

$$A(x_1, y_1), B(x_2, y_2), O(0, 0)$$

$$\text{则 } |\vec{OA}|^2 = x_1^2 + y_1^2$$

$$|\vec{OB}|^2 = x_2^2 + y_2^2$$

$$|\vec{AB}|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\begin{aligned} \cos < \vec{A}, \vec{B} > &= \frac{|\vec{OA}|^2 + |\vec{OB}|^2 - |\vec{AB}|^2}{2\sqrt{|\vec{OA}||\vec{OB}|}} \\ &= \frac{x_1^2 + y_1^2 + x_2^2 + y_2^2 - x_1^2 - 2x_1x_2 - x_2^2 - y_1^2 + 2y_1y_2 - y_2^2}{2\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}} \\ &= \frac{x_1x_2 + y_1y_2}{\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}} \end{aligned}$$

则:

$$|A||B| \cos < \vec{A}, \vec{B} > = \frac{x_1x_2 + y_1y_2}{\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}} \times \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)} = x_1x_2 + y_1y_2$$