Computer Science

CSC380: Principles of Data Science

Wrap-up 1

Kyoungseok Jang

Outline

- Data Science Ethics and Fairness
- Course Recap
- Additional Resources
- Final Exam Overview

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Data Science Ethics

The movement to hold AI accountable gains more steam

First-in-US NYC law requires algorithms used in hiring to be “audited” for bias.

KHARI JOHNSON, WIRED.COM – 12/5/2021, 6:10 AM

ars technica

[Article Link]

As Data Science / AI / ML become more standard,
we need to address fairness and ethics…

State of Michigan's mistake led to man filing bankruptcy

Paul Egan Detroit Free Press

The secret bias hidden in mortgage-approval algorithms

By EMMANUEL MARTINEZ and LAUREN KIRCHNER/The Markup August 25, 2021

Senators Question Regulators About Tenant Screening Oversight

Facebook's race-blind
...around hate speech
...e expense of Black
...v documents show

- Washington Post

ExamSoft's remote bar exam sparks privacy and facial recognition concerns

- Venture Beat

Data Science Ethics

- NYC adopted law requiring audits of algorithms used in hiring
- White house proposes an AI bill of rights to disclose when AI makes decisions with societal impact
- EU lawmakers require inspection of AI deemed high-risk
- Analysis of automated hiring software found to be biased to appearance, software program used to create resume, accent, or whether applicants have a bookshelf in the background
- Photo ID software works well for white men—black women, not so much

Data Science Ethics

Aspects of ethics include…

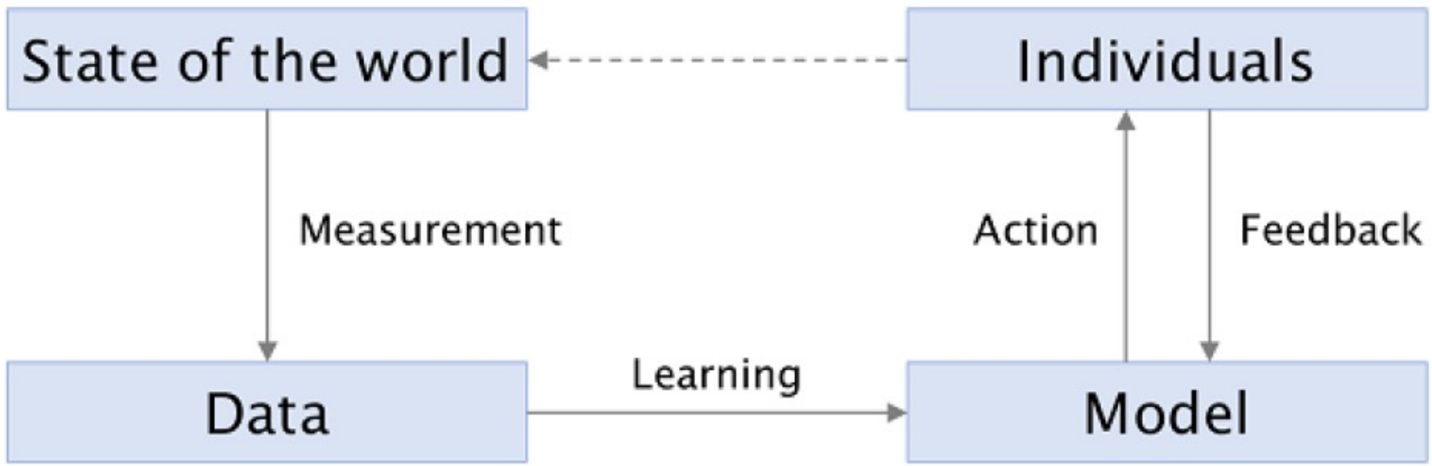
Security Who has access to the data?

Privacy Can data be used to identify individuals?

Fairness Are predictions biased across groups?

Transparency Do users know what they are consenting to? Are model decisions interpretable?

Impacts of Data Science



It is rare for data science to *not* involve people in some way

- Of the top 30 recent Kaggle competitions in 2021, 14 involve making decisions that directly affect people
- An additional 5 have obvious indirect affect on people
- Only 9 had no obvious impact on people

[Source: Brocas et al. “Fairness and ML”]

Data Science Fairness

Fairness issues can arise from biases in the data…

- Are there observable biases in the data?
- Can we correct for them?

- Differences in the distributions of training / test data?
- Can we detect these differences and avoid / correct them?

Training data reflect disparities, distortions, and biases from the real world and measurement process…

For each model a data scientist should ask… Does learning the model preserve, mitigate, or exacerbate these disparities?

Example Machine translation "She is a doctor" reverse translates to "He is a doctor" in many languages due to data biases.

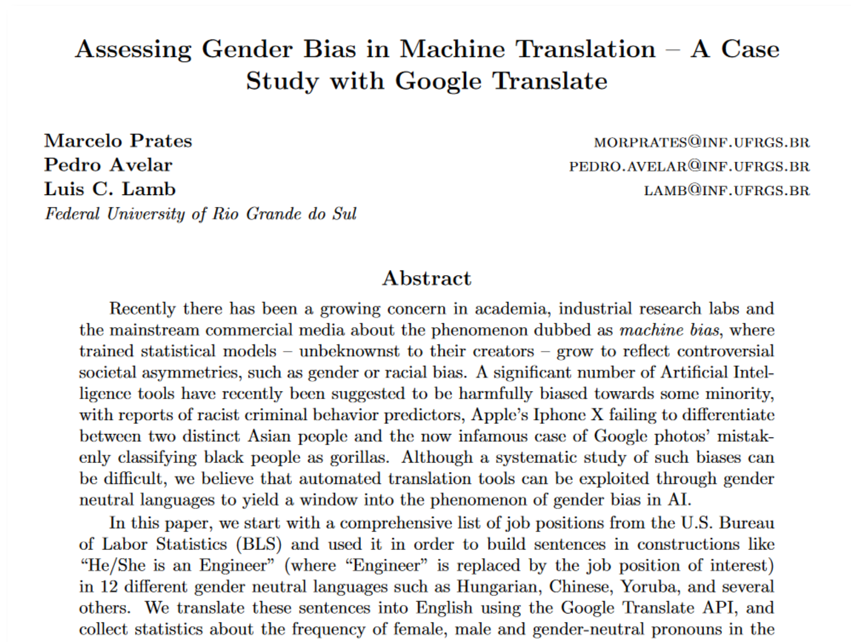
Data Science Ethics

A real-live example of dataset bias…

https://translate.google.com/

Exhibits gender bias in many languages…

…largely the result of using highly-parameterized neural networks with inadequate training data



Assessing Gender Bias in Machine Translation – A Case Study with Google Translate

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Abstract

Recently there has been a growing concern in academia, industrial research labs and the mainstream commercial media about the phenomenon dubbed as *machine bias*, where trained statistical models – unbeknownst to their creators – grow to reflect controversial societal asymmetries, such as gender or racial bias. A significant number of Artificial Intelligence tools have recently been suggested to be harmfully biased towards some minority, with reports of racist criminal behavior predictors, Apple's Iphone X failing to differentiate between two distinct Asian people and the now infamous case of Google photos' mistakenly classifying black people as gorillas. Although a systematic study of such biases can be difficult, we believe that automated translation tools can be exploited through gender neutral languages to yield a window into the phenomenon of gender bias in AI.

