MSBA 7004
Operations Analytics

Class 2-2: Process Flow Analysis (III)

Inventory & Little's Law

2023

Process Analysis (III) Outline

- Utilization and Utilization Profiles
- Inventory Build-Up Diagram (Short run)
- Little's Law (Long run)

Capacity Rate vs. Throughput Rate

- Both the *capacity rate* and the *throughput rate* measure the ***output rate*** of a process

Capacity rate: Maximum possible output rate

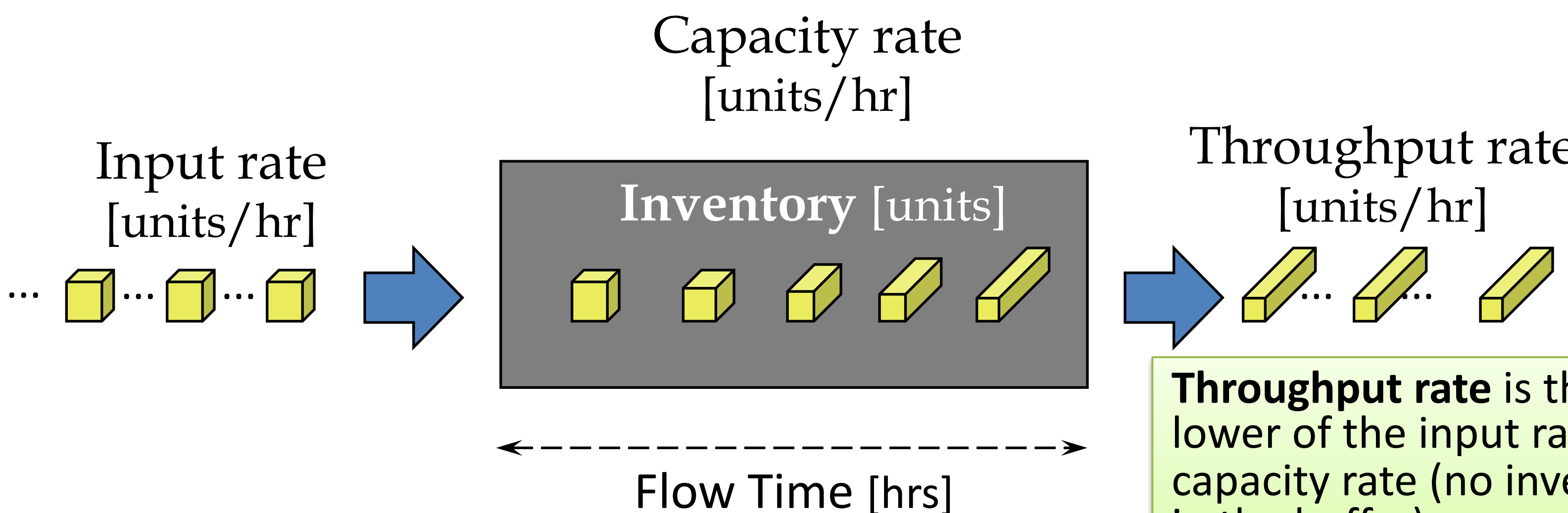
Throughput rate: Actual output rate

- The ***throughput*** (or ***flow***) ***rate*** depends on both:

Capacity rate

Input rate (also called the **arrival rate**)
Rate at which flow units arrive at the process
e.g. Arrival of customers (demand rate), or Raw material arrival into a factory (planned or scheduled arrivals)

Fundamental Process Characteristics



Throughput rate is the lower of the input rate and capacity rate (no inventory in the buffer)

Throughput Rate:

What if Input rate < Capacity rate?

