MSBA7003 Quantitative Analysis Methods

Tutorial 03

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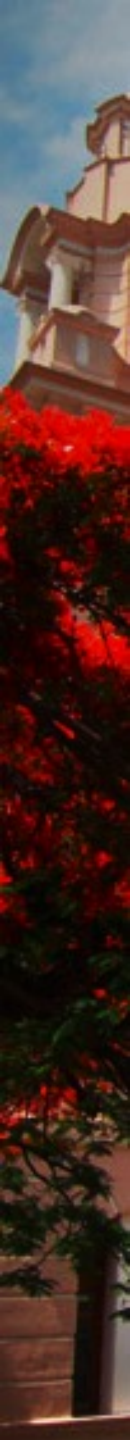
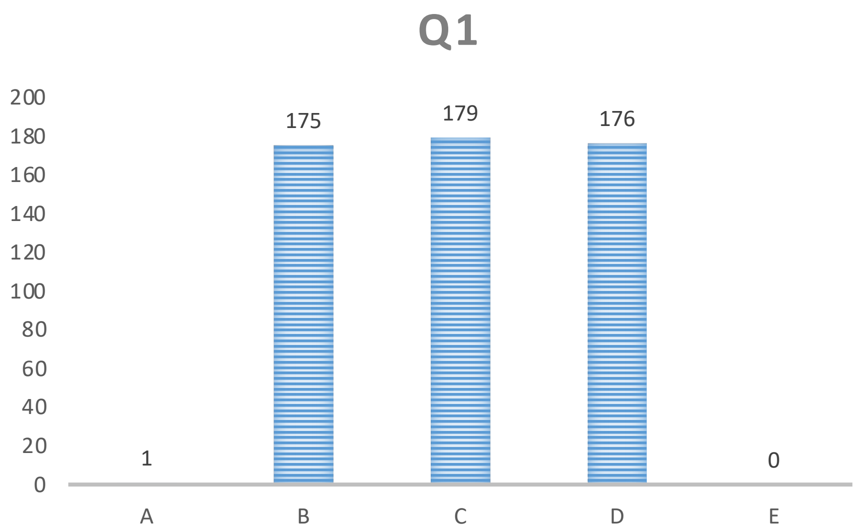
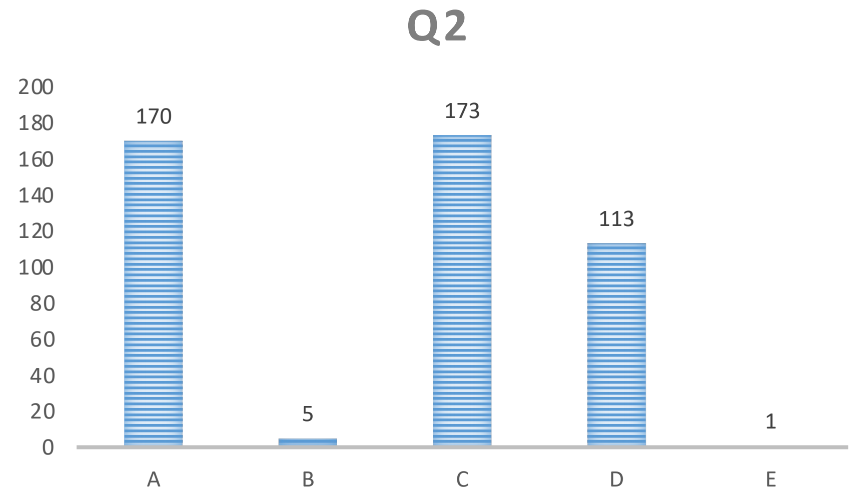
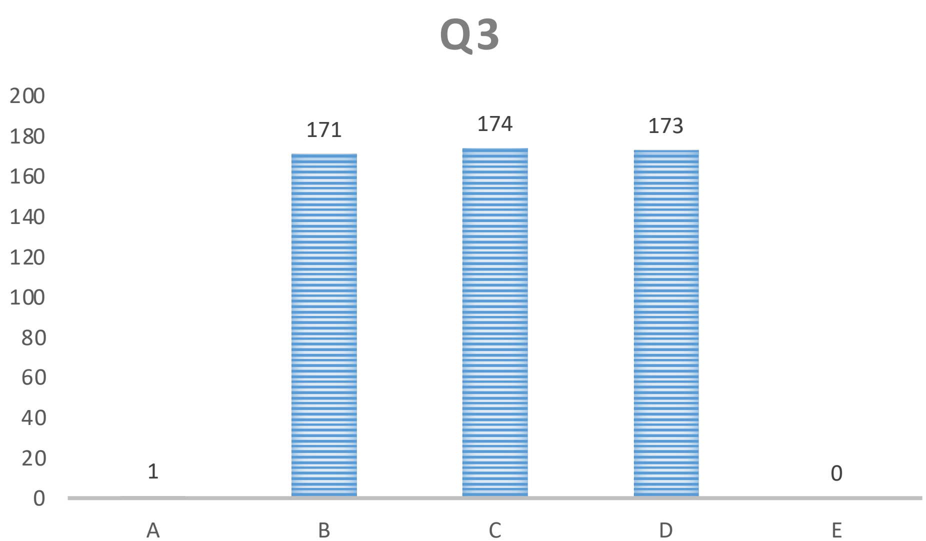


