# Lecture 14:

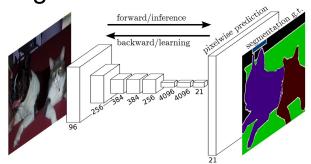
# **Videos** Unsupervised Learning

#### **Administrative**

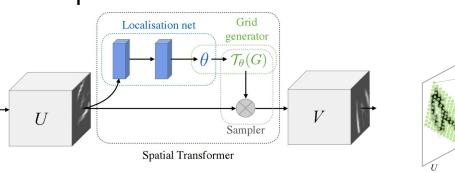
- Everyone should be done with Assignment 3 now
- Milestone grades will go out soon

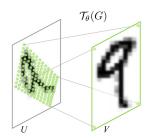
#### Last class

#### Segmentation

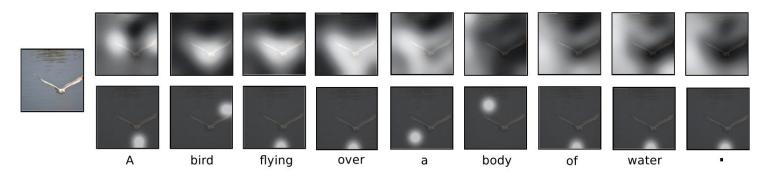


#### **Spatial Transformer**



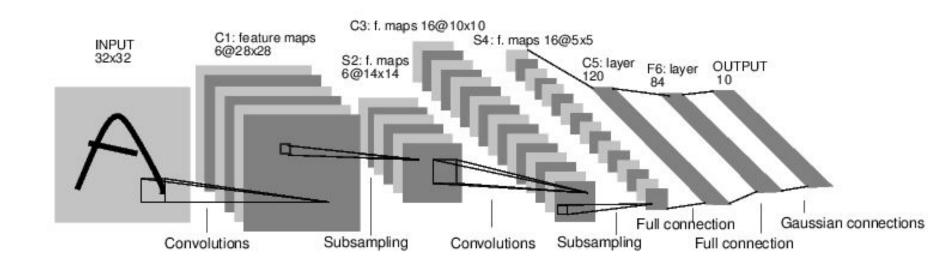


#### **Soft Attention**



# Videos

### ConvNets for images

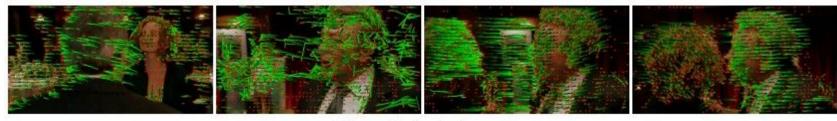


#### Feature-based approaches to Activity Recognition

Dense trajectories and motion boundary descriptors for action recognition Wang et al., 2013

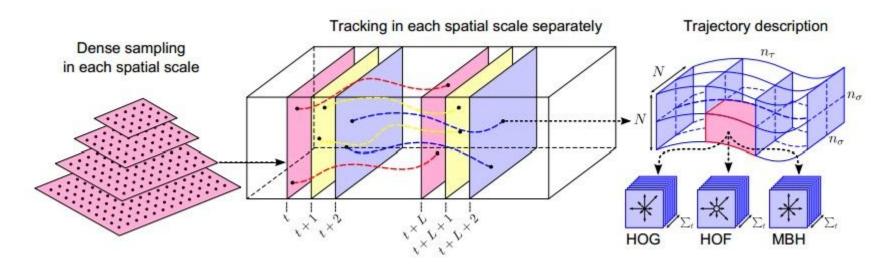
**Action Recognition with Improved Trajectories** Wang and Schmid, 2013

(code available!)



Dense trajectories

#### Dense trajectories and motion boundary descriptors for action recognition *Wang et al., 2013*

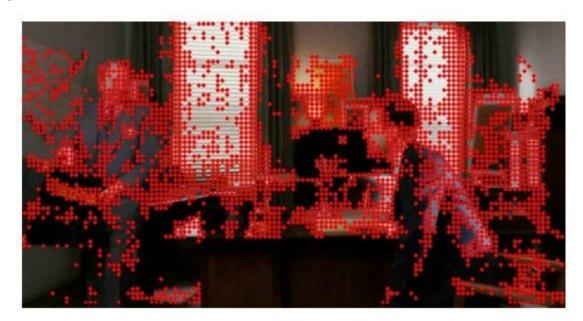


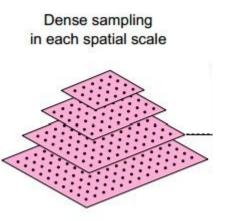
detect feature points

track features with optical flow

extract HOG/HOF/MBH features in the (stabilized) coordinate system of each tracklet

#### Dense trajectories and motion boundary descriptors for action recognition *Wang et al., 2013*





detected feature points

[J. Shi and C. Tomasi, "Good features to track," CVPR 1994] [Ivan Laptev 2005]

#### Dense trajectories and motion boundary descriptors for action recognition Wang et al., 2013

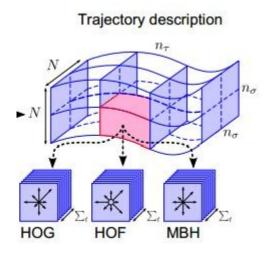


track each keypoint using optical flow.

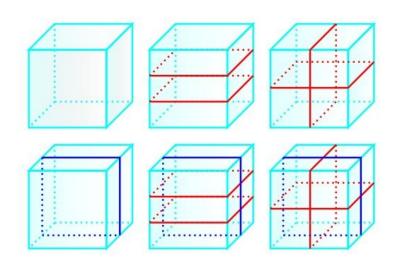
[G. Farnebäck, "Two-frame motion estimation based on polynomial expansion," 2003]

[T. Brox and J. Malik, "Large displacement optical flow: Descriptor matching in variational motion estimation," 2011]

#### Dense trajectories and motion boundary descriptors for action recognition *Wang et al.*, 2013



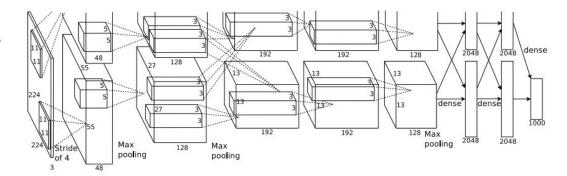
Extract features in the local coordinate system of each tracklet.



Accumulate into histograms, separately according to multiple spatio-temporal layouts.

#### Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4

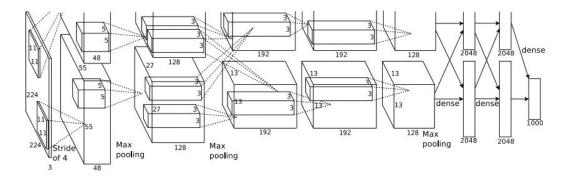
=>

Output volume [55x55x96]

Q: What if the input is now a small chunk of video? E.g. [227x227x3x15]?

#### Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4

=>

Output volume [55x55x96]

Q: What if the input is now a small chunk of video? E.g. [227x227x3x15]?

A: Extend the convolutional filters in time, perform spatio-temporal convolutions!

E.g. can have 11x11xT filters, where T = 2..15.

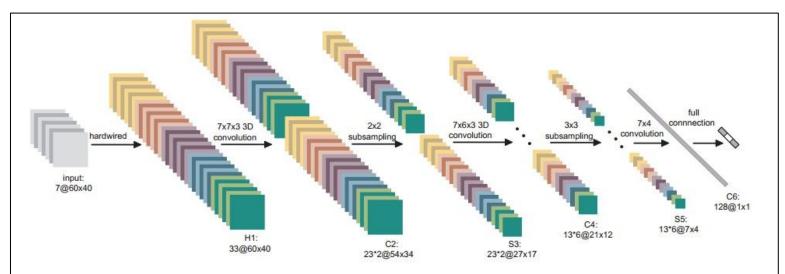
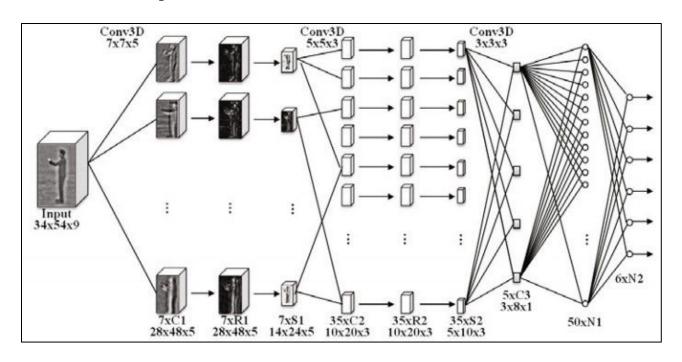


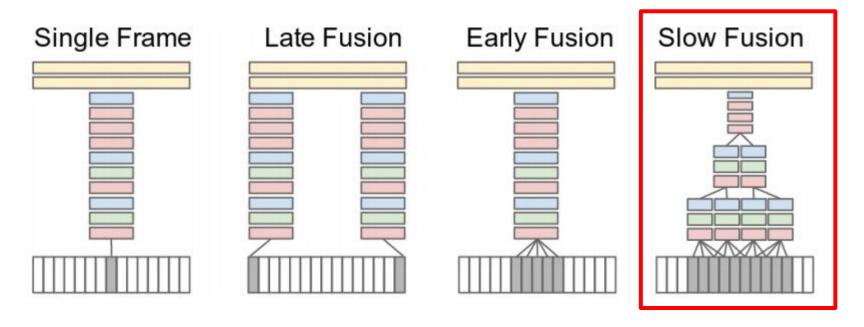
Figure 3. A 3D CNN architecture for human action recognition. This architecture consists of 1 hardwired layer, 3 convolution layers, 2 subsampling layers, and 1 full connection layer. Detailed descriptions are given in the text.

