

# MSBA 7004

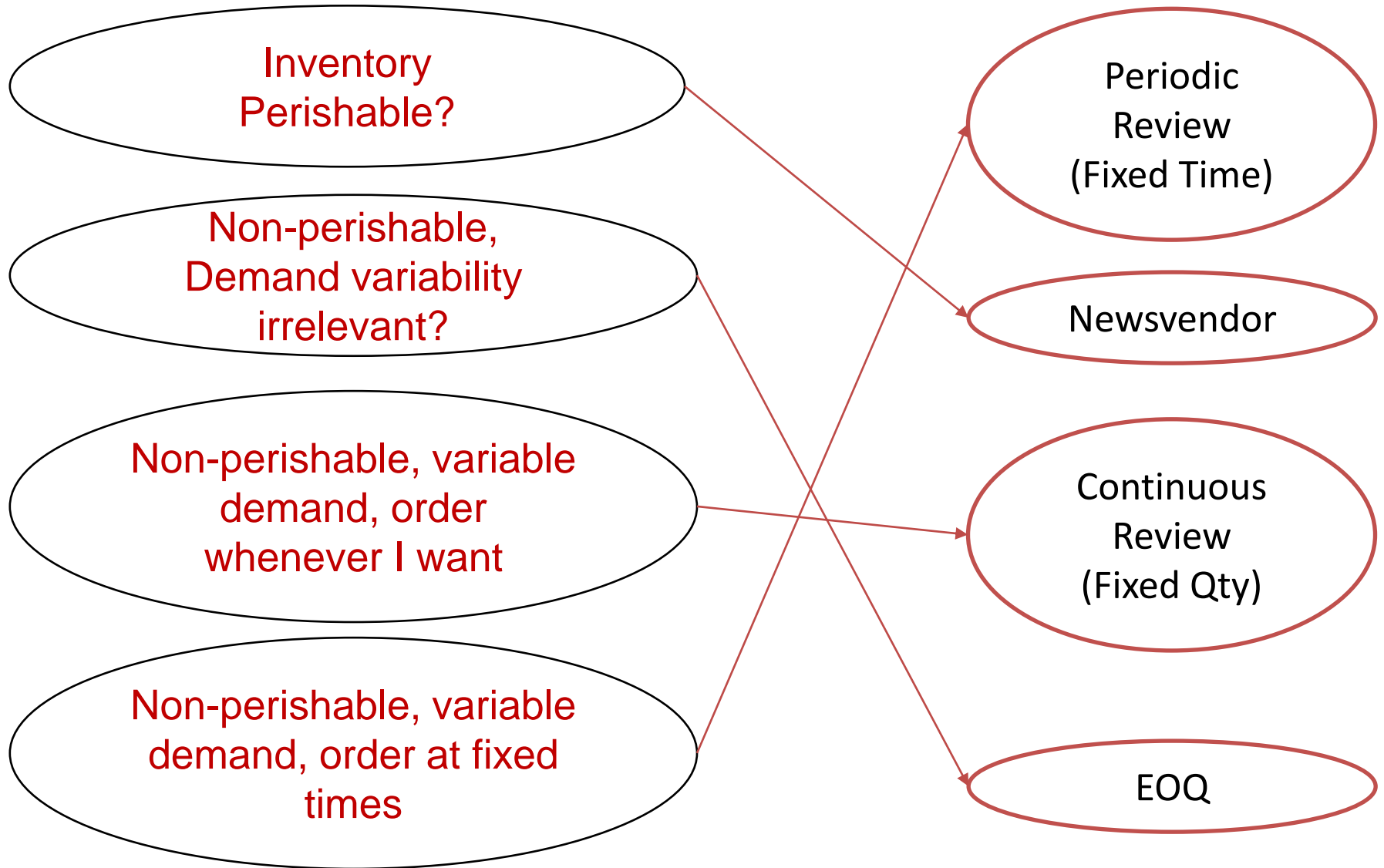
## Operations Analytics

Second-half course recap  
2023

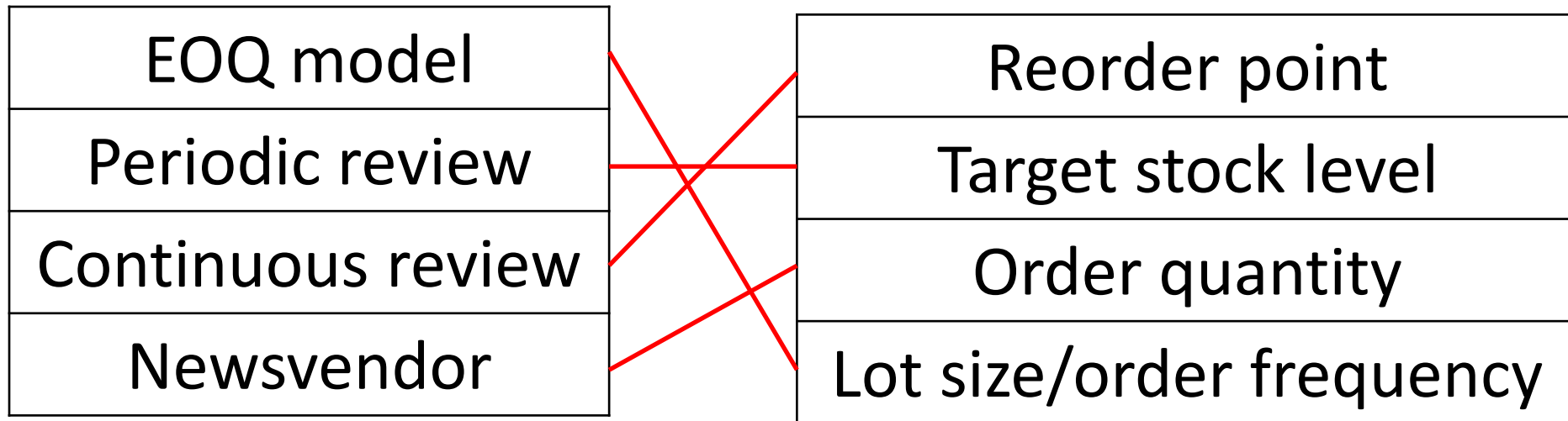
# Management Options

- Managing supply
  - Capacity management
    - Process (bottleneck) analysis
    - Queueing analysis
  - Inventory and supply chain management
  - Project management
- Managing demand
  - Revenue management

# Inventory management

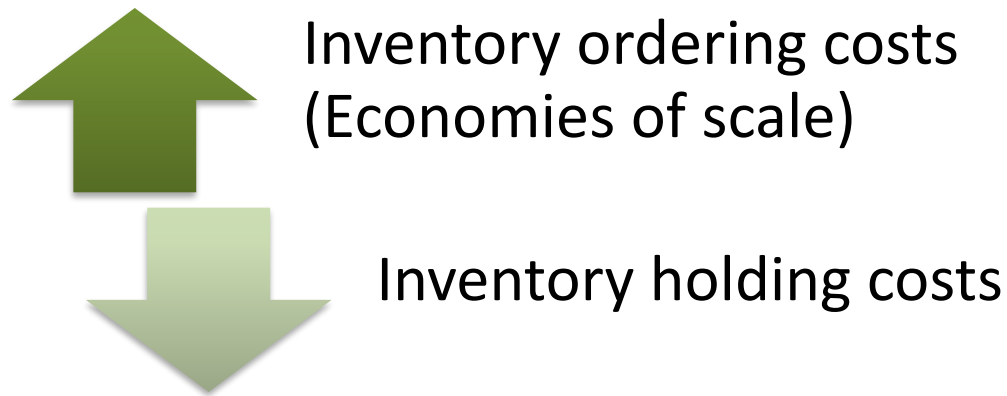


