第十三章 非正弦周期电路

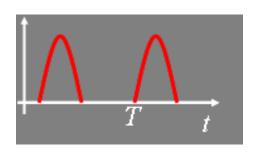
- § 13.1 非正弦周期信号
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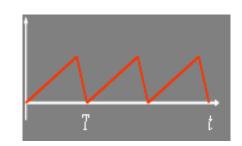
§ 13.1 非正弦周期信号

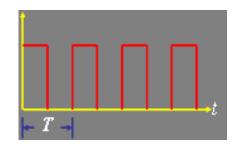
非正弦周期交流信号的特点:

- 1) 不是正弦波
- 2) 按周期规律变化,满足: f(t) = f(t + kT)

(k=0, 1, 2.....),式中T为周期。







§ 13.1 非正弦周期信号

采用谐波分析法,实质上就是通过应用数学中傅里叶级数展开方法,将非正弦周期信号分解为一系列不同频率的正弦量之和,再根据线性电路的叠加定理,分别计算在各个正弦量单独作用下电路中产生的同频率正弦电流分量和电压分量,最后,把所得分量按时域形式叠加得到电路在非正弦周期激励下的稳态电流和电压。

电工技术中所遇到的非正弦周期电流、电压信号多能满足展开成傅里叶级数的条件,因而能分解成如下傅里叶级数形式:

$$f(t) = a_0 + \sum_{k=1}^{\infty} \left[a_k \cos(k\omega_1 t) + b_k \sin(k\omega_1 t) \right]$$

也可表示成:

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_{km} \cos(k\omega_1 t + \phi_k)$$

以上两种表示式中系数之间关系为:

$$A_{0} = a_{0}$$

$$A_{km} = A_{km} \cos \phi_{k}$$

$$b_{k} = -A_{km} \sin \phi_{k}$$

$$\phi_{k} = \arctan \frac{-b_{k}}{a}$$

上述系数可按下列公式计算:

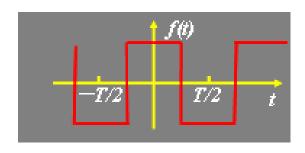
$$A_0 = a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(k\omega_1 t) d(\omega_1 t)$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(k\omega_1 t) d(\omega_1 t)$$

$$(k=1, 2, 3....)$$

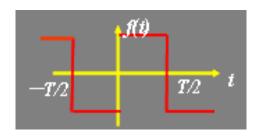
1、偶函数



$$f(t) = f(-t)$$

$$b_k = 0$$

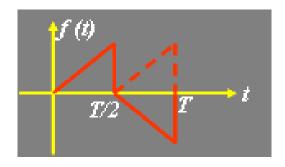
2、奇函数



$$f(t) = -f(t)$$

$$a_k = 0$$

3、奇谐波函数



$$f(t) = -f(t + \frac{T}{2})$$

$$a_0 = a_{2k} = b_{2k} = 0$$

4、偶函数镜对称

$$a_0 = a_{2k} = b_k = 0$$

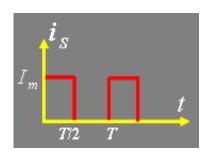
5、奇函数镜对称

$$a_0 = a_k = b_{2k} = 0$$

例13-1、把图示周期性方波电流分解成傅里叶级数。

解:

$$i_s(t) = \begin{cases} I_m & 0 < t < \frac{T}{2} \\ 0 & \frac{T}{2} < t < T \end{cases}$$



各次谐波分量的系数为:

$$I_{0} = \frac{1}{T} \int_{0}^{T} i_{s}(t) dt = \frac{1}{T} \int_{0}^{T/2} I_{m} dt = \frac{I_{m}}{2}$$

$$b_{k} = \frac{1}{\pi} \int_{0}^{2\pi} i_{s}(\omega t) \sin(k\omega t) d(\omega t) = \frac{I_{m}}{\pi} \left[-\frac{1}{k} \cos(k\omega t) \right]_{0}^{\pi} = \begin{cases} 0 & k \text{ 为偶数} \\ \frac{2I_{m}}{k\pi} & k \text{ 为奇数} \end{cases}$$

