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Black, Derman and Toy Model

This is an arbitrage-free interest rate model that generate expected rate path that perfectly fit the current term-structure.

Using the given sigmas and D(t), by the formula:

$$D(0.5) = \frac{1}{1 + \frac{r_0}{2}}$$

$$D(1.0) = \left(\frac{0.5}{1 + \frac{r_u}{2}} + \frac{0.5}{1 + \frac{r_d}{2}} \right) \frac{1}{1 + \frac{r_0}{2}}$$

$$where\ r_u = r_d \exp\left(2\sigma\sqrt{\tau}\right)$$

And we can generalize for further t = 1.5, 2.0, … to do the forward induction using the given D(t).

And by the model rate we can generate the expected 6-month rate in the future, comparing with the current forward rates (at t = 0) f(0.5,0.5). f(1.0,0.5)…, we can see the expected rates generated by the model is higher than the forward rates. I think it is because this model is an arbitrage free model, and it is risk-neutral pricing model. And the differences can be explained by the difference between the risk appetite in the real world and the risk-neutral world.

More details can be found in the output.xlsx and the source code file.