解:

(1) 由 $\rho^2=x^2+y^2,\rho\cos\theta=x$

故$C:\rho^2=2\sqrt{2}\cos\theta$

$$x^2+y^2=2\sqrt{2}x$$

即:$(x-\sqrt{2})^2+y^2=2$

(2)

设 $P(x,y),M(2\sqrt{2}\cos^2\theta,2\sqrt{2}\sin\theta\cos\theta)$

由题意: $\overrightarrow{AM}=(\sqrt{2}\cos 2\theta+\sqrt{2}-1,\sqrt{2}\sin 2\theta)$

则 $\overrightarrow{AP}=(x-1,y)=\sqrt{2}\overrightarrow{AM}=(2\cos 2\theta+2-\sqrt{2},2\sin 2\theta)$

故: $C_1:\begin{cases}x=2\cos 2\theta-\sqrt{2}+3\\ y=2\sin 2\theta\end{cases}$($\theta$ 为参数)

由（1）得: C 为圆，圆心在 $(\sqrt{2},0)$, 半径为 $\sqrt{2}$

$C_1:(x-3+\sqrt{2})^2+y^2=4$ 也是圆

故 C_1 圆心在 $(3-\sqrt{2},0)$, 半径为 2

两圆圆心距 $d=|\sqrt{2}-3+\sqrt{2}|=3-2\sqrt{2}<r_1-r_C=2-\sqrt{2}$

故 C 内含于 C_1