MSBA7003 Quantitative Analysis Methods

Tutorial 02

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2023 – 2024

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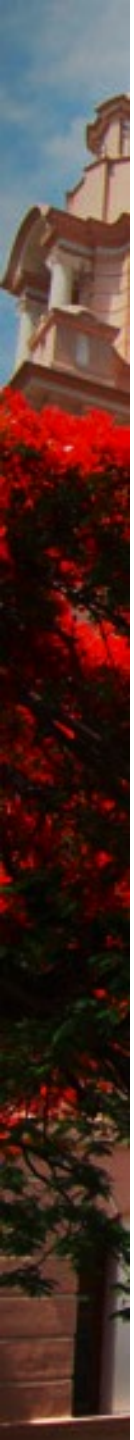


Agenda

- **Solutions of Assignment 01**
- **After-Class Exercises of Session 03**
- **Examples of Monte Carlo Simulation (with Python code)**
  - **The Monty Hall Problem**
  - **Buffon's Needle Problem Experiment**

Assignment 1 Q1

The joint probability distribution between two random variables X and Y is given below.

Joint Probabilities	Y = 1	Y = 2	Y = 3
X = 0	0.1	0.2	0.3
X = 1	0.2	0.1	0.1

Which of the following statement(s) is(are) true?

(A) X = 0 and Y = 1 are unconditionally independent.

(B) The event of X = 0 is less likely to occur given Y = 2 than given Y = 1.

(C) Pr(X = 1 | Y > 1) = 2/7.

(D) Suppose we randomly and independently draw two pairs of (X,Y): (X1,Y1) and (X2,Y2). We only see the values of Y. Then Pr(X1 + X2 = 0 | Y1 = 2 and Y2 = 3) = 1/2.

(E) None of the above.

Assignment 1 Q2

Suppose you can see a lawn out of your window. The lawn may be wet or dry in the afternoon. The state depends on the whether it rained earlier today and the previous operation of the sprinkler on the lawn. We are given the conditional probability of the afternoon lawn being wet given the previous operation of the sprinkler and whether it rained earlier today as below:

Previous operation of the sprinkler		On **B1**	On B1	Off **B0**	Off **B0**
Whether it rained earlier today		Yes **C1**	No **C0**	Yes **C1**	No **C0**
Lawn state	Wet **A1**	0.99	0.9	0.8	0.0
	Dry **A0**	0.01	0.1	0.2	1.0

We also know that the operation of the sprinkler is affected by the weather probabilistically in the following way in terms of conditional probabilities:

Whether it rained earlier today		Yes **C1**	No **C0**
Sprinkler	On **B1**	0.1	0.4
	Off **B0**	0.9	0.6

Assignment 1 Q2

In addition, our prior belief is that it rains with a probability of 0.2 every day.

Pr(C1) = 0.2

Which of the following statement(s) is(are) true?

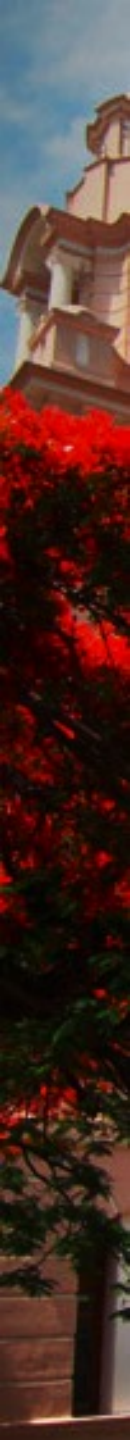
(A) If you know that the sprinkler was on earlier today, the probability that the lawn will be wet in the afternoon is 0.2*0.99 + 0.8*0.9 = 0.918.

(B) If you know that it rained in the morning, the probability that the lawn will be wet in the afternoon is 0.1*0.99 + 0.9*0.8 = 0.819.

(C) If you see in the afternoon that the lawn is wet, the probability that it rained before is (0.99 + 0.8)/(0.99 + 0.9 + 0.8 + 0) = 0.6654.

