MSBA 7004
Operations Analytics

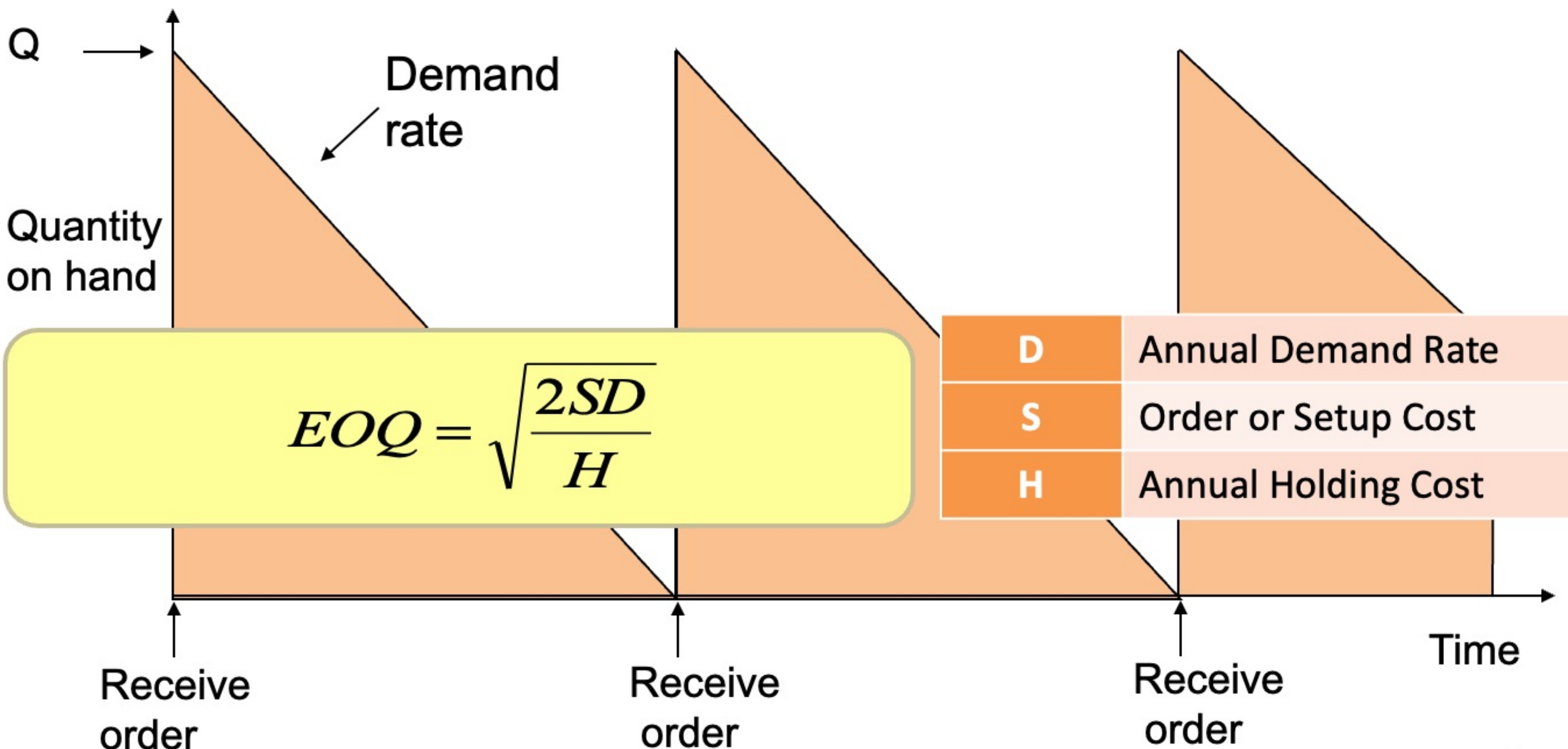
Class 6-2: Inventory Analysis (II)
Lead Time and Demand Uncertainty,
Continuous vs. Periodic Review Models
2023

Learning Objectives

- Understand the impact of <u>lead time</u> and <u>demand uncertainty</u> in inventory models
- Understand <u>continuous review</u> (or fixed-order-quantity) and <u>periodic review</u> (or fixed-time-period) models