**Match** the inventory model with the decision.



# Economic Order Quantity (EOQ) Model

- How much to order each time?
- The Economic Order Quantity (EOQ) balances



- Assumptions
  - Known annual demand, constant demand rate
  - No uncertainty (in demand)

# True or False?

The motivation of the EOQ model is to match the demand with the right quantity of supply.

# Economic Order Quantity

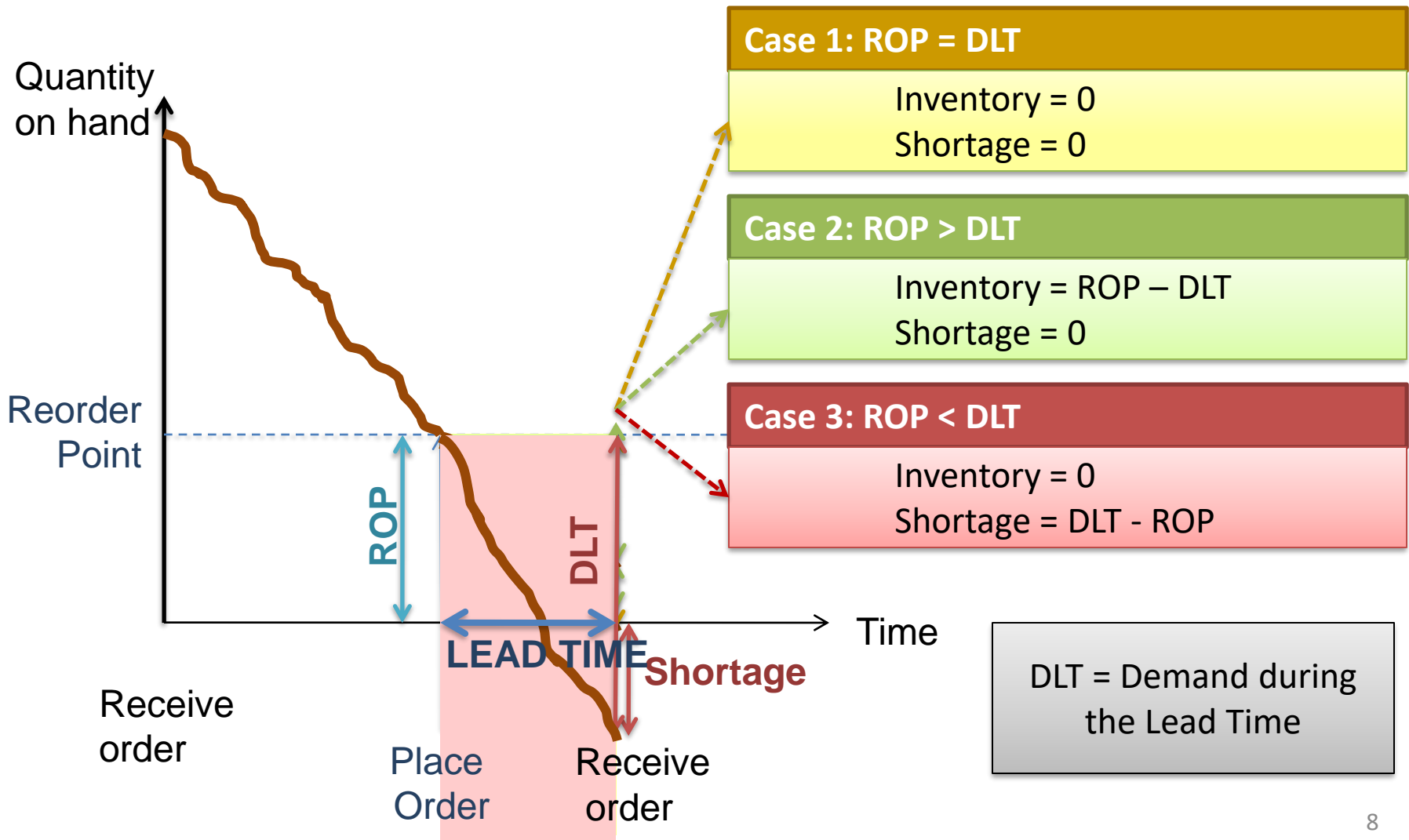
$$\text{Total Cost} = TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