[3D Convolutional Neural Networks for Human Action Recognition, Ji et al., 2010]

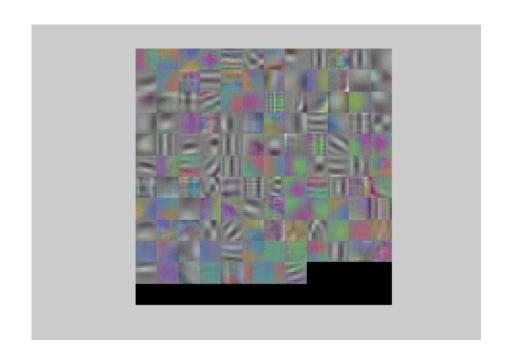


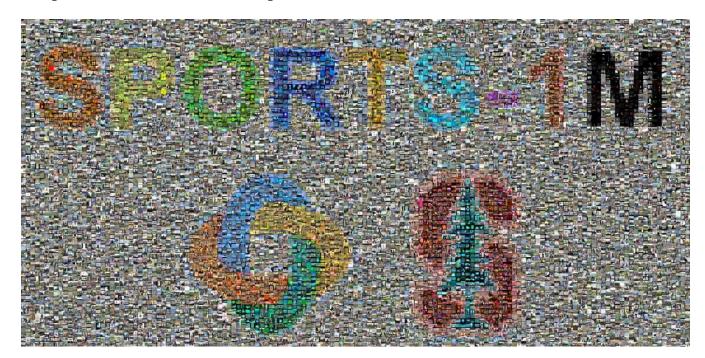
Sequential Deep Learning for Human Action Recognition, Baccouche et al., 2011

spatio-temporal convolutions; worked best.



Learned filters on the first layer

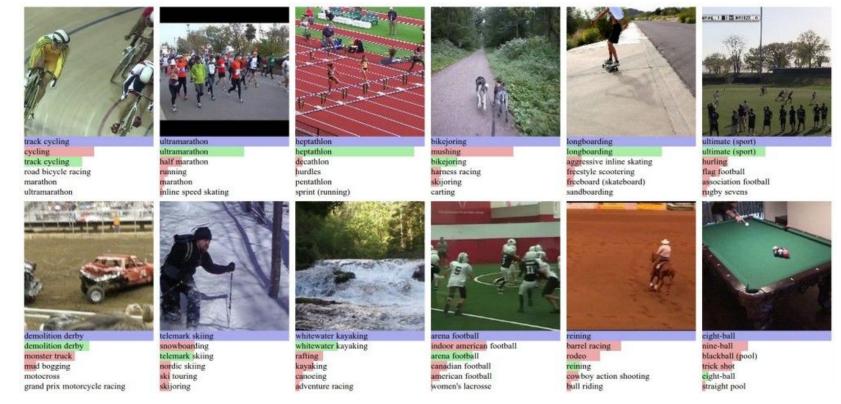




1 million videos 487 sports classes

		D 0 1
Clip Hit@1	Video Hit@1	Video Hit@5
	55.3	####
41.1	59.3	77.7
42.4	60.0	78.5
30.0	49.9	72.8
38.1	56.0	77.2
38.9	57.7	76.8
40.7	59.3	78.7
41.9	60.9	80.2
41.4	63.9	82.4
	41.1 42.4 30.0 38.1 38.9 40.7 41.9	- 55.3 41.1 59.3 42.4 60.0 30.0 49.9 38.1 56.0 38.9 57.7 40.7 59.3 41.9 60.9

The motion information didn't add all that much...



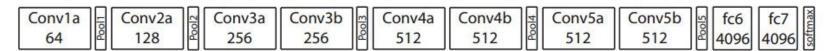
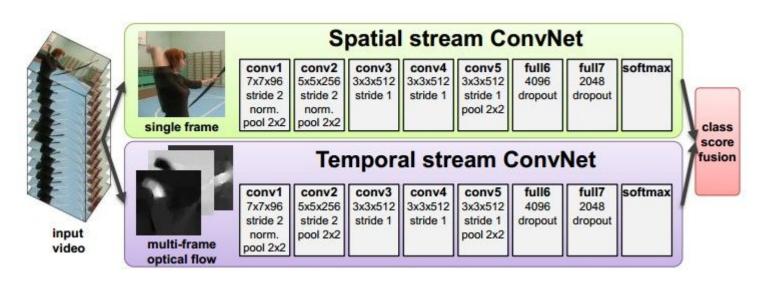


Figure 3. C3D architecture. C3D net has 8 convolution, 5 max-pooling, and 2 fully connected layers, followed by a softmax output layer. All 3D convolution kernels are  $3 \times 3 \times 3$  with stride 1 in both spatial and temporal dimensions. Number of filters are denoted in each box. The 3D pooling layers are denoted from pool1 to pool5. All pooling kernels are  $2 \times 2 \times 2$ , except for pool1 is  $1 \times 2 \times 2$ . Each fully connected layer has 4096 output units.

3D VGGNet, basically.

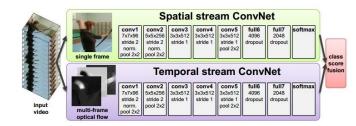
[Learning Spatiotemporal Features with 3D Convolutional Networks, Tran et al. 2015]



(of VGGNet fame)

[Two-Stream Convolutional Networks for Action Recognition in Videos, Simonyan and Zisserman 2014]

[T. Brox and J. Malik, "Large displacement optical flow: Descriptor matching in variational motion estimation," 2011]



Spatial stream ConvNet	73.0%	40.5%
Temporal stream ConvNet	83.7%	54.6%
Two-stream model (fusion by averaging)	86.9%	58.0%
Two-stream model (fusion by SVM)	88.0%	59.4%

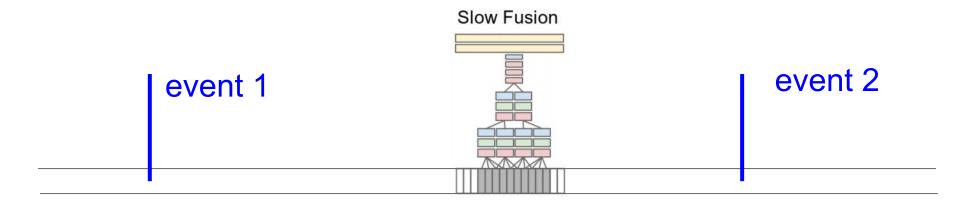
Two-stream version works much better than either alone.

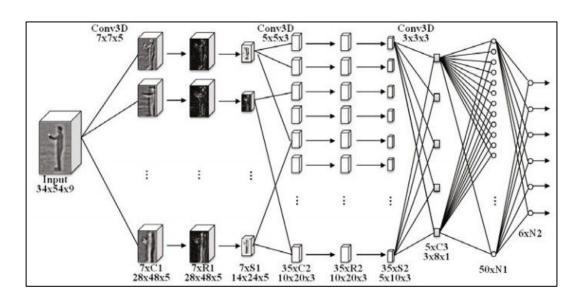
[Two-Stream Convolutional Networks for Action Recognition in Videos, **Simonyan** and Zisserman 2014]

[T. Brox and J. Malik, "Large displacement optical flow: Descriptor matching in variational motion estimation," 2011]

All 3D ConvNets so far used local motion cues to get extra accuracy (e.g. half a second or so)

Q: what if the temporal dependencies of interest are much much longer? E.g. several seconds?

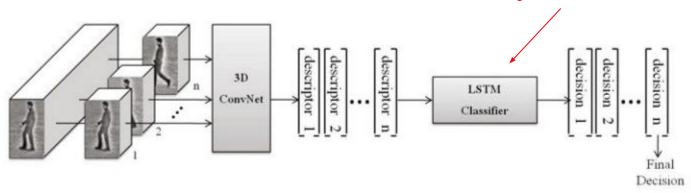




(This paper was way ahead of its time. Cited 65 times.)

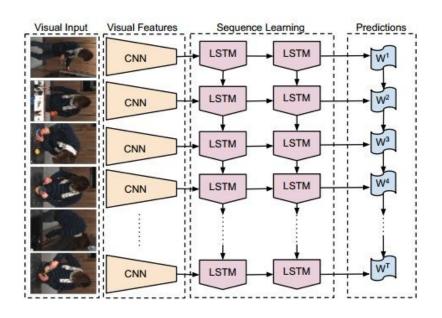
Sequential Deep Learning for Human Action Recognition, Baccouche et al., 2011

LSTM way before it was cool

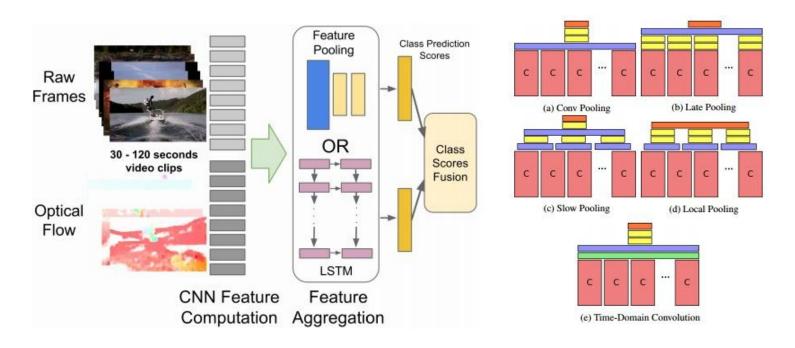


(This paper was way ahead of its time. Cited 65 times.)

Sequential Deep Learning for Human Action Recognition, Baccouche et al., 2011



[Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al., 2015]



[Beyond Short Snippets: Deep Networks for Video Classification, Ng et al., 2015]

# Summary so far

We looked at two types of architectural patterns:

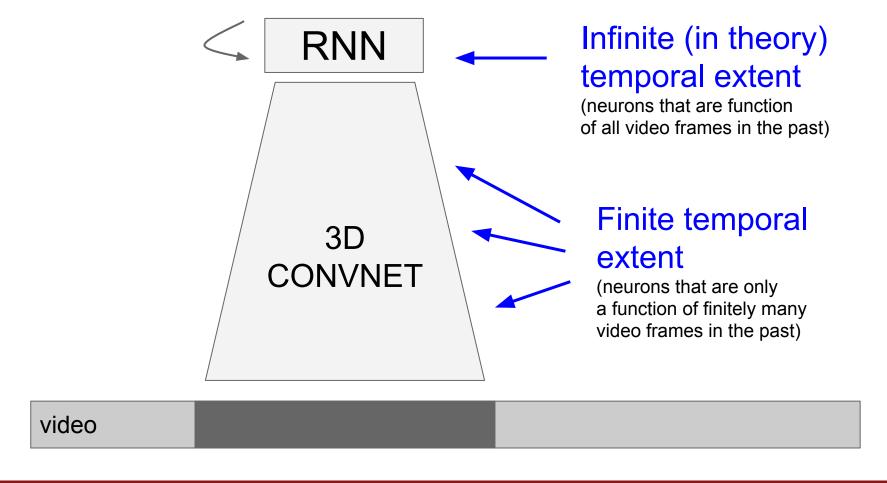
- 1. Model temporal motion locally (3D CONV)
- 2. Model temporal motion globally (LSTM / RNN)
- + Fusions of both approaches at the same time.