Cycle Stocks: Tradeoff between fixed costs and holding costs

Profile of Inventory Level over Time

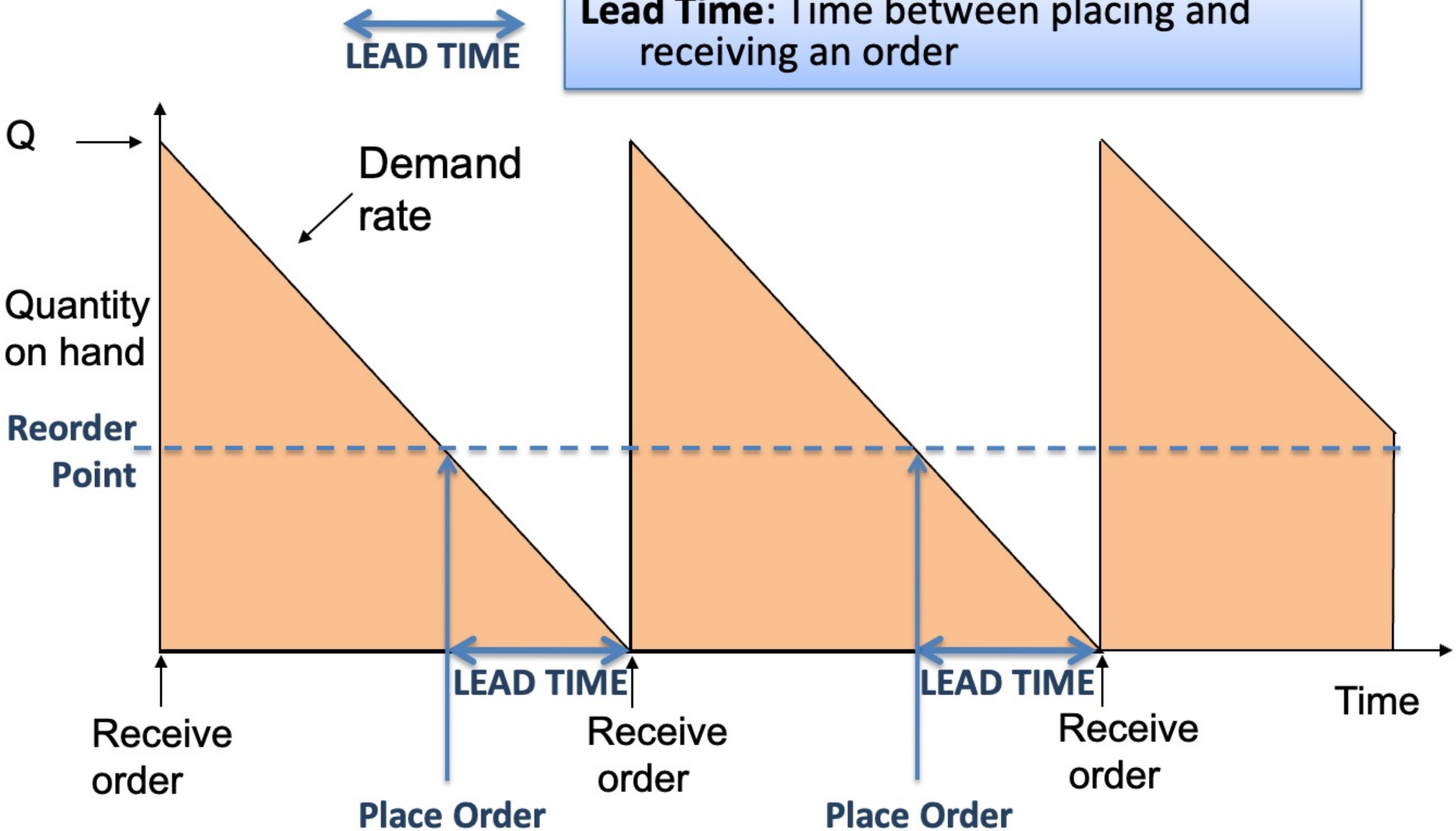


$$EOQ = \sqrt{\frac{2SD}{H}}$$

D	Annual Demand Rate
S	Order or Setup Cost
H	Annual Holding Cost

EOQ Model: With a "Lead Time"

Lead Time: Time between placing and receiving an order



Q-r (EO*Q*, *R*OP) Model: EOQ with Reorder Point

$$EOQ = \sqrt{\frac{2SD}{H}}$$

D	(Annual) Demand Rate
S	Order or Setup Cost
H	(Annual) Holding Cost

Reorder Point: ROP = D * LT

D	Demand Rate
LT	Lead Time

Valid provided LT < Cycle Time
We need a reorder point due to the lead time

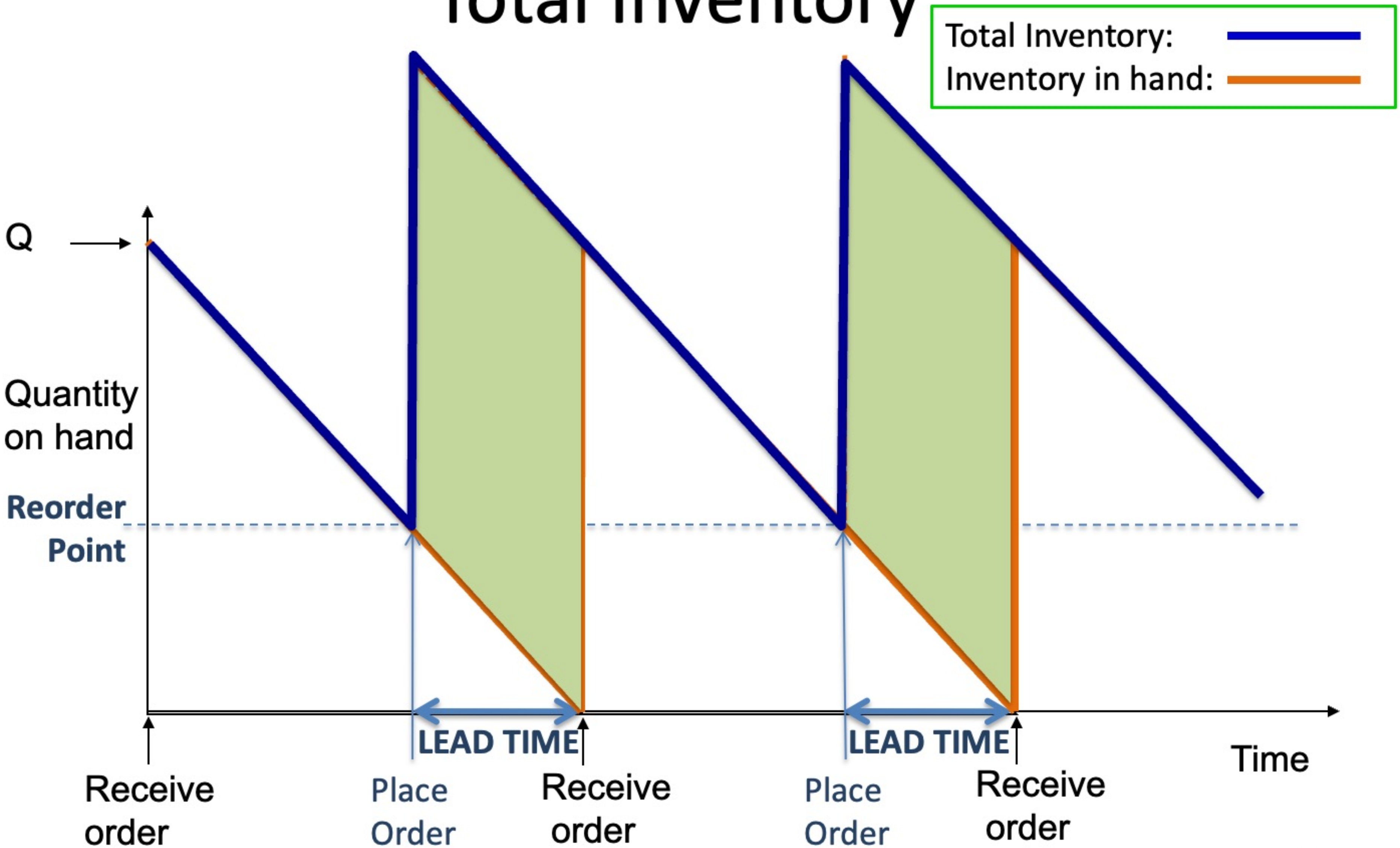
Practice Problem: QMH (1)

Queen Marry Hospital consumes 100 boxes of bandages per week. The price of bandages is $70 per box. The hospital operates 52 weeks per year. The cost of processing an order is $60, and the cost of holding one box for a year is 15% of the value of the material. What is the EOQ and the corresponding total cost?

If the lead-time is one-half week, what should be the reorder point?