D	Annual Demand Rate
S	Order or Setup Cost
H	Annual Holding Cost

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

$$TC_{OPT} = \sqrt{2SDH}$$

# Demand During Lead Time: Uncertain





# ROP and Normal Demand

Suppose weekly demand has a normal distribution:  $N(m, \sigma^2)$

Suppose lead time is  $k$  weeks.

Then, DLT has normal distribution  $N(m_{LT}, \sigma_{LT}^2)$

with mean  $m_{LT} = k \cdot m$  and standard deviation  $\sigma_{LT} = \sqrt{k} \cdot \sigma$

Given ROP, CSL is

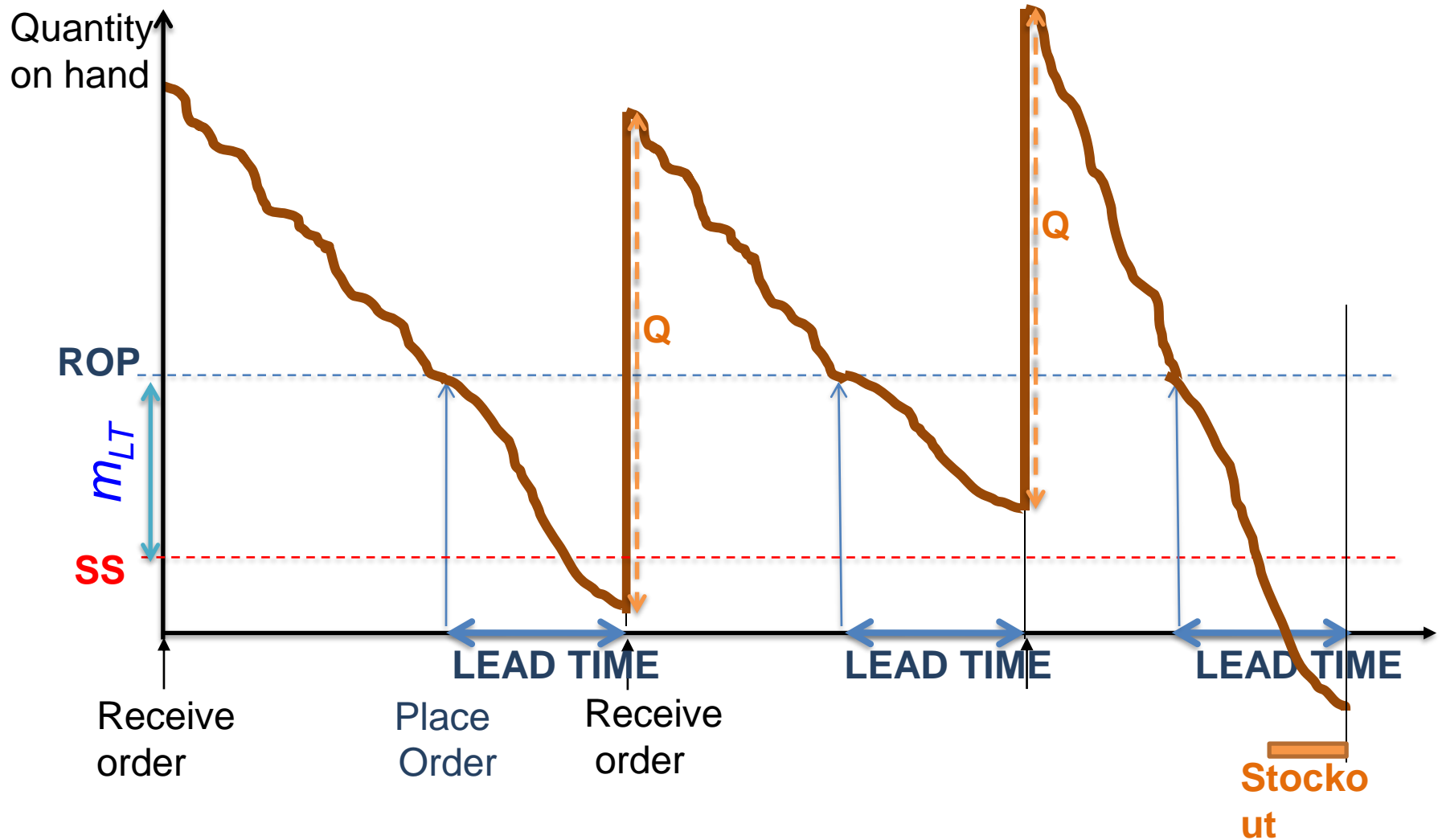
$$P[DLT \leq ROP] = P[N(0,1) \leq z] = \text{normsdist}(z) \quad \text{where} \quad z = \frac{ROP - m_{LT}}{\sigma_{LT}}$$

