MSBA 7004
Operations Analytics

Tutorial 3

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Learning objectives

- Reflect Inventory Analysis and some important concepts
- Reflect EOQ model and Q-r model
- Reflect continuous and periodic review system
- Analyze inventory analysis related practice problems

Warm-up game: reflection of definitions

- True [T/F] Inventory costs include ordering/setup(fixed), holding(carrying) and shortage(opportunity) cost
- True [T/F] Annual total cost includes annual setup cost and annual holding cost
- True [T/F] The EOQ model assumes that annual demand is deterministic and occurs at a constant rate.
- True [T/F] Pipeline inventory only exists in time window of duration LT in each inventory cycle duration
- False [T/F] The motivation of the EOQ model is to match the demand with the right quantity of supply.

Warm-up game: reflection of definitions

Economic Order Quantity(EOQ) Model answers the question: **How much to order each time**.

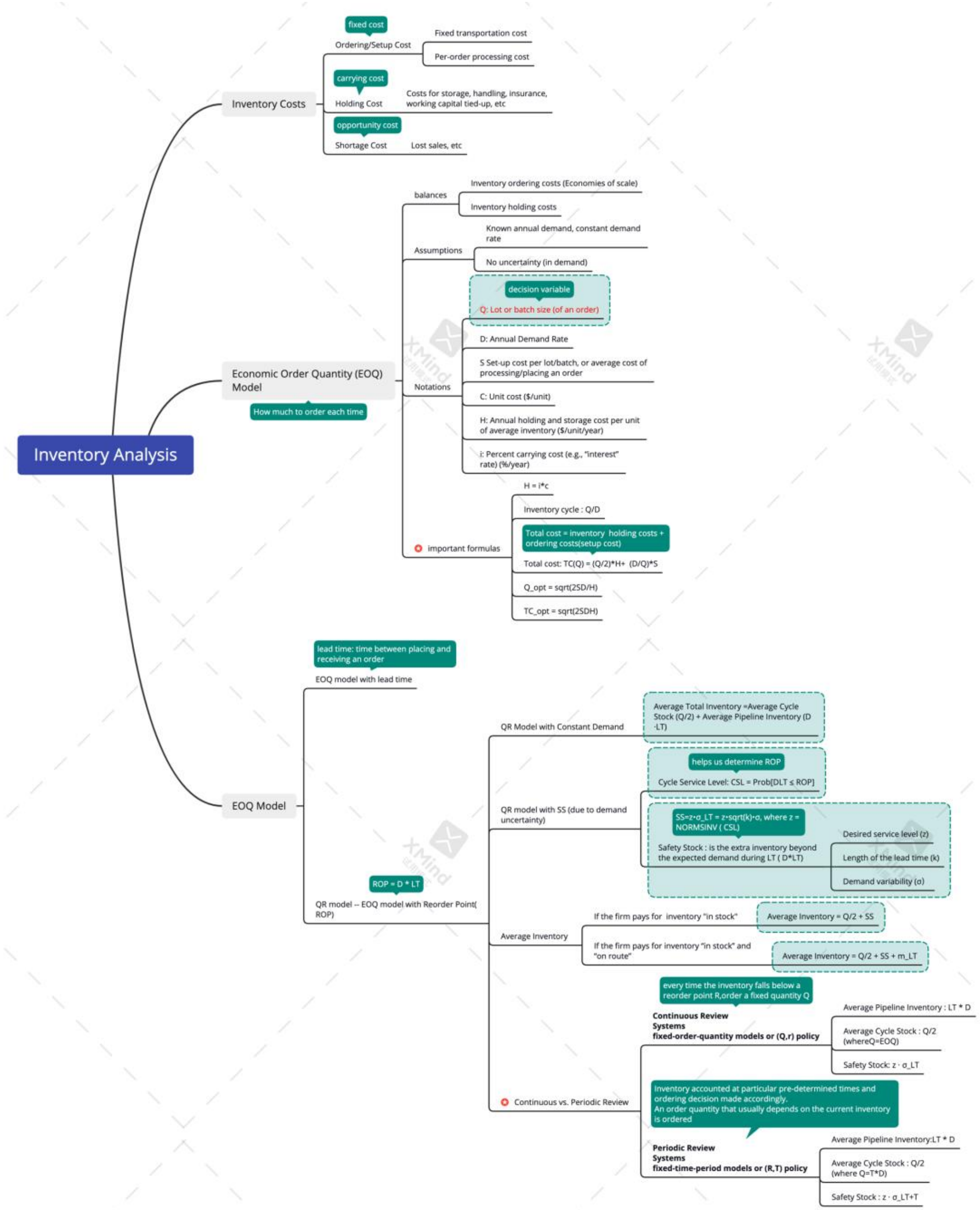
Economic Order Quantity(EOQ) Model balances

