statistical model (cleavage)

Assumption: 1 max principle stress σ_1.

2 weakest link postulate.

3 plastic deformation $\Rightarrow$ cleavage fracture.

Beremin Model

$$P_R = 1 - \exp\left[\frac{-\int_{PZ} \sigma_1^m \cdot dv/V_0}{\sigma_0^m} \right] \quad (1)$$

$$= 1 - \exp\left[-\frac{\sigma_w^m}{\sigma_0^m} \right]$$

σ_w, weibull stress. σ_0, scale parameter.

m, weibull modulus

★ V_0, mean volume occupied by each microcrack.

small scale yielding $\Longrightarrow$

$$P_R = 1 - \exp\left(- \frac{B \cdot K_I^4 \cdot \sigma_{ys}^{m-4} \cdot C_{m,n}}{V_0 \cdot \sigma_0^m} \right) \quad (2)$$

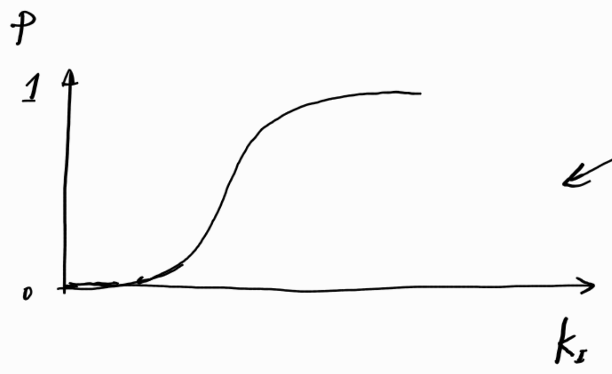
$C_{m,n}$, numerical coefficient

σ_{ys}, yield stress.

Note. for a given stress intensity of K_I,

the probability of clearage is P_R.

$\Rightarrow P(K_{Ic} < K_I) = P_R \Rightarrow K_{Ic}$ satisfies weibull distribution of $m_k = 4$.



Cumulative distribution fuction of Weibull Distribution.

[2

Under small scale yielding condition.

$P_R = f(BK_I^4)$, the same P_R for specimens of different thickness

$\Rightarrow$ Scaling Rule : $(BK_{IC}^4)_1 = (BK_{IC}^4)_2$

The Problem

If Take $0 < \sigma_1 < \sigma_{ys}$, the $P_R \neq 0$ (2)

But, According to the Assumption 3 (plastic $\Rightarrow$ cleavage)

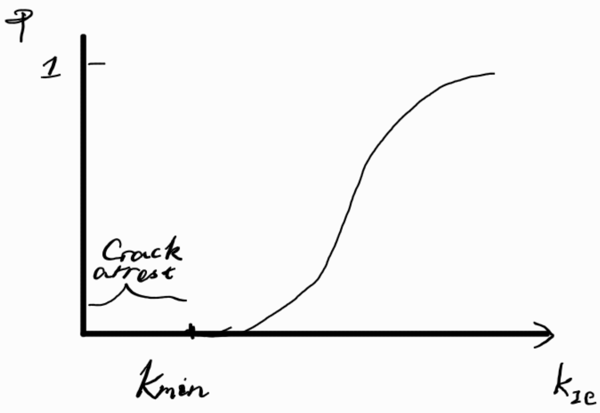
$P_R = 0$ (if $\sigma_1 < \sigma_{ys}$)

$\Rightarrow$ Contradictory!

$\Rightarrow$ Ambigous calibration of σ_0 and m.

The Master Curve approach

Modify $\Rightarrow$ $P_R = 1 - \exp\left[-\frac{B}{B_0}\left(\frac{K_I - K_{min}}{K_0 - K_{min}}\right)^4\right]$ (3)



Based on equation (4),

Wallin (1985),

$$P_R = 1 - \exp\left[-\left(\frac{K_{Jc} - K_{min}}{K_0 - K_{min}}\right)^4\right] \quad (4-0)$$

ASME Testing standard, $K_0 = 20\ MPa\sqrt{m}$ for Ferritic steel.

