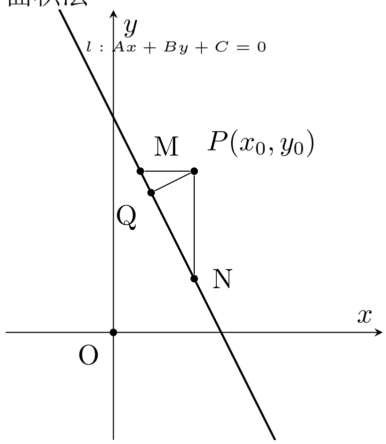
点到直线距离公式推导（AB 均不为 0 ）：

法一：　面积法



作 $PM \perp y$ 轴交 l 于 M,
作 $PN \perp x$ 轴交 l 于 N.
则:$M(x_M, y_0), N(x_0, y_N)$ 满足直线方程

$$\begin{cases} Ax_M + By_0 + C = 0 \\ Ax_0 + By_N + C = 0 \end{cases} \tag{1}$$

$x_M = \frac{-By_0 - C}{A}$,$y_N = \frac{-Ax_0 - C}{B}$,
$PM = \frac{Ax_0 + By_0 + C}{B}, PN = \frac{Ax_0 + By_0 + C}{A}$,
$MN = (Ax_0 + By_0 + C)\sqrt{(\frac{1}{A})^2 + (\frac{1}{B})^2}$.
故关于点到直线距离 PQ, 有:
$S_{\triangle PMN} = PM \times PN = MN \times PQ$

$$\begin{aligned} \Rightarrow PQ &= \frac{PM \times PN}{MN} \\ &= |\frac{(Ax_0 + By_0 + C)(Ax_0 + By_0 + C)}{AB(Ax_0 + By_0 + C)\sqrt{(\frac{1}{A})^2 + (\frac{1}{B})^2}}| \\ &= \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

法二：　交点法

直线 $PQ \perp l$, 故设 $PQ: Bx - Ay + D = 0$.
又 $P \in PQ \Rightarrow D = -Bx_0 + Ay_0$. 故垂足 $Q(x, y)$ 满足:

$$\begin{cases} Ax + By + C = 0 \\ Bx - Ay - Bx_0 + Ay_0 = 0 \end{cases}$$

$①\times B - ②\times A:$

$ABx + B^2y + BC - ABx + A^2y + ABx_0 - A^2y_0 = 0$

$(A^2 + B^2)y = A^2y_0 - ABx_0 - BC$

$\Rightarrow y = \dfrac{A^2y_0 - ABx_0 - BC}{A^2 + B^2}$

$①\times A + ②\times B:$

$A^2x + ABy + AC + B^2x - ABy - B^2x_0 + ABy_0 = 0$

$(A^2 + B^2)y = B^2x_0 - ABy_0 - AC$

$\Rightarrow x = \dfrac{B^2x_0 - ABy_0 - AC}{A^2 + B^2}$

$$\begin{aligned} \text{则: } PQ &= \sqrt{(x - x_0)^2 + (y - y_0)^2} \\ &= \sqrt{(\frac{A^2x_0 + ABy_0 + AC}{A^2 + B^2})^2 + (\frac{ABx_0 + B^2y_0 + BC}{A^2 + B^2})^2} \\ &= \frac{|Ax_0 + By_0 + C|}{A^2 + B^2} \times \sqrt{A^2 + B^2} \\ &= \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

法三： 向量法

l 的一个方向向量为 $(-B, A)$, 一个法向量 $\vec{n}$ 为 (A, B)

$Q(x_1, y_1)$ 为 l 上的任意一点

则有 $Ax_1 + By_1 + C = 0$

由向量内积公式,

$$
\begin{aligned}
d =& |\overrightarrow{QP}||\cos<\overrightarrow{QP},\vec{n}>| \\
=& \frac{|\overrightarrow{QP}\cdot\vec{n}|}{|\vec{n}|} \\
=& \frac{|(x_0-x_1,y_0-y_1)(A,B)|}{\sqrt{A^2+B^2}} \\
=& \frac{|(Ax_0-Ax_1+By_{-}By_1)(A,B)|}{\sqrt{A^2+B^2}} \\
=& \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}
\end{aligned}
$$

法四： 三角函数法

记 l 的倾角为 α

$M(x_M,y_0)$ ，$RQ\perp y$ 轴，与 l 交于 M

则: $x_M=\frac{n-By_0-C}{A}$

$$|x_M-x_0|=|\frac{Ax_0+BY_0+C}{A}|$$

又有:$\frac{1}{sin^2\alpha}=tan^2\alpha+1$

$$sin\alpha=\sqrt{\frac{1}{1+tan^2\alpha}}$$

$$sin\alpha=\sqrt{\frac{1}{1+(-\frac{B}{A})^2}}$$

$$sin\alpha=\frac{A}{\sqrt{A^2+B^2}}$$

故:$d=|x_M-x_0||\sin\alpha|$

$$d=|\frac{A(Ax_0+BY_0+C)}{A\sqrt{A^2+B^2}}|$$

$$d=\frac{|Ax_0+BY_0+C|}{\sqrt{A^2+B^2}}$$