MSBA 7004
Operations Analytics

Tutorial 4

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Learning Objectives

- Reflect newsvendor problem and marginal analysis
- Practice problems
- Review problems

SL* Example: Newsvendor Problem

- A newsvendor stocks newspapers to sell that day
- Trade-offs:
  - If stocks too few newspapers, misses potential sales.
  - If stocks too many newspapers, money wasted on unsold newspapers.

“Rocky”

How many newspapers should be stocked?

Assumptions

- One single sales period/season (i.e. inventory is not carried from period to period)
- Demand is uncertain
- Has to decide inventory quantity before demand realizes.

Solving the Newsvendor Problem

- Rocky pays $0.5 for each paper, and sells for $1.5

- Daily newspaper demand distribution:

Demand:	87	88	89	90	91	92	93
Probability:	0.03	0.07	0.2	0.4	0.2	0.07	0.03

- If Rocky buys 87 papers, profit = $_**87**___

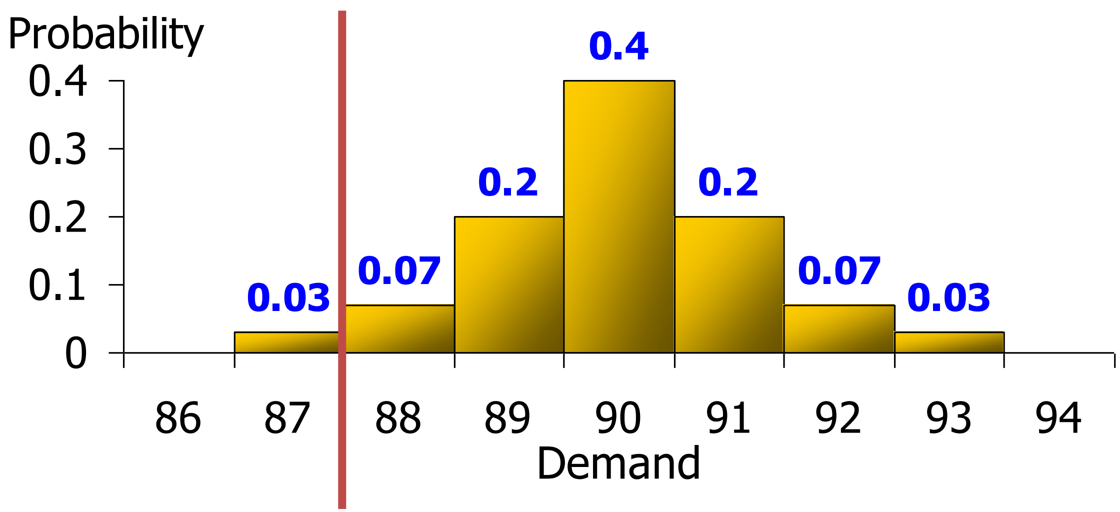
- Should Rocky buy 88? Let's see…

Marginal Analysis: 88th paper

- With probability **0.03**, the 88th paper will not be sold, and it costs Rocky $**0.5**
- With probability **0.97**, the 88th paper will be sold and brings Rocky profit of $**1.0**
- Cost < Benefit

0.03*$0.5=$0.015 0.97*$1.0=$0.97

- Should Rocky buy the 88th paper?

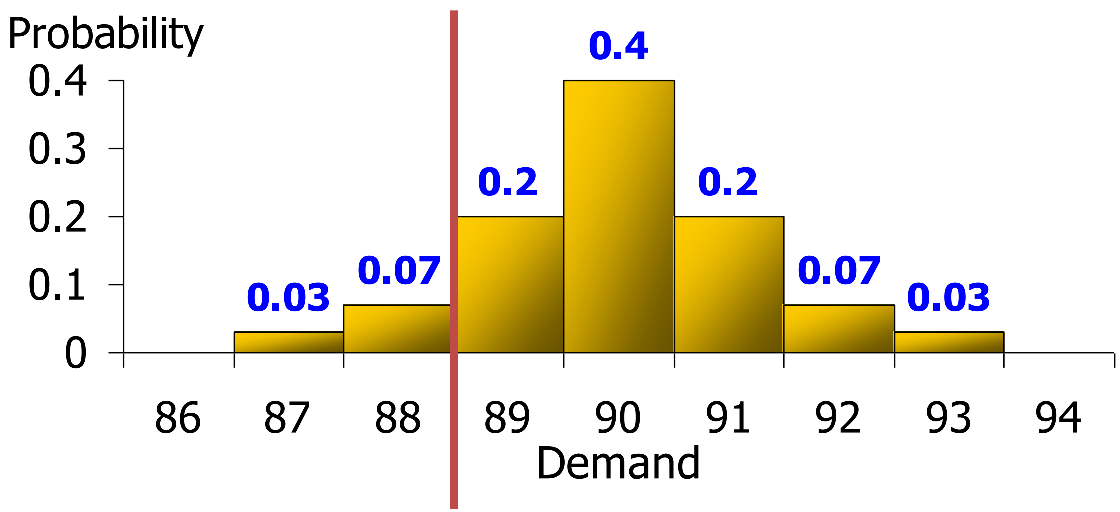


Marginal Analysis: 89th paper

- With probability **0.10**, the 89th paper will not be sold, and it costs Rocky $**0.5**
- With probability **0.90**, the 89th paper will be sold and brings Rocky profit of $**1.0**
- Cost < Benefit

0.10*$0.5=$0.05 0.90*$1.0=$0.90

- Should Rocky buy the 89th paper?

