Recommender System (MSBA 7027)

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2023


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Good luck and thanks for helping!

Recommender Systems

- Overview
- Content-based
- Collaborative filtering
  - User-based Collaborative filtering
  - Item-based Collaborative filtering
- Latent-factor model

Recommender Systems: Overview

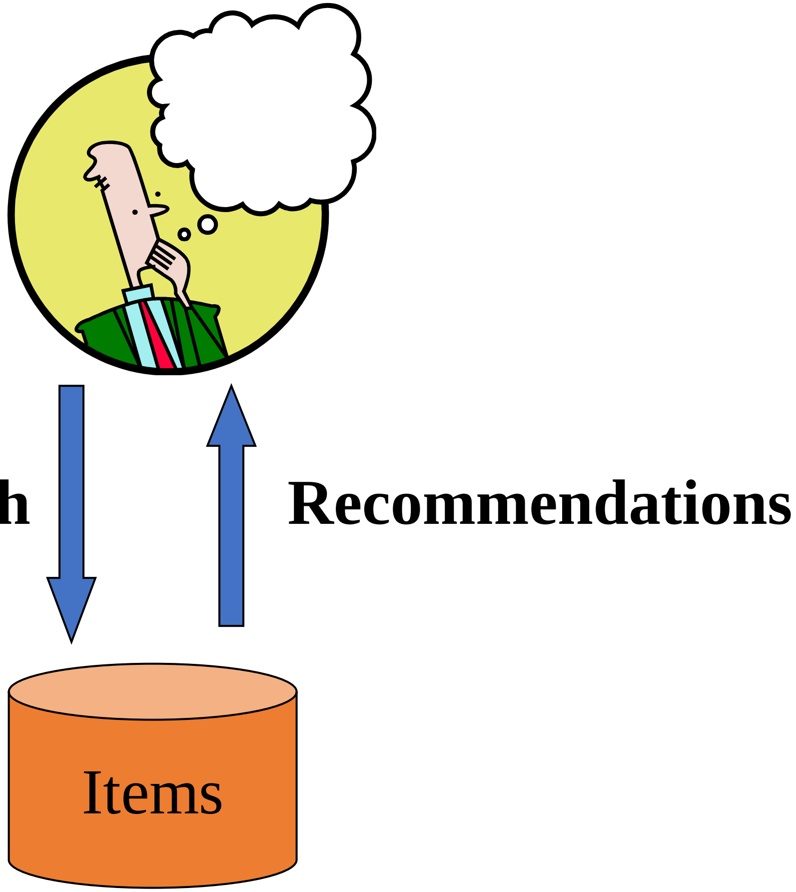
- Imagine a situation: user interacts with large catalog of items

Recommender Systems: Overview

- Imagine a situation: user interacts with large catalog of items

- Key feature: #items large, e.g. millions/billions
- Moreover, user doesn't know exactly what he/she wants

- Two ways user interacts with platform
  - Search
  - Recommendation



Recommender Systems: Overview

- **Why recommendation becomes so important in the last 10 years**
  - From an era of scarcity to an era of abundance

- Past: Era of scarcity
  - E.g. Shelf space is a scarce commodity for traditional retailers (like Walmart)

- Now: Era of abundance
  - E.g. Amazon/Taobao
  - Web enables near-zero-cost dissemination of information about products