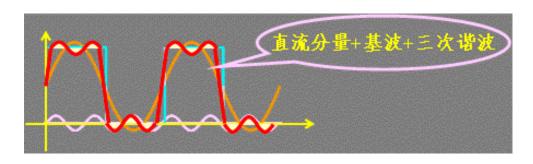
$$a_k = \frac{1}{\pi} \int_0^{2\pi} i_s(\omega t) \cos(k\omega t) d(\omega t) = \frac{I_m}{\pi} \cdot \frac{1}{k} \sin(k\omega t) \Big|_0^{\pi} = 0$$

$$A_k = \sqrt{a_k^2 + b_k^2} = b_k = \frac{2I_m}{k\pi}$$
 (k为奇数)

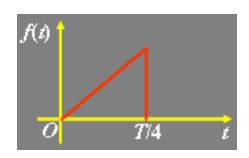
$$\phi_k = \arctan \frac{-b_k}{a_k} = -\frac{\pi}{2}$$

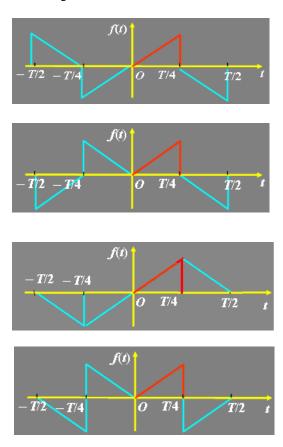
i。的傅里叶级数展开式为:

$$i_s = \frac{I_m}{2} + \frac{2I_m}{\pi} (\sin at + \frac{1}{3}\sin 3at + \frac{1}{5}\sin 5at + \cdots)$$



例13-2、给定函数f(t)的部分波形如图所示。为使f(t)的傅里叶级数中只包含如下的分量: (1)正弦分量; (2)余弦分量; (3)正弦奇次分量; (4)余弦奇次分量。试画出f(t)的波形。





§ 13.3 有效值、平均值和平均功率

一、三角函数的性质

1、正弦、余弦函数在一个周期内的积分为0,即:

$$\int_0^{2\pi} \sin k a t d(a t) = 0 \qquad \int_0^{2\pi} \cos k a t d(a t) = 0$$

 $2 \times \sin^2 \times \cos^2$ 在一个周期内的积分为 π ,即:

$$\int_0^{2\pi} \sin^2 k \, a t d\left(a t\right) = \pi \qquad \int_0^{2\pi} \cos^2 k \, a t d\left(a t\right) = \pi$$

3、三角函数的正交性如下式所示:

$$\int_0^{2\pi} \cos k \omega t \cdot \sin p \, \omega t d(\omega t) = 0$$

$$\int_0^{2\pi} \cos k \omega t \cdot \cos p \, \omega t d(\omega t) = 0 \qquad (k \neq p)$$

$$\int_0^{2\pi} \sin k \omega t \cdot \sin p \, \omega t d(\omega t) = 0$$

§ 13.3 有效值、平均值和平均功率

二、非正弦周期函数的有效值

设非正弦周期电流可以分解为傅里叶级数:

$$i(t) = I_0 + \sum_{k=1}^{\infty} I_{km} \cos(k\omega t + \varphi_k)$$

代入有效值的定义式中有:

$$I = \sqrt{\frac{1}{T} \int_0^T i^2 \left(\cot \right) \! d\left(t \right)} = \sqrt{\frac{1}{T} \int_0^T \! \left[I_0 + \sum_{k=1}^\infty I_{k\!m} \cos\!\left(k \cot + \varphi_k \right) \right]^2 \! d\left(t \right)}$$

§13.3 有效值、平均值和平均功率

利用上述三角函数的性质, 上式中 i 的展开式平方后将含有下列各项:

$$\begin{split} &\frac{1}{T} \int_{0}^{T} I_{0}^{2} dt = I_{0}^{2} \\ &\frac{1}{T} \int_{0}^{2\pi} I_{km}^{2} \cos^{2}(k \omega_{l} t + \varphi_{K}) d(\omega t) = I_{km}^{2} \\ &\frac{1}{T} \int_{0}^{2\pi} 2I_{0} \cos(k \omega_{l} t + \varphi_{K}) d(\omega t) = 0 \\ &\frac{1}{T} \int_{0}^{2\pi} 2I_{km} \cos(k \omega_{l} t + \varphi_{K}) I_{pm} \cos(p \omega_{l} t + \varphi_{p}) d(\omega t) = 0 \quad k \neq p \end{split}$$