$\boxed{K_0}$ Median cleavage value of K_{Jc} ($P = 50\%$), 1T specimans ($B = 25mm$)

$$\overline{K_{Jc,1T}} = 30 + 70\exp[0.019(T - T_0)], \quad (4-1)$$

T_0 is a reference Temperature when $\overline{K_{Jc,1T}} = 100 MPa\sqrt{m}$.

$$50\% = 1 - \exp\left[-\left(\frac{\overline{K_{Jc,1T}} - K_{min}}{K_{0,1T} - K_{min}}\right)^4\right]$$

$$\Rightarrow \overline{K_{Jc,1T}} = K_{min} + (K_{0,1T} - K_{min})(\ln 2)^{\frac{1}{4}}, \quad (4-2)$$

$$(4-1) + (4-2) \Rightarrow K_{0,1T} - K_{min} = \frac{30 + 70\exp[0.019(T - T_0)] - 20}{(\ln 2)^{\frac{1}{4}}}$$

$$= 11 + 77\exp[0.019(T - T_0)], \quad (4-3)$$

$$(4-3)\ (4-0) \Rightarrow ☆\ K_{Jc,1T}(P_R) = 20 + [-\ln(1-P)]^{\frac{1}{4}}\{11 + 77\exp[0.019(T - T_0)]\} \quad (5)$$

Once T_0 is known, (5) is fully determined!

Note, (5) Need SSY (small scale yielding) condition.

Scaling rule: According to equation (3)

$$B_1(K_{Jc,B1} - K_{min})^4 = B_2(K_{Jc,B2} - K_{min})^4$$

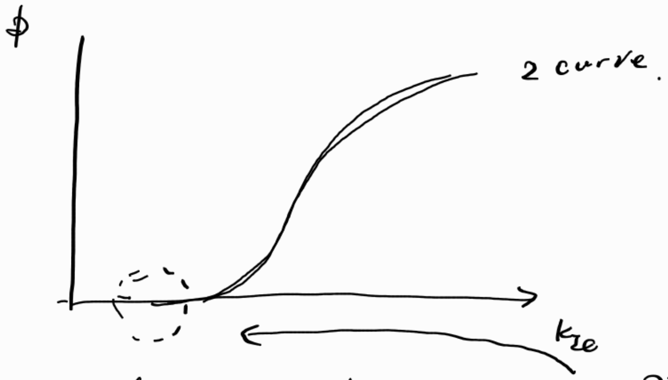
Problem.

3 parameter Weibull Distribution

(K_0, K_{min}, m)

Different combination of (K_0, K_{min}, m)

the cummulative Distribution are really close



But, the severe Problem is. [Different K_{min}].

* Mis predicting the lower bound K_{min}

The Prometey Unified Curve Model

1 the simplified Model

clearage fracture dominated by microcrack nucleation.

Assumption. the critical stress σ_d for nucleate, follow Weibull Distribution.

$$P(\sigma_d) = 1 - \exp\left[- \left(\frac{\sigma_d - \sigma_{th}}{\sigma_0} \right)^m \right]$$

3 parameter: σ_{th}, σ_0, m

$P(\sigma_d)$: the probability of finding a carbide strengh less than σ_d in a unit cell.

$$P_{1nuc} = P(\sigma_d > Max\,\sigma_{nuc}) = 1 - P_{nuc}$$

$$= \exp\left[-\left(\frac{Max\,\sigma_{nuc,i} - \sigma_{th}}{\sigma_0}\right)^m\right]$$

$$P_R = 1 - \prod P_{1nuc,i}$$

$$= 1 - \exp\left[-\frac{\sum_{i=1}^{k}(Max\,\sigma_{nuc,i} - \sigma_{th})^m}{\sigma_0^m}\right]$$

$Max\,\sigma_{nuc,i}$ = from the beginning load to current moment, the maximum of nucleation driving force.

$\boxed{\text{The comprehensive model}}$ cleavage controlled by micro crack nucleation or propogation.

☆ Also check this idea on Github homepage /beremin model.pdf

$$P = 1 - \prod_{i=1}^{k}\left\{1 - \sum_{i}^{k}\left[P_{nuc,i}(\varepsilon_{p,n}) - P_{nuc,i}(\varepsilon_{p,n-1})\right] \cdot Max\,P_{prop,i}(\varepsilon_{p,n}, \varepsilon_{p,N})\right\}$$

$P_{nuc,i}$, the probability of nucleation at the accumulated plastic strain $\varepsilon_{p,n}$, at $\varepsilon_{p,N} \Rightarrow P = 1$.