- Gives rise to the “Long tail” phenomenon

Long Tail

Imagine a graph

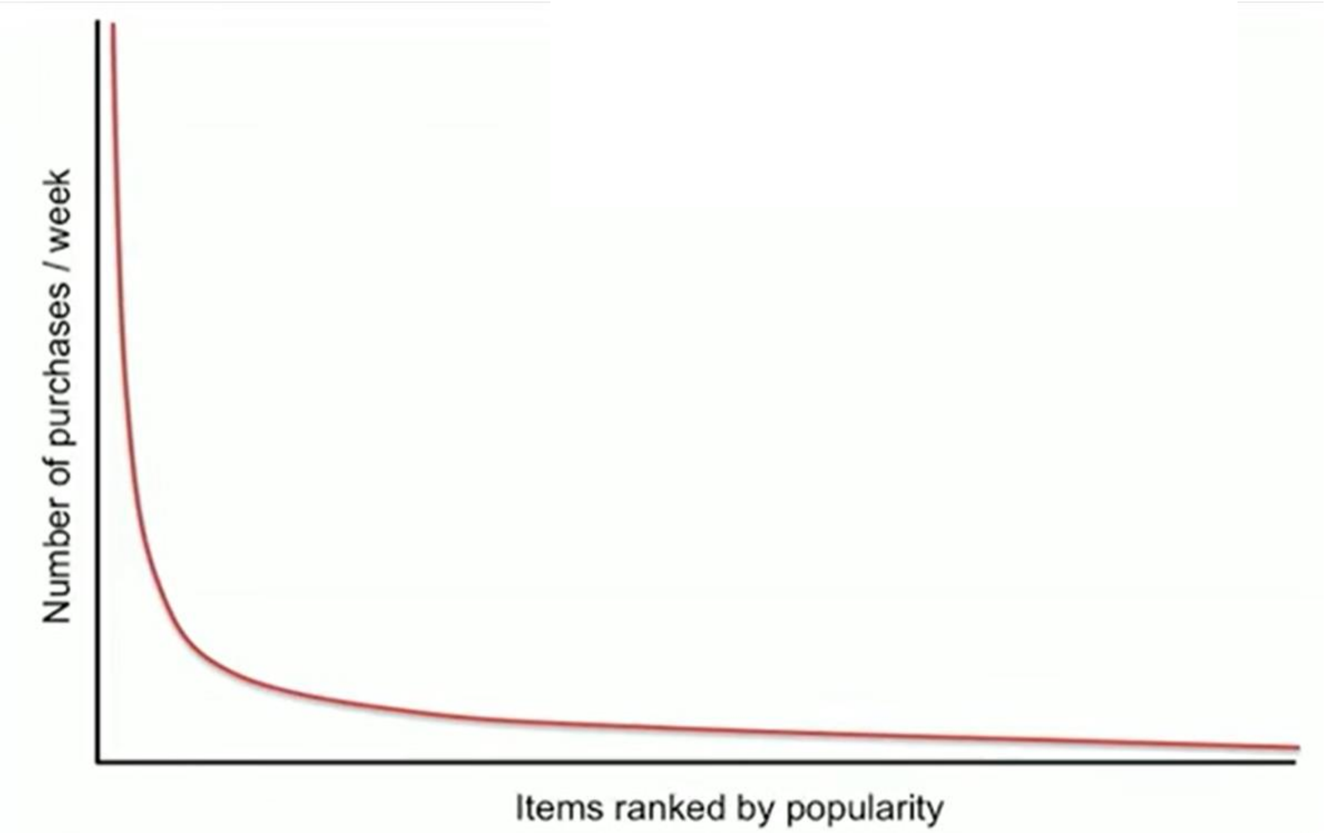
- X: items from a catalog (ranked by popularity)
  - Decreasing levels of popularity (left: more popular)
- Y: Actual popularity

Curve

- steep fall initially
  - Very few super-popular items
- Then slope becomes less steep
  - Never really reach the x-axis

Certain item

- purchase frequency not very high, but also not very low
  - e.g. once a week
- Not worthwhile for physical retailer to stock this item



(Left: more popular)