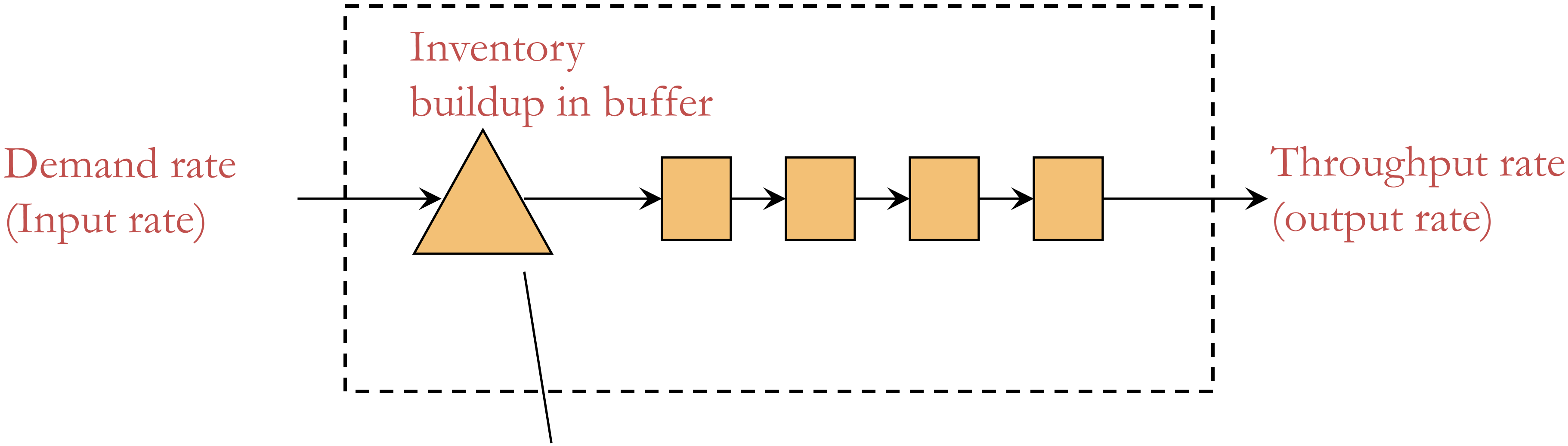
Throughput Rate:

What if Input rate > Capacity rate?

Flow time = average time for a unit to move through the process

Inventory = total number of flow units present in the process

Buffers in a process flow diagram



Waiting areas or buffers holding inventory

- Do not add value. Do not have a capacity.

The Funnel Analogy (continuous time)

When	Results?
Input Rate > Capacity Rate	
Input Rate < Capacity Rate, Inventory =0	
Input Rate < Capacity Rate, Inventory >0	

The Funnel Analogy (continuous time)

When	Results?
Input Rate > Capacity Rate	Output Rate =Capacity Rate, Inventory ↑
Input Rate < Capacity Rate, Inventory =0	Output Rate = Input Rate, Inventory = 0
Input Rate < Capacity Rate, Inventory >0	Output Rate = Capacity Rate, Inventory ↓

Utilization

$$\text{Utilization} = \frac{\text{Throughput Rate}}{\text{Capacity Rate}} = \frac{\text{Actual output rate}}{\text{Maximum output rate}} \leq 100\%$$

- Utilization gives us information about "excess capacity"
- The utilization of each resource in a process can be presented with a ***utilization profile*** How busy you are?

Resource	Capacity Rate (units/hour)	Input Rate (units/hour)	Utilization
1	6	4	66.67%
2	7	4	57.14%
3	8	4	50.00%
4	6	4	66.67%
5	5	4	80.00%

- What is the optimal utilization of a resource?

Implied Utilization

- To capture the idea that there may be excess demand in the short-run, *another* measure of utilization is often useful

$$\boxed{\text{Implied Utilization} = \frac{\text{Input Rate}}{\text{Capacity Rate}}}$$

Compare: $$\boxed{\text{Utilization} = \frac{\text{Throughput Rate}}{\text{Capacity Rate}}}$$

- Implied utilization also allows us to capture the idea of **overtime**
  - Organizations often budget for a fixed amount of capacity, and work overtime to meet excess demand

Operational Challenge: Mismatch between demand and supply

In any process, the input and output rates will vary *over time*

- When is capacity wasted (or utilization <1)?

  capacity rate > input rate and inventory=0

- When does inventory build up?

  capacity rate < input rate (the rest of input goes to the buffer and becomes inventory)

- Both are mismatches

- Operations is about matching supply and demand

Matching the Input and Capacity Rates

- For a variety of reasons, a perfect match is not possible. What are some of these reasons?
- Case I. Long-run mismatch

  Why? What can we do?

  Increase capacity rate
- Case II. Short-run mismatch

  Why? What can we do?

  Demand variability

  Increase capacity rate, better forecast, reduce variance (affect demand side)

Examples: ?

Restaurant (Happy Hour), Call centers (long waiting during day vs. short waiting at night), parking lots (monthly users vs hourly users)

Previous Capstone Project: Improving Parking Lot Utilization Rate

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Parking Services

Sino Parking Services Limited is one of the major car park operators in Hong Kong. Over the years, we have introduced user-friendly, cashless payment methods, automated access systems, and Hong Kong's first parking loyalty programme.

