MSBA 7004
Operations Analytics

Class 6-1: Inventory Analysis (I)
Introduction, Economic Order Quantity (EOQ)
2023

Learning Objectives

- Introduce the concept of inventory management
- Understand the analytical approaches used in modeling and making inventory decisions
- Understand the logic behind the economic order quantity (EOQ) model

Basic Outline

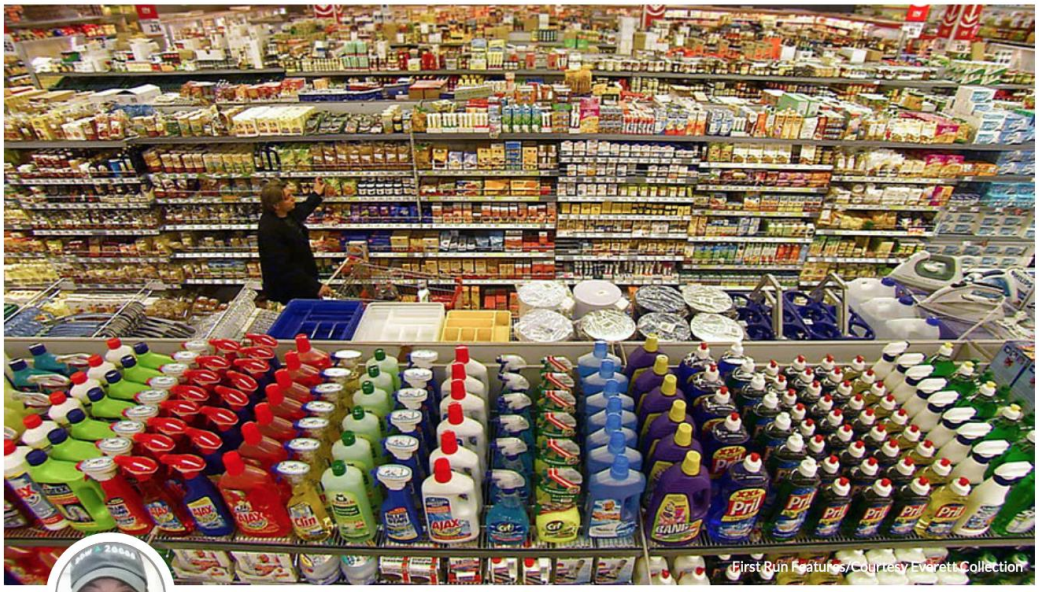
- What is inventory (in a supply chain)?
  - Why is inventory important?
  - What are the reasons for holding or not holding inventory?
  - How much inventory should we hold?
- Inventory management
  1. How much to order?
  - Economic order quantity (EOQ) model

Keep Track of Your Inventory

Grocery stores carry 40,000 more items than they did in the 1990s

Published: June 17, 2017 7:57 a.m. ET

Here's how much grocery shopping has changed over the last three decades



A decade ago you couldn't find gluten-free anything.