§ 13.3 有效值、平均值和平均功率

这样可以求得 i 的有效值为:

$$I = \sqrt{I_0^2 + \sum_{k=1}^{\infty} \frac{I_{km}^2}{2}} = \sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 \cdots}$$

由此得到结论: 周期函数的有效值为直流分量及各次谐波分量有效值平方和的方根。此结论可以推广用于其他非正弦周期量。

§13.3 有效值、平均值和平均功率

三、非正弦周期函数的平均值

设非正弦周期电流可以分解为傅里叶级数:

$$i(t) = I_0 + \sum_{k=1}^{\infty} I_{km} \cos(k \omega_1 t + \varphi_k)$$

则其平均值定义为:

$$I_{AV} = \frac{1}{T} \int_0^T |i(t)| dt$$

$$I_{AV} = \frac{1}{T} \int_0^T |I_m \cos(k\omega_l t)| d(t) = 0.637 I_m = 0.898 I$$

§ 13.3 有效值、平均值和平均功率

四、非正弦周期交流电路的平均功率

设任意一端口电路的非正弦周期电流和电压可以分解为傅里叶级数:

$$u(t) = U_0 + \sum_{k=1}^{\infty} U_{km} \cos(k\omega t + \varphi_{uk})$$

$$i(t) = I_0 + \sum_{k=1}^{\infty} I_{km} \cos(kat + \varphi_{ik})$$

则一端口的平均功率为:

$$P = \frac{1}{T} \int_0^T u \cdot i dt$$

§ 13.3 有效值、平均值和平均功率

不同频率的正弦电压与电流乘积的上述积分为零(即不产生平均功率);同频的正弦电压、电流乘积的上述积分不为零。

$$P = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos \varphi_k = U_0 I_0 + U_1 I_1 \cos \varphi_1 + U_2 I_2 \cos \varphi_2 + U_3 I_3 \cos \varphi_3 + \cdots$$

$$= P_0 + P_1 + P_2 + \dots$$

$$\varphi_k = \varphi_{uk} - \varphi_{ik}$$

由此得出结论:

非正弦周期电流电路的平均功率=直流分量的功率+各次谐波的平均功率。

根据以上讨论可得非正弦周期电流电路的计算步骤如下:

- (1)把给定电源的非正弦周期电流或电压作傅里叶级数分解,将非正弦周期量展开成若干频率的谐波信号,高次谐波取到哪一项为止,要根据所需准确度的高低而定;
- (2)利用直流和正弦交流电路的计算方法,对直流和各次谐波激励分别计算其响应;
 - (3)应用叠加定理将以上计算结果转换为瞬时值迭加。

非正弦周期交流电路的计算 § 13.4

例13-3、电路如上图所示,电流源为下图所示的方波信号。求电流源的输出 电压 u_0 。 已知: $R = 20\Omega$ L = 1mH C = 1000pF $I_m = 157\mu A$ $T = 6.28\mu s$

$$i_s = \frac{I_m}{2} + \frac{2I_m}{\pi} (\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots)$$

直流分量
$$I_0 = \frac{I_m}{2} = \frac{157}{2} = 78.5 \mu A$$

基波最大值
$$I_{1m} = \frac{2I_m}{\pi} = \frac{2 \times 157}{3.14} = 100 \mu A$$

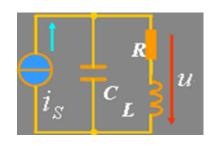
三次谐波最大值

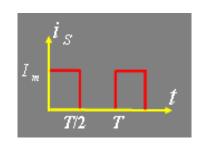
$$I_{3m} = \frac{1}{3}I_m = 33.3\mu A$$

五次谐波最大值

$$I_{5m} = \frac{1}{5}I_m = 20\mu A$$

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{6.28 \times 10^{-6}} = 10^6 \, rad \, / \, s$$





非正弦周期交流电路的计算 § 13.4

$$I_{S0} = 78.5 \ \mu A$$

$$I_{s0} = 78.5 \quad \mu \text{A} \qquad i_{s1} = 100 \sin 10^6 t \quad \mu \text{A}$$