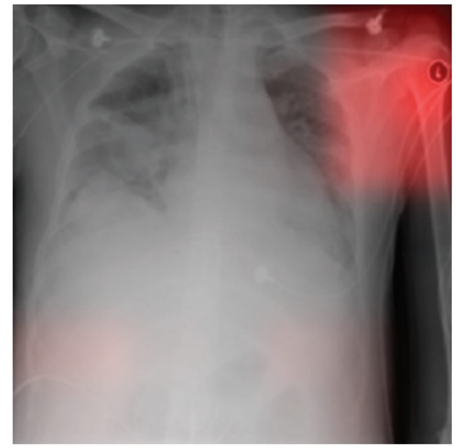
In this paper, we start with a comprehensive list of job positions from the U.S. Bureau of Labor Statistics (BLS) and used it in order to build sentences in constructions like "He/She is an Engineer" (where "Engineer" is replaced by the job position of interest) in 12 different gender neutral languages such as Hungarian, Chinese, Yoruba, and several others. We translate these sentences into English using the Google Translate API, and collect statistics about the frequency of female, male and gender-neutral pronouns in the

Data Science Ethics

- Short-cut learning

- E.g.) X-ray scan
  - Trained a classifier, but turns out it works well on some hospital, and works poorly on other hospital
  - Turns out, the deep neural network classifier learned other things than symptoms! (detecting hospital-specific metal token, posture of the X-ray picture)

- Shows why interpretability is important



Shortcut Learning in Deep Neural Networks

Robert Geirhos[1,2,*,§], Jörn-Henrik Jacobsen[3,*], Claudio Michaelis[1,2,*], Richard Zemel[†,3], Wieland Brendel[†,1], Matthias Bethge[†,1] & Felix A. Wichmann[†,1]

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Data Science Fairness

Example We are building a system to screen mortgage applications. Suppose we collect training data from two demographic groups: 85% White and 15% Black

- Predictive accuracy on the held-out validation set is 95%
- Only 5% error
- Should we sign off on the system as good?

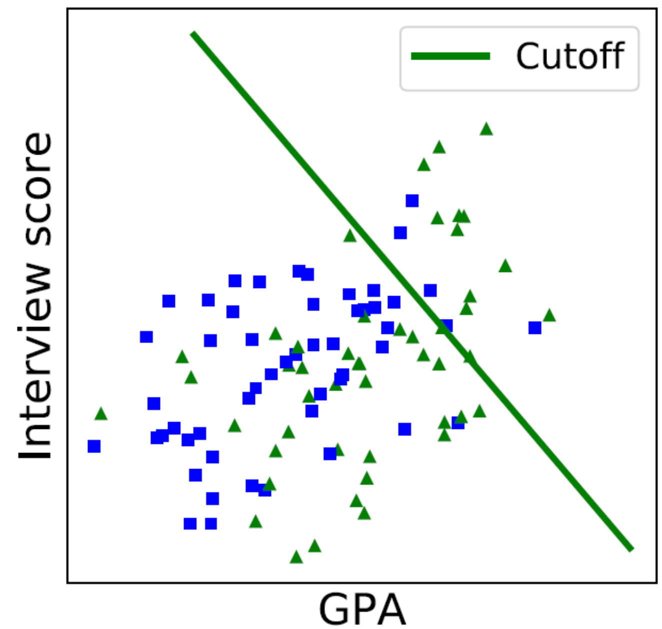
With 5% total error can have up to 66% error on the underrepresented group. Need to report error by each group and aim for 95% accuracy in any group.

Data Science Fairness

Example You are building a system for college admissions based on GPA and interview score (obviously a toy example)

- Fit a least squares regression model
- Model does not account for two demographic groups (blue / green)
- Does this make it fair? (fairness-as-blindness)

Admission rate much lower for blue cohort

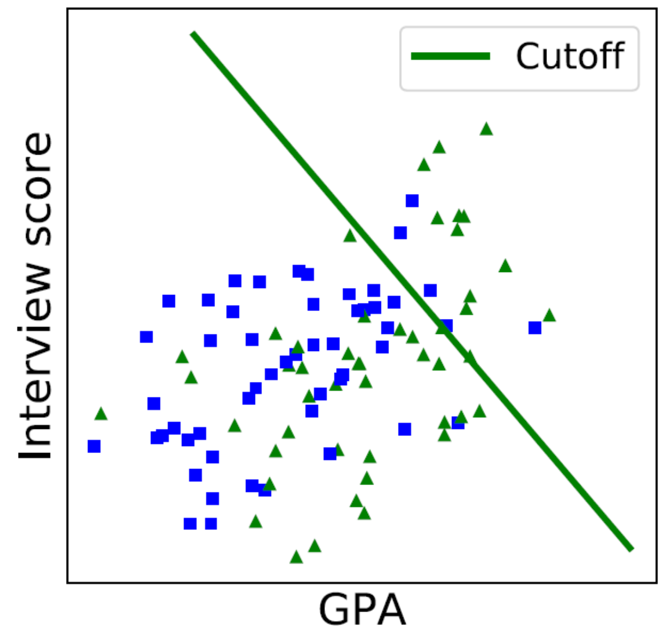


[Source: Brocas et al. “Fairness and ML”]

Data Science Fairness

How to address this behavior?

- GPA correlates with group—omit it as a predictor?
  - Would dramatically impact accuracy
- Pick separate cutoffs (fit separate model) for each group
  - No longer blind to demographics
  - What is the goal for picking cutoffs? Same admission rates?
- Could optimize for diversity among selected candidates
  - Measuring similarity is non-trivial



[Source: Brocas et al. “Fairness and ML”]

Classification Fairness Criteria

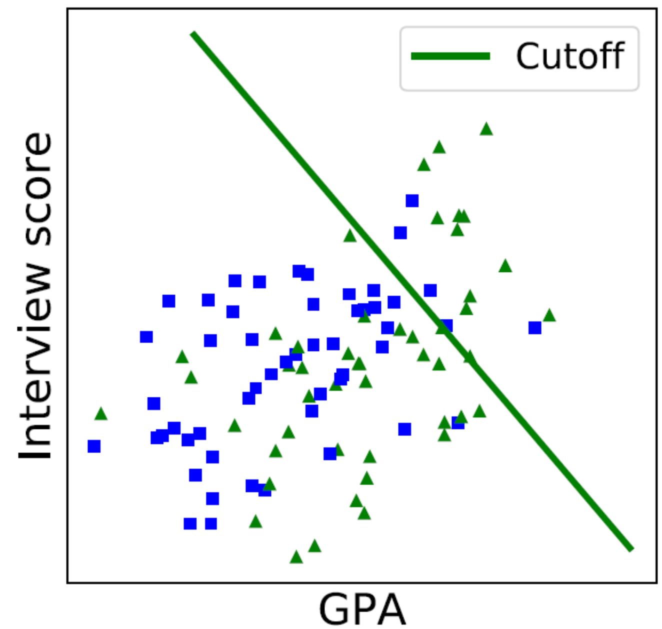
Let *A* be a sensitive attribute, target variable *Y*, and classifier prediction *R.*

Example In our admissions case,

A : Demographic group

R : Prediction of admission

Y : Actual acceptance outcome



Classification Fairness Criteria

Independence	Separation	Sufficiency
$R \perp A$	$R \perp A \mid Y$	$Y \perp A \mid R$

Independence The prediction and attribute are independent

Example The probability of predicting admission doesn't differ across demographic groups,

$$P(R \mid A = a) = P(R \mid A = b)$$

Demographic parity, statistical parity, group fairness, disparate impact

Classification Fairness Criteria

Independence	Separation	Sufficiency
$R \perp A$	$R \perp A \mid Y$	$Y \perp A \mid R$

Separation Score and attribute are conditionally independent, given the classifier decision

Example There is no relationship between prediction and attribute within accepted / non-accepted groups,

$$P(R \mid Y = 1, A = a) = P(R \mid Y = 1, A = b)$$

$$P(R \mid Y = 0, A = a) = P(R \mid Y = 0, A = b)$$

Classification Fairness Criteria

Independence	Separation	Sufficiency
$R \perp A$	$R \perp A \mid Y$	$Y \perp A \mid R$

