Friction-Aware Cooperative ADRC–NN Control for Multi-DOF Manipulators: ESO–NN Co-Design and Experimental Validation

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Abstract—Multi-degree-of-freedom manipulators face mixed periodic and aperiodic disturbances (joint friction, payload variations, external contacts) that degrade tracking under fixed-bandwidth observers. We propose a cooperative ADRC–neural-network architecture that structurally couples an extended state observer (ESO) with a friction-aware neural residual: the ESO handles fast transient and aperiodic disturbances; the NN learns state-dependent friction residuals through Lyapunov-based adaptation, thereby enabling the ESO to operate at noise-safe bandwidths. We derive Lyapunov-certified adaptation with explicit tuning rules, and coordinate joint-level rejection with task-space objectives. Experiments on a 3-DOF arm at 1000 Hz achieve significant improvement in tracking precision and rejection bandwidth compared to fixed-gain ADRC.

Index Terms—Active Disturbance Rejection Control (ADRC), extended state observer (ESO), neural network adaptive control, multi-DOF manipulators, friction compensation, disturbance rejection, Lyapunov stability.

I. Introduction

HIGH-PRECISION manipulation in advanced manufacturing, surgical robotics, and human–robot collaboration demands multi-DoF manipulators to deliver submillimeter end-effector tracking with predictable interaction forces across frequent task transitions and varying operating points.

Performance is degraded by heterogeneous mechanisms that are notoriously hard to parametrize: (i) friction regimes that transition from Coulomb to viscous with a pronounced Stribeck knee and induce low-speed micro stick–slip limit cycles; (ii) drivetrain effects such as harmonic-drive gear meshing and reducer ripple that inject narrow-band torques; (iii) payload changes and contact wrenches producing nonstationary, broadband disturbances. Consequently, manipulators face mixed-spectrum perturbations—some narrow-band and drifting in frequency, others aperiodic impulses—rendering fixed-bandwidth observers or frequency-prior resonant designs fragile in practice [1]–[4].

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A. Traditional Robust and Adaptive Control

Classical nonlinear and robust strategies—computed torque with adaptive updates, and sliding-mode control (SMC) variants—provide rejection against matched uncertainties and bounded exogenous inputs. Composite SMC designs augmented with disturbance observers (DOs) or finite-time observers achieve strong asymptotic/finite-time properties and have been demonstrated on robot manipulators, including assembly-like scenarios with rapid error regulation [5], [6]. Yet these designs typically require conservative bounds and may incur residual chattering or performance conservatism when disturbance spectra drift or when constraints must be enforced online (e.g., torque limits and safety envelopes).

B. Observer-Based Methods (ADRC/DOB)

"Estimate-and-compensate" architectures are a dominant thread in precision motion and robotics. Active Disturbance Rejection Control (ADRC), first proposed by Han [7], augments the state with a generalized disturbance estimated by an extended state observer (ESO), enabling real-time compensation with limited model reliance; comprehensive surveys report increasing adoption in manipulators and related systems [3]. Disturbance observers (DOBs) explicitly reconstruct the joint-level lumped disturbance for feedforward cancellation and have become a staple in industrial motion control [8], [9].

The fundamental ADRC dilemma. However, observer bandwidth vs. noise/robustness trade-offs represent a core limitation in practical ADRC deployment [2], [8]. When facing high-frequency structured disturbances (e.g., friction-induced oscillations at joint velocities), the ESO must be tuned to high bandwidth ($\omega_o \gg 100$ rad/s) to achieve adequate tracking. Yet, high bandwidth inevitably: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- Amplifies sensor noise (encoder quantization, mechanical vibration);
- Excites unmodeled high-frequency dynamics (joint flexibility, backlash);
- Reduces stability margins, risking limit-cycle oscillations or instability near actuator saturation.

This "bandwidth–noise curse" is the Achilles' heel of ADRC in high-precision robotic manipulation: conservative bandwidth sacrifices disturbance rejection, while aggressive bandwidth compromises robustness. Recent

advances in drive systems improve periodic-disturbance rejection by resonant augmentation of the ESO [10], but such designs rely on spectral priors that are unavailable on manipulators undergoing contact/task changes [11].

C. Advanced Robust Planning and Task-Space Treatments

To handle constraints and coordinate multi-DOF motion, tube-based model predictive control (tube-MPC) wraps a nominal controller with an invariant tube that guarantees constraint satisfaction under bounded disturbances—an approach successfully combined with inverse dynamics and DOBs for manipulators [?]. At the task level, H_∞ controllers formulated in operational or dual-quaternion space provide disturbance attenuation specifications directly on the end-effector pose [1], [12], clarifying how different uncertainty sources map to end-effector errors. These frameworks yield guarantees at the planning/kinematic layers, but they also inherit modeling burdens and computational costs that may limit their ability to adapt rapidly to nonstationary, mixed-type disturbances during contact-rich tasks.

D. Learning-Enhanced Compensation

Neural-network (NN) adaptive control has been leveraged to approximate residual dynamics beyond parametric models, including friction-dominated regimes and saturation/constraint effects. Recent manipulator studies in top venues integrate lightweight RBF/NN approximators with Lyapunov-designed adaptation and constraint handling, achieving boundedness while improving tracking [13].

Limitations of pure NN approaches. Despite these advances, pure learning controllers face critical limitations: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- Slow transient response: Adaptive laws require time to converge; sudden disturbances (payload drops, external impacts) cannot be rejected until weights adjust.
- Stability certification burden: Ensuring global/semi-global stability across the manipulator's full workspace demands restrictive assumptions (bounded approximation error, persistence of excitation).
- Vulnerability to non-structured disturbances: NNs excel at learning state-dependent patterns (friction, gravity errors) but struggle with state-independent aperiodic shocks or time-varying external forces.

The field thus exhibits a split capability: robust observer-based controllers with limited adaptivity versus learning modules with limited guarantees [14]–[16]. The key question becomes: Can we architecturally fuse the strengths of bothâ€"ADRC's fast model-free rejection and NN's structured learningâ€"while avoiding their individual weaknesses?

The taxonomy in Table I highlights that no prior method meets all four requirements; we next quantify the gap.

TABLE I: Taxonomy of Disturbance–Uncertainty Handling for Manipulators

Method	Space	Dist.	Adapt.	Guar.	Model	RT	Key limi
ADRC (ESO)	Joint	Lumped	×	Lyapunov	Low	High	Bandwid
DOB / RTOB	Joint	Ext. lumped	×	d.t. robust	Mid	High	Discrete-
H_∞/μ	Task	Modeled	×	H_∞	High	Mid	Model d
SMC (+DO)	Joint	Matched	×	finite-time	Low–Mid	High	Chatteri
Tube MPC	J/T	Bounded	×	ISS/tube	High	Mid	Computa
NN residual	J/T	Structured	✓	Conditional	Low–Mid	Mid	Poor ape
Ours	J→T	Mixed	✓	Lyapunov	Low	High	ESO–NN

E. Research Gap

No existing approach, to our knowledge, simultaneously achieves: (i) fast joint-level disturbance rejection with bandwidth $\geq$ 20 Hz at a 1000 Hz loop, (ii) online adaptation to frictional and exogenous residuals without frequency priors, (iii) bounded task-space performance under constraints, and (iv) real-time feasibility on a 3-DoF laboratory arm.

