Failure of metals I

Beremin model and Weibull stress

Determine the Possibility of Cleavage Fracture.

Probability in V_0: $P(\sigma) = \int_{a_c(\sigma)}^{\infty} p(a)\,da$, Griffith equ. $a_c = \frac{2E\gamma_s}{\pi\sigma^2}$

P_R in V (many V_0): $P_R = 1 - \exp[-\frac{V}{V_0}P(\sigma)] \approx 1-[1-P(\sigma)]^{\frac{V}{V_0}}$, when $\frac{V}{V_0} \gg 1$.

if. $p(a) = \gamma a^{-\beta}$.

then., $\int_{a_0}^{\infty} p(a) = 1$

$\Rightarrow \beta > 1.,\ \frac{\gamma}{\beta-1}\cdot a_0^{1-\beta} = 1$ (1)

In V_0, $P(\sigma) = \int_{a_c}^{\infty} \gamma a^{-\beta}\,da = \frac{\gamma}{1-\beta}\cdot(\infty^{1-\beta} - a_c^{1-\beta})$

$= \frac{-\gamma}{1-\beta}\ a_c^{1-\beta}$.

$\overset{(1)}{\Longrightarrow} (\frac{a_c}{a_0})^{1-\beta} = \left(\frac{\frac{2E\gamma_s}{\pi\sigma^2}}{\frac{2E\gamma_s}{\pi\sigma_0^2}}\right)^{1-\beta} = (\frac{\sigma_0}{\sigma})^{2-2\beta}$

$= (\frac{\sigma}{\sigma_0})^{2\beta-2}$.

In V: $P_R = 1 - \exp[\frac{-V}{V_0}P(\sigma)]$

$= 1 - \exp[\frac{-V}{V_0}(\frac{\sigma}{\sigma_0})^{2\beta-2}]$, σ_0 is measured in V_0, $\sigma_0 = \sqrt{\frac{2E\gamma_s}{a_0}}$ [initial fracture resistance]

Or, suppose, in V_0, the distribution is,

$P(\text{size} > a) = \exp[-(\frac{a-a_u}{a_0})^N]$

$\Rightarrow P_R = 1 - \exp\{-\frac{V}{V_0}\exp[-(\frac{a_c-a_u}{a_0})^N]$

$= 1 - \exp\{-\frac{V}{V_0}\exp[-\left(\frac{(1/\sigma_c)^2 - (1/\sigma_u)^2}{(1/\sigma_0)^2}\right)^N]$

if σ is not uniform in V

then $P(\sigma)$ is different in different dV.

so. $1-P_R = \cancel{\Sigma} \cancel{1-P_1 dV}$

$(1-P_1)^{\frac{dV_1}{V_0}} (1-P_2)^{\frac{dV_2}{V_0}} \dots$

$$\ln(1-P_R) = \frac{dV_1}{V_0}\ln(1-P_1) + \frac{dV_2}{V_0}\ln(1-P_2) + \dots$$

$$= \frac{\sum_1^{\infty} \ln(1-P_1)\, dV_1}{V_0}$$

$$\Rightarrow P_R = 1 - e^{[\int \ln(1-P)\, dV]/V_0}$$

if $\frac{\int P dV}{V_0} \gg 1$ $\quad \int \ln(1-P)\, dV \approx \int -P\, dV$

$$P_R = 1 - e^{(\int P dV)/V_0}$$

$$P = \int_{a_c}^{\infty} P(a)\, da = \left(\frac{\sigma}{\sigma_0}\right)^m$$

$$\Rightarrow P_R = 1 - \exp\left(\frac{-\int_{PZ} \sigma^m dV}{\sigma_0^m V_0}\right) = 1 - \exp\left[\left(\frac{\left(\frac{1}{V_0} \int_{PZ} \sigma^m dV\right)^{\frac{1}{m}}}{\sigma_0}\right)^m\right]$$

$$= 1 - \exp\left[\left(\frac{\sigma_w}{\sigma_0}\right)^m\right]$$, σ_w = weibull stress

Need to verify !

Sometimes. introduce a threshold.

Like. $P_R = 1 - \exp\left[-\left(\frac{\sigma_w - \sigma_{w\min}}{\sigma_u - \sigma_{u\min}}\right)^m\right]$

Note !

the <u>physical meaning</u> of $P(a)$ is not clear.