Sufficiency Outcome and attribute are independent given the model prediction

Example There is no relationship between whether someone is admitted and their demographic group within predictions

$$P(Y \mid R = 1, A = a) = P(Y \mid R = 1, A = b)$$

$$P(Y \mid R = 0, A = a) = P(Y \mid R = 0, A = b)$$

Data Science Fairness

In short… there is a lot to say on ethics and fairness… and much can be quantified rigorously…

FAIRNESS AND MACHINE LEARNING

Limitations and Opportunities

Solon Barocas, Moritz Hardt, Arvind Narayanan

https://fairmlbook.org/

Outline

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Probability and Statistics

- We’ve learned many definitions from this part.
- I want you to remember at least these concepts.
  - How to calculate (joint) probability and expectation
  - How to calculate conditional probability and expectation
  - Understand independence and how to prove/disprove it
- I will submit 9 problems for this part. They are from:
  - Last year’s final
  - Our midterm
  - A few new problems
- The following problems are candidates, and I will remove or split some of them to ensure that you can solve all the problems within the given time.
  - No totally new problems!

Problem 1(a)

- modification of the following problem (our midterm)

1. Suppose we have two independent random variables $X \sim \text{Uniform}(\{1,2\})$ and $Y \sim \text{Uniform}(\{1,2,3\})$.
(a) (7 points) Compute the probability mass function of random variable $Z = X \cdot Y$.
(b) (3 points) Compute $\mathbb{E}[Z]$.

- Possible difference
  - X and Y might follow different distributions. (Y~Unif({0,2})? X~Ber(0.7)?)
  - Z might be different (Z=X+Y, Z=X-Y, Z=X^2 + Y^2…)

Problem 2

- modification of the following problem (last final 3)

- Suppose we throw a biased six-sided die twice in a row. We know the distribution of this die – let X be the outcome of this die, then P(X=1)=0.2, P(X=2)=0.1, P(X=3)=0.3, P(X=4)=0.2, P(X=5)=0.1, P(X=6)=0.1. Let S be the sum of both throws. Calculate P(S=5).

- Possible difference
  - Distribution of X might be different (biased die?)
  - Maybe I can ask for a different probability (S=5 and the first throw is even? S=7?)

Problem 3

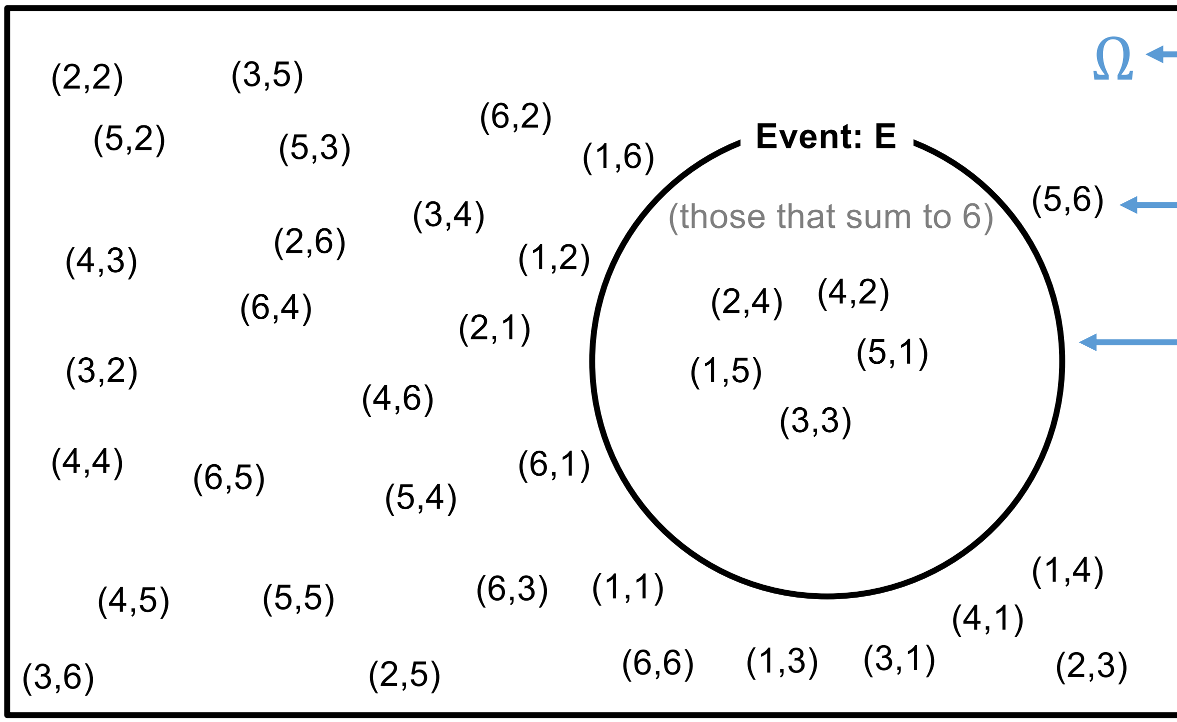
- Modification of the following problem:
- Suppose that we have three events A, B, and C, and we know the following facts:
  - P(A)=0.5, P(B)=0.2, P(C)=0.1
  - Events A and B are independent.
  - Event C is disjoint from A and B.

Now what is the probability of $P(A \cup B \cup C)$?

- Possible difference:
  - number of events, relationship, numbers…

Random Events and Probability

What is the probability of having two outcomes of fair dices sum to 6?



<u>Outcome space</u> consists of all possible outcomes

Each **<u>outcome</u>** $\omega \in \Omega$.

<u>Event</u> is a collection of outcomes and a subset of the outcome space

$E \subseteq \Omega$

Note: outcome = sample

Random Events and Probability

But, what is probability, really?

(e.g., can explain the probability of seeing an event when throwing two dice)

Mathematicians have found a set of conditions that 'makes sense'.

- Probability is a map P. ⇒ i.e., takes in an event, spits out a real value
- P must map events to a real value in interval [0,1].
- P is a (valid) **probability distribution** if it satisfies the following **axioms of probability**,
    1. For any event E, $\mathrm{P(E)} \geq 0$
    2. $P(\Omega) = 1$
    3. For any *finite* or *countably infinite* sequence of disjoint events $E_1, E_2, E_3, \ldots$

$$P\left(\bigcup_{i\geq 1} E_i\right) = \sum_{i\geq 1} P(E_i)$$

disjoint: intersection is empty

Random Events and Probability

- Many properties follows (i.e., can be proved mathematically)

$$\begin{aligned}
\mathbb{P}(\emptyset) &= 0 \\
A \subset B &\implies \mathbb{P}(A) \leq \mathbb{P}(B) \\
0 \leq \mathbb{P}(A) &\leq 1 \\
\mathbb{P}(A^c) &= 1 - \mathbb{P}(A) \\
A \bigcap B = \emptyset &\implies \mathbb{P}\left(A \bigcup B\right) = \mathbb{P}(A) + \mathbb{P}(B).
\end{aligned}$$

E.g., throw a die. A= getting 1, B=getting an odd number

E.g., A= getting 1, B=getting 3 or 5

Random Events and Probability

Law of total probability*:* Let A be an event. For any events $\mathrm{B}_1, \mathrm{B}_2, \ldots$ that partitions Ω, we have

$$P(A) = \sum_i P(A \cap B_i)$$

Example Roll two fair dice. Let X be the outcome of the first die. Let Y be the sum of both dice. What is the probability that both dice sum to 6 (i.e., Y=6)?