(D) If you see in the afternoon that the lawn is wet, the probability that the sprinkler was on before is (0.8*0.1*0.99 + 0.2*0.4*0.9)/(0.8*0.1*0.99 + 0.2*0.4*0.9 + 0.8*0.9*0.8 + 0.2*0.6*0) = 0.2.

(E) None of the above.



Assignment 1 Q3

- ABC Inc. is considering launching a new product and there are three options: product X, product Y, and do nothing. Product X requires an initial investment of $15 million and product Y requires $5 million. The gross profit (before subtracting the initial investment) that can be generated by each product depends on the market condition. If the market is strong, product X can generate a gross profit of $100 million and product Y can generate $20 million; if the market is weak, product X will lead to a loss of $80 million and product Y will cause a loss of $9 million (before subtracting the initial investment). The company's prior belief of a strong market is 60%.
- To get a better understanding of the market before deciding the choice of the new product, the manager hired a consulting firm to conduct market research. According to historical data, this consulting firm can successfully predict a strong market in 65% cases and can correctly predict a weak market in 55% cases.

Assignment 1 Q3

- ABC Inc. is considering launching a new product and there are three options:

ALTERNATIVE	STATE OF NATURE	
	STRONG MARKET (profits in million $)	WEAK MARKET (profits in million $)
Product X	100-15 = 85	-80-15 = -95
Product Y	20-5 = 15	-9-5 = -14
Do nothing	0	0
Probability	0.6	0.4

- To get a better understanding of the market before deciding the choice of the new product, the manager hired a consulting firm to conduct a market research. According to historical data, this consulting firm can successfully predict a strong market in 65% cases and can correctly predict a weak market in 55% cases.

Assignment 1 Q3

Based on this information, which of the following statement(s) is(are) true?

(A) If the consulting firm is not hired, it is better not to introduce any product.

(B) If there is not historical data about the consulting firm (i.e., we don't know the probabilities of accurate predictions), $38 million should be paid at most to hire the company.

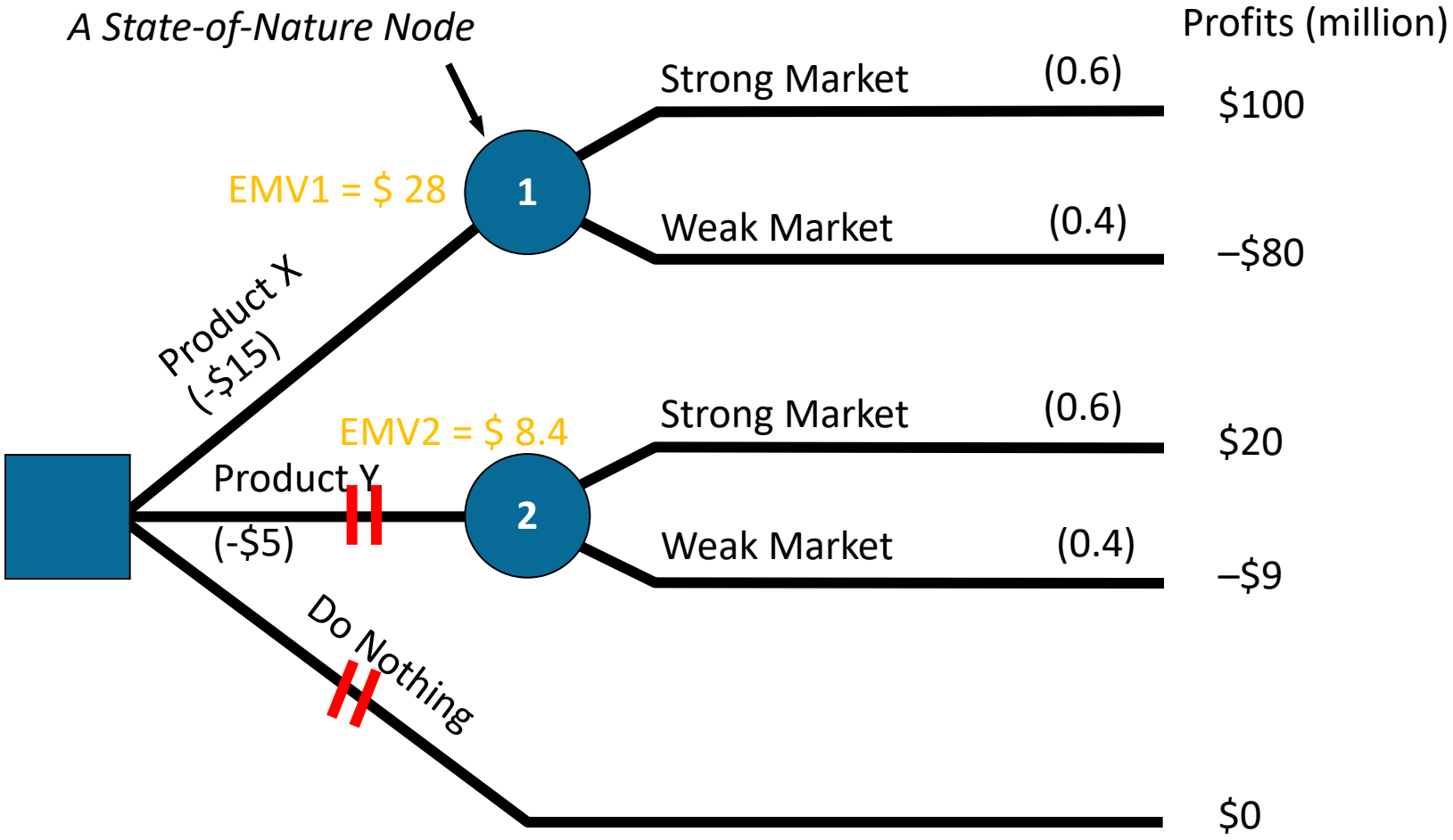
(C) If the consulting firm gave an unfavorable report, ABC should introduce product X.

