\&})$$

Let $(x_k, y_k) = [k] P$ for $k \in \{1 \dots 4\}$. Define each coordinate of $(x_S, y_R) = [1 + s_0 + 2 \cdot s_1] P$ as a linear combination of s_0, s_1 , and $s_{\&}$:

$$\text{let } x_S = x_1 + (x_2 - x_1) \cdot s_0 + (x_3 - x_1) \cdot s_1 + (x_4 + x_1 - x_2 - x_3) \cdot s_{\&}$$

$$\text{let } y_R = y_1 + (y_2 - y_1) \cdot s_0 + (y_3 - y_1) \cdot s_1 + (y_4 + y_1 - y_2 - y_3) \cdot s_{\&}$$

We implement the conditional negation as $(2 \cdot y_R) \times (s_2) = (y_R - y_S)$. After substitution of y_R this becomes:

$$(2 \cdot (y_1 + (y_2 - y_1) \cdot s_0 + (y_3 - y_1) \cdot s_1 + (y_4 + y_1 - y_2 - y_3) \cdot s_{\&})) \times (s_2) = (y_1 + (y_2 - y_1) \cdot s_0 + (y_3 - y_1) \cdot s_1 + (y_4 + y_1 - y_2 - y_3) \cdot s_{\&} - y_S)$$

Then we substitute x_S into the Montgomery addition constraints from § A.3.3.4 ‘*Affine-Montgomery arithmetic*’ on p. 201, as follows:

$$\begin{aligned}(x_1 + (x_2 - x_1) \cdot s_0 + (x_3 - x_1) \cdot s_1 + (x_4 + x_1 - x_2 - x_3) \cdot s_{\&} - x_A) \times (\lambda) &= (y_S - y_A) \\ (B_{\mathbb{M}} \cdot \lambda) \times (\lambda) &= (A_{\mathbb{M}} + x_A + x_1 + (x_2 - x_1) \cdot s_0 + (x_3 - x_1) \cdot s_1 + (x_4 + x_1 - x_2 - x_3) \cdot s_{\&} + x_C) \\ (x_A - x_C) \times (\lambda) &= (y_C + y_A)\end{aligned}$$

(In the sapling-crypto implementation, linear combinations are first-class values, so these substitutions do not need to be done “by hand”.)

For the first addition in each segment, both sides are looked up and substituted into the Montgomery addition, so the first lookup takes only 2 constraints.

When these hashes are used in the circuit, the first 6 bits of the input are fixed. For example, in the Merkle tree hashes they represent the layer number. This would allow a precomputation for the first two windows, but that optimization is not done in **Sapling**.

The cost of a Pedersen hash over ℓ bits (where ℓ includes the fixed bits) is as follows. The number of chunks is $c = \text{ceiling}\left(\frac{\ell}{3}\right)$ and the number of segments is $n = \text{ceiling}\left(\frac{\ell}{3 \cdot 63}\right)$.

The cost is then:

- $2 \cdot c$ constraints for the lookups;
- $3 \cdot (c - n)$ constraints for incomplete additions on the *Montgomery curve*;
- $2 \cdot n$ constraints for Montgomery-to-*ctEdwards* conversions;
- $6 \cdot (n - 1)$ constraints for *ctEdwards* additions;

for a total of $5 \cdot c + 5 \cdot n - 6$ constraints. This does not include the cost of boolean-constraining inputs.

In particular,

- for the Merkle tree hashes $\ell = 516$, so $c = 172$, $n = 3$, and the cost is 869 constraints;
- when a Pedersen hash is used to implement part of a Pedersen commitment for NoteCommit^{Sapling} (§ 5.4.8.2 ‘*Windowed Pedersen commitments*’ on p. 94), $\ell = 6 + \ell_{\text{value}} + 2 \cdot \ell_{\mathbb{J}} = 582$, $c = 194$, and $n = 4$, so the cost of the hash alone is 984 constraints.

A.3.3.10 Mixing Pedersen hash

A mixing *Pedersen hash* is used to compute ρ from cm and pos in § 4.16 ‘*Computing ρ values and Nullifiers*’ on p. 56. It takes as input a *Pedersen commitment* P , and hashes it with another input x .

Let $\mathcal{J}^{\text{Sapling}}$ be as defined in § 5.4.1.8 ‘*Mixing Pedersen Hash Function*’ on p. 79.

We define $\text{MixingPedersenHash} : \{0 \dots r_{\mathbb{J}} - 1\} \times \mathbb{J} \rightarrow \mathbb{J}$ by:

$$\text{MixingPedersenHash}(P, x) := P + [x] \mathcal{J}^{\text{Sapling}}.$$

This costs 92 constraints for a scalar multiplication (§ A.3.3.7 ‘*Fixed-base Affine-ctEdwards scalar multiplication*’ on p. 203), and 6 constraints for a *ctEdwards* addition (§ A.3.3.5 ‘*Affine-ctEdwards arithmetic*’ on p. 202), for a total of 98 constraints.

A.3.4 Merkle path check

Checking each layer of a Merkle authentication path, as described in § 4.9 ‘*Merkle Path Validity*’ on p. 47, requires to:

- boolean-constrain the path bit specifying whether the previous node is a left or right child;
- conditionally swap the previous-layer and sibling hashes (as \mathbb{F}_r elements) depending on the path bit;
- unpack the left and right hash inputs to two sequences of 255 bits;
- compute the Merkle hash for this node.

The unpacking need not be canonical in the sense discussed in § A.3.2.1 ‘*[Un]packing modulo r_s* ’ on p. 197; that is, it is *not* necessary to ensure that the left or right inputs to the hash represent integers in the range $\{0 \dots r_s - 1\}$. Since the root of the Merkle tree is calculated outside the circuit using the canonical representations, and since the *Pedersen hashes* are *collision-resistant* on arbitrary *bit-sequence* inputs, an attempt by an adversarial prover to use a *non-canonical* input would result in the wrong root being calculated, and the overall path check would fail.

For each layer, the cost is $1 + 2 \cdot 255$ boolean constraints, 2 constraints for the conditional swap (implemented as two selection constraints), and 869 constraints for the Merkle hash (§ A.3.3.9 ‘*Pedersen hash*’ on p. 205), for a total of 1380 constraints.

Non-normative note: The conditional swap $(a_0, a_1) \mapsto (c_0, c_1)$ could be implemented in only one constraint by substituting $c_1 = a_0 + a_1 - c_0$ into the uses of c_1 . The **Sapling** circuit does not use this optimization.

A.3.5 Windowed Pedersen Commitment

We construct *windowed Pedersen commitments* by reusing the Pedersen hash implementation described in § A.3.3.9 ‘*Pedersen hash*’ on p. 205, and adding a randomized point:

$$\text{WindowedPedersenCommit}_r(s) = \text{PedersenHashToPoint}(\text{"Zcash_PH"}, s) + [r] \text{FindGroupHash}^{\mathbb{J}^{(r)*}}(\text{"Zcash_PH"}, \text{"r"})$$

This can be implemented in:

- $5 \cdot c + 5 \cdot n - 6$ constraints for the Pedersen hash applied to $\ell = 6 + \text{length}(s)$ bits, where $c = \text{ceiling}\left(\frac{\ell}{3}\right)$ and $n = \text{ceiling}\left(\frac{\ell}{3 \cdot 63}\right)$;
- 750 constraints for the fixed-base scalar multiplication;
- 6 constraints for the final *ctEdwards* addition.

When `WindowedPedersenCommit` is used to instantiate `NoteCommitSapling`, the cost of the Pedersen hash is 984 constraints as calculated in § A.3.3.9 ‘*Pedersen hash*’ on p. 205, and so the total cost in that case is 1740 constraints. This does not include the cost of boolean-constraining the input s or the randomness r .

A.3.6 Homomorphic Pedersen Commitment

The *windowed Pedersen commitments* defined in the preceding section are highly efficient, but they do not support the homomorphic property we need when instantiating `ValueCommit`.

In order to support this property, we also define *homomorphic Pedersen commitments* as follows:

$$\text{HomomorphicPedersenCommit}_{\text{rcv}}^{\text{Sapling}}(D, v) = [v] \text{FindGroupHash}^{\mathbb{J}^{(r)*}}(D, \text{"v"}) + [\text{rcv}] \text{FindGroupHash}^{\mathbb{J}^{(r)*}}(D, \text{"r"})$$

In the case that we need for ValueCommit, v has 64 bits¹⁵. This value is given as a bit representation, which does not need to be constrained equal to an integer.

ValueCommit can be implemented in:

- 750 constraints for the 252-bit fixed-base multiplication by rcv ;
- 191 constraints for the 64-bit fixed-base multiplication by v ;
- 6 constraints for the *ctEdwards* addition

for a total cost of 947 constraints. This does not include the cost to boolean-constrain the input v or randomness rcv .

