

(I) 证明:

设直线 $AB: x = ky + \frac{1}{2}, l_1: y = y_1, l_2: y = y_2$

则: $A: (\frac{y_1^2}{2}, y_1), B: (\frac{y_2^2}{2}, y_2), R: (-\frac{1}{2}, \frac{y_1+y_2}{2}), F: (\frac{1}{2}, 0), P: (-\frac{1}{2}, y_1), Q: (-\frac{1}{2}, y_2)$

则: $k_{RA} = \frac{y_1 - \frac{y_1+y_2}{2}}{\frac{y_1^2}{2} - (-\frac{1}{2})} = \frac{y_1 - y_2}{y_1^2 + 1}, k_{QF} = \frac{0 - y_2}{\frac{1}{2} - (-\frac{1}{2})} = -y_2$

且 y_1, y_2 满足:

$$\begin{cases} C: y^2 = 2x \\ AB: x = ky + \frac{1}{2} \end{cases} \quad (1)$$

故有: $y^2 - 2ky - 1 = 0$

则: $y_1 y_2 = -1 \Rightarrow y_2 = -\frac{1}{y_1}$

故: $k_{RA} = \frac{y_1 + \frac{1}{y_1}}{y_1^2 + 1} = \frac{y_1^2 + 1}{y_1(y_1^2 + y_1)} = \frac{1}{y_1} = -y_2 = k_{QF}$

证毕

(II) 解:

易知 $S_{\triangle PQF} = \frac{1}{2}(y_1 - y_2)[\frac{1}{2} - (-\frac{1}{2})] = \frac{y_1 - y_2}{2}$

记 AB 与 x 轴交于 D 点, 设 $AB: x = ky + b$, 则 $D: (b, 0)$

故 $S_{\triangle ABF} = \frac{1}{2}|b - \frac{1}{2}||y_1 - y_2|$

由 $S_{\triangle PQF} = 2S_{\triangle ABF} \Rightarrow \frac{y_1 - y_2}{2} = |b - \frac{1}{2}||y_1 - y_2|$ 则: $|b - \frac{1}{2}| = \frac{1}{2}$

故 $b = 1$ 或 $b = 0$ (舍去)

且 y_1, y_2 满足:

$$\begin{cases} C: y^2 = 2x \\ AB: x = ky + 1 \end{cases} \quad (3)$$

故有 $y^2 - 2ky - 2 = 0$

则: $y_1 y_2 = -2, y_1 + y_2 = 2k$

则: $\frac{y_1^2}{2} + \frac{y_2^2}{2} = \frac{1}{2}[(y_1 + y_2)^2 - 2y_1 y_2] = 2k^2 + 2$

故 AB 中点的轨迹方程为 $x = k^2 + 1 = y^2 + 1$, 即 $y^2 = x - 1$