Given CSL, ROP satisfying  $CSL = P[DLT \leq ROP]$  is

$$ROP = m_{LT} + z \cdot \sigma_{LT} \quad \text{where} \quad z = \text{normsinv}(CSL)$$

Safety Stock (SS)

# ROP and SS



# Average Inventory

If the firm pays for inventory  
“in stock”

$$\text{Average Inventory} = Q/2 + SS$$

↑ “Default” case in this course  
(when there is no other explanation)

If the firm pays for inventory  
“in stock” and “on route”

$$\text{Average Inventory} = Q/2 + SS + m_{LT}$$

Pipeline Inventory

# Order Size for Fixed-Time-Period Model

Assume Normal demand uncertainty

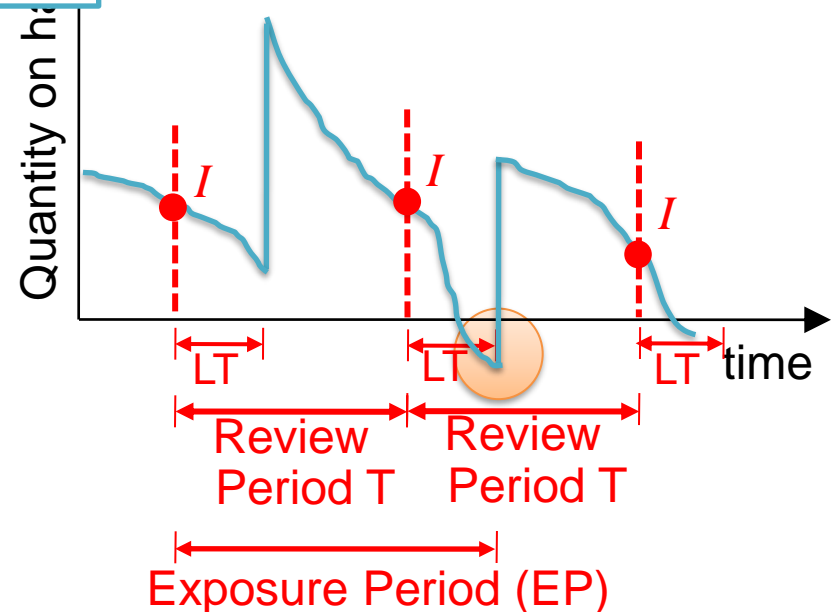
Given Cycle service level (CSL), we can find  $z = \text{normsinv}(\text{CSL})$

**Target Level or Target “Inventory Position”**  
 $= (LT + T) * D + z\sigma_{LT+T}$

$\sigma_{LT+T}$  Standard deviation of demand during the **exposure period**

**Order Quantity**  
 $= \text{Target Level} - I$

<b>I</b>	Current inventory
<b>T</b>	Review Period (time between reviews)
<b>EP</b>	<b>Exposure Period</b> (= LT+T)



# Practice problems

1. Kristen believes they are currently holding too much inventory for some ingredients, and she decides to optimize the inventory policy for whole wheat flour first using what she just learned from the Operations Analytics course. Kristen and her roommate bake cookies 7 days a week all year round. They get deliveries of flour from Flour Foods Co. each Friday and pay for the food upon delivery. They must place orders on Monday to ensure the Friday delivery (i.e., order lead time is 4 days). Kristen has observed that weekly demand for whole wheat flour is normally distributed with a mean of 66 lbs and a standard deviation of 6.6 lbs. Because she is a regular customer, Flour Foods Co. waives delivery charges. However, each package of flour is 10 lbs (i.e., order size must be multiples of 10 lbs).
- Suppose that on Monday, Oct 18, Kristen checked her whole wheat flour inventory and found that she had 83 lbs in stock. How much whole wheat flour should she order to maintain a 99% service level?

