# MSBA 7004
# Operations Analytics

Mid Course recap

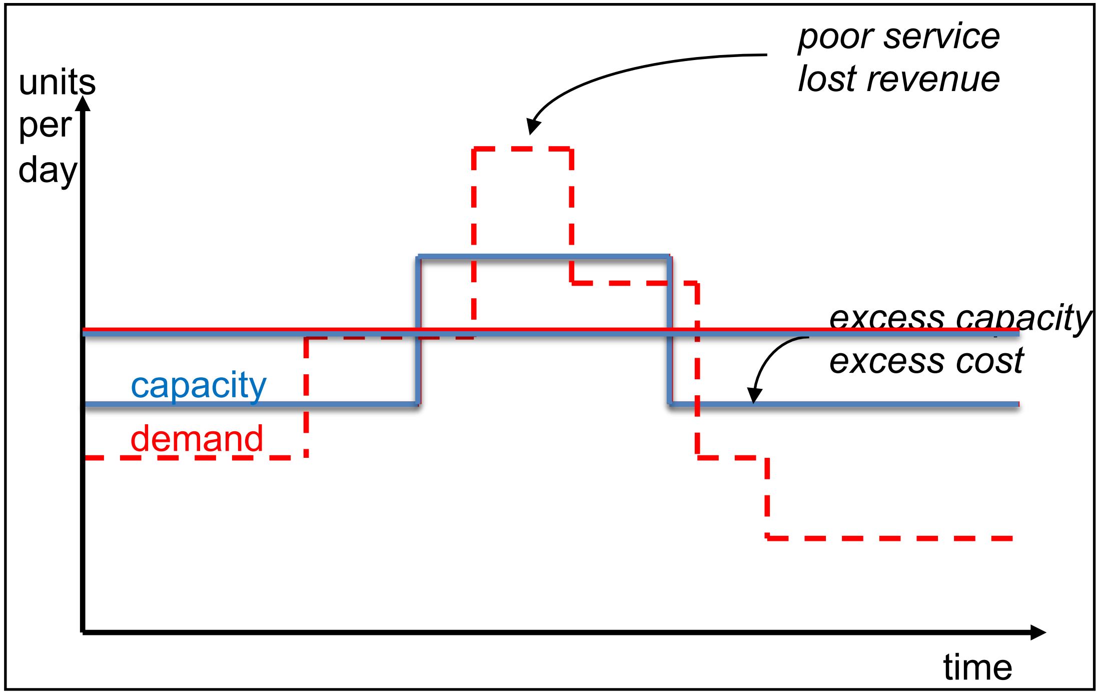
2023

# Why do we need Operations Management (OM)?

- Matching Supply with Demand

- Variability hits

- Demand fluctuates




**What should we do?**

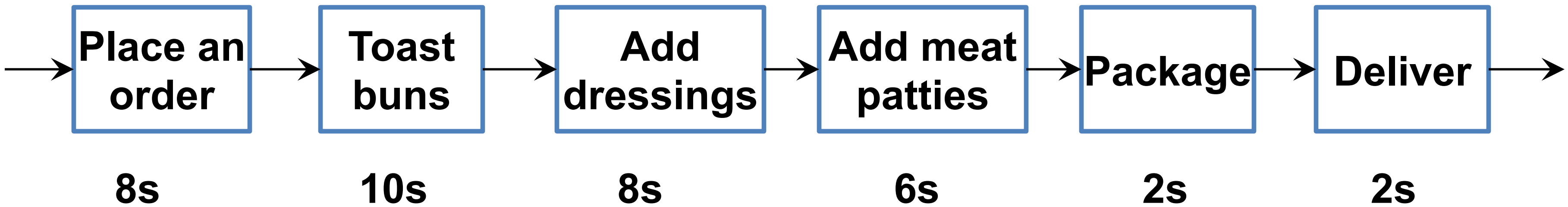
# Management Options

- Managing supply
  - **Capacity management**
    - **Process (bottleneck) analysis**
    - **Queueing analysis**
  - Inventory and supply chain management
  - **Project management**

- Managing demand
  - Revenue management

# Linear Flow Chart

- Flow unit: An order (each order = one burger)
- Tasks and sequences
- Flow time of each task




- Determine which resources are used in each task
  - Could indicate resources along each task
  - Swim-lane diagram or Gantt chart may be better

# Process Analysis (I) Review

- Process mapping basics
  - Key steps in process analysis
  - Get feedback and validate maps
- Process analysis basics (3 keywords)

**(Theoretical) Flow time** (or **Throughput Time**):
(Min) Length of time a unit spends in the system

**Capacity Rate** of a **Resource** or **Process:**
Max rate at which units can flow through a resource or process

**Bottleneck**:
Resource with the slowest capacity rate in a process determines the capacity rate of a process

# Improving Flow Time

- **Critical path:** The longest path in the process flowchart.
- **Critical activities:** Activities on a critical path.
- Theoretical flow time = Value-adding flow time (on the critical path)