Inventory ordering costs vs. **Inventory holding costs**

Cycle service level measures **the reliability of the system**

Safety Stock is **the extra inventory beyond the expected demand during lead time/exposure period.**

Summary of the concepts and formulas



Inventory Costs

- Reasons for not holding inventory:
  - Holding (Storage) cost
  - Opportunity cost of capital growth
  - Waste
- Inventory Costs:
  - Ordering/Setup Cost (Fixed Cost)
  - Holding (Storage) Cost
  - Shortage Cost (Opportunity Cost)

EOQ model (fixed-order-quantity)

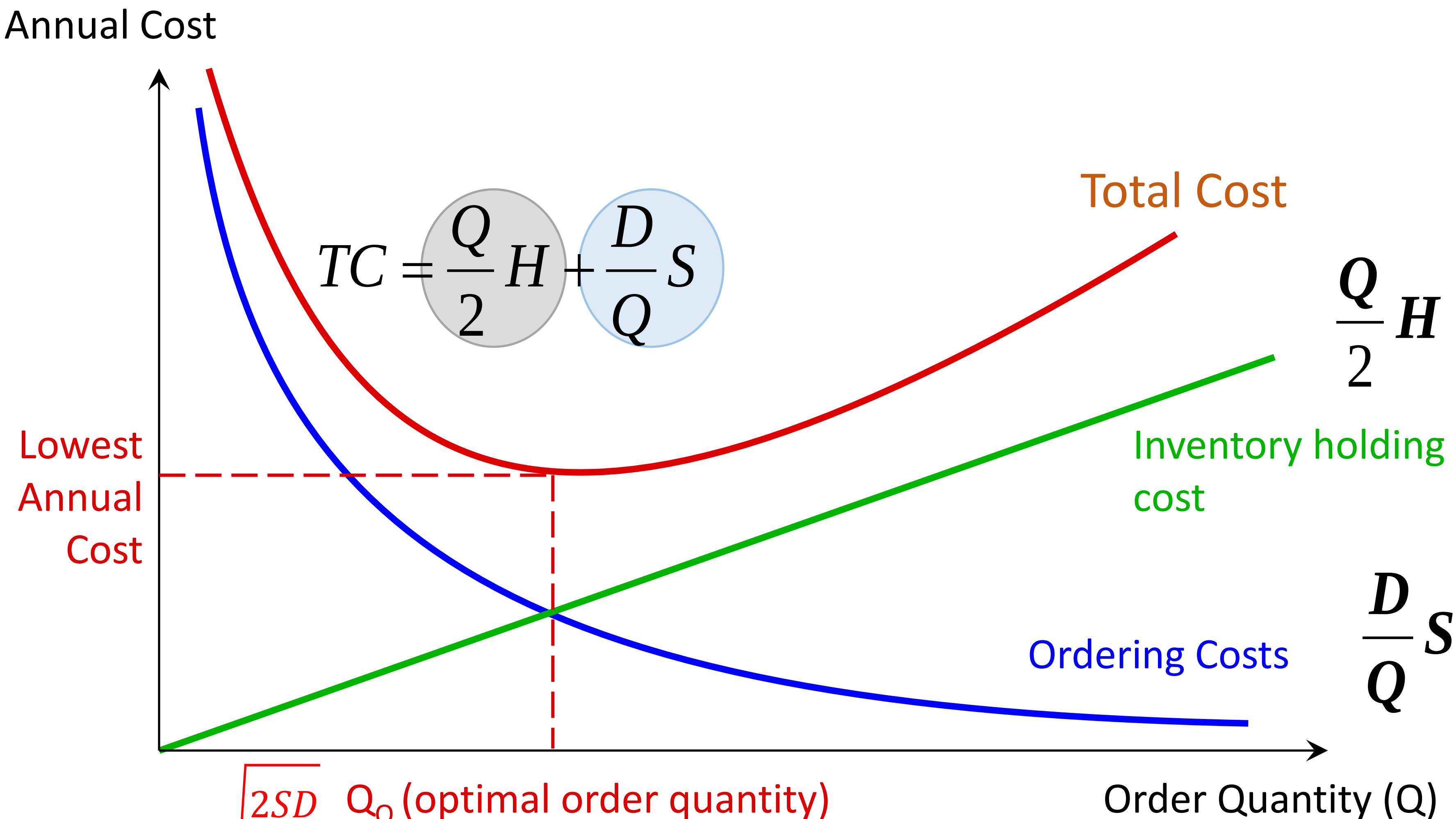
- Trade-off #1: How much to order each time?
- Objective: To find the optimal order frequency, which makes a balance between minimizing holding cost and set up cost