A.3.7 BLAKE2s hashes

BLAKE2s is defined in [ANWW2013]. Its main subcomponent is a “ G function”, defined as follows:

$$G : \{0..9\} \times \{0..2^{32}-1\}^{[4]} \rightarrow \{0..2^{32}-1\}^{[4]}$$

$$G(a, b, c, d, x, y) = (a'', b'', c'', d'') \text{ where}$$

$$a' = (a + b + x) \bmod 2^{32}$$

$$d' = (d \oplus a') \ggg 16$$

$$c' = (c + d') \bmod 2^{32}$$

$$b' = (b \oplus c') \ggg 12$$

$$a'' = (a' + b' + y) \bmod 2^{32}$$

$$d'' = (d' \oplus a'') \ggg 8$$

$$c'' = (c' + d'') \bmod 2^{32}$$

$$b'' = (b' \oplus c'') \ggg 7$$

The following table is used to determine which message words the x and y arguments to G are selected from:

$$\sigma_0 = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]$$

$$\sigma_1 = [14, 10, 4, 8, 9, 15, 13, 6, 1, 12, 0, 2, 11, 7, 5, 3]$$

$$\sigma_2 = [11, 8, 12, 0, 5, 2, 15, 13, 10, 14, 3, 6, 7, 1, 9, 4]$$

$$\sigma_3 = [7, 9, 3, 1, 13, 12, 11, 14, 2, 6, 5, 10, 4, 0, 15, 8]$$

$$\sigma_4 = [9, 0, 5, 7, 2, 4, 10, 15, 14, 1, 11, 12, 6, 8, 3, 13]$$

$$\sigma_5 = [2, 12, 6, 10, 0, 11, 8, 3, 4, 13, 7, 5, 15, 14, 1, 9]$$

$$\sigma_6 = [12, 5, 1, 15, 14, 13, 4, 10, 0, 7, 6, 3, 9, 2, 8, 11]$$

$$\sigma_7 = [13, 11, 7, 14, 12, 1, 3, 9, 5, 0, 15, 4, 8, 6, 2, 10]$$

$$\sigma_8 = [6, 15, 14, 9, 11, 3, 0, 8, 12, 2, 13, 7, 1, 4, 10, 5]$$

$$\sigma_9 = [10, 2, 8, 4, 7, 6, 1, 5, 15, 11, 9, 14, 3, 12, 13, 0]$$

The Initialization Vector is defined as:

$$\text{IV} : \{0..2^{32}-1\}^{[8]} := [0x6A09E667, 0xBB67AE85, 0x3C6EF372, 0xA54FF53A, \\ 0x510E527F, 0x9B05688C, 0x1F83D9AB, 0x5BE0CD19]$$

¹⁵It would be sufficient to use 51 bits, which accomodates the range $\{0..MAX_MONEY\}$, but the **Sapling** circuit uses 64.

The full hash function applied to an 8-byte personalization string and a single 64-byte block, in sequential mode with 32-byte output, can be expressed as follows.

Define BLAKE2s-256 : $(p : \mathbb{B}^{[8]}) \times (x : \mathbb{B}^{[64]}) \rightarrow \mathbb{B}^{[32]}$ as:

```

let PB :  $\mathbb{B}^{[32]}$  =  $[32, 0, 1, 1] \parallel [0x00]^{20} \parallel p$ 
let  $[t_0, t_1, f_0, f_1] : \{0..2^{32}-1\}^{[4]} = [0, 0, 0, 0xFFFFFFFF, 0]$ 
let  $h : \{0..2^{32}-1\}^{[8]} = [LEOS2IP_{32}(PB_{4 \cdot i .. 4 \cdot i + 3}) \oplus IV_i \text{ for } i \text{ from } 0 \text{ up to } 7]$ 
let  $m : \{0..2^{32}-1\}^{[16]} = [LEOS2IP_{32}(x_{4 \cdot i .. 4 \cdot i + 3}) \text{ for } i \text{ from } 0 \text{ up to } 15]$ 
let mutable  $v : \{0..2^{32}-1\}^{[16]} \leftarrow h \parallel [IV_0, IV_1, IV_2, IV_3, t_0 \oplus IV_4, t_1 \oplus IV_5, f_0 \oplus IV_6, f_1 \oplus IV_7]$ 

for  $r$  from 0 up to 9:
  set  $(v_0, v_4, v_8, v_{12}) \leftarrow G(v_0, v_4, v_8, v_{12}, m_{\sigma_{r,0}}, m_{\sigma_{r,1}})$ 
  set  $(v_1, v_5, v_9, v_{13}) \leftarrow G(v_1, v_5, v_9, v_{13}, m_{\sigma_{r,2}}, m_{\sigma_{r,3}})$ 
  set  $(v_2, v_6, v_{10}, v_{14}) \leftarrow G(v_2, v_6, v_{10}, v_{14}, m_{\sigma_{r,4}}, m_{\sigma_{r,5}})$ 
  set  $(v_3, v_7, v_{11}, v_{15}) \leftarrow G(v_3, v_7, v_{11}, v_{15}, m_{\sigma_{r,6}}, m_{\sigma_{r,7}})$ 

  set  $(v_0, v_5, v_{10}, v_{15}) \leftarrow G(v_0, v_5, v_{10}, v_{15}, m_{\sigma_{r,8}}, m_{\sigma_{r,9}})$ 
  set  $(v_1, v_6, v_{11}, v_{12}) \leftarrow G(v_1, v_6, v_{11}, v_{12}, m_{\sigma_{r,10}}, m_{\sigma_{r,11}})$ 
  set  $(v_2, v_7, v_8, v_{13}) \leftarrow G(v_2, v_7, v_8, v_{13}, m_{\sigma_{r,12}}, m_{\sigma_{r,13}})$ 
  set  $(v_3, v_4, v_9, v_{14}) \leftarrow G(v_3, v_4, v_9, v_{14}, m_{\sigma_{r,14}}, m_{\sigma_{r,15}})$ 

return LEBS2OSP256(concat $\mathbb{B}$ ( $[I2LEBSP_{32}(h_i \oplus v_i \oplus v_{i+8}) \text{ for } i \text{ from } 0 \text{ up to } 7]$ ))

```

In practice the message and output will be expressed as *bit sequences*. In the **Sapling** circuit, the personalization string will be constant for each use.

Each 32-bit exclusive-or is implemented in 32 constraints, one for each bit position $a \oplus b = c$ as in §A.3.1.5 ‘*Exclusive-or constraints*’ on p. 197.

Additions not involving a message word, i.e. $(a + b) \bmod 2^{32} = c$, are implemented using 33 constraints and a 33-bit equality check: constrain 33 boolean variables $c_0..32$, and then check $\sum_{i=0}^{i=31} (a_i + b_i) \cdot 2^i = \sum_{i=0}^{i=32} c_i \cdot 2^i$.

Additions involving a message word, i.e. $(a + b + m) \bmod 2^{32} = c$, are implemented using 34 constraints and a 34-bit equality check: constrain 34 boolean variables $c_0..33$, and then check $\sum_{i=0}^{i=31} (a_i + b_i + m_i) \cdot 2^i = \sum_{i=0}^{i=33} c_i \cdot 2^i$.

For each addition, only $c_0..31$ are used subsequently.

The equality checks are batched; as many sets of 33 or 34 boolean variables as will fit in a \mathbb{F}_{r_s} field element are equated together using one constraint. This allows 7 such checks per constraint.

Each G evaluation requires 262 constraints:

- $4 \cdot 32 = 128$ constraints for \oplus operations;
- $2 \cdot 33 = 66$ constraints for 32-bit additions not involving message words (excluding equality checks);
- $2 \cdot 34 = 68$ constraints for 32-bit additions involving message words (excluding equality checks).

The overall cost is 21006 constraints:

- $10 \cdot 8 \cdot 262 - 4 \cdot 2 \cdot 32 = 20704$ constraints for 80 G evaluations, excluding equality checks (the deduction of $4 \cdot 2 \cdot 32$ is because v is constant at the start of the first round, so in the first four calls to G , the parameters b and d are constant, eliminating the constraints for the first two XORs in those four calls to G);
- $\text{ceiling}\left(\frac{10 \cdot 8 \cdot 4}{7}\right) = 46$ constraints for equality checks;
- $8 \cdot 32 = 256$ constraints for final $v_i \oplus v_{i+8}$ operations (the h_i words are constants so no additional constraints are required to exclusive-or with them).

This cost includes boolean-constraining the hash output bits (done implicitly by the final \oplus operations), but not the message bits.

Non-normative notes:

- The equality checks could be eliminated entirely by substituting each check into a boolean constraint for c_0 , for instance, but this optimization is not done in **Sapling**.
- It should be clear that BLAKE2s is very expensive in the circuit compared to elliptic curve operations. This is primarily because it is inefficient to use \mathbb{F}_{r_s} elements to represent single bits. However Pedersen hashes do not have the necessary cryptographic properties for the two cases where the *Spend circuit* uses BLAKE2s. While it might be possible to use variants of functions with low circuit cost such as MiMC [AGRRT2017], it was felt that they had not yet received sufficient cryptanalytic attention to confidently use them for **Sapling**.

A.4 The Sapling Spend circuit

The **Sapling** Spend *statement* is defined in § 4.18.2 ‘*Spend Statement (Sapling)*’ on p. 59.

