# Homework

武國寧

## 1 在指定的區間上把下列函數展開稱為傅立葉級數

(1) $f(x) = x, (i)(-\pi, \pi), (ii)(0, 2\pi)$

(2) $f(x) = x^2, (i)(-\pi, \pi), (ii)(0, 2\pi)$

## 2 把函數 $f(x)$ 展開成傅立葉級數

$$f(x) = \begin{cases} -\frac{\pi}{4}, & -\pi < x < 0 \\ \frac{\pi}{4}, & 0 \le x < \pi \end{cases}$$

並推出下列結果：

(1) $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$

(2) $\dfrac{\pi}{3} = 1 + \dfrac{1}{5} - \dfrac{1}{7} - \dfrac{1}{11} + \dfrac{1}{13} + \dfrac{1}{17} + \cdots$

(3) $\dfrac{\sqrt{3}}{6}\pi = 1 - \dfrac{1}{5} + \dfrac{1}{7} - \dfrac{1}{11} + \dfrac{1}{13} - \dfrac{1}{17} + \cdots$

## 3 求下列函數 $f(x)$ 的傅立葉級數展式

(1) $f(x) = \dfrac{\pi - x}{2}, x \in (0, 2\pi)$

(2) $f(x) = \sqrt{1 - \cos x}, x \in (-\pi, \pi)$

(3) $f(x) = ax^2 + bx + c, (i)x \in (-\pi, \pi), (ii)x \in (0, 2\pi)$

(4) $f(x) = chx, x \in (-\pi, \pi)$