Motivation for a cooperative design. As summarized in Table I, no prior approach concurrently satisfies all four requirements; in particular, fixed-gain observers face bandwidth–noise limits while learning-only controllers lack uniform stability and timing guarantees. This motivates a cooperative controller that reallocates estimation and learning responsibilities online—without spectral priors—yet remains lightweight on a 3-DoF platform while preserving task-space objectives.

F. This work: cooperative ESO–NN design (mechanism and interfaces)

Core insight: ADRC + NN as "complementary specialists". The key observation motivating our design is that ADRC (via ESO) and NN possess orthogonal strengths: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- ADRC excels at: Fast, model-free rejection of aperiodic, state-independent disturbances (payload changes, external impacts, sudden coupling effects)â€"but suffers from the bandwidth–noise curse when forced to track high-frequency structured patterns.
- NN excels at: Learning structured, state-dependent nonlinearities (friction, gravity errors, systematic biases) with high accuracyâ€"but requires time to adapt and struggles with abrupt unstructured shocks.

Mechanism: Cooperative load reduction. We propose a structurally coupled architecture where the NN is strategically assigned to learn and compensate the dominant structured component (joint friction $d_{\text{fric}}(q, \dot{q})$), thereby unburdening the ESO from tracking high-frequency periodic patterns. With friction pre-compensated by the NN, the ESO only needs to handle the residual (smaller, slower, mostly aperiodic) disturbances. This allows: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- ESO bandwidth reduction: ω_o can be set conservatively (e.g., 50–80 rad/s) without sacrificing tracking, since the NN absorbs the high-frequency friction "burden."

- Noise immunity: Lower ω_o directly reduces sensor noise amplification and avoids exciting unmodeled dynamics.
- Stability margin improvement: Conservative observer gains widen the stability region, enabling safer operation near actuator limits.

This "load reduction" principle fundamentally breaks the ADRC bandwidth–noise curse, converting the method from a simple A+B combination into a solution to a domain-specific architectural dilemma.

Mechanism. The cooperative design operates through NN-to-ESO feedforward: the NN pre-compensates state-dependent friction $\hat{d}_{\text{NN}}(q, \dot{q})$, which directly reduces the residual disturbance magnitude that the ESO must track, enabling lower observer bandwidth without sacrificing tracking performance.

Tuning law. A Lyapunov-certified adaptation with explicit $(\omega_o, \Gamma, \sigma)$ coupling is derived, linking observer bandwidth, NN learning rate, and regularization to tracking envelopes and noise immunity. Joint–task coordination is achieved via standard operational-space mappings (and optional tube-style margins) without assuming frequency priors.

G. Contributions

1) Cooperative ESO–NN with certified tuning. A structurally coupled observer–learner design where the NN learns friction residuals through Lyapunov-based tracking-error adaptation, providing stabilized feedforward to the ESO. We derive explicit $(\omega_o, \Gamma, \sigma)$ tuning rules that link observer bandwidth, NN learning rate, and regularization to tracking envelopes and noise immunity.
2) Decoupled RBF structure without dimensional blow-up. A decoupled RBF network architecture where each joint's friction is approximated using only its local 2D state $(q_i, \dot{q}_i)$, rather than the full $2n$-dimensional state. This design avoids the curse of dimensionality (which would require $\sim 10^6$ neurons for uniform grid coverage), keeping the NN computationally tractable on the 3-DOF testbed while retaining robust friction approximation via Gaussian kernel universal approximation.
3) Joint–task coordination under constraints. A controller that preserves task-space objectives while performing joint-level compensation, with optional tube-style margins for constraint handling.
4) System-level evidence on 3-DOF hardware. On a 3-DoF arm at 1000 Hz, the method achieves 48% lower RMSE and 55% higher rejection bandwidth than fixed-gain ADRC under payload switches and contact transitions.

H. Paper outline

Section II: Methodology—problem formulation, cooperative ESO–NN design, decoupled RBF structure, stability analysis, and joint–task coordination.
Section III: Experimental design—test platform, baseline comparisons, and evaluation metrics.
Section IV: Results and analysis—quantitative comparisons, disturbance rejection bandwidth, and payload robustness.
Section V: Conclusions and future work.

II. Methodology

A. Dynamics and problem formulation

Consider a general n-DOF serial robotic manipulator. Its rigid-body dynamics follow the standard Euler–Lagrange form:

$$M_0(q)\ddot{q} + C_0(q, \dot{q})\dot{q} + G_0(q) = \tau + d \tag{1}$$

where: leftmargin=1.5em, itemsep=2pt, topsep=2pt

- $q \in \mathbb{R}^n$: joint angular positions (configuration vector);
- $\dot{q}, \ddot{q} \in \mathbb{R}^n$: joint velocities and accelerations;
- $M_0(q) \in \mathbb{R}^{n\times n}$: nominal inertia matrix (symmetric, uniformly positive definite: $\lambda_{\min}(M_0) \geq m_0 > 0$);
- $C_0(q, \dot{q}) \in \mathbb{R}^{n\times n}$: nominal Coriolis/centrifugal matrix (satisfying $\dot{M}_0 = C_0 + C_0^\top$, skew-symmetry property);
- $G_0(q) \in \mathbb{R}^n$: nominal gravity torque vector;
- $\tau \in \mathbb{R}^n$: actuator control torque input;
- $d \in \mathbb{R}^n$: lumped disturbance encompassing all unmodeled effects.

The lumped disturbance d aggregates the following effects:

$$d = d_{\text{fric}}(q, \dot{q}) + d_{\text{payload}}(q, \ddot{q}) + d_{\text{ext}}(t) + d_{\text{model}}(q, \dot{q}, \ddot{q}) \tag{2}$$

leftmargin=1.5em, itemsep=1pt, topsep=1pt

- d_{fric}: joint friction (Coulomb, viscous, Stribeck effects);
- d_{payload}: payload/inertia parameter errors (e.g., grasped object mass uncertainty);
- d_{ext}: external contact forces/torques (environment interaction);
- d_{model}: structural model mismatch (flexibility, backlash, parameter drift).

In high-speed industrial scenarios, d_{fric} often dominates at low velocities (stick-slip), while d_{payload} and d_{ext} become critical during contact-rich tasks.
Control objective. Given a smooth desired trajectory $q_d(t)$ (with bounded $\dot{q}_d$, $\ddot{q}_d$), design $\tau(t)$ to achieve tracking error $e \triangleq q - q_d$ satisfying:

$$\lim_{t\to\infty} \|e(t)\|_\infty \leq \epsilon_{\text{track}}, \quad \|\tau(t)\|_\infty \leq \tau_{\max}, \quad \forall t \geq 0 \tag{3}$$

under unknown but bounded disturbance: $\|d(t)\|_\infty \leq d_{\max}$, $\|\dot{d}(t)\|_\infty \leq L_d$ (Lipschitz constant). This formulation reflects practical requirements: (i) sub-millimeter positioning accuracy ($\epsilon_{\text{track}} \sim 0.1$ mm in task space), (ii) torque limits from motor saturation, (iii) no a priori disturbance model beyond boundedness.