# Process Analysis: Multiple Flow Units

- How to identify the bottleneck when there are multiple flow units
  – $Unit\ load = total\ amount\ of\ time\ required\ to\ finish\ the\ combination\ of\ flow\ units$
  – $Capacity\ rate = \frac{Number\ of\ resources}{Unit\ load}$
  – Bottleneck is the resource with the lowest capacity rate, and the bottleneck changes when the "product mix" changes

# Process Analysis: Multiple Flow Units

| Resource | Unit Load (minutes/unit) | | | |
|---|---|---|---|---|
| | Product A | Product B | Product C | 1A+2B+2C |
| 1 | 2.5 | 2.5 | 2.5 | 12.5 |
| 2 | 1.5 | 2 | 2.5 | 10.5 |
| 3 | 12 | 0 | 0 | 12 |
| 4 | 0 | 3 | 3 | 12 |
| 5 | 3 | 3 | 3 | 15 |

- When multiple flow units go through a process, the “product mix” needs to be considered while determining the unit load and the capacity
- The bottleneck depends on the product mix

# Capacity Rate vs. Throughput Rate

- Both the *capacity rate* and the *throughput rate* measure the ***output rate*** of a process

**Capacity rate:** Maximum possible output rate

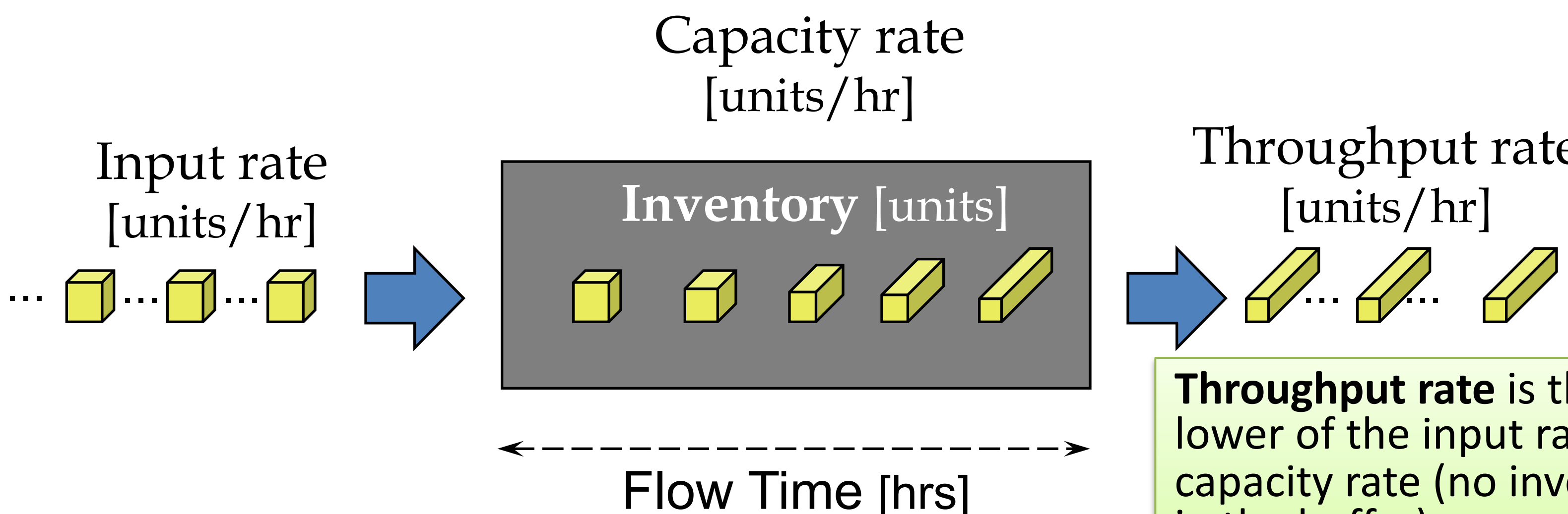
**Throughput rate:** Actual output rate

- The ***throughput*** (or ***flow***) ***rate*** depends on both:

**Capacity rate**

**Input rate** (also called the **arrival rate**)
Rate at which flow units arrive at the process
e.g. Arrival of customers (demand rate), or Raw material arrival into a factory (planned or scheduled arrivals)

# Fundamental Process Characteristics




**Throughput rate** is the lower of the input rate and capacity rate (no inventory in the buffer)

Throughput Rate:

What if Input rate < Capacity rate?