The primary input is

$$\begin{aligned} &(\text{rt}^{\text{Sapling}} : \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]}, \\ &\text{cv}^{\text{old}} : \text{ValueCommit}^{\text{Sapling}}.\text{Output}, \\ &\text{nf}^{\text{old}} : \mathbb{B}^{[\ell_{\text{PRFntSapling}}/8]}, \\ &\text{rk} : \text{SpendAuthSig}^{\text{Sapling}}.\text{Public}), \end{aligned}$$

which is encoded as $8 \mathbb{F}_{r_s}$ elements (starting with the fixed element 1 required by Groth16):

$$[1, \mathcal{U}(\text{rk}), \mathcal{V}(\text{rk}), \mathcal{U}(\text{cv}^{\text{old}}), \mathcal{V}(\text{cv}^{\text{old}}), \text{LEBS2IP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}(\text{rt}^{\text{Sapling}}), \text{LEBS2IP}_{254}(\text{nf}_{\star 0 \dots 253}^{\text{old}}), \text{LEBS2IP}_2(\text{nf}_{\star 254 \dots 255}^{\text{old}})]$$

where $\text{nf}_{\star}^{\text{old}} = \text{LEOS2BSP}_{\ell_{\text{PRFntSapling}}}(\text{nf}^{\text{old}})$.

The auxiliary input is

$$\begin{aligned} &(\text{path} : \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}][\text{MerkleDepth}^{\text{Sapling}}]}, \\ &\text{pos} : \{0 \dots 2^{\text{MerkleDepth}^{\text{Sapling}}} - 1\}, \\ &\text{g}_d : \mathbb{J}, \\ &\text{pk}_d : \mathbb{J}, \\ &\text{v}^{\text{old}} : \{0 \dots 2^{\ell_{\text{value}} - 1}\}, \\ &\text{rcv}^{\text{old}} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}, \\ &\text{cm}^{\text{old}} : \mathbb{J}, \\ &\text{rcm}^{\text{old}} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}, \\ &\alpha : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}, \\ &\text{ak} : \text{SpendAuthSig}^{\text{Sapling}}.\text{Public}, \\ &\text{nsk} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}). \end{aligned}$$

$\text{ValueCommit}^{\text{Sapling}}.\text{Output}$ and $\text{SpendAuthSig}^{\text{Sapling}}.\text{Public}$ are of type \mathbb{J} , so we have cv^{old} , cm^{old} , rk , g_d , pk_d , and ak that represent Jubjub curve points. However,

- cv^{old} will be constrained to an output of $\text{ValueCommit}^{\text{Sapling}}$;
- cm^{old} will be constrained to an output of $\text{NoteCommit}^{\text{Sapling}}$;
- rk will be constrained to $[\alpha] \mathcal{G}^{\text{Sapling}} + \text{ak}$;
- pk_d will be constrained to $[\text{ivk}] \text{g}_d$

so cv^{old} , cm^{old} , rk , and pk_d do not need to be explicitly checked to be on the curve.

In addition, nk_{\star} and p_{\star} used in **Nullifier integrity** are compressed representations of Jubjub curve points. **TODO:** explain why these are implemented as § A.3.3.2 ‘*ctEdwards [de]compression and validation*’ on p. 200 even though the statement spec doesn’t explicitly say to do validation.

Therefore we have g_d , ak , nk , and p that need to be constrained to valid Jubjub curve points as described in § A.3.3.2 ‘*ctEdwards [de]compression and validation*’ on p. 200.

In order to aid in comparing the implementation with the specification, we present the checks needed in the order in which they are implemented in the sapling-crypto code:

Check	Implements	Cost	Reference
ak is on the curve <i>TODO: FIXME also decompressed below</i>	ak : SpendAuthSig ^{Sapling} .Public	4	§ A.3.3.1 on p. 200
ak is not <i>small-order</i>	Small order checks	16	§ A.3.3.6 on p. 203
$\alpha\star : \mathbb{B}^{\ell_{\text{Sapling}}^{\text{scalar}}}$	$\alpha : \{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
$\alpha' = [\alpha\star] \mathcal{G}^{\text{Sapling}}$	Spend authority	750	§ A.3.3.7 on p. 203
rk = $\alpha' + \text{ak}$		6	§ A.3.3.5 on p. 202
inputize rk <i>TODO: not ccteddecompress-validate => wrong count</i>	rk : SpendAuthSig ^{Sapling} .Public	392?	§ A.3.3.2 on p. 200
nsk $\star : \mathbb{B}^{\ell_{\text{Sapling}}^{\text{scalar}}}$	nsk : $\{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
nk = [nsk \star] $\mathcal{H}^{\text{Sapling}}$	Nullifier integrity	750	§ A.3.3.7 on p. 203
ak \star = repr _J (ak : J)	Diversified address integrity	392	§ A.3.3.2 on p. 200
nk \star = repr _J (nk) <i>TODO: spec doesn't say to validate nk since it's calculated</i>	Nullifier integrity	392	§ A.3.3.2 on p. 200
ivk \star = I2LEBSP ₂₅₁ (CRH ^{ivk} (ak, nk)) †	Diversified address integrity	21006	§ A.3.7 on p. 209
g _d is on the curve	g _d : J	4	§ A.3.3.1 on p. 200
g _d is not <i>small-order</i>	Small order checks	16	§ A.3.3.6 on p. 203
pk _d = [ivk \star] g _d	Diversified address integrity	3252	§ A.3.3.8 on p. 204
v $\star^{\text{old}} : \mathbb{B}^{[64]}$	v ^{old} : $\{0 \dots 2^{64} - 1\}$	64	§ A.3.1.1 on p. 196
rcv $\star : \mathbb{B}^{\ell_{\text{Sapling}}^{\text{scalar}}}$	rcv : $\{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
cv = ValueCommit ^{rcv} (v ^{old})	Value commitment integrity	947	§ A.3.6 on p. 208
inputize cv		?	
rcm $\star : \mathbb{B}^{\ell_{\text{Sapling}}^{\text{scalar}}}$	rcm : $\{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
cm = NoteCommit ^{Sapling} _{rcm} (g _d , pk _d , v ^{old})	Note commitment integrity	1740	§ A.3.5 on p. 208
cm _u = Extract _{J(r)} (cm)	Merkle path validity	0	
rt' is the root of a Merkle tree with leaf cm _u , and authentication path (path, pos \star)		32 · 1380	§ A.3.4 on p. 208
pos \star = I2LEBSP _{MerkleDepth^{Sapling}} (pos)		1	§ A.3.2.1 on p. 197
if v ^{old} ≠ 0 then rt' = rt ^{Sapling}		1	§ A.3.1.2 on p. 197
inputize rt ^{Sapling}		?	
ρ = MixingPedersenHash(cm ^{old} , pos)		98	§ A.3.3.10 on p. 207
ρ \star = repr _J (ρ) <i>TODO: spec doesn't say to validate ρ since it's calculated</i>	Nullifier integrity	392	§ A.3.3.2 on p. 200
nf ^{old} _{nk\star} = PRF ^{nfSapling} _{nk\star} (ρ \star)		21006	§ A.3.7 on p. 209
pack nf ^{old} _{0..253} and nf ^{old} _{254..255} into two \mathbb{F}_r inputs	input encoding	2	§ A.3.2.1 on p. 197

† This is implemented by taking the output of BLAKE2s-256 as a *bit sequence* and dropping the most significant 5 bits, not by converting to an integer and back to a *bit sequence* as literally specified.

Note: The implementation represents α^* , nsk^* , ivk^* , rcm^* , rcv^* , and v^{old}_* as *bit sequences* rather than integers. It represents nf as a *bit sequence* rather than a *byte sequence*.

A.5 The Sapling Output circuit

The **Sapling** Output *statement* is defined in §4.18.3 ‘*Output Statement (Sapling)*’ on p. 60.

The primary input is

$$\begin{aligned} &(\text{cv}^{\text{new}} : \text{ValueCommit}^{\text{Sapling}}.\text{Output}, \\ &\text{cm}_u : \mathbb{B}^{[\ell_{\text{Merkle}}^{\text{Sapling}}]}, \\ &\text{epk} : \mathbb{J}), \end{aligned}$$

which is encoded as 6 \mathbb{F}_{r_s} elements (starting with the fixed element 1 required by Groth16):

$$[1, \mathcal{U}(\text{cv}^{\text{new}}), \mathcal{V}(\text{cv}^{\text{new}}), \mathcal{U}(\text{epk}), \mathcal{V}(\text{epk}), \text{LEBS2IP}_{\ell_{\text{Merkle}}^{\text{Sapling}}}(\text{cm}_u)]$$

The auxiliary input is

$$\begin{aligned} &(\text{g}_d : \mathbb{J}, \\ &\text{pk}^*_d : \mathbb{B}^{[\ell_d]}, \\ &\text{v}^{\text{new}} : \{0 \dots 2^{\ell_{\text{value}}} - 1\}, \\ &\text{rcv}^{\text{new}} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}, \\ &\text{rcm}^{\text{new}} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}, \\ &\text{esk} : \{0 \dots 2^{\ell_{\text{scalar}}^{\text{Sapling}}} - 1\}) \end{aligned}$$

$\text{ValueCommit}^{\text{Sapling}}.\text{Output}$ is of type \mathbb{J} , so we have cv^{new} , epk , and g_d that represent Jubjub curve points. However,

- cv^{new} will be constrained to an output of $\text{ValueCommit}^{\text{Sapling}}$,
- epk will be constrained to $[\text{esk}] \text{g}_d$

so cv^{new} and epk do not need to be explicitly checked to be on the curve.