(a) 直流分量 I_{S0} 单独作用时

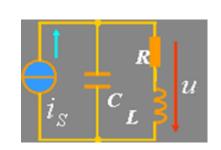
$$U_0 = RI_{S0} = 20 \times 78.5 \times 10^{-6} = 1.57 \text{ mV}$$

(b) 基波单独作用时

$$\frac{1}{\omega_1 C} = \frac{1}{10^6 \times 1000 \times 10^{-12}} = 1k\Omega \qquad \omega_1 L = 10^6 \times 10^{-3} = 1k\Omega$$

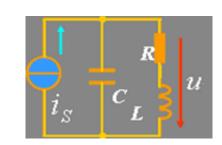
$$Z(\boldsymbol{\omega_{1}}) = \frac{(R + jX_{I}) \cdot (-jX_{C})}{R + j(X_{I} - X_{C})} \approx \frac{X_{I}X_{C}}{R} = \frac{L}{RC} = 50k\Omega$$

$$\dot{U}_1 = \dot{I}_1 \cdot Z(\omega_1) = \frac{100 \times 10^{-6}}{\sqrt{2}} \cdot 50 \times 10^3 = \frac{5000}{\sqrt{2}} mV$$



(c) 三次谐波单独作用时

$$\frac{1}{3\omega_{\!_{1}}C} = \frac{1}{3\times10^{6}\times1000\times10^{-12}} = 0.33\,\mathrm{K}\Omega \qquad 3\omega_{\!_{1}}L = 3\times10^{6}\times10^{-3} = 3k\Omega$$



$$Z(3\omega_1) = \frac{(R + jX_{I3})(-jX_{C3})}{R + j(X_{I3} - X_{C3})} = 374.5 \angle -89.19^{\circ}\Omega$$

$$\dot{U}_{3} = \dot{I}_{s3} \cdot Z(3\omega_{1}) = \frac{33.3 \times 10^{-6}}{\sqrt{2}} \times 374.5 \angle -89.19^{\circ} = \frac{12.47}{\sqrt{2}} \angle -89.19^{\circ} \, mV$$

(d) 五次谐波单独作用时

$$\frac{1}{5\omega_{1}C} = \frac{1}{5\times10^{6}\times1000\times10^{-12}} = 0.2 K\Omega \qquad 5\omega_{1}L = 5\times10^{6}\times10^{-3} = 5k\Omega$$
$$Z(5\omega_{1}) = \frac{(R+jX_{LS})(-jX_{CS})}{R+j(5X_{LS}-X_{CS})} = 208.3\angle -89.53^{\circ}\Omega$$

$$\dot{U}_{5} = \dot{I}_{5s} \cdot Z(5\omega_{1}) = \frac{20 \times 10^{-6}}{\sqrt{2}} \cdot 208.3 \angle -89.53 = \frac{4.166}{\sqrt{2}} \angle -89.53^{*} m V$$

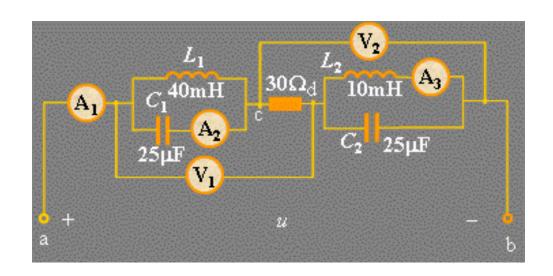
把各次谐波分量计算结果的瞬时值迭加:

$$u = U_0 + u_1 + u_3 + u_5$$

 $\approx 1.57 + 5000 \sin at$
 $+ 12.47 \sin(3at - 89.2^{\circ})$
 $+ 4.166 \sin(5at - 89.53^{\circ}) \ mV$

例13-4、求图(a)所示电路中各表读数(有效值)。

已知:
$$u = 30 + 120\cos 1000t + 60\cos(2000t + \frac{\pi}{4})V$$
.



