The Inn at Penn hotel has 270 rooms with standard queen-size beds and two rates: a full price of \$150 and a discount price of \$80. To receive the discount price, a customer must purchase the room at least two weeks in advance (this helps to distinguish between leisure travelers, who tend to book early, and business travelers, who value the flexibility of booking late). For a particular Tuesday night, the hotel estimates that the demand from leisure travelers could fill the whole hotel while the demand from business travelers is distributed normally with a mean of 80 rooms and a standard deviation of 25.

a. Find the optimal protection level for full-price rooms (the number of rooms to be protected from sale at a discount price).

$$c_u = 150 - 80 = 70$$
$$c_o = 80$$
$$CR = \frac{70}{150} = 0.47$$
$$1 - CR = 0.53$$
$$z = -0.08$$
$$S = 80 + 25 * (-0.08) = 78$$

b. The Sheraton declared a fare war by slashing business travelers' prices down to \$120. The Inn at Penn had to match that fare to keep demand at the same level. Does the optimal protection level increase, decrease, or remain the same?

$$c_u = 120 - 80 = 40$$
$$c_o = 80, CR = \frac{40}{120} = 0.33$$
$$1 - CR = 0.67$$
$$z = -0.44$$
$$S = 80 + 25 * (-0.44) = 69$$

Decreases. The lower price for business travelers leads to a lower critical ratio and hence to lower protection level, i.e., it is less valuable to protect rooms for the full fare.

Due to customer no-shows, The Inn at Penn hotel is considering implementing overbooking. The discount fare is \$180. The forecast of no-shows is Poisson with a mean of 15.5. The probabilities for Poisson(15.5) are as follows:

Y	F(Y)	Y	F(Y)	Y	F(Y)
8	0.0288	14	0.4154	20	0.8944
9	0.0552	15	0.5170	21	0.9304
10	0.0961	16	0.6154	22	0.9558
11	0.1538	17	0.7052	23	0.9730
12	0.2283	18	0.7825	24	0.9840
13	0.3171	19	0.8455	25	0.9909

The Inn is sensitive about the quality of service it provides alumni, so it estimates the cost of failing to honor a reservation is \$310 in lost goodwill and explicit expenses.

a. What is the optimal overbooking limit; that is, the maximum reservations above the available 140 rooms that The Inn should accept?

$$c_u = 180$$
$$c_o = 310$$
$$CR = \frac{180}{310 + 180} = 0.3673$$
$$S = 14$$

b. If The Inn accepts 150 reservations, what is the probability The Inn will not be able to honor a reservation?

$P(\text{No-show} \leq 9) = 0.0552$

c. If The Inn accepts 155 reservations, what is the probability The Inn will be fully occupied?
P(No-show <= 15) = 0.5170