$$\mathrm{P}(A, B) := P(A \cap B)$$

$$p(Y=6) = \sum_{x=1}^{6} p(Y=6, X=x)$$

$$= p(Y=6, X=1) + p(Y=6, X=2) + \ldots + p(Y=6, X=6)$$

$$= p(X' = 5, X = 1) + p(X' = 4, X = 2) + \cdots + p(X' = 0, X = 6)$$

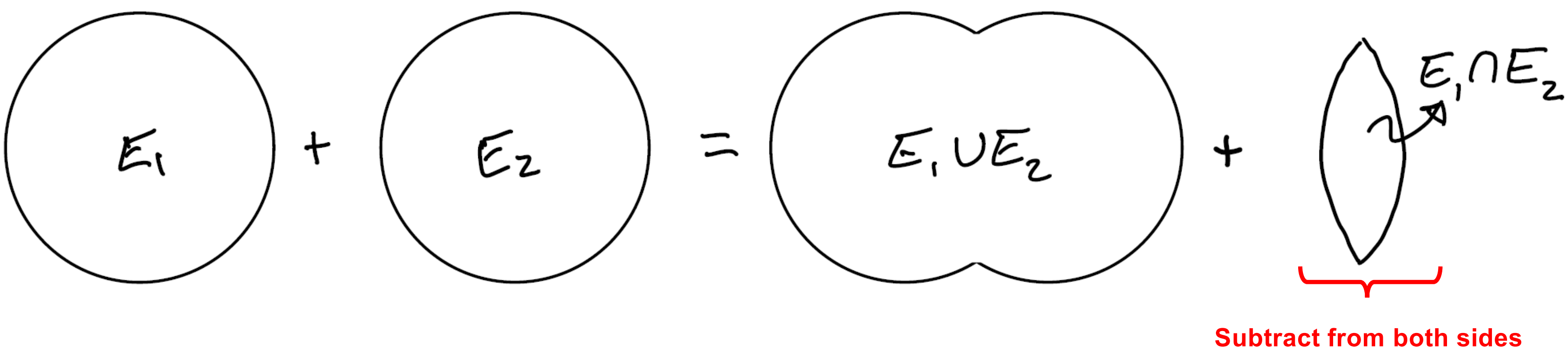
(X': the outcome of the second die)

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + 0 = \frac{5}{36}$$

Lemma: (inclusion-exclusion rule) *For any two events* E_1 *and* E_2,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Graphical Proof:



Problem 4

- Modification of the following problem

6. Suppose we have two random variables $X \sim \text{Bernoulli}(0.5)$ and $Y = 0.5X + 0.5$.
(a) (2 points) Compute $\mathbf{E}[X]$ and $\mathbf{E}[Y]$.
(b) (4 points) Compute $\text{Var}(X)$ and $\text{Var}(Y)$
(c) (4 points) Compute $\mathbf{E}[XY]$ and $\text{Cov}(X, Y)$.

- Possible change:
  - Maybe change X and Y to something else (possible scenario: I can introduce additional Z~Bernoulli(0.5) and say Y=X+Z)
  - Maybe remove some unnecessary sub-problems.

Problem 5

- Modification of the following problem

3. Suppose $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$, $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$, and $X_3 \sim \mathcal{N}(\mu_3, \sigma_3^2)$ are three independent random variables. Let $\hat{\mu} = \frac{1}{3}\sum_{i=1}^{3} X_i$. What distribution does $\hat{\mu}$ follow? Fully specify the parameters of the distribution. Show your work and reasoning.

- Possible change:
  - Number of random variables
  - Maybe I can use explicit numbers instead of μ_i and σ_i

Problem 7(b)

- Modification of the following problem (last year final)

7. Suppose we have placed two advertisements next to each other in a website. A user can either click both, click one of them, or not click at all. Let $A \in \{1, 0\}$ and $B \in \{1, 0\}$ be the random variables indicating whether each ad is clicked (1) or not(0). They follow the following joint probability table.

	$B = 1$	$B = 0$
$A = 1$	$1/8$	$3/8$
$A = 0$	$3/8$	$1/8$

(i) Compute E[B|A=1]

(ii) Is A and B independent or not? Justify your answer formally.

Possible changes: numbers, questions (like, compute E[A|B=1]), but I will ask (ii) definitely.

Independence

- Informally, given two events A and B, they are **independent** if the probability of A is not affected by whether B is true or false (and vice versa)
  - E.g., A = "die1=1" and B="die2=1" are independent.

    ⟹ we know that the probability of die1 being 1 would not be changed just because die2=1.

- Mathematically, this can be written as $P(A|B) = P(A)$ or $P(B|A) = P(B)$.

- E.g., A = "die1=6" and B="two dice sum to 6" are **not independent**.

  ∵ intuitively, when B is true, A can never happen! So, P(A|B)=0 but P(A) = 1/6.

- E.g., A = "die1=1" and B="two dice sum to 6" are **not independent**.

  ∵ P(A) = 1/6 = 0.166... . However, P(A|B) = 1/5 = 0.2

More examples

- Q: A = “die1=1” and B=“two dice sum to 5”. Independent?

No

$\because$ P(A) = 1/6 , P(A|B) = 1/4 = .25

- Q: A = “die1=even” and B=“two dice sum to 5”. Independent?

Yes

$\because$ P(A) = 1/2 , P(A|B) = 2/4 = 1/2

Independence

[Def] Two events A and B are **independent** if

$$P(A, B) = P(A)P(B)$$

$A \perp B$ means A and B are independent

"joint probability is product of two marginal probabilities"

=> note: symmetric!

(skipping the following..)

Also, a set of events $\{A_i \in \mathcal{F}\}_{i=1}^{n}$ (n can be ∞) are **mutually independent** if

for every $J \subseteq \{1, \ldots, n\}$, we have $P\left(\cap_{i \in J} A_i\right) = \prod_{i \in J} P(A_i)$

(∃ a notion of 'pairwise' independence, but not much useful, so we omit it here)

Independence

- Ex) recall two fair dice
  - We took it for granted that P((1,1)) is 1/36.
  - But why is it true, really?
  - To be rigorous,

$$P(die1 = 1, die2 = 1) = P(die1 = 1)P(die2 = 1) = \frac{1}{6} \cdot \frac{1}{6}$$

  due to independence.

  or, …= P(die1=1 | die2=1) * P(die2=1) = P(die1=1) * P(die2=1)

- E.g., two biased coin **<u>C1</u>** and **<u>C2</u>**. Suppose P(C1=H) = 0.3 and P(C2=H) = 0.4. Compute the probability of P(C1=H,C2=T).

0.3·0.6 = 0.18

quiz candidate

Independence

Definition *Two random variables* X *and* Y *are* *independent* *given if and only if*

$$p(X = x, Y = y) = p(X = x)p(Y = y)$$

for all values x and y, and we say $X \perp Y$.

