MAT2040-T13 Homework 1

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Question 1.

1.1)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{2}$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ -1 & 1 & 2 & 2 \\ 1 & 1 & -1 & 1 \end{array}\right] \tag{3}$$

1.2)

$$\begin{aligned} [A|b] &\xrightarrow[R_3 \to -R_1+R_3]{R_2 \to R_1+R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & -1 & -2 & -2 \end{array}\right] \xrightarrow{R_3 \to 3R_3+R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & 0 & -3 & -1 \end{array}\right] \\ &\xrightarrow{R_3 \to -\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \xrightarrow[R_2 \to -3R_3+R_2]{R_1 \to -R_3+R_1} \left[\begin{array}{ccc|c} 1 & 2 & 0 & \frac{8}{3} \\ 0 & 3 & 0 & 4 \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \\ &\xrightarrow[R_1 \to -2R_2+R_1]{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \end{aligned} \tag{4}$$

$$S = \begin{bmatrix} 0 \\ \frac{4}{3} \\ \frac{1}{3} \end{bmatrix} \tag{5}$$

Question 2.

$$
\left[\begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 2 & 3 & 1 & 0 & 1 & -3 \\ 3 & 4 & 2 & 1 & 2 & -1 \end{array}\right]
\xrightarrow[R_3\to -\frac{3}{4}R_1+R_3]{R_2\to -\frac{1}{2}R_1+R_2}
\left[\begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -1 & -\frac{1}{2} \\ 0 & \frac{1}{4} & -\frac{1}{4} & -\frac{5}{4} & -1 & \frac{11}{4} \end{array}\right]
$$
$$
\xrightarrow{\substack{R_2\to 2R_2 \\ R_3\to 4R_3 \\ R_3\to R_3-R_2}}
\left[\begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 0 & 1 & -1 & -3 & -2 & -1 \\ 0 & 0 & 0 & -2 & -2 & 12 \end{array}\right]
\xrightarrow{\substack{R_3\to -\frac{1}{2}R_3 \\ R_2\to R_2+3R_3 \\ R_1\to R_1-3R_3}}
\left[\begin{array}{ccccc|c} 4 & 5 & 3 & 0 & 1 & 13 \\ 0 & 1 & -1 & 0 & 1 & -19 \\ 0 & 0 & 0 & 1 & 1 & -6 \end{array}\right]
$$
$$
\xrightarrow{\substack{R_1\to R_1-5R_2 \\ R_1\to \frac{1}{4}R_1}}
\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -1 & 27 \\ 0 & 1 & -1 & 0 & 1 & -19 \\ 0 & 0 & 0 & 1 & 1 & -6 \end{array}\right] \tag{6}
$$

$$
S = \begin{bmatrix} -2x_3 + x_5 + 27 \\ x_3 - x_5 - 19 \\ x_3 \\ -x_5 - 6 \\ x_6 \end{bmatrix} \tag{7}
$$

Question 3.

In this question, we could use partition matrix:

$$
AB = \left[\begin{array}{cc|cc} 1 & 0 & a & b \\ 0 & 1 & c & d \end{array}\right] \left[\begin{array}{cc} -a & -b \\ -c & -d \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right] = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{8}
$$

$$
BA = \left[\begin{array}{cc} -a & -b \\ -c & -d \\ \hline 0 & 0 \\ -1 & 1 \end{array}\right] \left[\begin{array}{cc|cc} 1 & 0 & a & b \\ 0 & 1 & c & d \end{array}\right] = \begin{bmatrix} -a & --b & -a^2-bc & -ab-bd \\ -c & -d & -ac-cd & -bc-d^2 \\ 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix} \tag{9}
$$

Question 4.

4.1)

$$
\begin{aligned}
A^2 - B^2 \\
= A \cdot A + B \cdot B \\
= A^T \cdot A^T - B^T \cdot B^T \\
= (A \cdot A)^T - (B \cdot B)^T \\
= ((A^2) - (B^2))^T
\end{aligned} \tag{10}
$$

, so it is symmetric.

4.2) Let LHS $= (A+B)(A-B)$ and RHS $= [(A+B)(A-B)]^T$, then

$$\text{RHS} = (A-B)^T(A+B)^T = (A^T-B^T)(A^T-B^T) = (A-B)(A+B) \neq \text{LHS} \tag{11}$$

, so it is not symmetric.

4.3) Let LHS $= ABA$ and RHS $= (ABA)^T$, then

$$\text{RHS} = (BA)^T A^T = A^T B^T A^T = ABA = \text{LHS} \tag{12}$$

, so it is symmetric.

4.4) Let LHS $= ABAB$ and RHS $= (ABAB)^T$, then

$$\text{RHS} = (AB)^T(AB)^T = B^T A^T B^T A^T = BABA \neq \text{LHS} \tag{13}$$

, so it is not symmetric.