

# MAT2040-T13 Homework 1

Yuming Zhou (121050081@link.cuhk.edu.cn)

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## Question 1.

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1.1)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix} \quad (1)$$

$$B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad (2)$$

$$[A|b] = \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ -1 & 1 & 2 & 2 \\ 1 & 1 & -1 & 1 \end{array} \right] \quad (3)$$

1.2)

$$\begin{aligned}
 [A|b] &\xrightarrow{\substack{R_2 \rightarrow R_1 + R_2 \\ R_3 \rightarrow -R_1 + R_3}} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & -1 & -2 & -2 \end{array} \right] \xrightarrow{R_3 \rightarrow 3R_3 + R_2} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & 0 & -3 & -1 \end{array} \right] \\
 &\xrightarrow{R_3 \rightarrow -\frac{1}{3}R_3} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 3 & 3 & 5 \\ 0 & 0 & 1 & \frac{1}{3} \end{array} \right] \xrightarrow{\substack{R_1 \rightarrow -R_3 + R_1 \\ R_2 \rightarrow -3R_3 + R_2}} \left[ \begin{array}{ccc|c} 1 & 2 & 0 & \frac{8}{3} \\ 0 & 3 & 0 & 4 \\ 0 & 0 & 1 & \frac{1}{3} \end{array} \right] \\
 &\xrightarrow{\substack{R_2 \rightarrow \frac{1}{3}R_2 \\ R_1 \rightarrow -2R_2 + R_1}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array} \right] \quad (4)
 \end{aligned}$$

$$S = \begin{bmatrix} 0 \\ \frac{4}{3} \\ \frac{1}{3} \end{bmatrix} \quad (5)$$

**Question 2.**

$$\begin{aligned}
& \left[ \begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 2 & 3 & 1 & 0 & 1 & -3 \\ 3 & 4 & 2 & 1 & 2 & -1 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow -\frac{1}{2}R_1 + R_2 \\ R_3 \rightarrow -\frac{3}{4}R_1 + R_3}} \left[ \begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -1 & -\frac{1}{2} \\ 0 & \frac{1}{4} & -\frac{1}{4} & -\frac{5}{4} & -1 & \frac{11}{4} \end{array} \right] \\
& \xrightarrow{\substack{R_2 \rightarrow 2R_2 \\ R_3 \rightarrow 4R_3 \\ R_3 \rightarrow R_3 - R_2}} \left[ \begin{array}{ccccc|c} 4 & 5 & 3 & 3 & 4 & -5 \\ 0 & 1 & -1 & -3 & -2 & -1 \\ 0 & 0 & 0 & -2 & -2 & 12 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow -\frac{1}{2}R_3 \\ R_2 \rightarrow R_2 + 3R_3 \\ R_1 \rightarrow R_1 - 3R_3}} \left[ \begin{array}{ccccc|c} 4 & 5 & 3 & 0 & 1 & 13 \\ 0 & 1 & -1 & 0 & 1 & -19 \\ 0 & 0 & 0 & 1 & 1 & -6 \end{array} \right] \\
& \xrightarrow{\substack{R_1 \rightarrow R_1 - 5R_2 \\ R_1 \rightarrow \frac{1}{4}R_1}} \left[ \begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -1 & 27 \\ 0 & 1 & -1 & 0 & 1 & -19 \\ 0 & 0 & 0 & 1 & 1 & -6 \end{array} \right] \quad (6)
\end{aligned}$$

$$S = \begin{bmatrix} -2x_3 + x_5 + 27 \\ x_3 - x_5 - 19 \\ x_3 \\ -x_5 - 6 \\ x_6 \end{bmatrix} \quad (7)$$

**Question 3.**