Throughput Rate:

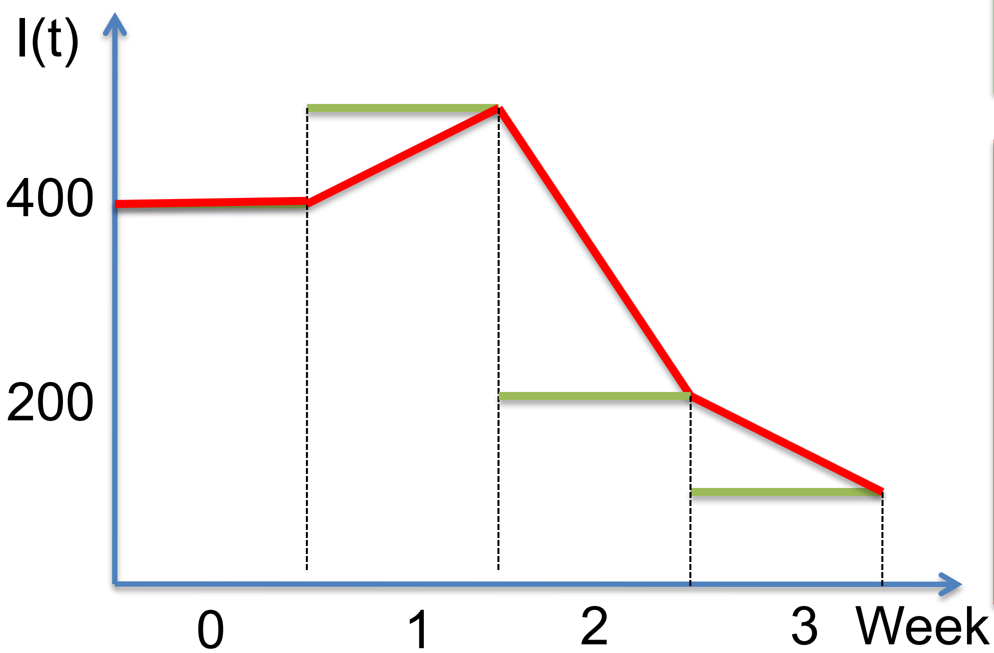
What if Input rate > Capacity rate?

**Flow time** = average time for a unit to move through the process

**Inventory =** total number of flow units present in the process

# Average Inventory

*Average inventory* depends on whether inventory is assumed to change in discrete steps, or continuously




Under the ***discrete*** assumption:

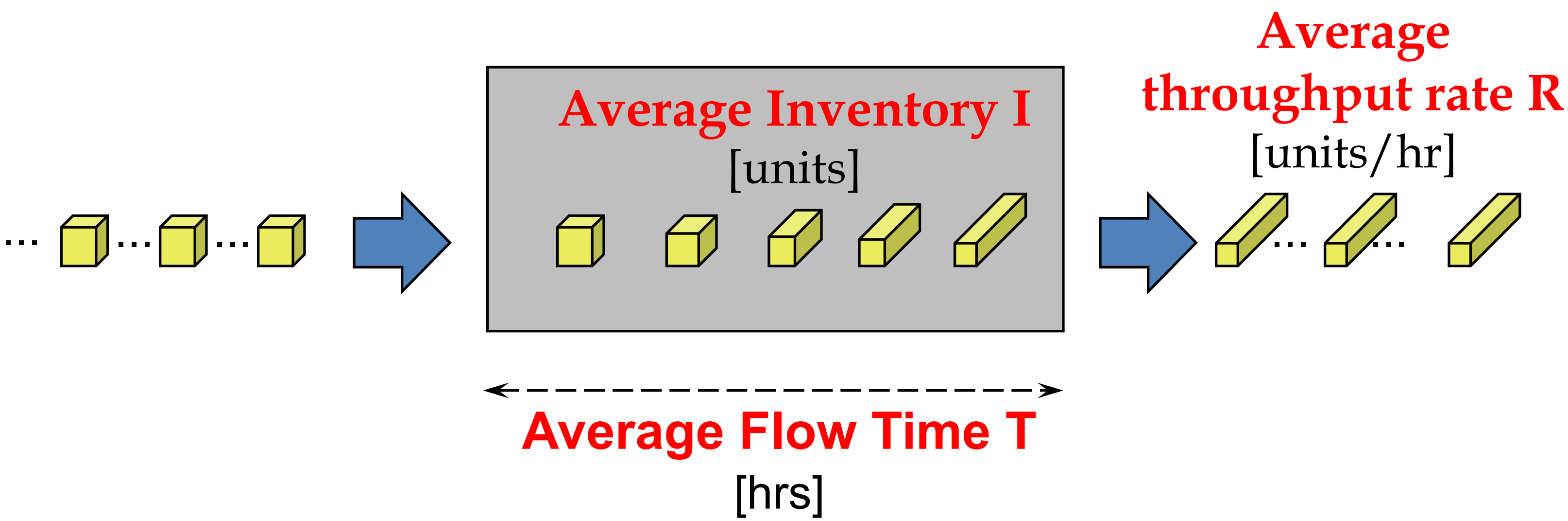
The average inventory over weeks 0 to 3 is 300

