MSBA7003 Quantitative Analysis Methods

Tutorial 01

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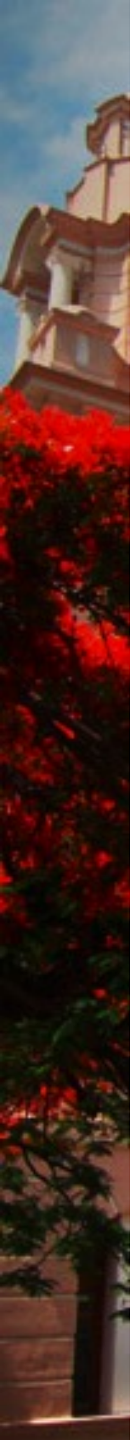

Class notes

- The template of answer sheet

					1
					2
					3
					4
					5
A	B	C	D	E	Number of the Question
Name:					
University Number:					

1					1
	1				2
		1			3
			1	1	4
1		1			5
A	B	C	D	E	Number of the Question
Name:	Zhang San				
University Nu	3030030300				
Please fill the name into cell "B8" with the surname name in front of the given name.					
Please fill the UID into cell "B9".					
Please fill in 1 in the corresponding cells according to chocies for each question.					
In this example, choices for each problems are as follows:					
For question 1, "A" is chosen.					
For question 2, "B" is chosen.					
For question 3, "C" is chosen.					
For question 4, "D" and "E" are chosen.					
For question 5, "A" and "C" are chosen.					

- Grading policy of assignment
  - The correct answer may consist of one or more options.
  - Full marks for the correct answer and zero otherwise.

- DO NOT
  - Need to submit a pdf file with your solutions
  - Zip the xlsx file
  - Submit the xlsm file (empty file when opening).
  - Trust ChatGPT

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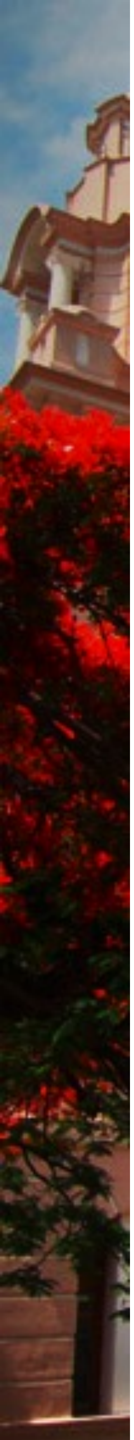


Tutorial

- Explanation of assignments and after-class exercise
- More exercises
  - Similar to in/after-class exercises and assignments
- Software
  - Python is most used
  - R, Excel(English Version)

Agenda

- Conditional Probability & Independence
  - Coin tosses examples
  - After-class exercises

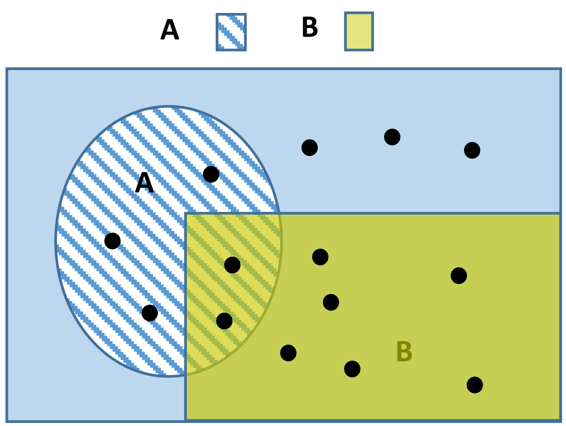
- Bayesian learning and decision making
  - Café du Donut

- Coding for Bayesian Update
  - New Product Pricing with Limited Data
  - Toss coins in Python