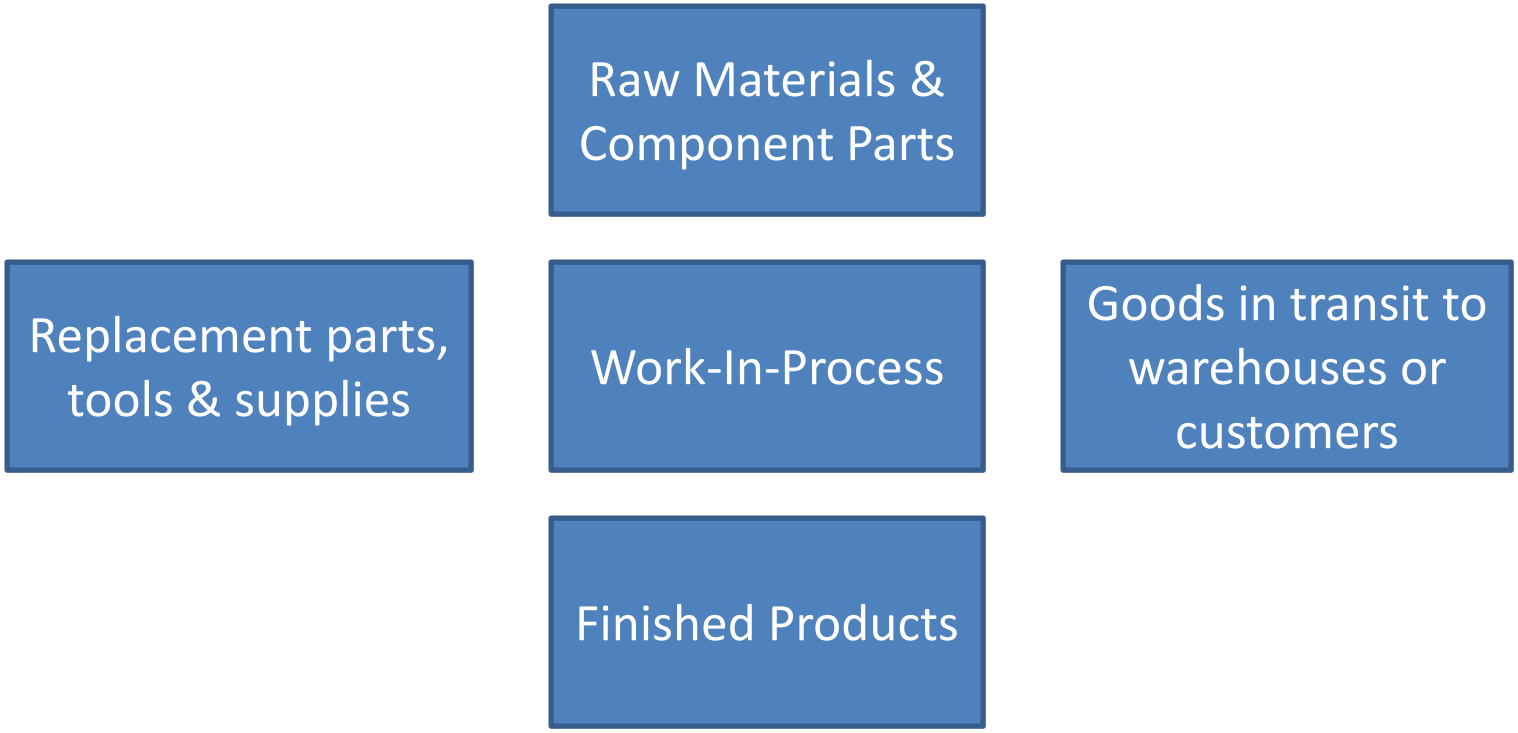
By ALESSANDRA MALITO
REPORTER

Grocery stores aren't just for your eggs and bread anymore.

The sheer number of food items has grown tremendously in the past two decades, said Michael Ruhlman, author of the new book "Grocery: The Buying and Selling of Food in America" (published by Abrams Press). Ruhlman, who

Inventory

- Definition: The stock of any item or resource used in an organization



Importance of Inventory
Balance Sheet: 2019.2 ~ 2020.1

	Wall Mart (Billion $)		Boeing* (Billion $)		General Motors* (Billion $)		Dell (Billion $)	
Cash & Short-Term Investments	9.5		10.0		22.4		10.0	
Account Receivable	6.3		12.5		33.4		17.4	
Inventories	44.4	71.8%	76.6	75.0%	10.4	13.9%	3.3	8.9%
Other Current Assets	1.6		3.1		4.6		6.2	
Total Current Assets	61.8	100%	102.2	100%	75.0	100%	36.9	100%
Other Assets	4.0		2.2		5.7		1.9	
Total Assets	236.5		133.6		228.0		118.9	