Under the ***continuous*** assumption:

The average inventory? "Area under the curve/#weeks"

# Little's Law

- Establishes a relationship between **average** inventory, **average** throughput rate, and **average** flow time

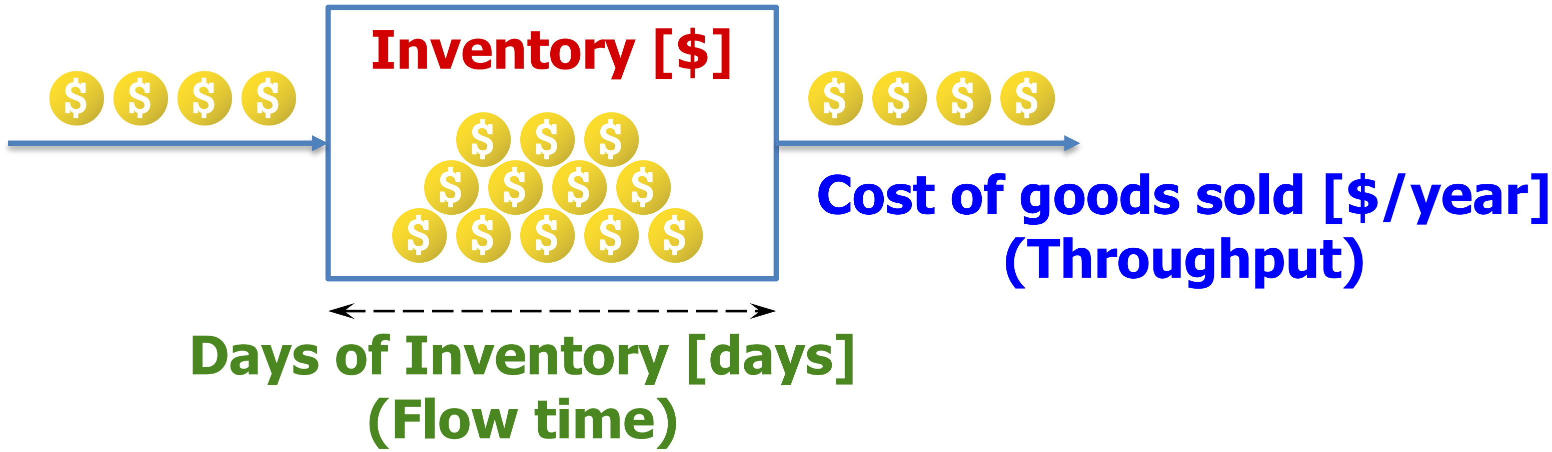


# Little's Law

- Throughput rate: 700 undergrads/year
- Flow time: 4 years
- Total # undergrads in HKU FBE?

$$I = R * T$$

# ***Days of Inventory*** & Inventory Turnover




**By Little's Law,**

$$\text{Days of Inventory} = \frac{365 \times \textbf{Inventory}}{\textbf{Cost of goods sold (Throughput)}}$$

$$\text{Inventory Turnover} = \frac{\textbf{Cost of goods sold (Throughput)}}{\textbf{Inventory}}$$

This measures the velocity of moving inventory.

# Little’s Law: Example - Retail

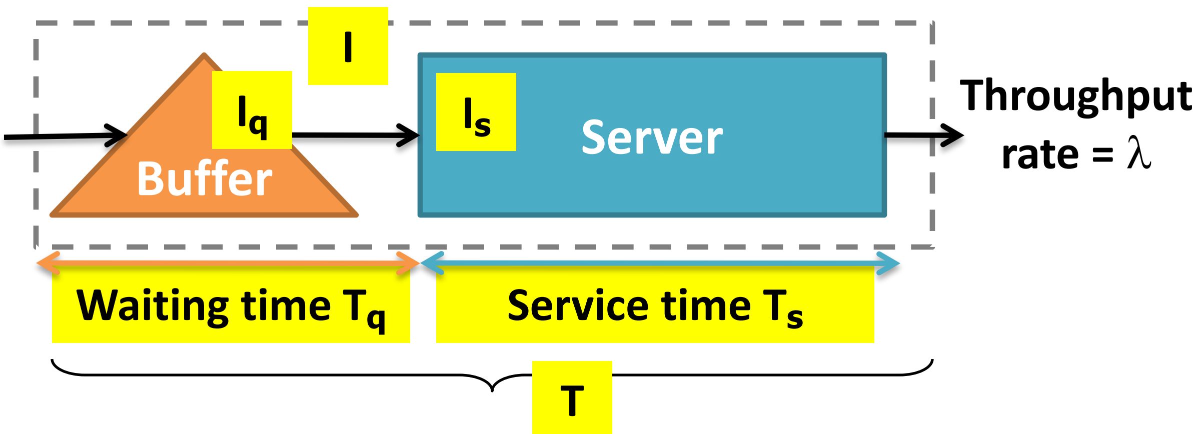
| ($ in billions) | Target (1/30/2020) | Wal-Mart (1/30/2020) |
|---|---|---|
| Net Sales | $78.11 | $523.96 |
| Cost of Goods Sold | $54.86/year | $394.61/year |
| Net Income | $3.28 | $14.88 |
| Inventories | $9.50 | $44.44 |
| Months of Inventory | 9.50/54.86*12<br>= 2.08 month => 5.8/yr | 44.44/394.61*12<br>= 1.35 month => ~~8.58/yr~~ 8.9/year |

