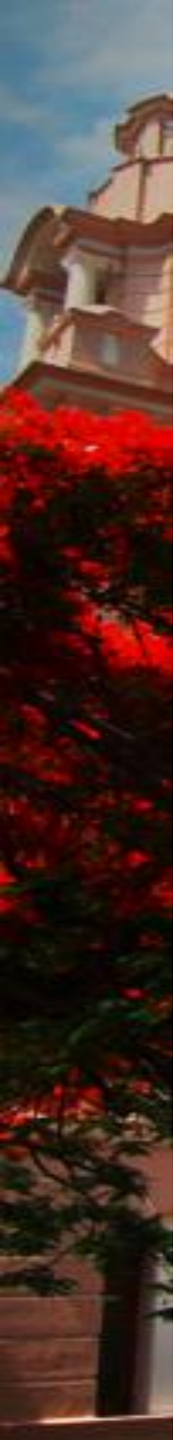
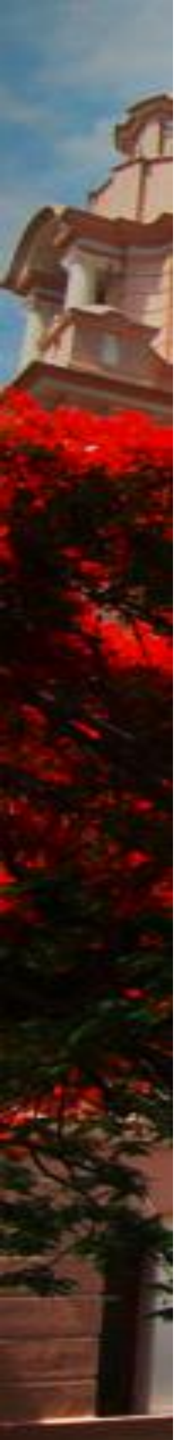
# MSBA7003 Quantitative Analysis Methods


# 06 Linear Programming II



# Agenda

- Worker Scheduling
- Portfolio Selection
- Transportation Planning
- Efficiency Evaluation
- Risk Management



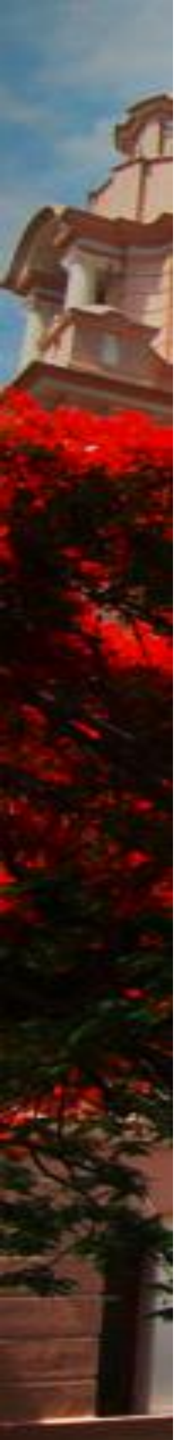
# Worker Scheduling

- A post office requires different numbers of full-time employees on different days of the week. Union rules states that each full-time employee must work five consecutive days and then receive two days off. The post office wants to meet its daily requirements using only full-time employees, while minimizing the total number of full-time employees on its payroll.

| Day of Week | Minimum Number of Employees Required |
|---|---|
| Monday | 17 |
| Tuesday | 13 |
| Wednesday | 15 |
| Thursday | 19 |
| Friday | 14 |
| Saturday | 16 |
| Sunday | 11 |

# Worker Scheduling

- Decision variables:
  - $X_1$: the number of employees whose first working day is Monday
  - $X_2$: the number of employees whose first working day is Tuesday
  - $X_3$: the number of employees whose first working day is Wednesday
  - $X_4$: the number of employees whose first working day is Thursday
  - $X_5$: the number of employees whose first working day is Friday
  - $X_6$: the number of employees whose first working day is Saturday
  - $X_7$: the number of employees whose first working day is Sunday
- Objective:
  - Minimize: $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$



# Worker Scheduling

- Constraints:
- Monday constraint: $X_1 + X_4 + X_5 + X_6 + X_7 \geq 17$
- Tuesday constraint: $X_1 + X_2 + X_5 + X_6 + X_7 \geq 13$
- Wednesday constraint: $X_1 + X_2 + X_3 + X_6 + X_7 \geq 15$
- Thursday constraint: $X_1 + X_2 + X_3 + X_4 + X_7 \geq 19$
- Friday constraint: $X_1 + X_2 + X_3 + X_4 + X_5 \geq 14$
- Saturday constraint: $X_2 + X_3 + X_4 + X_5 + X_6 \geq 16$
- Sunday constraint: $X_3 + X_4 + X_5 + X_6 + X_7 \geq 11$
- Non-negativity: $X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 0$

# Worker Scheduling

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Worker Scheduling Model | | | | | | | | | | | |
| | | | | | | | | | | | |
| Decision Variables | | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total No. of FT Employees | | |
| No. of FT Employees: | | 6.333333333 | 3.333333333 | 2 | 7.333333333 | 0 | 3.333333333 | 0 | | | 22.33333 |
| | | | | | | | | | | | |
| Constraints | | | | | | | | | | | RHS |
| | Monday | 1 | | | 1 | 1 | 1 | 1 | 17 | >= | 17 |
| | Tuesday | 1 | 1 | | | 1 | 1 | 1 | 13 | >= | 13 |
| | Wednesday | 1 | 1 | 1 | | | 1 | 1 | 15 | >= | 15 |
| | Thursday | 1 | 1 | 1 | 1 | | | 1 | 19 | >= | 19 |
| | Friday | 1 | 1 | 1 | 1 | 1 | | | 19 | >= | 14 |
| | Saturday | | 1 | 1 | 1 | 1 | 1 | | 16 | >= | 16 |
| | Sunday | | | 1 | 1 | 1 | 1 | 1 | 12.66667 | >= | 11 |