Assignment 2







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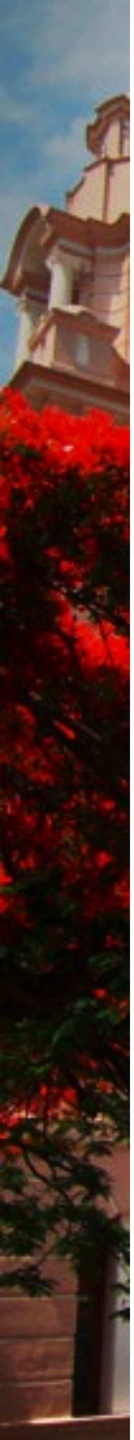
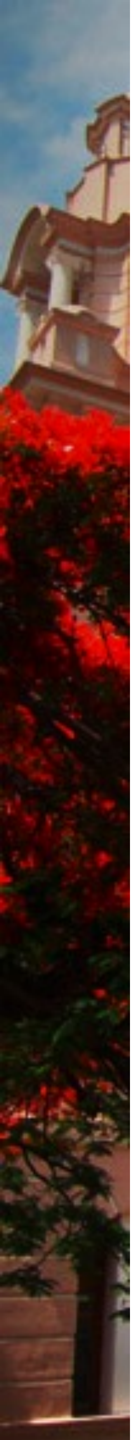


Agenda

- **Solutions to Assignment 2**
  - Q1 – Inventory Management
  - Q2 -- Simulation-Based Decision Making
  - Q3 – UCB1 strategy
- **In-Class Exercise in Session 06**
  - Pinevalley Bank
  - Salesperson
- **Linear Programming**
  - Distributing Goods through a Distribution Network



Assignment 2 Q1

Dumoor Appliance Center sells and services several brands of major appliances. Past sales for a particular model of the refrigerator have resulted in the following probability distribution for demand:

Demand per Week	0	1	2	3	4
Probability	0.10	0.20	0.30	0.35	0.05

The replenishment lead time, in weeks, is described by the following distribution:

Lead Time (weeks)	1	2	3
Probability	0.25	0. 40	0.35

Assignment 2 Q1

- Based on cost considerations as well as storage space, the company has decided to order 15 units each time. The shipping cost for each order is $30. The holding cost is $5 per week per unit that is left in inventory at the end of the week. The stock-out cost is $40 per unit. The company has decided to place an order whenever there are only two or fewer refrigerators left at the end of the week. No order can be placed when refrigerators are being shipped on the way. Simulate 10 weeks of operation for Dumoor Appliance by hand assuming that there are currently 5 units in inventory. You must use the random numbers listed in the table below to generate demand and lead time values, respectively.