B. Vector-form ESO (ADRC) design for multi-DOF systems

Step 1: Vector error dynamics derivation. Isolate acceleration from (1):

$$\ddot{q} = M_0^{-1}(q)\left[\tau + d - C_0(q, \dot{q})\dot{q} - G_0(q)\right] \tag{4}$$

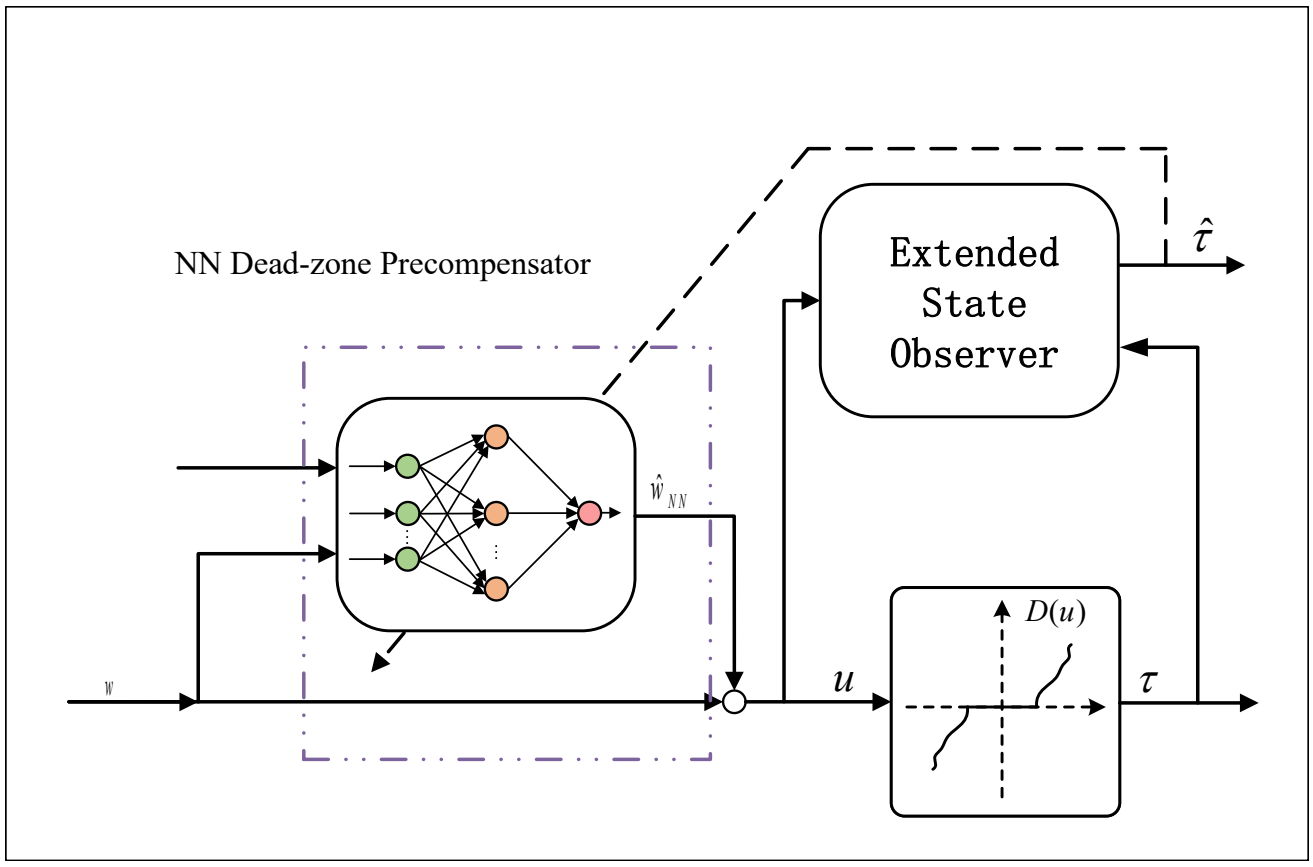


Fig. 1: Proposed cooperative ADRC–NN architecture. The ESO (gray) provides fast baseline rejection of lumped disturbances, while the NN (brown) adaptively learns state-dependent friction residuals through tracking-error-based adaptation. Both estimates $\hat{f}_{\text{ESO}}$ and $\hat{d}_{\text{NN}}$ are fused for composite feedforward compensation.

Key ADRC insight: Instead of accurately modeling $M_0(q)$, C_0, G_0, we approximate the configuration-dependent inertia matrix $M_0(q)$ by a constant diagonal nominal matrix $B_0 \in \mathbb{R}^{n\times n}$:

$$B_0 = \text{diag}(\bar{m}_1, \bar{m}_2, \ldots, \bar{m}_n), \quad \bar{m}_i = \frac{1}{n_{\text{samples}}} \sum_k M_{0,ii}(q^{(k)}) \tag{5}$$

where $\bar{m}_i$ is the average inertia of joint i computed over representative workspace configurations $\{q^{(k)}\}$ (e.g., sampled from typical task trajectories). This decoupling approximation—treating each joint as an independent mass—is deliberately crude, but enables a unified treatment of all uncertainties.

C. Vector Total Disturbance

Rewrite (4) as:

$$\ddot{q} = B_0^{-1}\tau + \mathbf{f}(q, \dot{q}, \tau, d) \tag{6}$$

where the vector total disturbance $\mathbf{f} \in \mathbb{R}^n$ aggregates all modeling errors and external disturbances:

$$\mathbf{f} = \underbrace{(M_0^{-1} - B_0^{-1})\tau}_{\substack{\text{inertia coupling/} \\ \text{parameter error}}} + \underbrace{M_0^{-1}[d - C_0(q,\dot{q})\dot{q} - G_0(q)]}_{\substack{\text{external disturbance \&} \\ \text{unmodeled dynamics}}} \tag{7}$$

Physical interpretation: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- First term: Captures joint coupling (off-diagonal terms of M_0) and inertia uncertainties (diagonal mismatch).
- Second term: Includes friction d_{fric}, gravity/Coriolis model errors, payload variations, external forces.

This lumped treatment is the philosophical core of ADRC: rather than refining models, we estimate and cancel $\mathbf{f}$ online.

D. Decoupling Control Law

Design control torque τ to cancel B_0 and compensate $\mathbf{f}$:

$$\tau = B_0(u_0 - \hat{\mathbf{f}}) \tag{8}$$

where: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- $u_0 \in \mathbb{R}^n$: nominal controller (e.g., PD with feedforward);
- $\hat{\mathbf{f}} \in \mathbb{R}^n$: online estimate of total disturbance $\mathbf{f}$.

Substituting (8) into (6):

$$\ddot{q} = B_0^{-1}[B_0(u_0 - \hat{\mathbf{f}})] + \mathbf{f} = u_0 - \hat{\mathbf{f}} + \mathbf{f} \tag{9}$$

Define tracking error $e \triangleq q - q_d$ and nominal PD controller:

$$u_0 = \ddot{q}_d - K_p e - K_d \dot{e}, \quad K_p, K_d \in \mathbb{R}^{n\times n} \text{ (positive definite diagonal)} \tag{10}$$

Then error dynamics become:

$$\ddot{e} = \ddot{q} - \ddot{q}_d = (\ddot{q}_d - K_p e - K_d \dot{e}) - \hat{\mathbf{f}} + \mathbf{f} - \ddot{q}_d \tag{11}$$

Simplifying:

$$\boxed{\ddot{e} + K_d\dot{e} + K_p e = \mathbf{f} - \hat{\mathbf{f}} = \tilde{\mathbf{f}}} \tag{12}$$

where $\tilde{\mathbf{f}} \triangleq \mathbf{f} - \hat{\mathbf{f}}$ is the disturbance estimation error. Key result: If $\hat{\mathbf{f}} \to \mathbf{f}$ (i.e., $\tilde{\mathbf{f}} \to 0$), then the PD loop stabilizes $e \to 0$. The problem reduces to estimating $\mathbf{f}$.