- To get integer solutions
  - Add integer constraints
  - Round the solutions to the nearest integers

# Portfolio Selection

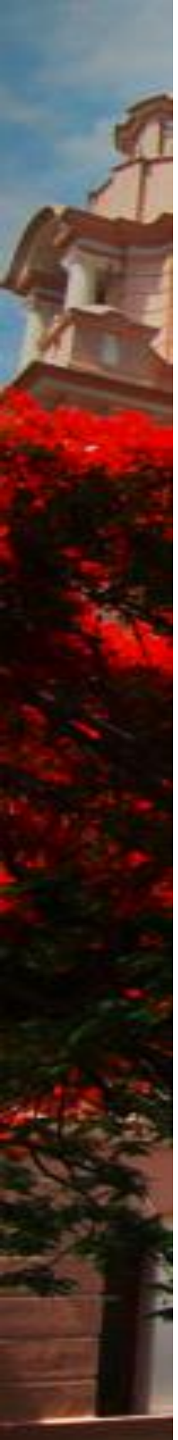
- The Heinlein and Krampf Brokerage firm is instructed by a client to invest $250,000, with guidelines:
  - Municipal bonds should constitute at least 20% of the investment
  - At least 40% of the investment should be placed in a combination of electronic firms, aerospace firms, and drug manufacturers
  - No more than 50% of the amount invested in municipal bonds should be placed in a high-risk, high-yield nursing home stock
- The goal is to maximize the projected return.

| Investment | Los Angeles municipal bonds | Thompson Electronics, Inc. | United Aerospace Corp. | Palmer Drugs | Happy Days Nursing Homes |
|---|---|---|---|---|---|
| Projected Return (%) | 5.3 | 6.8 | 4.9 | 8.4 | 11.8 |

# Portfolio Selection

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Investment | LA Municipal Bonds | Thompson Electronics, Inc. | United Aerospace Corp. | Palmer Drugs | Happy Days Nursing Homes | | | |
| 2 | Projected Return (%) | 5.3 | 6.8 | 4.9 | 8.4 | 11.8 | | | |
| 3 | Amount | 50000 | 0 | 0 | 175000 | 25000 | | | |
| 4 | | | | | | | | | |
| 5 | Total Return | 20300 | | | | | | | |
| 6 | | | | | | | | | |
| 7 | Constraints | | | | | | | | |
| 8 | | 1 | 1 | 1 | 1 | 1 | 250000 | <= | 250000 |
| 9 | | 1 | | | | | 50000 | >= | 50000 |
| 10 | | | 1 | 1 | 1 | | 175000 | >= | 100000 |
| 11 | | | | | | 1 | 25000 | <= | 25000 |

Question: How to set cells I9 and I10?



# Portfolio Selection: Sensitivity Report

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | Amount LA Municipal Bonds | 50000 | 0 | 0.053 | 0.014 | 0.406 |
| $C$3 | Amount Thompson Electronics, Inc. | 0 | -0.016 | 0.068 | 0.016 | 1E+30 |
| $D$3 | Amount United Aerospace Corp. | 0 | -0.035 | 0.049 | 0.035 | 1E+30 |
| $E$3 | Amount Palmer Drugs | 175000 | 0 | 0.084 | 0.034 | 0.009333333 |
| $F$3 | Amount Happy Days Nursing Homes | 25000 | 0 | 0.118 | 0.028 | 0.034 |

