

解: 设 $M(\frac{m^2}{2p}, m), N(\frac{n^2}{2p}, n)$

(1) $MD \perp x$ 轴, $\frac{m^2}{2p} = p$ 由抛物线性质, $|MF| = p + \frac{p}{2} = 3$

故 $p = 2, C: y^2 = 4x$ 焦点 $F(1, 0)$

(2) 直线 MN 的斜率不存在时, AB 的斜率也不存在, 此时 $\alpha - \beta = 0$

故由题意设 $MN: y = k(x - 1)$, 且 $k \neq 0$

$$\begin{cases} y = k(x - 1), \\ y^2 = 4x \end{cases}$$

消去 x 得 $\frac{y^2}{4} - \frac{y}{k} - 1 = 0$

故 $mn = -4, m + n = \frac{4}{k}$

$$K_{MD} = \frac{m}{\frac{m^2}{4} - 2}$$

则 $MD: y = \frac{1}{\frac{m}{4} - \frac{2}{m}}(x - 2)$

$$\begin{cases} y = \frac{1}{\frac{m}{4} - \frac{2}{m}}(x - 2), \\ y^2 = 4x \end{cases}$$

消去 x 得: $y^2 - (m - \frac{8}{m})y - 8 = 0$

$y_1 y_2 = -8$, 故 $A(\frac{16}{m^2}, -\frac{8}{m})$

同理 $B(\frac{16}{n^2}, -\frac{8}{n})$

故 $k_{AB} = \frac{\frac{8}{n} - \frac{8}{m}}{16(\frac{1}{n^2} - \frac{1}{m^2})} = \frac{-1}{2(\frac{1}{m} + \frac{1}{n})} = \frac{-mn}{2(m+n)} = \frac{k}{2}$

$$\begin{aligned} l_{AB} &= \frac{-mn}{2(m+n)}(x - \frac{16}{m^2}) - \frac{8}{m} \\ &= \frac{-mn}{2(m+n)}x + \frac{8n}{m(m+n)} - \frac{8}{m} \\ &= \frac{-mnx - 16}{2(m+n)} \\ &= \frac{k}{2}(x - 4) \end{aligned}$$

$y = 0$ 时 $-\frac{8}{m} \cdot \frac{2(m+n)}{mn} = x - \frac{16}{m^2} \Rightarrow x = \frac{-16}{mn} = 4$

故 α, β 同是第一象限角或第二象限角, 则 $\alpha - \beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

此时 $\tan x$ 单调递增

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{k - \frac{k}{2}}{1 + \frac{k^2}{2}} = \frac{1}{\frac{2}{k} + k}$$

显然 $k < 0$ 时, $\tan(\alpha - \beta) < 0, \alpha - \beta < 0$

$k > 0$ 时, $\tan(\alpha - \beta) = \frac{1}{\frac{2}{k} + k} \leq \frac{1}{2\sqrt{2}}$

$k = \sqrt{2}$ 时取等号, 此时 $\tan(\alpha - \beta)$ 取得最大值, $\alpha - \beta$ 也取得最大值

此时 m 满足 $m^2 - 2\sqrt{2}m - 4 = 0$, 取 $m = \sqrt{2} + \sqrt{6}$

则 $A(8 - 4\sqrt{3}, 2\sqrt{2} - 2\sqrt{6})$

故 $k_{AB} = \frac{\sqrt{2}}{2}$, $AB: y = \frac{\sqrt{2}}{2}(x - 8 + 4\sqrt{3}) + 2\sqrt{2} - 2\sqrt{6} = \frac{\sqrt{2}}{2}x - 2\sqrt{2}$