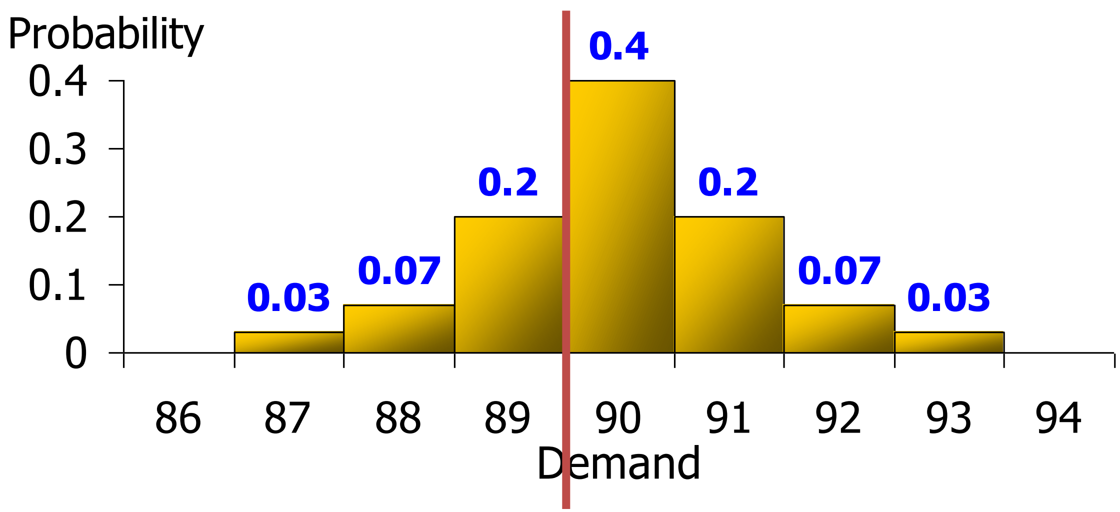


Marginal Analysis: 90th paper

- With probability **0.30**, the 90th paper will not be sold, and it costs Rocky $**0.5**
- With probability **0.70**, the 90th paper will be sold and brings Rocky profit of $**1.0**
- Cost < Benefit

0.30*$0.5=$0.15 0.70*$1.0=$0.70

- Should Rocky buy the 90th paper?

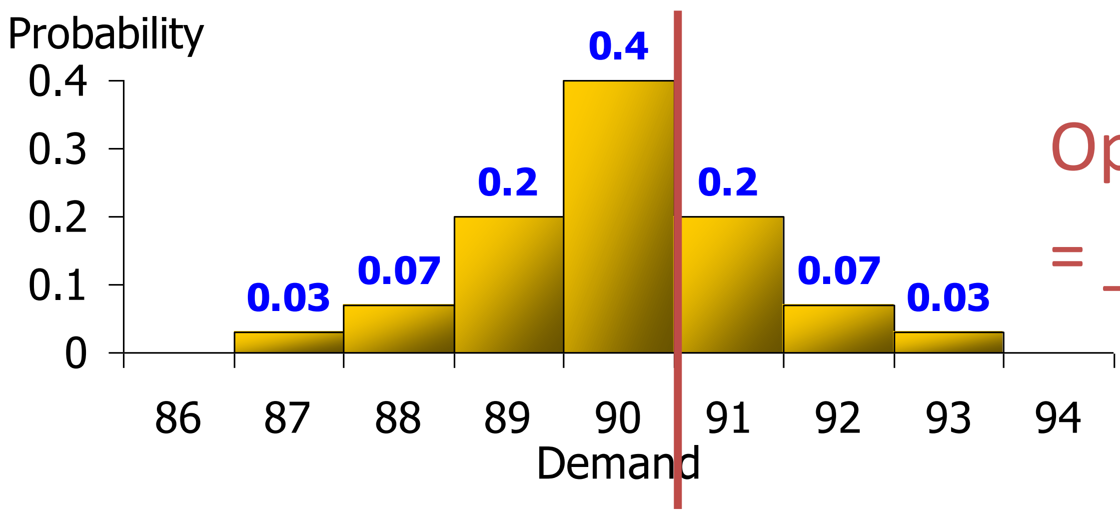


Marginal Analysis: 91th paper

- With probability **0.70**, the 91th paper will not be sold, and it costs Rocky $**0.5**
- With probability **0.30**, the 91th paper will be sold and brings Rocky profit of $**1.0**
- Cost > Benefit

0.70*$0.5=$0.35 0.30*$1.0=$0.30

- Should Rocky buy the 91th paper?



Optimal quantity = **90**

Marginal Analysis: Generalized

Notation:

C_o = Cost of over-stocking a unit (or marginal cost)

C_u = Cost of under-stocking a unit (or marginal benefit)

- Buy the $Q+1^{st}$ unit, as long as its cost < benefit:

$$P(D \leq Q)\, C_o \quad < \quad P(D > Q)\, C_u$$

or equivalently:

$$P(D \leq Q) \quad < \quad C_u / (C_u + C_o)$$

- Stop at Q^* when cost > benefit (inequality reverses):

$$P(D \leq Q^*) \quad \geq \quad C_u / (C_u + C_o)$$

- Q^* is the smallest quantity such that

$$P(D \leq Q^*) \geq \underbrace{C_u / (C_u + C_o)}_{\text{Critical Ratio}}$$

How to Interpret C_u and C_o? Important!

C_u , C_o are the *per unit opportunity costs*.

You should compute C_u in this way:

Suppose the actual demand D is larger than my current stock S. If I could return to the past and order one unit more, then it increases my total profit (or reduces my cost) by C_u.

You should compute C_o in this way:

Suppose the actual demand D is smaller than my current stock S. I could return to the past and order one unit less, then it increases my total profit (or reduces my cost) by C_o.

Example 1

Crazy Jo runs a tube rental on Huron River (Class 0.5 Rapids). He currently leases tubes from a dealer in Hong Kong at a cost of $10 per day. On Saturdays, he picks up the tubes and drives to a launching point on the river, where he rents tubes to white-water enthusiasts for $30 per day. Crazy Jo records the Saturday demand for tubes and finds the experience for the past 20 Saturdays below:

Demand	10	11	12	13	14	15	16	17	18	19	20
Frequency(days)	1	1	2	2	2	3	3	2	2	1	1

How many tubes should Crazy Jo lease from the dealer?

C_o = **10**

C_u = **30-10 = 20**

SL* = **20/(10+20) = 2/3**

Example 1

Demand (D)	Frequency	Probability	Q	SL P (D ≤ Q)
10	1	1/20=0.05	10	0.05
11	1	1/20=0.05	11	0.1
12	2	2/20=0.10	12	0.2
13	2	2/20=0.10	13	0.3
14	2	2/20=0.10	14	0.4
15	3	3/20=0.15	15	0.55
16	3	3/20=0.15	16	0.7
17	2	2/20=0.10	17	0.8
18	2	2/20=0.10	18	0.9
19	1	1/20=0.05	19	0.95
20	1	1/20=0.05	20	1.00