(D) The expect value of the consulting firm's report is about $3.12 million.

(E) None of the above.

Assignment 1 Q3

- When consulting firm is not hired,



Without research: EMV= $13

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Assignment 1 Q3

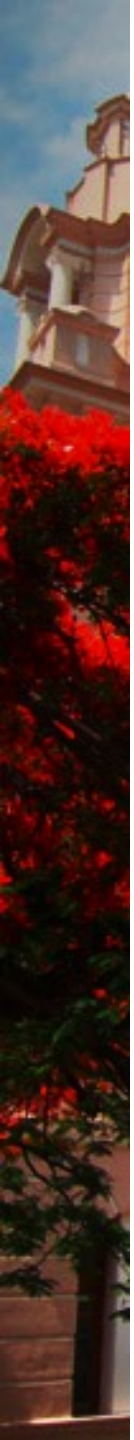
(A) If the consulting firm is not hired, it is better not to introduce any product.

Launching product X can get the best net profit, and EMV = $13

(B) If there is not historical data about the consulting firm (i.e., we don't know the probabilities of accurate predictions), $38 million should be paid at most to hire the company.

Considering the best payoff of each market

EVwPI = 0.6*(100-15)+0.4*0 = 51, so EVPI = EVwPI-EMV = $38.



Notes for Predictive Power /Accuracy Rate

- *According to historical data, this consulting firm can successfully predict a strong market in 65% cases and can correctly predict a weak market in 55% cases.*

- P(Stro |Fav) or P(Fav|Stro)?

- Think about the example of fortune tellers about Donald Trump.

- Prior belief is developed from private information and thus is usually useful.
- Historical data is obtained from other products, not the current one.
- The useful information we can obtain from data is the predictive power of ABC's survey.
- Accuracy Rate is the percentage of correct predictions for a given dataset.

Use of Historical Data

- For the current market:

Joint Prob.	Consulting Firm: Pos	Consulting Firm: Neg	Marginal
Market: Strong	0.65*0.6=0.39	0.35*0.6=0.21	0.6
Market: Weak	0.45*0.4=0.18	0.55*0.4=0.22	0.4
Marginal	0.57	0.43	

- P(Stro|Pos) heavily depends on the prior belief (strong market rate).
- The prior belief will be left unused if we calculate P(Pos|Stro) from the historical data.