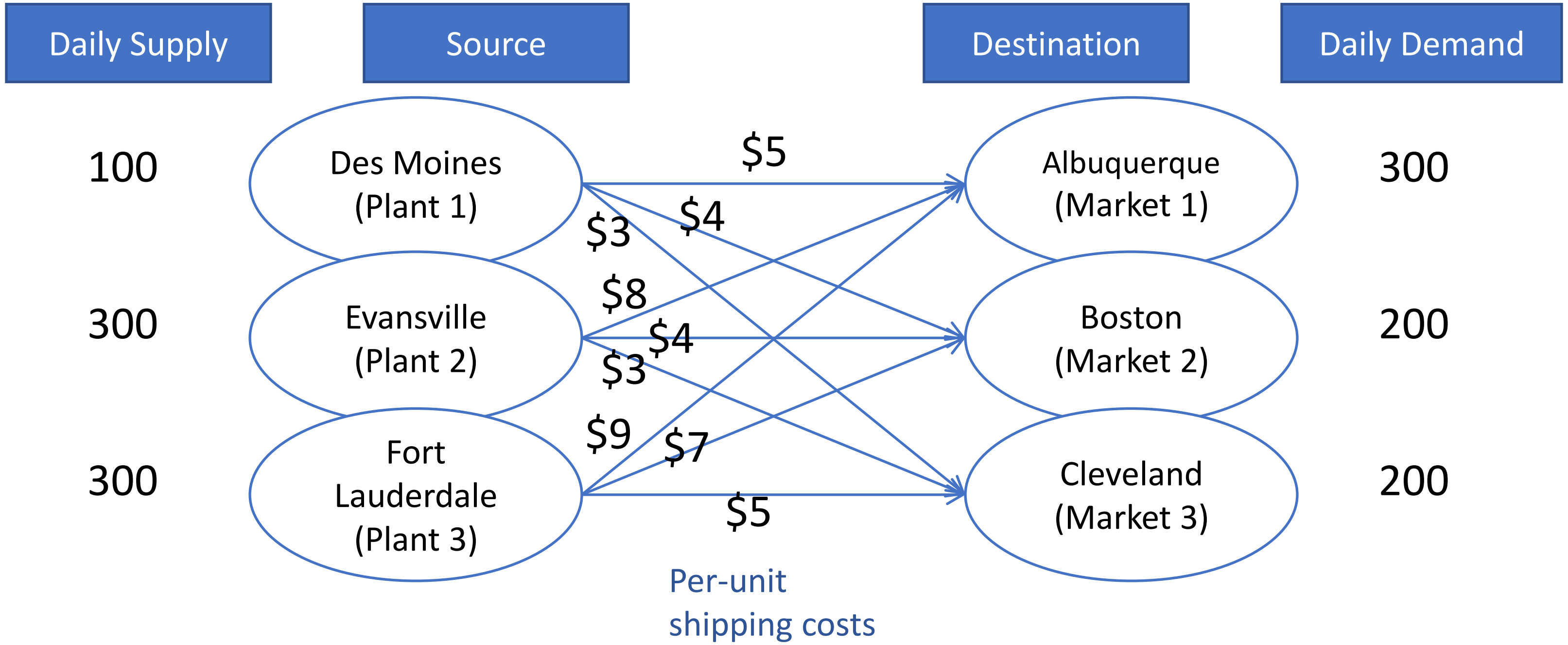
Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $G$8 | Total Amount Constraint | 250000 | 0.0812 | 250000 | 1E+30 | 250000 |
| $G$9 | Municipal Bond Constraint | 50000 | -0.014 | 0 | 50000 | 50000 |
| $G$10 | E.A.D. Constraint | 175000 | 0 | 0 | 75000 | 1E+30 |
| $G$11 | Nursing Home Constraint | 25000 | 0.034 | 0 | 75000 | 25000 |

Question: What is the marginal return rate?

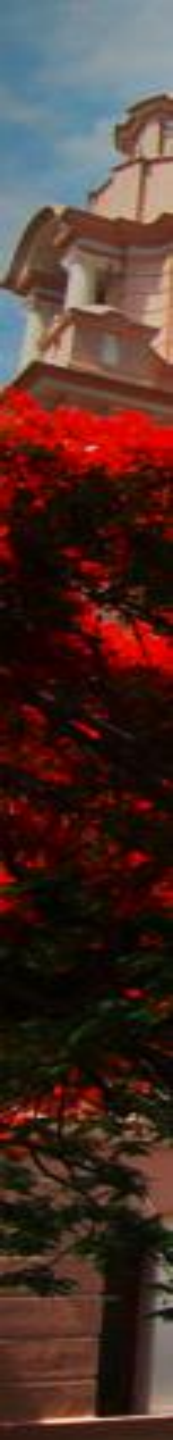
# Transportation Problem

- The Executive Furniture Corporation is faced with the following transportation problem and is trying to minimize the daily transportation cost. How to optimize the shipping plan, while the demand must be satisfied?



# Transportation Problem

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Executive Furniture Corporation** | | | | | |
| 2 | | | | | | |
| 3 | Source | | Des Moines | Evansville | Fort Lauderdale | Demand Sum |
| 4 | Destination | Albuquerque | 100 | 0 | 200 | 300 |
| 5 | | Boston | 0 | 200 | 0 | 200 |
| 6 | | Cleveland | 0 | 100 | 100 | 200 |
| 7 | | Supply Sum | 100 | 300 | 300 | |
| 8 | | | | | | |
| 9 | Model Parameters | | Des Moines | Evansville | Fort Lauderdale | Demand |
| 10 | | Albuquerque | $ 5.00 | $ 8.00 | $ 9.00 | 300 |
| 11 | | Boston | $ 4.00 | $ 4.00 | $ 7.00 | 200 |
| 12 | | Cleveland | $ 3.00 | $ 3.00 | $ 5.00 | 200 |
| 13 | | Supply | 100 | 300 | 300 | |
| 14 | | | | | | |
| 15 | Total Cost | $ 3,900.00 | | | | |



# In-Class Exercise

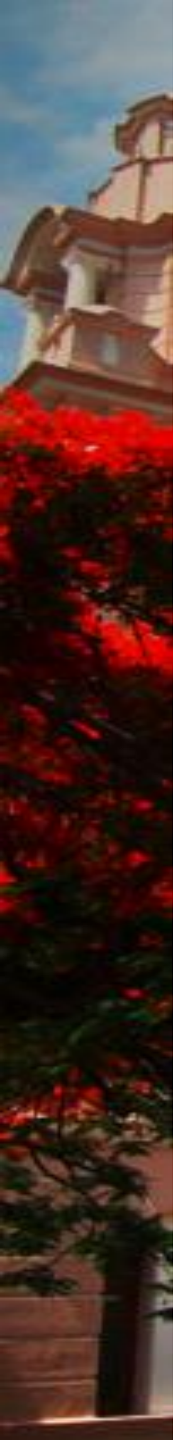
- The following table shows the sensitivity analysis of the transportation problem. If you can expand the supply by 100, which plant would you choose?

