

MSBA7003 Quantitative Analysis Methods

Assignment 1 (Due September 18 at 23:55. Please submit your solutions with the template.)

Q1.

The joint probability distribution between two random variables X and Y is given below.

Joint Probabilities	Y = 1	Y = 2	Y = 3
X = 0	0.1	0.2	0.3
X = 1	0.2	0.1	0.1

Which of the following statement(s) is(are) true?

(A) X = 0 and Y = 1 are unconditionally independent.

(B) The event of X = 0 is less likely to occur given Y = 2 than given Y = 1.

(C) $\Pr(X = 1 \mid Y > 1) = 2/7$.

(D) Suppose we randomly and independently draw two pairs of (X,Y): (X₁,Y₁) and (X₂,Y₂). We only see the values of Y. Then $\Pr(X_1 + X_2 = 0 \mid Y_1 = 2 \text{ and } Y_2 = 3) = 1/2$.

(E) None of the above.

Q2.

Suppose you can see a lawn out of your window. The lawn may be wet or dry in the afternoon. The state depends on the whether it rained earlier today and the previous operation of the sprinkler on the lawn. We are given the conditional probability of the afternoon lawn being wet given the previous operation of the sprinkler and whether it rained earlier today as below:

Previous operation of the sprinkler		On	On	Off	Off
Whether it rained earlier today		Yes	No	Yes	No
Lawn state	Wet	0.99	0.9	0.8	0.0
	Dry	0.01	0.1	0.2	1.0

We also know that the operation of the sprinkler is affected by the weather probabilistically in the following way in terms of conditional probabilities:

Whether it rained earlier today		Yes	No
Sprinkler	On	0.1	0.4
	Off	0.9	0.6

In addition, our prior belief is that it rains with a probability of 0.2 every day.

Which of the following statement(s) is(are) true?

(A) If you know that the sprinkler was on earlier today, the probability that the lawn will be wet in the afternoon is $0.2 \cdot 0.99 + 0.8 \cdot 0.9 = 0.918$.

(B) If you know that it rained in the morning, the probability that the lawn will be wet in the afternoon is $0.1 \cdot 0.99 + 0.9 \cdot 0.8 = 0.819$.

(C) If you see in the afternoon that the lawn is wet, the probability that it rained before is $(0.99 + 0.8) / (0.99 + 0.9 + 0.8 + 0) = 0.6654$.

(D) If you see in the afternoon that the lawn is wet, the probability that the sprinkler was on before is $(0.8 \cdot 0.1 \cdot 0.99 + 0.2 \cdot 0.4 \cdot 0.9) / (0.8 \cdot 0.1 \cdot 0.99 + 0.2 \cdot 0.4 \cdot 0.9 + 0.8 \cdot 0.9 \cdot 0.8 + 0.2 \cdot 0.6 \cdot 0) = 0.2$.

(E) None of the above.

Q3.

ABC Inc. is considering launching a new product and there are three options: product X, product Y, and do nothing. Product X requires an initial investment of \$15 million and product Y requires \$5 million. The gross profit (before subtracting the initial investment) that can be generated by each product depends on the market condition. If the market is strong, product X can generate a gross profit of \$100 million and product Y can generate \$20 million; if the market is weak, product X will lead to a loss of \$80 million and product Y will cause a loss of \$9 million (before subtracting the initial investment). The company's prior belief of a strong market is 60%.

To get a better understanding of the market before deciding the choice of the new product, the manager hired a consulting firm to conduct market research. According to historical data, this consulting firm can successfully predict a strong market in 65% cases and can correctly predict a weak market in 55% cases.

Based on this information, which of the following statement(s) is(are) true?

(A) If the consulting firm is not hired, it is better not to introduce any product.

(B) If there is not historical data about the consulting firm (i.e., we don't know the probabilities of accurate predictions), \$38 million should be paid at most to hire the company.

(C) If the consulting firm gave an unfavorable report, ABC should introduce product X.

(D) The expect value of the consulting firm's report is about \$3.12 million.

(E) None of the above.