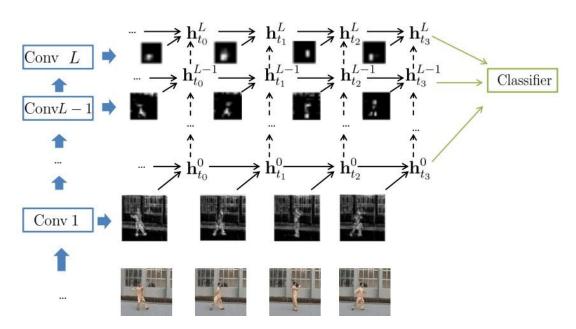
### Summary so far

We looked at two types of architectural patterns:

- 1. Model temporal motion locally (3D CONV)
- 2. Model temporal motion globally (LSTM / RNN)
- + Fusions of both approaches at the same time.

There is another (cleaner) way!





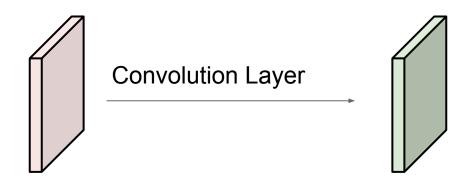
Beautiful:

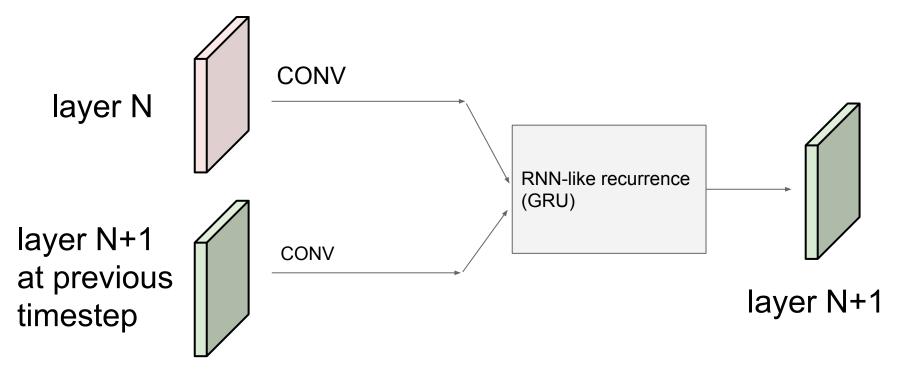
All neurons in the ConvNet are recurrent.

$$\begin{split} \mathbf{z}_t^l &= \sigma(\mathbf{W}_z^l * \mathbf{x}_t^l + \mathbf{U}_z^l * \mathbf{h}_{t-1}^l), \\ \mathbf{r}_t^l &= \sigma(\mathbf{W}_r^l * \mathbf{x}_t^l + \mathbf{U}_r^l * \mathbf{h}_{t-1}^l), \\ \tilde{\mathbf{h}}_t^l &= \tanh(\mathbf{W}^l * \mathbf{x}_t^l + \mathbf{U} * (\mathbf{r}_t^l \odot \mathbf{h}_{t-1}^l), \\ \mathbf{h}_t^l &= (1 - \mathbf{z}_t^l) \mathbf{h}_{t-1}^l + \mathbf{z}_t^l \tilde{\mathbf{h}}_t^l, \end{split}$$

Only requires (existing) 2D CONV routines. No need for 3D spatio-temporal CONV.

#### Normal ConvNet:





#### Recall: RNNs

$$h_t = f_W(h_{t-1}, x_t)$$

#### **GRU**

$$\mathbf{z}_{t} = \sigma(\mathbf{W}_{z}\mathbf{x}_{t} + \mathbf{U}_{z}\mathbf{h}_{t-1}),$$

$$\mathbf{r}_{t} = \sigma(\mathbf{W}_{r}\mathbf{x}_{t} + \mathbf{U}_{r}\mathbf{h}_{t-1}),$$

$$\tilde{\mathbf{h}}_{t} = \tanh(\mathbf{W}\mathbf{x}_{t} + \mathbf{U}(\mathbf{r}_{t} \odot \mathbf{h}_{t-1}))$$

$$\mathbf{h}_{t} = (1 - \mathbf{z}_{t})\mathbf{h}_{t-1} + \mathbf{z}_{t}\tilde{\mathbf{h}}_{t},$$

#### Vanilla RNN

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

LSTM 
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \text{tanh} \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$
$$h_t^l = o \odot \tanh(c_t^l)$$

#### Recall: RNNs

$$h_t = f_W(h_{t-1}, x_t)$$

#### **GRU**

$$\mathbf{z}_{t} = \sigma(\mathbf{W}_{z}\mathbf{x}_{t} + \mathbf{U}_{z}\mathbf{h}_{t-1}),$$

$$\mathbf{r}_{t} = \sigma(\mathbf{W}_{r}\mathbf{x}_{t} + \mathbf{U}_{r}\mathbf{h}_{t-1}),$$

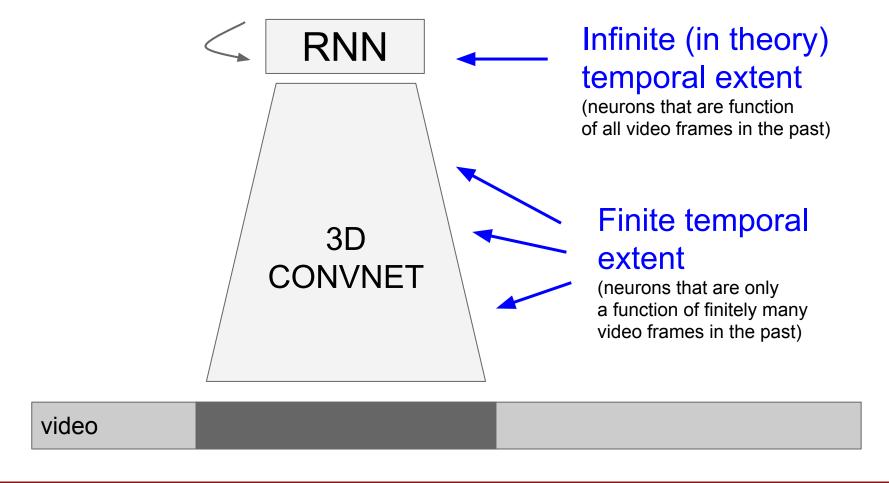
$$\tilde{\mathbf{h}}_{t} = \tanh(\mathbf{W}\mathbf{x}_{t} + \mathbf{U}(\mathbf{r}_{t} \odot \mathbf{h}_{t-1}))$$

$$\mathbf{h}_{t} = (1 - \mathbf{z}_{t})\mathbf{h}_{t-1} + \mathbf{z}_{t}\tilde{\mathbf{h}}_{t},$$

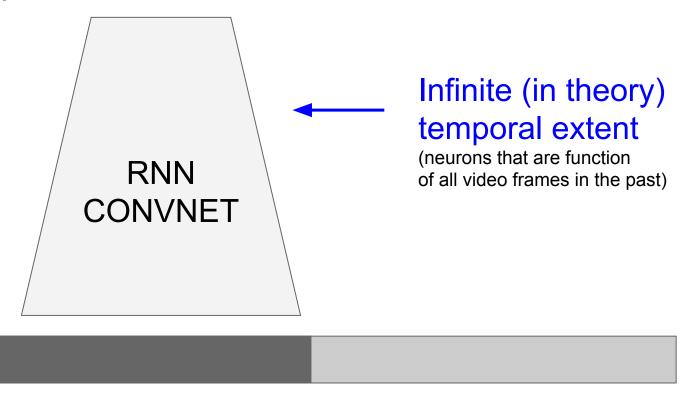
#### Matrix multiply



$$\begin{split} \mathbf{z}_t^l &= \sigma(\mathbf{W}_z^l * \mathbf{x}_t^l + \mathbf{U}_z^l * \mathbf{h}_{t-1}^l), \\ \mathbf{r}_t^l &= \sigma(\mathbf{W}_r^l * \mathbf{x}_t^l + \mathbf{U}_r^l * \mathbf{h}_{t-1}^l), \\ \tilde{\mathbf{h}}_t^l &= \tanh(\mathbf{W}^l * \mathbf{x}_t^l + \mathbf{U} * (\mathbf{r}_t^l \odot \mathbf{h}_{t-1}^l), \\ \mathbf{h}_t^l &= (1 - \mathbf{z}_t^l) \mathbf{h}_{t-1}^l + \mathbf{z}_t^l \tilde{\mathbf{h}}_t^l, \end{split}$$



#### i.e. we obtain:



# Summary

- You think you need a Spatio-Temporal Fancy Video ConvNet
- STOP. Do you really?
- Okay fine: do you want to model:
  - <u>local motion?</u> (use 3D CONV), or
  - global motion? (use LSTM).
- Try out using Optical Flow in a second stream (can work better sometimes)
- Try out GRU-RCN! (imo best model)

## **Unsupervised Learning**

### **Unsupervised Learning Overview**

- Definitions
- Autoencoders
  - Vanilla
  - Variational
- Adversarial Networks

## Supervised vs Unsupervised

#### **Supervised Learning**

**Data**: (x, y) x is data, y is label

**Goal**: Learn a *function* to map x -> y

**Examples:** Classification, regression, object detection, semantic segmentation, image captioning, etc

## Supervised vs Unsupervised

#### **Supervised Learning**

**Data**: (x, y) x is data, y is label

**Goal**: Learn a *function* to map x -> y

**Examples:** Classification, regression, object detection, semantic segmentation, image captioning, etc

#### **Unsupervised Learning**

**Data**: x

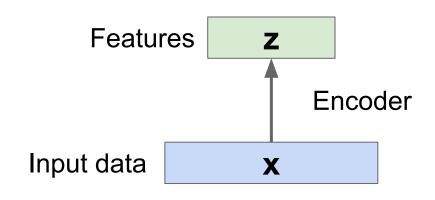
Just data, no labels!