$D = 66$  lbs. per week

$\sigma = 6.6$  lbs. for weekly demand

$SL = 99\%$

$T = 1$  week

$LT = \frac{4}{7}$  week

$Inventory = 83$  lbs.

Thus:

$$z = \text{normsinv}(SL) = 2.33$$

$$\sigma_{LT+T} = \sqrt{LT + T} \cdot \sigma = 8.27$$

$$\text{Target: } (LT + T) \cdot D + z\sigma_{LT+T} = 122.96$$

$$\text{Order Quantity: } Target - Inventory = 39.96 \text{ lbs.}$$

Therefore, Kristen should order 4 packages of flour to maintain a 99% service level.

# Practice Problems

2. Kristen also orders paper take-out bags with her logo printed on them. Daily demand for take-out bags is normally distributed with a mean of 50 bags and a standard deviation of 20 bags. Kristen places a new order for bags whenever she is running low. Kristen's printer (company) charges her \$15 per order for print setup independent of order size. Bags are printed in batches of 100, and priced at \$5 per batch (i.e., 5 cents each bag). It takes 5 days for an order to be printed and delivered. The only holding cost is the opportunity cost of capital, which is estimated to be 30% per year. Assume 360 days per year.

- a) What is the optimal order quantity per order for Kristen?
- b) On average, how often does Kristen need to order take-out bags? That is, if Kristen places an order once every  $X$  days, what is  $X$ ?
- c) If Kristen wants to make sure the bags do not run out with 99% probability during the order lead time, what is her optimal reorder point?

**a**

$$D = 0.5 \times 360 \text{ batches per year}$$

$$S = \$15$$

$$C = \$5$$

$$I = 30\%$$

Hence,  $Q_{OPT} = \sqrt{\frac{2SD}{I \cdot C}} = 40 \text{ batches.}$

**b.**

$$X = \frac{Q}{D} = 80 \text{ days}$$

**c.**

$$SL = 99\%$$

$$k = 5 \text{ days}$$

Average daily demand  $m = 0.5 \text{ batches}$

$$\sigma = 0.2 \text{ batches for daily demand}$$

$$m_{LT} = km = 2.5 \text{ batches}$$

$$\sigma_{LT} = \sqrt{k} \cdot \sigma = 0.45$$

$$z = \text{normsinv}(SL) = 2.33$$

Hence, to maintain a 99% service level,  $ROP = m_{LT} + z\sigma_{LT} = 3.54 \text{ batches.}$



# How much inventory should you hold?

- Trade-off #1: How much to order each time?



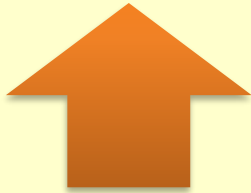
Inventory ordering costs  
(Economies of scale)



Inventory holding costs

Economic Order  
Quantity (EOQ)  
Model

- Trade-off #2: How much to hold each time?



Cost of running out



Cost of having excess  
inventory

Newsvendor  
Model

# What is the Best Service Level (SL\*) ?

- Trade-off:

	Inventory Holding Cost (Overage Cost)	Loss of Revenue & Goodwill (Underage Cost)
High Service Level	High	Low
Low Service Level	Low	High

# How to Interpret $C_u$ and $C_o$ ?

$C_u$ ,  $C_o$  are the unit opportunity costs.

You should compute  $C_u$ :

Suppose the actual demand  $D$  is larger than my current stock  $S$ . If I could return to the past and order one unit more, then it increases my profit (or reduces my cost) by  $C_u$ .

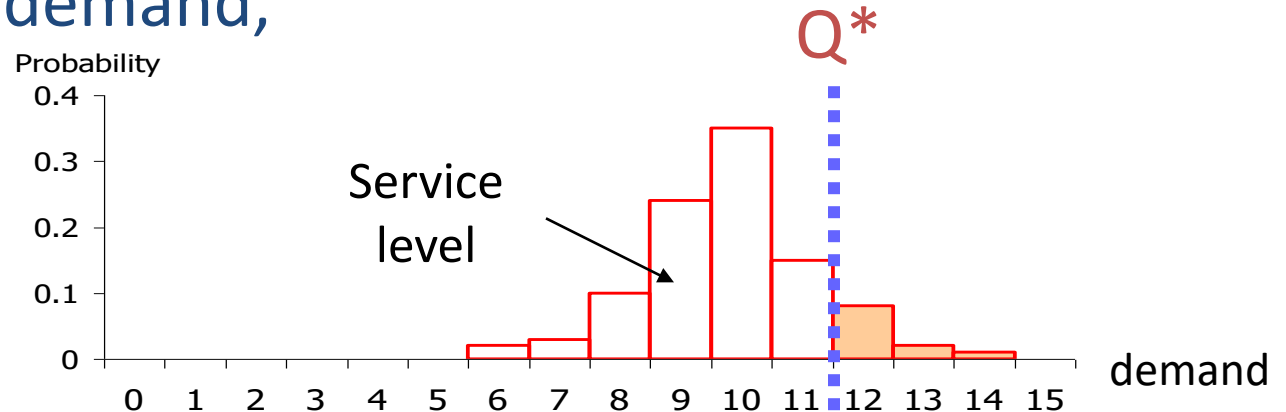
You should compute  $C_o$ :

Suppose the actual demand  $D$  is smaller than my current stock  $S$ . I could return to the past and order one unit less, then it increases my profit (or reduces my cost) by  $C_o$ .