D	Annual Demand Rate
Q	Lot or batch size (of an order)
S	Set-up cost per lot/batch, or average cost of processing/placing an order
C	Unit cost ($/unit)
H	Annual holding and storage cost per unit of average inventory ($/unit/year)
i	Percent carrying cost (e.g., “interest” rate) (%/year)

Usually, H = i*C

- Total Cost = TC(Q) = $\frac{Q}{2}H + \frac{D}{Q}S$
- $Q_{OPT} = \sqrt{\frac{2SD}{H}}$
- $\mathrm{TC}_{OPT} = \sqrt{2SDH}$

Goal: Minimize Total Cost



$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

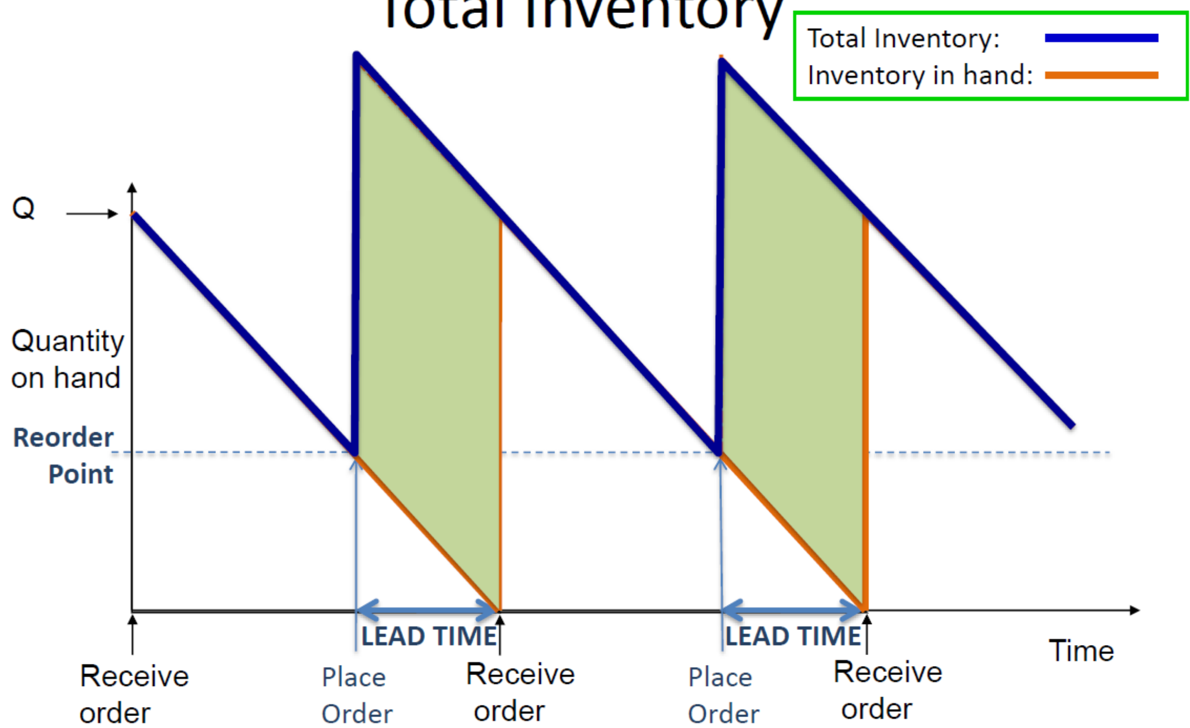
$$TC_{OPT} = \sqrt{2SDH}$$

Q-r (EOQ, ROP) model: with constant demand

- EOQ with Reorder Point
- Lead Time: Time between placing and receiving an order
- Reorder Point:
  - ROP = D * LT

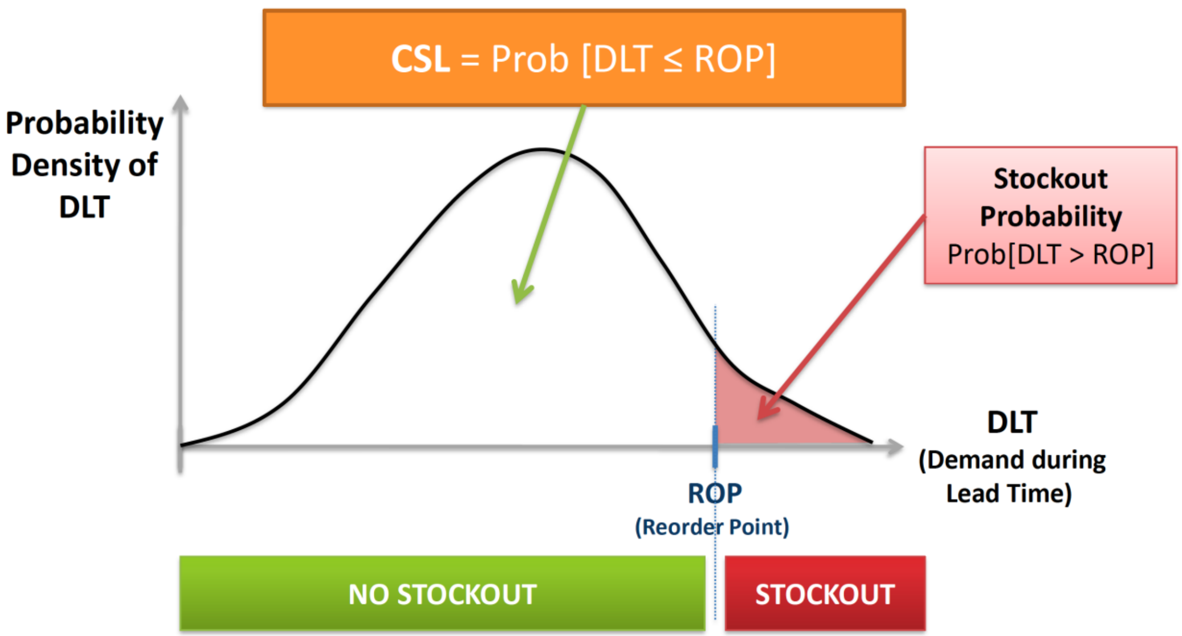
- Average Total Inventory = Average Cycle Stock (Q/2) + Average Pipeline Inventory (D * LT)

Cycle Stock, Pipeline Inventory, and Total Inventory



Q-r (EOQ, ROP) model: with uncertain demand

- Cycle Service Level (CSL)
- CSL = Prob[DLT ≤ ROP]