(1) 当直流分量 $u_0 = 30V$ 作用于电路时

 L_1 、 L_2 短路, C_1 、 C_2 开路

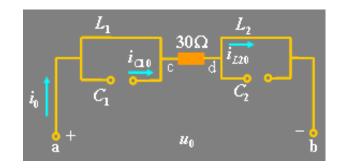
$$L_1$$
 L_2 V_2 L_2 I_2 $I_30\Omega_d$ $I_30\Omega_d$ I_4 I_5 $I_$

$$I_{1(0)} = I_{3(0)} = \frac{30}{30} = 1A$$
, $I_{2(0)} = 0$, $U_{1(0)} = U_{2(0)} = 30V$

(2) 基波作用于电路

$$\omega L_1 = 1000 \times 40 \times 10^{-3} = 40 \Omega$$
 $\omega L_2 = 1000 \times 10 \times 10^{-3} = 10 \Omega$

$$\frac{l}{\omega C_1} = \frac{l}{\omega C_2} = \frac{1}{1000 \times 25 \times 10^{-6}} = 40 \Omega$$

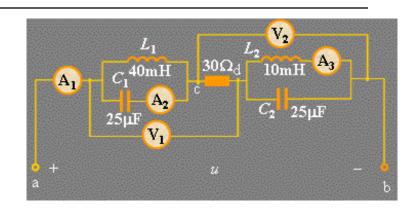


 L_1 、 C_1 对基波发生并联谐振

$$I_{3(1)} = I_{1(1)} = 0$$

$$I_{1(1)} = 0 \qquad \dot{U}_{1(1)} = \frac{120}{\sqrt{2}} \angle 0^{\circ} V \qquad \dot{U}_{2(1)} = 0 \qquad \dot{I}_{2(1)} = j\omega_{1}C_{1}\dot{U}_{1} = \frac{120 \angle 0^{\circ}}{\sqrt{2} \cdot (-j40)} = \frac{3}{\sqrt{2}} \angle 90^{\circ} A$$

(3) 二次谐波 u_2 =60 $\cos(2000t+\pi/4)$ V 作用于电路



$$2\omega L_1 = 2000 \times 40 \times 10^{-3} = 80\Omega, \quad 2\omega L_2 = 2000 \times 10 \times 10^{-3} = 20\Omega$$

$$\frac{l}{2\omega C_1} = \frac{l}{2\omega C_2} = \frac{1}{2000 \times 25 \times 10^{-4}} = 20\Omega$$

 L_2 、 C_2 对二次谐波发生并联谐振

$$I_{1(2)} = I_{2(2)} = 0$$
 $U_{1(2)} = 0$ $U_{1(2)} = 0$ $U_{2(2)} = U_2 = \frac{60}{\sqrt{2}} \angle 45^{\circ}V$

$$\dot{I}_{3(2)} = \frac{\dot{U}_2}{j2\omega L_2} = \frac{60\angle 45^{\circ}}{\sqrt{2} \cdot j20} = \frac{3}{\sqrt{2}} \angle -45^{\circ} A$$

电流表

$$A_1=1A$$

$$A_2 = \sqrt[3]{\sqrt{2}} = 2.12A$$

$$A_3 = \sqrt{1^2 + (3/\sqrt{2})^2} = 2.35A$$

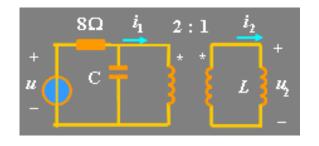
电压表

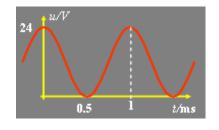
$$V_1 = \sqrt{30^2 + (120/\sqrt{2})^2} = 90V$$

$$V_2 = \sqrt{30^2 + (60/\sqrt{2})^2} = 52.0V$$

例13-5、左图所示电路中,已知电源u(t)是周期函数,波形如右图所示,

 $L = \frac{1}{2\pi} mH$, C=125/ μ F。求: 理想变压器原边电流 i_1 (t)及输出电压 u_2 的有效值。





$$\omega = 2\pi / T = 2\pi \times 10^3 \, rad / s$$

$$u(t) = 12 + 12\cos(\omega t)$$

直流分量 $u_0 = 12V$ 作用于电路时,电容开路、电感短路

$$i_{1(0)} = 12/8 = 1.5A$$

$$u_{2(\theta)} = 0$$

 $u_1 = 12\cos(\omega t)$ 作用于电路时

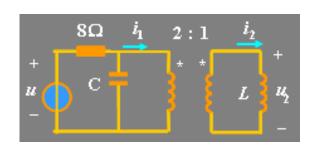
$$X_C = \frac{1}{\omega C} = \frac{\pi}{2\pi \times 10^3 \times 125 \times 10^{-6}} = 4\Omega$$