Inventory Policy	Quantity (Q)：	15	Trigger (r)：	2	Costs:	Holding	Stock-Out	Shipping
						5	40	30

- Continuous inventory monitoring: (r, Q) policy

Assignment 2 Q1

Week	Order Received	Total Available	R.N.	Demand	Sales	Lost Sales	Ending Inventory	Place Order	R.N.	Lead Time
1	0	5	0.52						0.56	
2			0.37						0.45	
3			0.82						0.07	
4			0.98						0.16	
5			0.96						0.48	
6			0.33						0.61	
7			0.50						0.31	
8			0.88						0.43	
9			0.90						0.28	
10			0.06						0.31	

Assignment 2 Q1

0. We add a column to formulate the weekend of order received if placed an order.

1. At the beginning of each week, we check if an order is received.

2. We calculate the # of total available items by adding the order received to the ending inventory in the last week.

3. We calculate the demand this week according to the random number.

4. We calculate the sales, lost sales, and ending inventory according to the demand and the total available units.

5. According to the ending inventory and whether there is an order on the way, we decide whether to place an order.

6. If we place an order, we calculate the lead time according to the random number.

7. We calculate the weekend of the order received if placed an order.

Assignment 2 Q1

Week	Order Received	Total Available	R.N.	Demand	Sales	Lost Sales	Ending Inventory	Place Order	R.N.	Lead Time	Weekend of Received
1	0	5	0.52	2	2	0	3	0	0.56	--	
2	0	3	0.37	2	2	0	1	1	0.45	2	4
3	0	1	0.82	3	1	2	0	0	0.07	--	
4	0	0	0.98	4	0	4	0	0	0.16	--	
5	1	15	0.96	4	4	0	11	0	0.48	--	
6	0	11	0.33	2	2	0	9	0	0.61	--	
7	0	9	0.5	2	2	0	7	0	0.31	--	
8	0	7	0.88	3	3	0	4	0	0.43	--	
9	0	4	0.9	3	3	0	1	1	0.28	2	11
10	0	1	0.06	0	0	0	1	0	0.31	--	

A)	total sales	19
B)	total lost sales	6
C)	total number of orders	2
D)	total (not average) cost of inventory holding, ordering, and stock-out costs	485

- Please refer to “Assignment_2_solutions.xlsx”.

Assignment 2 Q2

- Three Hills provides power to a large city through a series of 200 electric generators. The company is concerned about generator failures because a breakdown costs about $75 per generator per hour.
- There is one repairperson onsite at any time and the hourly wage is $30. Management team wants to evaluate the service maintenance cost and the machine breakdown cost.
- There are two important uncertain factors.
    - On the one hand, time between successive breakdowns for any generator follows an exponential distribution with a rate of once every 400 hours or λ= 1/400. In theory, if there are n working generators, the total number of breakdowns in one hour follows a Binomial distribution with maximum number n and success probability 1 – exp(- λ).
    - On the other hand, the number of broken-down generators that can be repaired by a repairperson in one hour ranges from zero to two according to the following distribution.

Assignment 2 Q2

# of generators repaired	Probability	Cumulative Probability
0	0.28	0.28
1	0.52	0.80
2	0.20	1.00

- The cost of breakdown cost of $75 would be incurred for an hour as long as a generator is not working at the beginning of the hour, regardless of whether it is fixed or not.
- The generator breakdown in this hour would incur the cost for the following hours until it is fixed.

Assignment 2 Q2

- Simulate the operations of the system for at least 10000 hours. Which of the following is(are) true? (Assume the default setting if not otherwise stated.)

- A) The average hourly total cost with a single repairperson is about \$110 ~ \$124.

- B) The average hourly total cost with two workers with the same skills is about \$120 ~ \$130.

- C) It is better to replace the repairperson with another one who receives the same salary and whose hourly repair rate is constantly one.

- D) Consider a second worker with the same skill. He can report for duty immediately when he is called and will get the same hourly wage only when he is working. Once he is called, he must work for 4 hours in a row and take a 2-hour break before he can be called again. Whenever there are 2 or more broken generators remaining unfixed at the beginning of an hour, the company calls him if he is available. The average hourly total cost will be about \$95 ~ \$100.

- Please refer to “Assignment_2_solutions.xlsx” or “Three_Hills_simulation.py”

Assignment 2 Q2

1. At the beginning of each hour, we first check and calculate the # of working generators by using the # of total generators - # of total failed in the last hour + # of repaired in the last hour
2. We calculate # of delayed = # of waiting in the last hour - # of repaired in the last hour
3. We calculate # of failed in this hour according to the Binomial distribution.
4. We calculate # of waiting in this hour = # failed in this hour+ # delayed in this hour
5. We calculate # of W1 and W2 can repair according to the given distribution.
6. The total # of repaired = min {# of worker(s) can repair in this hour, #waiting}.
7. Calculate the Breakdown costs.