Therefore we have only g_d that needs to be constrained to a valid Jubjub curve point as described in §A.3.3.2 ‘*ctEdwards [de]compression and validation*’ on p. 200.

Note: pk^*_d is *not* checked to be a valid compressed representation of a Jubjub curve point.

In order to aid in comparing the implementation with the specification, we present the checks needed in the order in which they are implemented in the sapling-crypto code:

Check	Implements	Cost	Reference
$v_{\star}^{\text{old}} : \mathbb{B}^{[64]}$	$v^{\text{old}} : \{0 \dots 2^{64} - 1\}$	64	§ A.3.1.1 on p. 196
$\text{rcv}_{\star} : \mathbb{B}^{[\ell_{\text{Sapling}}^{\text{scalar}}]}$	$\text{rcv} : \{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
$\text{cv} = \text{ValueCommit}_{\text{rcv}}^{\text{Sapling}}(v^{\text{old}})$	Value commitment integrity	947	§ A.3.6 on p. 208
inputize cv		?	
$g_{\star d} = \text{repr}_{\mathbb{J}}(g_d : \mathbb{J})$	Note commitment integrity	392	§ A.3.3.2 on p. 200
g_d is not <i>small-order</i>	Small order checks	16	§ A.3.3.6 on p. 203
$\text{esk}_{\star} : \mathbb{B}^{[\ell_{\text{Sapling}}^{\text{scalar}}]}$	$\text{esk} : \{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
$\text{epk} = [\text{esk}_{\star}] g_d$	Ephemeral public key integrity	3252	§ A.3.3.8 on p. 204
inputize epk		?	
$\text{pk}_{\star d} : \mathbb{B}^{[\ell_{\mathbb{J}}]}$	$\text{pk}_{\star d} : \mathbb{B}^{[\ell_{\mathbb{J}}]}$	256	§ A.3.1.1 on p. 196
$\text{rcm}_{\star} : \mathbb{B}^{[\ell_{\text{Sapling}}^{\text{scalar}}]}$	$\text{rcm} : \{0 \dots 2^{\ell_{\text{Sapling}}^{\text{scalar}}} - 1\}$	252	§ A.3.1.1 on p. 196
$\text{cm} = \text{NoteCommit}_{\text{rcm}}^{\text{Sapling}}(g_d, \text{pk}_{\star d}, v^{\text{old}})$	Note commitment integrity	1740	§ A.3.5 on p. 208
pack inputs		?	

Note: The implementation represents esk_{\star} , $\text{pk}_{\star d}$, rcm_{\star} , rcv_{\star} , and v_{\star}^{old} as *bit sequences* rather than integers.

B Batching Optimizations

B.1 RedDSA batch validation

The reference validation algorithm for RedDSA signatures is defined in § 5.4.7 ‘RedDSA, RedJubjub, *and* RedPallas’ on p. 90.

Let the RedDSA parameters \mathbb{G} (defining a subgroup $\mathbb{G}^{(r)}$ of order $r_{\mathbb{G}}$, a cofactor $h_{\mathbb{G}}$, a group operation $+$, an additive identity $\mathcal{O}_{\mathbb{G}}$, a bit-length $\ell_{\mathbb{G}}$, a representation function $\text{repr}_{\mathbb{G}}$, and an abstraction function $\text{abst}_{\mathbb{G}}$); $\mathcal{P}_{\mathbb{G}} : \mathbb{G}$; $\ell_{\mathbb{H}} : \mathbb{N}$; $\mathbb{H} : \mathbb{B}^{\mathbb{Y}^{[N]}} \rightarrow \mathbb{B}^{\mathbb{Y}^{[\ell_{\mathbb{H}}/8]}}$; and the derived hash function $\mathbb{H}^{\circledast} : \mathbb{B}^{\mathbb{Y}^{[N]}} \rightarrow \mathbb{F}_{r_{\mathbb{G}}}$ be as defined in that section.

Implementations **MAY** alternatively use the optimized procedure described in this section to perform faster validation of a batch of signatures, i.e. to determine whether all signatures in a batch are valid. Its input is a sequence of N *signature batch entries*, each of which is a (*validating key*, *message*, *signature*) triple.

Let LEOS2BSP, LEOS2IP, and LEBS2OSP be as defined in § 5.1 ‘*Integers, Bit Sequences, and Endianness*’ on p. 71.

Define $\text{RedDSA.BatchEntry} := \text{RedDSA.Public} \times \text{RedDSA.Message} \times \text{RedDSA.Signature}$.

Define $\text{RedDSA.BatchValidate} : (\text{entry}_{0..N-1} : \text{RedDSA.BatchEntry}^{[N]}) \rightarrow \mathbb{B}$ as:

For each $j \in \{0..N-1\}$:

Let $(\text{vk}_j, M_j, \sigma_j) = \text{entry}_j$.

Let \underline{R}_j be the first ceiling $(\ell_{\mathbb{G}}/8)$ bytes of σ_j , and let \underline{S}_j be the remaining ceiling $(\text{bitlength}(r_{\mathbb{G}})/8)$ bytes.

Let $R_j = \text{abst}_{\mathbb{G}}(\text{LEOS2BSP}_{\ell_{\mathbb{G}}}(\underline{R}_j))$, and let $S_j = \text{LEOS2IP}_{8 \cdot \text{length}(\underline{S}_j)}(\underline{S}_j)$.

Let $\underline{\text{vk}}_j = \text{LEBS2OSP}_{\ell_{\mathbb{G}}}(\text{repr}_{\mathbb{G}}(\text{vk}_j))$.

Let $c_j = \text{H}^{\oplus}(\underline{R}_j || \underline{\text{vk}}_j || M_j)$.

Choose random $z_j : \mathbb{F}_{r_{\mathbb{G}}}^* \xleftarrow{\mathbb{R}} \{1..2^{128} - 1\}$.

Return 1 if

- for all $j \in \{0..N-1\}$, $R_j \neq \perp$ and $S_j < r_{\mathbb{G}}$; and
- $[h_{\mathbb{G}}] \left(- \left[\sum_{j=0}^{N-1} (z_j \cdot S_j) \pmod{r_{\mathbb{G}}} \right] \mathcal{P}_{\mathbb{G}} + \sum_{j=0}^{N-1} [z_j] R_j + \sum_{j=0}^{N-1} [z_j \cdot c_j \pmod{r_{\mathbb{G}}}] \text{vk}_j \right) = \mathcal{O}_{\mathbb{G}}$.

otherwise 0.

The z_j values **MUST** be chosen independently of the *signature batch entries*.

Non-normative note: It is also acceptable to sample each z_j from $\{0..2^{128} - 1\}$, since the probability of obtaining zero for any z_j is negligible.

The performance benefit of this approach arises partly from replacing the per-signature scalar multiplication of the base $\mathcal{P}_{\mathbb{G}}$ with one such multiplication per batch, and partly from using an efficient algorithm for multiscalar multiplication such as Pippenger’s method [Bernstein2001] or the Bos–Coster method [deRoos1995], as explained in [BDLSY2012, section 5].

Note: Spend authorization signatures (§ 5.4.7.1 ‘*Spend Authorization Signature (Sapling and Orchard)*’ on p. 93) and binding signatures (§ 5.4.7.2 ‘*Binding Signature (Sapling and Orchard)*’ on p. 93) use different bases $\mathcal{P}_{\mathbb{G}}$. It is straightforward to adapt the above procedure to handle multiple bases; there will be one $-\left[\sum_j (z_j \cdot S_j) \pmod{r_{\mathbb{G}}}\right] \mathcal{P}$ term for each base \mathcal{P} . The benefit of this relative to using separate batches is that the multiscalar multiplication can be extended across a larger batch.

B.2 Groth16 batch verification

The reference verification algorithm for Groth16 proofs is defined in § 5.4.10.2 ‘Groth16’ on p. 109. The batch verification algorithm in this section applies techniques from [BFJISV2010, section 4].

Let $q_{\mathbb{S}}, r_{\mathbb{S}}, \mathbb{S}_{1,2,T}^{(r)}, \mathbb{S}_{1,2,T}^{(r)*}, \mathcal{P}_{\mathbb{S}_{1,2,T}}, \mathbf{1}_{\mathbb{S}}$, and $\hat{e}_{\mathbb{S}}$ be as defined in § 5.4.9.2 ‘BLS12-381’ on p. 99.

Define $\text{MillerLoop}_{\mathbb{S}} : \mathbb{S}_1^{(r)} \times \mathbb{S}_2^{(r)} \rightarrow \mathbb{S}_T^{(r)}$ and $\text{FinalExp}_{\mathbb{S}} : \mathbb{S}_T^{(r)} \rightarrow \mathbb{S}_T^{(r)}$ to be the Miller loop and final exponentiation respectively of the $\hat{e}_{\mathbb{S}}$ pairing computation, so that:

$$\hat{e}_{\mathbb{S}}(P, Q) = \text{FinalExp}_{\mathbb{S}}(\text{MillerLoop}_{\mathbb{S}}(P, Q))$$

where $\text{FinalExp}_{\mathbb{S}}(R) = R^t$ for some fixed t .

Define $\text{Groth16}_{\mathbb{S}}.\text{Proof} := \mathbb{S}_1^{(r)*} \times \mathbb{S}_2^{(r)*} \times \mathbb{S}_1^{(r)*}$.