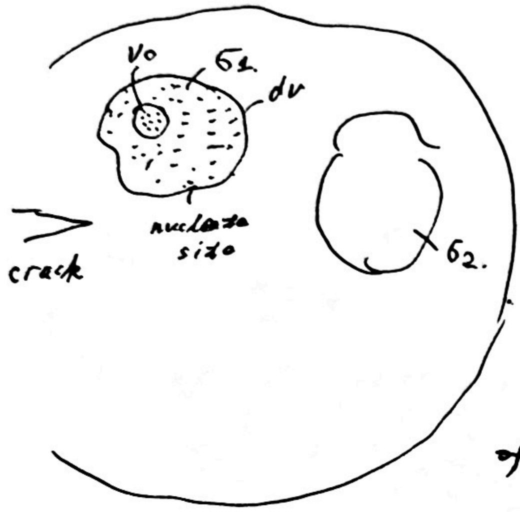
it's very complex in real. complicated situation.

Latti

Modified Model of Beremin Model,

— Bordet · Model.

(consider plastic strain, cause crack nucleate.)



We consider the nucleation of crack is depent on t, because the main crack is propogating. so the state changes $\Rightarrow$ nucleation of new crack.

so we define the Probobility of Nucleation, $P_n(t)$.

so, $P_n(t)$ in dV can derive $P_{cleavage}(t)$ in dV,

by $P_{clea} = P_{propagate} \, N_{site} \, dP_n(t) / N_{site}$

$$= P_{pro} \cdot dP_n(t)$$

$$dP_n(t) = \frac{N_r}{N_0} \cdot \frac{\sigma_y}{\sigma_y^0} \cdot \frac{d\varepsilon_{eq}^p}{\varepsilon_{eq}^{p,0}}$$

N_r = N. sites remaining, N_0 = N. sites originally

$$= (1-P_n) \cdot \frac{\sigma_y}{\sigma_y^0} \cdot \frac{d\varepsilon_{eq}^p}{\varepsilon_{eq}^{p,0}}$$

$$P_n(t) = 1 - \exp\left(-\frac{\sigma_y}{\sigma_{y0}} \frac{\varepsilon_{eq}^p}{\varepsilon_{eq}^{p,0}}\right),$$

$$P_R = 1 - \exp[\ln(1-P_{clea}) \, dV/V_0]$$

if $\int dV/V_0 \to \infty$ $\approx 1 - \exp\{ \cancel{\ln(} - P_{cleav} \, dV/V_0 \}$

$$= 1 - \exp\left(-\int_{P_Z} \int_0^t P_{pro} \, dP_n \cdot dV/V_0\right)$$

suppose that, the distribution of crack is $p(a) = \gamma a^{-\beta}$.

as mentioned before,

$$f_{pro} = \int_{a_0}^{\infty} p(a) \cdot da$$

$$= \left(\frac{\sigma_1}{\sigma_0}\right)^m \quad , \quad m = 2\beta - 2.$$

or, modified with threshold value

$$f_{pro} = (\sigma_1^m - \sigma_{th}^m) / \sigma_0^m - \sigma_{th}^{0\,m}$$

$$-\iint f_{prog} \cdot df_R \frac{dV}{V_0} =$$

$$\cancel{\ln f_R} \cdot \cancel{dt} - \int_{f_Z (\sigma_1 > \sigma_{th})} \int_0^{t (\text{or } \varepsilon_{eq}^p)} \frac{\sigma_1^m - \sigma_{th}^m}{\sigma_0^m - \sigma_{th}^{0\,m}} (1 - f_R(t)) \cdot \frac{\sigma_y}{\sigma_y^0} \frac{d\varepsilon_{eq}^p}{\varepsilon_{eq}^{q,0}} \frac{dV}{V_0}$$

$$\Rightarrow f_R = 1 - \exp\left\{ \frac{-\iint_{f_Z}^{t} (\sigma_1^m - \sigma_{th}^m)(1 - f_R(t)) \cdot \frac{\sigma_y}{\sigma_{y_0}} \cdot \frac{d\varepsilon_{eq}^p}{\varepsilon_{eq}^{q,0}} \frac{dV}{V_0}}{\iint_{f_Z,0}^{t} \sigma_0^m - \sigma_{th}^{0\,m}} \right\}$$

$$f_R = 1 - \exp\left[-\frac{(\sigma_w^*)^m}{(\sigma_0^*)^m} \right]$$