Assignment 2 Q2 Option D)

- How to formulate whether Work 2 is working?
- If in the last hour, W2 is working:
  - If he has worked for 4 hours → NO
  - If he had not worked for 4 hours→ YES
- ELSE (in the last hour, W2 is not working):
  - If at the beginning of this hour, there are 2 or more broken generators remaining unfixed
    - If he has taken a 2-hour break→ YES
    - If he has not taken a 2-hour break → NO
  - Else (at the beginning of this hour, there are NO 2 or more broken generators remaining unfixed) → NO

EXCEL Formulation for ‘W2 work’ in hour i

IF(J_{i-1} =1,IF(SUM(J_{i-4}: J_{i-1})=4,0,1),IF(C_i >=2,IF(SUM(J_{i-2} : J_{i-1})=0,1,0),0))

Pinevalley Bank Example (After-class exercise in Session 03)

- How to formulate the available time for Counter 2?
- If at the arrival time of the current customer (i), the last customer (i-1) has not finished service.
  - & in the arrival time of the next one customer (i+1), the last customer has not finished service.
  - & in the arrival time of the next two customer (i+2), the last customer has not finished service.
- The C2 Avail Time = max { the third customer’s arrival time, the completion time of C2 if opened in the previous time}

EXCEL Formulation for ‘C2 Avail Time’ in the arrival of customer i

IF(AND(C9<J8,C10<J8,C11<J8),MAX(C11,IF(H8=2,J8,IF(H7=2,J7,0))),99999999)

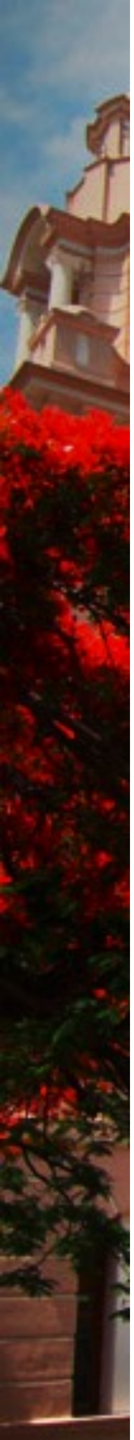
Assignment 2 Q3

- We are solving a dynamic decision-making problem for a project for which the final outcome is either success or failure. In the process of building a search tree with the Monte Carlo Tree Search algorithm to maximize the success rate (V), after the 22nd round of selection-expansion-simulation-backpropagation is finished, the first 4 rows of the table that stores the search tree are given below. The UCB1 selection strategy is used, and the constant C = 1/2.

Index	Parent	Child	Type	Note	n	V	UCB
1	-	{2, 3, 4}	Decision	Root	22	0.2727 $\frac{6}{22}$	-
2	1	{5, ⋯}	State	Option1-1	6	0.1667 $\frac{1}{6}$	0.8844
3	1	{⋯}	State	Option1-2	4	0.0000 $\frac{0}{4}$	0.8791
4	1	{⋯}	State	Option1-3	12	0.4167 $\frac{5}{12}$	0.9242

- The upper confidence bound (UCB) for node i (an option) is given by

$$B_i = V_i + \sqrt{\frac{\ln N_i}{n_i}}.$$



Assignment 2 Q3

- A) The 22nd search surely started from node 3.
- B) The 23rd search will surely start from node 4.
- C) If the 23rd search is successful, then the UCB values for nodes 2, 3, and 4 will all increase.
- D) The 24th search may start from node 2.
- E) None of the above.

Assignment 2 Q3

Index	Parent	Child	Type	Note	n	V	UCB
1	–	{2, 3, 4}	Decision	Root	22	0.2727	–
2	1	{5, …}	State	Option1–1	6	0.1667	0.8844
3	1	{…}	State	Option1–2	4	0.0000	0.8791
4	1	{…}	State	Option1–3	12	0.4167	0.9242

$\frac{6}{22}$ $\frac{1}{6}$ $\frac{0}{4}$ $\frac{5}{12}$

- If the 23rd research is successful, then n_i of node 4 would increase to 13, the n_i of nodes 2 and 3 would remain the same, and the value of N_i would increase to 23, then UCB values of node 2, 3 and 4 increase to 0.8896, 0.8853 and 0.9527 respectively.