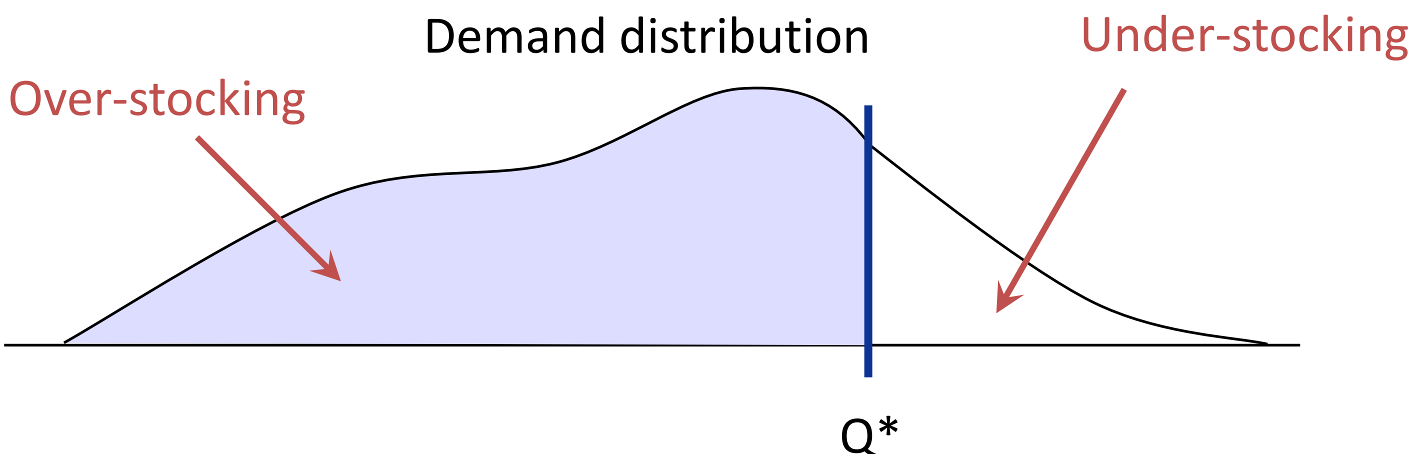
20

Q*= 16

Demand with Continuous Distribution

- The best Q* can be found directly from

$$P(D \leq Q^*) = C_u / (C_u+C_o)$$



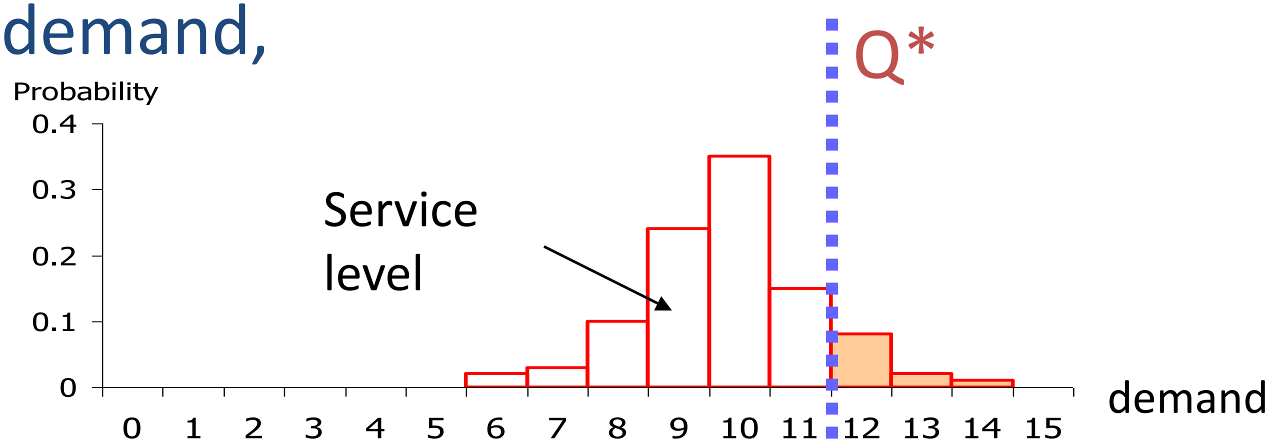
Q* is the smallest quantity such that
$P(D \leq Q^*) = SL^* = C_u / (C_u+C_o)$

Best Service Level (SL*) = Critical Ratio

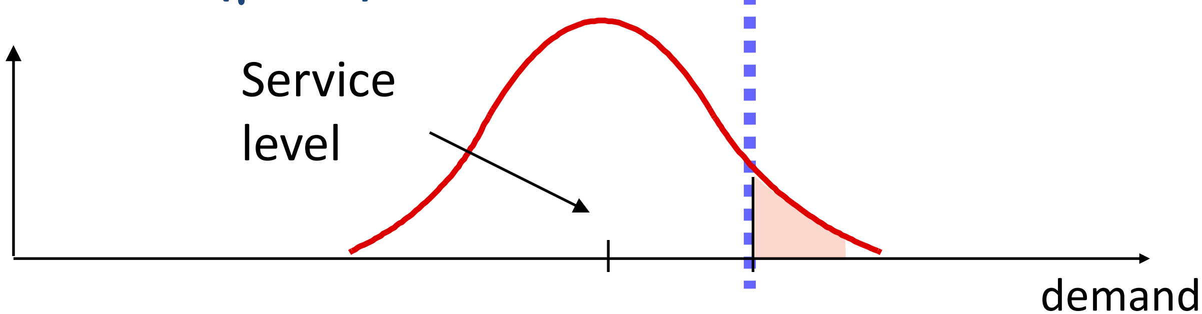
How to Compute Q* Given Critical Ratio

- With discrete demand,

Q* = smallest quantity such that $SL \geq C_u / (C_u + C_o)$



- With normal demand N(μ, σ)



z = NORM.S.INV($C_u / (C_u + C_o)$)

Q* = NORM.INV(Cu / (Cu+Co),μ,σ)

$$Q^* = \mu + z\,\sigma$$

Example 2

- During the Final Four, demand for basketball jerseys is expected to be normally distributed with mean 1,000 and standard deviation 300. Each jersey costs \$30. The selling price is \$50 per jersey. How many jerseys should be ordered?

