

# Deep Cyclic Group Networks

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[Project Website] <https://github.com/zcfan-tw/vectorNNtoolbox>



## Motivation

An increasing number of multi-dimensional data

- Multispectral image denoising
- Multispectral image segmentation
- EEG-based emotion recognition

Vector-valued networks are proposed to learn mutual associations among different dimensions (Dimensionality  $N$ ) in multi-dimensional data

- Deep complex neural network [1-2] ( $N=2$ )
- Vector-product neural network [3] ( $N=3$ )
- Deep quaternion neural network [4-5] ( $N=4$ )

From Previous works

- Scalar-valued neurons become vector-valued neurons
- Inputs, outputs, weights and biases are vector values

Drawbacks of previous works

- The dimensionality  $N$  in these models is fixed, such that they can not process arbitrary- $N$  dimensional data

Deep cyclic group network (DCGN)

- Dimensionality  $N$  is an arbitrary positive integer
- Vector-valued neurons with cyclic group algebras
  - Circular convolution
- Kernel maps and feature maps are three-way tensors
- Derivation of backpropagation learning algorithm, weight initialization, and batch normalization

## Proposed Model

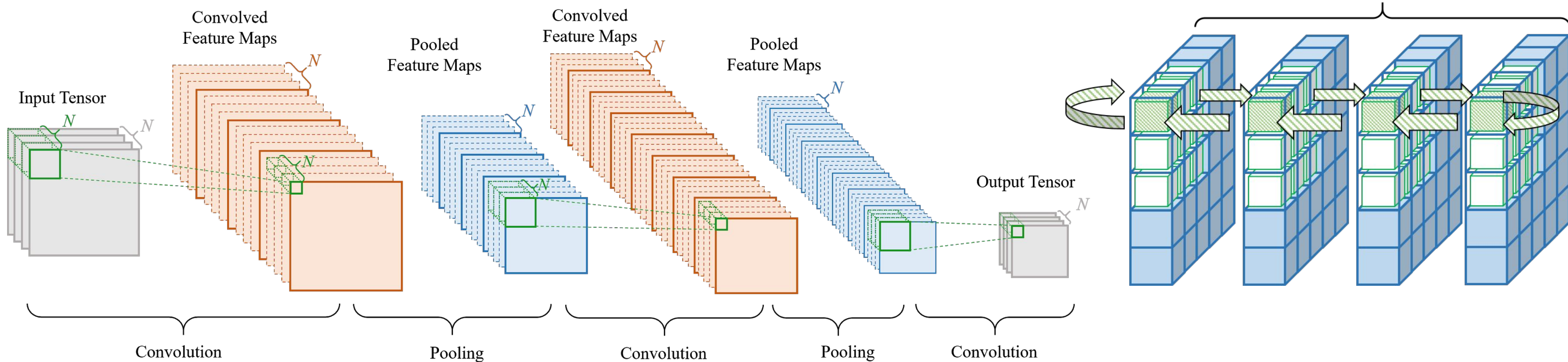


Illustration of the proposed DCGN model for regression problems

- Orange squares, blue squares and green squares are convolved features, pooled features and kernel maps respectively
- The feature and kernel maps are three-way tensors
  - When  $N$  is 1, the model reduces to conventional CNN
- All feature maps across all the layers have the same dimensionality  $N$
- The kernel map can learn the association between the elements across the  $N$  dimensions

