(I) $g(x) = e^x(\cos x - \sin x) = -\sqrt{2}e^x \sin(x - \frac{\pi}{4})$

故 $f(x)$ 的递增区间为 $[2k\pi + \frac{\pi}{4}, 2k\pi + \frac{5\pi}{4}]$

故 $f(x)$ 的递减区间为 $[2k\pi - \frac{3\pi}{4}, 2k\pi + \frac{\pi}{4}]$

(II) 记 $h(x) = f(x) + g(x)(\frac{\pi}{2} - x)$

则 $h'(x) = g(x) + g'(x)(\frac{\pi}{2} - x) - g(x)$

$= -2e^x \sin x(\frac{\pi}{2} - x)$

$x \in [\frac{\pi}{4}, \frac{\pi}{2}]$ 时,$h'(x) \leqslant 0$

故 $h(x)$ 单调递减, 且 $h(\frac{\pi}{2}) = 0$

故此时 $h(x) \geqslant 0$

(III) x_n 为 $f(x) - 1 = 0$ 的零点，即 $e^{x_n} \cos x_n = 1$

$x \in (2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2})$ 则 $x - 2n\pi \in (\frac{\pi}{4}, \frac{\pi}{2})$

x_n 越大 e_n^x 也越大，则 $\cos x_n$ 越小, 则 $x_n - 2n\pi$ 越大

$f(x_n - 2n\pi) = e^{x_n - 2n\pi} \cos(x_n - 2n\pi) = e^{-2n\pi}$

由 II 得 $f(x - 2n\pi) + g(x_n - 2n\pi)(\frac{\pi}{2} - x + 2n\pi) \geqslant 0$

考察 $g(x), g'(x) = -2e^x \sin x, x \in (\frac{\pi}{4}, \frac{\pi}{2})$ 时, $g'(x) < 0$

故 $g(x)$ 单调递减, $n > 1$ 时,$g(x_n - 2n\pi) < g(x_0) < 0$

$$
\begin{aligned}
\text{故 } \frac{\pi}{2} - x + 2n\pi &\leqslant \frac{-e^{-2n\pi}}{g(x_n - 2n\pi)} \\
&< \frac{e^{-2n\pi}}{-g(x_0)} \\
&= \frac{e^{-2n\pi}}{e^{x_0}(sinx_0 - cosx_0)} \\
&< \frac{e^{-2n\pi}}{\sin x_0 - \cos x_0}
\end{aligned}
$$