Graph Rectifiability Summer Research with Lisa Naples

Daily Report

By

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1.1 Basic Set Theory

Definition 1.1 (Countable) *An infinite set A is countable if its elements can be listed in the form $x_1, x_2, ...$ with every element of appearing at a specific place in the list; otherwise, the set is uncountable*

Definition 1.2 (Open) *$A \subset \mathbb{R}^n$ is open if, $\forall x \in A$, $\exists B(x, r) \in A$ where $r > 0$.*

Definition 1.3 (Closed) *$A \subset \mathbb{R}^n$ is closed if, whenever $\{x_k\} \in A$, $x_k \to x \in \mathbb{R}^n$, then $x \in A$.*

Definition 1.4 (Closure) *$\bar{A}$ is the intersection of all the closed sets containing a set A.*

Definition 1.5 (Interior) *$int(A)$ is the union of all open sets contained in A.*

Definition 1.4 and 1.5 shows that The *closure* of A is thought of as the **smallest closed set** containing A, and the *interior* as the **largest open set** contained in A.

Definition 1.6 (Boundary) $\partial A = \bar{A} \setminus int(A)$

Theorem 1.1 $x \in \partial A \Leftrightarrow \forall r > 0, B(x, r) \cap A \neq \emptyset, B(x, r) \cap A^C \neq \emptyset$

Definition 1.7 (Dense) *Set B is a dense in A if $A \subset \bar{B}$, that is, if there are points of B arbitrarily close to each point of A.*

Definition 1.8 (Compact) *A is compact if any collection of open sets that covers A has a finite subcollection which also covers A.*

Theorem 1.2 *A compact subset of $\mathbb{R}^n$ is both closed and bounded.*

Theorem 1.3 *The intersection of any collection of compact sets is compact.*

Definition 1.9 (Connected) *$A \subset \mathbb{R}^n$ is connected if there not exists open sets U and V s.t. $A \in U \cap V$ with disjoint and nonempty $A \cap U$ and $A \cap V$.*

Definition 1.10 (Connected Component) *The connected component of x is the largest connected subset of A containing a point x.*

Definition 1.11 (Disconnect) *The set A is totally disconnected if the connected component of each point consists of just that point.*

The definition of *disconnect* also can be as: $\exists$ open sets U and V s.t. $x \in U, y \in V$ and $A \subset U \cap V$.

Definition 1.12 (Borel Set) *Borel Sets is the smallest collection fo subsets of $\mathbb{R}^n$ with the following properties:*

1. *Every open set and every closed set is a Borel set.*
2. *The union of every finite or countable collection of Borel sets is a Borel set, and the intersection of every finite or countable collection of Borel sets is a Borel set.*

In short, Any set that can be constructed using a sequence of countable unions or intersections starting with the open sets or closed sets will certainly be Borel.

1.2 Functions and Limits

Definition 1.13 (Congruence) *The transformation $S : \mathbb{R}^n \to \mathbb{N}^n$ is congruence or isometry if it preserves distances i.e. if $|S(x) - S(y)| = |x - y|$ for $x, y \in \mathbb{R}^n$*

Special cases include *translations*, which are of the form $S(x) = x + a$ and have the effect of shifting points parallel to the vector a, *rotations* which have a centre a such that $|S(x) - a| = |x - a|$ for all x (for convenience, we also regard the identity transformation given by $I(x) = x$ as a rotation) and *reflections*, which maps points to their mirror images in some $(n-1)$ -dimensional plane. A congruence that may be achieved by a combination of a rotation and a translation, that is, does not involve reflection, is called a *rigid motion* or *direct congruence*. A transformation $S : \mathbb{R}^n \to \mathbb{R}^n$ is a *similarity* of *ratio* or *scale* $c > 0$ if $|S(x) - S(y)| = c|x - y|$ for all x, y in $\mathbb{R}^n$. A similarity transforms sets into geometrically similar ones with all lengths multiplied by the factor c.

Definition 1.14