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $C$7 | Supply Sum Des Moines | 100 | -4 | 100 | 200 | 0 |
| $D$7 | Supply Sum Evansville | 300 | -2 | 300 | 100 | 0 |
| $E$7 | Supply Sum Fort Lauderdale | 300 | 0 | 300 | 1E+30 | 0 |
| $F$4 | Albuquerque Demand Sum | 300 | 9 | 300 | 0 | 200 |
| $F$5 | Boston Demand Sum | 200 | 6 | 200 | 0 | 100 |
| $F$6 | Cleveland Demand Sum | 200 | 5 | 200 | 0 | 100 |

# In-Class Exercise

- Reconsider the Transportation Problem.
- Suppose each source can supply more than its capacity by outsourcing at price $p_i = 3$, and demand can be unsatisfied with a revenue loss of $r_j = 10$ per unit. What is the optimal outsourcing and transportation plan? Formulate the LP.



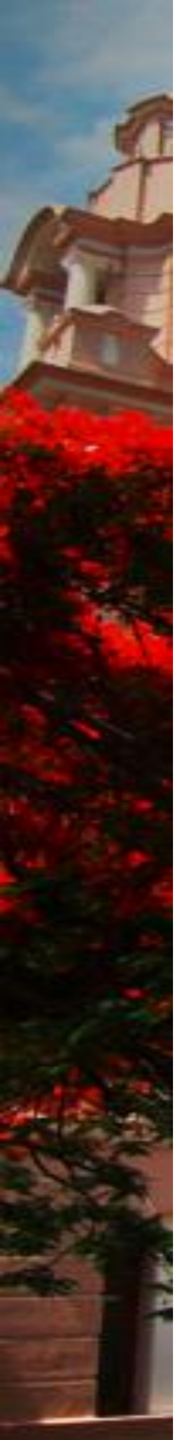
# In-Class Exercise

- Reconsider the Transportation Problem.
- Suppose each source can supply more than its capacity by outsourcing at price $p_i = 3$, and demand can be unsatisfied with a revenue loss of $r_j = 10$ per unit. What is the optimal outsourcing and transportation plan? Solve the problem.

# Efficiency Evaluation

- How to compare and evaluate the efficiencies of different individuals or business units?
- Absolute efficiency = Output value / Input value
- If we cannot directly measure the values of outputs and inputs, then we cannot obtain the absolute efficiency.

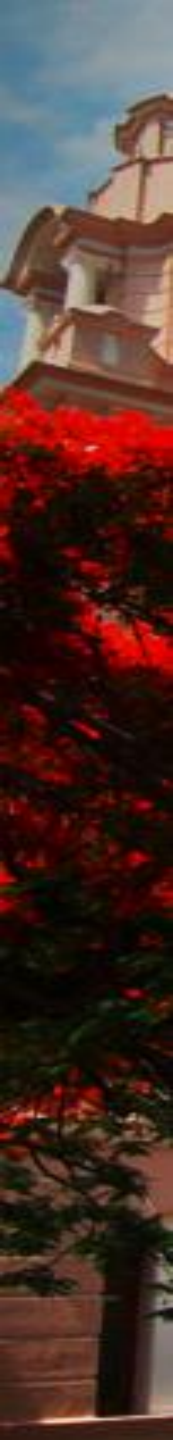
- Pinevalley Bank has three different branches. The management board now wants to evaluate the operational efficiency of each branch. Each branch has inputs from three aspects: 1) the number of workers, 2) the size of the business hall (in 100 sq. m.), and 3) consumption of stationary supplies (1000 dollars per month). The outputs also have three dimensions: 1) monthly new accounts, 2) monthly new loans, and 3) monthly new deposit value.
- Due to multiple inputs and outputs, the branch efficiency cannot be measured directly.



# Pinevalley Bank

- Pinevalley Bank branch information for a given month:

| Branch | Input #1: # of Workers | Input #2: Business Hall Size (100 sq. m.) | Input #3: Stationary Supplies ($1000) | Output #1: New Accounts | Output #2: **Number of New** Loan | Output #3: New Deposit Value (10 mils) |
|---|---|---|---|---|---|---|
| A | 27 | 3.5 | 15 | 11 | 9 | 1 |
| B | 15 | 2 | 10 | 10 | 8 | 0.4 |
| C | 43 | 7 | 39 | 24 | 20 | 5 |

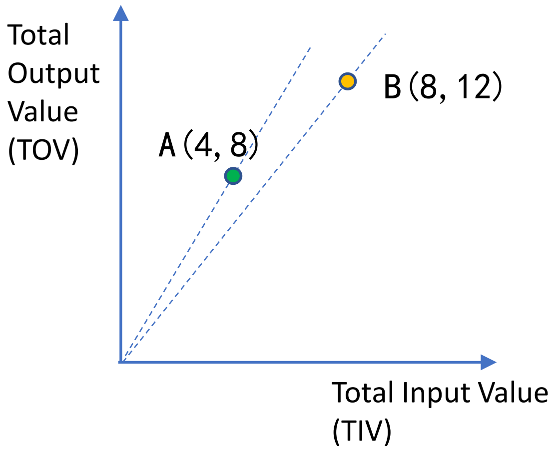


# Efficiency Evaluation

- There are two possible assumptions:
- 1. <u>Constant returns to scale</u>: For each unit, every dollar of input value will always create the same amount of output value.
  - Group A has 4 workers and can process 8 units of product per hour. Group B has 8 workers and can process 12 units per hour. Group B is relatively less efficient, because Group A can process 16 units if we double the number of workers.
- 2. <u>Non-constant returns to scale</u>: For each unit, the return to scale may change, but the performance of different units can be linearly combined.
  - Group A has 4 workers and can process 8 units of product per hour. Group B has 8 workers and can process 12 units per hour. Neither group is necessarily inefficient. If Group C has 6 workers and can process 9 units per hour, then Group C is relatively less efficient, because the average efficiency of A and B is higher (i.e., 6 workers process 10 units per hour).

