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## ECE 358 Project 2: CSMA/CD Performance Evaluation

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## 1 1-Persistent CSMA/CD Protocol

The 1-persistent CSMA/CD protocol was simulated with our code for N (number of packets): 20, 40, 60, 80, 100 and A (average packet rate/s) of 7, 10, and 20. With these values, we attained the graphs and data below with regard to efficiency and throughput.

### 1.1 Efficiency

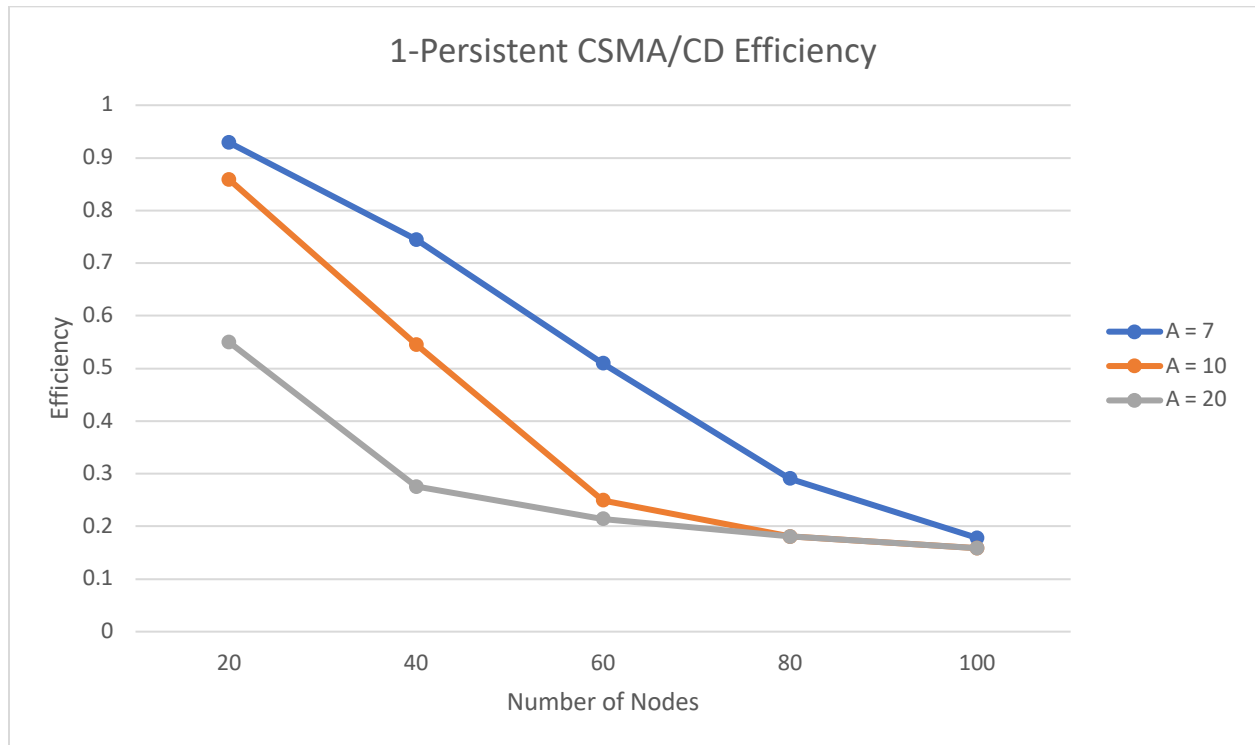


Figure 1: Efficiency of 1-Persistent CSMA/CD Protocol

The efficiency of this protocol is a decreasing linear relationship for increasing N (as seen with A = 7). This is expected as when you add more nodes that are trying to transmit, there are bound to be more collision as more nodes are trying to transmit at the same time. If there are more collisions, then the efficiency of the protocol decreases as the equation for efficiency is the following:

$$Efficiency = \frac{\# \text{ of Successful Transmissions}}{\# \text{ of Total Transmission}}$$

However, as you increase the value of A and keep the range of N constant, the relationship becomes more of an inversely proportional curve (as seen with A = 10 and A = 20). This is due to the fact that when the average packets per second sent for each node increases, the nodes are all trying to send more and more packets so the nodes will have higher collision counters and will be in exponential backoff for longer and longer times. This makes it such that almost all the nodes have a different exponential backoff timer and so the rate at which collisions increase will decrease. This in turn will make the rate at which efficiency drops will slow down as you increase the number of nodes.