# Newsvendor logic

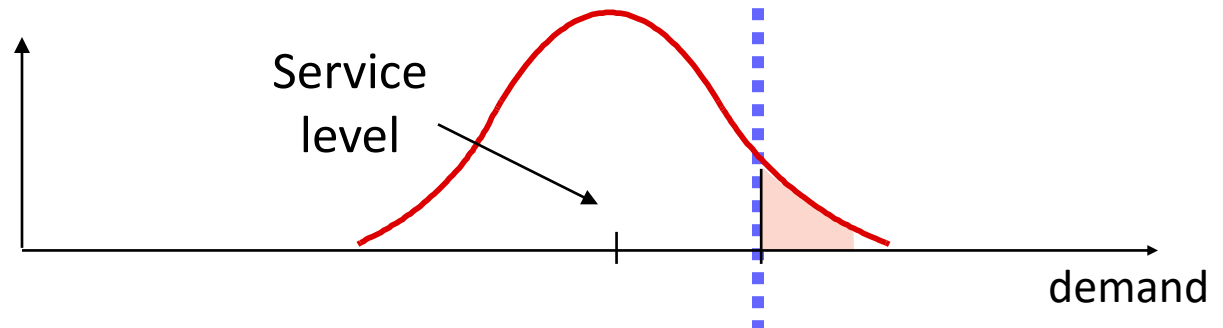
- With discrete demand,

$Q^*$  = smallest quantity such that  $SL \geq C_u / (C_u + C_o)$



- With normal demand  $N(\mu, \sigma)$

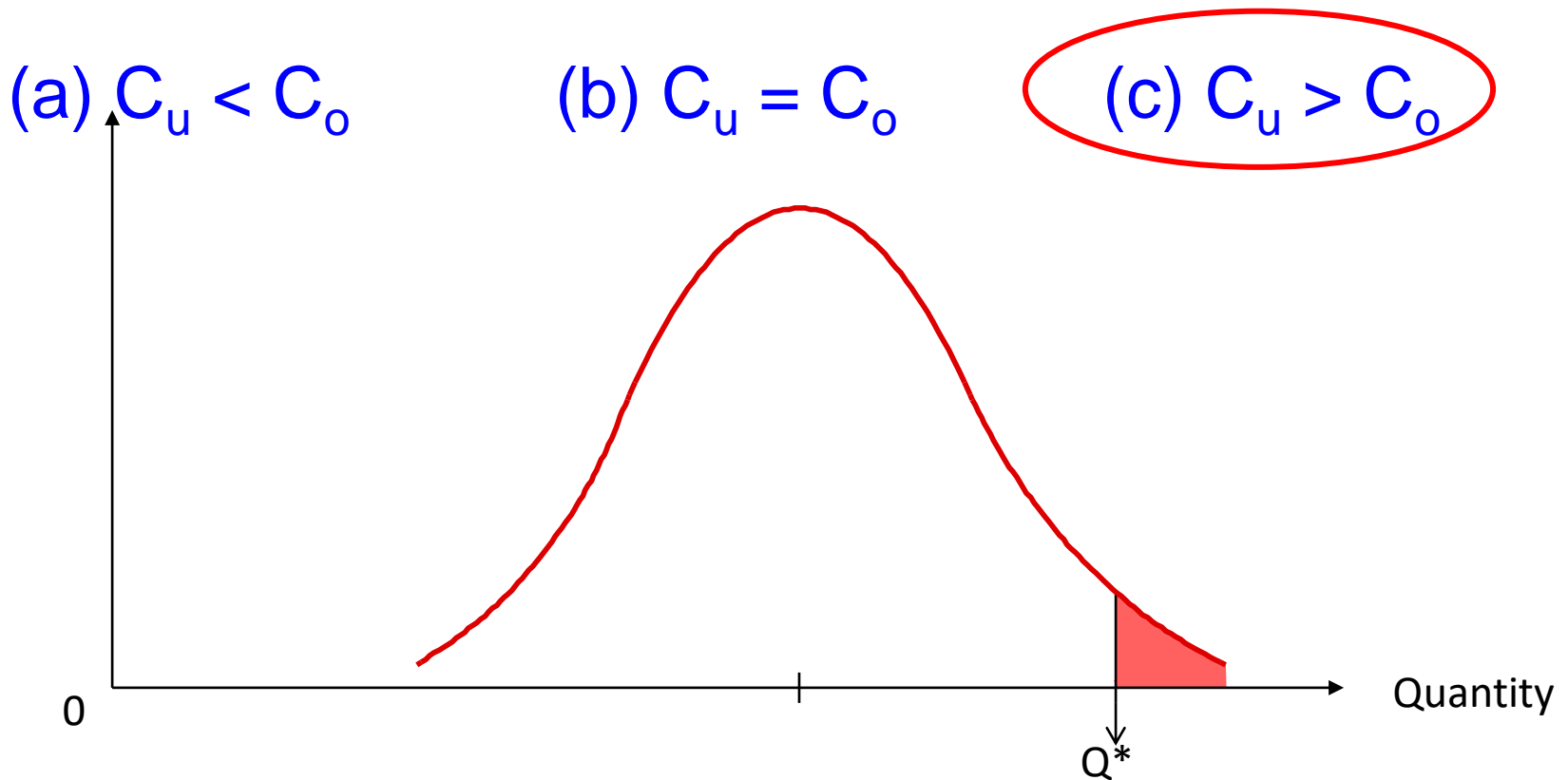
$Q^*$  = the quantity such that  $SL = C_u / (C_u + C_o)$