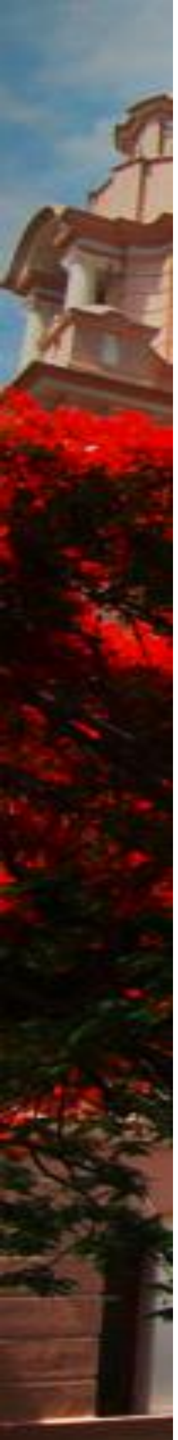
# Data Envelopment Analysis (DEA)

- Constant Returns to Scale:




- The larger the slope, the higher the efficiency.

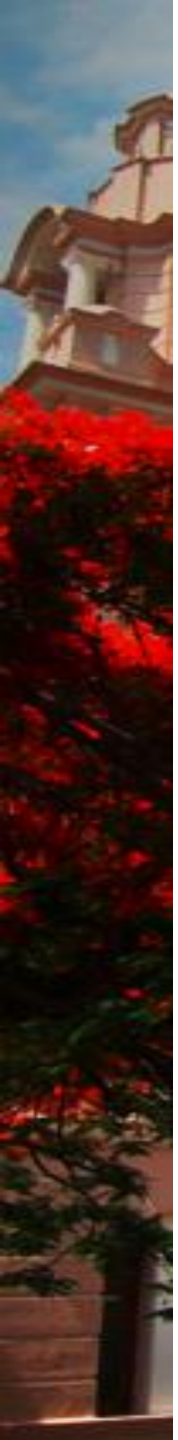
- When there are multiple inputs and outputs, the goal is to find a vector of input weights and a vector of output weights which can make a unit look as good as possible in terms of the ratio of TOV/TIV. Given these weights, if a unit is not the best among all the units, then it is inefficient.
- Constraints: 1) weights cannot be negative; 2) the ratio of TOV/TIV cannot be bigger than 1; 3) the TIV of the focal unit is normalized to 1.



# DEA: Pinevalley Bank

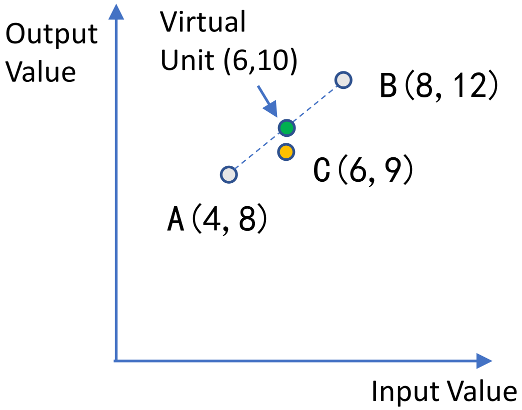
- Constant Returns to Scale:

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Branch | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 | Output 3 | | Branch | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 | Output 3 |
| 2 | A | 27 | 3.5 | 15 | 11 | 9 | 1 | | A | 27 | 3.5 | 15 | 11 | 9 | 1 |
| 3 | B | 15 | 2 | 10 | 10 | 8 | 0.4 | | B | 15 | 2 | 10 | 10 | 8 | 0.4 |
| 4 | C | 43 | 7 | 39 | 24 | 20 | 5 | | C | 43 | 7 | 39 | 24 | 20 | 5 |
| 5 | Weights | 0 | 0 | 0.066667 | 0 | 0.071667 | 0.233333 | | Weights | 0.066667 | 0 | 0 | 0.1 | 0 | 0 |
| 6 | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | |
| 8 | Branch A Evaluation: | | | | | | | | Branch B Evaluation: | | | | | | |
| 9 | Branch | TOV | | TIV | | Efficiency | | | Branch | TOV | | TIV | | Efficiency | |
| 10 | A | 0.878333 | <= | 1 | Fixed as 1 | 88% | | | A | 1.1 | <= | 1.8 | | 61% | |
| 11 | B | 0.666667 | <= | 0.666667 | | 100% | | | B | 1 | <= | 1 | Fixed as 1 | 100% | |
| 12 | C | 2.6 | <= | 2.6 | | 100% | | | C | 2.4 | <= | 2.866667 | | 84% | |



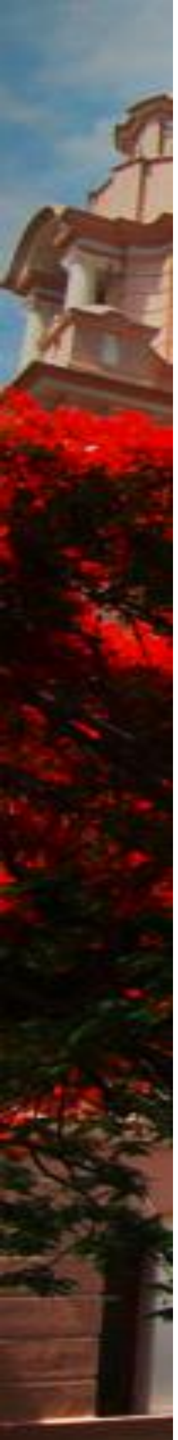
# Data Envelopment Analysis (DEA)

- Non-Constant Returns to Scale:




- After matching the input, the higher the output, the higher the efficiency.

