

(1) 解:

$$a_2 = 3a_1 - 4 = 5, a_3 = 3a_2 - 8 = 7$$

故猜想 $a_n = 2n + 1$

证明:(一般猜想后证明只需证明其正确性, 既满足所给递推式, 但保险起见, 还是从递推式推导出通项, 既是证明, 又是推导)

$$a_{n+1} = 3a_n - 4n$$

$$a_{n+1} - 2(n+1) + 2 = 3a_n - 6n$$

$$a_{n+1} - 2(n+1) - 1 = 3a_n - 6n + 3 = 3(a_n - 2n - 1)$$

故 $\{a_n - 2n - 1\}$ 是公比为 3, 首项为 $a_1 - 2 - 1 = 0$ 的等比数列, 即全为 0 的数列

$$\text{故 } a_n - 2n - 1 = 0 \Rightarrow a_n = 2n + 1$$

(2) 解:

$$2^n a_n = 2^n 2n + 2^n$$

$$S_n = \sum_{i=1}^n [2^i (2i + 1)]$$

$$2S_n = \sum_{i=1}^n [2^{i+1} (2i + 1)]$$

$$= \sum_{i=1}^n [2^{i+1} (2i + 3)] - 2 \sum_{i=1}^n 2^{i+1}$$

$$= S_{n+1} - 2 \times (2 + 1) - 2(2^{n+2} - 4)$$

$$= S_n + 2^{n+1} (2n + 3) - 2^{n+3} - 2$$

$$S_n = 2^{n+1} 2n + 3 \times 2^{n+1} - 4 \times 2^{n+1} - 2$$

$$= 2^{n+1} 2n - 2^{n+1} - 2$$