A $\text{Groth16}_{\mathbb{S}}$ proof comprises a tuple $(\pi_A, \pi_B, \pi_C) : \text{Groth16}_{\mathbb{S}}.\text{Proof}$.

Verification of a single Groth16_S proof against an instance encoded as $a_{0..ℓ} : \mathbb{F}_{r_S}^{[\ell+1]}$ requires checking the equation

$$\hat{e}_S(\pi_A, \pi_B) = \hat{e}_S(\pi_C, \Delta) \cdot \hat{e}_S\left(\sum_{i=0}^{\ell} [a_i] \Psi_i, \Gamma\right) \cdot Y$$

where $\Delta = [\delta] \mathcal{P}_{S_2}$, $\Gamma = [\gamma] \mathcal{P}_{S_2}$, $Y = [\alpha \cdot \beta] \mathcal{P}_{S_T}$, and $\Psi_i = \left[\frac{\beta \cdot u_i(x) + \alpha \cdot v_i(x) + w_i(x)}{\gamma} \right] \mathcal{P}_{S_1}$ for $i \in \{0..ℓ\}$ are elements of the verification key, as described (with slightly different notation) in [Groth2016, section 3.2].

This can be written as:

$$\hat{e}_S(\pi_A, -\pi_B) \cdot \hat{e}_S(\pi_C, \Delta) \cdot \hat{e}_S\left(\sum_{i=0}^{\ell} [a_i] \Psi_i, \Gamma\right) \cdot Y = \mathbf{1}_S.$$

Raising to the power of random $z \neq 0$ gives:

$$\hat{e}_S([z] \pi_A, -\pi_B) \cdot \hat{e}_S([z] \pi_C, \Delta) \cdot \hat{e}_S\left(\sum_{i=0}^{\ell} [z \cdot a_i] \Psi_i, \Gamma\right) \cdot Y^z = \mathbf{1}_S.$$

This justifies the following optimized procedure for performing faster verification of a batch of Groth16_S proofs. Implementations **MAY** use this procedure to determine whether all proofs in a batch are valid.

Define a type $\text{Groth16}_S.\text{BatchEntry} := \text{Groth16}_S.\text{Proof} \times \text{Groth16}_S.\text{PrimaryInput}$ representing *proof batch entries*.

Define $\text{Groth16}_S.\text{BatchVerify} : (\text{entry}_{0..N-1} : \text{Groth16}_S.\text{BatchEntry}^{[N]}) \rightarrow \mathbb{B}$ as:

For each $j \in \{0..N-1\}$:

Let $((\pi_{j,A}, \pi_{j,B}, \pi_{j,C}), a_{j,0..ℓ}) = \text{entry}_j$.

Choose random $z_j : \mathbb{F}_{r_S}^* \xleftarrow{R} \{1..2^{128} - 1\}$.

Let $\text{Accum}_{AB} = \prod_{j=0}^{N-1} \text{MillerLoop}_S([z_j] \pi_{j,A}, -\pi_{j,B})$.

Let $\text{Accum}_{\Delta} = \sum_{j=0}^{N-1} [z_j] \pi_{j,C}$.

Let $\text{Accum}_{\Gamma,i} = \sum_{j=0}^{N-1} (z_j \cdot a_{j,i}) \pmod{r_S}$ for $i \in \{0..ℓ\}$.

Let $\text{Accum}_Y = \sum_{j=0}^{N-1} z_j \pmod{r_S}$.

Return 1 if

$$\text{FinalExp}_S\left(\text{Accum}_{AB} \cdot \text{MillerLoop}_S(\text{Accum}_{\Delta}, \Delta) \cdot \text{MillerLoop}_S\left(\sum_{i=0}^{\ell} [\text{Accum}_{\Gamma,i}] \Psi_i, \Gamma\right)\right) \cdot Y^{\text{Accum}_Y} = \mathbf{1}_S,$$

otherwise 0.

The z_j values **MUST** be chosen independently of the *proof batch entries*.

Non-normative note: It is also acceptable to sample each z_j from $\{0..2^{128} - 1\}$, since the probability of obtaining zero for any z_j is negligible.

The performance benefit of this approach arises from computing two of the three Miller loops, and the final exponentiation, per batch instead of per proof. For the multiplications by z_j , an efficient algorithm for multiscalar multiplication such as Pippenger's method [Bernstein2001] or the Bos-Coster method [deRoos1995] may be used.

Note: Spend proofs (of the *statement* in §4.18.2 '*Spend Statement (Sapling)*' on p.59) and output proofs (of the *statement* in §4.18.3 '*Output Statement (Sapling)*' on p.60) use different verification keys, with different parameters Δ , Γ , Y , and $\Psi_{0..ℓ}$. It is straightforward to adapt the above procedure to handle multiple verification keys; the accumulator variables Accum_{Δ} , $\text{Accum}_{\Gamma,i}$, and Accum_Y are duplicated, with one term in the verification equation for each variable, while Accum_{AB} is shared.

Neglecting multiplications in $\mathbb{S}_T^{(r)}$ and \mathbb{F}_{r_s} , and other trivial operations, the cost of batched verification is therefore

- for each proof: the cost of decoding the proof representation to the form $\text{Groth16}_{\mathbb{S}}.\text{Proof}$, which requires three point decompressions and three subgroup checks (two for $\mathbb{S}_1^{(r)*}$ and one for $\mathbb{S}_2^{(r)*}$);
- for each successfully decoded proof: a Miller loop; and a 128-bit scalar multiplication by z_j in $\mathbb{S}_1^{(r)}$;
- for each verification key: two Miller loops; an exponentiation in $\mathbb{S}_T^{(r)}$; a multiscalar multiplication in $\mathbb{S}_1^{(r)}$ with N 128-bit scalars to compute Accum_{Δ} ; and a multiscalar multiplication in $\mathbb{S}_1^{(r)}$ with $\ell + 1$ 255-bit scalars to compute $\sum_{i=0}^{\ell} [\text{Accum}_{\Gamma, i}] \Psi_i$;
- one final exponentiation.

B.3 Ed25519 batch validation

The reference validation algorithm for Ed25519 signatures is defined in § 5.4.6 ‘Ed25519’ on p. 88.

[Canopy onward] Implementations **MAY** alternatively use the optimized procedure described in this section to perform faster validation of a batch of signatures, i.e. to determine whether all signatures in a batch are valid. The correctness of this procedure is dependent on the Ed25519 validation changes made for the **Canopy network upgrade** in [ZIP-215] (in particular the change to use the cofactor variant of the validation equation). The input is a sequence of N *signature batch entries*, each of which is a (*validating key*, message, signature) triple.

Let ℓ , B , $\text{abstBytes}_{\text{Ed25519}}$, and $\text{reprBytes}_{\text{Ed25519}}$ be as defined in § 5.4.6 ‘Ed25519’ on p. 88.

Let LEOS2IP be as defined in § 5.1 ‘Integers, Bit Sequences, and Endianness’ on p. 71.

SHA-512 is defined in § 5.4.1.1 ‘SHA-256, SHA-256d, SHA256Compress, and SHA-512 Hash Functions’ on p. 73.

Define $\text{Ed25519.BatchEntry} := \text{Ed25519.Public} \times \text{Ed25519.Message} \times \text{Ed25519.Signature}$.

Define $\text{Ed25519.BatchValidate} : (\text{entry}_{0..N-1} : \text{Ed25519.BatchEntry}^{[N]}) \rightarrow \mathbb{B}$ as:

For each $j \in \{0..N-1\}$:

Let $(A_j, M_j, \sigma_j) = \text{entry}_j$.

Let \underline{R}_j be the first 32 bytes of σ_j , and let \underline{S}_j be the remaining 32 bytes.

Let $R_j = \text{abstBytes}_{\text{Ed25519}}(\underline{R}_j)$, and let $S_j = \text{LEOS2IP}_{256}(\underline{S}_j)$.

Let $\underline{A}_j = \text{reprBytes}_{\text{Ed25519}}(A_j)$.

Let $c_j = \text{LEOS2IP}_{512}(\text{SHA-512}(\underline{R}_j \parallel \underline{A}_j \parallel M_j))$.

Choose random $z_j : \mathbb{F}_{\ell}^* \xleftarrow{\mathbb{R}} \{1..2^{128}-1\}$.

Return 1 if

- for all $j \in \{0..N-1\}$, $R_j \neq \perp$ and $S_j < \ell$; and
- $[8] \left(- \left[\sum_{j=0}^{N-1} (z_j \cdot S_j) \pmod{\ell} \right] B + \sum_{j=0}^{N-1} [z_j] R_j + \sum_{j=0}^{N-1} [z_j \cdot c_j \pmod{\ell}] A_j \right) = \mathcal{O}_{\text{Ed25519}}$,

otherwise 0.

The z_j values **MUST** be chosen independently of the *signature batch entries*.

Non-normative note: It is also acceptable to sample each z_j from $\{0..2^{128}-1\}$, since the probability of obtaining zero for any z_j is negligible.

The performance benefits of this approach are the same as for § B.1 ‘RedDSA *batch validation*’ on p. 215.