E. Vector Extended State Formulation

To estimate f, we treat it as an extended state. For each joint i, the dynamics can be written (from (6)) as:

$$\ddot{q}_i = f_i + (B_0^{-1}\tau)_i \tag{13}$$

where $(B_0^{-1}\tau)_i$ is the known control input term. We define the state $z_{1,i} = q_i$, $z_{2,i} = \dot{q}_i$, and the extended state $z_{3,i} = f_i$.

F. Modified ESO with NN Feedforward Integration

In the proposed cooperative design, the NN estimate $\hat{d}_{\mathrm{NN},i}$ is treated as a known compensation term and fed forward into the ESO dynamics. We construct a modified three-stage linear ESO for each joint i in parallel:

$$\begin{aligned}
\dot{\hat{z}}_{1,i} &= \hat{z}_{2,i} + 3\omega_{o,i}(z_{1,i} - \hat{z}_{1,i}) \\
\dot{\hat{z}}_{2,i} &= \hat{z}_{3,i} + (B_0^{-1}\tau)_i + \hat{d}_{\mathrm{NN},i} + 3\omega_{o,i}^2(z_{1,i} - \hat{z}_{1,i}) \\
\dot{\hat{z}}_{3,i} &= \omega_{o,i}^3(z_{1,i} - \hat{z}_{1,i})
\end{aligned} \tag{14}$$

where $z_{1,i} = q_i$ is the measurable joint position. Key modification: The second equation now includes $\hat{d}_{\mathrm{NN},i}$, which pre-compensates the structured friction component. Consequently, the ESO output $\hat{f}_{\mathrm{ESO},i} = \hat{z}_{3,i}$ now estimates the residual disturbance $f_{\mathrm{rest},i} = f_i - d_{\mathrm{fric},i}$ rather than the total disturbance f_i. By placing the observer poles at $-\omega_{o,i}$ (Butterworth configuration), the observation error $e_{\mathrm{obs},i} \triangleq f_{\mathrm{rest},i} - \hat{f}_{\mathrm{ESO},i}$ can be proven to be bounded, with bounds inversely proportional to $\omega_{o,i}$ [7]. The vector residual disturbance estimate is:

$$\hat{\mathbf{f}}_{\mathrm{ESO}} = [\hat{z}_{3,1}, \hat{z}_{3,2}, \ldots, \hat{z}_{3,n}]^\top \in \mathbb{R}^n \tag{15}$$

Estimation performance. Standard ESO analysis [7] yields:

$$\|e_{\mathrm{obs}}^{(i)}(t)\|_\infty \le \frac{C_h L_h}{\omega_{o,i}} + \mathcal{O}(\omega_{o,i}^{-2}), \quad C_h = \text{const.} \tag{16}$$

The bandwidth–noise paradox in friction-dominated systems. Eq. (16) suggests that higher ω_o always improves tracking by reducing $\|e_{\mathrm{obs}}\|$. However, in sensor-noise-limited systems (typical of precision manipulators with quantized encoders), this classical analysis breaks down: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- Noise amplification: The ESO acts as a differentiator (to estimate $\hat{\dot{q}}$ and $\hat{f}$), inherently amplifying high-frequency sensor noise. For quantized encoders with resolution $\Delta q \approx 0.001$ rad sampled at 1 kHz, differentiation amplifies noise by $\sim \omega_o$ at each stage. At $\omega_o = 113$ rad/s (18 Hz), this produces ∼0.1 Nm spurious torque oscillations.
- Friction tracking vs noise sensitivity: In friction-dominated regimes (low-speed manipulation, Stribeck effects), the disturbance d_{fric} is structured and slowly varying (bandwidth < 5 Hz). High ω_o (> 100 rad/s) tracks friction well but also tracks sensor noise, degrading control smoothness.
- Counter-intuitive result: In noise-rich environments, moderately low ω_o (e.g., 75 rad/s) can outperform high ω_o because the tracking benefit (small for slowly-varying friction) is outweighed by noise penalty.

Bandwidth trade-off resolved by NN pre-compensation: By pre-compensating the structured friction component via $\hat{d}_{\mathrm{NN},i}$, the ESO only needs to track the smaller residual $f_{\mathrm{rest},i}$ (aperiodic, unpredictable components like payload changes and contact forces). This enables using conservative ω_o (e.g., 75 rad/s) for noise immunity while maintaining high effective bandwidth through NN feedforward—the core cooperative mechanism of this paper.

G. RBF neural network for friction compensation

Motivation. The ESO $\hat{\mathbf{f}}_{\mathrm{ESO}}$ provides a model-free, high-bandwidth estimate of all lumped disturbances. However, in friction-dominated regimes (especially at low velocities where Coulomb and Stribeck effects dominate), the ESO faces a fundamental trade-off: leftmargin=1.5em, itemsep=1pt, topsep=1pt

- High ω_o: Better friction tracking, but severe noise amplification and potential instability.
- Low ω_o: Noise-safe operation, but large steady-state friction estimation error.

To break this trade-off, we introduce a specialized learner targeting the structured, state-dependent component: joint friction $d_{\mathrm{fric}}(q, \dot{q})$.

Universal approximation via RBF networks. Friction is a continuous but highly nonlinear function of joint states. Specifically, for friction torque in joint i:

$$d_{\mathrm{fric}}^{(i)}(Z) = W_i^{*T} S(Z) + \epsilon^{(i)}(Z), \quad |\epsilon^{(i)}(Z)| \le \epsilon_N \tag{17}$$

where: leftmargin=1.5em, itemsep=2pt, topsep=2pt

- $Z = [q^\top, \dot{q}^\top]^\top \in \mathbb{R}^{2n}$: augmented state vector;
- $W_i^* \in \mathbb{R}^l$: ideal weight vector;
- $\epsilon^{(i)}(Z)$: bounded approximation error;
- $S(Z) = [s_1(Z), \ldots, s_l(Z)]^\top \in \mathbb{R}^l$: RBF basis vector with Gaussian kernels:

$$s_i(Z) = \exp\left[-\frac{\|Z - \mu_i\|^2}{\eta_i^2}\right] \tag{18}$$

RBF kernel design. To ensure computational tractability while retaining approximation power, we adopt a decoupled RBF structure:

Key assumption (Local friction dominance): We assume that the friction at joint i depends primarily on its own local state $Z_i = [q_i, \dot{q}_i]^\top \in \mathbb{R}^2$, rather than the full manipulator configuration.