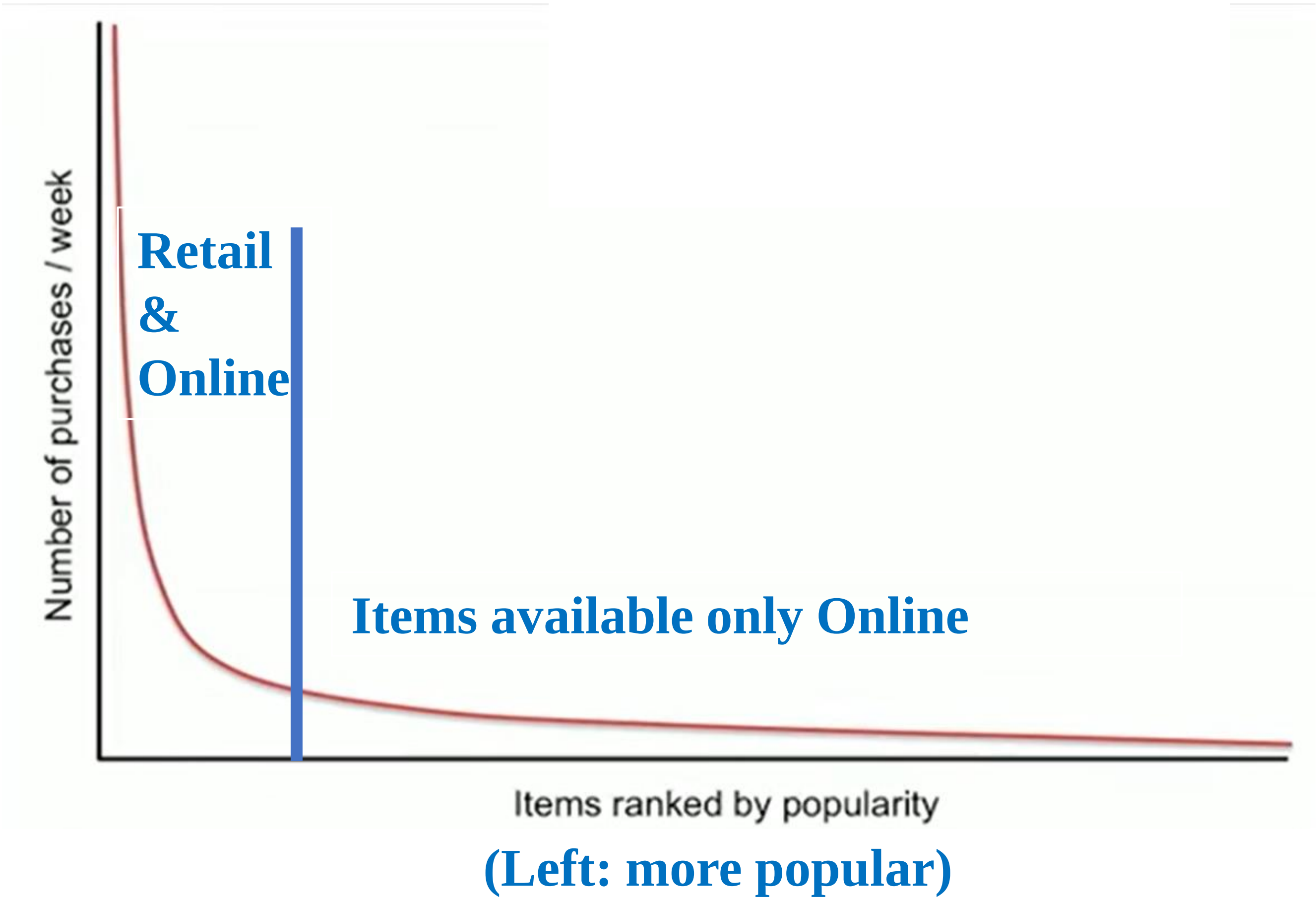
Long Tail

- **Cutoff point**
  - Left: retail & online
  - Right: only online

- **Phenomenon see in**
  - Movies, music, products, etc

- **Area under the curve** to the right may be larger than to the left

- **Problem:** too many items, how to let users know them



Long Tail

- **Problem:** too many items, how to let users know them

- Need a **better way** to let users find them easily
  - Recommendation systems

- **Examples**
  - Books, movies, music
  - News articles
  - People (friend recommendations on Facebook, LinkedIn, and Twitter)