## 1.2 Throughput

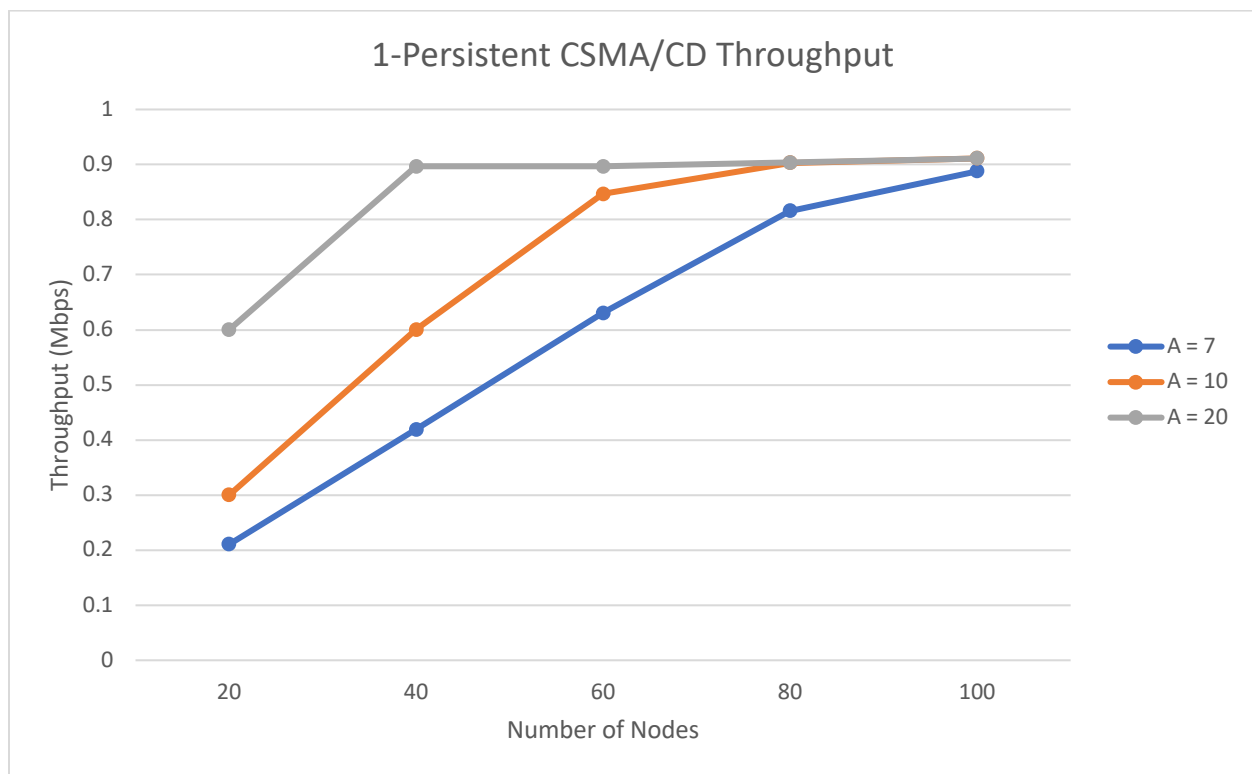


Figure 2: Throughput of 1-Persistent CSMA/CD Protocol

The throughput of the protocol has the relationship above. The equation for throughput is

$$\text{Throughput} = \frac{\# \text{ Successfully Transmitted Packets} * \text{Length of Packet}}{\text{Simulation Time}}$$

For each value of average rate of packets per second ( $A$ ), the throughput increases at a logarithmic relationship plateauing at around 0.9 Mbps. This is expected because the overall number of packets to be sent is much higher as the packet arrival rate is increased. Even though more collisions occur at higher packet arrival rates, the simulation time remains constant, but the number of successfully transmitted packets will increase due to the sheer number of packets generated. This also indicates that at higher arrival rates, the bus will almost always be busy.

## 2 Non-persistent CSMA/CD Protocol

The Non-persistent CSMA/CD protocol was simulated with our code for  $N$  (number of packets): 20, 40, 60, 80, 100 and  $A$  (average packet rate/s) of 7, 10, and 20. With these values, we attained the graphs and data below with regard to efficiency and throughput. This protocol is identical to the 1-persistent CSMA/CD protocol except that when a node senses that the medium is busy, it waits an exponential back-off before sensing the medium again.