D	100*52 per year
S	$60
C	$70
i	15%
H	i*C = (0.15)(70)=10.50

Cycle Stock, Pipeline Inventory, and Total Inventory



Accounting for *Pipeline* Inventory

- Pipeline inventory is the inventory “on route”

Average Pipeline Inventory

$$D \cdot LT$$

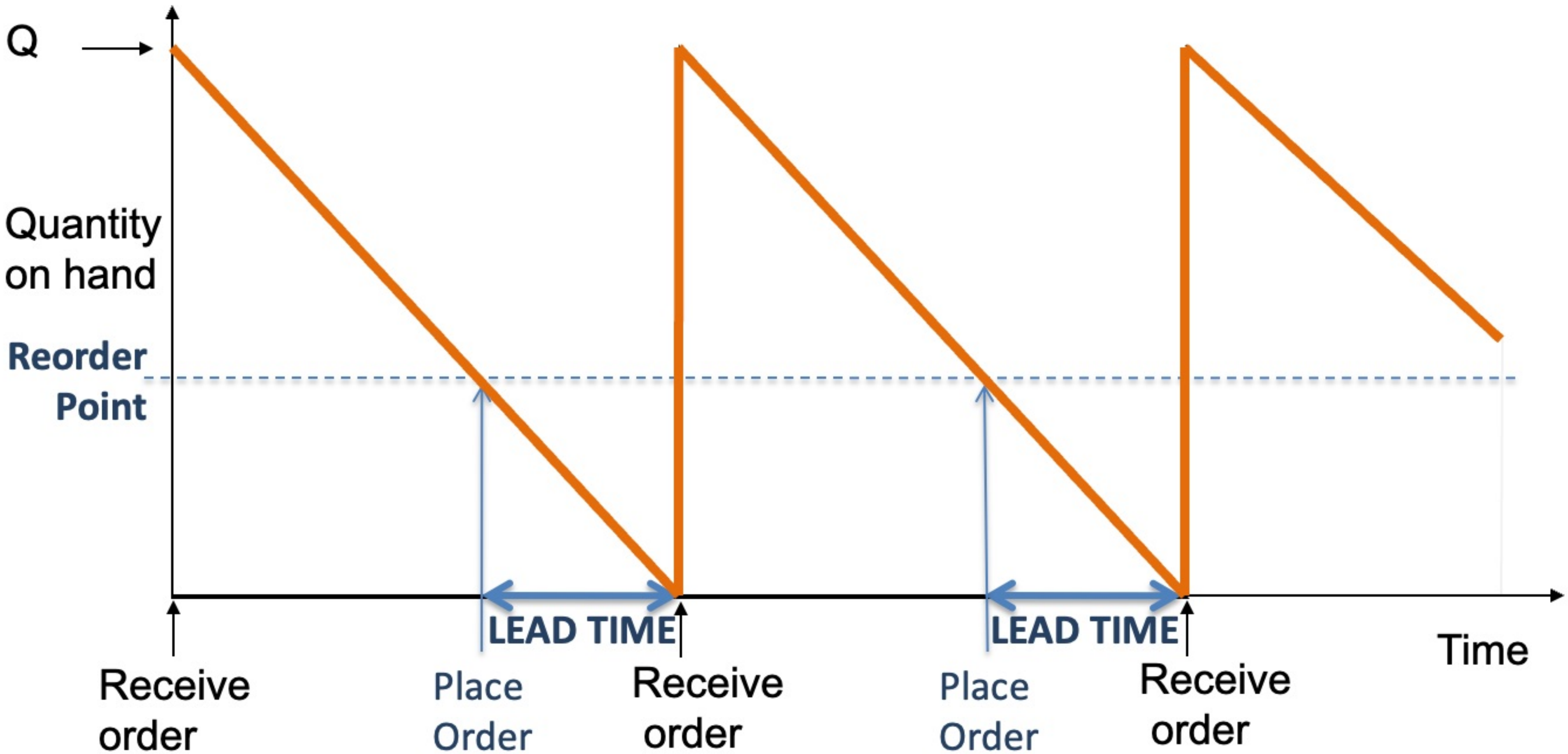
In each inventory cycle of duration Q/D, you have pipeline inventory of amount Q only in the time window of duration LT. In other periods, pipeline inventory is 0. Your average pipeline inventory: $\frac{\mathrm{LT} * \mathrm{Q} + \left(\frac{Q}{D} - LT\right) * 0}{Q/D}$=D*LT

- <u>If</u> the buyer pays for inventory when it is *shipped* from the seller, then the buyer needs to consider the cost of the pipeline inventory

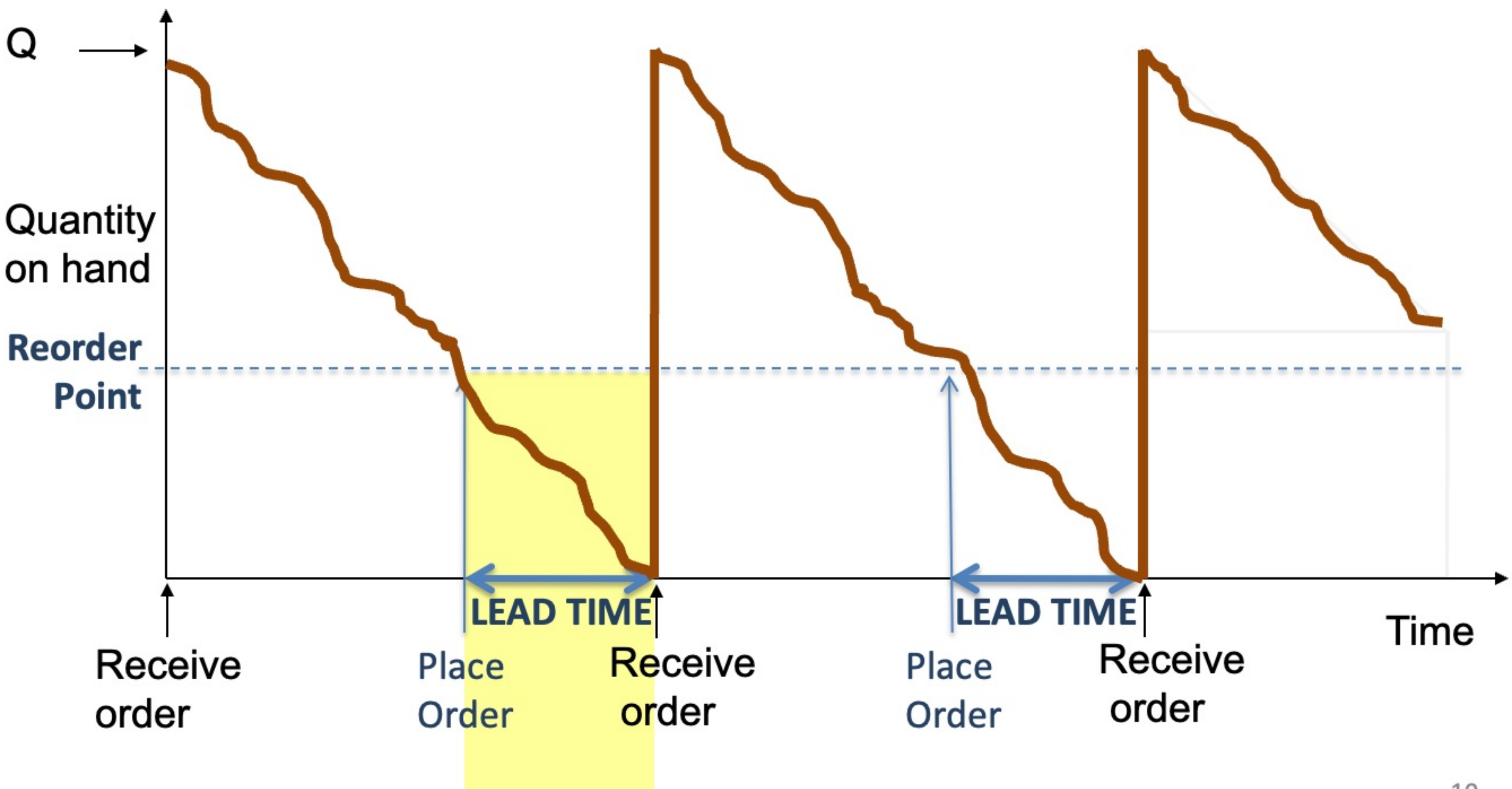
Average Total Inventory = **Average Cycle Stock (Q/2)** + **Average Pipeline Inventory (D ·LT)**