Types of Recommendations

- **Manually selected**
  - List of favorites
  - Lists of “essential” items

- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads

- **Tailored to individual users**
  - E.g. Amazon, Netflix, Pandora
  - Our focus here

Formal Model

- ***X*** = set of **Customers**
- ***I*** = set of **Items**
- **Utility function *u*: *X* × *I* → *R***
  - ***R***: rating
  - ***R*** ∈ an ordered set
    - e.g., **1-5** stars

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Key Problems

- **1) Gathering known ratings for matrix**
  - Different ways of collecting the data in the utility matrix

- **2) Infer unknown ratings from the known ones**
  - Different ways of extrapolating unknown ratings for a user

Let’s talk about each in detail

Key Problems

- **1) Gathering Known Ratings**
- **Explicit**
  - Ask people to rate items
  - Given incentives for people to rate items

- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - Ways to learn about low ratings

Key Problems

- **2) Infer Unknown Ratings**
- Suppose we have gathered enough ratings, how to extrapolate

- Popular approaches to recommender systems:
  - Content-based
  - Collaborative Filtering
    - User-based
    - Item-based
  - Latent-factor based

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Content-Based

- **Main Idea:** Recommend items to customer x similar to previous items rated highly by x

- **Examples:**
- Movie recommendations
  - Recommend movies with same actor(s), director, genre, …
- Book recommendations
  - Recommend books with same author(s), topic, …
- News recommendations
  - Recommend other news with “similar” content

Content-Based

- For each item, create an **item profile**

- Item profile involves a set (vector) of features
- For example
  - Movies: movie types, actor(s), title, director,…
  - Text: Set of “important” words in document

- How to pick important features?
  - Common techniques from text mining: TF-IDF (Term freq * Inverse Doc Freq)

Content-Based

- TF-IDF (for text mining, Optional)
  - TF: count of term in the doc / document length
  - IDF: log (#documents / #doc that contains the term)
  - TF-IDF score: TF * IDF

E.g.

- She is pretty.
- He is handsome.
- Both he and she are experts in machine-learning.

	d1	d2	d3
handsome	0	(1/3)*log(3/1)	0
She	(1/3)*log(3/2)	0	(1/8)*log(3/2)

Content-Based

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Content-Based

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1
Features							
x_1	0.8	1	0.9	0.1	0.9	0.8	0.01
x_2	0.1	0.01	0.02	1	0.3	0.5	0.7

e.g. x_1: romance,
x_2: action
/
x_1: seriousness,
x_2: female-oriented

- For user *i* & movie *j*, predict rating as: $\theta^{(i)\prime} x^{(j)}$
- $\theta^{(i)}$ will be learnt

e.g. User 1 (with para $\theta^{(1)}$) & movie 2 ($x^{(2)} = (1, 0.01)'$)

Content-Based

Decision variable: $\{\theta^{(i)}\}, i = 1, 2, 3, \ldots$

$$\min \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{i} \sum_{k} \left(\theta_k^{(i)}\right)^2$$

$\theta^{(i)}$: user *i*'s para
$x^{(j)}$: movie *j*'s para
$w^{(i,j)}$: value = 1 if user *i* has rated movie *j*
$r^{(i,j)}$: rating value (movie *j* rated by user *i*)

This is just a linear regression problem!

Content-Based

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Features							
x_1	0.8	1	0.9	0.1	0.9	0.8	0.01
x_2	0.1	0.01	0.02	1	0.3	0.5	0.7