Assignment 1 Q3

- When the consulting firm is hired, the belief is updated based on the consulting report:

Joint Prob.	Consulting Firm: Pos	Consulting Firm: Neg	Marginal
Market: Strong	0.65*0.6=0.39	0.35*0.6=0.21	0.6
Market: Weak	0.45*0.4=0.18	0.55*0.4=0.22	0.4
Marginal	0.57	0.43	

- If Consulting Firm: Negative:
  - P(Market: Strong|Consult Firm: Negative) = 0.21/0.43=0.4884 and
  - P(Market: Weak|Consult Firm: Negative) = 0.5116.
- EMV of launching product X is 100*0.4884-80*0.5116 -15 = -7.088,
- EMV of launching product Y is 20*0.4884-9*0.5116 -5 = 0.1636.
- Therefore, given the negative results from the consulting firm, ABC should introduce product Y(EMV = 0.1636).

Assignment 1 Q3

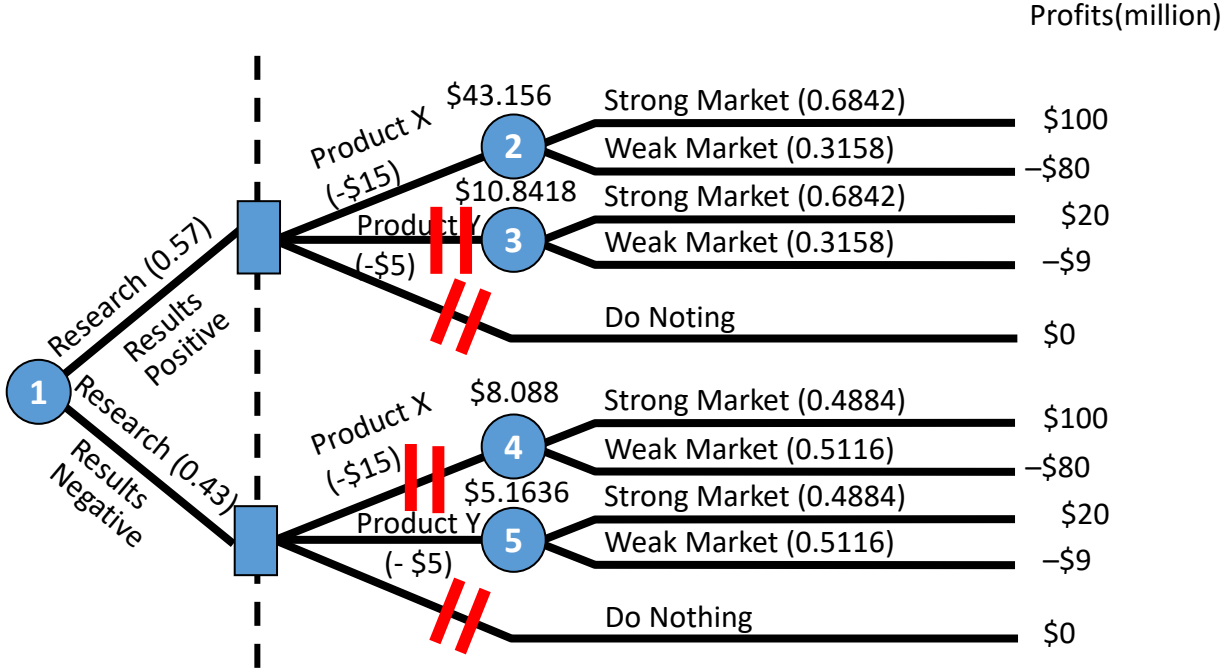
- The belief is updated based on the consulting report:

Joint Prob.	Consulting Frim: Pos	Consulting Frim: Neg	Marginal
Market: Strong	0.65*0.6=0.39	0.35*0.6=0.21	0.6
Market: Weak	0.45*0.4=0.18	0.55*0.4=0.22	0.4
Marginal	0.57	0.43	