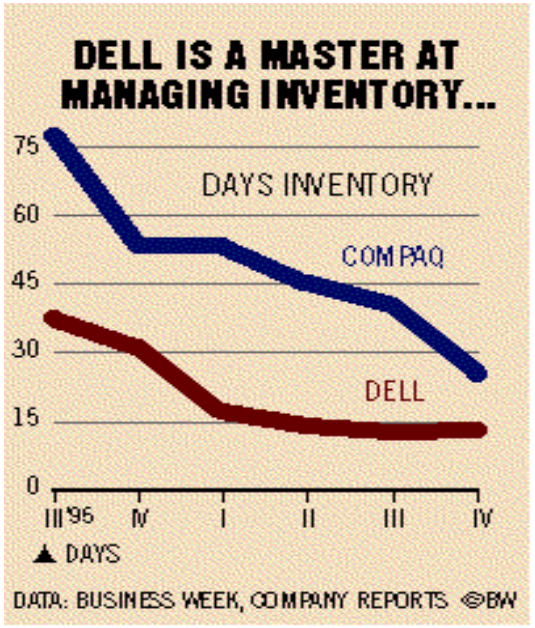
Source: https://www.wsj.com
* 2019.1 ~ 2019.12

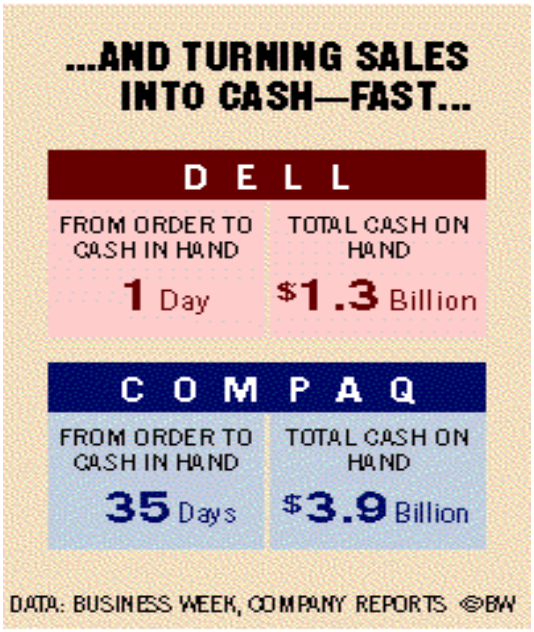
Importance of Inventory Management

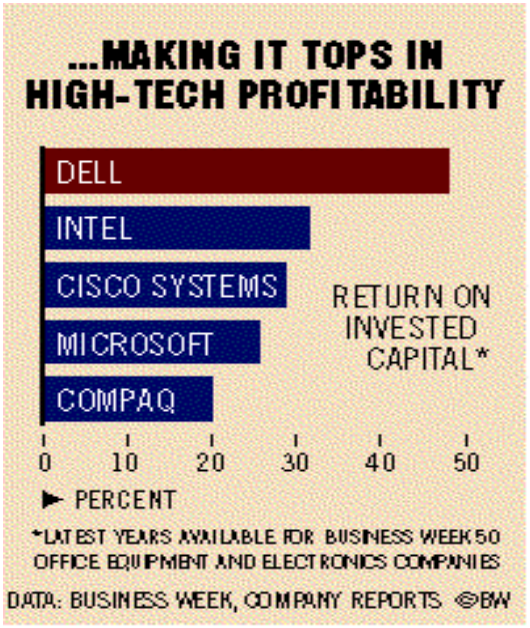
- **Natural-gas** futures soared 15% Thursday after U.S. inventory data slightly eased concerns about the possibility of a storage glut (*Wall Street Journal, Sept. 11, 09*) In August 2022, U.S. natural gas prices spiked to a 14-year high due to low inventory levels and high demand(CNN, Aug. 17, 22)
- **Fruit growers** were blessed with excellent weather this year. But that hasn't translated into a great year for the province's cherry and blueberry growers, as a bumper crop has flooded the market and pushed down prices. (*The Vancouver Sun, Aug. 09*)
- The **Ford** assembly plant in Oakville and 3,000 workers will remain idle this week because of a parts shortage from a supplier in India. (*Toronto Star, Oct. 27, 09*)
- **Nintendo** is experiencing Switch shortages due to low supply of critical components, like memory chips that store data and liquid-crystal displays, which Apple is gobbling up for its own products.. (*Fortune, May 31, 17*)

