

$f(x), g(x)$ 均为增函数, $\phi(x) = f(x)g(x)$

记 $\frac{f(x_1)-f(x_2)}{x_1-x_2} = \Delta(f)$

$$\begin{aligned}
 \Delta(\phi) &= \frac{\phi(x_1) - \phi(x_2)}{x_1 - x_2} \\
 &= \frac{f(x_1)g(x_1) - f(x_2)g(x_2)}{x_1 - x_2} \\
 &= \frac{f(x_1)g(x_1) - f(x_1)g(x_2) + f(x_1)g(x_2) - f(x_2)g(x_2)}{x_1 - x_2} \\
 &= \frac{f(x_1)[g(x_1) - g(x_2)] + g(x_2)[f(x_1) - f(x_2)]}{x_1 - x_2} \\
 &= f(x_1)\Delta(g) + g(x_2)\Delta(f)
 \end{aligned}$$