**Goal**: Learn some *structure* of the data

**Examples**: Clustering, dimensionality reduction, feature learning, generative models, etc.

### **Unsupervised Learning**

- Autoencoders
  - Traditional: feature learning
  - Variational: generate samples
- Generative Adversarial Networks: Generate samples

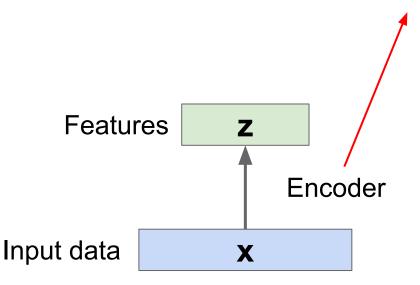


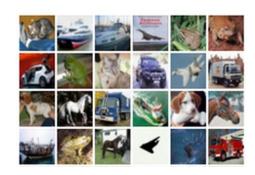


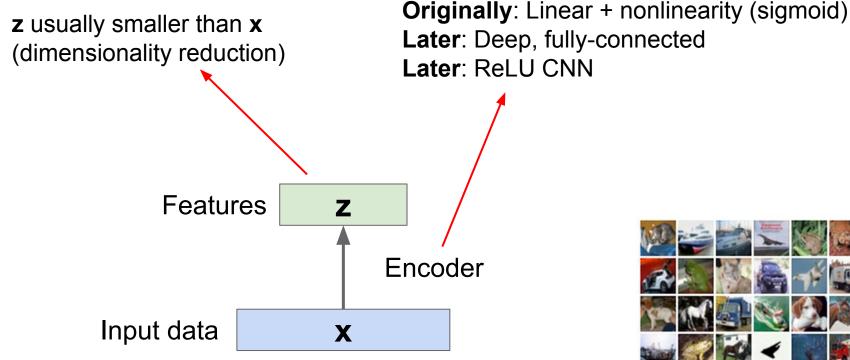
**Originally**: Linear + nonlinearity (sigmoid)

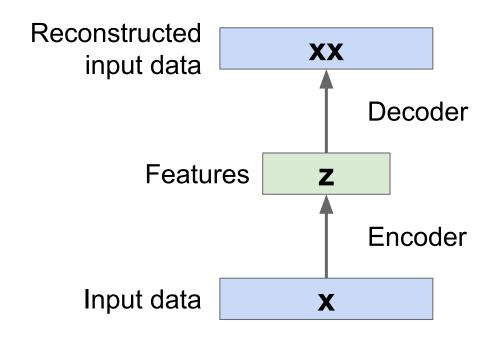
Later: Deep, fully-connected

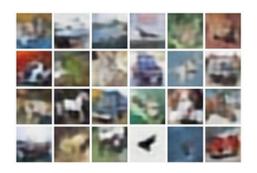
Later: ReLU CNN









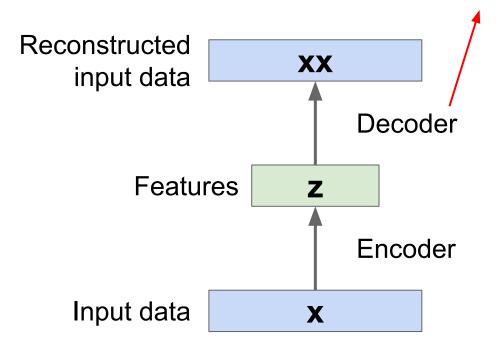




Originally: Linear + nonlinearity (sigmoid)

Later: Deep, fully-connected

Later: ReLU CNN (upconv)





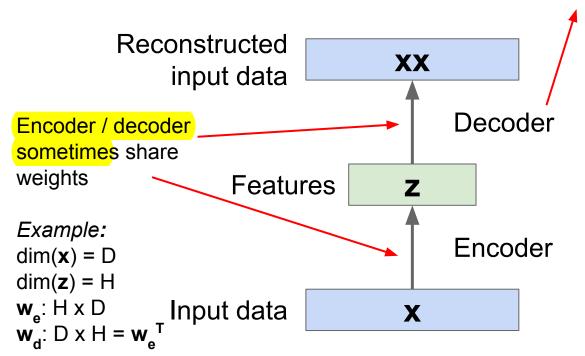
**Encoder**: 4-layer conv **Decoder**: 4-layer upconv



Originally: Linear + nonlinearity (sigmoid)

Later: Deep, fully-connected

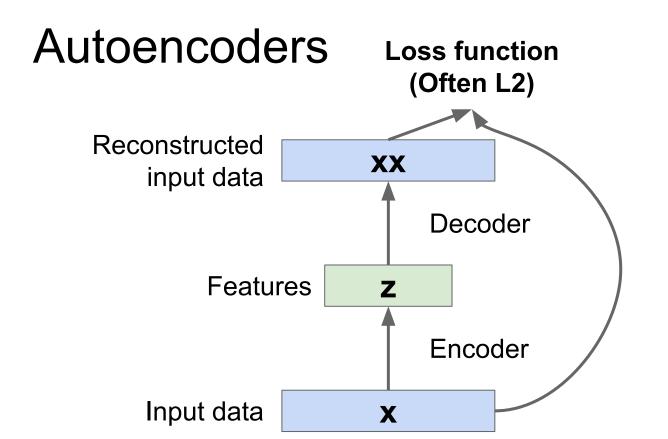
Later: ReLU CNN (upconv)

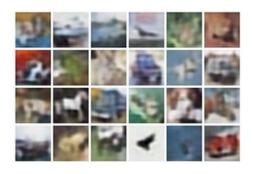




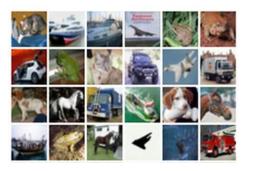
Train for reconstruction with no labels!

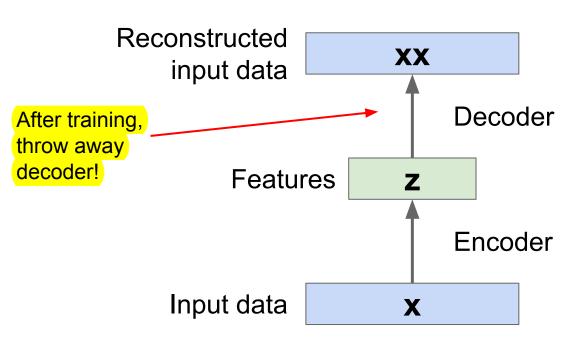


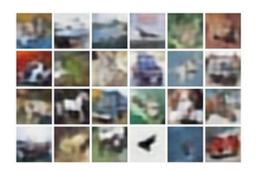




Train for reconstruction with no labels!



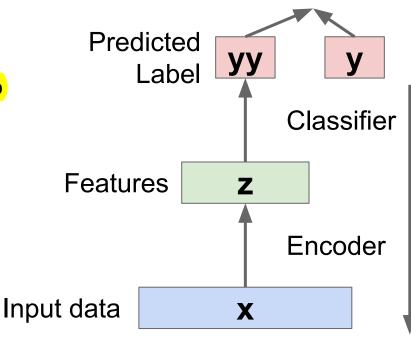






Loss function (Softmax, etc)

Use encoder to initialize a supervised model



bird plane dog deer truck

Train for final task (sometimes with small data)



Fine-tune

jointly with

classifier

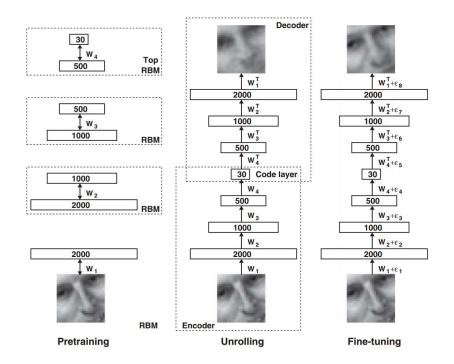
encoder

## Autoencoders: Greedy Training

In mid 2000s layer-wise pretraining with Restricted Boltzmann Machines (RBM) was common

#### Training deep nets was hard in 2006!

It is difficult to optimize the weights in nonlinear autoencoders that have multiple hidden layers (2–4). With large initial weights, autoencoders typically find poor local minima; with small initial weights, the gradients in the early layers are tiny, making it infeasible to train autoencoders with many hidden layers. If



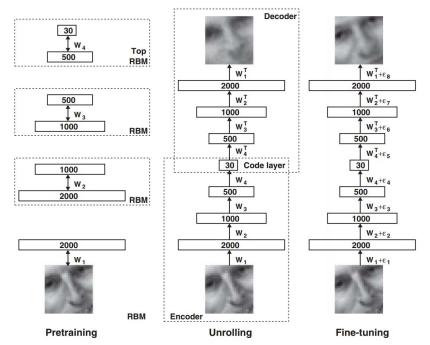
Hinton and Salakhutdinov, "Reducing the Dimensionality of Data with Neural Networks", Science 2006

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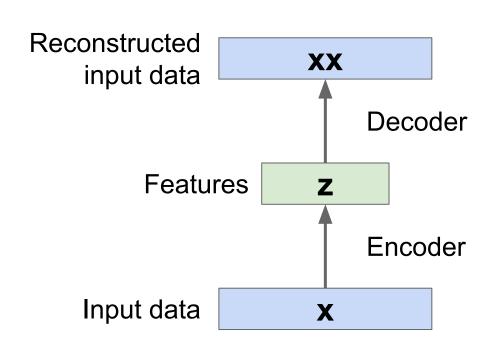
It is difficult to optimize the weights in nonlinear autoencoders that have multiple hidden layers (2–4). With large initial weights, autoencoders typically find poor local minima; with small initial weights, the gradients in the early layers are tiny, making it infeasible to train autoencoders with many hidden layers. If



Not common anymore

With ReLU, proper initialization, batchnorm, Adam, etc easily train from scratch

Hinton and Salakhutdinov, "Reducing the Dimensionality of Data with Neural Networks", Science 2006

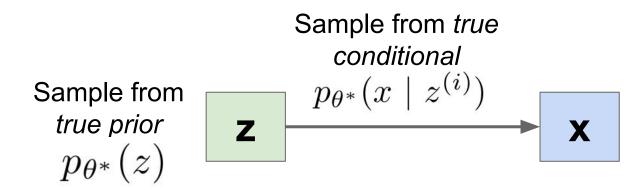


Autoencoders can reconstruct data, and can learn features to initialize a supervised model

Can we generate images from an autoencoder?

