In-Class Exercise

- Consider a high school. Suppose we know the following conditional distributions of Y (the gender) and a prior marginal distribution of X (the class number):

$f(x,y)$	$X=1$	$X=2$	$X=3$	f_Y
$Y=1\|X$	0.75	0.5	0.4	?
$Y=2\|X$	0.25	0.5	0.6	?
f_X	1/3	1/3	1/3	

- For a random student, what is the probability of Y = 1 (marginal probability)?
  f_(Y=1)=f(Y=1|X)×f_(X=1)+f(Y=1|X)×f_(X=2)+f(Y=1|X)×f_(X=3)=0.55
- For a random student, what is the probability of X = 1 (i.e., from class 1) given Y = 1?
  f_(X=1|Y=1)= f_(Y=1|X=1)f_(X=1)/f_(Y=1)=0.4545
- Suppose we fix the value of X by a random draw: only one class is on the playground. We do not know X. If the first student has Y = 1, then what is the probability of X = 1? What is the probability of the second student having Y=1 again?
  f_(X=1|Y=1)=f_(Y=1|X=1)f_(X=1)/f_(Y=1)=0.4545

  Since Y is independently sample and X is fixed sampled, so f(Y=1|Y=1) = f(Y=1|X)×f_(X=1|Y=1)+f(Y=1|X)×f_(X=2|Y=1)+f(Y=1|X)×f_(X=2|Y=1)+ = 0.75*0.4545+0.5*0.3030+0.4*0.2424=0.5893

  For reference, the calculated conditional and marginal probabilities are as follows:

$f(x,y)$	X=1	X=2	X=3	f_Y	$f_{Y\|Y=1}$
$Y=1\|X$	0.75	0.5	0.4	0.55	0.5893
$Y=2\|X$	0.75	0.5	0.6	0.45	0.4107
f_X	1/3	1/3	1/3		
$f_{X\|Y=1}$	0.4545	0.3030	0.2424		

After-Class Exercise

- Suppose $P(A)=0.4$ and $P(B)=0.6$. Are A and B mutually exclusive?

  Not sure. A and B can have overlap.
- Suppose A and B are mutually exclusive and $P(A)=0.4$. Then $(B)=?$

  Not sure. A and B may not be collectively exhaustive.
- Suppose A and B are mutually exclusive. In addition, $P(A)=0.4$ and $P(B)=0.6$. Suppose C and D are also mutually exclusive and collectively exhaustive. Further, $P(C|A)=0.2$ and $P(D|B)=0.4$. What are $P(C)$ and $P(D)$?

	A	B	
C	x	0.6 – y	0.08 + 0.6 – 0.24 = 0.44
D	0.4 – x	y	0.4 – 0.08 + 0.24 = 0.56
	0.4	0.6	

$$P(C|A) = 0.2 = \frac{x}{0.4} \;=>\; x = 0.08$$

$$P(D|B) = 0.4 = \frac{y}{0.6} \;=>\; y = 0.24$$

- There are two fortune tellers, A & B. According to historical data, A's predictions were correct in 90% cases, while B's predictions were correct only in 30% cases. Now, without communicating with each other, both A & B predict that Donald Trump will be elected again after four years. Without any information, your prior belief about Donald Trump being elected again is 0.5. Now knowing A & B's predictions, what should be your corrected belief?

  Pr(A: win & B: win & DT win) = Pr(A: win & B: win | DT win)*Pr(DT win) = 0.9*0.3*0.5

  Pr(A: win & B: win & DT lose) = Pr(A: win & B: win | DT lose)*Pr(DT lose) = 0.1*0.7*0.5

  Pr(DT win | A: win & B: win) = 0.27/(0.27 + 0.07) = 27/34.

- When a man passes the airport security check, they discover a bomb in his bag. He explains. "Statisticians show that the probability of a bomb being on an airplane is 1/10,000. However, the chance that there are two bombs on one plane is 1/10,000,000. So, I am much safer …"

  Suppose the statisticians are right and it is impossible to have more than two bombs on an airplane. Do you agree with the man?

  No. The probability of having a second bomb given there is at least a bomb is about 1/1,000.

- If event A and B are independent given event C happens, then A and B are also independent given C does not happen.

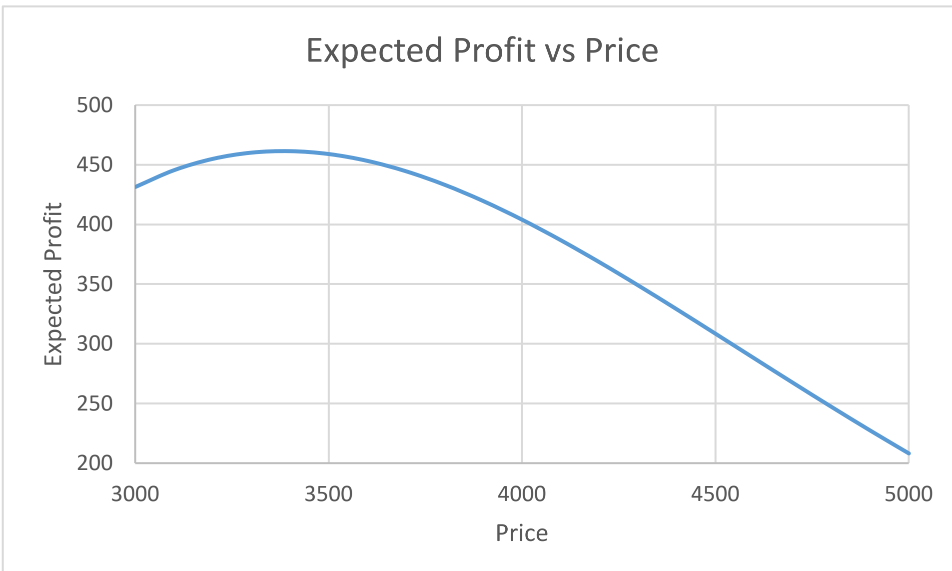
  Not true. Let A = raining tomorrow, B = local people income increased, and C = Hong Kong. Given in the city of Hong Kong, A and B are independent because the economy of Hong Kong does not rely on the agriculture industry. But globally, A and B are not independent.

- In the introductory example, what is the best price to set if the first customer did not buy? Consider three possible means of WTP: $2,000, $3,500, and $5,000.

  We first update the probability of each scenario and compute the expected profit for a new price. Note that, when the price is changed, the conditional probability of buying will change accordingly.

		Scenario1	Scenario2	Scenario3		Price	4000
	Mean	2000	3500	5000		Cost	2000
	Prior	0.333333333	0.333333333	0.333333333		Std	1000
Conditional Probabilities							
	Buy	0.022750132	0.308537539	0.841344746			
	Leave	0.977249868	0.691462461	0.158655254			
Joint Probabilities							
	Buy	0.007583377	0.102845846	0.280448249			
	Leave	0.325749956	0.230487487	0.052885085	<= observation		New Price
	Posterior	0.534785599	0.378392649	0.086821751			3000
Conditional probabilities under the new price				Expected profit under the new price			
	Buy	0.158655254	0.691462461	0.977249868			431.3374

Using a data table in Excel, we can find out that the optimal price now is about $3,400.



Please find the model and calculations in the Excel file.