解:

(I) $f'(x) = -2\alpha\sin 2x + (\alpha-1)(-\sin x) = -4\alpha\sin x\cos x - \alpha\sin x + \sin x$;

(II) $(|f(x)| = |\alpha\cos 2x + (\alpha-1)(\cos x+1)| \leqslant a + 2(a-1) = 3a-2)$

$f(x) = 2\alpha cos^2(x) - \alpha + \alpha cosx + \alpha - cosx - 1$

$$= 2\alpha(\cos x + \frac{\alpha-1}{4\alpha})^2 - \frac{\alpha^2-2\alpha+1}{8a} - 1$$

$$= 2\alpha(\cos x + \frac{\alpha-1}{4\alpha})^2 - \frac{\alpha^2+6\alpha+1}{8a}$$

记 $t = \cos x$, 则 $t \in [-1,1], f(x) = g(t) = 2\alpha(t - \frac{1-\alpha}{4\alpha})^2 - \frac{\alpha^2+6\alpha+1}{8a}$

可知 $g(t)$ 为关于 t 的二次函数, 开口向上, 极小值 $A_1 = -\frac{\alpha^2+6\alpha+1}{8a}$

由 $t = \cos x \in [-1,1]$, 若 $\frac{1-\alpha}{4\alpha} \in [-1,1]$, 即 $\alpha \geqslant \frac{1}{5}$ 时: 极值在区间内

$\cos x = 1$ 时:$g(1) = 3\alpha - 2, \cos x = -1$ 时:$g(-1) = \alpha$

则:

(i) $0 < \alpha \leqslant \frac{1}{5}$ 时,:

$|g(t)|$ 的最大值在端点取得

又 $|g(-1)| - |g(1)| = \alpha - 2 + 3\alpha = 4\alpha - 2 < 0$

故此时 $A = |g(1)| = |3\alpha - 2| = 2 - 3\alpha$

(ii) $\alpha > \frac{1}{5}$ 时:

A 为 $|A_1|, |g(1)|, |g(-1)|$ 中的最大值

由 $a > 0$, 故 $|A_1| - |g(-1)| = \frac{\alpha^2+6\alpha+1}{8\alpha} - \alpha$

$$= \frac{-7\alpha^2+6\alpha+1}{8\alpha}$$

$$= \frac{(7\alpha+1)(1-\alpha)}{8\alpha}$$

(1) $a \geqslant 1$ 时:

$|g(-1)| \geqslant |A_1|$

$|g(1)| - |g(-1)| = 3\alpha - 2 - \alpha = 2\alpha - 2 \geqslant 0$

故此时 $A = 3\alpha - 2$

(2) $a < 1$ 时:

$|g(-1)| < |A_1|$

$$
\begin{aligned}
|A_1| - |g(1)| &= \frac{\alpha^2+6\alpha+1}{8a} - 2 + 3\alpha \\
&= \frac{25\alpha^2-10\alpha+1}{8\alpha} \\
&= \frac{(5\alpha-1)^2}{8\alpha} \geqslant 0
\end{aligned}
$$

故此时 $A = |A_1| = \frac{\alpha^2+6\alpha+1}{8\alpha}$

$$
综上,A = \begin{cases} 2-3\alpha & ,\alpha \leqslant \frac{1}{5} \\ \frac{\alpha^2+6\alpha+1}{8\alpha} & ,\frac{1}{5} < \alpha < 1 \\ 3\alpha-2 & ,\alpha \geqslant 1 \end{cases}
$$

(III) 证明:

$|f'(x)| = |-2\alpha\sin 2x - (\alpha-1)\sin x| < 2\alpha + |\alpha-1|$

(a) $\alpha \geqslant 1$ 时:

$|f'(x)| - 2A < 3\alpha - 1 - 6\alpha + 4 = 3 - 3\alpha \leqslant 0$

(b) $\frac{1}{5} < \alpha < 1$ 时:

$$
\begin{aligned}
|f'(x)| - 2A &< \alpha + 1 - \frac{\alpha^2+6\alpha+1}{4\alpha} \\
&= \frac{3\alpha^2-2\alpha-1}{4\alpha} \\
&= \frac{(3\alpha+1)(\alpha-1)}{4\alpha} < 0
\end{aligned}
$$

(c) $0 < \alpha \leqslant \frac{1}{5}$ 时:

$|f'(x)| - 2A < \alpha + 1 - 4 + 6\alpha = 7\alpha - 3 < 0$

综上，$|f'(x)| \leqslant 2A$