Accurately matching demand with supply is the key challenge

- WSJ, Aug. 93: **Dell Computer** stock plunges. The company was sharply off in forecast of demand resulting in inventory write-downs.
- BW 1997:





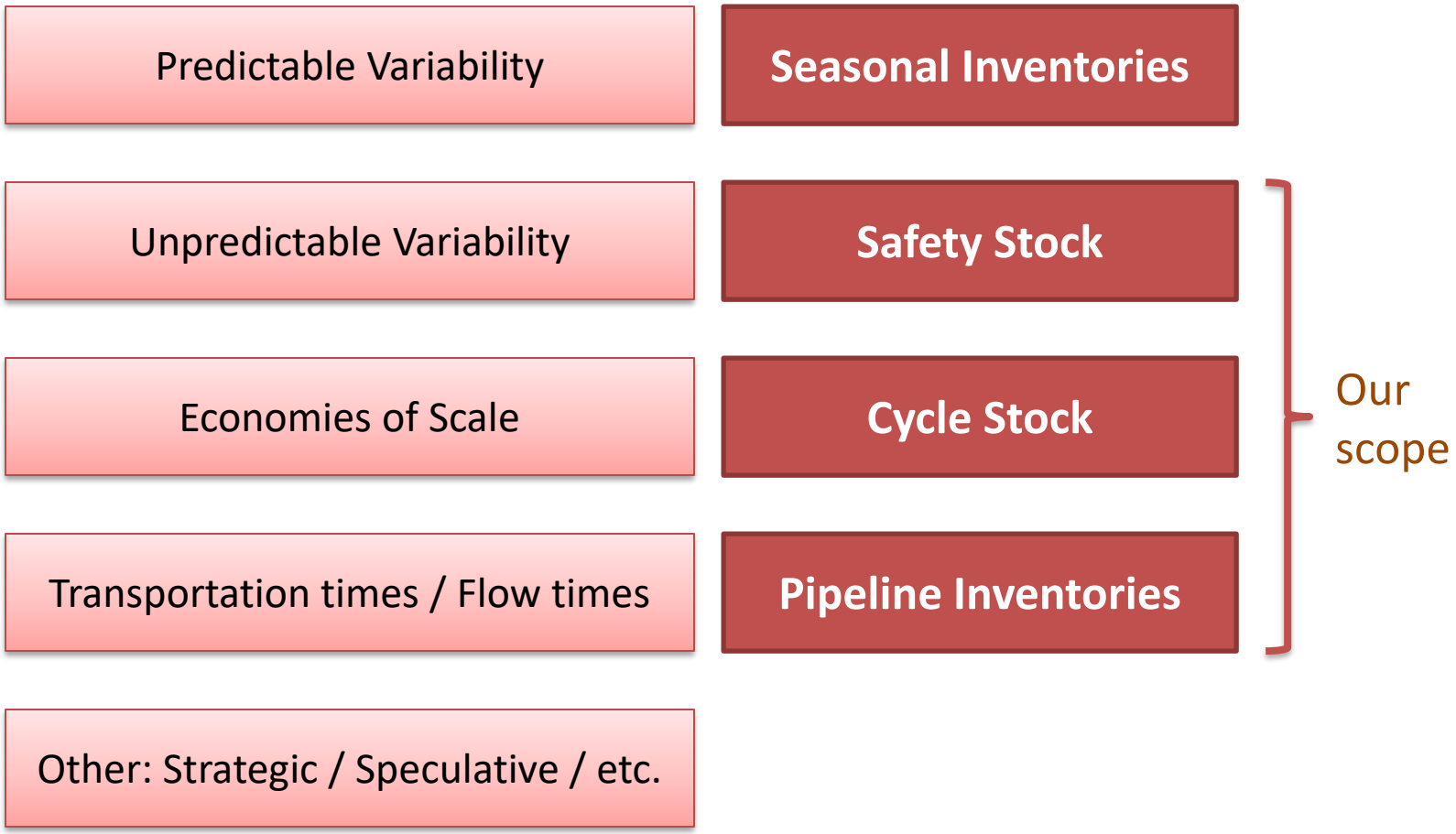


Inventory and the Economy

- Total business inventories in the US is $2.55 trillion

  (Source: U.S. Census Bureau, Oct 2023)