List of Theorems and Lemmata

Theorem 5.4.1	The encoding function $\langle \cdot \rangle$ is injective	78
Lemma 5.4.2	An injectivity property for Sinsemilla	80
Theorem 5.4.3	Collision resistance of SinsemillaHash and SinsemillaHashToPoint	81
Theorem 5.4.4	A \perp output from SinsemillaHashToPoint yields a nontrivial discrete log relation	82
Theorem 5.4.5	Uncommitted ^{Sapling} is not in the range of NoteCommit ^{Sapling}	94
Theorem 5.4.6	Uncommitted ^{Orchard} is not in the range of NoteCommit ^{Orchard}	97
Lemma 5.4.7	Let $P = (u, v) \in \mathbb{J}^{(r)}$. Then $(u, -v) \notin \mathbb{J}^{(r)}$	102
Theorem 5.4.8	\mathcal{U} is injective on $\mathbb{J}^{(r)}$	102
Theorem A.2.1	$(0, 0)$ is the only point with $y = 0$ on certain <i>Montgomery curves</i>	196
Theorem A.3.1	Correctness of a constraint system for range checks	199
Theorem A.3.2	Exceptional points (ctEdwards \rightarrow Montgomery)	201
Theorem A.3.3	Exceptional points (Montgomery \rightarrow ctEdwards)	201
Theorem A.3.4	Distinct- x theorem	201
Theorem A.3.5	Concerning addition inputs in the Pedersen circuit	206

Index

account, 43	Bech32, 36, 111, 115, 117, 149, 158, 170
Action circuit, 31, 48, 62, 97, 124, 150–154	Bech32m, 111, 115–117, 149, 154
Action description, 8, 9, 14–16, 19 , 20–21, 27, 41 , 42, 44, 45, 47, 53, 54, 56, 65–68, 70, 110, 120, 123, 124, 127, 144, 149, 154, 156, 158	bellman, 109 , 117
Action statement, 19, 34, 42, 45, 47, 55–57, 61 , 62, 63, 124, 127, 154, 156–158	best valid block chain, 16 , 17, 64, 67, 176
Action transfer, 17, 19 , 20–21, 27, 41, 42, 45, 49, 53, 55, 124, 127, 141, 156, 158	big-endian, 98, 100, 106, 109, 110, 117, 131, 199
activation block, 17, 22, 90, 109, 149	bilateral consensus rule change, 118
activation block height, 118 , 137, 163	binding (commitment scheme), 23, 29 , 30, 52, 54, 94–97, 141, 142, 144, 145, 151, 164, 171, 176
ALL CAPS , 7	binding signature, 92, 144
anchor, 18 , 19–20, 39, 40, 42, 58, 59, 62, 126, 149	binding signature (Orchard), 20, 27, 49, 53 , 54–55, 120
authenticated one-time symmetric encryption, 25 , 26, 86	binding signature (Sapling), 19, 27, 49, 51 , 52–54, 119, 120, 172
auxiliary input, 33 , 34, 46, 47, 49, 56, 58–62, 156, 169	binding signature scheme, 90, 93
Base58Check, 111–113, 135, 137	bit sequence, 10, 11, 35, 37, 47, 48, 60–62, 64, 66, 71, 73, 92, 149, 151, 153, 160, 161, 166, 172, 195, 208, 210, 214, 215
BCTV14, 34, 39, 49, 55, 108 , 109, 119, 125, 146, 150, 159, 166, 170, 195	Bitcoin , 1, 7 , 8, 17, 20, 21, 27, 49, 50, 111, 112, 117, 119, 120, 122–124, 128–131, 138, 139, 146, 150, 156, 161, 176, 178
	Bitcoin Core, 123, 146, 150

block, 16–22, 34, 39, 40, 42, 44, 50, 57, 66–68, 109, 117, 118, 122, 124, **128**, 129–133, 137, 138, 150, 152, 154, 163, 165, 176, 177
 block chain, 8, 9, 13–15, **16**, 17–18, 22, 57, 64, 67, 70, 71, 108–110, 123, 129, 133, 138, 141, 157, 163, 165, 174, 176
 block chain reorganization, 64, 67, 118
 block hash, **22**, 109, 149
 block header, 16, 73, **128**, 129–133, 155, 173, 176, 178
 block height, **16**, 17, 21, 57, 66–69, 118–120, 122–126, 129, 132–137, 152–154, 159, 166
 block subsidy, **21**, 118, 122, 133, 136, 151, 160, 178
 block target spacing, **131**, 133, 137
 block timestamp, **128**
 block version number, **128**, 129–130, 165, 175
Blossom, 1, 7, 49, **118**, 128–130, 133, 136, 137, 163–165
 BLS12-381, 34, **99**, 103, 109, 145, 171
 BN-254, 34, **97**, 100, 109
 Bulletproofs, 53
 byte sequence, 10, 71, 74, 86, 91, 92, 110, 111, 154, 156, 159, 172, 196, 214

Canopy, 1, 7, 15, 21, 39, 43, 49, 50, 63, 67, 68, 89, 90, 115, **118**, 121–124, 128–130, 135–137, 148, 149, 153, 159–162, 218
 chain value pool, **8**, 115, 147, 150
 chain value pool balance (deferred development fund), **57**
 chain value pool balance (Orchard), **57**
 chain value pool balance (Sapling), **57**, 159
 chain value pool balance (Sprout), **57**, 159
 chain value pool balance (transparent), **57**
 chunk (of a Pedersen hash input), **78**
 coinbase transaction, 16, 17, **21**, 50, 121–124, 129, 134–137, 150–153, 155, 156, 159–161, 171, 174, 178
 coins (in Zerocash), **8**
 collision resistance, 22, 23, 25, 32, 74–81, 84, 85, 94, 96, 140–143, 145, 146, 157, 173, 179, 208
 Commit^{ivk} randomness, 37, 116
 commitment scheme, 22, **29**, 30–32, 37, 81, 94–97, 141–143, 145, 151, 164, 170, 176
 commitment trapdoor, 13, 14, **29**, 30, 40, 41, 44, 45, 53, 96, 140, 148, 170
 complete twisted Edwards affine coordinates, 66, 94, **195**, 196, 202, 203
 complete twisted Edwards compressed encoding, **101**, 113, 114, 199
 complete twisted Edwards elliptic curve, 11, 44, 89, **100**, 101–102, 164, 195, 200, 201, 203, 205
 consensus branch, **123**, 150
 consensus branch ID, 49, 50, 120, 121, 149, 156
 consensus rule change, 118
 coordinate extractor, **32**, 159, 174
CryptoNote, 9, 178
 ctEdwards, **11**, 100, 204, 205, 207–209

 Decentralized Anonymous Payment scheme, 1, **7**
 Decisional Diffie–Hellman Problem, 33, 76, 83, 142, 151, 155
 default diversified payment address, **36**, **38**, 103
 deferred development fund chain value pool, 8, 57
 Discrete Logarithm Problem, 32, 63, 77, 79–81, 141, 155, 205
 distinct- x criterion, 174, **202**, 205
 diversified base, 24, **36**, 44, 45, 65, 76
 diversified payment address, **12**, 13–14, 36, 38, 46, 47, 76, 77, 142, 146, 155, 165, 167
 diversified transmission key, 14, **36**, **38**, 41, 42, 44, 45, 65, 126, 127
 diversifier, 14, 24, 25, **36**, 38, 76, 77, 86, 113, 155
 diversifier index, **38**, 155
 diversifier key, 12, 25, 37, 38, 116, 156
 double-spend, 9, 16, 18–21, 63, 141, 145, 147, 175
 dummy note, **45**, 46–47, 139–141, 156, 158, 159

 ECDSA, 27, 111, 169
 Ed25519, 27, 73, 88–90, 121, 149, 154, 159, 161, 162, 171, 174, 178, 218
 ephemeral private key, 41, 42, 44, 65, 126, 127
 ephemeral public key, 26, 63, 65, 143
 Equihash, 1, 23, 84, 128, 129, **130**, 131, 146, 165, 169, 175, 177, 178
 expanded spending key, **12**
 extended spending key, 12, 164

 family of group hashes into a subgroup, **32**
 Founders’ Reward, **21**, 121, 122, 133, 134, 136, 165, 171, 175, 177, 178
 full node, **174**
 full validator, **16**, 17, 21, 90, 109, 129, 149, 174
 full viewing key, 9, **12**, 13, 36, 38, 41, 42, 46, 47, 56, 57, 70, 83, **114**, 115, **116**, 127, 151, 152, 170
 funding stream, **21**, 121, 122, 124, 133, **136**, 137–138, 159, 165

genesis block, **16**, 22, 122, **128**, 129, 136, 138, 159, 176

Groth16, 34, 39, 49, 52, 55, **109**, 119, 125, 145, 165, 166, 168, 170, 195, 212, 214, 216

group hash, **32**, 102, 107, 157

Halo 2, 20, 34, 49, 54, 55, 79, **110**, 146, 150

halo2, 34

halving, 118, 136, 137

hash function, 18, 23, 24, 35, 73, 77–80, 82–85, 89, 90, 144, 171

hash value (of a Merkle tree node), **20**, 47, 48, 74, 97, 155

hash-to-curve, 107

Heartwood, 1, 7, 49, **118**, 122–124, 128–130, 137, 148, 153, 159, 162, 163

hiding (commitment scheme), **29**, 30, 38, 94–97, 141, 142, 151, 176

Hierarchical Deterministic Wallet, 12

homomorphic Pedersen commitment, **95**, 208

Human-Readable Part, **113**, 114, 117

in-band, 143, 178

incoming viewing key, **12**, 13, 23, 31, 35, 36, 38, 57, 63, 64, 69, 75, 77, 111, **112**, **114**, 115, **116**, 144, 150, 151, 154, 155, 175

incremental Merkle tree, **20**, 47, 48, 74, 77, 79, 163, 179

index (of a Merkle tree node), **20**, 48

internal node (of a Merkle tree), **48**

iso-Pallas, 105, 107, 108

iso-Vesta, 105, 107, 108

JoinSplit circuit, 117

JoinSplit description, 8, 14–16, **18**, 21, 27, **38**, 39, 42, 43, 45, 46, 50, 63, 64, 70, 109, 119, 122, 123, 125, 139, 140, 144, 159, 169, 174, 175