Assumption. $P_{nuc} = 1 - \exp\left[-\left(\frac{\sigma_{nuc} - \sigma_{th}}{\sigma_0}\right)^m\right]$

$$P_{prop} = 1 - \exp\left[-\left(\frac{\sigma_1 \cdot \sqrt{1 + \beta\exp(-\partial\varepsilon_p)}}{\sigma_c}\right)^{\xi}\right]$$

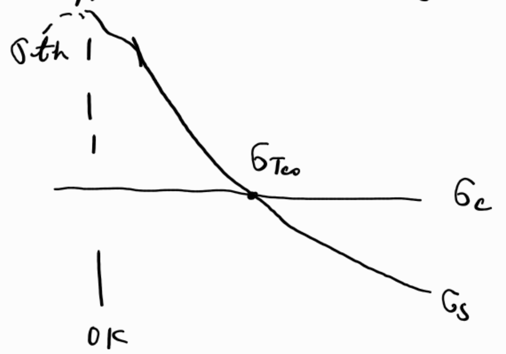
Problems

1. P_{nuc} : According to P_{nuc}, if $\sigma_{nuc} < \sigma_{th}$, $P_{nuc} = 0$.

But this result is contradictory to the physical fact

(Ductile – Brittle transition)

Suppose $\sigma_{th} \approx \sigma_{ys}(0K)$, $[0K, T_{c0}]$ brittle.



$\sigma_{T_{c0}} < \sigma_{th}$
- 1. should not cleavage According to equation
- 2. cleavage [real physics]

2. Φ_{prop} . lack of threshod value

Newly Developed Model

The same idea with Ref Acta Materialia – Failure of metals 1.

In classic Beremin Model. Micro crack in V_0 obey specific distribution (Power law or Weibull Distribution).

But in new model. suppose the distribution is not known.

$$P_f = 1 - \exp\left\{ \int_{PZ} \ln\left[1 - P(V_0)\right] \frac{dV}{V_0} \right\}$$

$$P(V_0) = \int_{\Omega=4\pi} \left(\int_{\sigma_{eq,0}}^{\sigma_{eq}} g(s)\, ds \right) d\Omega \Big/ (4\pi)$$

$g(s)$, probability density function respect to critical strength S.

Actually, $P(V_0) = \int_{\Omega^*} \int_{a_0}^{a_c} g(a)\, da\, d\Omega \,/ (4\pi)$. $K = Y\sigma\sqrt{\pi a}$

$\xrightarrow{\text{equivalent}}$ $P(V_0) = \int_{\Omega} \int_{\sigma} g(s) ds\, d\Omega / (4\pi)$

$$\ln(1-P_R) = \int_{PZ} \ln[1-P(V_0)] \frac{dV}{V_0}$$

$$= \frac{V_{PZ}}{V_0} \int_{PZ} \ln[1-P(V_0)] \frac{dV}{V_{PZ}}.$$

$$r_{PZ}(\theta) \propto \left(\frac{k_I}{\sigma_{YS}}\right)^2$$

$$V_{PZ} \propto B \cdot \int_0^{2\pi} d\theta \int_0^{r_{PZ}} r\, dr \propto B\left(\frac{k_I}{\sigma_{YS}}\right)^4.$$

$$\Rightarrow \quad -\ln(1-P_R) \propto B\left(\frac{k_I}{\sigma_{YS}}\right)^4 \left\{ \frac{-1}{V_0} \int_{PZ} \ln[1-P(V_0)] \frac{dV}{V_{PZ}} \right\}.$$

$$\Rightarrow \quad \frac{\ln(1/(1-P_R))}{B(k_I/\sigma_{YS})^4} = f(k_{Imin}, k_I) \propto \left\{-\frac{1}{V_0}\int_{PZ} \ln[1-P(V_0)]\frac{dV}{V_{PZ}}\right\}$$

Introduce k_{Imin}, $g(s)$.

Not a simple weibull Distribution of modulus 4.

How to detect $g(s)$?

Sometimes, $f(k_I, k_{Imin})$ is gained from data fitting, but the method make the physical meaning lost.