Conditional Probability

The idea of conditioning

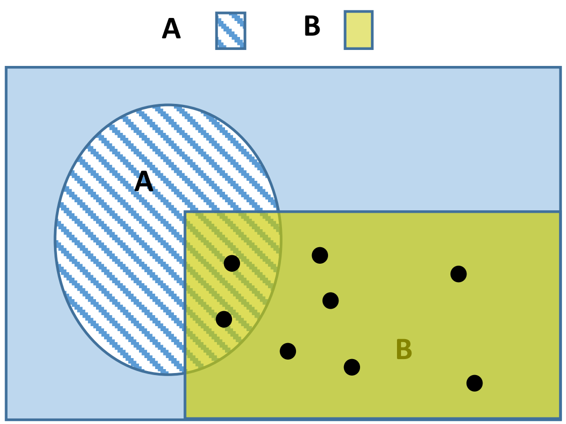
Assume 14 equally likely outcomes



$$Pr(A) = \frac{5}{14} \quad Pr(B) = \frac{8}{14}$$

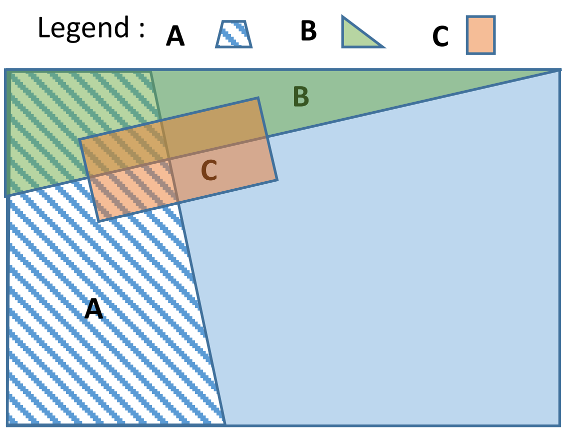
Use new information to revise a model

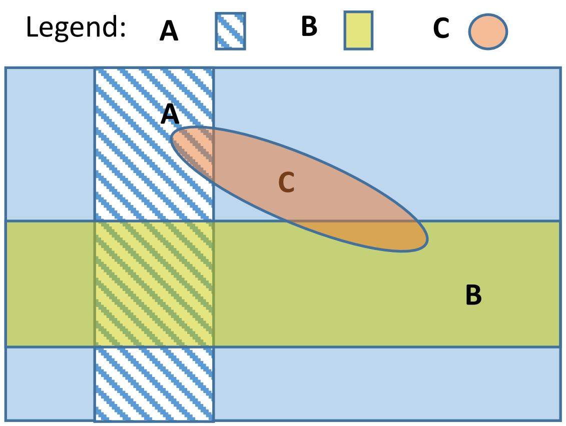
If told B occurred:



$$Pr(A|B) = \frac{2}{8} \quad Pr(B|B) = 1$$

Conditional independence



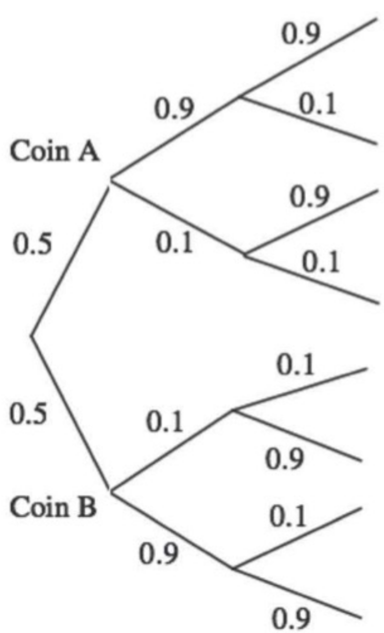


Conditional independent

Assume A and B are independent ($A \perp B$), A and B are not independent if we are given C occurred.

Conditioning may affect independence

- Two unfair coins, A and B:
  - Pr (H|coin A) = 0.9, Pr (H|coin B) = 0.1
- Choose either coin with equal probability



Compare :

- Pr(toss 11 = H)

- Pr (toss 11 = H|first 10 tosses are heads)

Independence vs. pairwise independence

- Two independent fair coin tosses
  - H_1: First toss is H
  - H_2 : Second toss is H

HH	HT
TH	TT

- C: the two tosses had the same result

H_1 = {HH, HT} , H_2 = {HH, TH}, C = {HH,TT}

Pr(H_1) = 1/2, Pr(H_2) = 1/2 , Pr(C)=1/2

Pr(H1∩C) = P(H1 ∩ H2) =Pr(H2 ∩C) = 1/4

Pr(H1 ∩ C ∩ H2) =Pr(HH) = 1/4

H_1, H_2, and C are pairwise independent, but not independent

After-Class Exercise

- Suppose $P(A) = 0.4$ and $P(B) = 0.6$. Are A and B mutually exclusive?

- Suppose A and B are mutually exclusive and $P(A) = 0.4$. Then $\mathrm{P}(B) = ?$

- Suppose A and B are mutually exclusive. In addition, $P(A) = 0.4$ and $P(B) = 0.6$. Suppose C and D are also mutually exclusive and collectively exhaustive. Further, $P(C|A) = 0.2$ and $P(D|B) = 0.4$. What are $P(C)$ and $P(D)$?

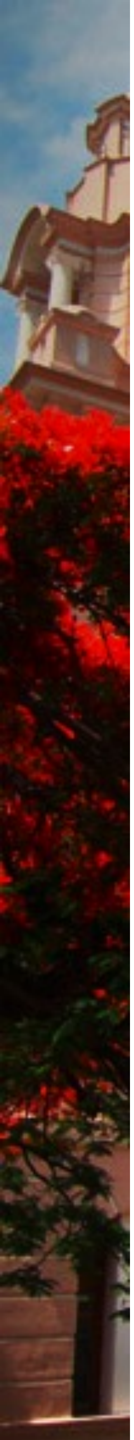
After-Class Exercise

- There are two fortune tellers, A & B. According to historical data, A's predictions were correct in 90% cases, while B's predictions were correct only in 30% cases.
- Now, without communicating with each other, both A & B predict that Donald Trump will be elected again after four years.
- Without any information, your prior belief about Donald Trump being elected again is 0.5. Now knowing A & B's predictions, what should be your corrected belief?