A Bayesian spin on an autoencoder - lets us generate data!

Assume our data  $\{x^{(i)}\}_{i=1}^N$  is generated like this:

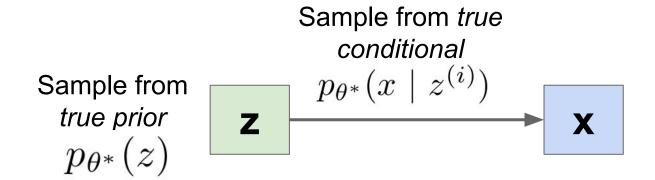


Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

A Bayesian spin on an autoencoder!

Assume our data  $\{x^{(i)}\}_{i=1}^{N}$  is generated like this:

Intuition: x is an image, z gives class, orientation, attributes, etc

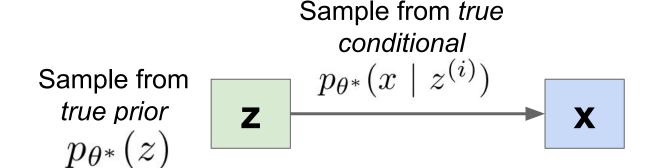


Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

A Bayesian spin on an autoencoder!

Assume our data  $\{x^{(i)}\}_{i=1}^N$  is generated like this:

Intuition: x is an image, z gives class, orientation, attributes, etc



**Problem**: Estimate  $\theta$  without access to latent states  $\gamma(i)$ !

Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

**Prior**: Assume  $p_{\theta}(z)$  is a unit Gaussian

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Conditional: Assume  $p_{\theta}(x \mid z)$  is a diagonal Gaussian, predict mean and variance with neural net

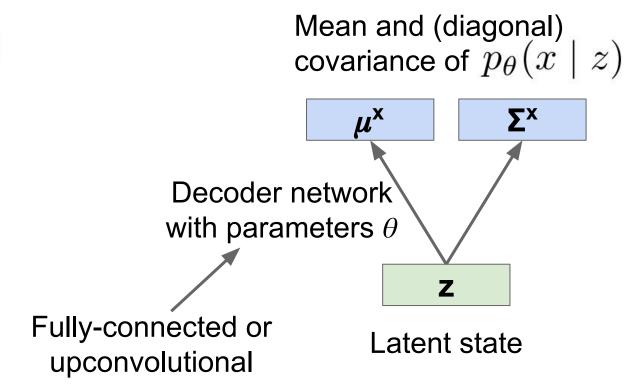
**Prior**: Assume  $p_{\theta}(z)$  is a unit Gaussian

Conditional: Assume  $p_{\theta}(x \mid z)$  is a diagonal Gaussian, predict mean and variance with neural net

Mean and (diagonal) covariance of  $p_{\theta}(x \mid z)$  $\sum_{X}$ Decoder network with parameters  $\theta$ Latent state

**Prior**: Assume  $p_{\theta}(z)$  is a unit Gaussian

Conditional: Assume  $p_{\theta}(x \mid z)$  is a diagonal Gaussian, predict mean and variance with neural net



By Bayes Rule the posterior is:

$$p_{\theta}(z \mid x) = \frac{p_{\theta}(x \mid z)p_{\theta}(z)}{p_{\theta}(x)}$$

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$$p_{\theta}(z \mid x) = \frac{p_{\theta}(x \mid z) p_{\theta}(z)}{p_{\theta}(x)}$$

Use decoder network =)
Gaussian =)
Intractible integral =(

By Bayes Rule the posterior is:

$$p_{\theta}(z \mid x) = \frac{p_{\theta}(x \mid z)p_{\theta}(z)}{p_{\theta}(x)}$$

Use decoder network =)
Gaussian =)
Intractible integral =(

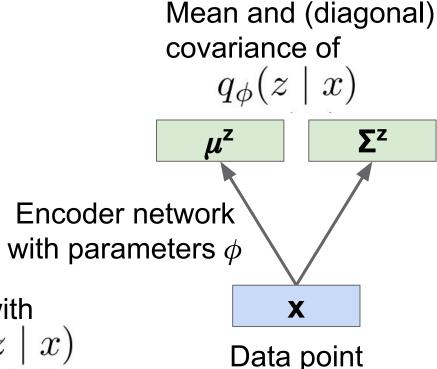
Mean and (diagonal) covariance of **Encoder network** with parameters  $\phi$ Data point

By Bayes Rule the posterior is:

$$p_{\theta}(z \mid x) = \frac{p_{\theta}(x \mid z) p_{\theta}(z)}{p_{\theta}(x)}$$

Use decoder network =)
Gaussian =)
Intractible integral =(

Approximate posterior with encoder network  $q_{\phi}(z \mid x)$ 



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$$p_{\theta}(z \mid x) = \frac{p_{\theta}(x \mid z) p_{\theta}(z)}{p_{\theta}(x)}$$

Use decoder network =)
Gaussian =)
Intractible integral =(

Approximate posterior with encoder network  $q_\phi(z\mid x)$ 

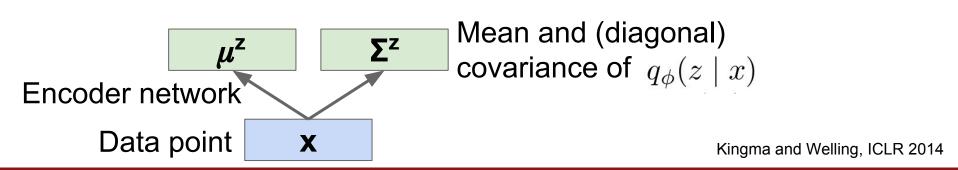
Kingma and Welling, ICLR 2014

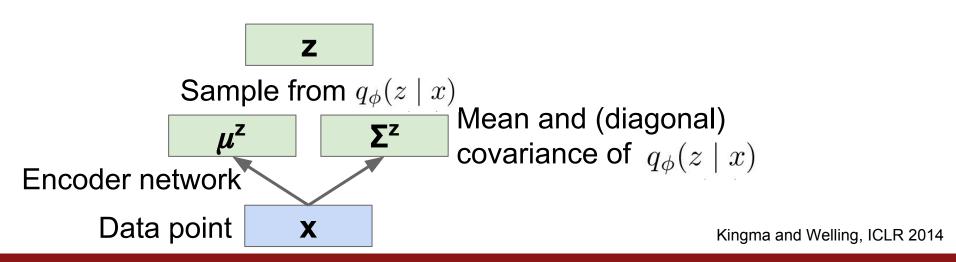
Fully-connected or convolutional  $q_{\phi}(z \mid x)$ Encoder network with parameters  $\phi$ 

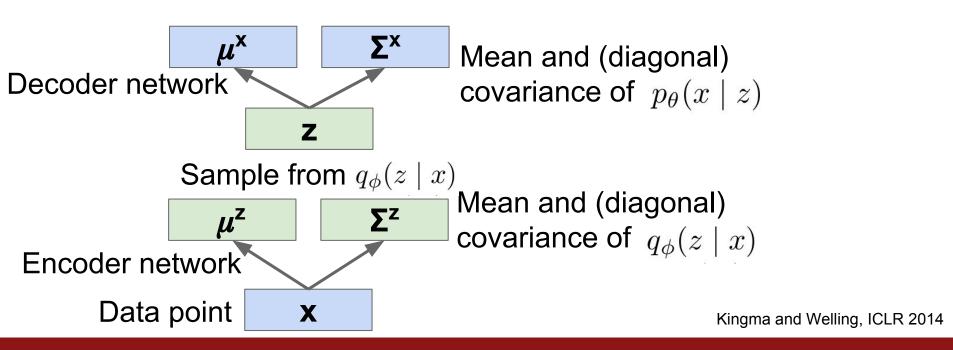
Data point

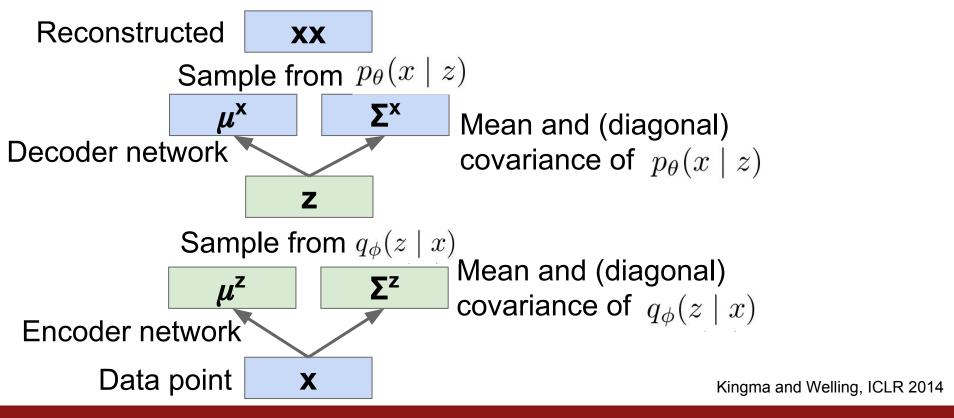
Mean and (diagonal)