### 2.1 Efficiency

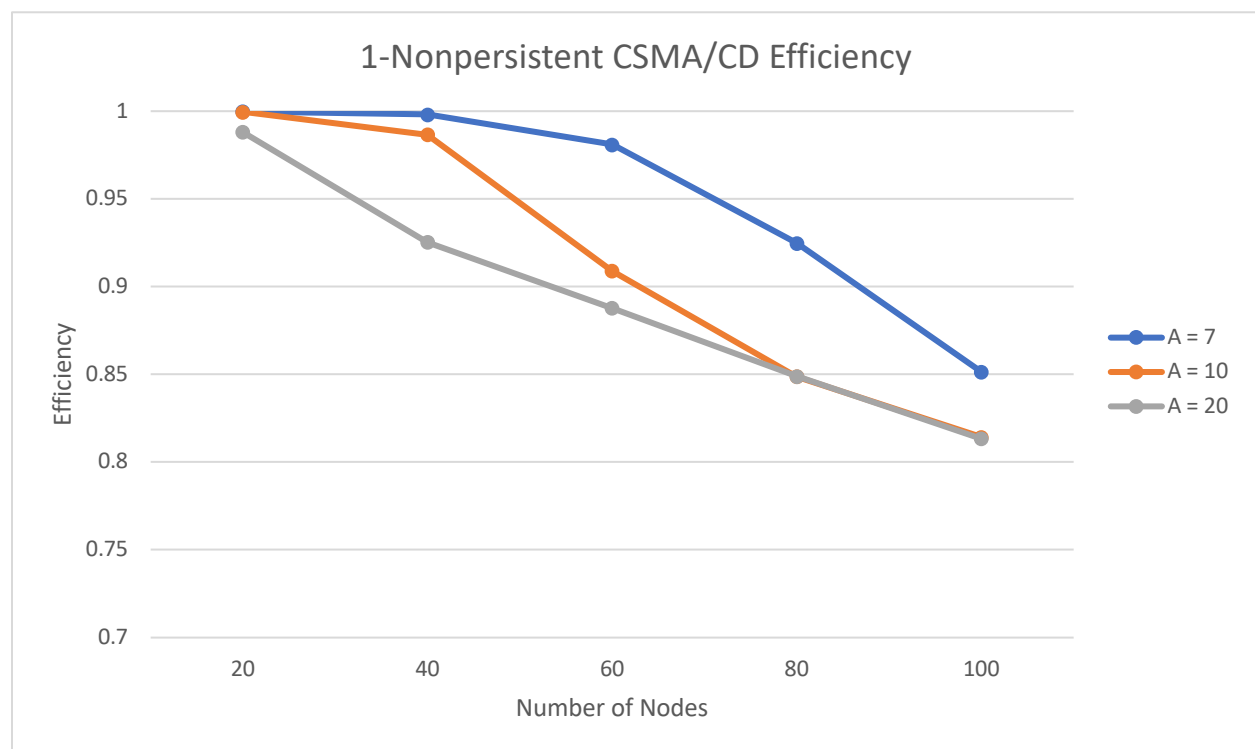


Figure 3: Efficiency of Non-persistent CSMA/CD Protocol

The efficiency of this protocol is much higher than the 1-persistent CSMA/CD protocol for all values of  $A$ . This is expected because adding an exponential back-off after nodes detect the medium as busy will buffer packets in the nodes queue. This buffering will decrease the number of collisions significantly and the next transmitted packet for each node must be after the medium becoming free. The general trend for varying values of  $A$  is similar to 1-persistent CSMA/CD protocol where higher values of  $A$  will have a lower efficiency than lower values of  $A$ . This is again due to the fact that there will be relatively more collisions when the rate of packets per second increases for each node as the exponential back-off is longer and longer after each collision. At higher number of nodes, this makes it such that almost all the nodes have a different exponential back-off timer and so the rate at which collisions increase will decrease. This in turn will make the rate at which efficiency drops will slow down.

