Vegetation Banding: Work in Progress

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1 Introduction

There are many instances in nature where vegetation growth in dryland environments displays a band-like pattern formation. Our goal is to understand a simple model between w the water concentration and b the biomass concentration.

We consider a two-species reaction-diffusion system as in [1]. We use the same simple cubic nonlinearity in addition to diffusive rates and an additional advective term in the water concentration.

$$\begin{aligned} w_t &= d_w w_{xx} + c w_x - b(1-b)(b-a) - \gamma w \\ b_t &= d_b b_{xx} + b(1-b)(b-a) + \gamma w \end{aligned} \tag{1.1}$$

Here, $\gamma > 0$, $a \in (0,1)$, $0 \leq c \ll 1$, and $0 < d_b \ll d_w$. We also generally set $g(b) = b(1-b)(b-a)$ for convenience. When $c = 0$, we have that setting $d_t = 0$ in looking for equilibria, we have that

$$d_w w_{xx} + d_b b_{xx} = 0 \implies d_w w + d_b b = \theta$$

for some constant θ. Substituting into (1.1) we have that

$$d_b b_{xx} - g(b) + \frac{\gamma}{d_w}(\theta - d_b b) = 0$$

Defining $v_\theta'(b) = -g(b) + \frac{\gamma}{d_w}(\theta - d_b b)$, we have that $d_b b_{xx} = -v_\theta'(b)$, which is a quartic polynomial in b. We can understand the dynamics of the equilibria, and want to analyze the heteroclinic orbit between the two saddle equilibria when $c \neq 0$.

References

[1] Madeleine Kotzagiannidis, Jeremiah Peterson, Joseph Redford, Arnd Scheel, and Qiliang Wu. *Stable pattern selection through invasion fronts in closed two-species reaction-diffusion systems*. Minneapolis, MN, 2011.