Q-r Model with Constant Demand

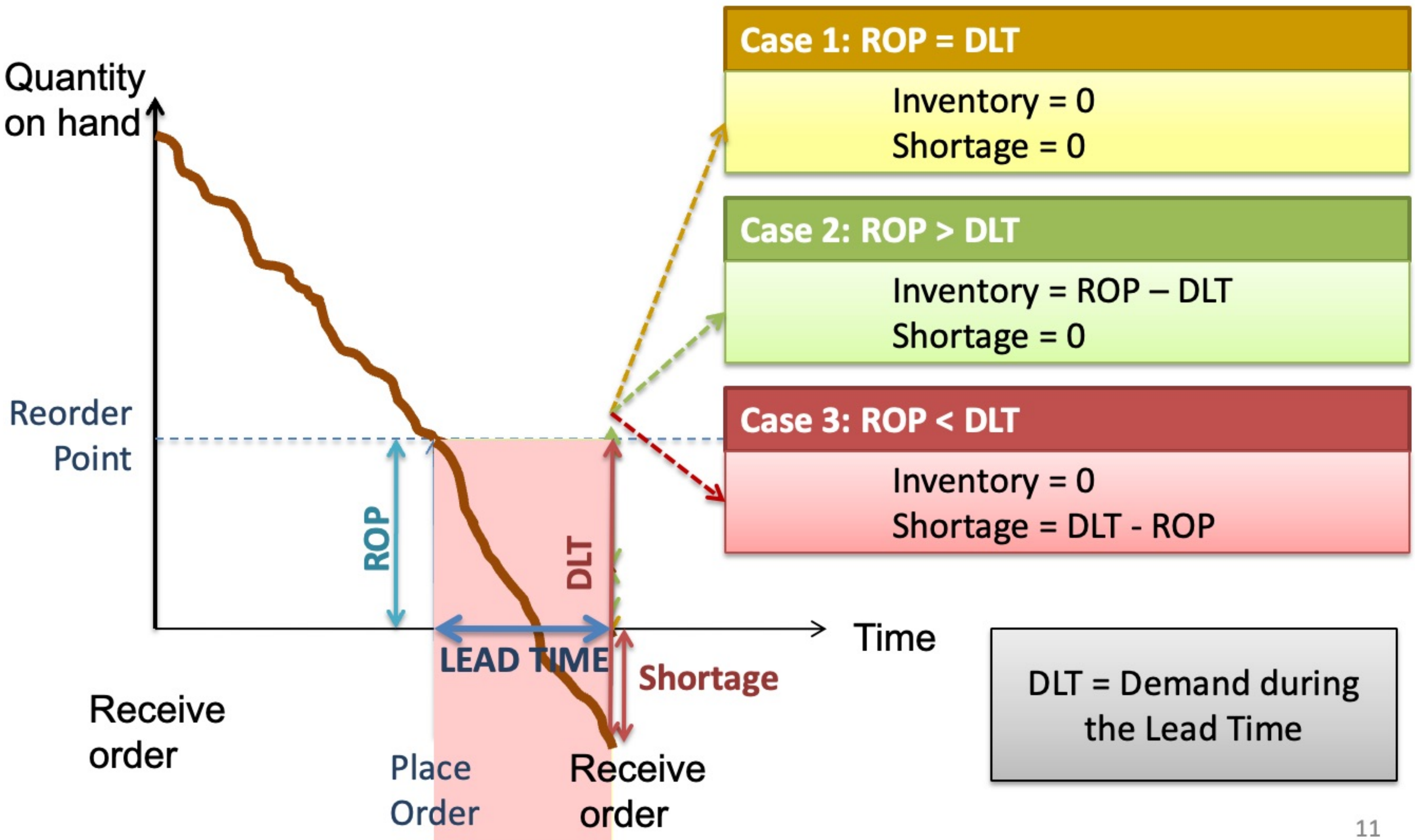
So far, we assumed demand is **constant** ...



… But what if demand is uncertain?



