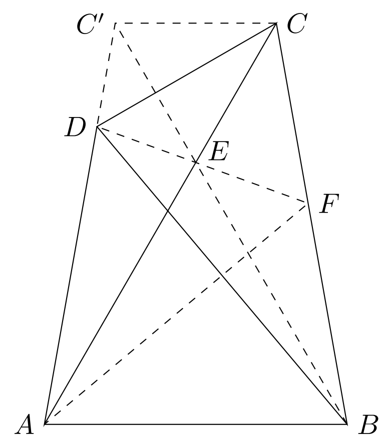
四边形 $ABCD$ 中, $\angle ABC = 50°, \angle DBC = 30°, \angle BAC = 60°, \angle DAC = 20°$. 求 $\angle ACD$

解:



过 C 作 $CC' \parallel AB$ 并交 AD 的延长线于 C' 连接 $C'B$

交 AC 于 E', 连接 DE 并延长, 交 BC 于 F, 连接 AF.

$\angle BAC' = 60° + 20° = 80°$

$\angle ABC = 50° + 30° = 80° = \angle BAC'$

又 $CC' \parallel AB$ 故 $ABCC'$ 为等腰梯形

据对称性, $\angle ABC' = \angle BAC = 60°$

故 $\triangle ECC', \triangle ABE$ 为正三角形, 则 $AE = AB$

$\triangle ABD$ 中:

$\angle ADB = 180° - 50° - 80° = 50° = \angle ABD$

故 $AD = AB = AE$

在 $\triangle ADE$ 中, $\angle ADE = \angle AED = 80°$

则 $\angle BDF = 30° = \angle DBF, \angle CDC' = 100°$

故 $\triangle BDF$ 为等腰三角形, $DF = BF$

又 $AF = AF, AD = AE$ 则 $\triangle AFD \cong \triangle AFB$

故 $\angle BAF = \frac{1}{2}\angle DAB = 40°$

$\triangle ABF$ 中 $\angle BFA = 180° - 40° - 80° = 60°$

$\triangle BEF$ 中 $\angle BEF = 180° - \angle BFE - \angle EBF = 40°$

而 $\angle AC'B = 180° - \angle C'AB - \angle C'BA = 40° = \angle BEF = \angle C'EF$

故 $\triangle C'DE$ 为等腰三角形, $C'D = DE$

又 $\triangle CC'E$ 为正三角形, $CC' = CE$

又 $CD = CD$, 故 $\triangle CC'D \cong \triangle CED$

而 $\angle C'CE = 180° - \angle CBA = 100°$

$\angle ACB = \angle AC'B = 40°$

故 $\angle ACD = \frac{1}{2}\angle C'CA = \frac{1}{2}(100° - 40°) = 30°$