1987	120^+	15000^+	380^+
ESTABLISHED	CAR PARKS	PARKING SPACES	ELECTRIC VEHICLE CHARGERS

Previous Capstone Project: Improving Parking Lot Utilization Rate

- Setting: Fixed and limited parking slots, serve two types of customers (monthly and hourly parking)

- Problem to solve:
  - Objective function: max ???

  = max ???
  - Constraint: Not interfering the use of monthly parking customers
  - Decision variable: ???

Previous Capstone Project: Improving Parking Lot Utilization Rate

- Setting: Fixed and limited parking slots, serve two types of customers (monthly and hourly parking)

- Problem to solve:
  - Objective function: max Revenue

  = max {Sum(Utilization of each parking slot)}

  - Constraint: Not interfering the use of monthly parking customers

  - Decision variable: Number of hourly parking to admit

So far, we have introduced the following Process Characteristics

Process Characteristics	Corresponding Question
Flow Units	What is my "product"?
Input rates and output rates	What is the demand for my process? What is my capacity?
Utilization	What is the proportion of time that the resource is being used?
Flow times (time spent in process)	How long does it take me to produce one "unit"?
Stocks (inventory build-up in buffer)	How much inventory (of flow units) do I need to hold? E.g., queue of patients at a hospital, cars at a dealership, or a-warehouse-full of materials.

Steps for Process Flow Analysis

- Process Mapping: Standard/Linear/Swim Lane/Gantt Chart
- Bottleneck Analysis: Identify *"flow units*", "theoretical flow time", "bottleneck resource", "capacity rate"
- Short-run analysis (inventory build-up diagram)
- Long-run analysis (Little's Law)

An example: Security screening at HKG

Time	Input rate (passengers/15 min slot)	Capacity rate (passengers/15 min slot)	Excess Demand	Excess Capacity
6:15	7	15	0	8
6:30	10	15	0	5
6:45	8	15	0	7
7:00	12	15	0	3
7:15	9	15	0	6
7:30	16	15	1	0
7:45	14	15	0	1
8:00	19	15	4	0
8:15	22	15	7	0
8:30	17	15	2	0
8:45	13	15	0	2
9:00	11	15	0	4
9:15	12	15	0	3
9:30	8	15	0	7
9:45	10	15	0	5
10:00	7	15	0	8
TOTAL	**195**	**240**		

Do we have enough capacity?

...but not at all times

Enough capacity for the shift ...

Data for a 4-hour shift in 15-min time slots: 7 arrive between 6:00 and 6:15 etc.

Short-run vs. Long-run Averages

- Since the input and output rates may vary over time, both the ***short-run average*** and the ***long-run average*** rates provide useful information.

- Long-run average input rate must be less than or equal to the long-run average capacity rate

Why?

- Long-run average *throughput rate*
  = Long-run average *input rate*

Why?

- Short-run average input rate can be greater than the short-run average capacity rate

But what would this lead to?

Assumption: All the input need to go through the process.

Security Screening Example Revisited

- What is the capacity rate?