## 2.2 Throughput

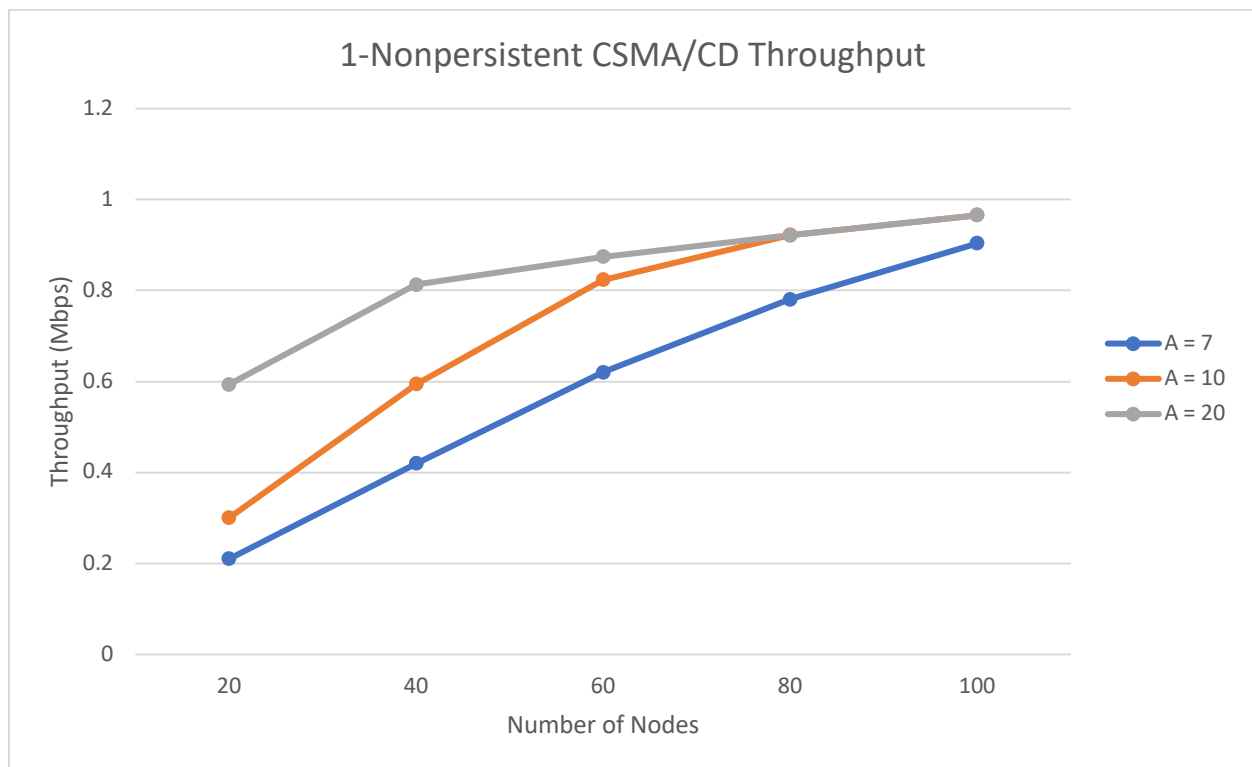


Figure 4: Throughput of Nonpersistent CSMA/CD Protocol

The throughput of the protocol has the relationship above. For each value of average rate of packets per second ( $A$ ), the throughput increases at a logarithmic relationship up to the value 1.0Mbps. This is expected because the overall number of packets to be sent is much higher as the

packet arrival rate is increased. Even though more collisions occur at higher packet arrival rates, the simulation time remains constant, but the number of successfully transmitted packets will increase due to the sheer number of packets generated. This also indicates that at higher arrival rates, the bus will almost always be busy. This is a similar trend to 1-persistence CSMA/CD protocol except that the throughput values are higher. As the efficiency of this protocol is a lot higher than 1-persistence CSMA/CD protocol, the throughput will also be higher as there are fewer collisions and therefore shorter backoff times making the nodes successfully send more packets during the simulation.

### 3 Source Code and Design Decisions

A class was written to generate exponential random variables whenever the class instance calls a `genValue` method. In this method, a uniform random variable is generated and uses it along with an inputted  $\lambda$  value to generate a single exponential random via the inverse method. Theoretically, exponential distributions should have an expected value of  $\frac{1}{\lambda}$  and a variance of  $\frac{1}{\lambda^2}$ . This is the same exponential random variable generator that was used in Lab 1. This class was used to generate the arrival times in the queue of each created node. The  $\lambda$  used for this distribution is the value of “A” or the average packet arrival rate since the average of a Poisson distribution is  $\lambda$ .

The `Packet` class was used again in for cleanliness of code and ease of understanding the flow of the simulation.

We created a new class called `Node` that would handle basic functions by the node and commonly used functions associated with it. The `genPacketArrivalEvents` function will populate the queue for the node with arrival times based on a Poisson distribution where the value of  $\lambda$  (average for the distribution) is the value of “A”. This function is called upon the object’s creation so that each `Node` will always be populated with a list of packets. There are also other class functions like `waitExponentialBackoff` and `waitExponentialBackoffMediumSensing` that add an exponential backoff to the arrival times of packets and `bufferPackets` that serve to shift the arrival times of all packets within a range of timestamps to a certain value. These functions make designing the simulator easier as we can just call the appropriate class functions when necessary. Lastly, there are helper functions that just serve to simplify the code like `removeFirstPacket`,

*getFirstPacketTimestamp*, *genExponentialBackoffTime* where it serves to just add a level of transparency to the code we write in the main simulator.

