MAT2040-T13 Homework 2

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Question 1.

$$
\begin{bmatrix} 2 & 4 \,\Big|\, 1 & 0 \\ 3 & 1 \,\Big|\, 0 & 1 \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & \frac{1}{3} \,\Big|\, 0 & \frac{1}{3} \\ 1 & 2 \,\Big|\, \frac{1}{2} & 0 \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & \frac{1}{3} \,\Big|\, 0 & \frac{1}{3} \\ 0 & \frac{5}{3} \,\Big|\, \frac{1}{2} & -\frac{1}{3} \end{bmatrix}
$$

$$
\rightarrow \begin{bmatrix} 1 & \frac{1}{3} \,\Big|\, 0 & \frac{1}{3} \\ 0 & \frac{1}{3} \,\Big|\, \frac{1}{10} & -\frac{1}{15} \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & 0 \,\Big|\, -\frac{1}{10} & \frac{6}{15} \\ 0 & 1 \,\Big|\, \frac{3}{10} & -\frac{1}{5} \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & 0 \,\Big|\, -\frac{1}{10} & \frac{2}{5} \\ 0 & 1 \,\Big|\, \frac{3}{10} & -\frac{1}{5} \end{bmatrix} \quad (1)
$$

Question 2.

$$
\begin{bmatrix} 1 & 4 & 3 \,\Big|\, 1 & 0 & 0 \\ -1 & -2 & 0 \,\Big|\, 0 & 1 & 0 \\ 2 & 2 & 3 \,\Big|\, 0 & 0 & 1 \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & 4 & 3 \,\Big|\, 1 & 0 & 0 \\ 0 & 2 & 3 \,\Big|\, 0 & 1 & 0 \\ 0 & 0 & -3 \,\Big|\, -\frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \rightarrow
$$

$$
\begin{bmatrix} 1 & 4 & 0 \,\Big|\, \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 2 & 0 \,\Big|\, \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \,\Big|\, \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & 4 & 0 \,\Big|\, \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \\ 0 & -4 & 0 \,\Big|\, -1 & 1 & 1 \\ 0 & 0 & 1 \,\Big|\, \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \rightarrow
$$

$$
\begin{bmatrix} 1 & 0 & 0 \,\Big|\, -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 \,\Big|\, \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 \,\Big|\, \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \quad (2)
$$

$$
S = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \begin{pmatrix} 12 \\ -12 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -\frac{8}{3} \end{pmatrix} \quad (3)
$$

Question 3.

$$
E_3 E_2 E_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad (4)
$$

Question 4.

To prove

$$
M = ABC \text{ is invertible} \quad (5)
$$

$$
\Longleftrightarrow
$$

$$
A, B, C \text{ are all invertible} \quad (6)
$$

From eq. (5) to eq. (6):

$$OK \tag{7}$$

From eq. (6) to eq. (5):

$$AA^{-1}B^{-1}BC^{-1}C = I^3 \tag{8}$$