Data point **x** 

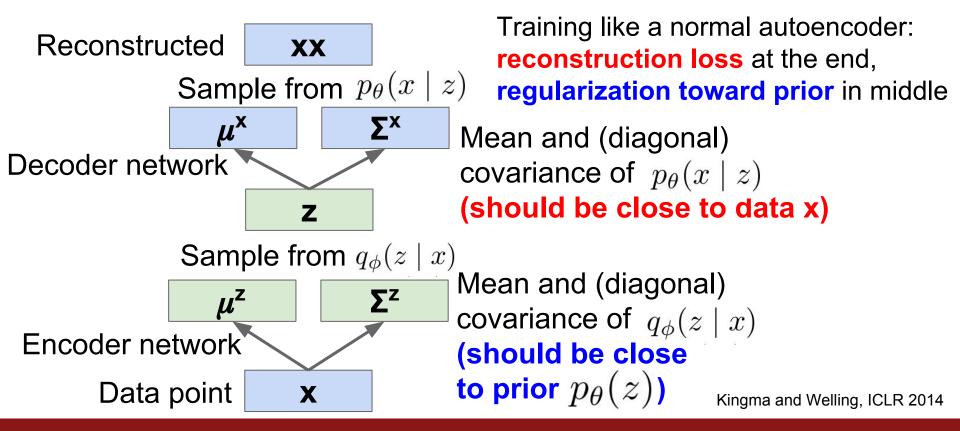








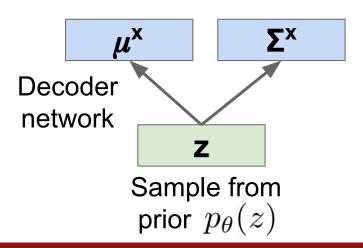
## Variational Autoencoder

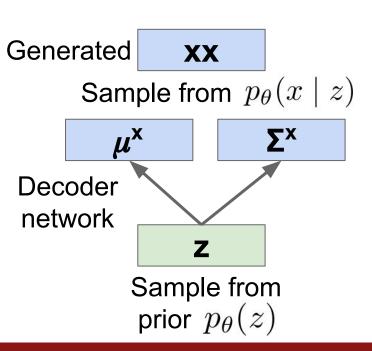


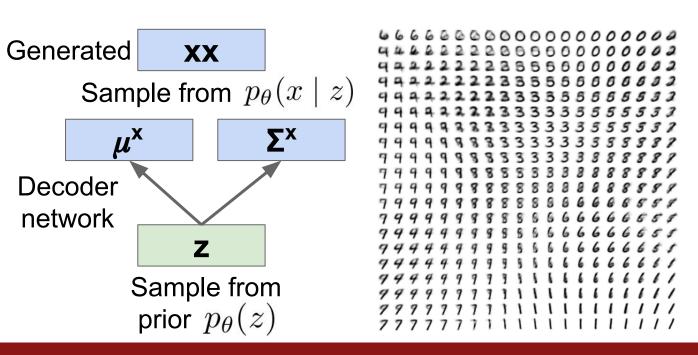
After network is trained:

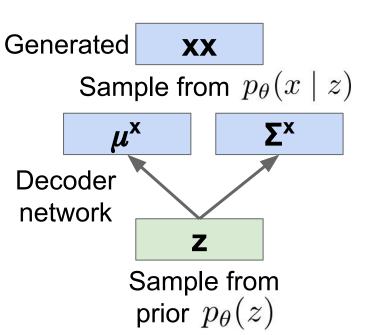
Z

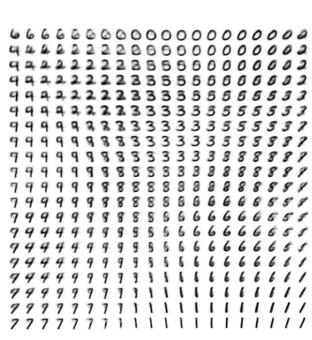
Sample from prior  $p_{\theta}(z)$ 













Diagonal prior on **z** => After network is trained: independent latent variables 00000000000000 Generated XX Sample from  $p_{\theta}(x \mid z)$ Decoder network Sample from prior  $p_{\theta}(z)$ 

# Variational Autoencoder: Math Maximum Likelihood?

$$\theta^* = \arg\max_{\theta} \prod_{i=1}^N p_{\theta}(x^{(i)})$$
 Maximize likelihood of dataset  $\{x^{(i)}\}_{i=1}^N$ 

Kingma and Welling, ICLR 2014

## Variational Autoencoder: Math Maximum Likelihood?

$$\begin{aligned} \theta^* &= \arg\max_{\theta} \prod_{i=1}^N p_{\theta}(x^{(i)}) \quad \text{Maximize likelihood of dataset } \left\{x^{(i)}\right\}_{i=1}^N \\ &= \arg\max_{\theta} \sum_{i=1}^N \log p_{\theta}(x^{(i)}) \quad \text{Maximize log-likelihood instead} \\ &\text{because sums are nicer} \end{aligned}$$

Kingma and Welling, ICLR 2014

## Variational Autoencoder: Math Maximum Likelihood?

$$\begin{split} \theta^* &= \arg\max_{\theta} \prod_{i=1}^N p_{\theta}(x^{(i)}) \quad \text{Maximize likelihood of dataset } \left\{x^{(i)}\right\}_{i=1}^N \\ &= \arg\max_{\theta} \sum_{i=1}^N \log p_{\theta}(x^{(i)}) \quad \text{Maximize log-likelihood instead because sums are nicer} \end{split}$$

$$p_{\theta}(x^{(i)}) = \int p_{\theta}(x^{(i)}, z) dz$$
 Marginalize joint distribution

Kingma and Welling, ICLR 2014

# Variational Autoencoder: Math Maximum Likelihood?

$$\begin{split} \theta^* &= \arg\max_{\theta} \prod_{i=1}^N p_{\theta}(x^{(i)}) \quad \text{Maximize likelihood of dataset } \left\{x^{(i)}\right\}_{i=1}^N \\ &= \arg\max_{\theta} \sum_{i=1}^N \log p_{\theta}(x^{(i)}) \quad \text{Maximize log-likelihood instead because sums are nicer} \end{split}$$

$$p_{ heta}(x^{(i)}) = \int p_{ heta}(x^{(i)},z)dz = \int p_{ heta}(x^{(i)}\mid z)p_{ heta}(z)dz$$
 Intractible integral =(

 $\log p_{\theta}(x^{(i)})$ 

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$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$
$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \qquad (\text{Bayes' Rule})$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

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$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)}|z)p_{\theta}(z)}{p_{\theta}(z|x^{(i)})} \right] \quad (\text{Bayes' Rule})$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)}|z)p_{\theta}(z)}{p_{\theta}(z|x^{(i)})} \frac{q_{\phi}(z|x^{(i)})}{q_{\phi}(z|x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)}|z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z|x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z|x^{(i)})}{p_{\theta}(z|x^{(i)})} \right] \quad (\text{Logarithms})$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

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$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Logarithms})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z)) + D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Bayes' Rule})$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Logarithms})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z)) + D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))$$

$$\mathcal{L}(x^{(i)}, \theta, \phi) \text{ "Elbow"}$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Bayes' Rule})$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Logarithms})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z)) + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))}_{\geq 0} \right]$$

$$\stackrel{\mathcal{L}(x^{(i)}, \theta, \phi) \text{ "Flhow"}}{} \stackrel{\text{"Flhow"}}{} \geq 0$$

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \qquad (\text{Bayes' Rule})$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \qquad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \qquad (\text{Logarithms})$$

$$= \underbrace{\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z))}_{\mathcal{L}(x^{(i)}, \theta, \phi)} + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))}_{\geq 0} \right]}_{\geq 0}$$

$$\log p_{\theta}(x^{(i)}) \ge \mathcal{L}(x^{(i)}, \theta, \phi)$$

### Variational lower bound (elbow)

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)}|z)p_{\theta}(z)}{p_{\theta}(z|x^{(i)})} \right] \quad (\text{Bayes' Rule})$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)}|z)p_{\theta}(z)}{p_{\theta}(z|x^{(i)})} \frac{q_{\phi}(z|x^{(i)})}{q_{\phi}(z|x^{(i)})} \right] \quad (\text{Multiply by constant})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)}|z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z|x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z|x^{(i)})}{p_{\theta}(z|x^{(i)})} \right] \quad (\text{Logarithms})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)}|z) \right] - D_{KL}(q_{\phi}(z|x^{(i)}) || p_{\theta}(z)) + D_{KL}(q_{\phi}(z|x^{(i)}) || p_{\theta}(z|x^{(i)})) \right]$$

$$\stackrel{\geq 0}{N}$$

 $\log p_{\theta}(x^{(i)}) \geq \mathcal{L}(x^{(i)}, \theta, \phi)$ Variational lower bound (elbow)  $\theta^*, \phi^* = \arg\max_{\theta, \phi} \sum_{i=1} \mathcal{L}(x^{(i)}, \theta, \phi)$ 

Training: Maximize lower bound

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$$\begin{split} \log p_{\theta}(x^{(i)}) &= \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \quad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z) \\ \textbf{Reconstruct} \\ \textbf{the input} &= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Bayes' Rule}) \\ \textbf{data} \\ &= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right] \quad (\text{Multiply by constant}) \\ &= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \quad (\text{Logarithms}) \\ &= \underbrace{\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z))}_{\mathcal{D}(x^{(i)}, \theta, \phi)} + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))}_{\geq 0} \\ &\geq 0 \\ \log p_{\theta}(x^{(i)}) \geq \mathcal{L}(x^{(i)}, \theta, \phi) \qquad \qquad \theta^{*}, \phi^{*} = \arg \max_{\theta, \phi} \sum_{i=1}^{N} \mathcal{L}(x^{(i)}, \theta, \phi) \end{split}$$

Variational lower bound (elbow)

Latent states should follow the prior

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[\log p_{\theta}(x^{(i)})\right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$
 Reconstruct

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x + z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right]$$

The input 
$$= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right]$$
 (Bayes' Rule)

data
$$= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right]$$
 (Multiply by constant)
$$= \mathbf{E}_z \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] + \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right]$$
 (Logarithms)

$$= \underbrace{\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z))}_{C(x^{(i)} \mid \theta \mid \phi)} + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z \mid x^{(i)}))}_{\geq 0}$$

$$\log p_{\theta}(x^{(i)}) > \mathcal{L}(x^{(i)}, \theta, \phi)$$

$$\mathcal{L}(x^{(i)}, \theta, \phi)$$
 "Elbow" 
$$\theta^*, \phi^* = \arg\max_{\theta, \phi} \sum_{i=1}^{N} \mathcal{L}(x^{(i)}, \theta, \phi)$$

**Latent states** should follow the prior

 $\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$  **Reconstruct** the input  $= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right]$  (Bayes' Rule) data

Sampling  $= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right]$  (Multiply by constant) with reparam.  $= \mathbf{E}_z \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] + \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right]$  (Logarithms)

the input 
$$= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right]$$

(Bayes
$$\frac{q_{\phi}(z)}{q_{\phi}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})}$$

Sampling 
$$= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(t)} \mid z)p_{\theta}}{p_{\theta}(z \mid x^{(t)})} \right]$$

trick

$$\mathbf{E}_z \left[ \log p_t(x^{(i)} \mid z) \right] - \mathbf{E}_z$$

$$\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z)) + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z \mid x^{(i)}))}_{N} + \underbrace{D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z \mid x^{(i)}))}_{N} \right]$$

$$\frac{1}{2} + \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right]$$

$$\geq 0$$

$$\log p_{\theta}(x^{(i)}) \ge \mathcal{L}(x^{(i)}, \theta, \phi)$$

$$\theta^*, \phi^* = \arg\max_{\theta, \phi} \sum_{i=1} \mathcal{L}(x^{(i)}, \theta, \phi)$$

Variational lower bound (elbow)

**Latent states** should follow the prior

 $\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$  **Reconstruct** the input  $= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right]$  (Bayes' Rule)

**Everything is** Gaussian,

Sampling  $= \mathbf{E}_z \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})} \right]$  (Multiply by constant) closed form solution! with reparam.  $= \mathbf{E}_z \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_z \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right]$  (Logarithms)

trick

reparam.