Our two main classes are *PersistentCSMASimulator* and *NonpersistentCSMASimulator* that solves questions 1 and 2 respectively. Each of the classes have variables *transmittedPackets* and *successfullyTransmittedPackets* in order to properly evaluate the performance of the protocol.

### 3.1 Persistent CSMA Design

*PersistentCSMASimulator* has three main stages – *createNodes*, *processPackets*, and *printResults*. The *createNodes* stage will create the appropriate number of nodes for the simulation and store it in a local variable.

```
# For each node, calculate when the packet arrives + check collision
transmissionSuccess = True
for rxNode in self.nodes:
    offset = abs(rxNode.getNodePosition() - txNode.getNodePosition())
    if (offset == 0):
        continue

    propagationDelay = offset * UNIT_PROPAGATION_DELAY
    firstBitArrivalTime = currentTime + propagationDelay
    lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY

    if rxNode.checkCollision(firstBitArrivalTime):
        rxNode.waitExponentialBackoff()
        self.transmittedPackets += 1
        transmissionSuccess = False

if not transmissionSuccess:
    txNode.waitExponentialBackoff()
else:
    self.successfullyTransmittedPackets += 1
    txNode.removeFirstPacket()
    self.bufferAllPacketsForBusy(currentTime, txNode)
```

Figure 5: Main logic within each packet transmission

The *processPackets* stage starts by taking the smallest arrival time within all the nodes and seeing if it collides with any other packets from any other nodes. The *transmittedPackets* counter is incremented regardless if there is a collision because the sender node is trying to send a packet. If there is a collision, then both the transmitter and receiver nodes will have exponential backoff and



the *transmittedPackets* variable will be incremented for each transmission along with the collision counter incrementing in each node.

```
# If packet arrival < arrival of transmitted first bit, bus appears to be idle
def checkCollision(self, firstBitArrivalTime):
    return self.getFirstPacketTimestamp() <= firstBitArrivalTime

def waitExponentialBackoff(self):
    self.collision_counter += 1
    self.collision_counter_medium = 0
    if self.collision_counter > COLLISION_LIMIT:
        self.removeFirstPacket()
    else:
        # Each node waits backoff time. Means we start waiting from our first packet time
        newArrivalTime = self.getFirstPacketTimestamp() + self.genExponentialBackoffTime()
        self.bufferPackets(0, newArrivalTime)
```

Figure 6: Functions to check Collision and apply Exponential Backoff

If a collision does not happen, then the *successfullyTransmittedPackets* counter will be incremented and the rest of the nodes will be checked to see if they had any packets that had arrival times between the timestamps of the first and last bits to arrive at each node. If a node has packets waiting to be sent between these timestamps, then their arrival timestamps would be updated to be the end of the transmission.

```
def bufferAllPacketsForBusy(self, currentTime, txNode):
    for node in self.nodes:
        offset = abs(node.getNodePosition() - txNode.getNodePosition())
        propagationDelay = offset * UNIT_PROPAGATION_DELAY
        firstBitArrivalTime = currentTime + propagationDelay
        lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY
        node.bufferPackets(firstBitArrivalTime, lastBitArrivalTime)
```

Figure 7: Function to buffer packet arrival times when no collisions occur

### 3.2 Non-Persistent CSMA Design

The NonpersistentCSMASimulator is very similar to the PersistentCSMASimulator except that when a node senses that the medium is busy, it doesn't just try to sense again immediately but instead waits for multiple exponential backoff time period. If the backoff fails, or exceeds the backoff counter limit of 10, the packet will be dropped, and the number of transmitted packets is incremented. This increment is needed because the dropped packet counts as a failed transmission.

```

def bufferAllPacketsForBusy(self, currentTime, txNode):
    for node in self.nodes:
        offset = abs(node.getNodePosition() - txNode.getNodePosition())
        propagationDelay = offset * UNIT_PROPAGATION_DELAY
        firstBitArrivalTime = currentTime + propagationDelay
        lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY
        if node.waitExponentialBackoffMediumSensing(firstBitArrivalTime, lastBitArrivalTime):
            self.transmittedPackets += 1