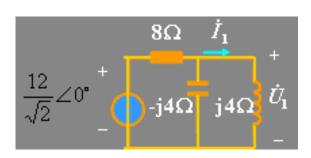
$$X_L = \omega L = 2\pi \times 10^3 \times \frac{1}{2\pi} \times 10^{-3} = 1\Omega$$

$$\dot{U}_{1(1)} = \dot{U} = \frac{12}{\sqrt{2}} \angle 0^{\circ}$$

$$\dot{I}_{1(1)} = \frac{\dot{U}}{j4} = \frac{12}{\sqrt{2} \cdot j4} = -j \frac{3}{\sqrt{2}} A$$

$$\dot{U}_{2(1)} = \frac{1}{n} \dot{U}_1 = \frac{6}{\sqrt{2}} \angle 0^{\circ} V$$





$$i_1(t) = 1.5 + 3\cos(\omega t - 90^{\circ})A$$

$$U_2 = \frac{6}{\sqrt{2}} = 4.243V$$

例13-6、求图示电路中 a、b 两端电压有效值 U_{ab} 、电流 i 及功率表的读数。

已知:
$$u_1(t) = 220\sqrt{2}\cos\omega tV$$
 $u_2(t) = 220\sqrt{2}\cos\omega t + 100\sqrt{2}\cos(3\omega t + 30^\circ)V$

60Ω

$$U_{ab} = \sqrt{440^2 + 100^2} = 451.22V$$

一次谐波作用时: $U_{ab(1)} = 440 \angle 0^{\circ} V$

$$\dot{I}_{(1)} = \frac{440}{60 + i20} = \frac{22}{3 + i} = 6.96 \angle -18.4^{\circ} A$$

三次谐波作用时:

$$\dot{U}_{ab(3)} = 100 \angle 30^{\circ} V$$

$$\dot{I}_{(3)} = \frac{100\angle 30^{\circ}}{60 + j60} = \frac{5\angle 30^{\circ}}{3 + j3} = 1.18\angle -15^{\circ} A$$

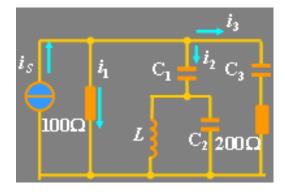
$$i(t) = 6.69\sqrt{2}\cos(\omega t - 18.4^{\circ}) + 1.18\sqrt{2}\cos(3\omega t - 15^{\circ})A$$

$$P = 220 \times 6.69 \cos 18.4^{\circ} = 1452.92W$$

例13-7、图示电路中已知 $i_s(t) = 5 + 20\cos 1000t + 10\cos 3000tA$ L=0.1H, $C_3=1\mu F$,电容 C_1 中只有基波电流,电容 C_3 中只有三次谐波电流,求 C_1 、 C_2 和各支路电流。

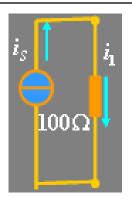
 C_1 中只有基波电流,说明L 和 C_2 对三次谐波发生并联谐振

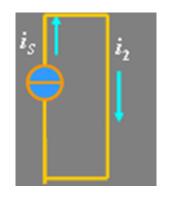
$$C_2 = \frac{1}{(3\omega)^2 L} = \frac{1}{9 \times 10^5} F$$

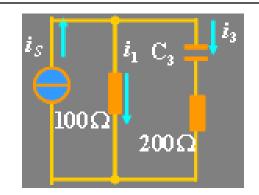


 C_3 中只有三次谐波电流,说明L、 C_1 、 C_2 对一次谐波发生串联谐振

$$\frac{1}{j\omega C_1} + \frac{-L/C_2}{j(\omega L - 1/\omega C_2)} = 0 C_1 = \frac{8}{9 \times 10^5} F$$