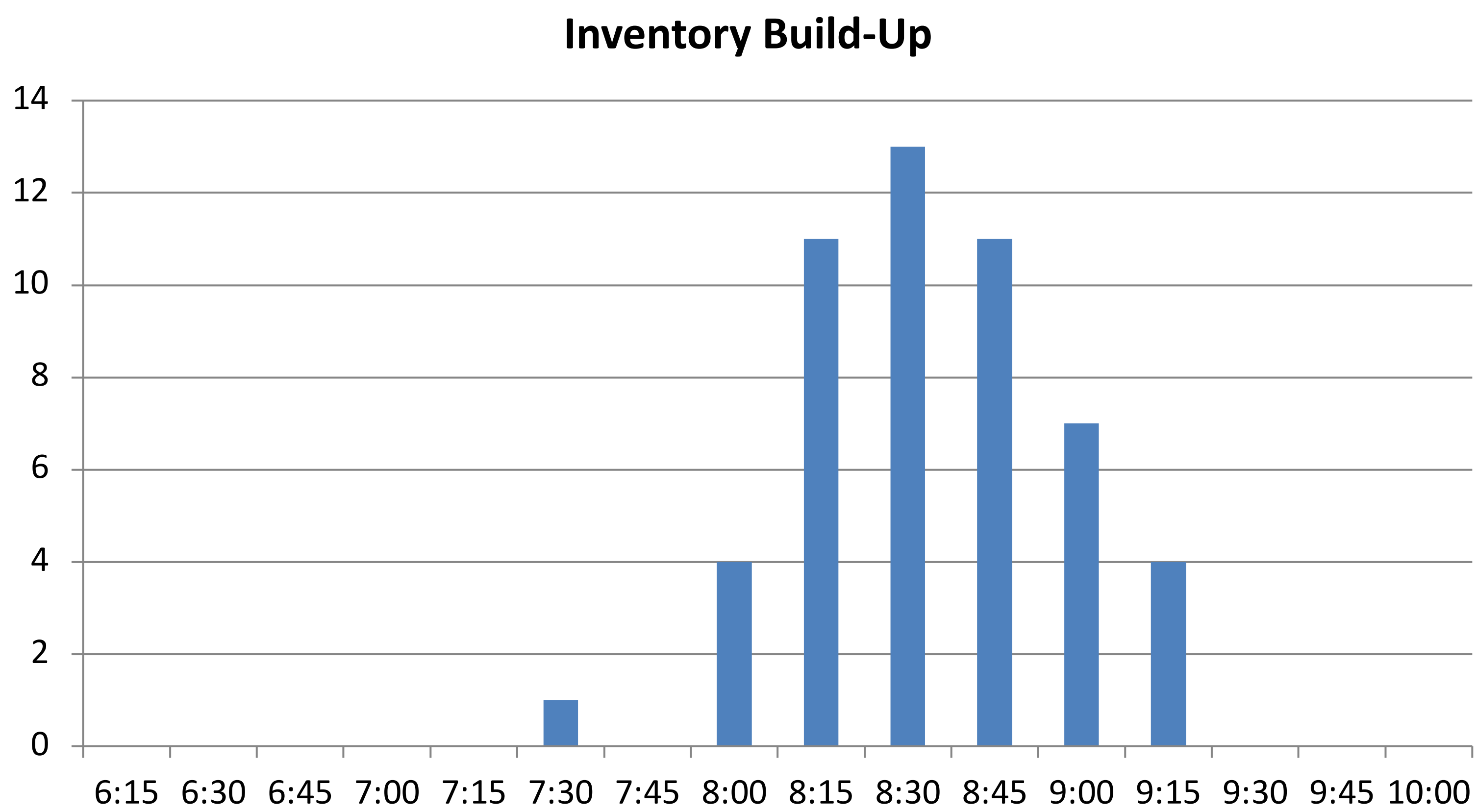
  Note: In this example, the capacity rate is given. In practice, it may not be obvious. Finding the capacity rate will involve drawing a process flow map, identifying activities, times, resources, etc, and finding the bottleneck

- What is the (average) size of the line?
- How long do passengers wait (flow time)?

Inventory Build-Up Diagram

Time	Input rate (passengers/15 min slot)	Capacity rate (passengers/15 min slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP
6:15	7	15	0	8	
6:30	10	15	0	5	
6:45	8	15	0	7	
7:00	12	15	0	3	
7:15	9	15	0	6	
7:30	16	15	1	0	
7:45	14	15	0	1	
8:00	19	15	4	0	
8:15	22	15	7	0	
8:30	17	15	2	0	
8:45	13	15	0	2	
9:00	11	15	0	4	
9:15	12	15	0	3	
9:30	8	15	0	7	
9:45	10	15	0	5	
10:00	7	15	0	8	
TOTAL	**195**	**240**			

Inventory Build-Up Diagram



- What is the “average inventory” in the **buffer**?

Calculating "Average Inventory"

Time	Input rate (passengers/15 min slot)	Capacity rate (passengers / 15 min slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP
6:15	7	15	0	8	**0**
6:30	10	15	0	5	**0**
6:45	8	15	0	7	**0**
7:00	12	15	0	3	**0**
7:15	9	15	0	6	**0**
7:30	16	15	1	0	**1**
7:45	14	15	0	1	**0**
8:00	19	15	4	0	**4**
8:15	22	15	7	0	**11**
8:30	17	15	2	0	**13**
8:45	13	15	0	2	**11**
9:00	11	15	0	4	**7**
9:15	12	15	0	3	**4**
9:30	8	15	0	7	**0**
9:45	10	15	0	5	**0**
10:00	7	15	0	8	**0**
	195	**240**			

Empty Buffer (No Queue)