- From now on, we will just write it down as $p(X, Y) = p(X)p(Y)$
- Property: X and Y are independent if and only if $p(X) = p(X|Y)$ (or $p(Y) = p(Y|X)$)

➢ N RVs are independent if

$$p(X_1, \dots, X_N) = \prod_{i=1}^{N} p(X_i)$$

(Again, for all the possible values $x_1, \dots, x_N$)

Moments of Random Variables

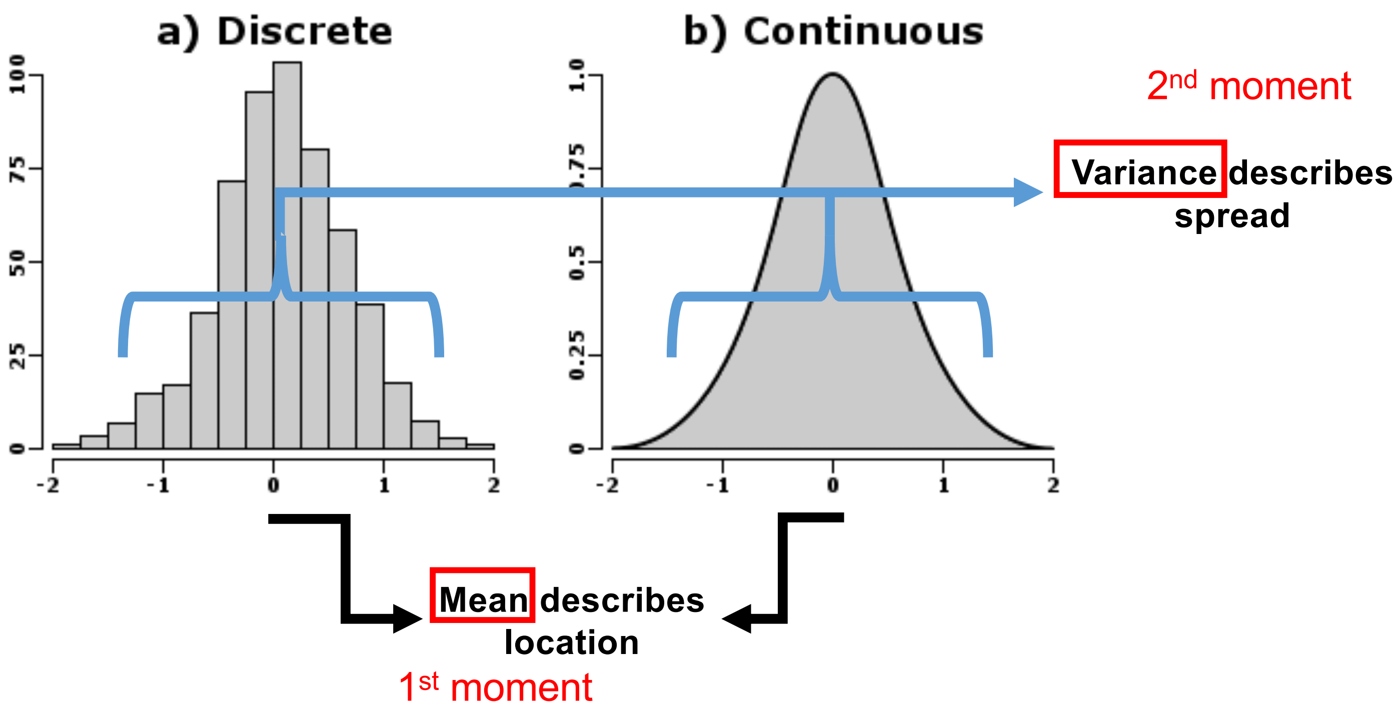
(informal introduction)

Properties of a RV are characterized by its distribution / PMF / PDF
But there are "summary" numbers capturing important characteristics
This is called "**moments**".

Moment ordinal	Moment			Cumulant	
	Raw	Central	Standardized	Raw	Normalized
1	Mean	0	0	Mean	N/A
2	–	Variance	1	Variance	1
3	–	–	Skewness	–	Skewness
4	–	–	(Non-excess or historical) kurtosis		Excess kurtosis

(Wikipedia)

Moments of Random Variables



Moments characterize properties of the distribution "shape"

Mean = Expectation = Expected Value

Definition *The <u>expectation</u> of a discrete RV X, denoted by $\mathbf{E}[X]$, is:*

(with PMF)

$$\mathbf{E}[X] = \sum_{x} x \cdot p(X = x)$$

Summation over all values in domain of X

- **<u>Effectively, a weighted average</u>**: *each outcome weighted by probability of occurring*

Some people call it **<u>average</u>** rather than **<u>mean</u>**, but I wouldn't.

$\Rightarrow$ average is a particular 'operator': $\frac{1}{|\mathcal{X}|}\sum_{x\in\mathcal{X}} x$

$\Rightarrow$ in data science, average is something about the data, not the distribution behind the data

Expected Value

Example Let X be the sum of two fair dice, compute $\mathbf{E}[X]$:

	count	prob.
2: (1,1)	1	1/36
3: (1,2), (2,1)	2	2/36
…	..	…
6: (1,5), (2,4), (3,3), (4,2), (5,1)	5	5/36
7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6	6/36
8: (2,6), (3,5), (4,4), (5,3), (6,2)	5	5/36
…	…	…
12: (6,6)	1	1/36

Expectation: $2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \ldots + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + \ldots + 12 \cdot \frac{1}{36}$ =7

Expected Value

Theorem (Linearity of Expectations) *For any finite collection of RVs $X_1, \cdots, X_n$ with finite expectations,*

$$\mathbf{E}\left[\sum_{i=1}^{N} X_i\right] = \sum_{i=1}^{N} \mathbf{E}[X_i]$$

E.g. for two RVs X and Y
$\mathbf{E}[X+Y] = \mathbf{E}[X] + \mathbf{E}[Y]$

you do not need an independence!

Example Throw two fair dice. What is the expected sum? Let X and Y be the outcome of the first and second die, respectively. Then,

$$\mathbf{E}[X+Y] = \mathbf{E}[X] + \mathbf{E}[Y] = 3.5 + 3.5 = 7$$

Expected Value

Theorem For any random variable X and constant c,

$$\mathbf{E}[cX] = c\mathbf{E}[X]$$

Example Let X and Y be the outcome of two fair dice, then:

$$\begin{aligned} \mathbf{E}[2(X+Y)] &= \mathbf{E}[2X] + \mathbf{E}[2Y] \\ &= 2\mathbf{E}[X] + 2\mathbf{E}[Y] \\ &= 2 \cdot 3.5 + 2 \cdot 3.5 = 14 \end{aligned}$$

Caveat: c has to be a constant, not a random variable!
E.g., X: outcome of a fair die, c: outcome of another fair die

Expected Value

Definition *The <u>conditional expectation</u> of a discrete RV X, given Y is:*

cf. $\mathbf{E}[X] = \sum_x x \cdot p(X = x)$

$$\mathbf{E}[X \mid Y = y] = \sum_x x\, p(X = x \mid Y = y)$$

Example Roll two fair dice. X_1: first die outcome, Y: sum of two dice

quiz candidate

$$\mathbf{E}[X_1 \mid Y = 5] = \sum_{x=1}^{4} x\, p(X_1 = x \mid Y = 5)$$

$$= \sum_{x=1}^{4} x \frac{p(X_1 = x, Y = 5)}{p(Y = 5)} = \sum_{x=1}^{4} x \frac{1/36}{4/36} = \frac{5}{2}$$

Conditional expectation follows properties of expectation (linearity, etc.)

Expected Value

Example: Two fair dice.

Y=outcome of die 1
X=sum of two dice

X|Y=1 ~ U{2,3,4,5,6,7} E[X|Y=1] = 4.5 $P(Y=1)=\frac{1}{6}$

X|Y=2 ~ U{3,4,5,6,7,8} E[X|Y=2] = 5.5 $P(Y=2)=\frac{1}{6}$

E[X|Y=3] = 6.5

… E[X|Y=4] = 7.5 …

E[X|Y=5] = 8.5

X|Y=6 ~ U{7,8,9,10,11,12} E[X|Y=6] = 9.5 $P(Y=6)=\frac{1}{6}$

$\mathbf{E}_X[X|Y]$ is a random variable:
$\mathbf{E}_X[X|Y] \sim U\{4.5, 5.5, 6.5, 7.5, 8.5, 9.5\}$

Expectation is 7
=> $\mathrm{E}_Y\left[\mathbf{E}_{\boldsymbol{X}}[X|Y]\right] = 7$
=> coincides with $\mathbf{E}[X]$ we computed before!

