ME 6510: Intermediate Heat Transfer

Autumn Semester, 2018
The Ohio State University
Project 2
Due Tuesday November 27, 2018 by 6 pm

In this project you will determine the temperature profiles near the entrance section of a pipe whose wall is maintained at a constant temperature, T_w, and eventually compare the resulting Nusselt number with analytical solution. You will also examine the effect of axial conduction on the Nusselt number. For simplicity, we will consider fully-developed laminar flow entering the pipe (without this assumption, one has to solve the full Navier-Stokes equations, and that is beyond the scope of this class). The inlet temperature profile is a plug profile ($T = T_{in}$) and will develop slowly into a fully-developed profile as you go further downstream.

In class, we discussed the case of negligible axial conduction, and found that the problem reduces to the so-called Graetz problem, whose solution can be obtained using separation of variables. The solution is summarized as follows:

x^+	Nu_x
0	∞
0.001	12.80
0.004	8.03
0.01	6.00
0.04	4.17
0.08	3.77
0.10	3.71
0.20	3.66
∞	3.66

where $x^+ = \dfrac{2(x/D)}{\mathrm{Re}\,\mathrm{Pr}}$, and $Nu_x = h_x D / k$. Note that the x coordinate starts from the inlet of the pipe. The pipe has a diameter D. The convective film heat transfer coefficient at location x is h_x.

Develop a program to solve the finite difference equations that you developed in HW5. You are free to use a solver of your choice. Use the following inputs for your calculations: D = 2 cm, T_w = 500K, T_{in} = 300K. The thermo-physical properties to be used are that of water: k = 0.6 W/m/k, ρ = 998 kg/m^3, ν = 10^{-6} m^2/s, and c = 4182 J/kg/K. Use a mass flow rate of 0.01 kg/s. In order to apply zero gradient boundary conditions at the outlet, the length of your computational domain must be substantially larger than the thermal entry length. In general, it is recommended that you choose a length that is at least twice the length at which fully developed thermal condition is reached. Note that $\left(\dfrac{x}{D}\right)_{fullydeveloped} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}$.

Reporting of Results

Write a brief (not to exceed 10 pages) report that has the following sections:

(1) Abstract: 100 words or less summarizing why and what you did, and what you found.

(2) Introduction: objectives, problem description
(3) Theory and Solution Method: state governing equations, assumptions, mathematical treatment for finite-difference equations *etc.*
(4) Results and Discussion:
   a. Plot non-dimensional temperature at the x^+ values shown in the table above. You may exclude 0 and ∞. All profiles should be plotted on the same graph and labeled.
   b. Tabulate values of Nu_x for the same x^+ locations, and compare with the values given above *with and without axial conduction*.
   c. Comment on your results. Your grade will be influenced by how insightful and thought-provoking this discussion is. You are welcome to include additional plots/tables to supplement your discussion.

(5) Major Conclusions

The entire report, except equations, must be typewritten. Equations may be hand-written. Derivation of finite-difference equations should not be part of the report, but should be kept handy in case you need to see the instructor for help. Your computer programs must be attached to the report in the form of an Appendix. If you do not submit the programs, your report will not be graded.

General Instructions

- Start planning as soon as the project is handed out. Do not wait until the last day.
- In order to do this successfully, you need to plan ahead. Discussion of your solution strategy with the instructor prior to code development is encouraged.
- Do not use hardwired values in your code. Use variables instead. Typical examples are geometric parameters, number of nodes *etc*.
- If you find that you have not made much progress in one week, seek advice from the instructor immediately.
- Make sure that your report has no spelling and grammatical errors. Proofread it carefully.