	C =	1					success	
	Index	Parent	Child	Type	Note	n	V	UCB
	1	-	{2,3,4}	Decision	Root	23	0.3043	-
	2	1	{5,…}	State	Option1-1	6	0.1667	0.8896
	3	1	{…}	State	Option1-2	4	0.0000	0.8854
	4	1	{…}	State	Option1-3	13	0.4615	0.9527

Assignment 2 Q3

Index	Parent	Child	Type	Note	n	V	UCB
1	–	{2, 3, 4}	Decision	Root	22	0.2727	–
2	1	{5, …}	State	Option1-1	6	0.1667	0.8844
3	1	{…}	State	Option1-2	4	0.0000	0.8791
4	1	{…}	State	Option1-3	12	0.4167	0.9242

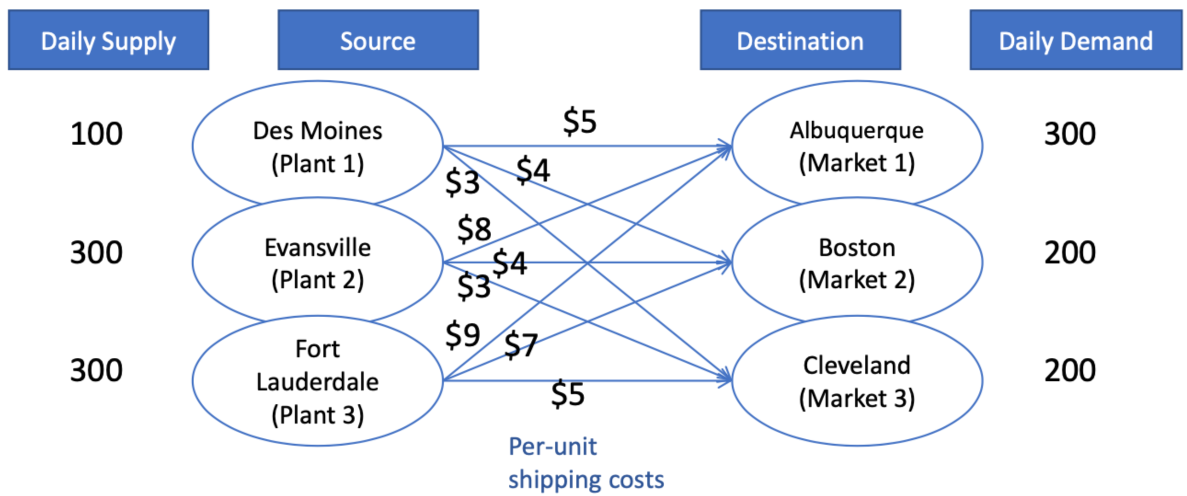
$\frac{6}{22}$ $\frac{1}{6}$ $\frac{0}{4}$ $\frac{5}{12}$

- If the 23 research is not successful, we can obtain the updated UBC values of nodes 2, 3, and 4 equal to 0.8896, 0.8853, and 0.8757, respectively.

C =	1					Failure	
Index	Parent	Child	Type	Note	n	V	UCB
1	-	{2,3,4}	Decision	Root	23	0.2609	-
2	1	{5,…}	State	Option1-1	6	0.1667	0.8896
3	1	{…}	State	Option1-2	4	0.0000	0.8854
4	1	{…}	State	Option1-3	13	0.3846	0.8757

Transportation Problem

- The Executive Furniture Corporation is faced with the following transportation problem and is trying to minimize the daily transportation cost. How to optimize the shipping plan, while the demand must be satisfied?



- Reconsider the Transportation Problem. Suppose each source can supply more than its capacity by outsourcing at price $p_i = 3$, and demand can be unsatisfied with a revenue loss of $r_j = 10$ per unit. What is the optimal outsourcing and transportation plan? Formulate and solve the LP.