$C_o =$ **30** $C_u =$ **20**

SL*= **20/(30+20) =0.4**

z = **-0.253** Q* = **1000-0.253*300 = 924.1**

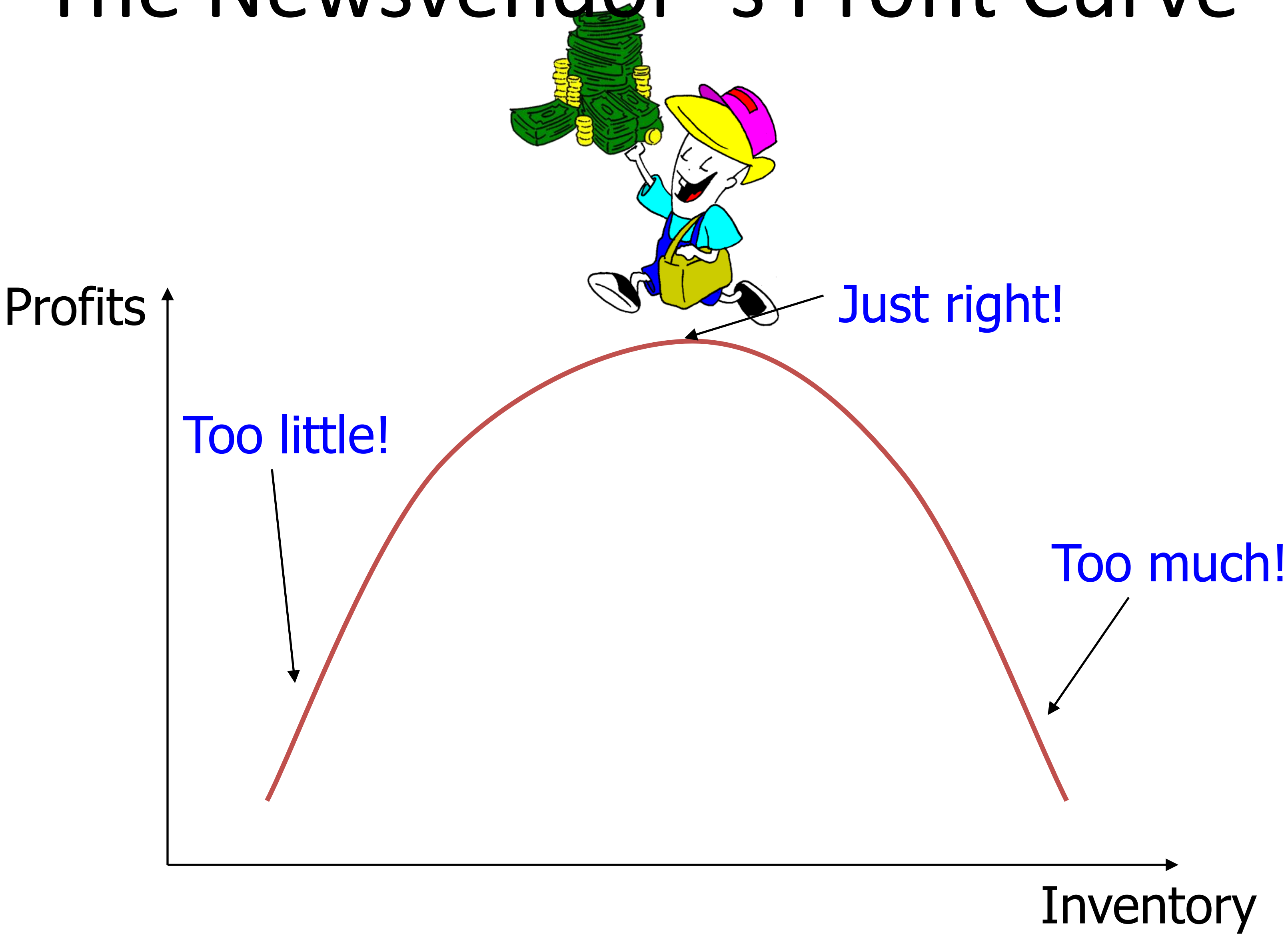
- What if the unsold jerseys can be sold off at half price?

$C_o =$ **30-25=5** $C_u =$ **20**

SL*= **20/(5+20) =0.8**

z = **0.8416** Q* = **1000+0.8416*300 = 1252.5**

The Newsvendor's Profit Curve



Practice Questions for today

Practice Problem 1

Happy Henry's car dealership sells a popular model EX3. Every month, a shipment of the cars is made to Happy Henry's. Lead time is negligible. Monthly demand for EX3 is distributed as N(60, 15). The average cost of holding an EX3 for one year is $1500. In case of shortage, customers are willing to wait, but there is an extra bookkeeping cost of $75 per customer and a loss-of-goodwill estimated to be $600 per customer. How many cars should Happy Henry's order up to every month (i.e., Target Stock Level)?

c_u = Good-will cost + bookkeeping = 600 + 75 = $675

c_o = Monthly holding cost = 1500 / 12 = $125

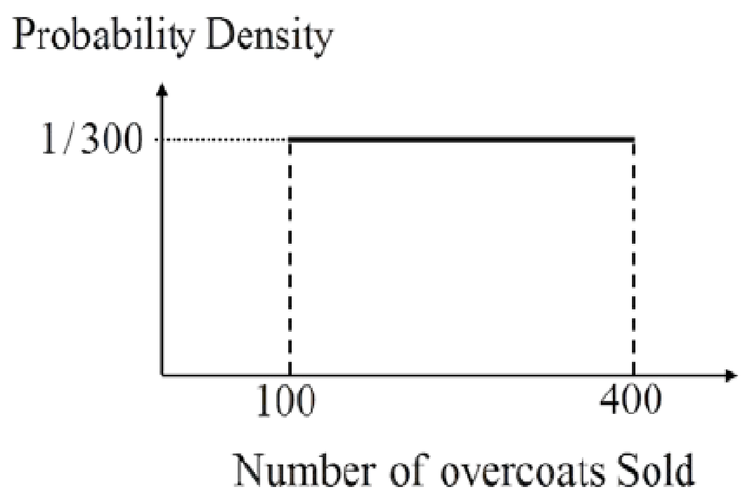
SL* = 675 / (675 + 125) = 0.8438

z = NORMSINV(0.8438) = 1.01

Target Stock Level = 60 + 15×1.01 = 75.15

Practice Problem 2: Fashion statement

Natalie Attired is a buyer for a men's fashion retail store. She will be ordering a new cloth overcoat from Paris for the fall fashion season. Based on her experience, she expects to sell at least 100 coats, and at most 400, but she feels that any number of sales in between is equally likely. Therefore, she estimates that her sales are uniformly distributed between 100 and 400:

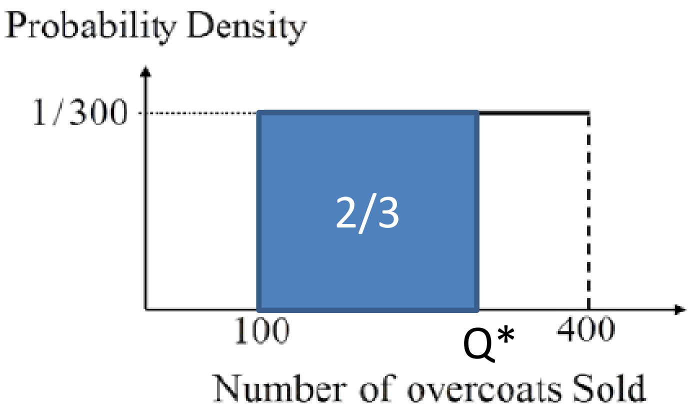


The total cost to the store is \$100 per coat, and the retail price is set at \$180. Any coats left over at the end of the season would be sold at \$60 each.

a) How many coats should Natalie buy if she wants to maximize profits?