If Wal-Mart carried the same months of inventory as Target, its inventories would be

I = R x T = (394.61/12 )*2.08 = 68.40

This would tie up about $23.96 billion extra in inventory.

# What are we trying to quantify?




Little's Law holds

$$I_q = \lambda T_q$$
$$I_s = \lambda T_s$$
$$I = \lambda T$$

## System Characteristics

| | |
|---|---|
| **Utilization** | $\rho$ (In a stable system, $\rho = \lambda/\mu \leq 100\%$) |
| **Safety Capacity** | $\mu - \lambda$ |

## Performance Measures

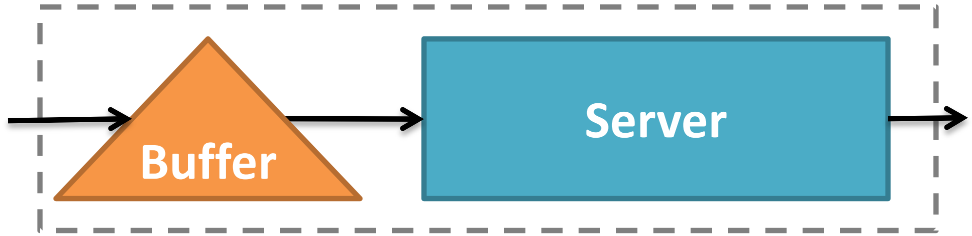
| | |
|---|---|
| $T_q$ | Average waiting time (in queue) |
| $I_q$ | Average queue length |
| $T_s$ | Average time spent at the server |
| $I_s$ | Average number of customers being served |
| $T=T_q+T_s$ | Average flow time (in process) |
| $I=I_q+I_s$ | Average number of customers in the process |

# Quick "Quiz"

**Assumption:** $\lambda \leq \mu$

**Service rate:** $\mu$ persons/min
(average capacity rate)

**Arrival rate:**
$\lambda$ persons/min
(average input rate)




Average throughput rate
$\lambda$ persons/min

- Average number of persons in the system:

$$l = l_q + l_s$$

- Question: $l_s$=??? (Express $l_s$ in terms of $\lambda$ and $\mu$)

- Answer: $l_s = \lambda T_s = \lambda/\mu$

# Pollaczek-Khinchin (PK) Formula: Single server

$$I_q \cong \frac{\rho^2}{1-\rho} \times \frac{C_a^2 + C_s^2}{2}$$

"=" for special cases

"≈" in general

| | |
|---|---|
| $I_q$ | Average queue length (excl. inventory in service) |
| $\rho$ | (Long run) Average utilization<br>= Average Throughput / Average Capacity = $\lambda / \mu$ |
| $C_a = \sigma\{a\}/E\{a\}$ | Coefficient of variation of inter-arrival times |
| $C_s = \sigma\{s\}/E\{s\}$ | Coefficient of variation of service times |

# Key Equation: VUT

Avg. Wait Time = Variability × Utilization × Service Time

$$\boxed{T_q \cong \left(\frac{{C_a}^2 + {C_s}^2}{2}\right) \times \left(\frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)}\right) \times s}$$

