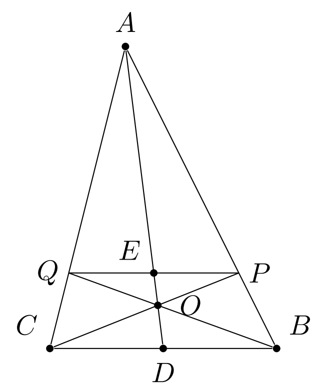
梯形 $PQCB$ 中, $PQ \mathbin{//} BC$, 延长 CQ, BP 相交与 A. 连接 PC, QB 交于 O, 连接 AO 并延长, 交 BC 于 D.

求证 D 为 BC 中点.

解:



易知 $\triangle APQ \sim \triangle ABC$

故设 $\frac{\overrightarrow{AQ}}{\overrightarrow{AC}} = \frac{\overrightarrow{AP}}{\overrightarrow{AB}} = \frac{\overrightarrow{PQ}}{\overrightarrow{BC}} = \lambda$

则 $\overrightarrow{QC} = (1-n)\overrightarrow{AC}$,

同时 $\triangle OPQ \sim \triangle OCB, \triangle OEP \sim \triangle ODC$

则 $\frac{\overrightarrow{OE}}{\overrightarrow{OD}} = \frac{\overrightarrow{OP}}{\overrightarrow{OC}} = \frac{\overrightarrow{QP}}{\overrightarrow{BC}} = \lambda$

$\overrightarrow{PO} = \lambda\overrightarrow{OC}$

$\overrightarrow{PC} = (1+\lambda)\overrightarrow{OC}$

$$\begin{aligned}\overrightarrow{OC} &= \frac{\overrightarrow{PB} + \overrightarrow{BC}}{1+\lambda} \\ &= \frac{(1-\lambda)\overrightarrow{AB} + \overrightarrow{AC} - \overrightarrow{AB}}{1+\lambda} \\ &= \frac{\overrightarrow{AC} - \lambda\overrightarrow{AB}}{1+\lambda}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AO} &= \overrightarrow{AC} - \overrightarrow{OC} \\ &= \frac{\lambda\overrightarrow{AC} + \lambda\overrightarrow{AB}}{1+\lambda} \\ &= \frac{\lambda}{1+\lambda}(\overrightarrow{AB} + \overrightarrow{AC})\end{aligned}$$

故有 μ, ν 使得 $\overrightarrow{AD} = (\frac{\mu\lambda}{1+\lambda})(\overrightarrow{AB} + \overrightarrow{AC})$

$$= \nu\overrightarrow{AB} + (1-\nu)\overrightarrow{AC}$$

解得 $\nu = \frac{1}{2}$, 故 D 为 BC 中点.