- Co = Cost of over-stocking a overcoat(marginal cost)=100-60=40
- Cu = Cost of under-stocking a overcoat(marginal benefit) =180-100=80

$$\text{Prob}\ (D \leq Q^*) = 80/(80+40) = 2/3$$



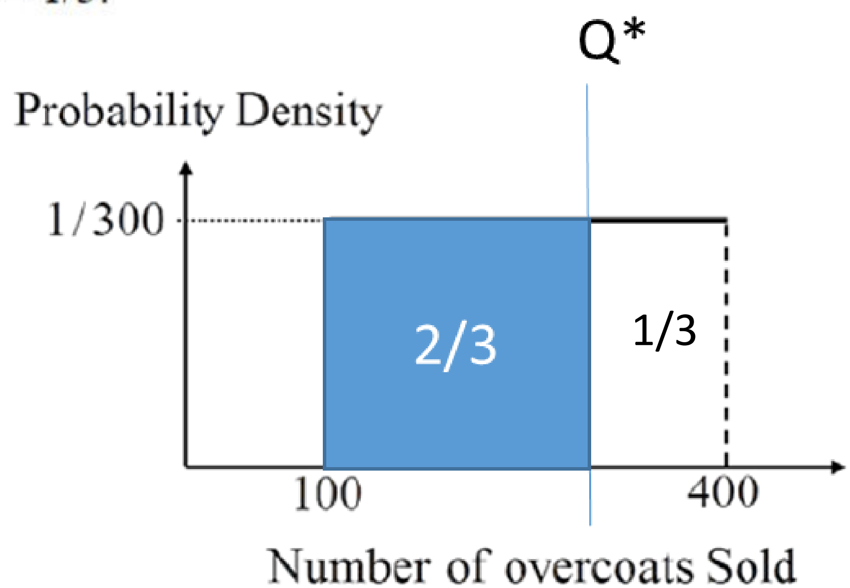
where 'D' is the unknown demand for coats. Since demand is uniformly distributed between 100 and 400, Q* is 2/3 of the way between these limits $\boxed{Q^* = 300.}$

For the remainder of this question, assume that Natalie buys the number of coats suggested in part (a):

b) What is the probability that the coats sell out? What is the probability that they do not sell out?

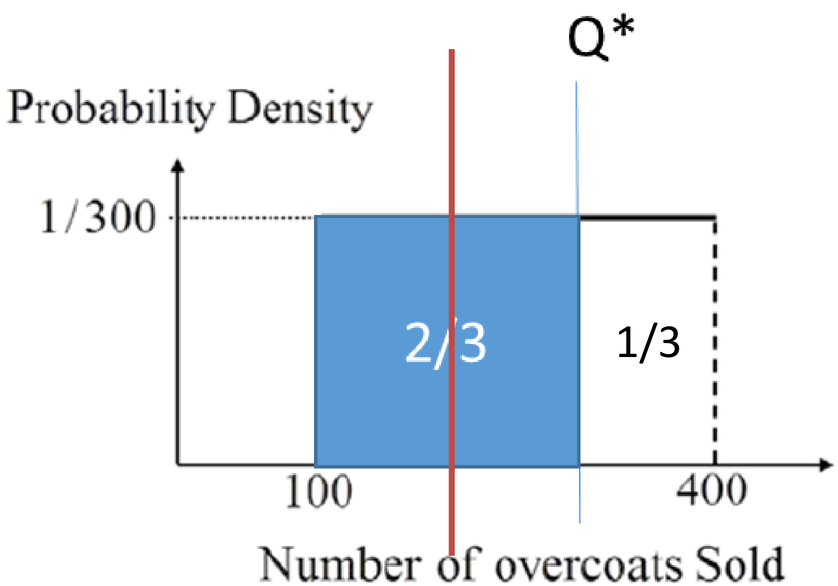
Prob (coats do not sell out) = Prob($D \leq 300$) = 2/3.

Prob (coats do sell out) = Prob ($D > 300$) = 1/3.



c) <u>Given that the coats do not sell out, what is the expected number of coats sold?</u>

Given that coats do not sell out, actual sales are uniformly distributed between 100 and 300. Therefore, the expected number sold is 200 coats.