- For each focal unit, the goal is to construct a virtual unit based on the linear combination of all existing units such that the virtual unit has lower or the same input values in all aspects. If the virtual unit can beat the focal unit in every aspect of output, then the focal unit is inefficient.
- Constraints: 1) weights cannot be negative and must sum up to 1; 2) the input values for the virtual unit cannot exceed the focal unit.



# DEA: Pinevalley Bank

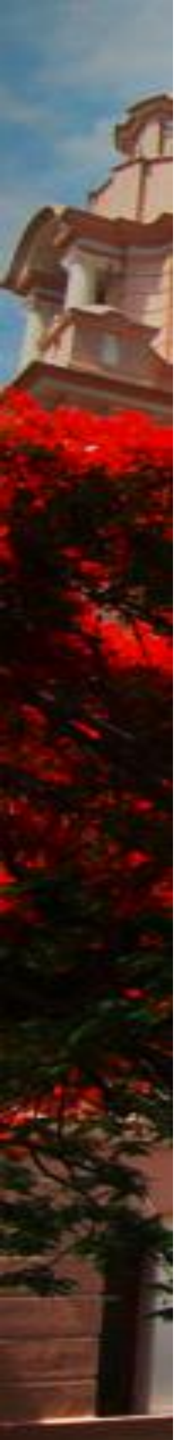
- Non-Constant Returns to Scale:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Branch A Evaluation: | | | | | | | |
| 2 | Branch | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 | Output 3 | Weights |
| 3 | A | 27 | 3.5 | 15 | 11 | 9 | 1 | 0 |
| 4 | B | 15 | 2 | 10 | 10 | 8 | 0.4 | 0.827586 |
| 5 | C | 43 | 7 | 39 | 24 | 20 | 5 | 0.172414 |
| 6 | Virtual | 19.82759 | 2.862069 | 15 | 12.41379 | 10.06897 | 1.193103 | Sum |
| 7 | | | | | | | Constraint 1: Weights sum up to 1 | 1 |
| 8 | | | | | | | | |
| 9 | Constraint 2: Lower inputs | | | | Constraint 3: Higher than k-multiple outputs | | | |
| 10 | | <= | <= | <= | >= | >= | >= | Multiple k |
| 11 | Branch A | 27 | 3.5 | 15 | 12.30651 | 10.06897 | 1.118774 | 1.118774 |

| | I | J | K | L | M | N | O | P | Q |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | Branch B Evaluation: | | | | | | | |
| 2 | | Branch | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 | Output 3 | Weights |
| 3 | | A | 27 | 3.5 | 15 | 11 | 9 | 1 | 0 |
| 4 | | B | 15 | 2 | 10 | 10 | 8 | 0.4 | 1 |
| 5 | | C | 43 | 7 | 39 | 24 | 20 | 5 | 0 |
| 6 | | Virtual | 15 | 2 | 10 | 10 | 8 | 0.4 | Sum |
| 7 | | | | | | | | Constraint 1: Weights sum up to 1 | 1 |
| 8 | | | | | | | | | |
| 9 | | Constraint 2: Lower inputs | | | | Constraint 3: Higher than k-multiple outputs | | | |
| 10 | | | <= | <= | <= | >= | >= | >= | Multiple k |
| 11 | | Branch B | 15 | 2 | 10 | 10 | 8 | 0.4 | 1 |

Branch Output × Multiple k

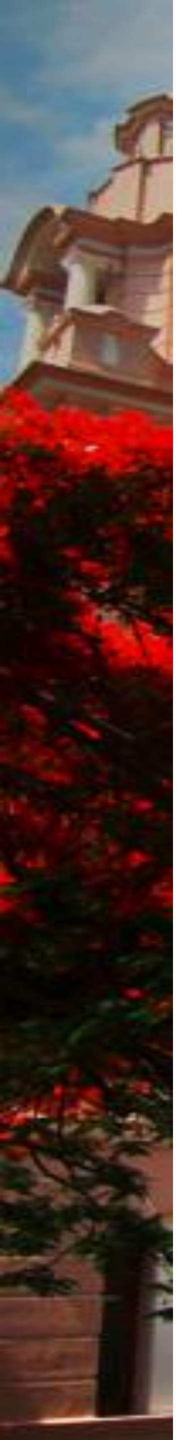
- Goal: maximize multiple k
- Focal branch efficiency = 1/k



# In-Class Exercise

- Your company HR plans to develop a model to identify inefficient salespeople. Given the following data, which worker(s) could be labelled as being inefficient?

| Salesperson | Inputs | | | Outputs | | |
|---|---|---|---|---|---|---|
| | Expense ($k) | Years of Experience | # of Days Working | Lead Conversion | Sales Value ($k) | # of Clients |
| Jimmy | 34 | 8 | 288 | 0.82 | 550 | 5 |
| Mike | 30 | 5 | 330 | 0.67 | 400 | 3 |
| Shirley | 52 | 4 | 340 | 0.92 | 680 | 4 |