JoinSplit proof, 46

JoinSplit signature, 27, 49, **50**, 178

JoinSplit signing key, **43**

JoinSplit statement, 9, 18, 34, 39, 46, 50, 55, 57, **58**, 108, 140–142, 145, 151, 170, 175, 195

JoinSplit transfer, 17, **18**, 19–21, 38, 39, 45, 49–51, 120, 125, 139, 141, 144, 145, 151, 176, 179

Jubjub, 15, 27, 32, 44, 51, 55, 60, 61, 67, 69, 76, 77, 85, 87, 90, 94, 95, **100**, 101, 103, 104, 114, 126, 142–144, 159, 161, 164, 169, 172, 174, 175, 195, 201, 203, 212, 214

key agreement scheme, **25**, 26, 35, 37, 63, 65, 86–88, 143

Key Derivation Function, **26**, 63, 65, 87, 88

key privacy, **9**, **26**, 76, 143, 144, 146, 165, 167

layer (of a Merkle tree), **20**, 48, 60, 62, 97, 154

leaf node (of a Merkle tree), 17, 21, **48**, 97

librustzcash, 69

libsnaek (Zcash fork), **108**, 109, 117, 145

linear combination, **195**, 197

little-endian, 94, 114, 116, 122, 131

Mainnet, 17, 21, 22, 69, 72, 73, 109, 111–114, 117, 118, 122, 123, 129, 130, 133, 134, 137, 138, 149, 161, 165, 177

MAY, 7, 17, 36, 38, 40, 41, 49, 53, 55, 56, 67, 90, 92, 96, 109, 111, 113, 148, 149, 215, 217, 218

median-time-past, **128**, 129, 163

memo field, **14**, 63, 65, 70, 139, 157, 158, 162, 174, 178

mempool, 67, 69, 160

Merkle path, 46, 47, **48**, 58, 59, 62, 205

Mimblewimble, 53

miner subsidy, **21**, 50, 133, 151, 178

monomorphism, **29**, 163

Montgomery affine coordinates, **195**, 196, 201, 202

Montgomery elliptic curve, 167, **195**, 196, 201, 204, 205, 207, 219

multi-party computation, 117

MUST, 7, 16–21, 39–45, 49, 50, 57, 60–64, 66, 67, 69, 89, 92, 109, 110, 112–116, 118, 119, 121–124, 126, 127, 129, 131, 135–137, 150–154, 156, 159, 161, 168, 216–218

MUST NOT, 7, 21, 40, 41, 67, 89, 98, 100, 101, 104, 108, 121–123, 129, 150, 152, 156, 169

network, 17, **21**, 22, 137

network upgrade, 15, 17, 22, 49, 50, 108, 109, 115, **118**, 124, 133, 137, 149, 150, 166, 218

node (of a Merkle tree), **20**, 47, 48

non-canonical (compressed encoding of a point), **31**, 40, 42, 67, 69, 92, 113, 122, 159, 160

non-canonical (encoding of a field element), 40, 41, 48, 155, 208

nonmalleability (of proofs), **34**

nonmalleability (of signatures), **27**

note, 8, 9, **13**, 14–16, 18, 19, 23, 24, 28, 38–43, 45–48, 50, 55–57, 63–67, 69–71, 85, 110, 124–127, 139–142, 144, 145, 148, 149, 151, 153, 156–159, 162, 172, 205

note commitment, 8, 14, **15**, 16, 18, 20, 21, 23, 39, 41, 42, 48, 56, 57, 64–66, 125–127, 140–142, 144, 145, 148, 151, 152, 157, 171, 175, 205

note commitment scheme, 30, 93, 94, 96, 144, 205
 note commitment tree, 15–18, **20**, 21, 47, 75, 120, 125, 126, 128, 129, 141, 152, 153, 173
 note plaintext, **14**, **15**, 41, 42, 63, 65, 71, 110, 123, 124, 126, 127, 144, 148, 157, 160, 161
 note plaintext lead byte, 44, 45, 123, 144, 153, 161
 note position, 8, 15, **20**, 56, 141
 note traceability set, **9**, 171
NU5, 1, 7, 12, 16, 17, 21, 22, 24, 34, 40, 49, 50, 63, 67–69, 91, 113, **118**, 121–124, 128–130, 141, 148, 149, 152–159
NU6, 1, 7, 17, 50, 57, 118, 122, 130, 138, 147
 nullifier, 8, 9, 13, 14, **16**, 18, 19, **21**, 23, 39, 40, 42, 45, 56, 57, 67, 70, 71, 81, 83, 85, 124–127, 140–143, 145, 147, 148, 157, 160, 172, 176
 nullifier deriving key, 8, 16, 37, 56, 67, 116
 nullifier private key, **12**
 nullifier set, 16–18, **21**, 64, 67

 one-time (authenticated symmetric encryption), **25**
 one-time (signature scheme), **27**
 open (a commitment), **29**
OPTIONAL, 7, 43
Orchard, 8, 9, 12–21, 24–27, 31, 33, 34, 37, 38, 44, 45, 47–49, 53, 55, 56, 63, 65–71, 74, 76, 79, 83, 85, 86, 93, 95, 103, 105, 106, 110, 111, 115–117, 120, 123, 124, 140–146, 148–151, 153–158
 Orchard balancing value, **53**, 54
 out-of-band, 63–66, 71
 outgoing cipher key, 41, 42, **65**, 84, 85, 126, 127, 156
 outgoing ciphertext, **65**, 69, 84, 85, 144, 145, 151, 156, 172
 outgoing viewing key, **12**, 35, 37, 38, 43–45, 65, 68, 114, 123, 153, 169, 170
 Output circuit, 117, 172, 204, 205
 Output description, 8, 9, 14, 15, **19**, 20, 27, 30, **40**, 41, 43, 44, 51–53, 65, 66, 68, 70, 96, 109, 119, 120, 122, 123, 126, 127, 144, 148, 159, 169, 173, 174
 Output statement, 19, 34, 41, 44, 52, **60**, 63, 109, 126, 169, 172
 Output transfer, 17, **19**, 27, 40, 51, 53, 139, 141
Overwinter, 1, 7, 49, 74, **118**, 121–124, 129, 165, 169, 170, 173–175

 packing, **197**
 Pallas, 16, 27, 32, 38, 42, 44, 48, 53, 55, 56, 62, 63, 79, 83, 88, 90, 95–97, **103**, 104–106, 108, 116, 127, 141–145, 151, 157
 partitioning oracle attack, **144**, 145, 151

 paying key, **13**, 46
 payment address, 111, 115, 137, 149, 155, 158
 Pedersen commitment, 19, 20, 52–54, 77, 79, **94**, 142, 172–174, 204, 207
 Pedersen hash, 32, **77**, 79, 94, 141, 143, 171–174, 205, 207, 208
 Pedersen value commitment, **19**, 139
 peer-to-peer protocol, 17, 43–45, 118, 121, 128, 138, 171, 174
 piece (of a Sinsemilla hash input), **80**
 PLONK, 146
 point at infinity, **103**
 positioned note, **15**, 56, 67, 141
 prescribed way (to pay a funding stream recipient), **137**
 prevout (previous output), 122, 150
 primary input, **33**, 39–42, 48, 56, 58–62, 154, 155, 168
 prime order (of a group element), 69, 148, 171, 205
 prime-order curve, 100, 108, 143
 prime-order group, 76, 81
 prime-order subgroup, 51, 76, 88, 113, 148, 160, 203
 private key, 8, 9, 12, **25**, 26, **27**, 31, 37, 38, 53, 65, 66, 69, 86, 112, 143, 155, 156
 proof authorizing key, 8, **12**, 35, 56, 84
 proof batch entry, **217**
 proof-of-work, 1, 130, 146
 proving key (for a zk-SNARK), **33**, 34, 117
 proving system (preprocessing zk-SNARK), 1, 7–9, 19, 20, **33**, 34, 108–110, 139, 146, 166, 167, 169, 178
 Pseudo Random Function, 16, 22, **24**, **25**, 35, 36, 73, 84, 85, 94, 142, 143, 145, 174
 Pseudo Random Permutation, **25**, **77**, 86
 public key, **13**, 25, 39, 41, 42, 64, 66, 76, 86, 112, 113, 115, 125–127, 143, 144