Variability effect

Utilization effect

Time scale

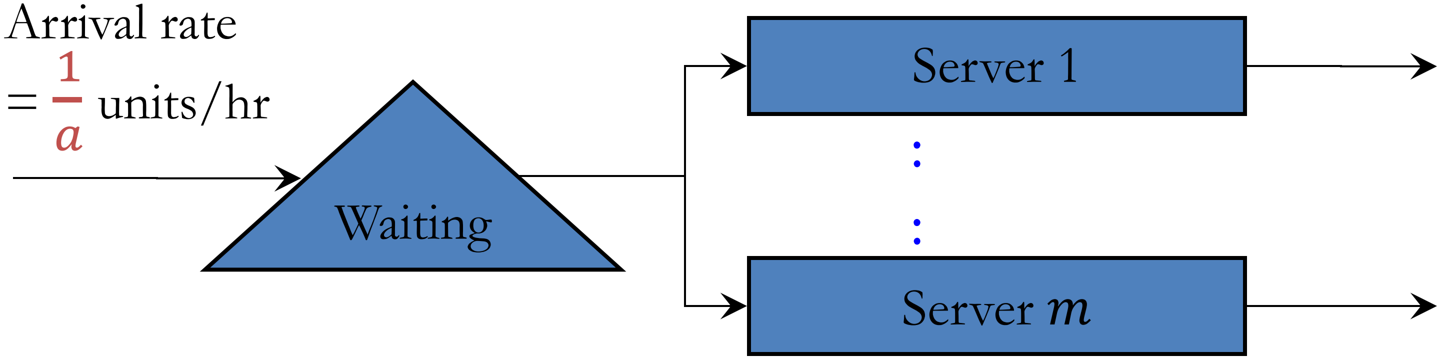
When $m = 1$ (single server), VUT Equation reduces to:

$$T_q \cong \left(\frac{{C_a}^2 + {C_s}^2}{2}\right) \times \left(\frac{u}{1-u}\right) \times s$$

# How Many are Waiting in Line on Average?

Capacity of each server $= \frac{1}{s}$ units/hr

Arrival rate
$= \frac{1}{a}$ units/hr




| Avg. number waiting | | Avg. number being served | | Avg. number in system |
|---|---|---|---|---|
| $I_q$ | $+$ | $I_s$ | $=$ | $I_{sys}$ |
| Avg. waiting time | | Avg. service time | | Avg. time in system |
| $T_q$ | $+$ | $\boldsymbol{s}$ | $=$ | $T_{sys}$ |

**Little's Law:** $I_q = \dfrac{T_q}{a}$ $\qquad I_s = \dfrac{s}{a}$ $\qquad I_{sys} = \dfrac{T_{sys}}{a}$

# Project management

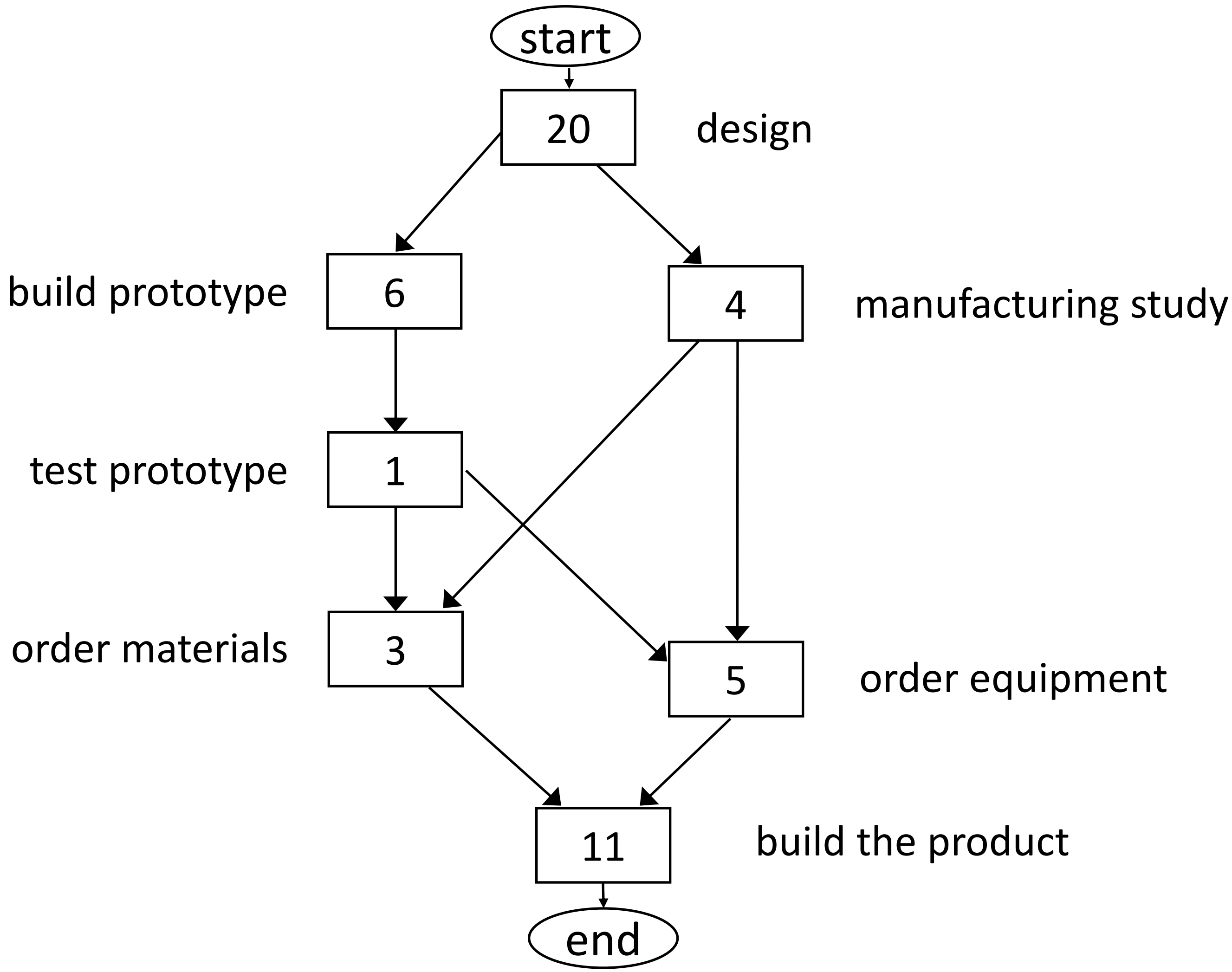
- Critical Path Method
  - Determine Project Duration
  - Cost effective way to reduce project duration

