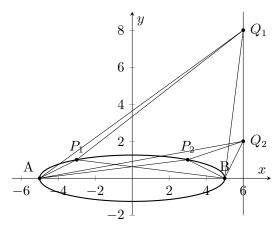解:

(1) 由题意:$e^2 = \frac{25-m^2}{25} = \frac{15}{16}$

$\Rightarrow m^2 = \frac{25}{16}$

故 $C : \frac{x^2}{25} + \frac{y^2}{\frac{25}{16}} = 1$

(2)

易知 $B : (5,0)$, 设 $P : (x_1, y_1), Q : (6, m)$

不妨设 $m > 0$, 则 $y_1 > 0$

则 $BQ : y = \frac{m-0}{6-5}(x-5) = m(x-5), |BQ| = \sqrt{1+m^2}$

由 $BQ \perp BP, BP : y = -\frac{1}{m}(x-5)$

则 $BP = \sqrt{(y_P - y_B)^2 + (x_P - x_B)^2}$

$\qquad = \sqrt{(1+m^2)(y_1)^2}$

又: $|BP| = |BQ| \Rightarrow (y_1)^2 = 1$

则 $y_1 = 1$

代入 C 得:$x^2 + 16 = 25 \Rightarrow x = \pm 3$

故 $P_1 : (-3, 1), P_2 : (3, 1)$

则 $k_{BP_1} = \frac{1}{-3-5} = -\frac{1}{m} \Rightarrow m = 8$

同理 $k_{BP_2} = \frac{1}{3-5} = -\frac{1}{m} \Rightarrow m = 2$

则:$P_1Q_1 : y = \frac{8-1}{6+3}(x+3) + 1 = \frac{7}{9}x + \frac{10}{3} \Rightarrow 7x - 9y + 30 = 0$

同理:$P_2Q_2 : y = \frac{2-1}{6-3}(x-3) + 1 = \frac{1}{3}x \Rightarrow x - 3y = 0$

则 $S\triangle AP_1Q_1 = \dfrac{1}{2}|P_1Q_1|d_{A-P_1Q_1}$

$$= \dfrac{1}{2}\sqrt{(8-1)^2+(6+3)^2} \times \dfrac{|-5\times 7 - 0 + 30|}{\sqrt{7^2+(-9)^2}}$$

$$= \dfrac{1}{2}\sqrt{49+81} \times \dfrac{5}{\sqrt{49+81}}$$

$$= \dfrac{5}{2}$$

同理 $S\triangle AP_2Q_2 = \dfrac{1}{2}|P_2Q_2|d_{A-P_2Q_2}$

$$= \dfrac{1}{2}\sqrt{(2-1)^2+(6-3)^2} \times \dfrac{|-5\times 1 - 0 + 0|}{\sqrt{1^2+3^2}}$$

$$= \dfrac{1}{2}\sqrt{1+9} \times \dfrac{5}{\sqrt{10}}$$

$$= \dfrac{5}{2}$$

综上 $\triangle APQ$ 的面积为 $\frac{5}{2}$