Step 1: find  $z$ ,  $z = \text{NORMSINV}(C_u / (C_u + C_o))$ , or use  $z$  table

Step 2: compute  $Q^*$ ,  $Q^* = \mu + z \sigma$

In the [newsvendor problem](#), suppose the demand distribution and the optimal stocking level  $Q^*$  are as shown. How do you think  $C_u$  and  $C_o$  compare?



# Example 1: Newsvendor Model with Normal Demand

For the academic year 2020-2021, demand for HKU T-shirts is normally distributed with mean 1000 and standard deviation 200.

Cost of shirts is \$10.

Selling price is \$15.

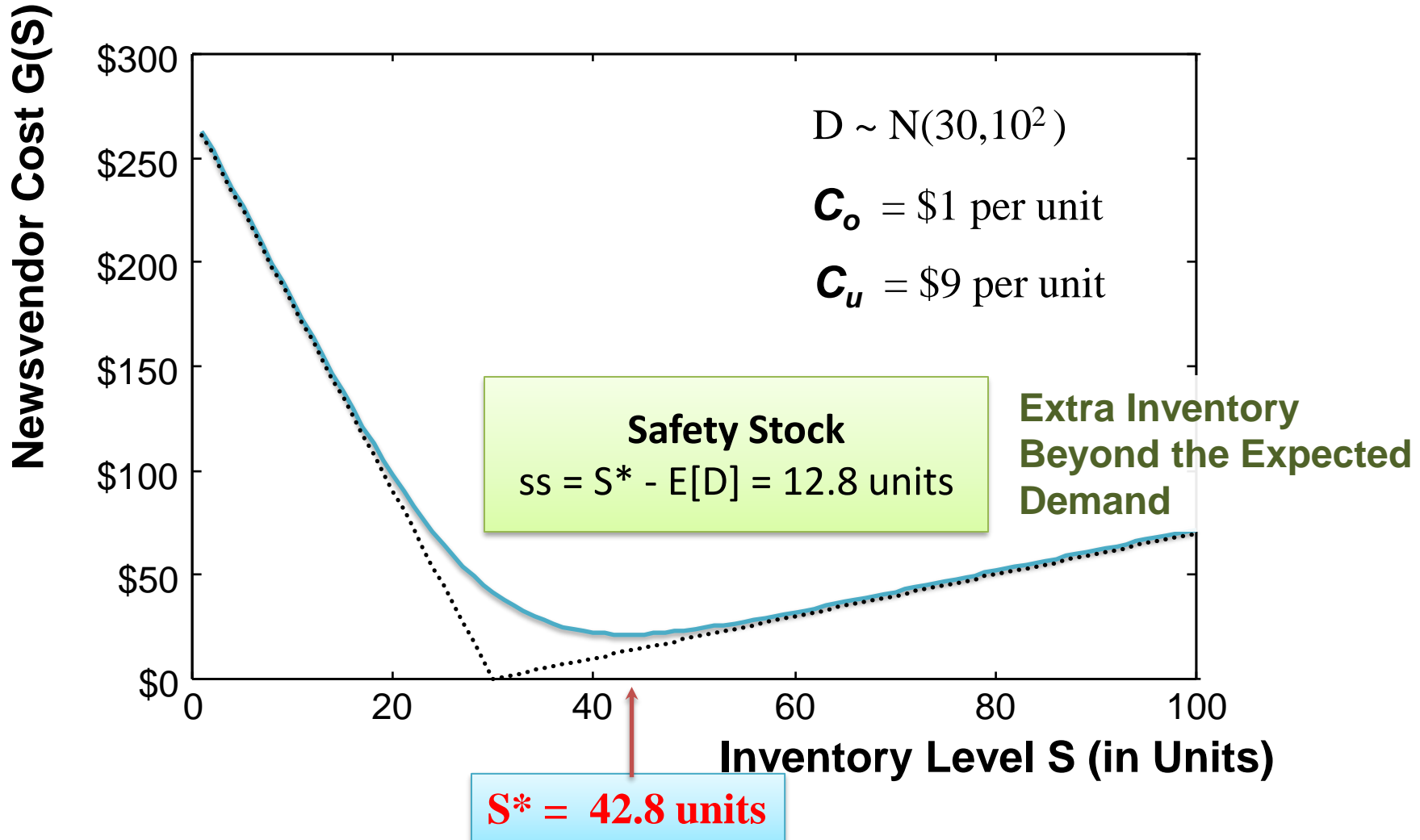
Unsold shirts can be sold off at \$8 in the summer of 2021.