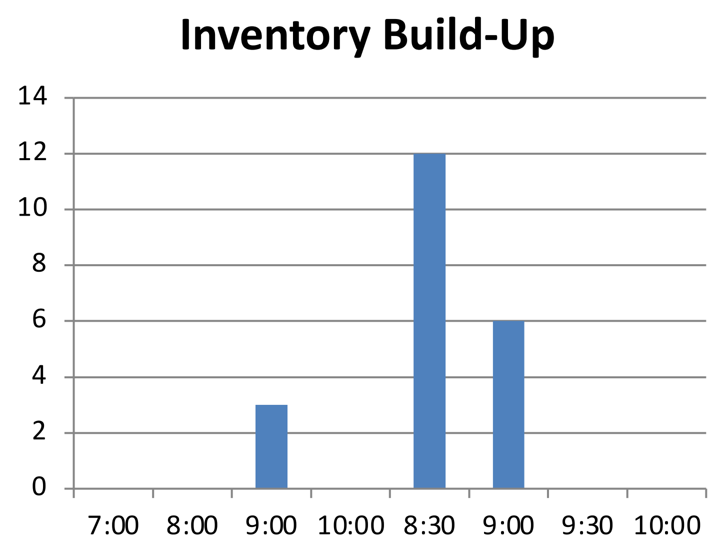
Buffer NOT empty

Average Inventory = 3.1875

Consider another example …

Time	Input rate (passengers/30 min slot)	Capacity rate (passengers / 30 min slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP
6:30	17	30	0	13	0
7:00	20	30	0	10	0
7:30	25	30	0	5	0
8:00	33	30	3	0	3
8:30	39	30	9	0	12
9:00	24	30	0	6	6
9:30	20	30	0	10	0
10:00	17	30	0	13	0
	195	240			2.625

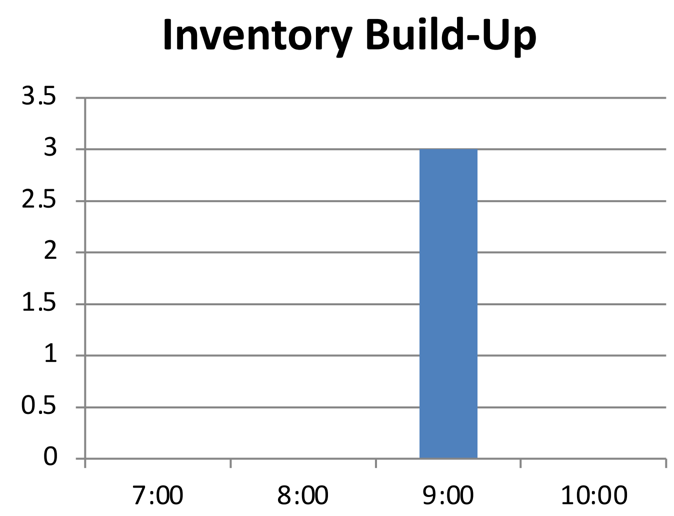
Average Inventory



… and another …

Time	Input rate (passengers/60 min slot)	Capacity rate (passengers / 60 min slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP
7:00	37	60	0	23	0
8:00	58	60	0	2	0
9:00	63	60	3	0	3
10:00	37	60	0	23	0
	195	240			0.75

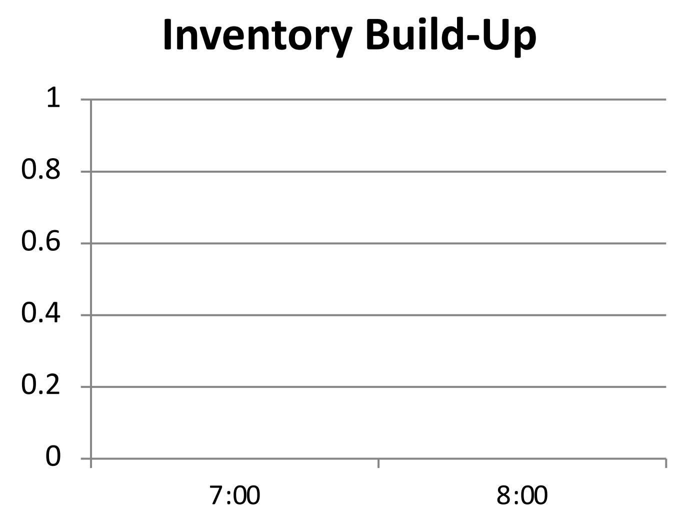
Average Inventory



… and another

Time	Input rate (passengers/2 hour slot)	Capacity rate (passengers / 2 hour slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP
8:00	95	120	0	25	0
10:00	100	120	0	0	0
	195	240			0

Average Inventory



Average Inventory changes as we select different time intervals in the inventory build-up diagram, why?