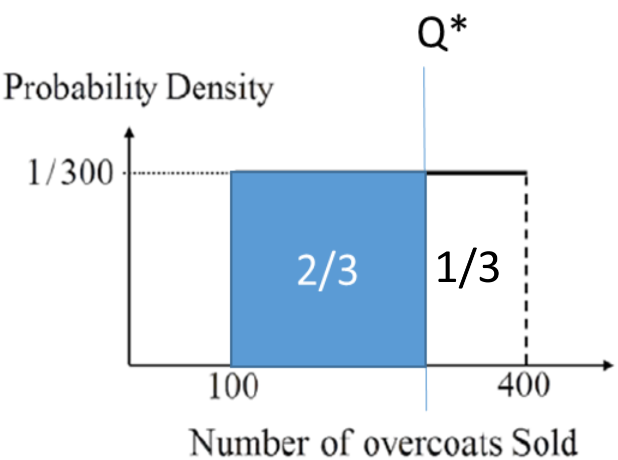


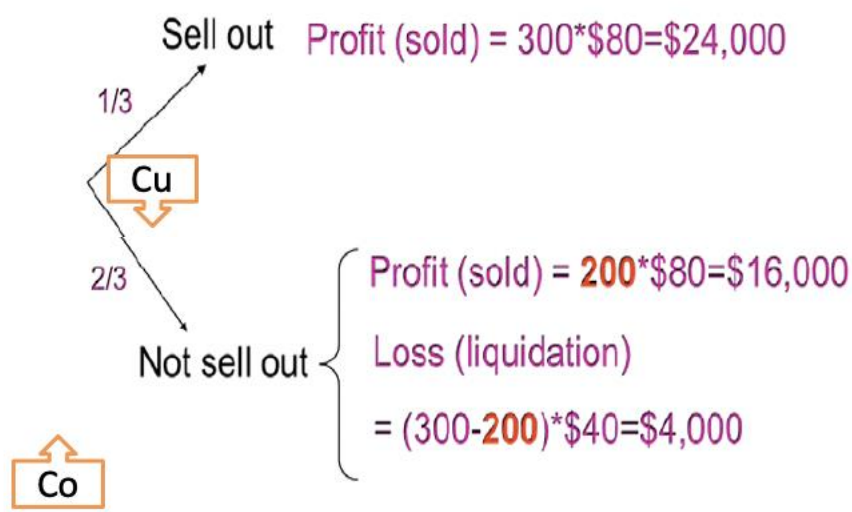
Expected sales | not sell out

Average = 200 coats

d) What is the expected profit from sales of this coat?

A probability tree might help:





Expectation profit = 1/3 * 24,000+2/3*($16,000-$4,000)

= $16,000

- Alternatively,
- Expected Profit = (p-c)*expected sales – (c-s)*expected excess inventory where p is retail

$$(p-c)\cdot E[\min(Q^*, D)] - (c-s)\cdot E[(Q^*-D)^+]$$

$p = 180,\ c = 100,\ s = 60$

$Q^* = 300 \quad D \sim U(100, 400)$

$$80\cdot\int_{100}^{400} \min(300, x)\,\frac{1}{300}\,dx - 40\cdot\int_{100}^{400} (300-x)^+\cdot\frac{1}{300}\,dx$$

$$= \frac{80}{300}\left[\int_{100}^{300} x\,dx + \int_{300}^{400} 300\,dx\right] - \frac{40}{300}\cdot\left[\int_{100}^{300} 300-x\,dx\right] + \cancel{\int_{300}^{400}}$$

$$= \frac{80}{300}\cdot\left[\frac{1}{2}x^2\Big|_{100}^{300} + 300\cdot 100\right] - \frac{40}{300}\cdot\left[300\cdot 200 - \frac{1}{2}x^2\Big|_{100}^{300}\right]$$

$$= \frac{80}{300}\left[\frac{1}{\cancel{2}}\cdot 400\cdot \cancel{2}00 + 300\cdot 100\right] - \frac{40}{300}\cdot\left[300\cdot 200 - \frac{1}{\cancel{2}}\,400\cdot 200\right]$$

$$= \frac{80\cdot 700\cancel{0}}{3\cancel{00}} - \frac{40}{3\cancel{00}}\cdot 200\cancel{00}$$

$56 - 8 = 48$

$$= \frac{56000 - 8000}{3} = 16000$$

Practice Problem 3:

- TVN is a television station that has 25 thirty-second advertising slots during each evening. The station is now selling advertising for the first few days in November. They could sell all the slots now for $4,000 each, but because in November 7 there will be an important sport event, the station may be able to sell slots to sport brands at the last minute for a price of $10,000 each. For now, assume that a slot not sold in advance and not sold at the last minute is worthless to TVN.
- To help make this decision, the sales force has created the following probability distribution for last minute sales:

Demand d	**Probability Prob(D = d)**	**Cumulative Prob(D ≤ d)**
8	0.00	0.00
9	0.05	0.05
10	0.10	0.15
11	0.15	0.30
12	0.20	0.50
13	0.10	0.60
14	0.10	0.70
15	0.10	0.80
16	0.10	0.90
17	0.05	0.95
18	0.05	1.00
19	0.00	1.00

a)Determine the overage cost and the underage cost per unit for last minute sales. How many slots should TVN sell *in advance*?

$C_u = \$10{,}000 - \$4{,}000 \; C_o = \$4{,}000$

$$p = \frac{C_u}{C_u + C_o} = \frac{6{,}000}{10{,}000} = 0.6$$

Therefore, TVN should sell 25 – 13 = 12 slots in advance.

Q* is the smallest quantity s.t. P(D≤Q) (SL ≥ CR)

b) Now suppose that if a slot is not sold in advance and is not sold at the last minute, it may be used for a promotional message worth \$2,500. Now how many slots should TVN sell in advance?

$C_u = \$10{,}000 - \$4{,}000 \; C_o = \$4{,}000 - 2{,}500$

$$p = \frac{C_u}{C_u + C_o} = \frac{6{,}000}{7{,}500} = 0.8$$

In this case, TVN should sell 25 – 15 = 10 slots in advance.

Practice Problem 4:

You operate a small tee-shirt silk screening company. Your local professional football team has just made it to the championship game, which will be played in one week. You are considering selling tee-shirts with “Champions” printed across the top of the shirt along with the local team’s name and mascot. If the team indeed wins, these shirts will sell extremely well, but if they lose, then there will clearly be no demand for these shirts. Unfortunately, you must print the shirts before the game because otherwise there will not be enough time to take advantage of the hype. You estimate that there is an *x*% chance the local team will win, and if they win, you figure your demand will be normally distributed with mean 15,000 and standard deviation 6,000. It costs $5.50 to buy and silk screen each shirt, you will sell these shirts for $12.50 each, and any leftover shirts (whether the team wins or loses) can be sold to a liquidator for $0.50 each.

(a) If x = 100 (i.e. x% = 100%), how many shirts should you print to maximize your expected profit?

Co = 5.5 – 0.5 = 5
Cu = 12.5 – 5.5 = 7
Critical ratio = Cu / (Cu + Co) = 7/12 = 0.583
z = 0.21 (from the table)
Because the demand is normally distributed (when x = 100), S* = 15,000 + 0.21 * 6,000 = 16,260

(b) If x = 40, how many shirts should you print to maximize your expected profit?

