

# Isabelle/HOL Exercises

## Logic and Sets

### Elimination of Connectives

In classical propositional logic, the connectives  $=$ ,  $\vee$ ,  $\neg$  can be replaced by  $\longrightarrow$ ,  $\wedge$ , *False*. Define corresponding simplification rules as lemmas and prove their correctness. (You may use automated proof tactics.)

```
lemma equiv_conel: "(A = B) = ((A  $\longrightarrow$  B)  $\wedge$  (B  $\longrightarrow$  A))"
  by iprover
```

```
lemma or_conel: "(A  $\vee$  B) = ( $\neg$  ( $\neg$  A  $\wedge$   $\neg$  B))"
  by blast
```

```
lemma not_conel: "( $\neg$  A) = (A  $\longrightarrow$  False)"
  by blast
```

What is the result of your translation for the formula  $A \vee (B \wedge C) = A$ ? (You can use Isabelle's simplifier to compute the result by using the simplifier's *only* option.)

Stating  $A \vee (B \wedge C) = A$  as a lemma and application of  
(*simp only: equiv\_conel or\_conel not\_conel*)

results in the simplified goal

$(A \longrightarrow \text{False}) \wedge ((B \wedge C \longrightarrow A) \wedge (A \longrightarrow B \wedge C) \longrightarrow \text{False}) \longrightarrow \text{False}.$