Approximation error involved when discrete time intervals are used; error propagates when the intervals are larger

Short-run and Long-run Averages

Short-Run Long-Run

- Can average input rate be less than the average output rate?
- Can average input rate be more than average output rate?
- Can average output rate be more than capacity rate?
- Can average input rate be more than capacity rate?

Assumption: All the input need to go through the process. Yes or No?

Short-run and Long-run Averages

	Short-Run	Long-Run
Can average input rate be less than the average output rate?	Yes	No
Can average input rate be more than average output rate?	Yes	No
Can average output rate be more than capacity rate?	No	No
Can average input rate be more than capacity rate?	Yes	No

In the long run…

Average Input Rate = Average Output Rate ≤ Average Capacity Rate (Assume all the input need to go through the process)

Estimating Process Measures

- Process measures change **over time**
  - Depending on the mismatch between input rate and capacity rate inventory builds up over time
- We are interested in *averages* of these quantities
- “Average” values of process measures can be misleading
- It is often convenient to assume <u>continuous</u> input and output processes

Definitions

- ***Instantaneous Flow Rates***

$R_i(t)$	The **input rate** to the process at time t
$R_o(t)$	The **output rate** of the process at time t
$\Delta R(t) = R_i(t) – R_o(t)$	**Instantaneous inventory accumulation** at time t

- ***Inventory Level***

I(t)	The *number of units (in buffer)* at time t

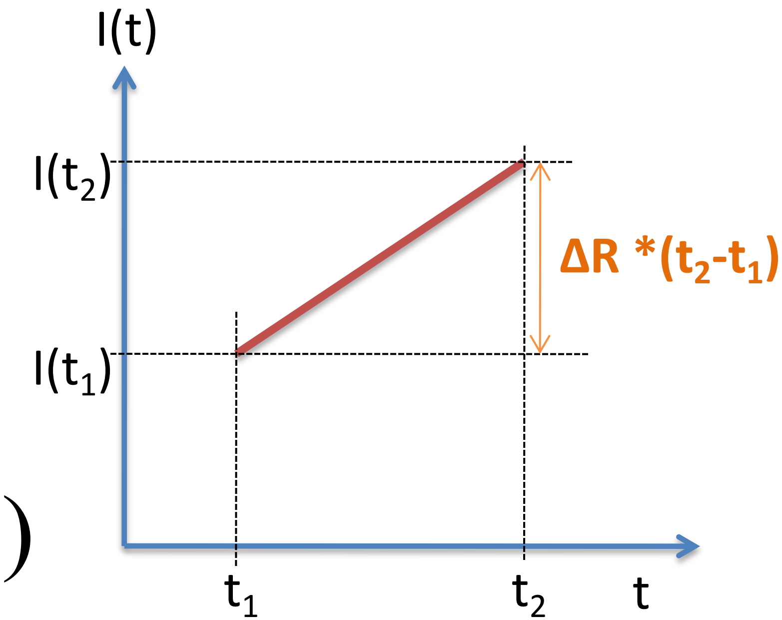
- ***Flow Time***

T(t)	The *time* that a unit which enters (leaves) the process at time t spends (has spent) within the process

This can be defined in many ways

Inventory and Flow Dynamics

- Let (t_1,t_2) denote an interval of time starting at t_1 and ending at t_2
- Suppose "instantaneous inventory accumulation" $\Delta R(t)$ is *constant* over (t_1,t_2) and equals ΔR. Then,