- P($D \leq S^*$) = P(D≤ S^*|win)*P(win)+ P(D≤ S^*|lose)*P(lose)
- (P(D≤ 0|lose)=1 Since when the team lose, the demand would be 0;)
- P($D \leq 0$) = P(D≤ 0|win)*0.4+ P(D≤ 0|lose)*0.6

  = P(D≤ 0|win)*0.4+ 0.6 ≥ 0.583

So the best strategy is to choose S* = 0

One frequently asked question:
if X changes and $P(D<=0)$ becomes less than the critical ratio, how should we calculate the Q*?
Say if X=60, then $P(D<=0) = 0.4 < 0.583$. In this case, can I use 0.183 (0.583 - 0.4) to calculate the z-score and use that z-score to get the Q*?

$$P(D \leq Q) = P(D \leq Q|win)*0.6 + P(D \leq Q|lose)*0.4$$
$$= P(D \leq Q|win)*0.6 + 0.4 = 0.583$$
$$P(D \leq Q|win) = (0.583 - 0.4)/0.6$$

Find the z-score

Compute Q*

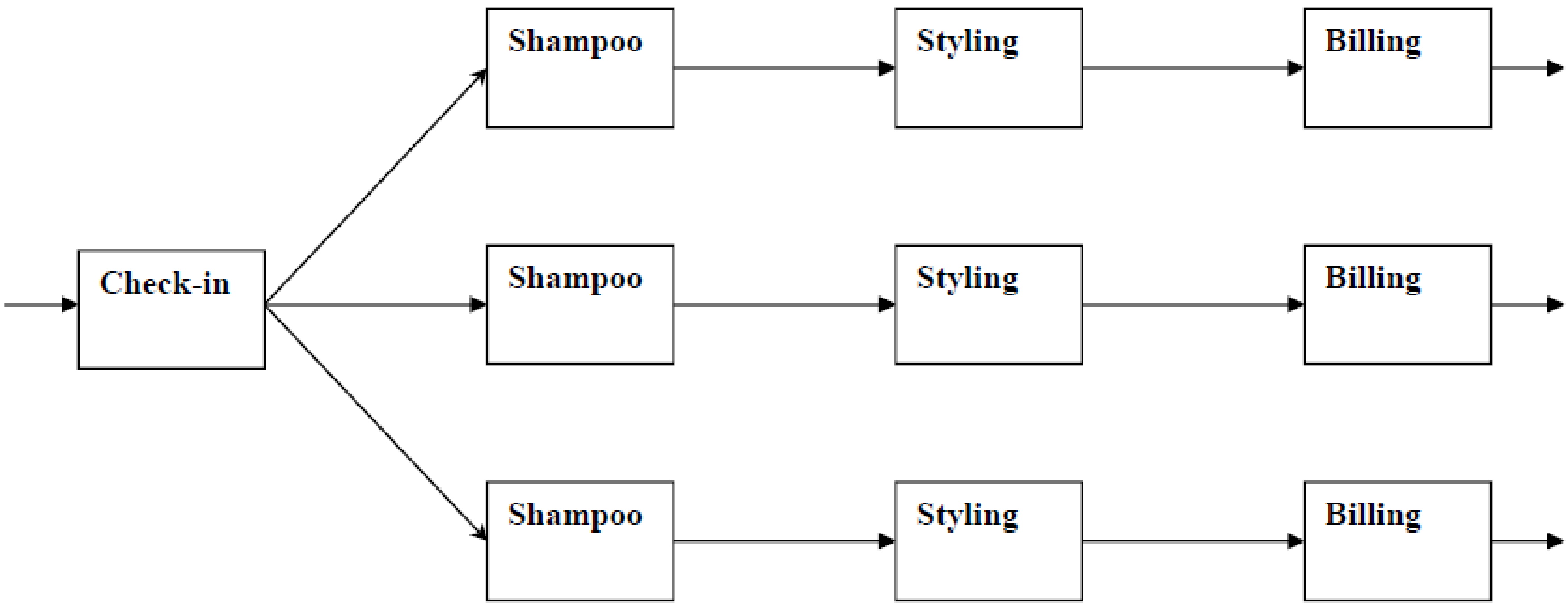
Review Practice Questions

Practice 1

- Three hairstylists, A, B, and C, run ‘Fantasia Hair Salon’ in the Kennedy Town area of Hong Kong. A’s younger sister Lulu works as the receptionist. They stay open from 7:00am to 9:00pm in order to accommodate as many people’s work schedules as possible. They perform only shampooing and hairstyling activities.

- On arrival, every customer first checks in with Lulu. This takes only 4 minutes. One of the three stylists then takes charge of the customer and performs all three activities (shampooing, styling, and billing) consecutively. On average, it takes 8 minutes to shampoo, 12 minutes to style the hair, and 4 minutes to bill the customer.

Process flow diagram



a) What is the number of customers that can be serviced per hour in this hair salon?

- *The capacity rate of Lulu in this case is **15 customers/hour***
- *The capacity rate of each stylist (Shampooing, Styling, Billing) is **2.5 customers/hour.***
- *Number of customers that can be served per hour is **3*2.5=7.5***

b) An operations specialist has suggested that the billing operation be transferred to Lulu. What would be the capacity rate of the hair salon if the hair salon accept this suggestion?

- *The capacity rate of Lulu (Checking + Billing) in this case is **7.5 customers/hour***
- *The capacity rate of each stylist is 3 customers/hour; **3*3=9 /hour***
- *Number of customers that can be served per hour is **7.5** since Lulu is now the bottleneck.*

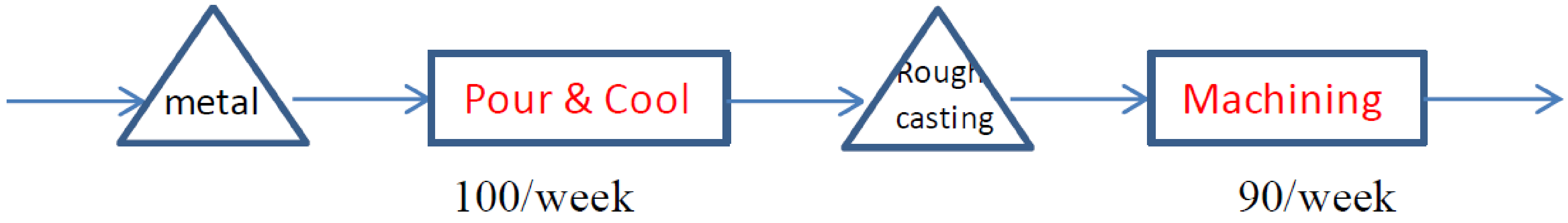
Practice 2

Mendon Metals produces precision metal castings (castings are shells for pumps, turbines, and other heavy-use equipment). The castings are manufactured in two steps. First, heated metal is poured into a mold and allowed to cool in a special storage area. Then, the rough casting is machined so that holes and seams are smooth. After pouring and cooling, 25% of the castings contain air bubbles and cannot be sold. Unfortunately, this defect is not discovered until after machining, when the air bubbles can be seen on the surface. This means that both good and defective castings are sent through the machining step. The statistics for the two processes are:

Stage	Maximum Capacity	% of Castings with Defects Introduced	Average Flow Time
Pouring and cooling	100/week	25%	3 weeks
Machining	90/week	0%	2 weeks

- The average casting is sold for about $250. The cost of materials used in each finished casting is $40 and other marginal costs (labor, energy) add up to $60 per casting. Finally, Mendon Metals can sell any good casting it can produce.