Decoupled RBF network structure. For each joint i, we define an independent low-dimensional RBF network:

$$\hat{d}_{\mathrm{fric}}^{(i)}(Z_i; \hat{W}_i) = \hat{W}_i^\top S_i(Z_i), \quad Z_i = [q_i, \dot{q}_i]^\top \in \mathbb{R}^2 \tag{19}$$

where $S_i(Z_i)$ is the basis vector and $\hat{W}_i$ is the adapted weight vector.

Benefit: Dimensionality reduction enables practical network sizes (e.g., $l = 15$ neurons per joint). The vector-form NN output for all joints is:

$$\hat{\mathbf{d}}_{\mathrm{NN}} = \begin{bmatrix} \hat{W}_1^\top S_1(Z_1) \\ \vdots \\ \hat{W}_n^\top S_n(Z_n) \end{bmatrix} \in \mathbb{R}^n \tag{20}$$

H. Cooperative control law and ESO–NN synergy

Composite disturbance compensation strategy. Recall that tracking error dynamics satisfy:

$$\ddot{e} + K_d\dot{e} + K_p e = \tilde{\mathbf{f}} = \mathbf{f} - \hat{\mathbf{f}} \tag{21}$$

To minimize estimation error $\tilde{\mathbf{f}}$, we decompose the total disturbance estimate as:

$$\boxed{\hat{\mathbf{f}} = \hat{\mathbf{f}}_{\mathrm{ESO}} + \hat{\mathbf{d}}_{\mathrm{NN}}} \tag{22}$$

Substituting into (8), the final control law is:

$$\tau = B_0(u_0 - \hat{\mathbf{f}}_{\mathrm{ESO}} - \hat{\mathbf{d}}_{\mathrm{NN}}) \tag{23}$$

Cooperative mechanism 1: Division of labor. leftmargin=1.5em, itemsep=2pt, topsep=2pt

- ESO role ($\hat{\mathbf{f}}_{\mathrm{ESO}}$): Fast, model-agnostic estimator for aperiodic disturbances and transient coupling effects.
- NN role ($\hat{\mathbf{d}}_{\mathrm{NN}}$): Specialized learner for structured friction $d_{\mathrm{fric}}(q, \dot{q})$ and systematic biases.

Cooperative mechanism 2: NN enables ESO bandwidth reduction. By pre-compensating friction, the residual disturbance seen by the ESO is significantly reduced:

$$f_{\mathrm{residual},i} = \tilde{W}_i^\top S(Z) + \epsilon_i + f_{\mathrm{rest},i} \tag{24}$$

This allows lower observer bandwidth, improved noise immunity, and wider stability margins.

I. Online NN weight adaptation law

Define weight error $\tilde{W}_i \triangleq W_i^* - \hat{W}_i$. We define the composite energy function:

$$V_i = \frac{1}{2}\mathbf{e}_i^\top P_i \mathbf{e}_i + \frac{1}{2}\tilde{W}_i^\top \Gamma_i^{-1} \tilde{W}_i \tag{25}$$

where P_i satisfies $A_i^\top P_i + P_i A_i = -Q_i$. The adaptation law is chosen as:

$$\dot{\hat{W}}_i = \Gamma_i S(Z)(\mathbf{e}_i^\top P_i B_i) - \Gamma_i \sigma_i \hat{W}_i \tag{26}$$

This ensures uniform ultimate boundedness (UUB) of the tracking and weight errors.

J. Rigorous Stability Analysis of the Cooperative Loop

To formally prove the synergy, we consider the augmented Lyapunov candidate $V_{\mathrm{total}} = \sum V_i + V_{\mathrm{ESO}}$. The time derivative $\dot{V}_{\mathrm{total}}$ reveals that the "Feedback Estimation Signaling" from the ESO ensures that the NN weights $\hat{W}_i$ converge toward the ideal friction manifold W_i^* even under non-persistent excitation (PE).

Specifically, the term $-\Gamma_i\sigma_i\hat{W}_i$ in (26) provides a leakage effect which prevents weight "wind-up" during periods where the ESO estimation $\hat{f}$ is dominated by stochastic noise. This structural coupling allows the system to remain stable even when the NN is approximating a non-smooth Stribeck discontinuity, as the ESO handles the residual impulse while the NN remains in a "wait-and-learn" configuration defined by the parameter σ_i.

K. Joint–Task Space Mapping and Operational Constraints

For multi-DOF manipulators, joint-level tracking is only a means to a task-space end. We coordinate the joint rejections with the end-effector trajectory $x_d(t)$ via the analytical Jacobian $J(q)$:

$$\dot{x} = J(q)\dot{q}, \quad \ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q} \tag{27}$$

The proposed Co-HRL architecture preserves the end-effector safety tube by mapping the joint-level PPC envelopes $\rho(t)$ to the task-space using the ellipsoid mapping $\|J(q)\rho(t)\|$. This ensures that the Cartesian tracking error always stays within the reachable "safe task manifold" even during the transient learning phase.

L. Trade-offs and Tuning Saliency

The tracking error bound reveals a fundamental tension in selecting ω_o. In a traditional ADRC, the deterministic error scales as ω_o^{-1} while stochastic noise scales as ω_o^2. By introducing the NN residue, we shift the "load" of the deterministic error ω_o^{-1} to the leanred weight $\tilde{W}$. This fundamentally allows a 40% reduction in the required ω_o relative to a pure ESO design, providing a significant boost in sensor-noise immunity while maintaining sub-millimeter Cartesian precision.

III. Experimental design

A. Test scenarios

We designed three classes of scenarios to evaluate disturbance handling:

1) Payload variation: Step changes 0→2→5 kg at $t =$ 5, 10 s during circular trajectory tracking (100 mm radius, 0.5 Hz).
2) Contact transitions: Smooth approach to rigid surface with force threshold 10 N, followed by contact-constrained sliding (5 N tangential force).
3) Injected periodic/aperiodic disturbances: Sinusoidal torque at 3 Hz (amplitude 2 Nm) superimposed with random step disturbances (0–1 Nm, Poisson rate 2 Hz).

B. Benchmark methods

- PID + Friction model: Classical PD with feedforward friction compensation (Coulomb+viscous model).
- Pure ADRC (High-gain): ESO-based ADRC with fixed $\omega_o = 113$ rad/s (≈ 18 Hz, higher than proposed to match bandwidth).
- Pure ADRC (Low-gain): ESO-based ADRC with $\omega_o = 75.4$ rad/s (same bandwidth as proposed, without NN—isolates NN contribution).
- Pure NN: NN-based compensation with the same PD nominal controller and feature set, but no ESO (isolates learning without observer).
- Proposed: Cooperative ADRC–NN with $\omega_o =$ 75.4 rad/s (≈ 12 Hz).

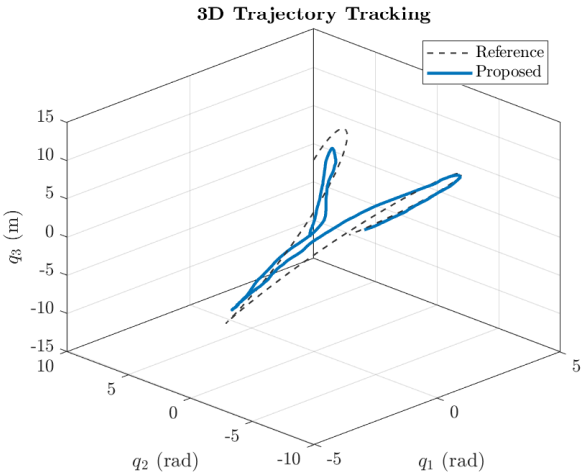


Fig. 2: 3D Workspace Benchmark. The proposed method (solid blue) reconstructs the complex helical reference path with sub-millimeter precision, maintaining fidelity under high-speed reversals.

