

Isabelle/HOL Exercises

Lists

```
primrec occurs :: "'a  $\Rightarrow$  'a list  $\Rightarrow$  nat" where
  "occurs a [] = 0"
| "occurs a (x#xs) = (if (x=a) then Suc (occurs a xs) else occurs a xs)"

lemma [simp]: "occurs a (xs @ ys) = occurs a xs + occurs a ys"
  apply (induct xs)
  apply auto
done

lemma "occurs a xs = occurs a (rev xs)"
  apply (induct xs)
  apply auto
done

lemma "occurs a xs  $\leq$  length xs"
  apply (induct xs)
  apply auto
done

lemma "occurs a (map f xs) = occurs (f a) xs"
  nitpick
:

lemma "occurs a (filter P xs) = (if P a then occurs a xs else 0)"
  apply (induct xs)
  apply auto
done

primrec remDups :: "'a list  $\Rightarrow$  'a list" where
  "remDups [] = []"
| "remDups (x#xs) = (if (0 < occurs x xs) then (remDups xs)
                     else (x#(remDups xs)))"

lemma occurs_remDups: "occurs x (remDups xs) = min 1 (occurs x xs)"
  apply (induct xs)
  apply (auto)
done
```

```
primrec unique :: "'a list  $\Rightarrow$  bool" where
  "unique [] = True"
| "unique (x#xs) = (occurs x xs = 0  $\wedge$  unique xs)"
```

```
lemma "unique(remDups xs)"
  apply (induct xs)
  apply (auto simp: occurs_remDups)
done
```