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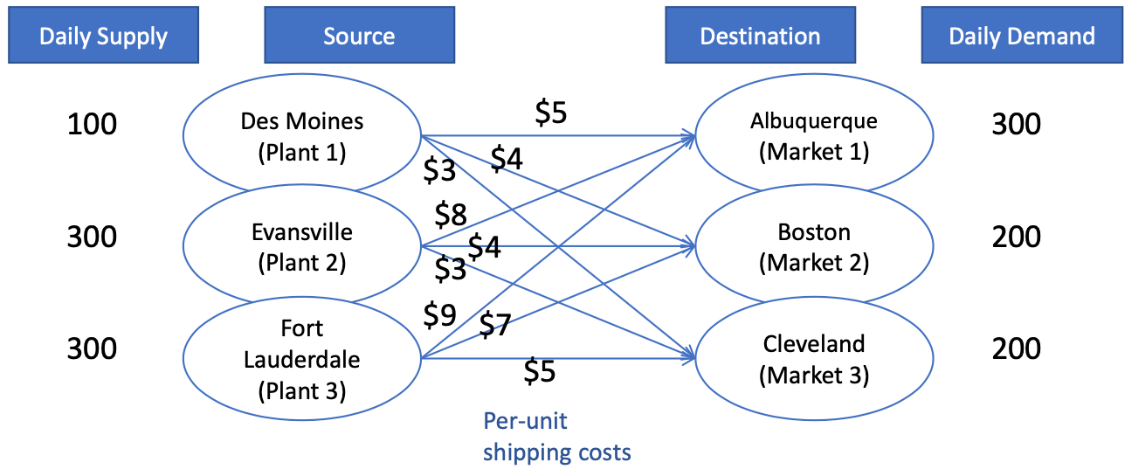
Transportation Problem



Suppose each source can supply more than its capacity by outsourcing at price $p_i = 3$, and demand can be unsatisfied with a revenue loss of $r_j = 10$ per unit.

- $\min\limits_{x_{ij} \geq 0} \quad \sum_{i,j} c_{ij} \cdot x_{ij} + \sum_i p_i \cdot \max\{0, \sum_j x_{ij} - s_i\} + \sum_j r_j \cdot \max\{0, d_j - \sum_i x_{ij}\}$
- Introduce $z_i \geq 0$ and $w_j \geq 0$ such that
  - $z_i \geq 0$ and $z_i \geq \sum_j x_{ij} - s_i$
  - $w_j \geq 0$ and $w_j \geq d_j - \sum_i x_{ij}$
  - Objective $= \sum_{i,j} c_{ij} \cdot x_{ij} + \sum_i p_i \cdot z_i + \sum_j r_j \cdot w_j$.

Transportation Problem

- $\min_{x_{ij}\geq 0} \quad \sum_{i,j} c_{ij}\cdot x_{ij} + \sum_i p_i \cdot \max\{0, \sum_j x_{ij} - s_i\} + \sum_j r_j \cdot \max\{0, d_j - \sum_i x_{ij}\}$
- Introduce $z_i \geq 0$ and $w_j \geq 0$ such that
  - $z_i \geq 0$ and $z_i \geq \sum_j x_{ij} - s_i$
  - $w_j \geq 0$ and $w_j \geq d_j - \sum_i x_{ij}$
  - Objective = $\sum_{i,j} c_{ij}\cdot x_{ij} + \sum_i p_i \cdot z_i + \sum_j r_j \cdot w_j$.

Executive Furniture Corporation (Transportation)							
	Outsourcing Quantity	600	0	0			
	Outsourcing Price	3	3	3			
	Source	Des Moines	Evansville	Fort Lauderdale	Ship to Sum		Lost Sales
Destination	Albuquerque	300	0	0	300		0
	Boston	200	0	0	200		0
	Cleveland	200	0	0	200		0
	Ship from Sum	700	0	0			
Model Parameters		Des Moines	Evansville	Fort Lauderdale	Demand		Loss per unit
	Albuquerque	$ 5.00	$ 8.00	$ 9.00	300		$ 10.00
	Boston	$ 4.00	$ 4.00	$ 7.00	200		$ 10.00
	Cleveland	$ 3.00	$ 3.00	$ 5.00	200		$ 10.00
	Supply	100	300	300	Demand Surplus		
	Supply Deficit	600	-300	-300	0		
					0		
Total Cost	$ 4,700.00				0		

- Please refer to “Solution to in-class exercise(Session 06).xlsx ”

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Salesperson

- Your company HR plans to develop a model to identify inefficient salespeople. Given the following data, which worker(s) could be labelled as being inefficient?