- Time-Cost Analysis
  - Behind or Ahead of schedule
  - Over or under budget

# More Example: Flow Time

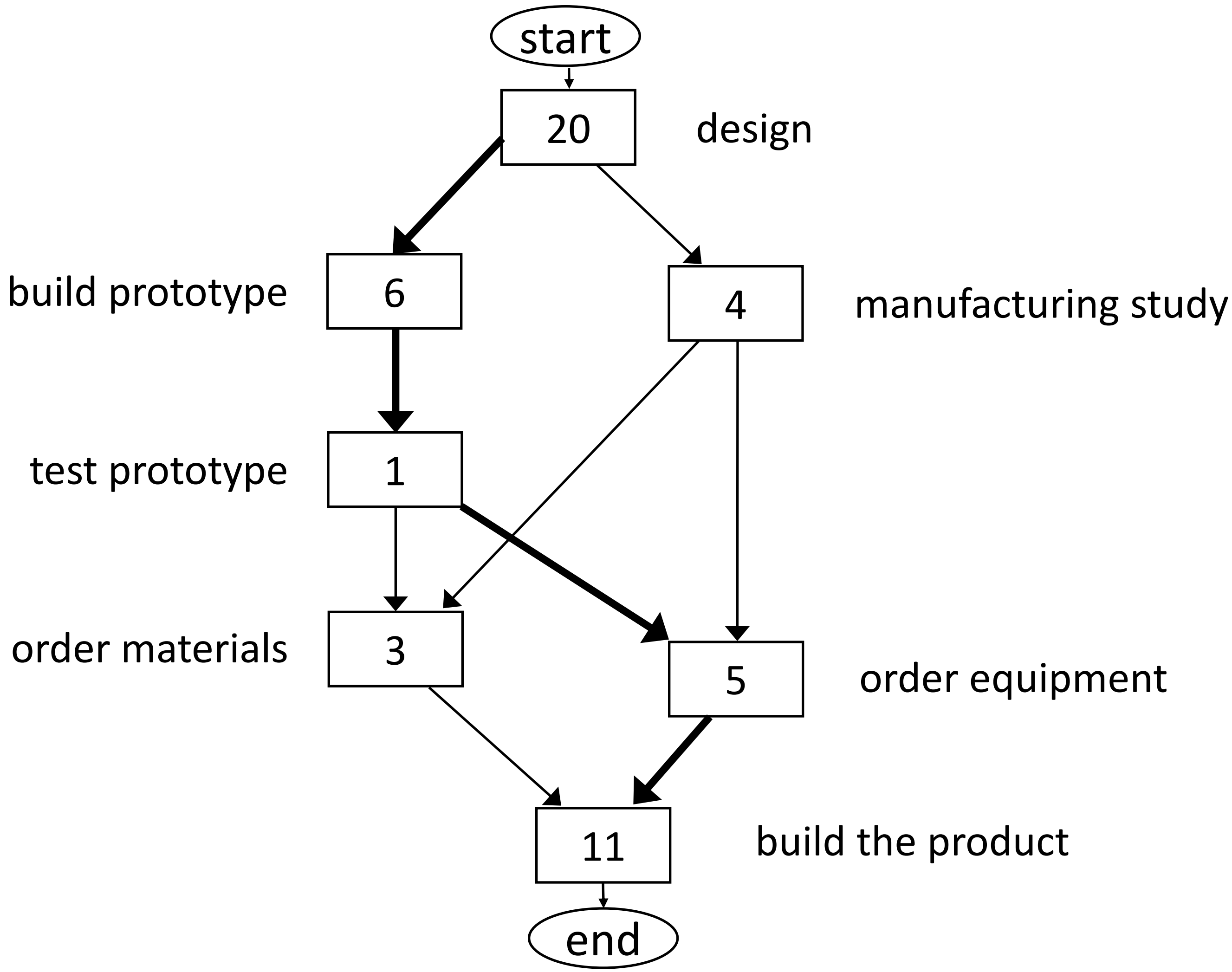
- Critical Path

Q: What is the theoretical flow time of the process?