Decision variable: $\theta^{(i)}$: user i's para

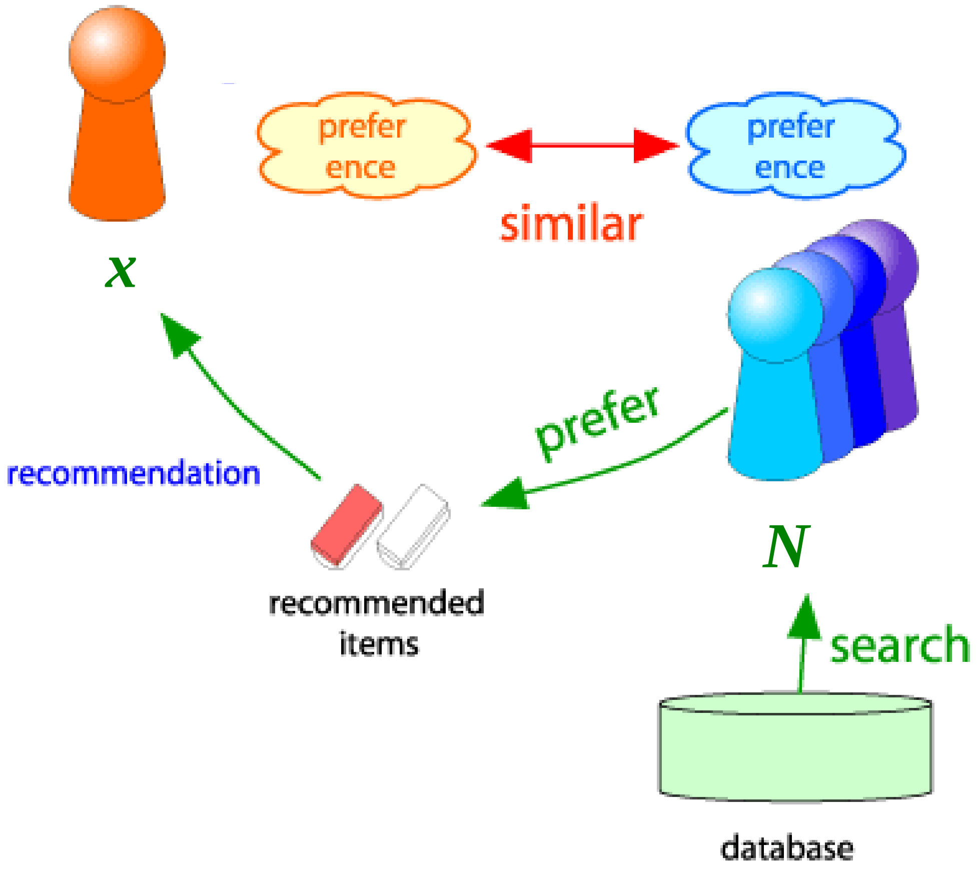
User 1: $\min\ [4 - \left(\theta_1^{(1)}, \theta_2^{(1)}\right) \cdot (0.8, 0.1)]^2 + [5 - \left(\theta_1^{(1)}, \theta_2^{(1)}\right) \cdot (0.1, 1)]^2 + [1 - \left(\theta_1^{(1)}, \theta_2^{(1)}\right) \cdot (0.9, 0.3)]^2$

User 2: $\min\ [5 - \left(\theta_1^{(2)}, \theta_2^{(2)}\right) \cdot (0.8, 0.1)]^2 + [5 - \left(\theta_1^{(2)}, \theta_2^{(2)}\right) \cdot (1, 0.01)]^2 + [4 - \left(\theta_1^{(2)}, \theta_2^{(2)}\right) \cdot (0.9, 0.02)]^2$

…

Collaborative Filtering

- Consider user ***x***
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



Motivating Example

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

- Consider user ***x*** and ***y*** with rating vectors r_x and r_y

- Need similarity metric s(x,y)
  - A & B seem more similar (M1)
  - A & C seem more dissimilar (M4 & M5)

- Want similarity metric to capture the intuition: s(A,B)>s(A,C)

Motivating Example: 1st Approach (Jaccard)

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

- $s_J(x, y) = |r_x \cap r_y| / |r_x \cup r_y|$
- $s_J(A, B) = 1/5$; $s_J(A, C) = 2/4$
  - $s_J(A, B) < s_J(A, C)$
- **Problem**: ignores rating values

Motivating Example: 2nd Approach (Cosine)