- If Consulting Firm: Positive :
  - P(Market: Strong|Consult Firm: Positive) = 0.39/0.57=0.6842 and
  - P(Market: Weak|Consult Firm: Positive) = 0.3158.
- EMV of launching product X is 100*0. 6842-80*0. 3158 -15 = 28.156,
- EMV of launching product Y is 20*0. 6842-9*0. 3158 -5 = 5.8418.
- Therefore, given the positive results from the consulting firm, ABC should introduce product X (EMV = 28.156) .

Assignment 1 Q3

- Decision Tree with sample information



- $EMV_{withoutresearch} = 13$
- Therefore, the value of consulting firm's report is

  0.57*(28.156 - 13) + 0.43*(0.1636 - 13) = 3.119 .

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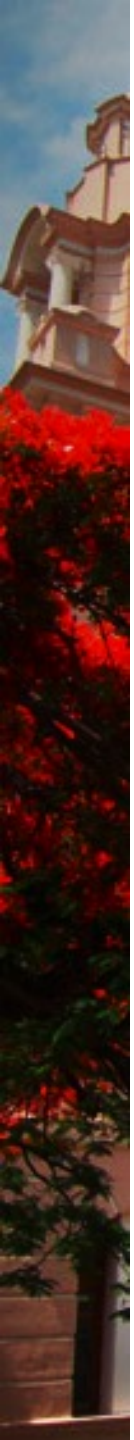
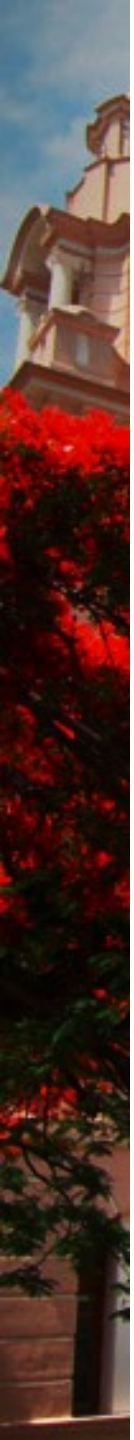


After-Class Exercise

- ***True or False?***
- Ken is good at shooting three-pointers. For each shot, the chance of success is 0.92. To simulate the total score after 10 shots, in total 10 random numbers should be generated.

After-Class Exercise

- ***True or False?***
- We want to simulate the operations of a restaurant. Assume customer dining time per table follows an exponential distribution with a mean of 1 hour. Currently, there are 20 tables of dining customers. Among them, the number of customers still eating after half an hour can be simulated by a binomial distribution.



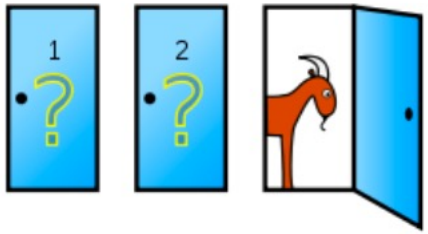
Pinevalley Bank

- The X branch of Pinevalley Bank has four service counters, but sometimes it is not necessary to open all four counters. Historical data shows that customers arrive every 10 minutes on average between 2 to 3 p.m. on a Thursday afternoon. The inter-arrival time follows exponential distribution. The service time for each customer also follows exponential distribution with a mean of 5 minutes. The question is, in order to make sure the average waiting time for each customer is less than 3 minutes, what is the minimum number of counters needed during this period of time?
- For Pinevally Bank, ***suppose only one window is always open and the second window is open only when the queue length is equal or longer than 3 people.*** Will the average waiting time for the customers satisfy the requirement?

- Please refer to “Pinevalley.xlsx”.