Salesperson	Inputs			Outputs		
	Expense ($k)	Years of Experience	# of Days Working	Lead Conversion	Sales Value ($k)	# of Clients
Jimmy	34	8	288	0.82	550	5
Mike	30	5	330	0.67	400	3
Shirley	52	4	340	0.92	680	4

- Please refer to “Solution to in-class exercise(Session 06).xlsx”

Constant Returns To Scale

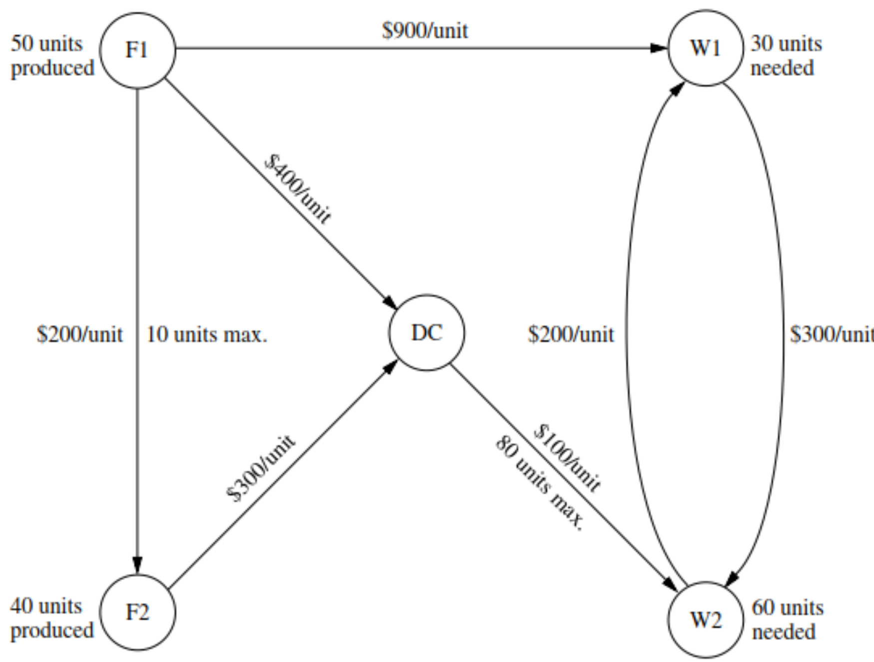
	Inputs			Outputs			
Salesperson	Expense ($k)	Years of Experience	# of Days Working	Lead Conversion	Sales Value ($k)	# of Clients	
Jimmy	34	8	288	0.82	550	5	
Mike	30	5	330	0.67	400	3	
Shirley	52	4	340	0.92	680	4	
Weights	0.0294118	0	0	0	0	0.2	Jimmy
	0.0237926	0.057244535	0	1.06761566	0.0007117	0	Mike
	0.0170168	0.028781513	0	0	0.0014706	0	Shirley
Jimmy Evaluation:							
Branch	TOV		TIV		Efficiency		
A	1	<=	1	Fixed as 1	100%		
B	0.6	<=	0.88235294		68%		
C	0.8	<=	1.52941176		52%		

Non-constant Returns To Scale

Jimmy Evaluation							
	Inputs			Outputs			
Salesperson	Expense ($k)	Years of Experience	# of Days Working	Lead Conversion	Sales Value ($k)	# of Clients	Weights
Jimmy	34	8	288	0.82	550	5	1
Mike	30	5	330	0.67	400	3	0
Shirley	52	4	340	0.92	680	4	1.3878E-16
Virtual	34	8	288	0.82	550	5	
				Constraint 1: Weights sum up to 1			1
Constraint 2: Lower inputs				Constraint 3: Higher than k-multiple outputs			
	<=	<=	<=	>=	>=	>=	Multiple k
Jimmy	34	8	288	0.82	550	5	1

Distributing Goods

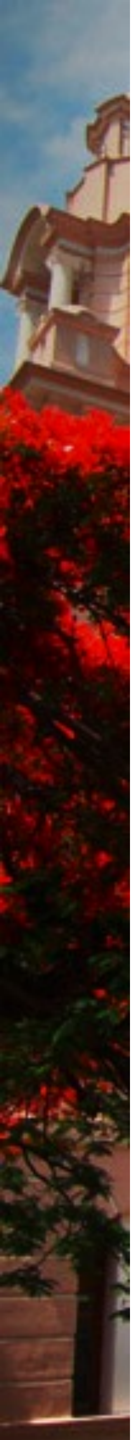
- The DISTRIBUTION UNLIMITED CO. will be producing the same new product at two different factories. Then the product must be shipped to two warehouses, where either factory can supply either warehouse.