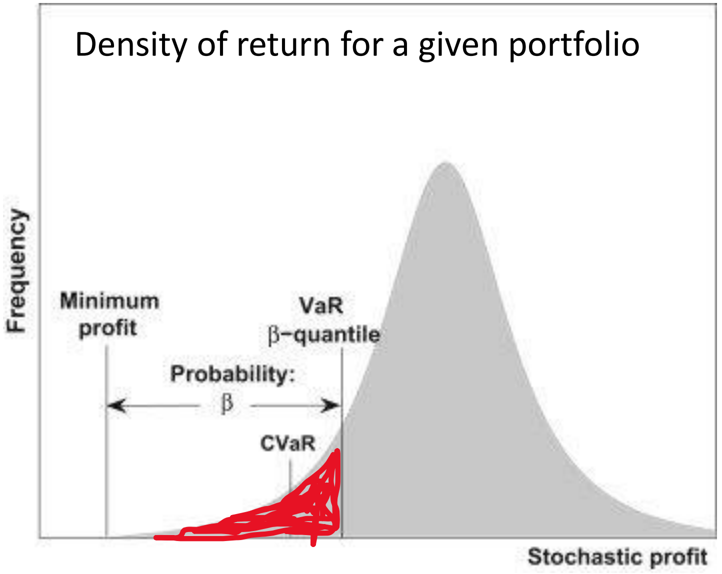
# Risk Management

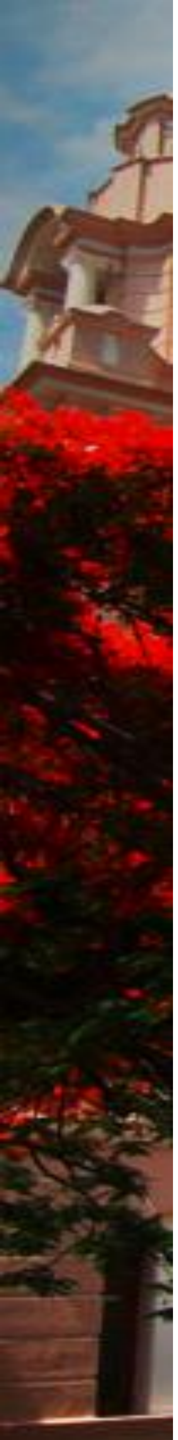
- The Heinlein and Krampf Brokerage firm is instructed by a client to invest $250,000 among the five securities. It is okay to hold cash. Assume that the return rate of each security is an independent normal random variable. The table below gives the expected return rate and the standard deviation of the return rate for each security.
- The goal is to maximize the conditional value at risk below the 10% quantile. How to allocate the money?

| Investment | Los Angeles municipal bonds | Thompson Electronics, Inc. | United Aerospace Corp. | Palmer Drugs | Happy Days Nursing Homes |
|---|---|---|---|---|---|
| Projected Return (%) | 5.3 | 6.8 | 4.9 | 8.4 | 11.8 |
| Standard Deviation | 1.2 | 4.4 | 5.2 | 12.0 | 19.9 |

# Risk Management

- In risk management, we care about the tail risk. For the target random return X, the **value at risk** (VaR) at level β is the β–quantile of X. The **conditional value at risk** (CVaR) falling short of an β–level VaR cutoff is the expected return of X given it is below the β–quantile.





# Risk Management

- How to maximize the β–level conditional value at risk?
- Suppose X(ε,D) depends on random factor ε and decision D.
  - Step 1: Obtain S sample values of ε and index them by s.
  - Step 2: Introduce auxiliary decision variables μ and $z_s$ for all s.
  - Step 3: Solve the following optimization problem.
    - Maximize $Y = \mu - \frac{\sum_s z_s}{S\beta}$
    - Subject to $z_s \geq \mu - X(\varepsilon_s, D)$ and $z_s \geq 0$ for all s.
  - Interpret Y as the β–level CVaR and μ as the β–level VaR.

# Risk Management

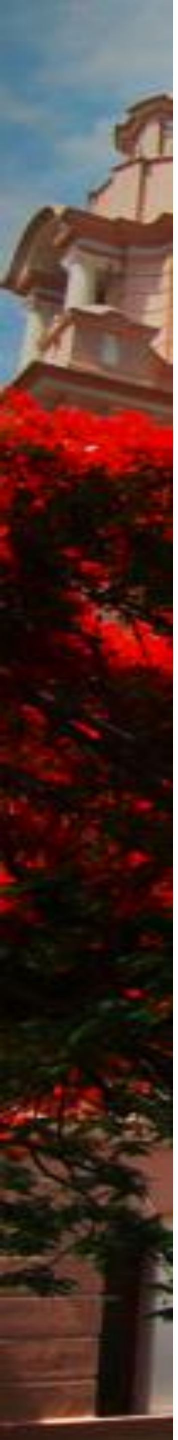
$$VaR - \frac{Avg(\max(F.S.,0))}{\beta}$$