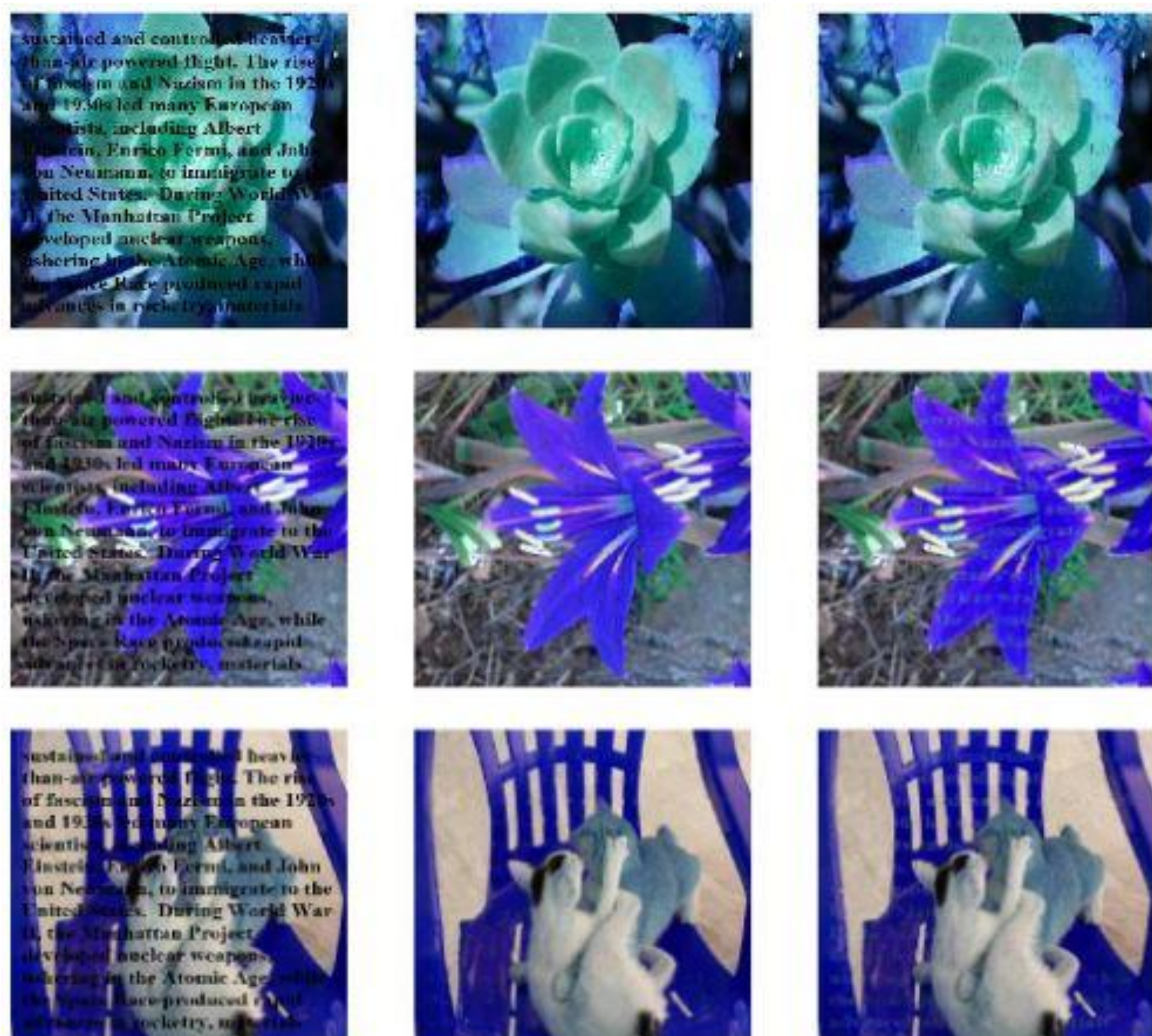
Illustration of the convolution procedure with circular convolution

- Each cube represents a scalar.
- Each matrix is a frontal slice of the tensor
- Each mode-3 fiber is an  $N$ -dimensional vector
- The feature map and the kernel map are composed of blue cubes and green cubes respectively
- Each mode-3 fiber of the kernel map performs circular convolution with a mode-3 fiber of the feature map, by rotating itself along the mode-3 dimension

## Experiments

**Color Image Denoising ( $N=3$ )**

- Contaminate images by texts with variant fonts
- Comparison between DCGN and CNN under similar number of parameters



