3.1.3

(c): $G = (V, \Sigma, R, S)$

$V = \{a, b, S\}$, $\Sigma = \{a, b\}$, $R = \{S \to a, S \to b,$

$S \to aSa, S \to bSb, S \to e\}$

3.1.4

(a): $G = (V, \Sigma, R, S)$

$V = \{a, b, S\}$, $\Sigma = \{a, b\}$, $R = \{S \to e,$

$S \to aS, S \to aSb\}$

(b): $G = (V, \Sigma, R, S)$

$V = \{a, b, c, d, A, B, S\}$, $\Sigma = \{a, b, c, d\}$

let: $m + n = p + q = x$, $n = x - m$, $p = x - q$

$\Rightarrow a^m b^{x-m} c^{x-q} x^{q} \Rightarrow a^{q} a^{m-q} b^{x-m} c^{x-m} c^{m-q} d^{q}$

$R = \{S \to e, S \to aSd, S \to A, A \to aAc,$

$A \to B, B \to bBc\}$

3.3.2

(c): $M = (K, \Sigma, \Gamma, \Delta, s, F)$

$K = \{q, r\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b\}$, $F = \{r\}$.

$\Delta = \{((q,e,e),(r,e)), ((q,a,e),(q,a)), ((q,b,e),(q,b)),$

$((q,a,e),(r,e)), ((q,b,e),(r,e)), ((q,a,a),(r,e)), ((q,b,b),(r,e))\}$

(d):

$M = (K, \Sigma, \Gamma, \Delta, S, F)$

$K = \{q\}$, $\Sigma = \{a, b\}$, $\Gamma = \{A, a, b\}$, $F = \{q\}$

$\Delta = \{((q,a,e),(q,A)), ((q,b,e),(q,b)),$

$((q,a,b),(q,a)), ((q,b,A),(q,a)), ((q,b,a),(q,e))\}$

3.4.1

$M = \{p, q, \{p, q\}, \{(,)\}, \{(,), S\}, \Delta\}$

$\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, SS)),$

$((q, e, S), (q, (S))), ((q, e, S), (q, e)), ((q, (, ()), (q, e)),$

$((q,),)), (q, e))\}$

$\Rightarrow (p, (()()), e) \vdash_M (p, (()()), S) \vdash_M (p, (()()), (S))$

$\vdash_M (p, ()()), S)) \vdash_M (p, ()()), SS)) \vdash_M (p, ()()), (S)S))$

$\vdash_M (p,)()), S)S)) \vdash_M (p,)()),)S)) \vdash_M (p, ()), S))$

$\vdash_M (p, ()), (S))) \vdash_M (p,)), S))) \vdash_M (p,)),))) \vdash_M$

$(p,),)) \vdash_M (p, e, e)$

3.5.1

(b)

$\{a,b\}^* - \{a^n b^n : n \geq 0\} = \overset{A}{\{a^m b^n : m,n \geq 0, m \neq n\}} \cup \overset{B}{\Sigma^* a \Sigma^* b \Sigma a^*}$

$\cup\, \Sigma^* b \Sigma^* a \Sigma^* b$ C

A, B, C are context free, so it is context free

(c)

$\{\cdots\} = \overset{A}{\{a^* b^n c^* d^n\}} \cup \overset{B}{\{a^m b^n c^p d^q : m+n = p+q\}}$

$\cup\, \{a^m b^* c^p d^* : m \leq p\}$ C

A, B, C are context free, so it is context free

(d)

$\{a,b\}^* - L = \overset{A}{a\Sigma^*} \cup \overset{B}{\Sigma^* a} \cup \overset{C}{\Sigma^* bb \Sigma^*} \cup \overset{d}{\{\Sigma^* b a^m b a^n :}$

$m+1 \neq n\}$

A, B, C, D are context free, so it is context free

3.5.2

(c)

assume L is context free

suppose there u, v, x, y, z. $W = uvxyz$

$|w| \geq k$, $|vxy| \leq k$, $vy \neq e$, $uv^nxy^nz \in L$

$\Rightarrow W = a^k b a^k b a^k b$

if v contains a, b. uv^nxy^nz contains 4 bs

if v and y each contain $a\,b$. uv^nxy^nz contain 5 bs

So suppose $v = a^n$, $y \in a^q$, $q < k$

$\Rightarrow uv^nxy^nz = a^{k+p} b a^{k+q} b a^k b$. X

3.5.14

(a)

$\{\cdots\} = \{a^m b^m c^p\} \cup \{a^m b^n c^n\} \cup \{a^m b^n c^m\}$: ✓

(b)

$\{\cdots\} = \{a^m b^n c^p : m \neq n\} \cup \{a^m b^n c^p : m \neq p\} \cup$

$\{a^m b^n c^p : n \neq p\}$: ✓

(c)

$\{\cdots\} = \{a^m b^m c^m\}$: by pumping theorem X

(d)

$\{\cdots\} = \{w \in \{a,b,c\}^* : a's \neq b's\} \cup \{w \in \{a,b,c\}^* : a's \neq c's\}$

$\cup \{w \in \{a,b,c\}^* : b's \neq c's\}$: ✓