Review: The Monty Hall Problem

- Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

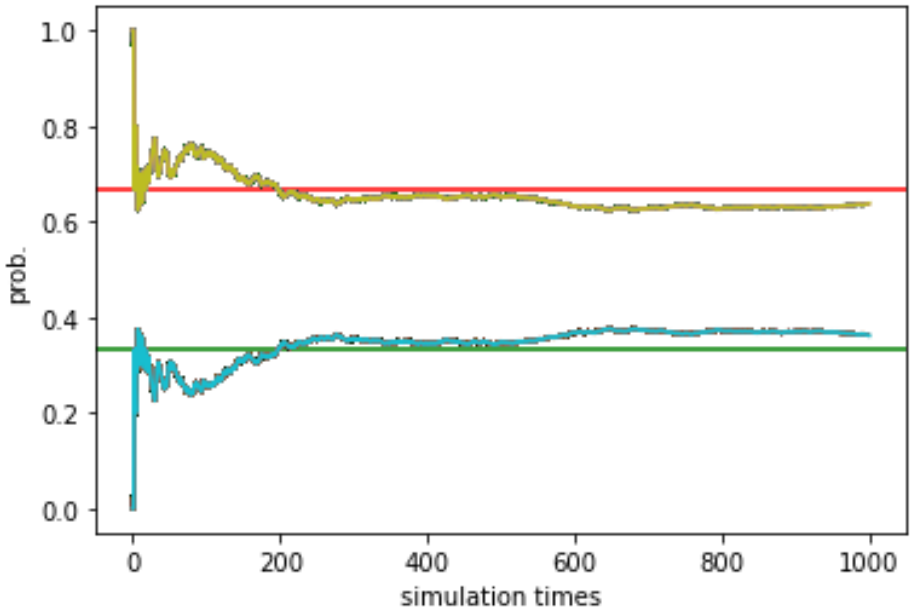


Solve with Bayes' Theorem

You picked door 1	The car is behind door 1	The car is behind door 2	The car is behind door 3	Marginal
Host opens door 1				
Host opens door 2				
Host opens door 3				
Marginal				

Solve with Monte Carlo Simulation

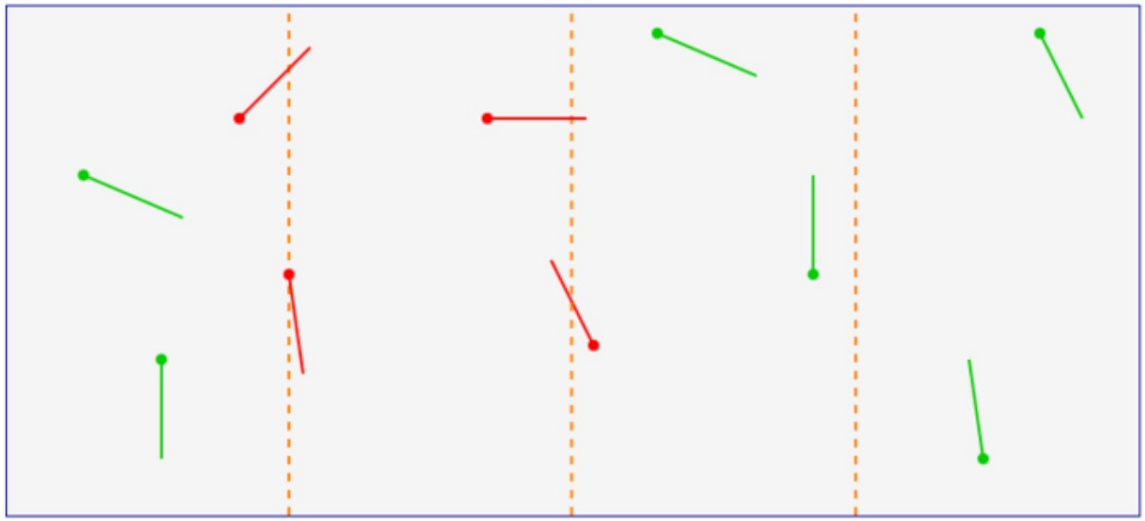
- Please refer to “monty_hall_problem.py”




```
Win prob. if you swith: 0.671
Win Prob. if you stick to your orginal choice: 0.329
```

Buffon's Needle Problem Experiment

- Buffon's Needle Problem refers to a question first posed by Georges-Louis Leclerc, Comte de Buffon: "Suppose we have a floor made of parallel strips of wood, each of the same width. If we drop a needle on the floor, what is the probability that the needle will land on a line between two strips?"