Demand During Lead Time: Uncertain



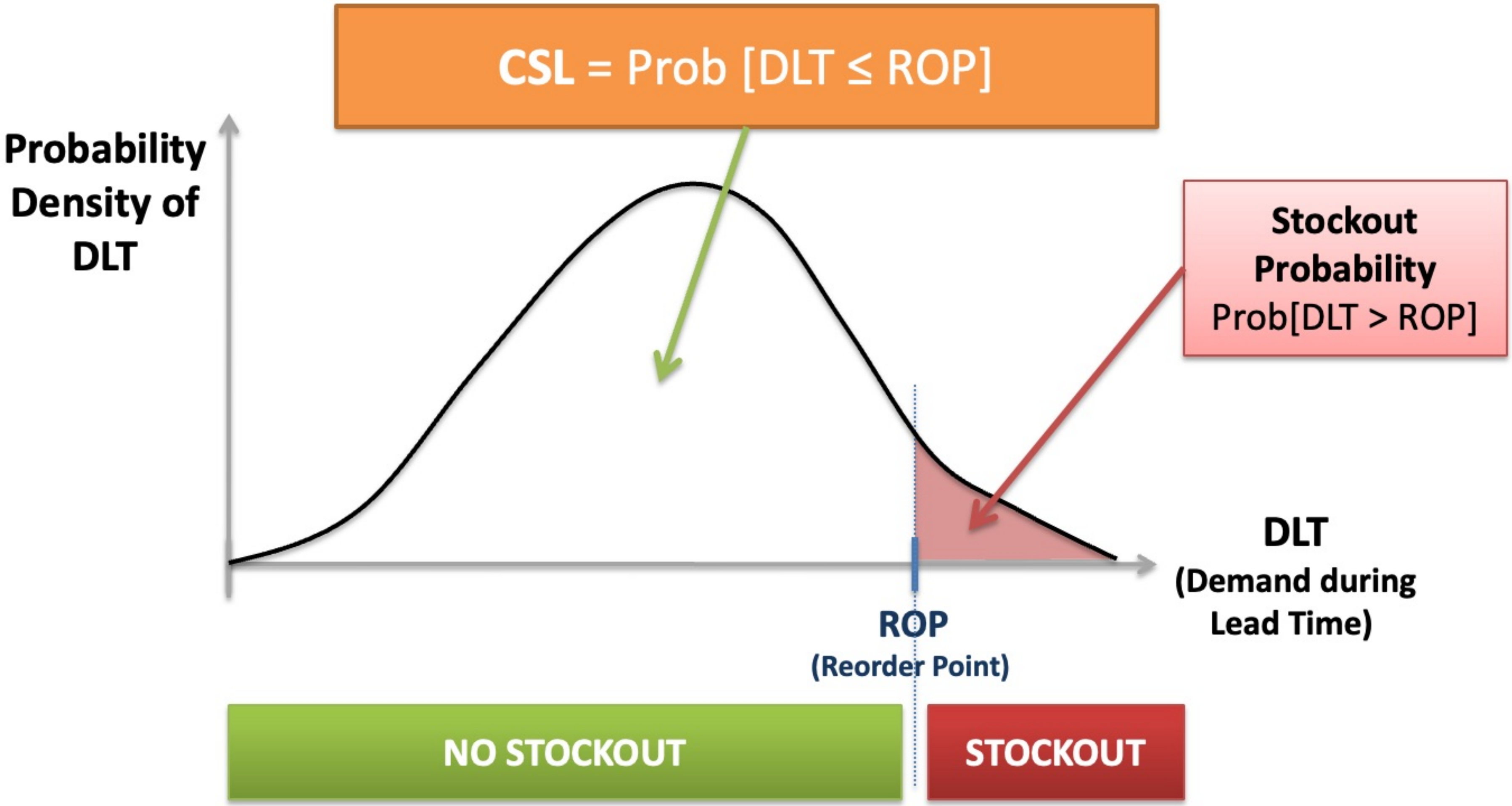
(Cycle) Service Level

- Shortage occurs if demand during lead time (DLT) exceeds reorder point (ROP)
- **Cycle Service Level** (CSL), or **Service Level** (SL)
  – Measure of reliability of the system
  – Probability of ***not*** stocking out in a cycle

CSL = Prob [DLT ≤ ROP]

- Desired service level helps us determine ROP
- Service level is a managerial decision
  – How often do you not want to stock out

How to Determine Reorder Point (ROP)?

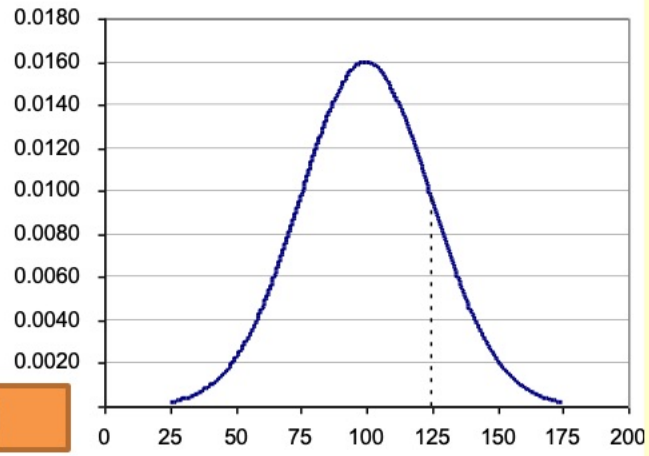


Normal Distribution Tutorial (1): Rescaling into Z-score

Start with
μ= 100,
σ= 25.

$D \sim N(\mu, \sigma^2)$

S = 125



P(D ≤ 125)

=

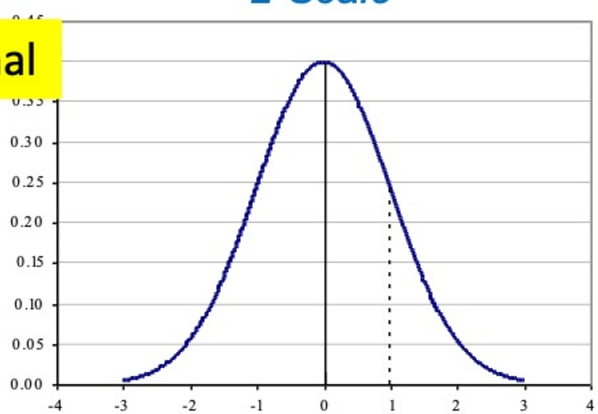
P(X ≤ 1)

z-Scale

Standard normal

$X \sim N(0,1)$

$$z = \frac{Q-\mu}{\sigma} = \frac{125-100}{25} = 1$$



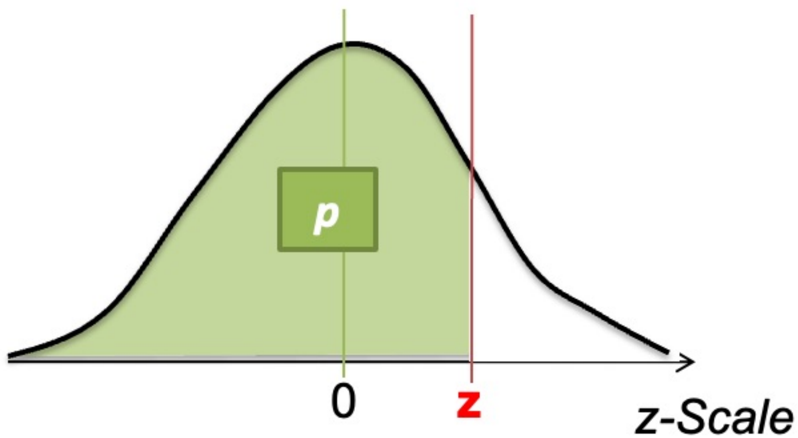
- Let Q be the order quantity, and (μ, σ) the parameters of the normal demand distribution
- *Prob*{demand is Q or lower} = *Prob*{the outcome of a standard normal is z or lower}, where

$$z = \frac{S-\mu}{\sigma} \quad \text{or} \quad S = \mu + z \times \sigma$$

- Look up *Prob*{the outcome of a standard normal is z or lower} in the Standard Normal Distribution Function Table, or NORMSDIST (excel), norm.cdf (scipy)

Normal Distribution Tutorial (2): Relationship between P and Z (using excel)

Consider the Standard Normal Distribution (in z-scale).