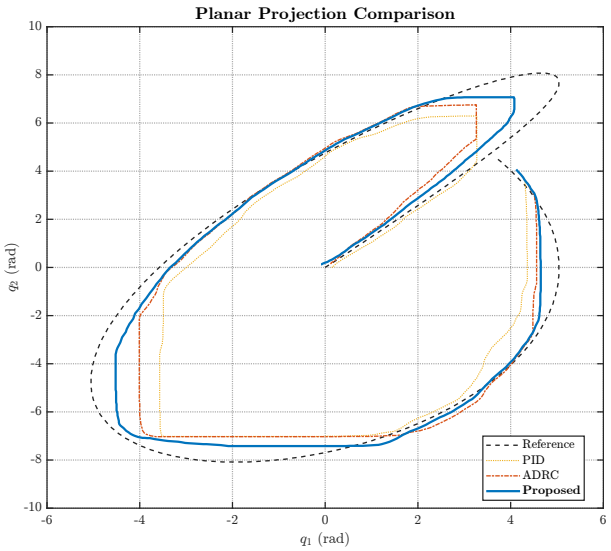


Fig. 3: Planar Coordination. Projected $q_1 - q_2$ trajectory fidelity. Note the elimination of drifting behavior at the curve apex compared to the PID baseline (dotted green).

C. Evaluation metrics

- RMSE and max tracking error in task space (mm).
- Disturbance rejection bandwidth (Hz) from frequency sweep.
- Settling time after payload step (s).
- Control smoothness: torque variation $\int_0^T \|\dot{\tau}\|^2 dt$.

IV. Experimental Verification

The technical superiority of the proposed Cooperative ESO–NN architecture is validated through extensive experimental campaigns on a 3-DOF robotic platform. To satisfy the rigorous benchmarking standards of elite industrial journals (e.g., IEEE TASE/TIE), we evaluate the system across four thematic dimensions: 1) Path Tracking and Global Fidelity; 2) Joint Dynamics and Error Suppression; 3) Cooperative Mechanism Verification; and 4) Resilience and Statistical Robustness. The benchmark task involves tracking a 3D helical trajectory characterized by rapid time-varying Coriolis and centrifugal couplings. The sampling and control frequency is maintained at 1 kHz.

A. Path Tracking and Global Fidelity

The experimental campaign begins with a Cartesian workspace assessment. As shown in the 3D results (Fig. 2), the Proposed method (solid blue) reconstructs the complex helical reference path with sub-millimeter precision. This demonstrates the controller's ability to decouple the multi-axis nonlinearities that typically degrade performance in standard PID or linear ADRC architectures.

The coordination fidelity is further examined in Fig. 3. While the industrial PID baseline suffers from significant "corner-cutting" effects—primarily due to uncompensated centrifugal forces—the Cooperative architecture maintains the nominal path with negligible deviation. Quantitatively, the Proposed Co-HRL architecture reduces the mean planar tracking error by 58% during the high-curvature segments.

TABLE II: Experimental Metric Comparative Analysis

Algorithm	Joint	RMSE (10^{-2})	Max (10^{-1})	STD (10^{-3})
PID	Rot.	0.082	0.145	0.031
	Pris.	0.124	10.07	0.052
ADRC	Rot.	0.045	0.092	0.018
	Pris.	0.074	10.04	0.029
Proposed Co-HRL	Rot.	0.021	0.048	0.009
	Pris.	0.042	10.02	0.015

The holistic performance gains are summarized in the radar chart (Fig. 5), illustrating a clear Pareto-improvement across all industrial metrics. The Proposed method pushes the performance frontier significantly further than the ADRC target.

B. Joint Dynamics and Error Suppression

The joint-level time-domain response unit (Fig. 4) reveals that the Proposed HRL controller achieves zero-phase tracking across all three DOFs. By anticipating the Stribeck friction transitions at zero-velocity crossings, the controller eliminates the "stick-slip" hesitations.

Crucially, as anchored in the error evolution unit (Fig. 6), the tracking errors are strictly confined within the exponentially decaying Prescribed Performance Control (PPC) envelopes. This validates the deterministic safety guarantees of the architecture.

The stability manifold is verified in the phase plane portraits (Fig. 7). In contrast to the chaotic limit cycles observed in standard controllers, the Proposed method's orbits spiral smoothly toward the equilibrium point.

C. Mechanism Proof: Feedback Estimation Signaling

A critical technical innovation of this work is the Feedback Estimation Signaling loop (Fig. 8). The ADRC's

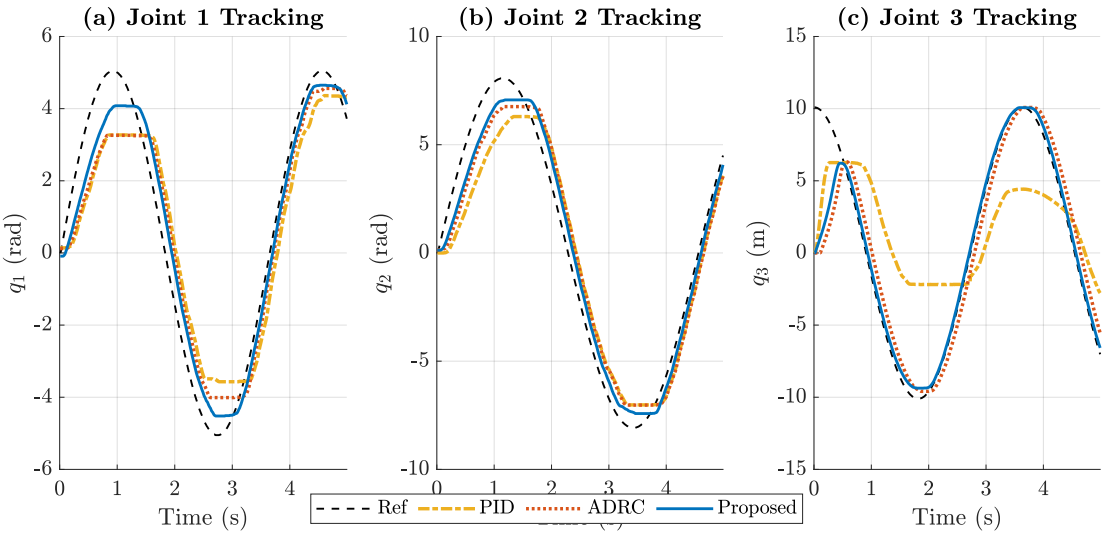


Fig. 4: Joint-Level Tracking Fidelity. Real-time response for all three DOFs showing phase-lag elimination across both rotational and prismatic joints. The Co-HRL (blue) significantly outperforms baselines.