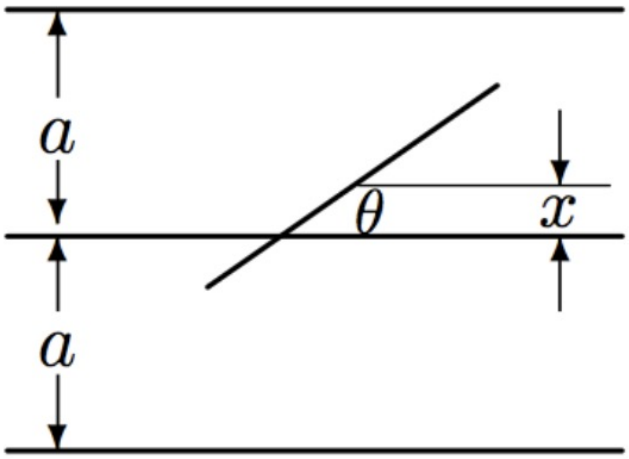
Solve with Calculus

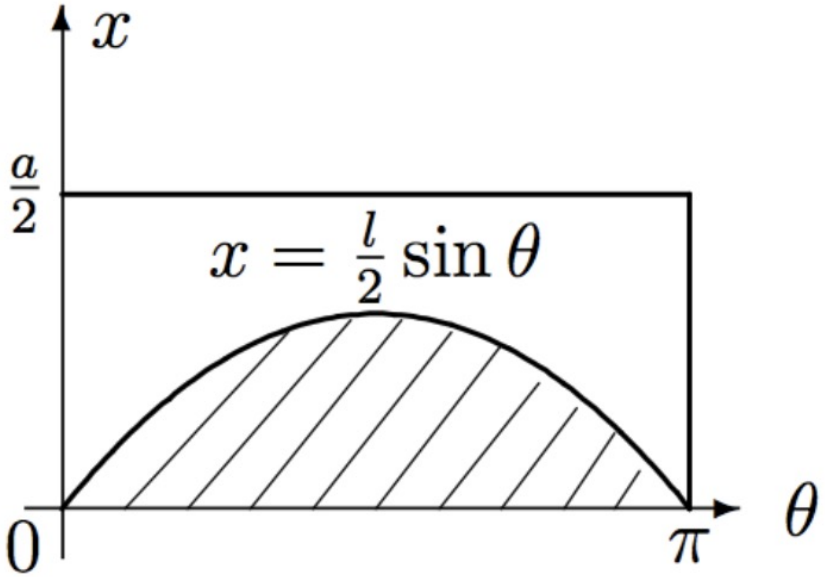
$l \leq a$

x: the distance between the midpoint of needle and the nearest strip

θ: the acngle between the needle and the bar

needle will land on a line between two strips when $x \leq \frac{l}{2}\sin\theta$





Solve with Monte Carlo Simulation

- Please refer to “buffon_needle_problem.py”

```python
#import required libraries
import numpy as np

def buffon(n,l,a):
    # n -- simulation times
    # l -- the length of needle
    # a -- distance between two strips
    k = 0
    # generating n random number x~U(0,π)
    theta = np.random.uniform(0, np.pi, n)
    # generating n random number x~U(0,a/2)
    x = np.random.uniform(0, a / 2, n)
    # k -- the number of land on a line
    k = np.sum(x <= l * np.sin(theta) / 2)
    prob = k/n
    pi = 2 * l * n / (a * k)
    print('prob.:',prob)
    print('pi:',pi)

#simulate 10**8 times
buffon(10**8,0.8,1)
```

$l = 0.8$

$a = 1$

use the property of numpy to avoid loop

```
Kristy/Documents/HKU/Cou
prob.: 0.50927851
pi: 3.141699421010323
```

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