Given z, compute probability p using **NORMSDIST**(z), norm.cdf(z).

Given p, compute z using **NORMSINV**(p), norm.ppf(p).

Adding Normal Distributions

Summing k distributions

If $$X_i \sim N(m_i, (\sigma_i)^2)$$

then $$\sum_{i=1}^{k} X_i \sim N\left(\sum_{i=1}^{k} m_i, \sum_{i=1}^{k} (\sigma_i)^2\right)$$

Summing k *identical* distributions

If $$X_i \sim N(m, \sigma^2)$$

then $$\sum_{i=1}^{k} X_i \sim N(km, (\sqrt{k}\sigma)^2)$$

Normal Distribution Tutorial (3): Relationship between P and Z (using Z-score chart)

For a z-score of 1.13 what is the service level?

z	0	0.01	0.02	0.03	0.04	0.05
0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943

ROP and Normal Demand

Suppose weekly demand has a normal distribution: $N(m, \sigma^2)$

Suppose lead time is *k* weeks.

Then, DLT has normal distribution $N(m_{LT}, {\sigma_{LT}}^2)$

with mean $m_{LT} = k \cdot m$ and standard deviation $\sigma_{LT} = \sqrt{k} \cdot \sigma$

Given ROP, CSL is

$$P[DLT \le ROP] = P[N(0,1) \le z] = normsdist\ (z)$$ where $$z = \frac{ROP - m_{LT}}{\sigma_{LT}}$$

Given CSL, *ROP* satisfying $\mathrm{CSL} = P[DLT \le \mathrm{ROP}]$ is

$$\mathrm{ROP} = m_{LT} + z \cdot \sigma_{LT}$$ where z=*normsinv*(CSL)

Safety Stock (SS)

Safety Stock and Normal Distribution

- Safety stock is the extra inventory beyond the expected demand during LT (D*LT)
- When demand has (unpredictable) uncertainty, safety stock is held to cushion against uncertainties
- Safety stock is based on $$SS = z \cdot \sigma_{LT} = z \cdot \sqrt{k} \cdot \sigma$$
  - Desired service level (z)
  - Length of the lead time (k)
  - Demand variability (σ)

Given CSL, *SS* achieving this CSL (under Normal demand) is

$SS = z \cdot \sigma_{LT}$ where z=*normsinv*(CSL)

Summary: Q-r model with SS (due to demand uncertainty)

- To compute Q: Use the EOQ Formula $EOQ = \sqrt{\frac{2SD}{H}}$
- ROP = $m_{LT} + SS$

$SS = z\sigma_{LT}$

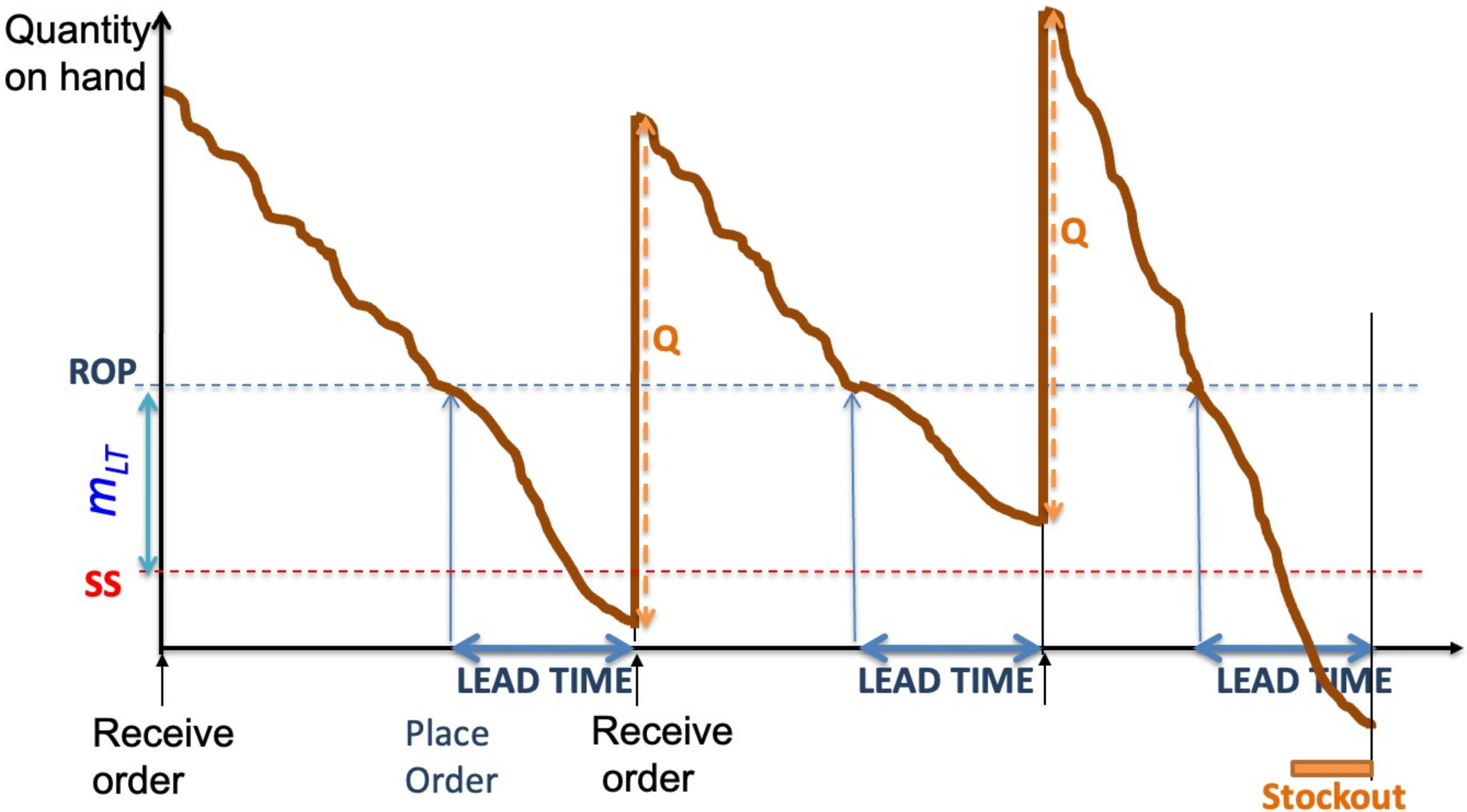
$\sigma_{LT} = \sqrt{k}\sigma$

k: Lead time in “unit time”