- In the aircraft industry, the Inventory to Net Working Capital ratio is about 80%. In the “electronic and other electrical equipment" industry, the ratio is about 62%
- For each dollar of GNP in the trade and manufacturing sector, about 40% worth was held in inventory.
- Average cost per year of inventory is 30 to 35% of its value

Why should you hold inventory?



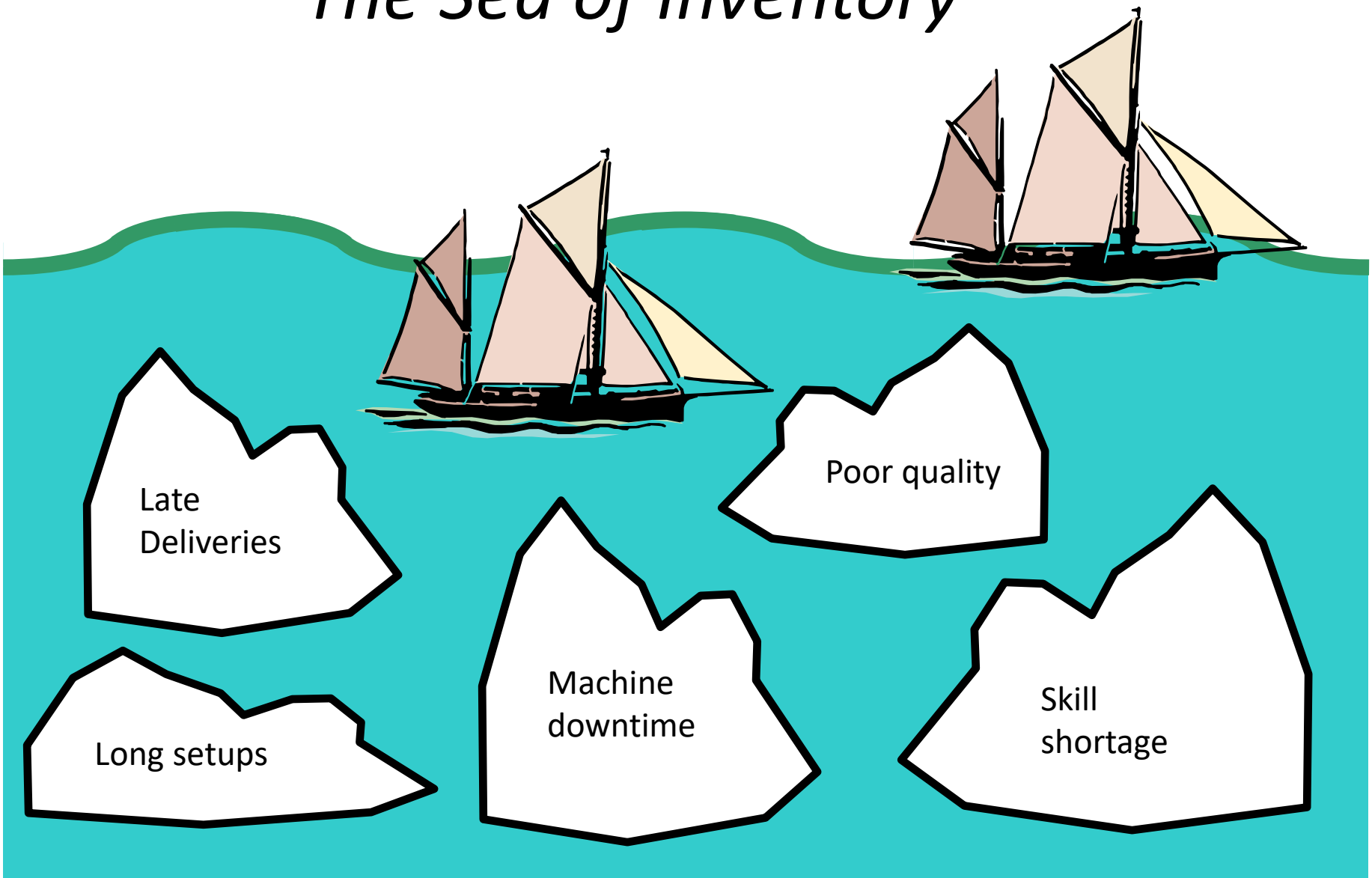
Why should you **not hold** inventory?

