

(I) $\frac{p}{2} = 1 \Rightarrow p = 2$

故准线 $x = -1$, 抛物线方程 $y^2 = 4x$

(II) 据对称性可设 A 在 x 轴上方, 但不能设直线 AB 的斜率 k 为正

故以常规设 k 做法, 难以区分 A, B , 而要求 Q 点坐标, 必须区分 A, B

故设 $A(t^2, 2t), t > 0, AB: y = \frac{2t}{t^2-1}(x-1)$

$$\begin{cases} y^2 = 4x \\ y = \frac{2t}{t^2-1}(x-1) \end{cases}$$

消去 x 得: $(t^2-1)y = 2t(\frac{y^2}{4} - 1)$

$$ty^2 - 2(t^2-1)x - 4t = 0$$

$$y_1 + y_2 = \frac{2(t^2-1)}{t} = 2(t - \frac{1}{t})$$

$$\text{故 } y_B = -\frac{2}{t}, x_B = \frac{1}{t^2}$$

又重心 $G(x_G, 0)$ 在 y 轴上, 故 $C(x_C, -\frac{2(t^2-1)}{t})$

$$\text{带抛物线 } \frac{4(t^2-1)^2}{t^2} = 4x \Rightarrow x = \frac{(t^2-1)^2}{t^2} = (t - \frac{1}{t})^2$$

$$\text{则 } C((t - \frac{1}{t})^2, -2(t - \frac{1}{t}))$$

$$x_G = \frac{t^2 + \frac{1}{t^2} + (1 - \frac{1}{t})^2}{3} = \frac{2(t^4 - t^2 + 1)}{3t^2}$$

$$k_{AC} = \frac{4t - \frac{2}{t}}{2 - \frac{1}{t^2}} = 2t, \text{ 直线 } AC: y - 2t = 2t(x - t^2)$$

$$y = 0 \text{ 时 } -2t = 2t(x - t^2) \Rightarrow x_G = t^2 - 1$$

由 Q 在 F 的右侧, 即 $t^2 - 1 > 1, t^2 > 2, t > \sqrt{2}$

$$|FG| = |\frac{2(t^4 - t^2 + 1)}{3t^2} - 1| = |\frac{2t^4 - 5t^2 + 2}{3t^2}|$$

$$|QG| = |t^2 - 1 - \frac{2(t^4 - t^2 + 1)}{3t^2}| = |\frac{t^4 - t^2 - 2}{3t^2}|$$

$$\frac{S_1}{S_2} = \frac{|\frac{2t^4 - 5t^2 + 2}{3t^2}| \cdot 2t}{|\frac{t^4 - t^2 - 2}{3t^2}| \cdot 2(t - \frac{1}{t})} = \frac{(2t^2 - 1)2t}{2(t^2 + 1)(t - \frac{1}{t})} = \frac{2t^4 - t^2}{t^4 - 1} = 2 - \frac{t^2 - 2}{t^4 - 1}$$

记 $u = t^2 - 2$, 则 $t^2 = u + 2, t^4 - 1 = u^2 + 4u + 3$

$$\text{则 } \frac{S_1}{S_2} = 2 - \frac{1}{u + \frac{3}{u} + 4} \geq 2 - \frac{1}{2\sqrt{3} + 4} = 2 - \frac{4 - 2\sqrt{3}}{4} = \frac{2 + \sqrt{3}}{2}$$

在 $u = \frac{3}{u} \Rightarrow u = \sqrt{3}$ 时取得 ” = ” 号, 此时 $t^2 = 2 + \sqrt{3}$

$$\text{此时 } x_G = \frac{2(7 + 4\sqrt{3} - 2 - \sqrt{3} + 1)}{3(2 + \sqrt{3})} = \frac{2(6 + 3\sqrt{3})}{3(2 + \sqrt{3})} = 2$$

故 $G(2, 0)$