### Normal Form Games

Multiplayer game where all players take an action simultaneously

#### Definition – Normal form game

A normal form game is a tuple G=(N,A,u)

- ullet N is a set of n number of players
- $A=A_1 \times \ldots \times A_n$  is an *action profile*, where  $A_i$  is the set of actions for player i. Thus, an action profile  $a=(a_1,\ldots,a_n)$  describes the simultaneous moves by all players.
- $u:A \to \mathbb{R}^N$  is a reward function that returns an N-tuple specifying the payoff each player receives in state S. This is called the utility for an action.
- · L'ach player receives a utility (aka remark) bused on their action and every other players actions
- · Goal of each player is to maximise their own utility (Rational)
  - · Players do non care about other players' utilities (self-Interested)

(an use a matrix/tuble to represent outcomes

· All players have perfect information

### Strategies.

Pure Strategy: Always pick the same action every time

Mixed Strategy: Each action has a fixed of chance of being chosen

### Strategy Dominance

Weakly Dominan Strategy

S, dominates Si if the utility received by S, against any other strategy is always greater than or equal to the utility

received by Sr against the same opponents.

Strongly dominant strategy

Same as weak version, but S, has to always have better utility

Uhan Sr (never equal)

A strategy that dominates all other strategies is known as a dominant strategy

Even if the opponent knows our strategy, they would not gain any advantage if it is a dominant strategy

Solving Normal Form Games

Best Response: Strategy maximising utility assuming we know what strategy the opponent will we

Wash Equilibrium! When all players have the best response against all other players.

1.e. If a player know was all other players' strategies are, they still will not want to change their convent strategy

#### Algorithm – Best response

**Input:** Normal form game G=(N,A,u), agent i, and strategy profile  $s_{-i}$  for agents other than i **Output:** Set of best responses

 $\begin{aligned} best\_response &= \emptyset \\ best\_response\_value &= -\infty \\ \text{For each } s_i &\in S_i \\ &\quad \text{if } u(s_i, s_{-i}) > best\_response\_value \text{ then} \\ &\quad best\_response &= \{s_i\} \\ &\quad \text{else if } u(s_i, s_{-i}) = best\_response\_value \text{ then} \\ &\quad best\_response &= best\_response \cup \{s_i\} \\ \text{return } best\_response \end{aligned}$ 

#### 1 Algorithm – Nash equilibria

Input: Normal form game G=(N,A,u)

Output: Set of Nash equilibria

 $nash\_equilibria = \emptyset$ 

For each  $s_1 \in A_1$ 

For each  $s_2 \in A_2$ 

 $\text{if } s_i \in BestResponse(i,s_j) \text{ and } s_j \in BestResponse(j,s_i) \text{ then }$ 

 $nash\_equilibria = nash\_equilibria \cup \{(s_i, s_j)\}$ 

 ${\tt return}\; nash\_equilibria$ 

## To find if an outcome is a nash equilibrium!

Start from that state

Check if any player can improve by picking a different action

Is no player can improve, then it is a nash equilibrium

# Expected Utility

Average Utility received from playing a pure strategy given the opporents strategies

## Indisserence

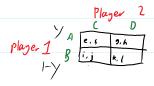
If two pure strategies have the same expected utility

# Mized Strategy Equilibria

A mixed Strategy which makes all opponents indifferent to their pure strategies.

Finding a mixed strategy equilibria.

(assuming 2 player 2action game)



We want to find Y: the probability of selecting oution A

1-> becomes the probability of selecting action B

U(c) = Yxq + (1-Y)x)

U(0) = Yxh + (1->)xl

Solve for Y and 1-Y

	Adversary Terminal 1 Terminal 2	
Terminal Defender	1 5,-3	-1, 1
	2 -5, 5	2,-1

Game 2:

ECLT = ECRT

807=40