σ: STD of demand per “time unit”

Mean demand during lead time
(mean demand per “time unit”)*(lead time in “unit time”)
=D*k

ROP and SS



Quantity on hand
ROP
m_{LT}
SS
Q
Q
LEAD TIME
LEAD TIME
LEAD TIME
Receive order
Place Order
Receive order
Stockout

Average Inventory

If the firm pays for inventory "in stock"	If the firm pays for inventory "in stock" and "on route"
Average Inventory = Q/2 + SS	Average Inventory = Q/2 + SS + m_{LT} Pipeline Inventory

↑ "Default" case in this course
(when there is no other explanation)

Practice Problem: QMH (2)

Demand is normally distributed with mean 100 boxes of bandages per week and a standard deviation 20 boxes. The lead time is one-half week.

What safety stock is necessary if the hospital uses a 97% service level?

What should be the reorder point?

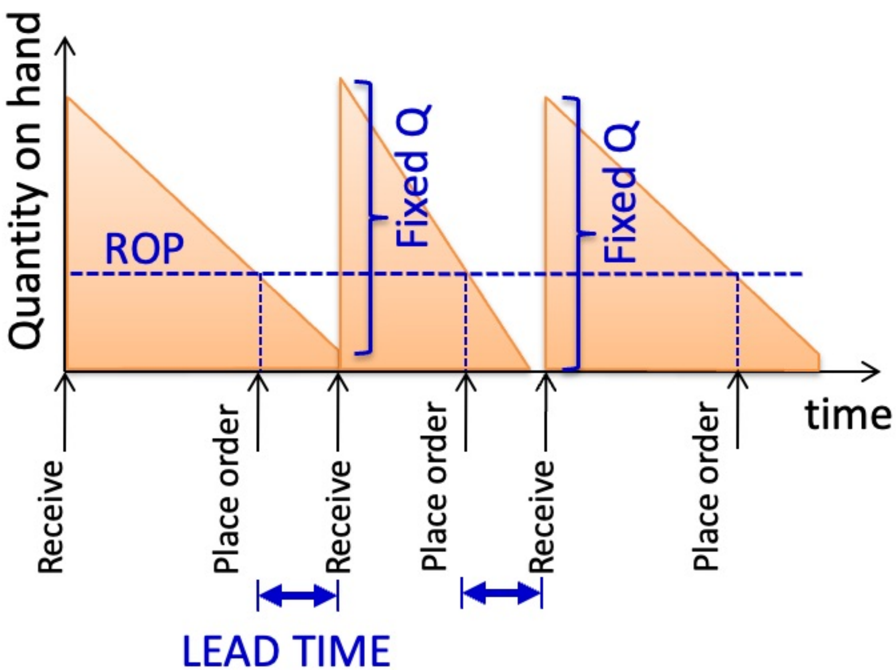
D	100 per week (average)
σ	20 for weekly demand
k	0.5 week
SL	97%

Continuous vs. Periodic Review

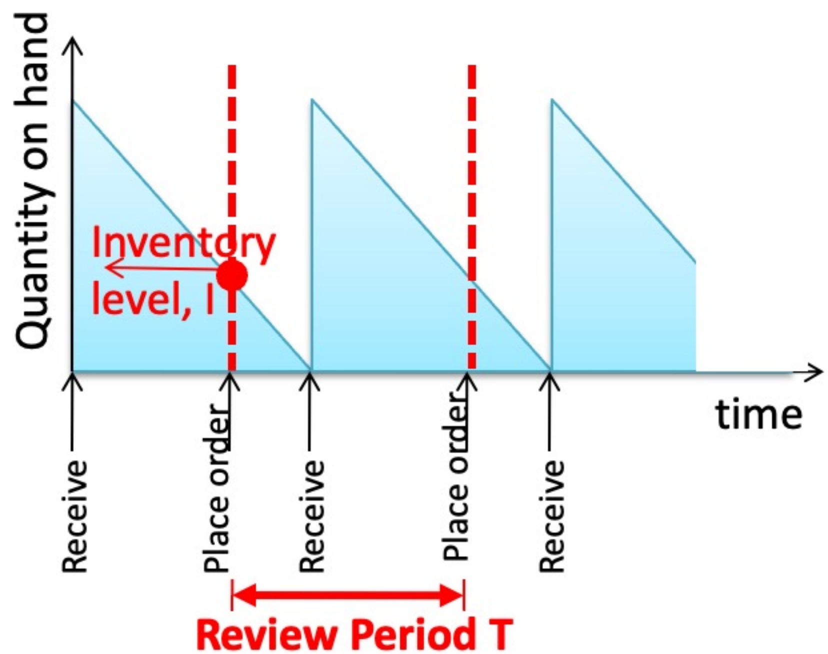
Continuous Review Systems	Periodic Review Systems
Event-triggered Example: every time the inventory falls below a reorder point **R**, order a fixed quantity **Q**	Time-triggered Inventory accounted at particular pre-determined times and ordering decision made accordingly An order quantity that usually depends on the current inventory-on-hand is ordered
Also called **fixed-order-quantity models or (Q,r) policy**	Also called **fixed-time-period models or (R,T) policy**

Continuous vs. Periodic Review

Fixed-Order-Quantity Model (Continuous Review)



Fixed-Time-Period Model (Periodic Review)