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

- $s_C(x, y) = \frac{r_x \cdot r_y}{|r_x||r_y|}$
- $s_C(A, B) = \frac{4 \cdot 5}{\sqrt{4^2+5^2+1^2}\sqrt{5^2+5^2+4^2}} = \frac{20}{\sqrt{42}\sqrt{66}} = 0.38$
- $s_C(A, C) = \frac{5 \cdot 2 + 1 \cdot 4}{\sqrt{4^2+5^2+1^2}\sqrt{2^2+4^2+5^2}} = \frac{14}{\sqrt{42}\sqrt{45}} = 0.32$
  - $s_C(A, B) > s_C(A, C)$, but we can do better

Motivating Example: 3^{rd} Approach (Centered-Cosine)

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Standardize each user's ratings

User/Movie	1	2	3	4	5	6	7
A	2/3			5/3	- 7/3		
B	1/3	1/3	- 2/3				
C				- 5/3	1/3	4/3	
D		1					- 1

Motivating Example: 3rd Approach (Centered-Cosine)

User/Movie	1	2	3	4	5	6	7
A	2/3			5/3	- 7/3		
B	1/3	1/3	- 2/3				
C				- 5/3	1/3	4/3	
D		1					- 1

- $s_{CC}(x, y) = \frac{r_x \cdot r_y}{|r_x||r_y|}$, where r_x, r_y are standardized ratings
- $s_{CC}(A, B) = 0.09$
- $s_{CC}(A, C) = -0.56$
  - $s_{CC}(A, B) > s_{CC}(A, C)$
  - Captures intuition better
    - Missing ratings treated as “average”
    - Remove bias for users who consistently rate high/low

Predicting ratings

- Goal: given user $\boldsymbol{x}$ and movie i, predict rating r_{xi}

- Two Popular Approaches:
  - User-based Collaborative Filtering
  - Item-based Collaborative Filtering

Predicting ratings

User-based collaborative filtering

- Let r_x be the vector of user *x*'s ratings
- Let *N* be the set of *k* users most similar to *x* who have rated movie *i*
- Prediction for user *x and* movie *i* :
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$
  - where s_{xy} is the similarity between user *x* and user *y*

Predicting ratings

Item-based collaborative filtering (Dual Approach)

- Let r_x be the vector of user *x*'s ratings
- Let *M* be the set of *k* movies (which user *x* has rated) most similar to *i*
- Prediction for user *x and* movie *i* :
  - $r_{xi} = \frac{\sum_{j \in M} s_{ij} \cdot r_{xj}}{\sum_{j \in M} s_{ij}}$
  - where s_{ij} is the similarity between movie *i* and movie *j*

Predicting ratings: Example

User/Movie	1	2	3	4	5	6
1	5	5	5		3	2
2		2	3	5		2
3	1	5	3	4	2	
4	2			3		4
5	4		5	5	4	2
6	3	2		4	3	
7	4	1	?	4	1	
8			1		2	3

Predicting ratings: Example

User/Movie	1	2	3	4	5	6
1	1	1	1		- 1	- 2
2		- 1	0	2		- 1
3	- 2	2	0	1	- 1	
4	- 1			0		1
5	0		1	-1	0	- 2
6	0	-1		1	0	
7	1.5	- 1.5	?	1.5	- 1.5	
8			- 1		0	1

Predicting ratings: Example

User-Based Collaborative Filtering

User/Movie	1	2	3	4	5	6
1	1	1	1		- 1	- 2
2		- 1	0	2		- 1
3	- 2	2	0	1	- 1	
4	- 1			0		1
5	0		1	-1	0	- 2
6	0	-1		1	0	
7	1.5	- 1.5	?	1.5	- 1.5	
8			- 1		0	1

Similarity
0.18
0.61
-0.32
-0.35
-0.2
0.7
1
0

$$r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$$

- 2 Nearest users: $r_{73} = \frac{0.61 \cdot 0 + 0.18 \cdot 1}{0.61 + 0.18} = 0.25$
- What will be the predicted rating? **(Ans: 2.5+0.25)**

