

For a filament of radius $r = r_0$ at axial position $z = z_0$ with current I_0 , the radial and axial magnetic flux densities are given by:

$$B_r(r, z) = \frac{\mu_0 I_0}{2\pi d} \frac{(z - z_0)}{r} \left[\frac{r^2 + r_0^2 + (z - z_0)^2}{(r - r_0)^2 + (z - z_0)^2} E(m) - K(m) \right]$$

$$B_z(r, z) = \frac{\mu_0 I_0}{2\pi d} \left[K(m) - \frac{r^2 - r_0^2 + (z - z_0)^2}{(r - r_0)^2 + (z - z_0)^2} E(m) \right]$$

and the poloidal flux is given by:

$$\psi(r, z) = \mu_0 I_0 d \left[\left(1 - \frac{m}{2}\right) K(m) - E(m) \right]$$

where

$$d^2 = (z - z_0)^2 + (r + r_0)^2,$$

$$m = \frac{4rr_0}{d^2},$$

and $K(m)$ and $E(m)$ are the complete elliptic integrals of the first kind:

$$K(m) = \int_0^{\pi/2} \frac{d\varphi}{(1 - m \sin^2 \varphi)^{1/2}}$$

and the second kind:

$$E(m) = \int_0^{\pi/2} (1 - m \sin^2 \varphi)^{1/2} d\varphi.$$