 Quadratic Arithmetic Program, **108**, 109, **195**
 quadratic constraint program, 7, 108, 109, 175, **195**, 197, 200, 201

 random oracle, 32, 33, 76, **77**, 81, 84, 103, 105, 108, 161, 170
 randomized Spend validating key, 172
 randomizer, **28**, 29, 55
 Rank 1 Constraint System, **195**
 raw encoding, 64, 66, **111**, 112–117, 160
 re-randomized, **28**, 29, 55, 90, 93

receiving key, 9, **12**, 175

RECOMMENDED, 7, 14, 36, 92, 111

represented group, **31**, 32, 90, 100, 103, 159, 171

represented pairing, **33**, 97, 99, 171

represented subgroup, **31**, 32, 33

root (of a Merkle tree), **20**, 48, 120, 125, 126, 128, 129, 153

RPC byte order, **22**, 117

Sapling, 1, 7–9, 12–21, 23–27, 30, 32, 34–36, 39, 43–49, 51, 53, 55, 56, 63, 65–71, 74–77, 84–86, 93, **95**, 100, 103, 108–111, 113, 114, 117, **118**, 119–126, 128–130, 136, 137, 139, 141–149, 152, 153, 155, 156, 158–175, 195, 197–200, 203–205, 207–212, 214

Sapling balancing value, **51**, 170

secp256k1, 27

segment (of a Pedersen hash input), **78**

serial numbers (in Zerocash), **8**

settled, **17**, 109, 150

SHA-256, **73**, 74, 84, 93, 94, 117, 141, 142, 151, 173

SHA-256d, **73**, 121, 128, 130, 131

SHA-512, **73**, 89, 161, 218

SHA256Compress, **73**, 74, 84, 94, 112, 141–143, 145, 173, 176

shielded, **8**, 18, 42–44, 69, 124, 139, 140

shielded input, **8**, 18, **19**, 46, 47

shielded output, **8**, **19**, 20, 63, 65, 123, 124, 155, 161

shielded payment address, 8, 9, **12**, 13–14, 23, 24, 44–47, 55, 65, 70, 75–77, 86, 111, **112**, **113**, **115**, 124, 136, 143, 156, 158, 160, 176, 178

shielded transfer, **8**

short Weierstrass affine coordinates, 38, 42, 48, 62, 66, 83, 97, 107, 108, 151

short Weierstrass compressed encoding, 115

short Weierstrass elliptic curve, **103**, 105, 107, 157

SHOULD, 7, 17, 43–45, 52, 54, 77, 108, 123, 129

SHOULD NOT, 7, 38, 165

side-channel, 23, 143, 144

SIGHASH algorithm, **49**, 50, 155

SIGHASH transaction hash, 40, 42, **49**, 52–55, 74, 119–121, 138, 139, 149, 156, 163, 169, 177

SIGHASH type, **49**, 50, 52, 54, 55, 151, 169, 179

signature batch entry, **215**, 216, **218**

signature scheme, 22, **26**, 27–29, 74, 88, 90, 93, 172

signature scheme with key monomorphism, **29**, 93, 163

signature scheme with re-randomizable keys, **28**, 35, 37, 55, 93

signing key, **26**, 27–29, 50, 52, 54, 170, 172

Sinsemilla commitment, **96**, 142

Sinsemilla hash, 96, 143

slanted text, **7**

small order (of a group element), 40, 41, 59, 60, 69, 148, 169, 171, 173, 203, 213, 215

spend authorization address key, **55**, 63

spend authorization private key, **55**

spend authorization randomizer, **56**, 167

spend authorization signature, **39**, 40, **41**, 42, 49, 51, 53, **55**, 56, 63, 124, 125, 127, 144, 167, 172

spend authorization signature scheme, 55, 90, **93**

Spend authorizing key, **12**, 35, 37, 84

Spend circuit, 48, 117, 148, 155, 172, 204, 205, 211

Spend description, 8, 16, **19**, 20–21, 27, 30, **39**, 40, 46, 51, 52, 55, 56, 70, 96, 109, 119, 120, 122–126, 148, 169, 173, 174

Spend proof, 48

Spend statement, 19, 23, 34, 40, 46, 52, 55–57, **59**, 60, 75, 109, 125, 154, 169, 171

Spend transfer, 17, **19**, 20–21, 27, 39, 49, 51, 53, 125, 139, 141

Spend validating key, 37, 116

spending authority, 9, 13, 36, 38, 56, 77

spending key, 8, 9, **12**, 13, 16, 24, 35–37, 46, 55, 56, 64, 70, 111, 112, **113**, **114**, **116**, 117, 140, 141, 145, 150, 154, 160, 173, 176, 178

Sprout, 7–9, 12–19, 21, 24, 26, 30, 34, 35, 42, 43, 46, 47, 49–51, 56, 63–65, 70, 71, 74, 109–113, 115, 117, **118**, 124, 125, 140–146, 148, 149, 152, 157, 158, 160–164, 166, 168–170, 172, 173

standard P2SH script, 135, **137**

standard redeem script, 135, **137**

statement, 19, 20, **33**, 55, 58, 145, 173, 195, 212, 214, 217

synthetic blinding factor, **53**

target threshold, 128, 131, 132, **133**

TAZ, 22

Testnet, 17, 22, 69, 72, 73, 111–114, 117, 122, 123, 129, 130, 133, 135, 137, 138, 149, 160, 161, 163, 164, 166, 175, 177

total input value, 122

total issued supply, **57**, 147

total output value, 122

transaction, 8, 9, 15–21, 27, 38–45, 49–56, 64, 66–71, 109, **119**, **120**, 121–127, 129, 139–141, 150, 153, 154, 156–159, 165, 169, 174, 179

transaction binding validating key, **51**, **54**, 122

- transaction fee, **21**, 122, 160
- transaction ID, **17**, 121, 124, 153
- transaction value pool (Orchard), **53**
- transaction value pool (Sapling), **51**
- transaction value pool (transparent), **17**, 18, 39, 50, 51, 53, 125, 152
- transaction version number, 49, 109, 119, 120, **121**, 123–126, 160, 175
- transactions, 9, 13, **17**, 18–19, 21, 27, 28, 38–41, 43–45, 49, 50, 55–57, 64, 67, 69, **74**, 110, 118, 122–124, 126, 128, 129, 141, 149–153, 155–157, 160, 168, 169
- transmission key, 9, **13**, 14, 26, 43, 63, 64, 69, 160
- transmitted note ciphertext, 66–68, 144
- transmitted note ciphertext (Orchard), 42, 45, 68, 127
- transmitted note ciphertext (Sapling), 41, 44, **66**, 67–71, 127
- transmitted notes ciphertext (Sprout), 39, 43, 63, **64**, 70, 125
- transparent, **8**, 9, 17, 19, 20, 50, 111, 119, 120, 122, 124, 139, 159, 165, 177
- transparent address, **111**, 134, 177
- transparent address (P2PKH), **111**, 115, 169, 177
- transparent address (P2SH multisig), 135, 137
- transparent address (P2SH), **111**, 115, 136, 137, 177
- transparent input, **17**, 18, 21, 49, 50, 119–122, 124, 150, 155, 156, 162
- transparent output, **17**, 18, 50, 119–122, 124, 134, 159, 162
- treestate, 15, 16, **17**, 18–21, 39, 40, 42, 128, 129, 169, 176, 179
- unified full viewing key, **115**, 116, 155
- unified incoming viewing key, **115**, 116, 155
- unified payment address, **111**, **115**, 149, 155, 156, 158
- Uniform Random String, 32, **33**, 170
- unpacking, **197**
- US-ASCII, 10, 117
- UTXO (unspent transaction output), 57, 122, 139, 150
- UTXO (unspent transaction output) set, 17, **20**, 57
- valid block chain, **16**, 17, 21, 124, 140
- valid Equihash solution, 129, 130, **131**
- validating key (for a signature scheme), **26**, 27–29, 38, 40, 42, 50, 53, 55, 89, 92, 93, 101, 111, 119, 121, 126, 127, 162, 170–172, 174, 215, 218
- value commitment, 8, 9, **19**, **20**, 27, 40–42, 45, 51–55, 65, 126, 127
- value commitment scheme, 52, 54
- verifying key (for a zk-SNARK), **33**, 34, 117
- version group ID, 121, **123**, 160
- Vesta, 32, 34, **103**, 104–105, 108, 145, 157
- weak PRF, **144**
- windowed, **94**
- windowed Pedersen commitment, **208**
- wtxid, **17**, 121, 153
- zatoshi, **13**, 14, 22, 51, 53, 72, 73, 122, 133, 135, 137
- Zcash**, **1**, 7–9, 12, 17, 18, 21, 22, 27, 33, 34, 49, 50, 71, 73, 74, 92, 100, 103, 106, 108–113, 117–120, 123, 124, 128, 130, 131, 137–148, 160, 161, 169, 176, 177, 179
- zcashd, 12, 17, 36, 69, 90, 131, 133, 142, 148, 150, 151, 159, 160, 162, 164, 165, 174, 175
- zebra, 17, 69
- ZEC**, **13**, 21, 22
- Zerocash**, **1**, **7**, 8, 23, 25, 50, 139–143, 145, 146, 148, 158, 175, 177–179
- zk-SNARK circuit, 77, 100, 103, 117
- zk-SNARK proof, 15, 16, 18–20, 27, **33**, 34, 39–42, 55, 56, 109, 120, 125–127, 141, 144, 147, 150, 151, 168, 169, 176