```

Figure 8: Function to apply exponential backoff for medium sensing

Each time that the node sees a transmission, another exponential backoff time period is added to the node's first packet arrival time. This is done through implementing a new function called *waitExponentialBackoffMediumSensing* function that will get called instead of the *bufferPackets* function at the end the *bufferAllPacketsForBusy* function which tells all other nodes that the medium is currently busy.

```

def waitExponentialBackoffMediumSensing(self, lowerLimit, upperLimit):
    if self.getFirstPacketTimestamp() >= lowerLimit and self.getFirstPacketTimestamp() <= upperLimit:
        newArrivalTime = self.getFirstPacketTimestamp()

        # Add a backoff for each time the node sees the bus being busy
        while newArrivalTime < upperLimit:
            self.collision_counter_medium += 1
            if self.collision_counter_medium > COLLISION_LIMIT:
                self.removeFirstPacketMediumSensing()
                return

            newArrivalTime += self.genExponentialBackoffTimeMediumSensing()

```

Figure 9: Function to Wait Exponential Backoff for Medium Sensing

Through these classes, we were able to attain the results from the previous section. The lab2.py script allows easy execution of the classes described above and its use guide is in the README.md file included.

## Source Code: Lab2.py

```
import numpy as np
import time

from PersistentCSMASimulator import PersistentCSMASimulator
from NonpersistentCSMASimulator import NonpersistentCSMASimulator
from ExponentialRandomVariableGenerator import ExponentialRandomVariableGenerator

def question_1():
    for A in [7, 10, 20]:
        for N in [20, 40, 60, 80, 100]:
            simulator = PersistentCSMASimulator(N, A).run()

def question_2():
    for A in [7, 10, 20]:
        for N in [20, 40, 60, 80, 100]:
            simulator = NonpersistentCSMASimulator(N, A).run()

# main
question_number = raw_input("Enter Question Number [1, 2] ")
question_number = int(question_number)

start_time = time.time()

if question_number == 1:
    question_1()
elif question_number == 2:
    question_2()
```

## Source Code: PersistentCSMASimulator.py

```
from __future__ import division
from Node import Node

SIMULATION_TIME = 1000 # 1000s

TRANSMISSION_RATE = 1000000 # 1 Mbps
PACKET_LENGTH = 1500 # assume all packets are the same length
TRANSMISSION_DELAY = PACKET_LENGTH / TRANSMISSION_RATE

DISTANCE_BETWEEN_NODES = 10
PROPAGATION_SPEED = (2/3) * 300000000
```

```
UNIT_PROPAGATION_DELAY = DISTANCE_BETWEEN_NODES /  
PROPAGATION_SPEED
```

```
class PersistentCSMASimulator:
```

```
    def __init__(self, numNodes, avgPacketArrivalRate):  
        self.nodes = []
```

```
        self.numNodes = numNodes  
        self.avgPacketArrivalRate = avgPacketArrivalRate
```

```
        # metrics  
        self.transmittedPackets = 0  
        self.successfullyTransmittedPackets = 0
```

```
    def run(self):  
        self.createNodes()  
        self.processPackets()  
        self.printResults()
```

```
    def createNodes(self):  
        for i in range(self.numNodes):  
            self.nodes.append(Node(i, self.avgPacketArrivalRate, SIMULATION_TIME))
```

```
    def bufferAllPacketsForBusy(self, currentTime, txNode):  
        for node in self.nodes:  
            offset = abs(node.getNodePosition() - txNode.getNodePosition())  
            propagationDelay = offset * UNIT_PROPAGATION_DELAY  
            firstBitArrivalTime = currentTime + propagationDelay  
            lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY  
            node.bufferPackets(firstBitArrivalTime, lastBitArrivalTime)
```

```
    def processPackets(self):  
        while True:  
            # get the sender node which has the smallest packet arrival time  
            txNode = min(self.nodes, key=lambda node: node.getFirstPacketTimestamp())  
  
            # update the currentTime  
            currentTime = txNode.getFirstPacketTimestamp()  
            if currentTime > SIMULATION_TIME:  
                break  
  
            # A packet is trying to be sent  
            self.transmittedPackets += 1  
  
            # For each node, calculate when the packet arrives + check collision  
            transmissionSuccess = True
```

```

for rxNode in self.nodes:
    offset = abs(rxNode.getNodePosition() - txNode.getNodePosition())
    if (offset == 0):
        continue

    propagationDelay = offset * UNIT_PROPAGATION_DELAY
    firstBitArrivalTime = currentTime + propagationDelay
    lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY

    if rxNode.checkCollision(firstBitArrivalTime):
        rxNode.waitExponentialBackoff()
        self.transmittedPackets += 1
        transmissionSuccess = False

    if not transmissionSuccess:
        txNode.waitExponentialBackoff()
    else:
        self.successfullyTransmittedPackets += 1
        self.bufferAllPacketsForBusy(currentTime, txNode)
        txNode.removeFirstPacket()

def printResults(self):
    print("===== RESULTS =====")
    print("Arrival Rate: {}, NumNodes: {}".format(self.avgPacketArrivalRate,
self.numNodes))
    print("Successfully Transmitted Packets:
{}".format(self.successfullyTransmittedPackets))
    print("Total Transmitted Packets: {}".format(self.transmittedPackets))
    print("Efficiency of CSMA/CD: {}".format((self.successfullyTransmittedPackets /
self.transmittedPackets)))
    print("Throughput of CSMA/CD: {} Mbps".format(((self.successfullyTransmittedPackets
* PACKET_LENGTH / 1000000) / SIMULATION_TIME)))

