

$$\begin{aligned}
& \text{引理: } C_m^n = C_{m-1}^{n-1} + C_{m-1}^n \\
& \sum_{j=i}^n C_j^i = C_i^i + C_{i+1}^i + \cdots + C_n^i \\
& \quad = C_{i+1}^{i+1} + C_{i+1}^i + \cdots + C_n^i \\
& \quad = C_{i+1}^{i+2} + C_{i+2}^i + \cdots + C_n^i \\
& \quad \dots \\
& \quad = C_{n+1}^{i+1} \\
P/A &= \sum_{i=1}^n \frac{1}{(1+r)^i} \\
&= \frac{\sum_{i=1}^n (1+r)^{i-1}}{(1+r)^n} \\
&= \frac{\sum_{i=1}^n \sum_{j=0}^{i-1} C_i^j r^j}{(1+r)^n} \\
&= \frac{\sum_{i=1}^{n-1} \sum_{j=i}^{n-1} C_j^i r^i}{(1+r)^n} \\
&= \frac{\sum_{i=0}^{n-1} C_n^{i+1} r^i}{(1+r)^n} \\
&= \frac{\sum_{i=0}^{n-1} C_n^{i+1} r^{i+1}}{r(1+r)^n} \\
&= \frac{\sum_{i=1}^n C_n^i r^i}{r(1+r)^n} \\
&= \frac{(1+r)^n - 1}{r(1+r)^n}
\end{aligned}$$

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