How many shirts should the HKU bookstore buy for the 2020-2021 academic year?

Mean demand	1000
STD of demand	200

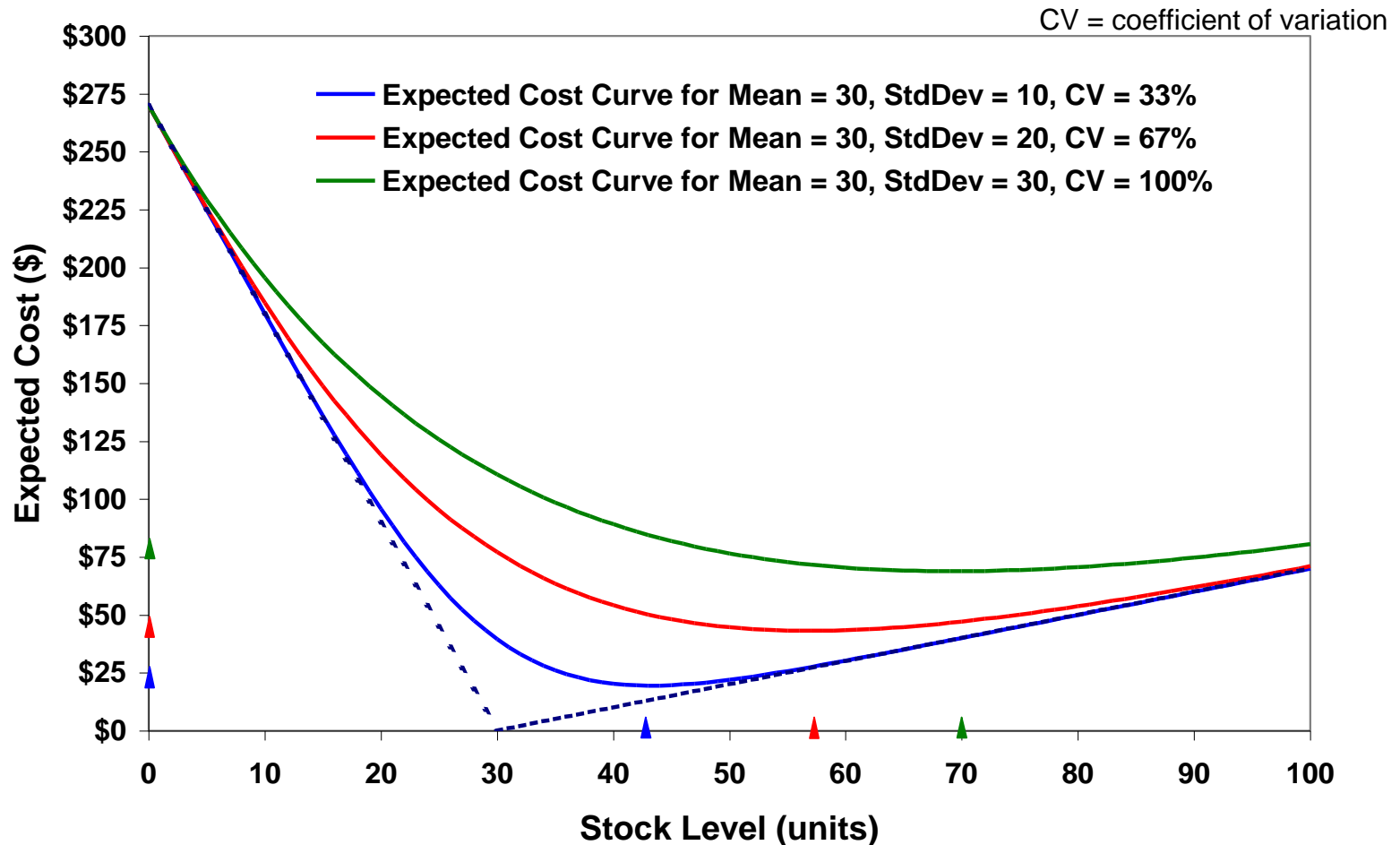
$c_o$	
$c_u$	

## Example 2: Newsvendor Model with Normal Demand



# Impact of Demand Variability

## Expected Cost Function





# Supply chain management

- Bullwhip effect
  - causes and ways to reduce the effect
- Supply chain coordination via contracts
  - Profit-sharing contract
  - Buyback contract

# Revenue management

Managing demand:

- Price-based RM
- Capacity-based RM

Applying newsvendor logic to:

- Reserving capacity
- Overbooking

# Traditional OM

- Production planning and scheduling
- Inventory management
- Warehouse & transportation



TOYOTA



# Modern OM

- Today OM studies many kinds of business processes.
- OM concerns both *manufacturing* and *service* industries.



瑪麗醫院  
Queen Mary Hospital