Predicting ratings: Example

Item-Based Collaborative Filtering

User/Movie	1	2	3	4	5	6
1	1	1	1		- 1	- 2
2		- 1	0	2		- 1
3	- 2	2	0	1	- 1	
4	- 1			0		1
5	0		1	-1	0	- 2
6	0	-1		1	0	
7	1.5	- 1.5	?	1.5	- 1.5	
8			- 1		0	1
Similarity	0.2	0.19	1	0.19	-0.28	-0.87

$$r_{xi} = \frac{\sum_{j \in M} s_{ij} \cdot r_{xj}}{\sum_{j \in M} s_{ij}}$$

- 2 Nearest movies: $r_{73} = \frac{0.2 \cdot 1.5 + 0.19(-1.5)}{0.2+0.19} = 0.03$
- What will be the predicted rating? **(Ans: 2.5+0.03)**

Latent-factor Recommendation system

Motivation: from Content-based (Quick Recap below)

Decision variable: $\{\theta^{(i)}\}, i = 1, 2, 3, \ldots$

$$\min \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{i} \sum_{k} \left(\theta_k^{(i)}\right)^2$$

$\theta^{(i)}$: user *i*'s para
$x^{(j)}$: movie *j*'s para
$w^{(i,j)}$: value = 1 if user *i* has rated movie *j*
$r^{(i,j)}$: rating value (movie *j* rated by user *i*)

Latent-factor Recommendation system

User/Movie	1	2	3	4	5	6	7
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					1

Features							
x_1	0.8	1	0.9	0.1	0.9	0.8	0.01
x_2	0.1	0.01	0.02	1	0.3	0.5	0.7

Decision variable: $\theta^{(i)}$: user i's para

User 1: $\min\ [4-\left(\theta_1^{(1)},\theta_2^{(1)}\right)\cdot(0.8,0.1)]^2+[5-\left(\theta_1^{(1)},\theta_2^{(1)}\right)\cdot(0.1,1)]^2+[1-\left(\theta_1^{(1)},\theta_2^{(1)}\right)\cdot(0.9,0.3)]^2$

User 2: $\min\ [5-\left(\theta_1^{(2)},\theta_2^{(2)}\right)\cdot(0.8,0.1)]^2+[5-\left(\theta_1^{(2)},\theta_2^{(2)}\right)\cdot(1,0.01)]^2+[4-\left(\theta_1^{(2)},\theta_2^{(2)}\right)\cdot(0.9,0.02)]^2$

…

Suppose it is hard to come up with features of the movies $x^{(j)}$,

What should we do?

Latent-factor Recommendation system

Suppose we do not have features of the movies $x^{(j)}$

We can still have an algorithm that learns $x^{(j)}$ by itself

Idea

$$\min \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{i} \sum_{k} \left(\theta_k^{(i)}\right)^2 + \lambda \sum_{j} \sum_{k} \left(x_k^{(j)}\right)^2$$

Given x, minimize θ
Given θ, minimize x

Latent-factor Recommendation system

More specifically

Fix $\{x^{(j)}\}$, $\quad min_{\{\theta^{(i)}\}} \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{i} \sum_{k} \left(\theta_k^{(i)}\right)^2$

Fix $\{\theta^{(i)}\}$, $\quad min_{\{x^{(j)}\}} \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{j} \sum_{k} \left(x_k^{(j)}\right)^2$

Latent-factor Recommendation system: Example

User/Movie	1	2
A	5	?
B	2	4

Features		
x_1		
x_2		

Goal: predict $r^{(1,2)}$

Latent-factor Recommendation system: Example

User/Movie	1	2
A	5	?
B	2	4

Features		
x_1	0.02	0.05
x_2	-0.04	0.03

Goal: predict $r^{(1,2)}$

Initialization: assign some random values $\{x^{(j)}\}$

Latent-factor Recommendation system: Example

User/Movie	1	2
A	5	?
B	2	4

Features		
x_1	0.02	0.05
x_2	-0.04	0.03

Goal: predict $r^{(1,2)}$

Given $\{x^{(j)}\}$ $$min_{\{\theta^{(i)}\}} \sum_i \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_i \sum_k \left(\theta_k^{(i)}\right)^2$$