- Let Q be the order quantity, and (μ, σ) the parameters of the normal demand distribution
- *Prob*{demand is Q or lower} = *Prob*{the outcome of a standard normal is z or lower}, where

$$z = \frac{S-\mu}{\sigma} \quad \text{or} \quad S = \mu + z \times \sigma$$

- Look up *Prob*{the outcome of a standard normal is z or lower} in the Standard Normal Distribution Function Table, or NORMSDIST (excel), norm.cdf (scipy)

Q-r (EOQ, ROP) model: with SS due to demand uncertainty

- ROP = $m_{LT} + SS = D * k + z\sigma_{LT} = D * k + z * \sqrt{k} * \sigma$

- Safety Stock is the extra inventory beyond the expected demand during LT (D*LT)
- SS is based on Desired service level (z), Length of lead time (k), and demand variability (σ)
- SS = $z * \sigma_{LT} = z \; * \; \sqrt{k} * \sigma$ where z = normsinv(CSL)
- To compute Q: same formula in EOQ

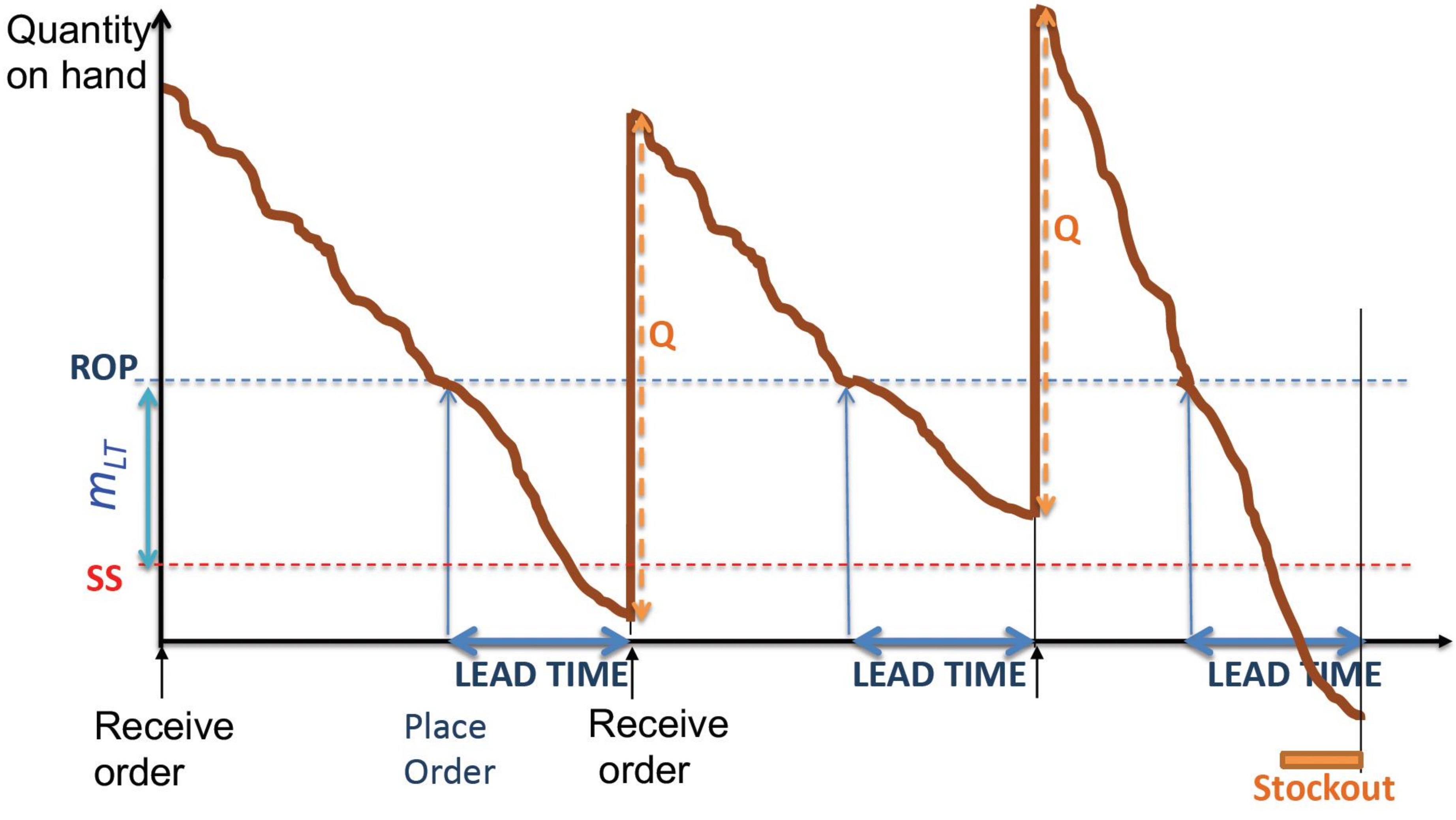
Average Inventory

If the firm pays for inventory "in stock"	If the firm pays for inventory "in stock" and "on route"
Average Inventory = Q/2 + SS	Average Inventory = Q/2 + SS + m_{LT}
	Pipeline Inventory

↑ "Default" case in this course
(when there is no other explanation)

Continuous Review

ROP and SS