 $= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z)) + D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z \mid x^{(i)})) \right]$  $\mathcal{L}(x^{(i)}, \theta, \phi)$ 

 $\log p_{\theta}(x^{(i)}) \ge \mathcal{L}(x^{(i)}, \theta, \phi)$ Variational lower bound (elbow)  $\theta^*, \phi^* = \arg\max_{\theta, \phi} \sum \mathcal{L}(x^{(i)}, \theta, \phi)$ 

### **Autoencoder Overview**

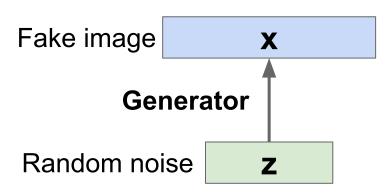
- Traditional Autoencoders
  - Try to reconstruct input
  - Used to learn features, initialize supervised model
  - Not used much anymore
- Variational Autoencoders
  - Bayesian meets deep learning
  - Sample from model to generate images

Can we generate images with less math?

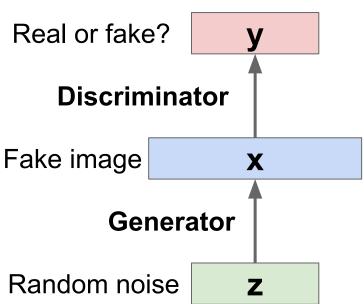
Random noise

Z

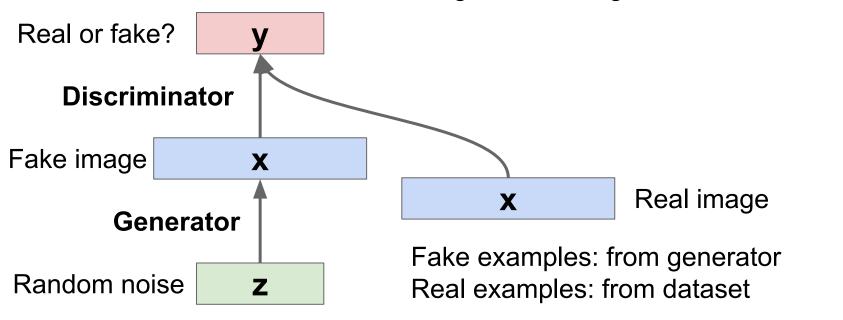
Can we generate images with less math?



Can we generate images with less math?

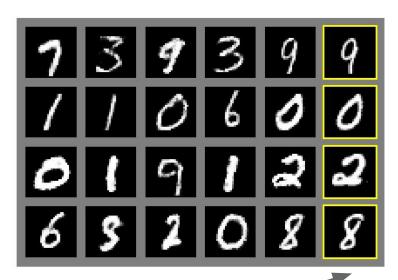


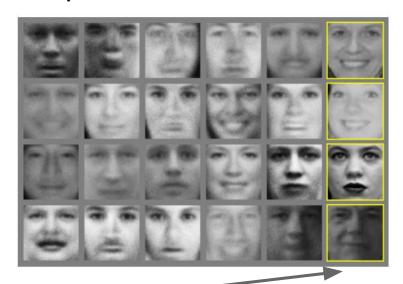
Can we generate images with less math?



Can we generate images with less math? Real or fake? Train generator and discriminator jointly **Discriminator** After training, easy to generate images Fake image Real image Generator Fake examples: from generator Random noise Real examples: from dataset

Generated samples





Nearest neighbor from training set

Goodfellow et al, "Generative Adversarial Nets", NIPS 2014

Generated samples (CIFAR-10)

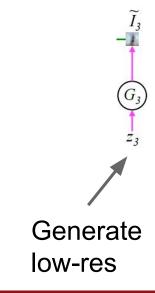




Nearest neighbor from training set

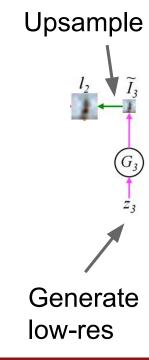
Goodfellow et al, "Generative Adversarial Nets", NIPS 2014

## Generative Adversarial Nets: Multiscale



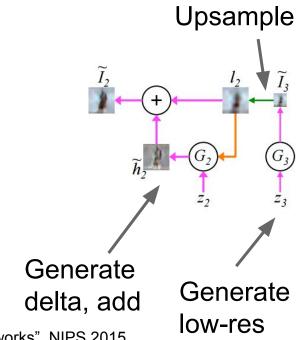
Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015

## Generative Adversarial Nets: Multiscale

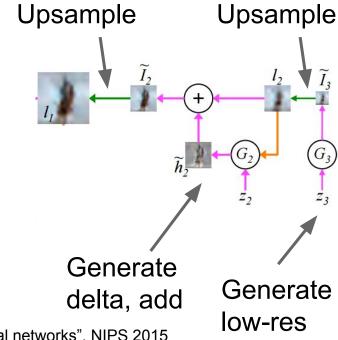


Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015

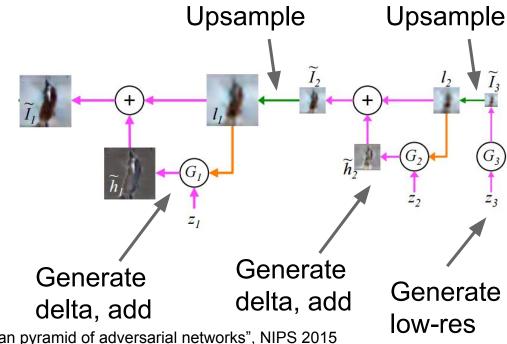
## Generative Adversarial Nets: Multiscale



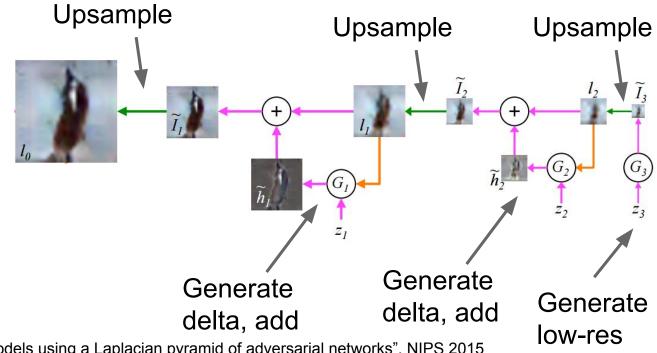
Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015



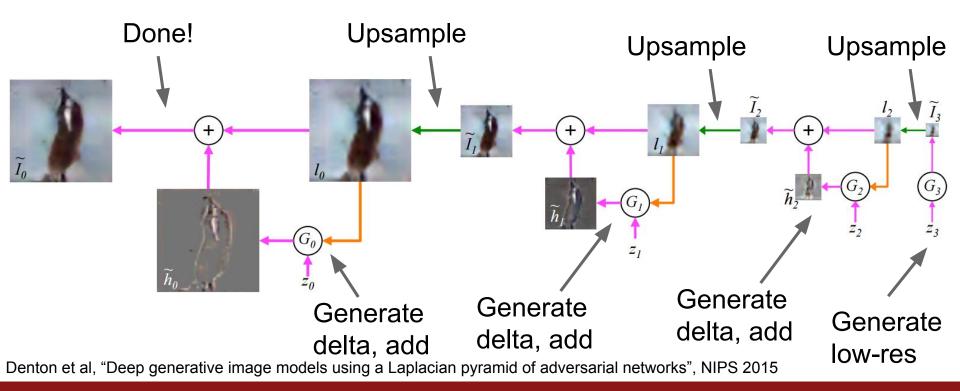
Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015

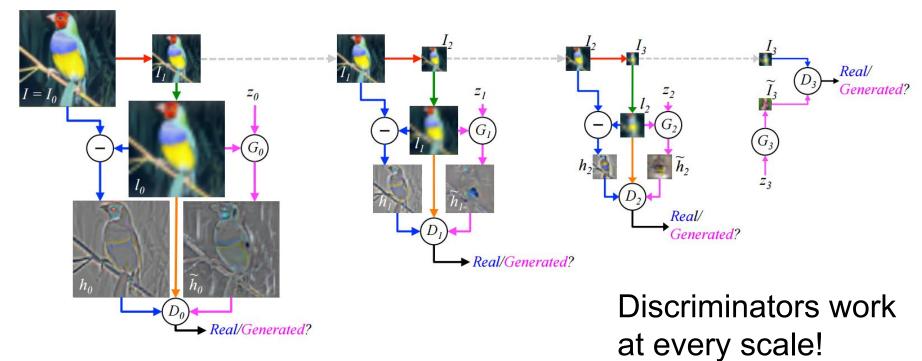


Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015



Denton et al, "Deep generative image models using a Laplacian pyramid of adversarial networks", NIPS 2015





Denton et al, NIPS 2015



Train separate model per-class on CIFAR-10

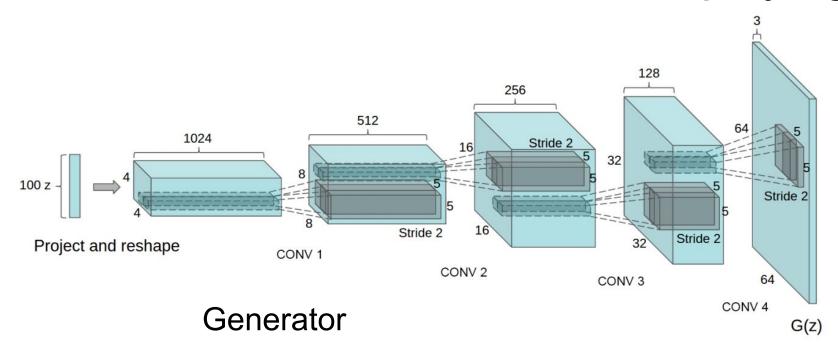
Denton et al, NIPS 2015

Generator is an upsampling network with fractionally-strided convolutions Discriminator is a convolutional network

Architecture guidelines for stable Deep Convolutional GANs

- Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator).
- Use batchnorm in both the generator and the discriminator.
- Remove fully connected hidden layers for deeper architectures.
- Use ReLU activation in generator for all layers except for the output, which uses Tanh.
- Use LeakyReLU activation in the discriminator for all layers.