Independence and Moments

Theorem: *If* $X \perp Y$ *then* $\mathbf{E}[XY] = \mathbf{E}[X]\mathbf{E}[Y]$ *.*

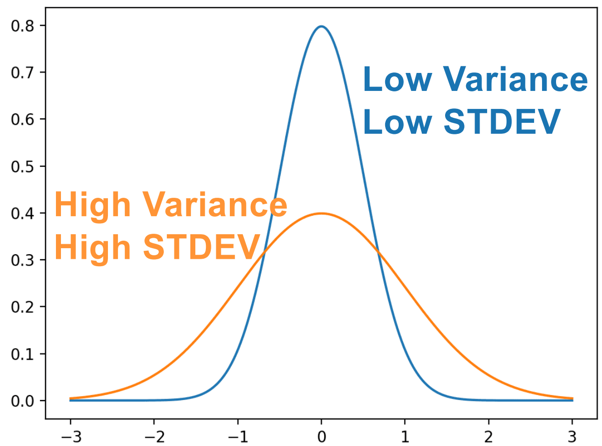
<u>Comparison</u>: $\mathbf{E}[X + Y] = \mathbf{E}[X] + \mathbf{E}[Y]$ regardless of independence!

Variance

Definition The variance of a RV X is defined as,

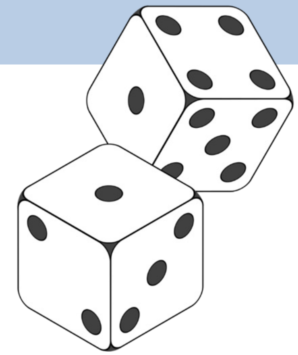
$$\mathbf{Var}[X] = \mathbf{E}[(X - \mathbf{E}[X])^2]$$

The standard deviation (STDEV) is $\sigma[X] = \sqrt{\mathbf{Var}[X]}$.



- *Describes the "spread" of a distribution*
- *Describes uncertainty of outcome*
- *STDEV is in original units (more intuitive), variance is in units²*
- *Variance is more mathematically useful than STDEV*

Variance



Example Let X be the result of a fair six-sided die.

The variance is then,

$$\begin{aligned}\mathrm{Var}(X) &= \sum_{i=1}^{6} \frac{1}{6}\left(i - \frac{7}{2}\right)^2 \\ &= \frac{1}{6}\left((-5/2)^2 + (-3/2)^2 + (-1/2)^2 + (1/2)^2 + (3/2)^2 + (5/2)^2\right) \\ &= \frac{35}{12} \approx 2.92.\end{aligned}$$

The STDEV is $\sqrt{\mathrm{Var}(X)} \approx 1.71$, which suggests we should expect outcomes to vary around the mean of 3.5 by $\pm$ 1.71

Variance

Lemma An equivalent form of variance is:

$$\mathbf{Var}[X] = \mathbf{E}[X^2] - (\mathbf{E}[X])^2$$

Proof

$$\mathbf{E}[(X - \mathbf{E}[X])^2] = \mathbf{E}[X^2 - 2X\mathbf{E}[X] + \mathbf{E}[X]^2] \quad \textbf{(Expand it)}$$

$$= \mathbf{E}[X^2] - 2\mathbf{E}[X]\mathbf{E}[X] + \mathbf{E}[X]^2 \quad \textbf{(Linearity of expectations)}$$

$$= \mathbf{E}[X^2] - 2\mathbf{E}[X]^2 + \mathbf{E}[X]^2 \quad \textbf{(Algebra)}$$

$$= \mathbf{E}[X^2] - \mathbf{E}[X]^2 \quad \textbf{(Algebra)}$$

Variance

- If c is a constant, $Var[cX] = c^2 Var[X]$

- Important that c has to be a constant here!

Independence and Moments

Recall that for any two RVs X and Y variance is not a linear function,

$$\mathbf{Var}[X+Y] = \mathbf{Var}[X] + \mathbf{Var}[Y] + 2\mathbf{Cov}(X,Y)$$

<u>**If X and Y are independent then they have zero covariance**</u>,

$$\mathbf{Cov}(X,Y) = 0$$

Thus, $$\mathbf{Var}[X+Y] = \mathbf{Var}[X] + \mathbf{Var}[Y]$$

And, for a collection of independent RVs $X_1, X_2, \ldots, X_N$ we have,

$$\mathbf{Var}\left(\sum_{i=1}^{N} X_i\right) = \sum_{i=1}^{N} \mathbf{Var}(X_i)$$

Q: Is variance is a linear operator under independence?

A: No! $Var(cX) \neq c\, Var(X)$ for a constant c. Rather, $Var(cX) = c^2 Var(X)$.

Covariance

Definition *The* *covariance* *of two RVs* X *and* Y *is defined as,*

$$\mathbf{Cov}(X,Y) = \mathbf{E}[(X - \mathbf{E}[X])(Y - \mathbf{E}[Y])]$$

Question *What is Cov(X,X)?*

Answer *Cov(X,X) = Var(X)*

Covariance

- A shortcut to compute covariance.
- $Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$

$$= E\big[XY - X \cdot E[Y] - Y \cdot E[X] + E[X]E[Y]\big]$$

$$= E[XY] - E[X]E[Y] - E[Y]E[X] + E[X]E[Y]$$

$$= E[XY] - E[X]E[Y]$$

- Safety check: $Cov(X, X) = E[XX] - E[X]E[X] = Var(X)$

Problem 6

- Modification of the following problem:
- You tossed a fair coin three times. Let X be the number of heads you observed after three tosses.
  - 1) What is the probability that you observe exactly one head?
  - 2) Compute $E[X^2]$.
  - 3) Given that you observed the result of the first coin was tail, what is the conditional expectation of X?
- Possible difference:
  - Number of tosses, question details, but for 1) I will ask probability, 2) I will ask expectation(or variance), 3) I will ask conditional expectation.
  - Maybe I can remove one sub-problem from here…

Problem 7(a)

- Modification of the following problem (last year final)

7. Suppose we have placed two advertisements next to each other in a website. A user can either click both, click one of them, or not click at all. Let $A \in \{1, 0\}$ and $B \in \{1, 0\}$ be the random variables indicating whether each ad is clicked (1) or not(0). They follow the following joint probability table.

	$B = 1$	$B = 0$
$A = 1$	$1/8$	$3/8$
$A = 0$	$3/8$	$1/8$

(i) Compute E[B|A=1]

(ii) Is A and B independent or not? Justify your answer formally.

Problem 8

- Modification of the following problem:(our midterm, last final 9)

8. A spam email detector has the following property:

- Given a spam email, it will raise an alarm with probability 0.9.
- Given a non-spam email, it will raise an alarm with probability 0.01.

In addition, suppose that 1 out of 10 emails is a spam email. Given an email, we use $S \in \{\text{True}, \text{False}\}$ to denote whether it is a spam, and use $R \in \{\text{True}, \text{False}\}$ to denote whether the spam detector raises an alarm.
(a) (5 points) Compute the joint probability distribution of (S, R).
(b) (5 points) Conditioned on that the spam detector raises an alarm, what is the probability that the email is a spam?

- I will only change numbers.

