P342-344

15：

Suppose that the Chomp board have n rows and n columns. Let P(n) be the statement that if a player has presented his opponent with a Chomp configuration consisting of just n cookies in the top row and n cookies in the leftmost column, then he can win the game.

We will prove ∀nP(n) by strong induction.

We know that P(1) is true, because the opponent is forced to take the poisoned cookie at his ﬁrst turn.

Let k ≥ 1 and assume that P(j) is true for all j ≤ k. We claim that P(k+ 1) is true.

It is the opponent’s turn to move. If he picks the poisoned cookie, then the game is over and he loses.

Otherwise, assume he picks the cookie in the top row in column j, or the cookie in the left column in row j, for some j with 2 ≤ j ≤ k +1. The ﬁrst player now picks the cookie in the left column in row j, or the cookie in the top row in column j respectively. This leaves the position covered by P(j−1) for his opponent, so by the inductive hypothesis, he can win.

35：

Let P(n) be the statement that if x1, x2, ..., xn are n distinct real numbers, then n−1 multiplications are used to find the product of these numbers no matter how parentheses are inserted in the product.

The basis case: P(1) is true because1−1=0 multiplications are required to find the product of x1,a product with only one factor.

Suppose that P(k) is true for 1≤k ≤n. The last multiplication used to ﬁnd the product of the n + 1 distinct real numbers x1, x2, ..., xn, xn+1 is a multiplication of the product of the first k of these numbers for some k and the product of the last n + 1 − k of them. By the inductive hypothesis, k−1 multiplications are used to find the product of k of the numbers, no matter how parentheses were inserted in the product of these numbers, and n−k multiplications are used to find the product of the other n + 1 − k of them, no matter how parentheses were inserted in the product of these numbers. Because one more multiplication is required to find the product of all n+1 numbers, the total number of multiplications used equals (k − 1) + (n − k)+ 1 = n.

Hence, P(n+1) is true.

P358-359

15:

Basis step: f0 \* f1 + f1 \* f2 = 0\*1 + 1\*1 = 1^2 = (f2)^ 2.

Inductive step: Assume that f0 \* f1 + f1 \* f2 +···+f2k−1 \* f2k = (f2k)^2. Then f0 \* f1 + f1 \* f2 +···+ f2k−1 \* f2k + f2k \* f2k+1 + f2k+1 \* f2k+2 = (f2k)^2 + f2k \* f2k+1 + f2k+1 \* f2k+2 = f2k \* (f2k + f2k+1) + f2k+1 \* f2k+2 = f2k \* f2k+2 + f2k+1 \* f2k+2 = (f2k +f2k+1) \* f2k+2 =(f2k+2)^2.

47:

a): Pm,m = Pm because a number bigger than m can’t be used in the partition.

b): when m = 1, it only has one partition that is 1;

when n =1, it only has one partition that is 1+1+…+1(m);

when n > m, because of a), Pm,n = Pm;

when n = m >1, Pm,m = Pm, because there only one partition that includes m, Pm,m = Pm,m-1 + 1;

when m > n > 1, because a partition of m into integers not exceeding n either does not use any ns and hence, is counted in Pm,n−1 or else uses an n and a partition of m−n, and hence, is counted in Pm−n,n. So Pm,n =Pm,n−1+Pm−n,n.