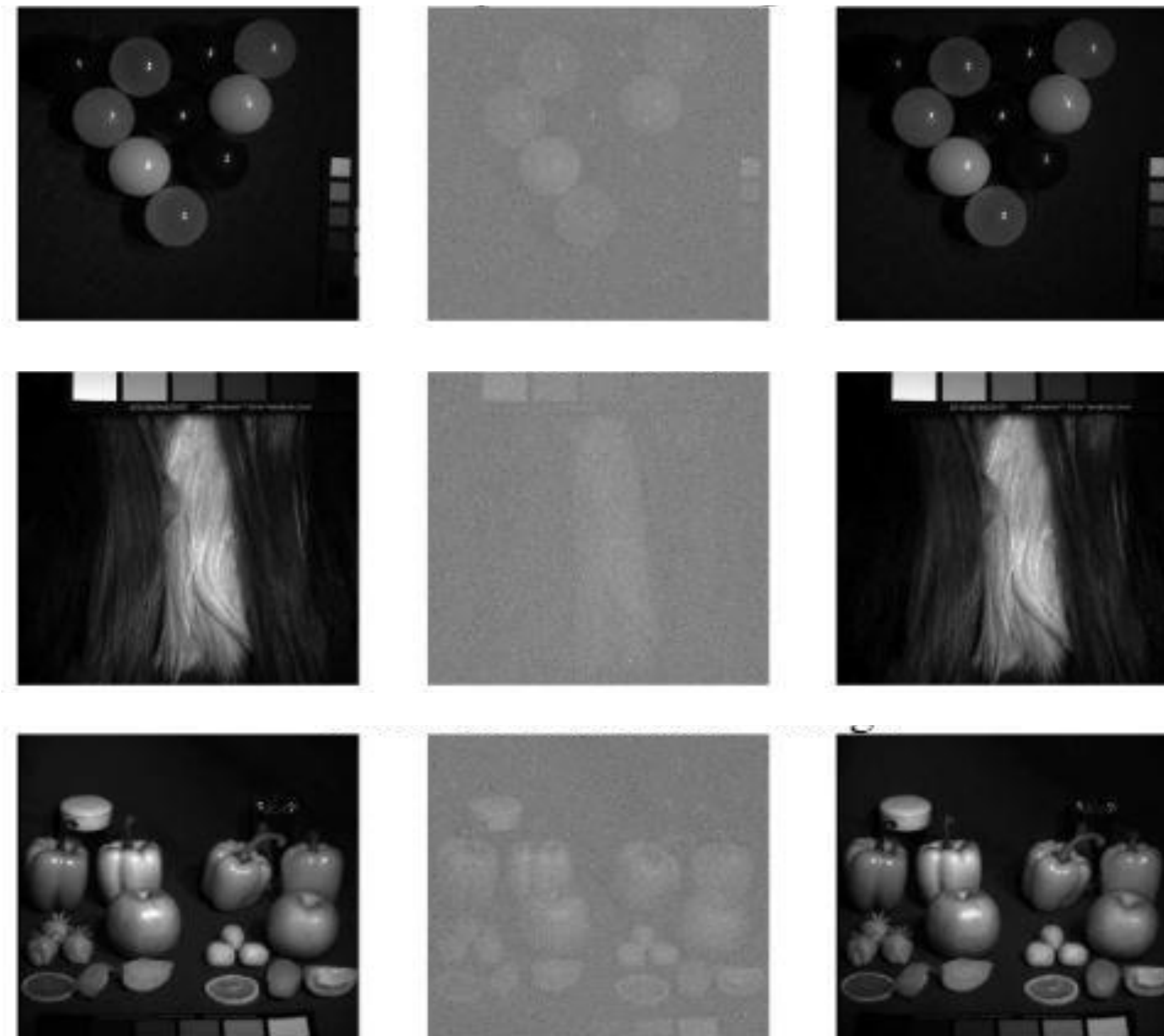
Noisy

DCGN

CNN

**Multispectral Image Denoising ( $N=5$ )**

- Add Gaussian noise with the specific sparsity and sigma value
- Sparsity: 10% Sigma: 200



Original

Noisy

DCGN

## Future Work

- Test DCGN on classification problems and datasets with even larger dimensions

## References

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- [2] T. Nitta, Complex-Valued Neural Networks: Utilizing High-Dimensional Parameters. Hershey, PA: Information Science Reference, 2009.
- [3] T. Nitta, "A backpropagation algorithm for neural networks based on a 3D vector product," in Proc. Int. Joint Conf. Neural Netw. (IJCNN), 1993, pp. 589–592.
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