Order Size for Fixed-Time-Period Model

Assume Normal demand uncertainty

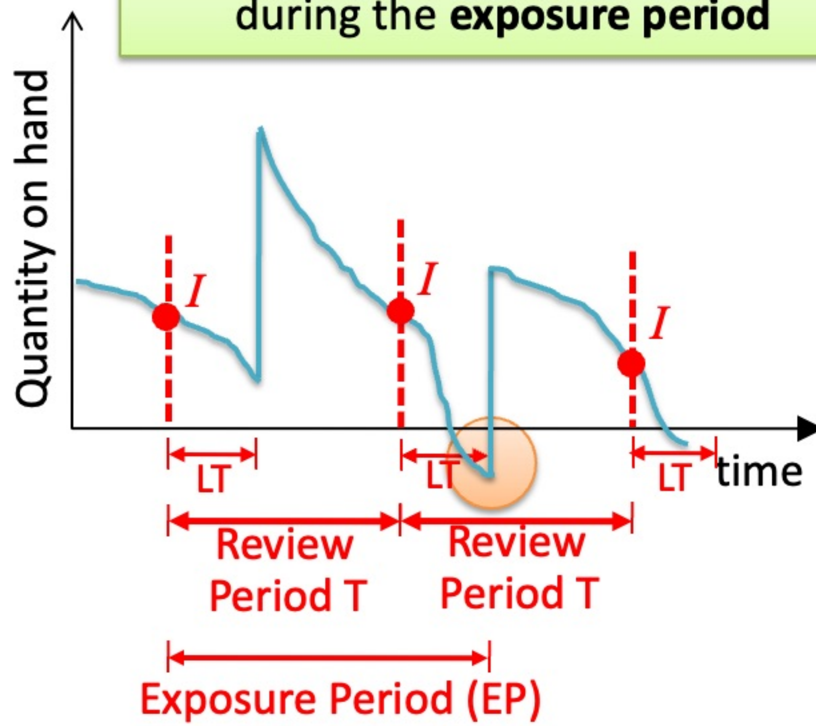
Given Cycle service level (CSL), we can find z=normsinv(CSL)

σ_{LT+T} Standard deviation of demand during the **exposure period**

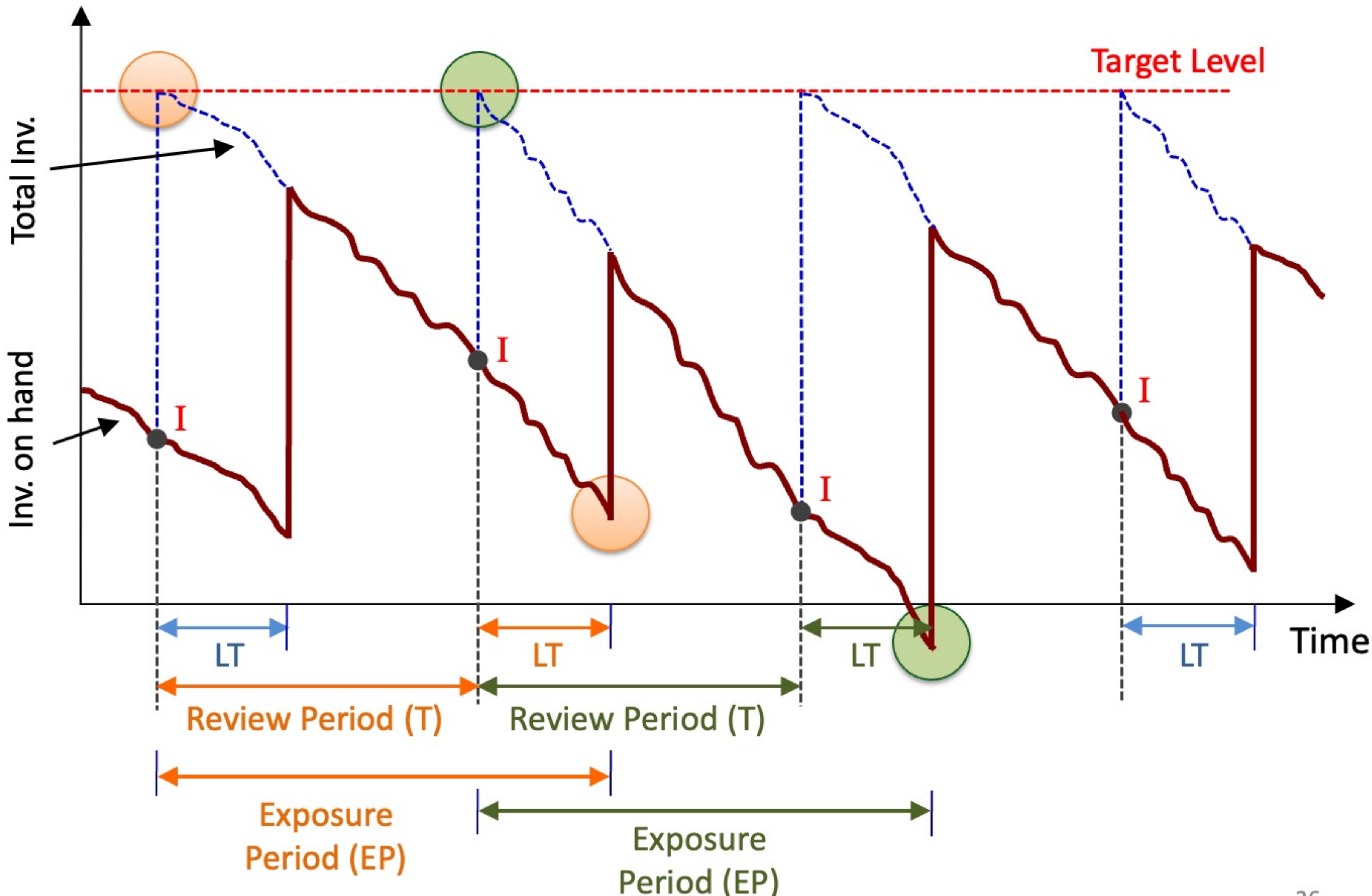
Target Level or "Inventory Position"
= (LT + T) * D + $z\sigma_{LT+T}$

Order Quantity
= Target Level - I

I	Current inventory level
T	Review Period (time between reviews)
EP	**Exposure Period** (= LT+T)



Fixed-Time-Period Model: Illustration



Example: Fixed-Time-Period Model

Average daily demand for a product is 20 units.

The review period is 30 days and lead time is 10 days.

Policy: Not stocking out 96% of the time.

At the beginning of the review period, there are 200 units in inventory.

The daily demand standard deviation is 4 units.

How many units should be ordered?

Average Inventory Comparison

	Fixed-Order-Quantity Model	Fixed-Time-Period Model
Average Pipeline Inventory	The average amount of inventory in transit (Think: Each unit sold must have come through the pipeline)	
Average Cycle Stock	The average amount of inventory that fluctuates over time because of the ordering cycle (Think: The amount of fluctuation ~= The amount ordered each cycle)	
Safety Stock	"Just in case" inventory	

LT: Lead Time

T: Review Period (Time between Reviews)

Total Cost Analysis

- Total Supply Chain Cost

  =Inventory Cost (Holding + Order) + Transportation Cost

- We can analyze annual supply chain cost

  Annual Inventory Cost + Annual Transportation Cost

- Can also look at supply chain cost per unit

  (Annual Inventory Cost + Annual Transportation Cost) divided by (Annual Sales)