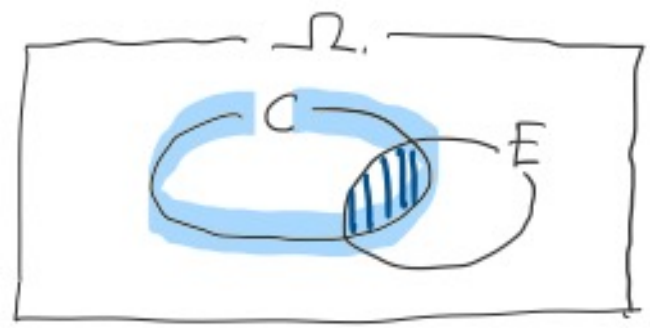
Problem 9

- Modification of the following problem:
- A box contains two coins. One fair coin and one two-headed coin. You picked a coin at random and tossed it.
  - You observed it lands heads up. What is the probability that the coin you chose was a fair coin?
  - You observed it lands heads up. What is the expected number of heads inside the box? (note: the fair coin has only one head, and the two-headed coin has two heads)
- Possible changes: number of coins, bias, question slightly…

Conditional Probability

- Two fair dice example:
  - Suppose I roll two dice secretly and tell you that one of the dice is 2. C
  - **<u>In this situation</u>**, find the probability of two dice summing to 6. E

- Turns out, such a probability can be computed by $\frac{P(E \cap C)}{P(C)}$
- It's like "zooming in" to the condition.
- This happens a lot in practice, so let's give it a notation:



$$P(E|C) := \frac{P(E \cap C)}{P(C)}$$

Say: probability of "E given C", "E conditioned on C"

"it's the ratio"

Conditional Probability

Chain rule

- $P(A \cap B) = P(A|B)P(B)$ ←just a rearrangement of definition
- $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$
- $P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1) \prod_{i=2}^{n} P(E_i | \cap_{j=1}^{i-1} E_j)$

valid for any ordering!

Law of total probability: If $A \in \mathcal{F}$ and $\{B_i \in \mathcal{F}\}_i$ partitions Ω, then

$$P(A) = \sum_i P(A, B_i) = \sum_i P(B_i)P(A|B_i)$$

$$= \sum_i P(A)P(B_i|A)$$

(trivially true by definition)

Shortcut:
P(A,B) := P(A ∩ B)

Conditional Probability

[W:Ex.6.9] The Public Health Department gives us the following information:

- A test for the disease yields a positive result 90% of the time when the disease is present

  P(test=+ | disease=Y) = 0.9

- A test for the disease yields a positive result 1% of the time when the disease is not present

  P(test=+ | disease=N) = 0.01

- One person in 1,000 has the disease.

  P(disease=Y) = 0.001

Q: What is the probability that a person with positive test has the disease?

P(disease=Y | test=+)

Conditional Probability

What we know:

P(test=+ | D=Y) = 0.9
P(test=+ | D=N) = 0.01
P(D=Y) = 0.001

$\Rightarrow$

P(test=- | D=Y) = 0.1
P(test=- | D=N) = 0.99
P(D=N) = 0.999

Question: P(D=Y | test=+)

$$= \frac{P(D = Y\ , test = +)}{P(test = +)}$$

$$P(test = +) = P(test = +, D = Y) + P(test = +, D = N)$$

$$P(test = +, D = Y) = P(test = +|\ D = Y)P(D = Y)$$

$$P(test = +, D = N) = P(test = +|\ D = N)P(D = N)$$

The answer is 0.0826…

CAVEAT: P(test=+ | D=Y) = 0.9 ≠ P(D=Y | test=+)

Also: P(D=Y) = 0.001 vs P(D=Y | test=+)

Expected Value

Definition *The* <u>*conditional expectation*</u> *of a discrete RV X, given Y is:*

cf. $\mathbf{E}[X] = \sum_x x \cdot p(X = x)$

$$\mathbf{E}[X \mid Y = y] = \sum_x x\, p(X = x \mid Y = y)$$

Example Roll two fair dice. X_1: first die outcome, Y: sum of two dice

quiz candidate

$$\mathbf{E}[X_1 \mid Y = 5] = \sum_{x=1}^{4} x\, p(X_1 = x \mid Y = 5)$$

$$= \sum_{x=1}^{4} x \frac{p(X_1 = x, Y = 5)}{p(Y = 5)} = \sum_{x=1}^{4} x \frac{1/36}{4/36} = \frac{5}{2}$$

Conditional expectation follows properties of expectation (linearity, etc.)

Problem 10

- I will change the question to some other example.

10. Suppose we would like to do an exit poll for a presidential election where we ask people coming out of poll stations who they have voted for. Due to resource constraints, we cannot ask every single person. Which sampling method is appropriate?

(1) Simple random sampling.

(2) Systematic sampling.

(3) Stratified sampling.

(4) Cluster sampling.

Problem Candidates

- These are my current (format) candidates for the first half, but I will talk with our TAs and will
  - 1) Remove some of the problems or sub-problems
  - 2) Split one problem into two
  - 3) Substitute some problems with candidates on the next slides.

Candidate 1

- This is Problem 1

1. Suppose we have two independent random variables $X \sim \text{Uniform}(\{1, 2\})$ and $Y \sim \text{Uniform}(\{1, 2, 3\})$.
(a) (7 points) Compute the probability mass function of random variable $Z = X \cdot Y$.
(b) (3 points) Compute $\mathbb{E}[Z]$.

- Adding some continuing problem…?

2. Continuing the previous problem,
(a) (5 points) Compute $\mathbb{P}(X = 2 \mid Z = 2)$.
(b) (5 points) Compute $\mathbb{E}\left[X \mid Z = 2\right]$.

Candidate 2

- Last year final

2. Suppose we throw a fair six-sided die twice in a row. Let A be a random variable representing the number on the first throw, and B be the number on the second throw. Let S be the sum of both throws. Compute $P(A = 2 \mid S = 2)$. 0

4. Under the same setup as the previous problem (the nonfair one), compute $P(A = 1 \mid S = 3)$. $0.02/0.04 = 0.5$

Possible modification: biases, detailed numbers…

Candidate 3

5. Suppose that the random variable X has the following distribution:

$$P(X = 0) = 0.3, P(X = 1) = 0.2, P(X = 2) = 0.5$$

Compute Var(X).

Possible modification: distribution, compute something else (E[X^3], E[X^2+X], …)

Problem 11~20

- Next week (May 2nd)

Outline

- Data Science Ethics and Fairness
- Course Recap
- Additional Resources
- Final Exam Overview