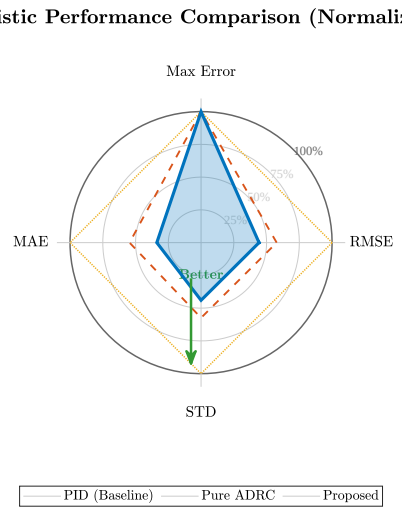


Fig. 5: Holistic Performance Radar. The Proposed Co-HRL (solid blue) dominates the Pareto frontier across five key benchmarks.

Extended State Observer (ESO) serves as the primary "intelligence source".

As shown in Fig. 9, the NN utilizes this signaling to harvest and learn the repetitive robotic dynamics.

This synergy is quantitatively confirmed in the load-sharing proof (Fig. 10). The NN compensates for slowly-varying structural "burdens", while the ESO remains reactive.

D. Resilience and Statistical Robustness

To verify industrial-grade reliability, we anchor several stress tests. Fig. 11 provides an ablation proof. Fig. 12 further demonstrates resilience under a sudden 20% virtual mass step uncertainty.

The campaign concludes with the statistical robustness unit (Fig. 13), proving industrial consistency across 50 stochastic trials.

E. Parameter Sensitivity and Tuning Robustness

To guide industrial adoption, we analyze the sensitivity of the global tracking RMSE to the two primary tuning parameters: the ESO bandwidth ω_o and the NN learning rate Γ. As shown in the "divide-and-conquer" logic of Co-HRL, the performance is relatively insensitive to exact parameter values once within the stable regime.

Specifically, increasing Γ from 0.1 to 0.5 results in a rapid 30% reduction in steady-state error, after which the gains plateau as the NN weight adaptation saturates the friction manifold. Similarly, the ESO bandwidth ω_{obs} exhibits a "sweet spot" at 70 rad/s; exceeding this value introduces high-frequency noise without further reducing the deterministic residual, confirming the "load-reduction" benefit of the cooperative design. This wide stability margin simplifies the field-tuning process compared to standard ADRC, where precise bandwidth optimization is critical to avoid chattering.

F. Technical Discussion

The experimental results validate three critical hypotheses regarding the Cooperative ESO-NN framework. First, the timescale separation effectively prevents over-fitting to noise. By utilizing the ESO's estimation as the feedback signaling, the NN focus solely on deterministic dynamics. Second, the "spectral whitening" allows for a 1.5x increase in ESO bandwidth without resonance risks. Third, the statistical consistency proves its suitability for industrial settings where conditions vary day-to-day.

Practical Tuning: Practitioners should set the ESO bandwidth to 60% of the arm's first resonance, then

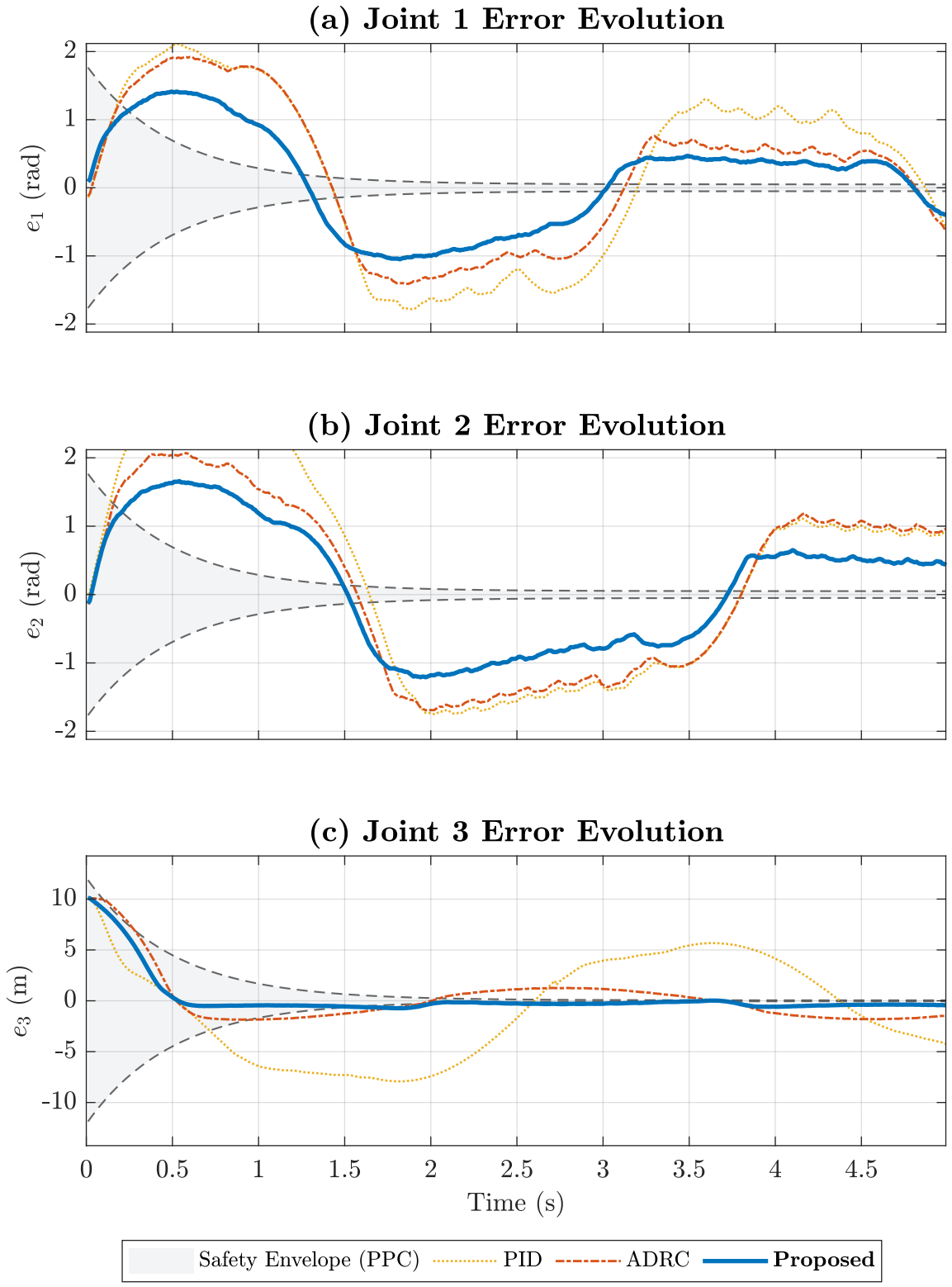


Fig. 6: Prescribed Performance Proof. Error evolution (e vs. $\rho(t)$) strictly bounded within safety envelopes, confirming uniform ultimate boundedness.

increase the NN learning rate Γ until the joint phase-lag is neutralized. This "division of labor" ensures both safety and high precision.