Radford et al, "Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks", ICLR 2016



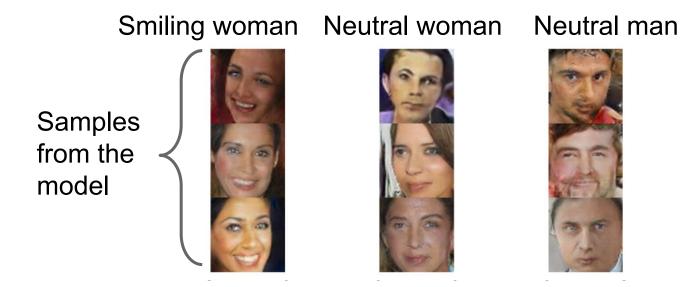
Radford et al, "Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks", ICLR 2016

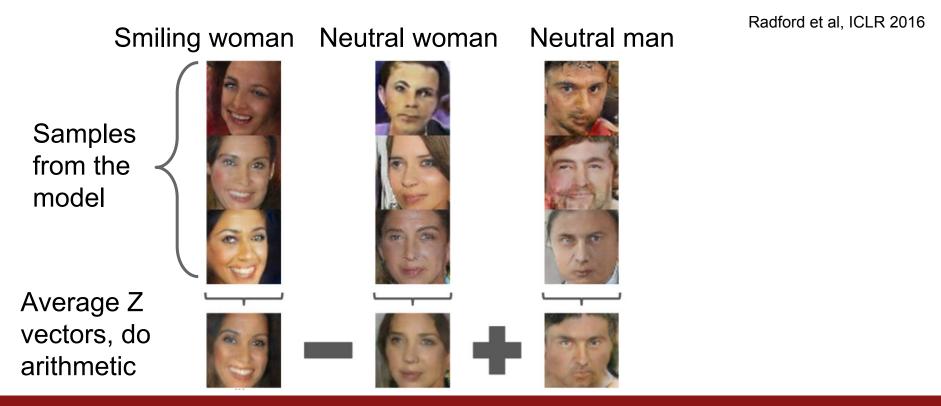
Samples from the model look amazing!



Interpolating between random points in laten space

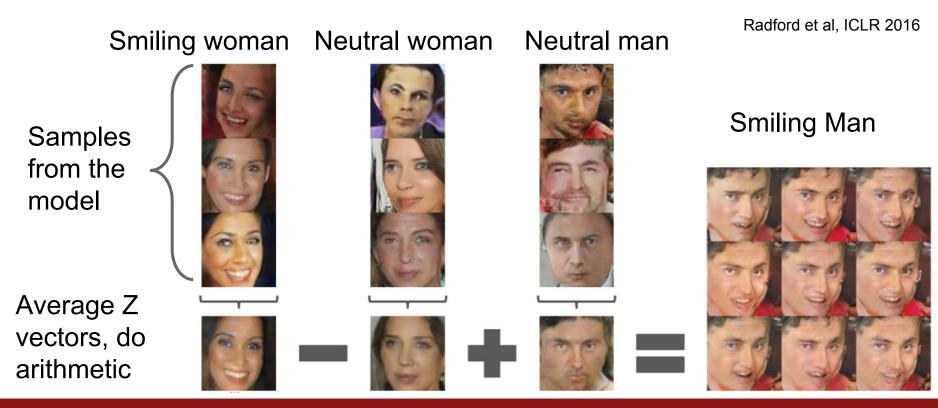






Fei-Fei Li & Andrej Karpathy & Justin Johnson

Lecture 14 - 120 29 Feb 2016



Glasses man No glasses man No glasses woman Radford et al.

Glasses man





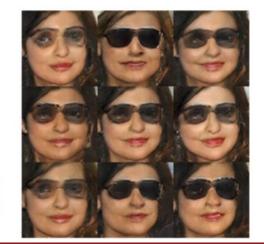






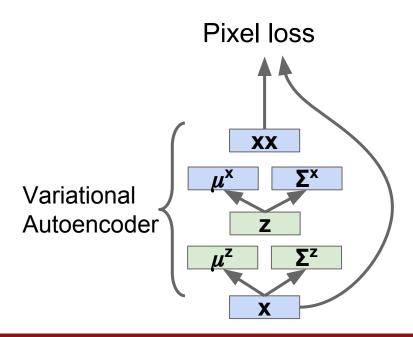




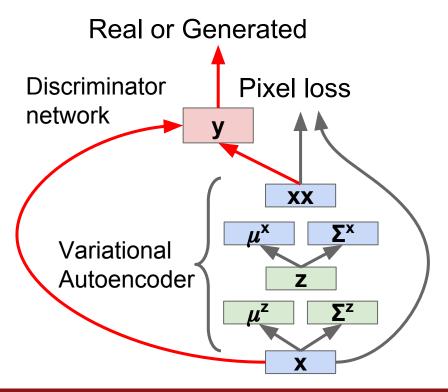




Dosovitskiy and Brox, "Generating Images with Perceptual Similarity Metrics based on Deep Networks", arXiv 2016

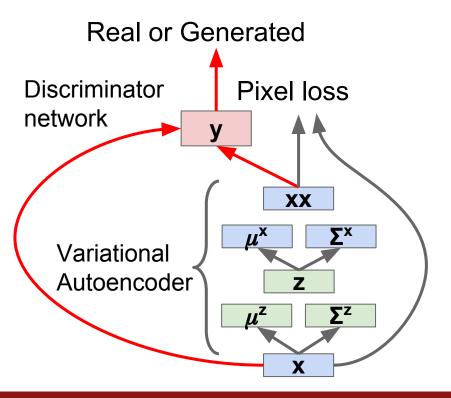


Dosovitskiy and Brox, "Generating Images with Perceptual Similarity arXiv 2016

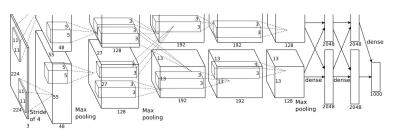


Metrics based on Deep Networks",

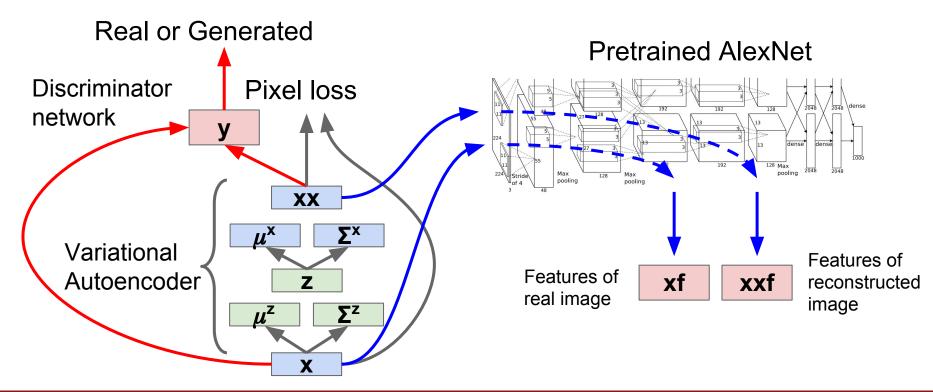
Dosovitskiy and Brox, "Generating Images with Perceptual Similarity Metrics based on Deep Networks", arXiv 2016



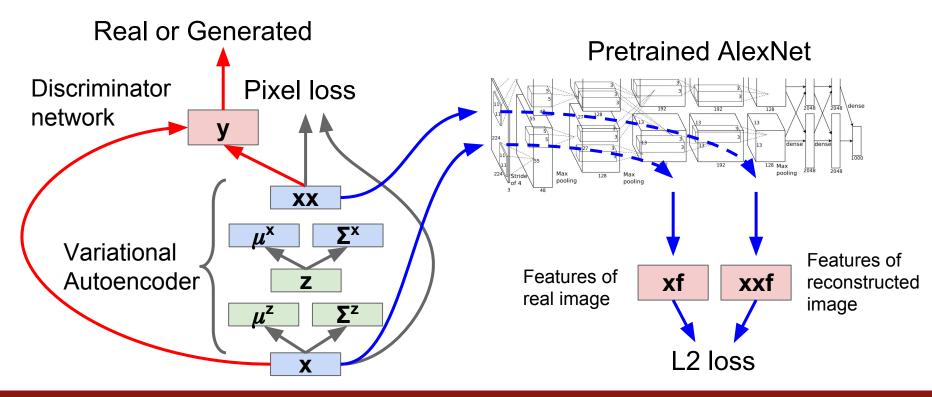
#### Pretrained AlexNet



Dosovitskiy and Brox, "Generating Images with Perceptual Similarity Metrics based on Deep Networks", arXiv 2016



Dosovitskiy and Brox, "Generating Images with Perceptual Similarity Metrics based on Deep Networks", arXiv 2016



Dosovitskiy and Brox, "Generating Images with Perceptual Similarity Metrics based on Deep Networks", arXiv 2016

Samples from the model, trained on ImageNet



## Recap

- Videos
- Unsupervised learning
  - Autoencoders: Traditional / variational
  - Generative Adversarial Networks
- Next time: Guest lecture from Jeff Dean