Periodic Review

Assume Normal demand uncertainty

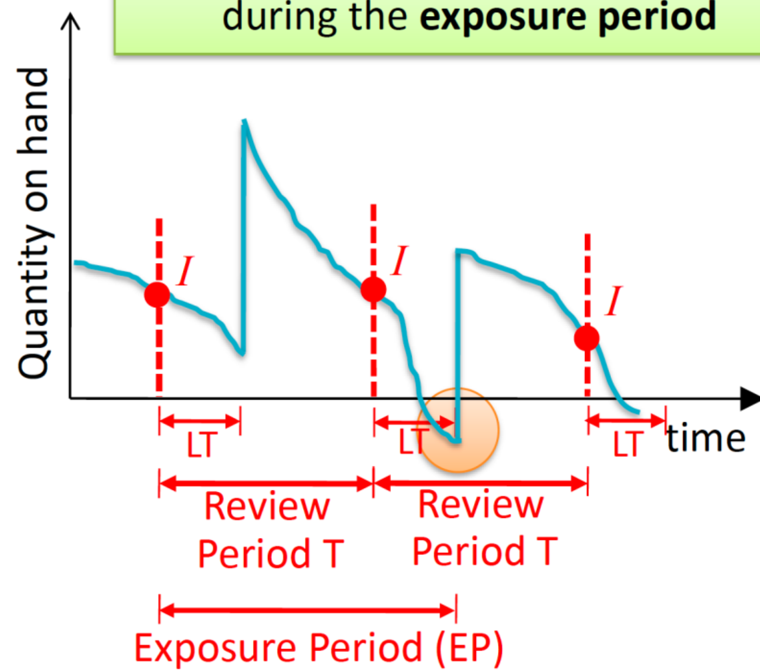
Target Level or "Inventory Position"
= (LT + T) * D + $z\sigma_{LT+T}$

Order Quantity
= Target Level - I

I	Current inventory level
T	Review Period (time between reviews)
EP	**Exposure Period** (= LT+T)

Given Cycle service level (CSL), we can find z=normsinv(CSL)

σ_{LT+T} Standard deviation of demand during the **exposure period**



Continuous Review vs. Periodic Review

	Continuous Review	**Periodic Review**
Exposure Period	EP = L	EP = T + L
When to order	Inventory hits Re-Order Point ROP = d L + SS	Fixed time: Every T units of time
How much to order	Fixed quantity: Order EOQ	Q = Target stock level - Inventory TSL = d (L + T) + SS
Average Cycle Inventory	Q / 2 = EOQ / 2	Q / 2 = d T / 2
Safety Stock	SS = z σ_{EP} = z σ_L [Normal]	SS = z σ_{EP} =z σ_{T+L} [Normal]
Average Pipeline Inventory	d L	d L
Average Inventory (owned by firm)	• If **pays for inventory "in stock" and "on route"** EOQ / 2 + SS + d L • If **pays for inventory "in stock"** : EOQ / 2 + SS	• If **pays for inventory "in stock" and "on route":** d T / 2 + SS + d L • If **pays for inventory "in stock":** d T / 2 + SS