Process flow diagram for this casting manufacturing process.



a) What is the maximum throughput of castings (including good and bad products) for the entire process? What stage is the bottleneck?

- *The capacity of the process is 90 castings /week. The bottleneck is the machining.*

b) Assume that the system is operating at its maximum throughput (as calculated in part b). What is the throughput of *good* castings?

- *90/week*(1-25%)=67.5 castings/week*

c) Calculate the average value of the materials in inventory (in $). (Suppose the material is worthy of $40 per casting before machining gets finished)

- *Flow time = sum of flow time for the two stages = 3 weeks +2 weeks =5 weeks*
- *Little's Law: Inventory = throughput rate* flow time = 90 castings/week*5 weeks = 450 castings Value of Inventory = 450 castings * 40$/casting =18000 $*

Practice 3

- A fishing fleet delivers fresh fish to a cannery factory, where the fish is canned. Suppose that daily input to the cannery follows the following pattern:

(1) 100 tons/hr. between 8 a.m. and 12 noon,

(2) 75 tons/hr. between 1 p.m. and 3 p.m.,

(3) 100 tons/hr. between 3 p.m. and 5 p.m.

- Note that there is no input between 12 noon and 1 p.m. The processing capacity of the cannery is:

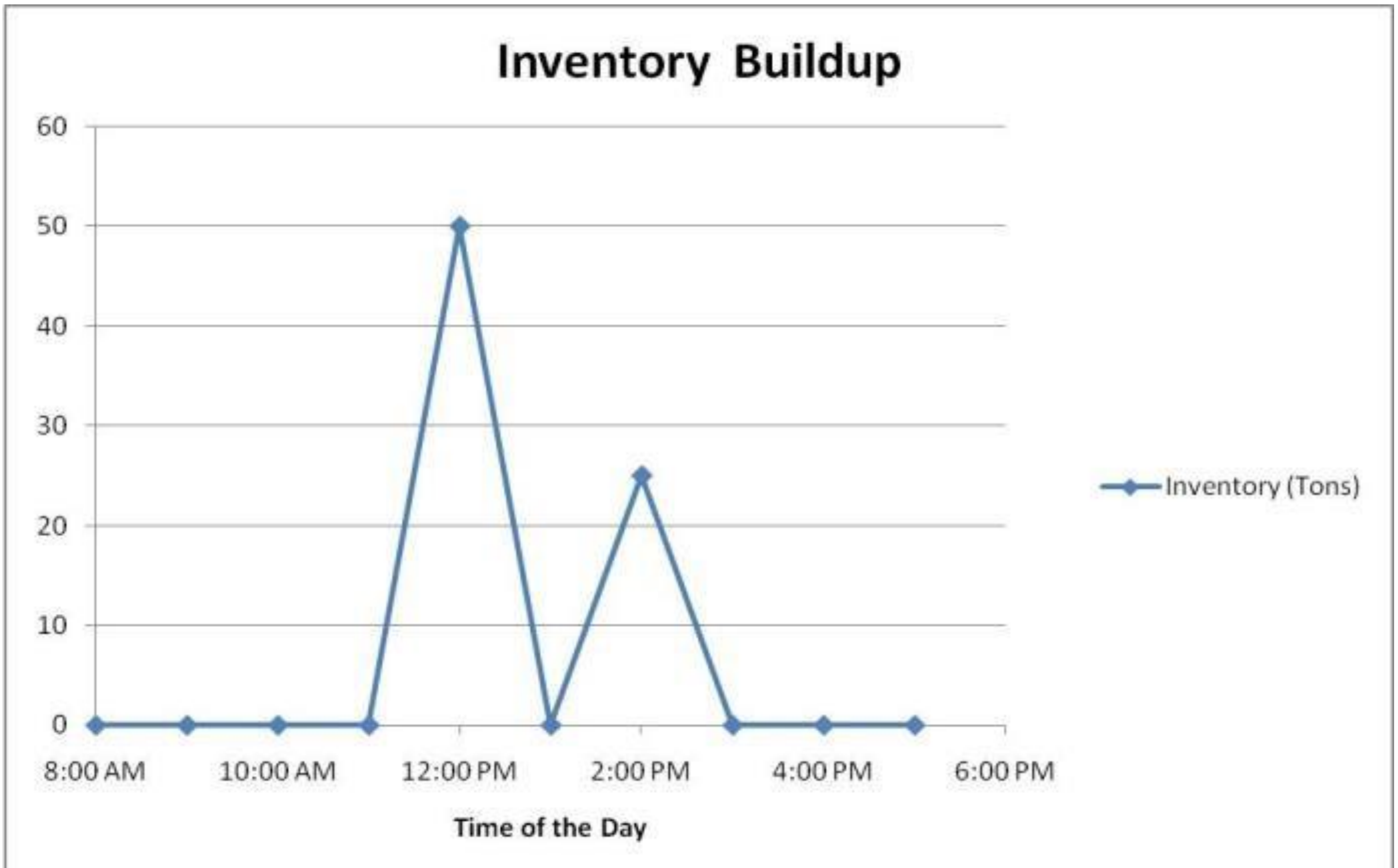
(1) 100 tons/hr. between 8 a.m. and 11 a.m.

(2) 50 tons/hr. between 11 a.m. and 2 p.m.

(3) 100 tons/hr. between 2 p.m. and 5 p.m.

Assume that there is unlimited freezer space for unprocessed fish. (Assuming Inventory builds up continuously.)

a) Draw an inventory buildup diagram that shows inventory of unprocessed fish in the freezer starting at 8 a.m. and ending at 5 p.m.



b) Consider a fish that enters the cannery between 8 a.m. and 12 at noon. What is the average time (in minutes) spent by such a fish in the freezer before being packed?

- Average time =
- [Area below the curve from 8 am – 1 pm (the last fish who enters 8 - 12 leaves at 1 pm)] divided by [tons of fishes enter during 8 – 12]
- = 50 ton*hours/400 tons = 1/8 hours = 7.5 minutes.

- All the best to your final exam!
- Hope those tutorials help!
- Thank you all!