Data Science Competitions

Competitions can be a great way to hone your skills…

CodaLab

MACHINE HACK

topcoder™

kaggle

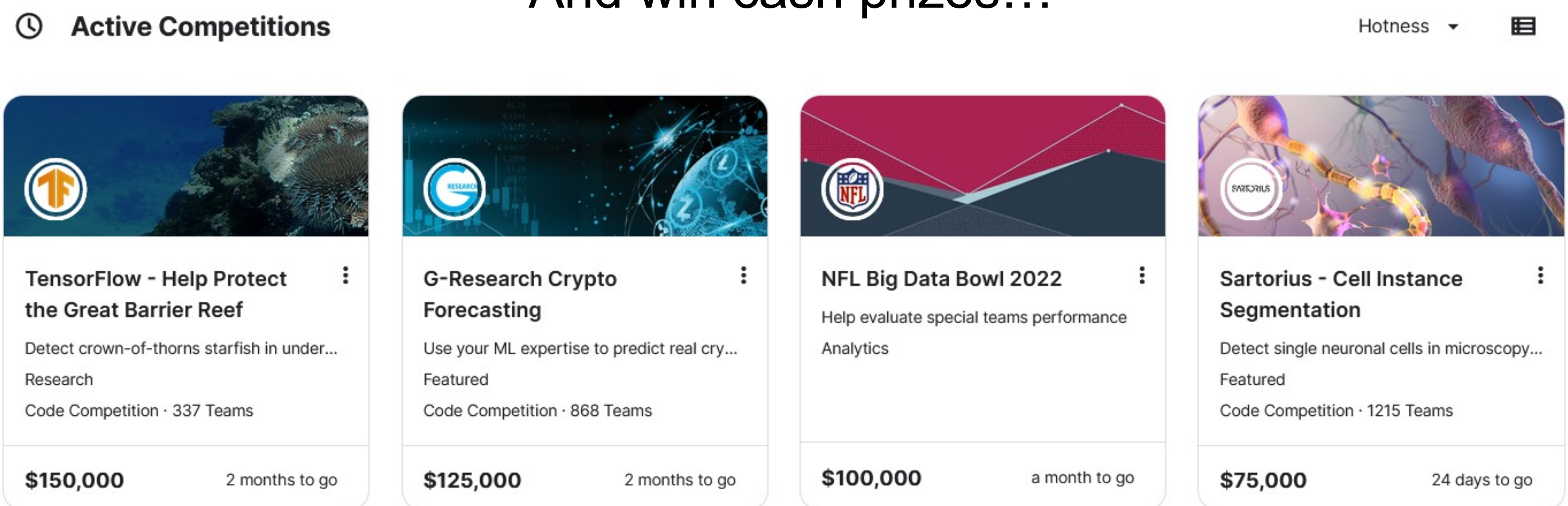
Analytics Vidhya

IDA
INTERNATIONAL
DATA ANALYSIS OLYMPIAD

DRIVENDATA

Data Science Competitions

And win cash prizes…



Can also be a great source for datasets to practice

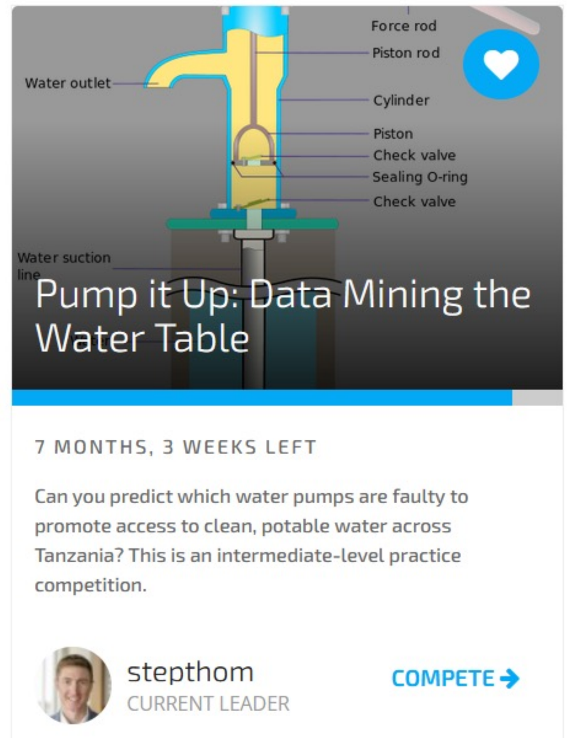
www.Kaggle.com

Data Science Competitions

Cash prizes aren't the only goal…

DRIVENDATA

- Focuses on social impact
- Challenges last 2-3 months
- Real-world predictive problems
  - Detecting hateful content online
  - Predicting disease spread
  - Predicting damage from earthquakes
  - …
- Submissions are released as open source



Additional Relevant Courses

- CSC 480 : Principles of Machine Learning
- CSC 444 : Introduction to Data Visualization
- ISTA 457 : Neural Networks
- ESOC 330 : Digital Dilemmas : Privacy, Property, and Access
- MATH 574M : Statistical Machine Learning

Videos

3Blue1Brown

- Accessible videos on a variety of math topics
- Nicely produced, engaging graphics
- A number of ML / Data Science / Statistics topics covered

Steve Brunton – YouTube Channel

- More detailed videos on math / engineering topics
- Good linear algebra and machine learning videos
- Associated book,

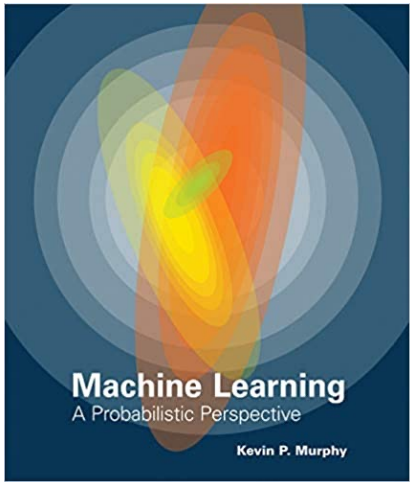
Data-Driven Science and Engineering : ML, Dynamical Systems, and control

Videos

MIT Open Courseware

- Lots of topics freely available
- Excellent Linear Algebra course by Prof. Gilbert Strang (YouTube lectures)
- All assignments and exams available online

Textbooks



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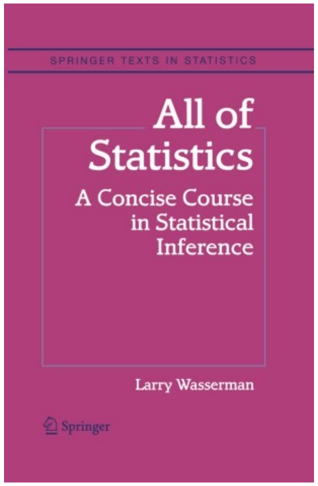

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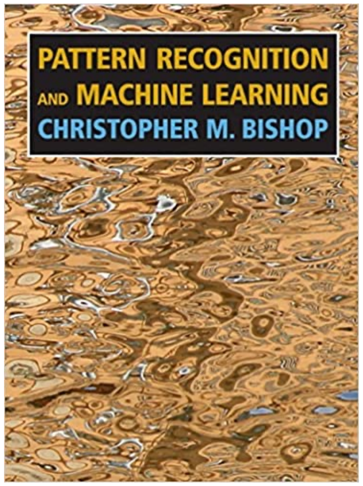
Textbooks



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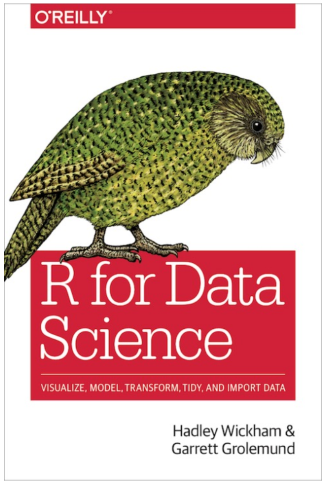

Wickham and Grolemund. “R for Data Science.” O’Reilly, 2016

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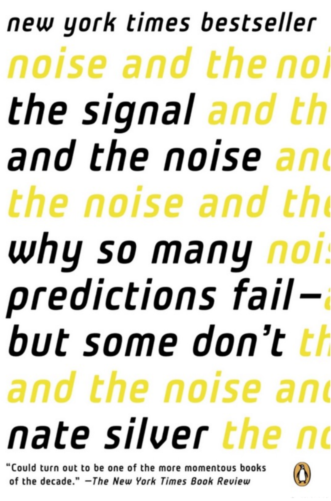
Non-Textbooks



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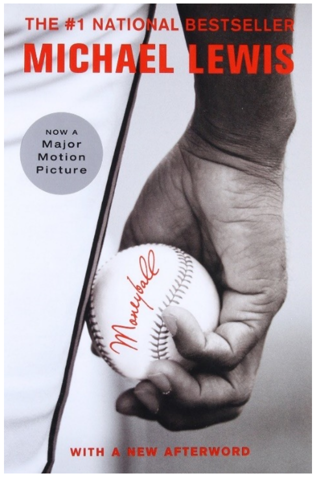

Lewis, M. "Moneyball." W. W. Norton, 2011

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Questions?