After-Class Exercise

- When a man passes the airport security check, they discover a bomb in his bag. He explains. “Statisticians show that the probability of a bomb being on an airplane is 1/10,000. However, the chance that there are two bombs on one plane is 1/10,000,000. So, I am much safer …”
- Suppose the statisticians are right and it is impossible to have more than two bombs on an airplane. Do you agree with the man?

- If event A and B are independent given event C happens, then A and B are also independent given C does not happen.



After-Class Exercise

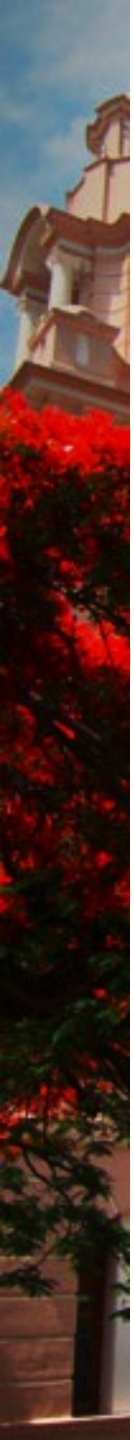
- If events A and B are dependent, it is impossible to find event C that is independent of A but not independent of B.

After-Class Exercise

- Suppose Chow Tai Fook introduced a new gold ring. Historical sales data of similar rings suggest that customers' willingness to pay (WTP) follows a normal distribution with a standard deviation of $1,000. However, the mean WTP of this new ring is unknown. It can be anywhere between $2,000 to $5,000.
- The introductory price for this ring is $4,000. The cost for this ring is $2,000. There is sufficient inventory.
- If the first customer who showed interested in this ring did not buy it, how should the price be adjusted afterwards?
- What is the best price to set if the first customer did not buy? Consider three possible means of WTP: $2,000, $3,500, and $5,000.
- Please find the model and calculations in the Excel file.

Café du Donut

- The Café buys donuts each day for $40 per carton of 20 dozen donuts. Any cartons not sold are thrown away at the end of the day. If a carton is sold, the revenue is $60.
- Different from the case in Session 2, the salesperson is faced with two kinds of demand situations and needs to decide order size each day.
- Suppose that the order size (Q) can only be between 6 and 7 due to the storage capacity and delivery capacity.



Café du Donut

- The salesperson’s initial belief is that two demand situations are equally likely.

DAILY DEMAND (CARTONS)	PROBABILITY UNDER LOW DEMAND	PROBABILITY UNDER HIGH DEMAND	MARGINAL PROBABILITY
4	0.25	0.05	0.15
5	0.20	0.10	0.15
6	0.15	0.10	0.125
7	0.15	0.15	0.15
8	0.10	0.15	0.125
9	0.10	0.20	0.15
10	0.05	0.25	0.15

- On the first day, should the order size be 6 or 7?

Café du Donut

- Monetary Payoff (Profit) Table

	D = 4	D = 5	D = 6	D = 7	D = 8	D = 9	D = 10	EMV
Q = 6	0	60	120	120	120	120	120	93
Q = 7	-40	20	80	140	140	140	140	87.5
Prob.	0.15	0.15	0.125	0.15	0.125	0.15	0.15	

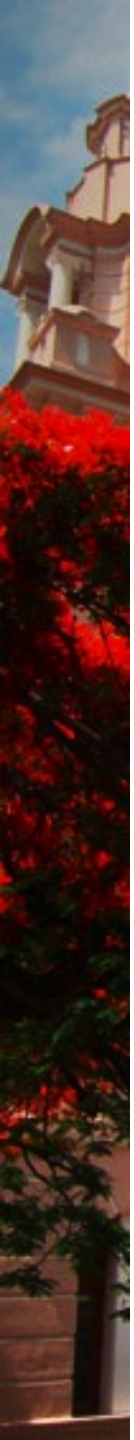
- If the salesperson finds that the demand in first day is 8, should he increase the order size from 6 to 7?
  - Here, we assume that the distribution of the demand situation on the second day is the same as the distribution of the situation on the first day.

Café du Donut

- The salesperson updates his belief:

	Low Demand	High Demand
$D_1 = 8$	0.1*0.5	0.15*0.5

DAILY DEMAND (CARTONS)	PROBABILITY UNDER LOW DEMAND	PROBABILITY UNDER HIGH DEMAND
4	0.25	0.05
5	0.20	0.10
6	0.15	0.10
7	0.15	0.15
8	0.10	0.15
9	0.10	0.20
10	0.05	0.25



Café du Donut

- The salesperson then updates monetary payoff (profit) table

	D = 4	D = 5	D = 6	D = 7	D = 8	D = 9	D = 10
Q = 6	0	60	120	120	120	120	120
Q = 7	-40	20	80	140	140	140	140
Prob.							

- The order size should not be increased.

Tossing Coins

- What is the probability of getting a head?
- Suppose there are three possible cases: 1/3, 1/2, and 2/3.

- What if we can toss the coin many times?
- Please refer to the “TossCoin.py”.