```

[Source Code: NonpersistentCSMASimulator.py](#)

```

from __future__ import division
from Node import Node

SIMULATION_TIME = 1000 # 1000s

TRANSMISSION_RATE = 1000000 # 1 Mbps
PACKET_LENGTH = 1500 # assume all packets are the same length
TRANSMISSION_DELAY = PACKET_LENGTH / TRANSMISSION_RATE

DISTANCE BETWEEN NODES = 10

```

```
PROPAGATION_SPEED = (2/3) * 300000000
UNIT_PROPAGATION_DELAY = DISTANCE_BETWEEN_NODES / PROPAGATION_SPEED
```

```
class NonpersistentCSMASimulator:
```

```
    def __init__(self, numNodes, avgPacketArrivalRate):
        self.nodes = []
```

```
        self.numNodes = numNodes
        self.avgPacketArrivalRate = avgPacketArrivalRate
```

```
        # metrics
        self.transmittedPackets = 0
        self.successfullyTransmittedPackets = 0
```

```
    def run(self):
        self.createNodes()
        self.processPackets()
        self.printResults()
```

```
    def createNodes(self):
        for i in range(self.numNodes):
            self.nodes.append(Node(i, self.avgPacketArrivalRate, SIMULATION_TIME))
```

```
    def bufferAllPacketsForBusy(self, currentTime, txNode):
        for node in self.nodes:
            offset = abs(node.getNodePosition() - txNode.getNodePosition())
            propagationDelay = offset * UNIT_PROPAGATION_DELAY
            firstBitArrivalTime = currentTime + propagationDelay
            lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY
            if node.waitExponentialBackoffMediumSensing(firstBitArrivalTime, lastBitArrivalTime):
                self.transmittedPackets += 1
```

```
    def processPackets(self):
        while True:
            # get the sender node which has the smallest packet arrival time
            txNode = min(self.nodes, key=lambda node: node.getFirstPacketTimestamp())
```

```
            # update the currentTime
            currentTime = txNode.getFirstPacketTimestamp()
            if currentTime > SIMULATION_TIME:
                break
```

```
            # A packet is trying to be sent
            self.transmittedPackets += 1
```

```
            # For each node, calculate when the packet arrives + check collision
            transmissionSuccess = True
            for rxNode in self.nodes:
                offset = abs(rxNode.getNodePosition() - txNode.getNodePosition())
                if (offset == 0):
                    continue
```

```

propagationDelay = offset * UNIT_PROPAGATION_DELAY
firstBitArrivalTime = currentTime + propagationDelay
lastBitArrivalTime = firstBitArrivalTime + TRANSMISSION_DELAY

if rxNode.checkCollision(firstBitArrivalTime):
    rxNode.waitExponentialBackoff()
    self.transmittedPackets += 1
    transmissionSuccess = False

if not transmissionSuccess:
    txNode.waitExponentialBackoff()
else:
    self.successfullyTransmittedPackets += 1
    self.bufferAllPacketsForBusy(currentTime, txNode)
    txNode.removeFirstPacket()

def printResults(self):
    print("===== RESULTS =====")
    print("Arrival Rate: {}, NumNodes: {}".format(self.avgPacketArrivalRate, self.numNodes))
    print("Successfully Transmitted Packets: {}".format(self.successfullyTransmittedPackets))
    print("Total Transmitted Packets: {}".format(self.transmittedPackets))
    print("Efficiency of CSMA/CD: {}".format((self.successfullyTransmittedPackets /
self.transmittedPackets)))
    print("Throughput of CSMA/CD: {} Mbps".format(((self.successfullyTransmittedPackets *
PACKET_LENGTH / 1000000) / SIMULATION_TIME)))