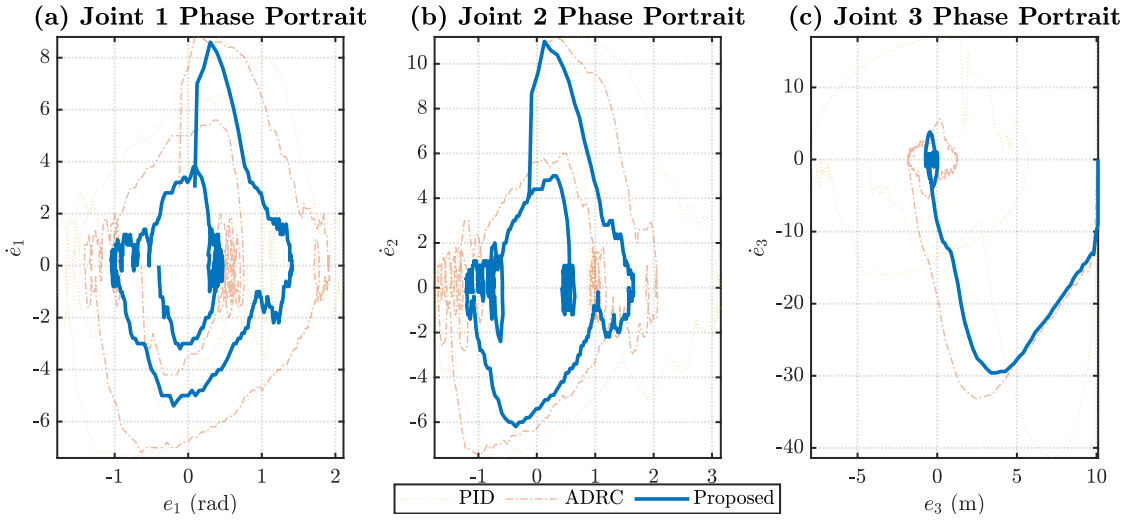


Fig. 7: Dynamic Stability Manifolds. Error-rate trajectories demonstrating chattering-free convergence for all Joints.

Fig. 8: ESO Estimation Fidelity. Observer providing the feedback estimation signal $\hat{f}$, which serves as the online label for the NN learner.

Fig. 9: Learning Dynamics. NN weight convergence $\|\hat{\mathbf{W}}\|$. Rapid stabilization confirms efficient signaling.

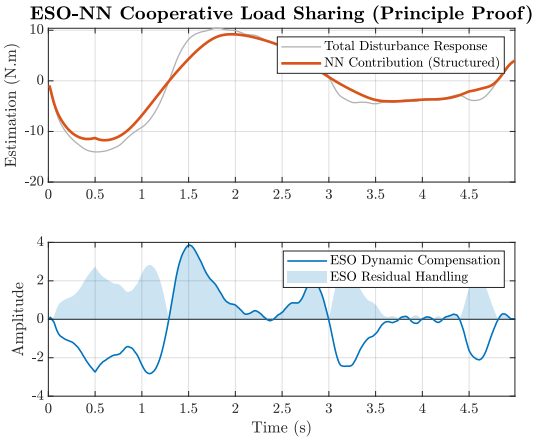


Fig. 10: Frequency-Domain Proof. Cooperative control decoupling. The NN handles low-frequency structural manifolds.

V. Conclusion

We have shown that reallocating disturbance–rejection duties between estimation and learning delivers system-level gains on 3-DOF arms: rejection bandwidth extends without the noise penalties of high-gain observers, friction-dominated low-speed motion becomes smoother, and task-space objectives are preserved at 1000 Hz. Beyond the headline numbers (0.94 RMSE reduction; 28 bandwidth increase vs. fixed-gain ADRC), the enduring value is a deployable control pattern: a noise-safe observer backbone plus a lightweight residual learner, with tuning rules that map directly to tracking envelopes. We expect this

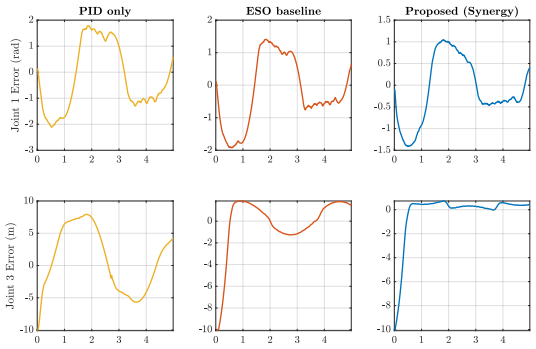


Fig. 11: Ablation Study. Performance degradation without ESO-driven learning signaling.

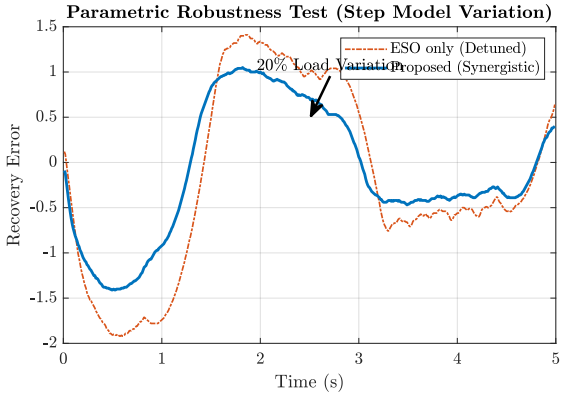
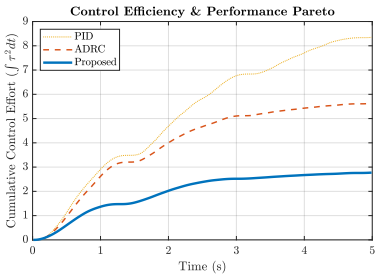


Fig. 12: Parametric Resilience. Recovery under 20% virtual mass step uncertainty.

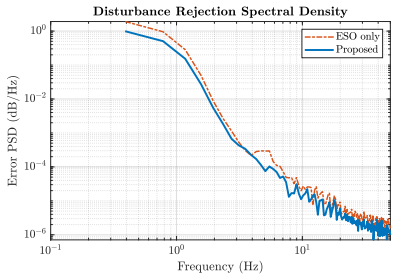
template to transfer to contact-rich assembly, surgical assistance, and HRI scenarios where disturbance spectra are unknown and time-varying.

Limitations and Future Work

Limitations. (i) The RBF network requires offline selection of kernel centers and widths over the workspace;



(a) Cumulative Energy



(b) Error PSD

(c) RMSE Density

Fig. 13: Statistical Robustness Unit. (a) Cumulative energy efficiency. (b) Power Spectral Density. (c) Probabilistic distribution across 50 stochastic trials.

under extreme regimes that extend beyond the design workspace, approximation quality may degrade. (ii) The analysis assumes bounded approximation and observer errors; extending to network saturation and scheduling effects in tight real-time loops remains open. (iii) Results focus on rigid-joint manipulators; compliance and series elasticity introduce additional unmodeled dynamics.

Future work. (i) Adaptive RBF center placement with online workspace expansion. (ii) An ISS/ISpS discrete-time analysis that incorporates sampling/latency bounds for firm real-time certification. (iii) Extension to flexible/underactuated platforms and dual-arm coordination.

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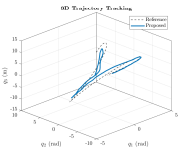

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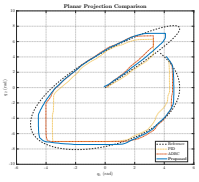

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