Practice Questions for today

Practice problem 1

- A Machine Shop makes a line of metal tables for customers. Some of these tables are carried in finished-goods inventory. A particular table has the following characteristics:

- Sales = 200 per year
- Setup cost = $1,000 per setup
- Carrying cost = 20 percent per year
- Item cost = $25

a) How many of these tables should be made in a single production lot?

Solutions

(a)

$$Q^* = \sqrt{\frac{2 \times S \times D}{i \times C}} = \sqrt{\frac{2 \times 1{,}000 \times 200}{0.2 \times 25}} \approx 283 \text{ tables}$$

b) How often should production be scheduled?

(b) Production should be scheduled every $Q^*/D = 1.41$ (approx. 516 days) or, equivalently, $D/Q^* = 0.71$ times a year.

Practice problem 2

- Every time a particular retailer places an order from a supplier in another country, the retailer incurs a fixed fee of \$100 (for shipping and customs charges). The purchase cost per item is \$1.50 and the holding cost for one item for one year would be 40% of this cost. The weekly demand at the retailer is 700 units. Assume 52 weeks a year in the following.

a) What is the optimal order quantity for the retailer that minimizes the total cost?

Solutions

(a) $i = 40\%$, $c = \$1.5$, $H = i*c = 0.6$, $S = \$100$, $D = 52*700 = 36{,}400/\text{yr}$

$Q* = \text{sqrt}(2SD/H) = \text{sqrt}(2*100*36400/0.6) = 3{,}483$

b) What is the total yearly inventory cost for the retailer?

(b) $\mathrm{TC(Q^*)} = \mathrm{HQ^*}/2 + \mathrm{SD/Q^*} = 0.6^*3{,}483/2 + 100^*36{,}400/3{,}483 = \$2{,}090$

c) Suppose the supplier is only willing to deliver orders in multiples of 5,000. By how much will this increase the retailer's yearly inventory cost?

(c) $TC(5000) = HQ/2 + SD/Q = 0.6*5{,}000/2 + 100*36{,}400/5{,}000 = \$2{,}228$

Extra cost $= 2228\text{-}2090 = 138$

d) If there is exactly a 5 day lead time on deliveries, what level should the on-hand inventory be when the retailer places an order (assume no uncertainty in demand)?

(d) $700 * 5/7 = 500$

e) Going back to the original problem described at the beginning, suppose the retailer now says that if you order 4,000 or more, they will charge you $1.10 per item. Assuming holding cost is still 40% of purchase cost, what is your optimal order quantity?

(e) $i = 40\%$, $c = \$1.1$, $H = i*c = 0.44$

$Q* = \text{sqrt}(2SD/H) = \text{sqrt}(2*100*36400/0.44) = 4{,}068$

Practice problem 3

- A local company produces a programmable EPROM (erasable programmable read-only memory) for several industrial clients. They have experienced a relatively flat demand of 60 units per week for the product. The accounting department has estimated that it costs \$50 to initiate a production run, each unit costs the company \$2 to manufacture, and the cost of holding is based on a 30% annual interest rate.

a) If each production run takes almost no time, determine the optimal quantity of each production, the average annual holding cost, and annual setup cost.

First, we compute H = IC=0.3*2 = 0.6 per unit per year.

S= 50, D =60*52= 3120 per year.

Then Q = sqr(2SD/H)= 721.11 (you may round up to 721 or 722).

At the optimal order quantity, average holding cost always equals to average setup cost, in this case, which is HQ/2=216.33.

b) Now suppose each production takes 2 weeks, and the firm uses a (Q,r) policy. What is the optimal order quantity? At what reorder point, the production run has to start? What is the maximum level of total inventory of the EPROMs (including the pipeline inventory)?