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Investment | LA Municipal Bonds | Thompson Electronics, Inc. | United Aerospace Corp. | Palmer Drugs | Happy Days Nursing Homes | | | |
| 2 | Projected Return (! | 5.3 | 6.8 | 4.9 | 8.4 | 11.8 | | | |
| 3 | Std. Dev. | 1.2 | 4.4 | 5.2 | 12 | 19.9 | | | |
| 4 | Amount | 199575.6429 | 29742.4907 | 14380.45178 | 2138.350525 | 4163.064143 | 250000 | <= | 250000 |
| 5 | | | | | | | Total | | Budget |
| 6 | Risk level | 0.1 | | | | | | | |
| 7 | VaR | 10866.2673 | | | | | | CVaR | 9272.797 |
| 8 | | | | | | | | | |
| 9 | Samples | 4.112747346 | (Formula) | | | | Return | Fall Short | max(F.S.,0) |
| 10 | 1 | 4.47765297 | 5.880269876 | 8.291370412 | 9.806066112 | 9.135484745 | 12467.58 | -1601.32 | 0 |
| 11 | 2 | 5.03164465 | 7.970612754 | 6.267024768 | 19.88598081 | 24.06953425 | 14741.08 | -3874.82 | 0 |
| 12 | 3 | 5.604775907 | 6.586877456 | -1.234999087 | -4.828227951 | 17.92521858 | 13610.26 | -2744 | 0 |
| 13 | 4 | 7.59336873 | 8.862532497 | 3.893762589 | 16.05722908 | 18.12586595 | 19448.34 | -8582.08 | 0 |
| 14 | 5 | 7.598586242 | 9.812093088 | 8.59909994 | 2.601053126 | 19.52748209 | 20188.44 | -9322.17 | 0 |
| 15 | 6 | 7.620537573 | -0.265197691 | -4.746650113 | 21.97369641 | -9.940557384 | 14503.31 | -3637.05 | 0 |
| 16 | 7 | 3.415595985 | 10.91611372 | -2.499272747 | -7.598720805 | 12.93991009 | 10080.22 | 786.0428 | 786.0428 |
| 17 | 8 | 6.469280201 | 6.154674034 | 6.661628806 | 19.63960275 | -0.542877121 | 16097 | -5230.73 | 0 |
| 18 | 9 | 4.356317932 | 2.107376452 | -2.946973687 | 3.368087083 | -0.947841411 | 8929.71 | 1936.557 | 1936.557 |
| 19 | 10 | 5.388686114 | 10.34400969 | 4.364457264 | 2.731717327 | 24.40779671 | 15533.23 | -4666.96 | 0 |
| 20 | 11 | 5.836060768 | 8.693605101 | 12.2370995 | 22.70722887 | 5.794269843 | 16719.58 | -5853.31 | 0 |
| 21 | 12 | 7.394897841 | 16.99356512 | 4.614023756 | 29.82006479 | 56.18285849 | 23452.83 | -12586.6 | 0 |
| 22 | 13 | 5.144383271 | 5.07544769 | 3.037473514 | 33.33453001 | -13.51784492 | 12363.36 | -1497.09 | 0 |
| 23 | 14 | 6.73846124 | 4.241574142 | -0.074488422 | 5.42366765 | 4.128025876 | 14986.99 | -4120.73 | 0 |
| 24 | 15 | 5.024158139 | 8.007901249 | 1.960806527 | 33.01246591 | -6.622314058 | 13120.95 | -2254.68 | 0 |
| 25 | 16 | 6.103893768 | 7.067400836 | -9.515196159 | 1.004876363 | -1.970584848 | 12855.03 | -1988.76 | 0 |
| 26 | 17 | 6.7895225 | 3.224704997 | 11.7619441 | 2.879567307 | 9.511172881 | 16658.29 | -5792.03 | 0 |

$X(\varepsilon_s, D)$

$\varepsilon_s$

$z_s$



# Risk Management

```python
# Prepare the data
sec_data = {
   'security': ['LA Municipal Bonds', 'Thompson Electronics', 'United Aerospace',
                'Palmer Drugs', 'Nursing Homes'],
   'return': [5.3, 6.8, 4.9, 8.4, 11.8],
   }
CovM = {
   'LA Municipal Bonds':   [1.44,     -0.528,    -1.248,    -4.32,      -9.552],
   'Thompson Electronics': [-0.528,    19.36,     9.152,     5.28,      17.512],
   'United Aerospace':     [-1.248,    9.152,     27.04,     18.72,     10.348],
   'Palmer Drugs':         [-4.32,     5.28,      18.72,     144,       119.4],
   'Nursing Homes':        [-9.552,    17.512,    10.348,    119.4,     396.01]
   }
budget = 250000; beta = 0.1; simulation = 1000
# Prepare data frames
from pandas import DataFrame
df_sec = DataFrame(data=sec_data)
df_sec.set_index(['security'], inplace=True) #This line replace the default index
securities = df_sec.index
df_cov = DataFrame(data=CovM, index=securities, columns=securities)
# Choleski decomposition of the Covariance Matrix
import numpy as np
Cov_Matrix = df_cov.to_numpy()
L = np.linalg.cholesky(Cov_Matrix)
```

The covariance matrix captures the correlation between any two securities.

The Choleski decomposition: $\Omega = L'L$

# Risk Management

```
# Define the model
from pulp import *
prob = LpProblem('Portfolio_Risk_Management',LpMaximize)
# Define decision variables
investment = LpVariable.dicts('Invested_Value',securities,lowBound=0)
VaR = LpVariable('VaR')
Z = LpVariable.dicts('Shortfall',range(simulation),lowBound=0)
# Define the objective
prob += VaR - lpSum([Z[i] for i in range(simulation)])/beta/simulation, 'CVaR'
# Define the constraints
from numpy import random
prob += lpSum([investment[j] for j in securities]) <= budget
for i in range(simulation):
    r = L.dot(random.normal(0,1,len(securities))) + df_sec['return'].to_numpy()
    prob += Z[i] >= (
        VaR - lpSum([investment[securities[j]]*r[j] for j in range(len(securities))])/100)
prob.solve()
print('The CVaR =',value(prob.objective))
```

Simulate the return rate vector:

$$r = L \cdot u + r_e$$

wherein $u$ is a vector of independent standard normal random numbers and $r_e$ is the vector of expected return rates.