$$\dot{I}_{1(0)} = 5A$$

$$I_{2(1)} = \frac{20}{\sqrt{2}} \angle 0^{\bullet} A$$

$$\dot{I}_{3(3)} = \frac{100 \times 10 / \sqrt{2}}{100 + 200 - j \cdot 10^{3} / 3} = \frac{30 / \sqrt{2}}{9 - j \cdot 10} = \frac{2.23}{\sqrt{2}} \angle 48^{\circ} A$$

$$\dot{I}_{1(3)} = \dot{I}_{s} - \dot{I}_{3(3)} = \frac{10}{\sqrt{2}} - \frac{30/\sqrt{2}}{9 - j10} = \frac{8.67}{\sqrt{2}} \angle -11^{\circ} A$$

$$i_1(t) = 5 + 8.67\cos(3000t - 11^0)A$$

$$i_2(t) = 20\cos 1000t A$$

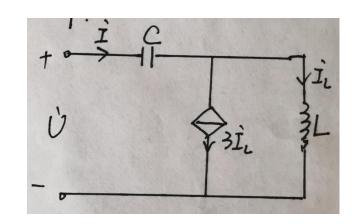
$$i_3(t) = 2.23\cos(3000t + 48^{\circ})A$$

$$\dot{v} = -j 2\hat{I} - \hat{I}$$

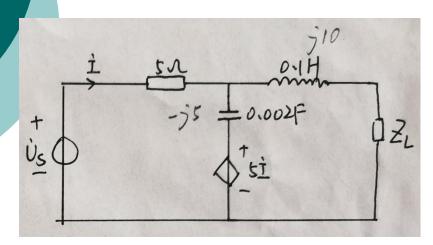
 $z = \frac{1}{2} = -1 - j 2$. λ .

$$\dot{U} = 2 \times 4\dot{I}_1 + \dot{j} = 2\dot{I}_1$$
 $\dot{Z} = \frac{\dot{U}}{4\dot{I}_1} = 2 + \dot{j} = 0.5 \, \text{N}$

2 下国际市电路的分临上沿板, 若能, 试花沿板水车



3. 下国所示电路, Che Us=100/0°V, w=100 rad/s, ZL取价值吸放设备大路, 并起Pmax.



$$X_{L} = jwL = jlu \Lambda$$

$$X_{C} = -jwC = -js\Lambda$$

$$V_{S} = S\dot{L} - jt\dot{L} + S\dot{L} \implies \dot{L} = \frac{lvo}{lo-js}$$

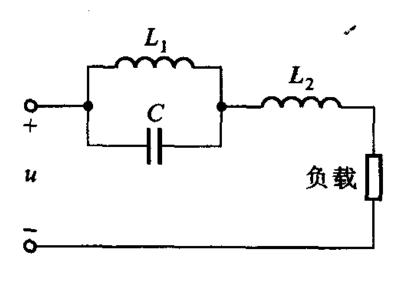
$$\dot{V}_{OC} = (s-js)\dot{L} = 60-j20$$

$$| \omega = s \hat{I} + j | o \hat{I}_{sc}|$$

$$| \hat{\sigma} | \hat{\sigma} |$$

作业

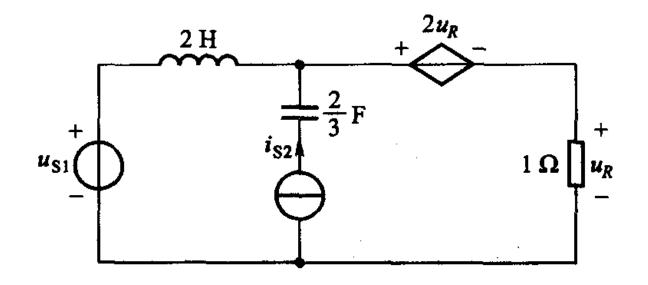
13—8 题 13-8 图所示为滤波电路,要求负载中不含基波分量,但 $4\omega_1$ 的谐波分量能全部传送至负载。如 $\omega_1=1~000~{\rm rad/s}$, $C=1~{\mu}$ F, 求 L_1 和 L_2 。



题 13-8 图

作业

题 13-11 图所示电路中 $u_{Si}=[1.5+5\sqrt{2}\sin(2t+90^\circ)]$ V,电流源电流 $i_{S2}=2\sin(1.5t)$ A。求 u_R 及 u_{Si} 发出的功率。



题 13-11 图