Lead time does not affect the optimal order quantity. So Q is still 721. Since the demand is flat, we could ignore the safety stock. (Consider a (Q,r) model with deterministic demand in the (Q,r) lecture slides).

So ROP=LT*D=120 units. When inventory hits to 120, the firm starts to order, at which time the total inventory reaches the peak, which is ROP(on hand inventory)+Q(pipeline inventory)=120+721=841.

Practice problem 4

- A local bakery store maintains a periodic inventory system for flour. An order is placed every three weeks and it takes one week for each order to arrive. The average weekly demand of flour is 600 pounds with a standard deviation of 100 pounds per week. Assume the demand is normally distributed.

(a) Currently the bakery store maintains a fixed-time-period inventory system with a target inventory level at 2,500 pounds. The target inventory level is chosen to cover the average demand during the exposure period plus some safety stock. How much safety stock is reserved under the current inventory policy? What is the corresponding service level (i.e., probability of no stock-out)?

Exposure period = 3 + 1 = 4 weeks

Average demand during exposure period = 600 * 4 = 2,400

Target level = 2500 = 2400 + SS

SS = 100

$\sigma(EP) = 100*\text{sqrt}(4) = 200$

$z = SS / \sigma(EP) = 0.5$

service level = 69% (from the table)

z	0	0.01	0.02	0.03	0.04	0.05
0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943

(b) The bakery store is considering switching to a fixed-order-quantity inventory system with continuous review. The ordering cost is $60 per order and annual holding costs are $0.1 per pound. Assume the store operates 50 weeks each year. What should be the economic order quantity for flour? What will be the average cycle stock?

$S = 60, H = 0.1, D = 600*50 = 30,000$
$Q^* = sqrt(2SD/H) = sqrt(2*60*30,000/0.1) = 6,000$

Average Cycle Stock = Q/2 = 3000

(c) The manager of the bakery store wants to guarantee a 97% service level for the exposure period of the fixed-time-period system or the lead-time period of the fixed-order-quantity system. What are the safety stock levels in these two inventory systems? Based on the safety stock levels, which system is better? Is there any consideration other than the safety stock levels when choosing the inventory system? (NORMSINV(0.97) = 1.89)

Fixed-time-period (FTP) model:

EP = 4 weeks, service level = 0.97, z = 1.89, $\sigma(EP) = 100*\text{sqrt}(4) = 200$

$SS = z * \sigma(EP) = 378$

Fixed-order-quantity (FOQ) model:

LT = 1 week, $\sigma(LT) = 100$

$SS = z * \sigma(LT) = 189$

So, SS(FTP) > SS(FOQ). Based on the SS, FOQ is better.

Other considerations:

FOQ model requires reviewing the inventory continuously, which is costly;
The (upstream) suppliers may only take orders periodically, so it may be out of the control of the bakery;

Practice problem 5

- A company currently has 200 units of a product on hand that it orders every two weeks (14 days) when the salesperson visits the premises. Demand for the product averages 20 units per day with a standard deviation of 5 units. Lead time for the product to arrive is seven days. Management has a 95% probability of not stocking out for this product. (NORMSINV(0.95) = 1.64)

The salesperson is due to come in late this afternoon when 180 units are left in stock (assuming 20 are sold today). How many units should be ordered?

$$z = \text{NORMSINV(SL)} = 1.64$$

$$\sigma_{LT+T} = \text{sqrt}(LT + T) \times \sigma = \sqrt{7 + 14} \times 5$$

$$\text{Target} = (LT + T) \times D + z\sigma_{LT+T} = (7 + 14) \times 20 + 1.64 \times \sqrt{7 + 14} \times 5 \approx 458 \text{ units}$$

$$Q = T - I = 458 - 180 = 278 \text{ units}$$

Key take-aways

- EOQ model and Q-r model
- Continuous review vs. periodic review
- Practice problems!