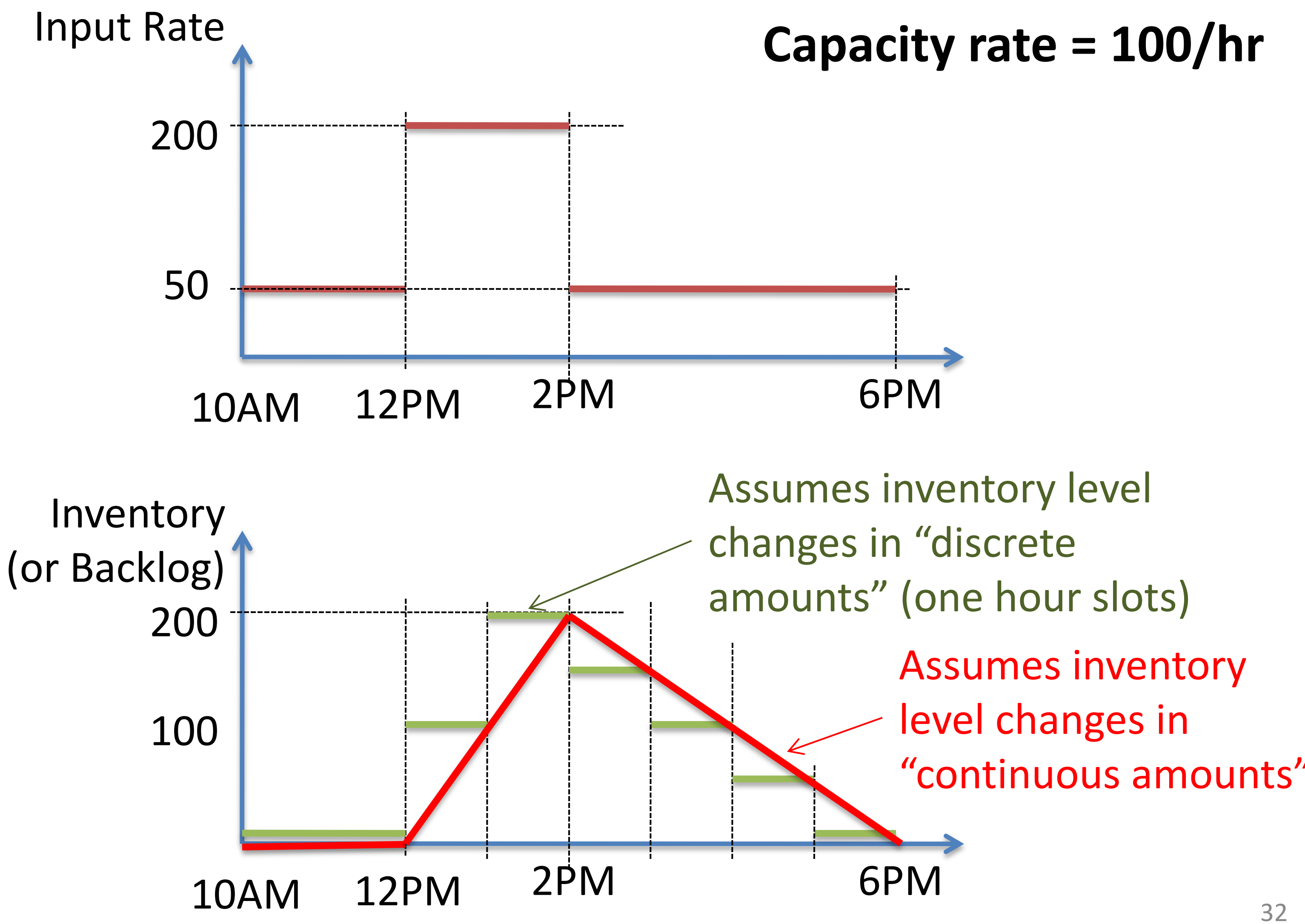


$$I(t_2) = I(t_1) + \Delta R \cdot (t_2 - t_1)$$

Ending Inventory | Starting Inventory | Change in Inventory

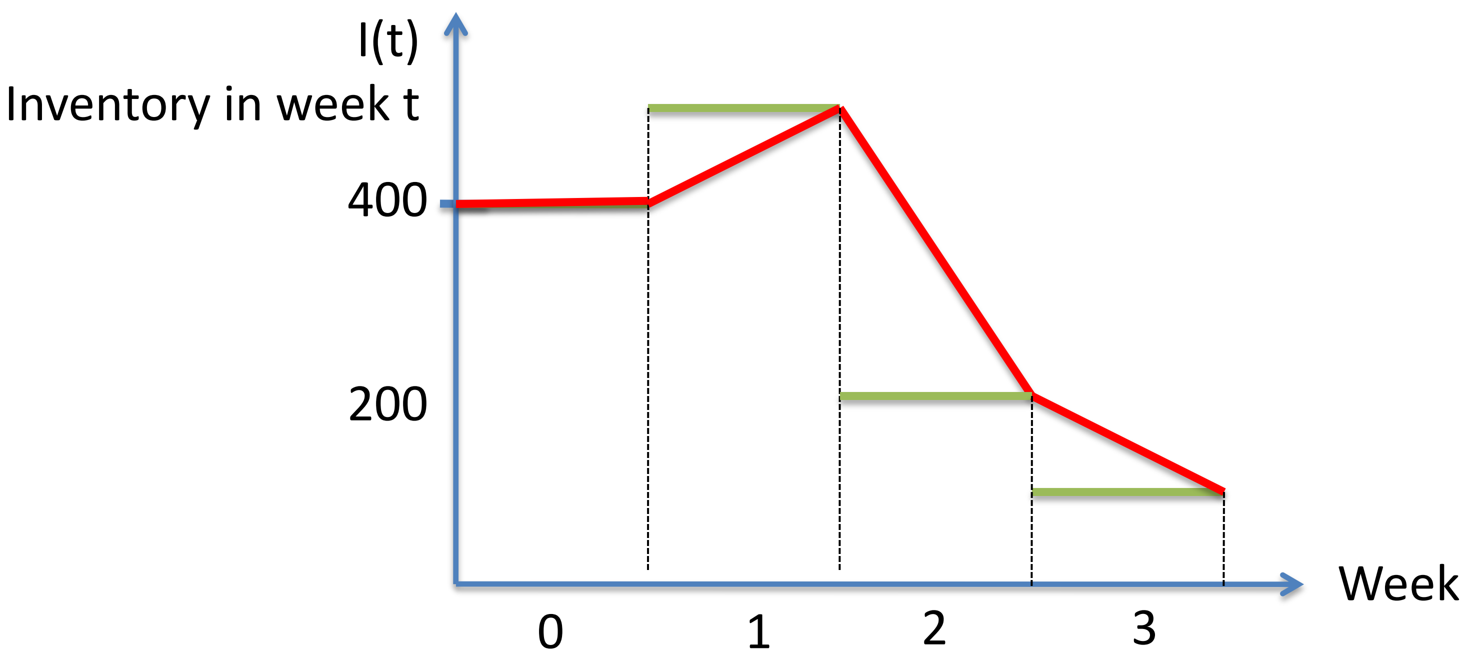
$$\boxed{\text{Average Inventory} = \frac{\text{Starting Inventory} + \text{Ending Inventory}}{2}}$$

Inventory Build-Up Diagram



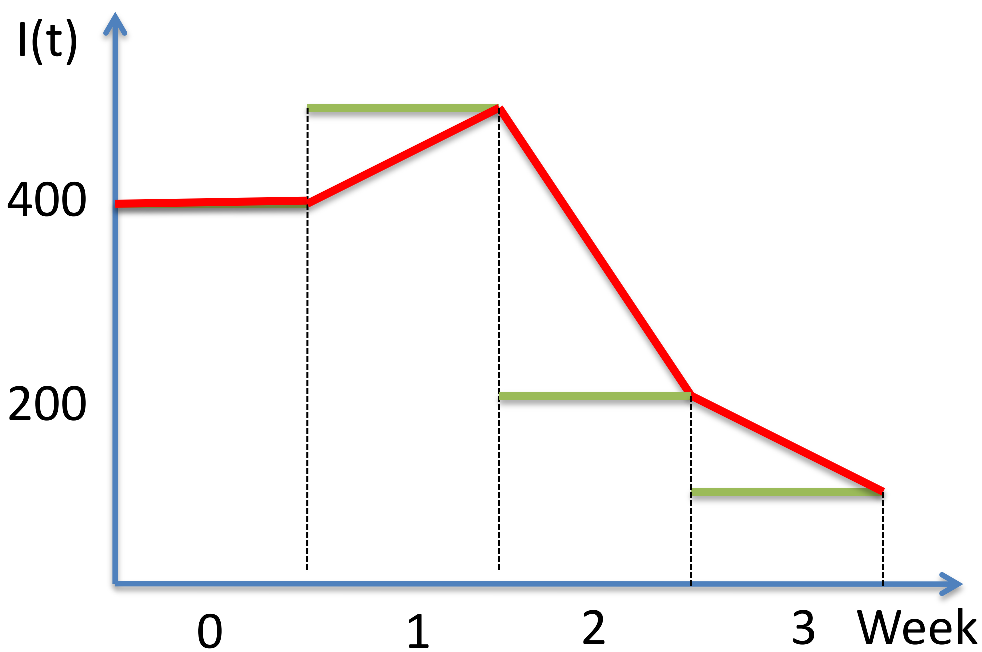
Another Inventory Build-Up Example

Week	Input Rate	Capacity Rate	Inventory
0			400
1	900	800	500
2	900	1200	200
3	900	1000	100



Average Inventory

Average inventory depends on whether inventory is assumed to change in discrete steps, or continuously



Under the ***discrete*** assumption:

The average inventory over weeks 0 to 3 is 300

Under the ***continuous*** assumption:

The average inventory? "Area under the curve/#weeks"