# More Example: Flow Time

- Critical Path



# Flow Time

Activity Times Summations

Path 1 = 41 days

Path 2 = 40 days

Path 3 = 43 days **Critical Path**

Path 4 = 38 days

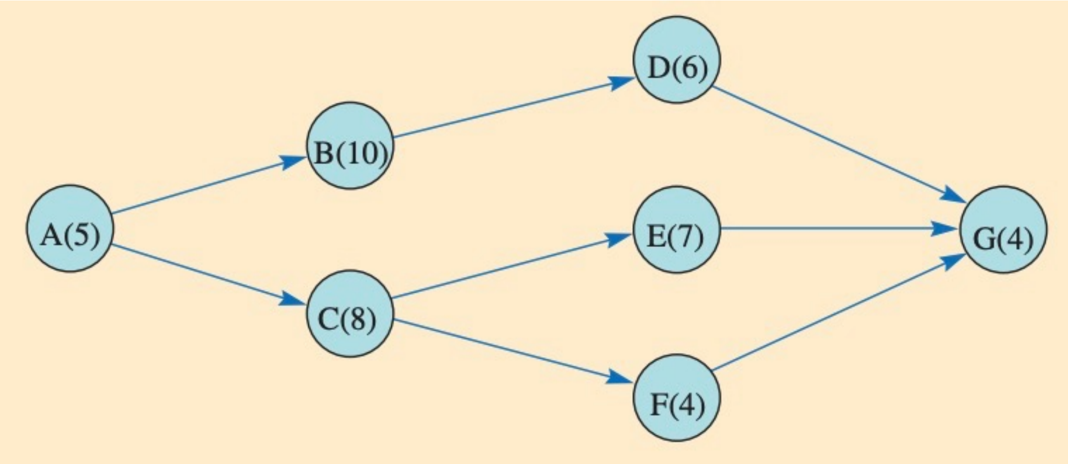
# Practice Problem

| Activity | Time (weeks) | Immediate Predecessors | Normal Cost | Crash Time | Crash Cost |
|---|---|---|---|---|---|
| A | 5 | None | $7000 | 3 | $13000 |
| B | 10 | A | 12000 | 7 | 18000 |
| C | 8 | A | 5000 | 7 | 7000 |
| D | 6 | B | 4000 | 5 | 5000 |
| E | 7 | C | 3000 | 6 | 6000 |
| F | 4 | C | 6000 | 3 | 7000 |
| G | 4 | D, E, F | 7000 | 3 | 9000 |

Determine the critical path and the project duration for the project.

Reduce the project completion time by three weeks in the most cost-effective way.

# Practice Problem

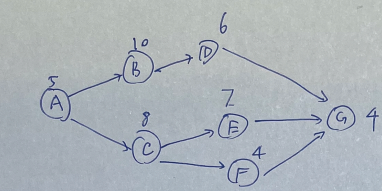



A-B-D-G, 25 weeks

First, reduce D by one week. Second, crash activity G by one week. Third, crash activity A by one week.

| Activity | Time (weeks) | Normal Cost | Crash Time | Crash Cost | Cost/week; Crash Limit |
|---|---|---|---|---|---|
| A | 5 | $7000 | 3 | $13000 | $3000;2 |
| B | 10 | 12000 | 7 | 18000 | 2000;3 |
| C | 8 | 5000 | 7 | 7000 | 2000;1 |
| D | 6 | 4000 | 5 | 5000 | 1000;1 |
| E | 7 | 3000 | 6 | 6000 | 3000;1 |
| F | 4 | 6000 | 3 | 7000 | 1000;1 |
| G | 4 | 7000 | 3 | 9000 | 2000;1 |

# Practice Problem




| | | after step 1. | after step 2 | after step 3. |
|---|---|---|---|---|
| A-B-D-G | 25 | 24 | 23 | 22 |
| A-C-E-G | 24 | 24 | 23 | 22 |
| A-C-F-G | 21 | 21 | 20 | 19. |

Step 1: Crashable Tasks.
A, B, D, G. D
Crash D by 1 week. (after step 1, D is non-crashable).

Step 2: Crashable Tasks: G
A or BC or BE or G.
Crash G by 1 week. (after step 2, G is non-crashable).

step 3: A or BC or BE A.
Crash A by 1 week.

Crash D 1 week, G 1 week, A 1 week.

# Thank You!

- Office Hour
- Monday 7:40 - 8:40 pm
- Thursday 4:40 – 5:40 pm
- KKL 1312

- Email: fengtian@hku.hk