- The decision to be made concerns how much to ship through each shipping lane. The objective is to minimize the total shipping cost.

Distributing Goods

- Decision variables
  - $x_{F1-F2}, x_{F1-DC}, x_{F1-W1}, x_{F2-DC}, x_{DC-W2}, x_{W1-W2}, x_{W2-W1}$: the amounts shipped through the respective lanes
- Constraints
  - Upper-bound constraints: $x_{F1-F2} \leq 10$ and $x_{DC-W2} \leq 80$
  - Net flow constraints:
    - Factory: amount shipped out - amount shipped in = amount produced
    - DC: amount shipped out = amount shipped in
    - Warehouse: amount shipped in - amount shipped out = amount needed
  - Nonnegativity constraints
- Objective
  - To minimize the total shipping cost



Distributing Goods

- Programming model

$$
\begin{aligned}
\text{Minimize} \quad Z = {} & 2x_{\text{F1-F2}} + 4x_{\text{F1-DC}} + 9x_{\text{F1-W1}} + 3x_{\text{F2-DC}} + x_{\text{DC-W2}} \\
& + 3x_{\text{W1-W2}} + 2x_{\text{W2-W1}},
\end{aligned}
$$

subject to the following constraints:

1. Net flow constraints:

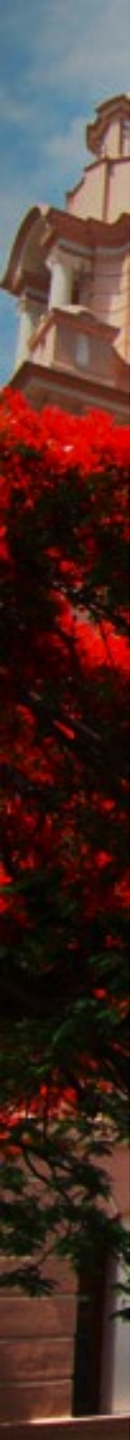
$$
\begin{array}{lllllllll}
x_{\text{F1-F2}} & + x_{\text{F1-DC}} & + x_{\text{F1-W1}} & & & & & = & 50 \text{ (factory 1)} \\
-x_{\text{F1-F2}} & & & + x_{\text{F2-DC}} & & & & = & 40 \text{ (factory 2)} \\
& - x_{\text{F1-DC}} & & - x_{\text{F2-DC}} & + x_{\text{DC-W2}} & & & = & 0 \text{ (distribution center)} \\
& & - x_{\text{F1-W1}} & & & + x_{\text{W1-W2}} & - x_{\text{W2-W1}} & = & -30 \text{ (warehouse 1)} \\
& & & & - x_{\text{DC-W2}} & - x_{\text{W1-W2}} & + x_{\text{W2-W1}} & = & -60 \text{ (warehouse 2)}
\end{array}
$$

2. Upper-bound constraints:

$$
x_{\text{F1-F2}} \le 10, \qquad x_{\text{DC-W2}} \le 80
$$

3. Nonnegativity constraints:

$$
\begin{gathered}
x_{\text{F1-F2}} \ge 0, \quad x_{\text{F1-DC}} \ge 0, \quad x_{\text{F1-W1}} \ge 0, \quad x_{\text{F2-DC}} \ge 0, \quad x_{\text{DC-W2}} \ge 0, \\
x_{\text{W1-W2}} \ge 0, \quad x_{\text{W2-W1}} \ge 0.
\end{gathered}
$$



Distributing Goods

- Final solution

$$x_{\text{F1-F2}} = 0, \quad x_{\text{F1-DC}} = 40, \quad x_{\text{F1-W1}} = 10, \quad x_{\text{F2-DC}} = 40, \quad x_{\text{DC-W2}} = 80,$$
$$x_{\text{W1-W2}} = 0, \quad x_{\text{W2-W1}} = 20.$$

- The resulting total shipping cost is $49,000.

	F1-F2	F1-DC	F1-W1	F2-DC	DC-W2	W1-W2	W2-W1		total shipping cost	
Decision Variables	0	40	10	40	80	0	20		490	
coeff.	2	4	9	3	1	3	2			
Constraints										
	1	1	1					50	=	50
	-1			1				40	=	40
		-1		-1	1			0	=	0
			-1			1	-1	-30	=	-30
					-1	-1	1	-60	=	-60
	1							0	<=	10
					1			80	<=	80

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