Latent-factor Recommendation system: Example

User/Movie	1	2
A	5	?
B	2	4

Features		
x_1	0.02	0.05
x_2	-0.04	0.03

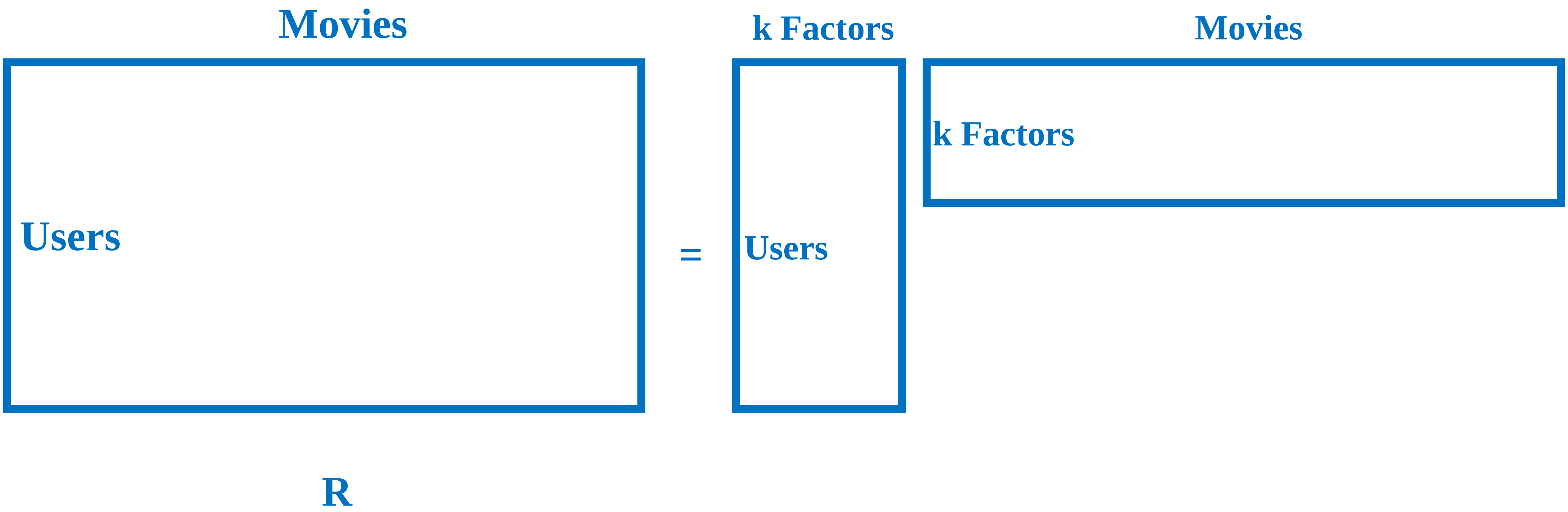
Goal: predict $r^{(1,2)}$

Given $\{\theta^{(i)}\}$

$$min_{\{x^{(j)}\}} \sum_{i} \sum_{j|w^{(i,j)}=1} (\theta^{(i)\prime} x^{(j)} - r^{(i,j)})^2 + \lambda \sum_{j} \sum_{k} \left(x_k^{(j)}\right)^2$$

Latent-factor Recommendation system

High level:



Each user and movie: represented by k-dim vector

Other Issues

Cold-start problem (for new user)

- Ask users to list their preferences (e.g. “tell me your 3 favorite movies”)
- Try to get user’s other info. (e.g. region, gender, browse history, etc.)
- Recommend popular & diverse items

1st-rater problem (for new item)

- Pay a group of people to rate them
- Content-based algo.