```

## Source Code: Node.py

```

from __future__ import division
from ExponentialRandomVariableGenerator import ExponentialRandomVariableGenerator
from Packet import Packet
from collections import deque
import random

COLLISION_LIMIT = 10
TRANSMISSION_RATE = 1000000 # 1 Mbps

class Node:
    def __init__(self, position, arrivalTimeLambda, simulationTime):
        self.queue = deque()
        self.position = position
        self.arrivalTimeLambda = arrivalTimeLambda
        self.simulationTime = simulationTime
        self.collision_counter = 0
        self.collision_counter_medium = 0

```

```

self.genPacketArrivalEvents()

def genPacketArrivalEvents(self):
    # create Arrival Time generator
    arrivalTimeGenerator =
ExponentialRandomVariableGenerator(lmbda=self.arrivalTimeLambda)

    # create arrival events for the simulation
    currentTime = 0
    while currentTime < self.simulationTime:
        # add inter-arrival time to arrive at current timestamp
        interArrivalTime = arrivalTimeGenerator.genValue()
        currentTime += interArrivalTime

        # add packet to queue
        self.queue.append(Packet(currentTime))

    # Checks if next packet is during a transmission. If next packet
    # Arrives before the sender's first bit arrives, bus appears to be idle
    def checkIfBusy(self, firstBitArrivalTime, lastBitArrivaltime):
        return firstBitArrivalTime < self.getFirstPacketTimestamp() and
self.getFirstPacketTimestamp() < lastBitArrivaltime

    # If packet arrival < arrival of transmitted first bit, bus appears to be idle
    def checkCollision(self, firstBitArrivalTime):
        return self.getFirstPacketTimestamp() <= firstBitArrivalTime

    def waitExponentialBackoff(self):
        self.collision_counter += 1
        self.collision_counter_medium = 0
        if self.collision_counter > COLLISION_LIMIT:
            self.removeFirstPacket()
        else:
            # Each node waits backoff time. Means we start waiting from our first packet time
            newArrivalTime = self.getFirstPacketTimestamp() +
self.genExponentialBackoffTime()
            self.bufferPackets(0, newArrivalTime)

    def waitExponentialBackoffMediumSensing(self, lowerLimit, upperLimit):
        if self.getFirstPacketTimestamp() >= lowerLimit and self.getFirstPacketTimestamp() <=
upperLimit:
            newArrivalTime = self.getFirstPacketTimestamp()

        # Add a backoff for each time the node sees the bus being busy
        while newArrivalTime < upperLimit:
            self.collision_counter_medium += 1

```



```

        if self.collision_counter_medium > COLLISION_LIMIT:
            self.removeFirstPacketMediumSensing()
            # return true is a packet was dropped
            return True

        newArrivalTime += self.genExponentialBackoffTimeMediumSensing()

        # Buffer arrival times to when busy becomes free
        self.bufferPackets(0, newArrivalTime)
        # return false is no packets were dropped
        return False

# Pushes packet timestamps to an upper limit given a range
def bufferPackets(self, lowerLimit, upperLimit):
    for packet in self.queue:
        if packet.timestamp >= lowerLimit and packet.timestamp <= upperLimit:
            packet.timestamp = upperLimit
        elif packet.timestamp > upperLimit:
            break

def genExponentialBackoffTime(self):
    # generate a random number between 0 and 2^i-1
    R = random.randint(0, (2**self.collision_counter) - 1)
    # random number * 512 bit-time
    backoff = R * 512 * (1.0 / TRANSMISSION_RATE)
    return backoff

def genExponentialBackoffTimeMediumSensing(self):
    # generate a random number between 0 and 2^i-1
    R = random.randint(0, (2**self.collision_counter_medium) - 1)
    # random number * 512 bit-time
    backoff = R * 512 * (1.0 / TRANSMISSION_RATE)
    return backoff

def removeFirstPacket(self):
    self.queue.popleft()
    self.collision_counter = 0
    self.collision_counter_medium = 0

def removeFirstPacketMediumSensing(self):
    self.queue.popleft()
    self.collision_counter_medium = 0

def getFirstPacketTimestamp(self):
    if self.queue:
        return self.queue[0].timestamp

```

```
    else:
        return float('inf')

    def getNodePosition(self):
        return self.position
```

[Source Code: Packet.py](#)

```
TRANSMISSION_RATE = 1000000 # 1 Mbps

class Packet:
    def __init__(self, length):
        self.length = length

    def getTransmissionTime(self):
        return self.length / TRANSMISSION_RATE
```