- Inventory increases certain costs such as
  - Holding (storage) cost
  - Opportunity cost of capital growth (interest etc.)
  - Waste (perishable products)

- Inventory hides problems
  - The Sea of Inventory

Reducing Waste

The Sea of Inventory

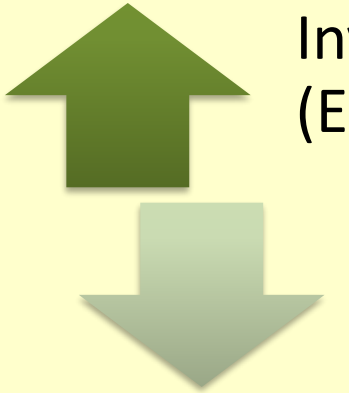


Inventory Costs

Ordering/Setup Cost (Fixed Cost)	• Fixed transportation cost • Per-order processing cost
Holding Cost (Carrying Cost)	• Costs for storage, handling, insurance, working capital tied-up, etc
Shortage Cost (Opportunity Cost)	• Lost sales, etc

How much inventory should you hold?

- Trade-off #1: How much to order each time?

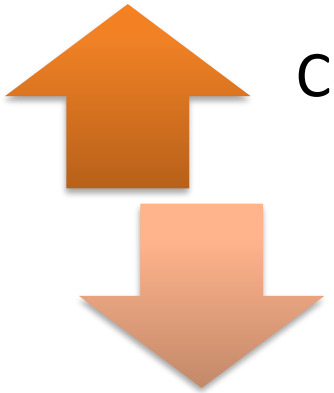


Inventory ordering costs (Economies of scale)

Inventory holding costs

Economic Order Quantity (EOQ) Model

- Trade-off #2: How much to store each time?



Cost of running out

Cost of having excess inventory

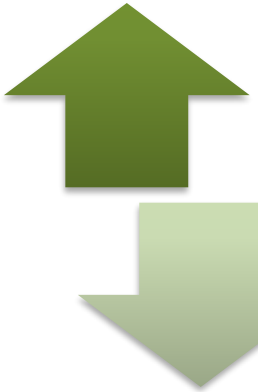
Newsvendor Model

Motivation for EOQ Model: ATM

- How much cash do you take out from ATM?
- How many rolls of toilet paper?
- Why not more or less?

Economic Order Quantity (EOQ) Model

- How much to order each time?
- The Economic Order Quantity (EOQ) balances



Inventory ordering costs
(Economies of scale)

Inventory holding costs

- <u>Assumptions</u>
  - Known annual demand, constant demand rate
  - No uncertainty (in demand)

Notations in EOQ Model

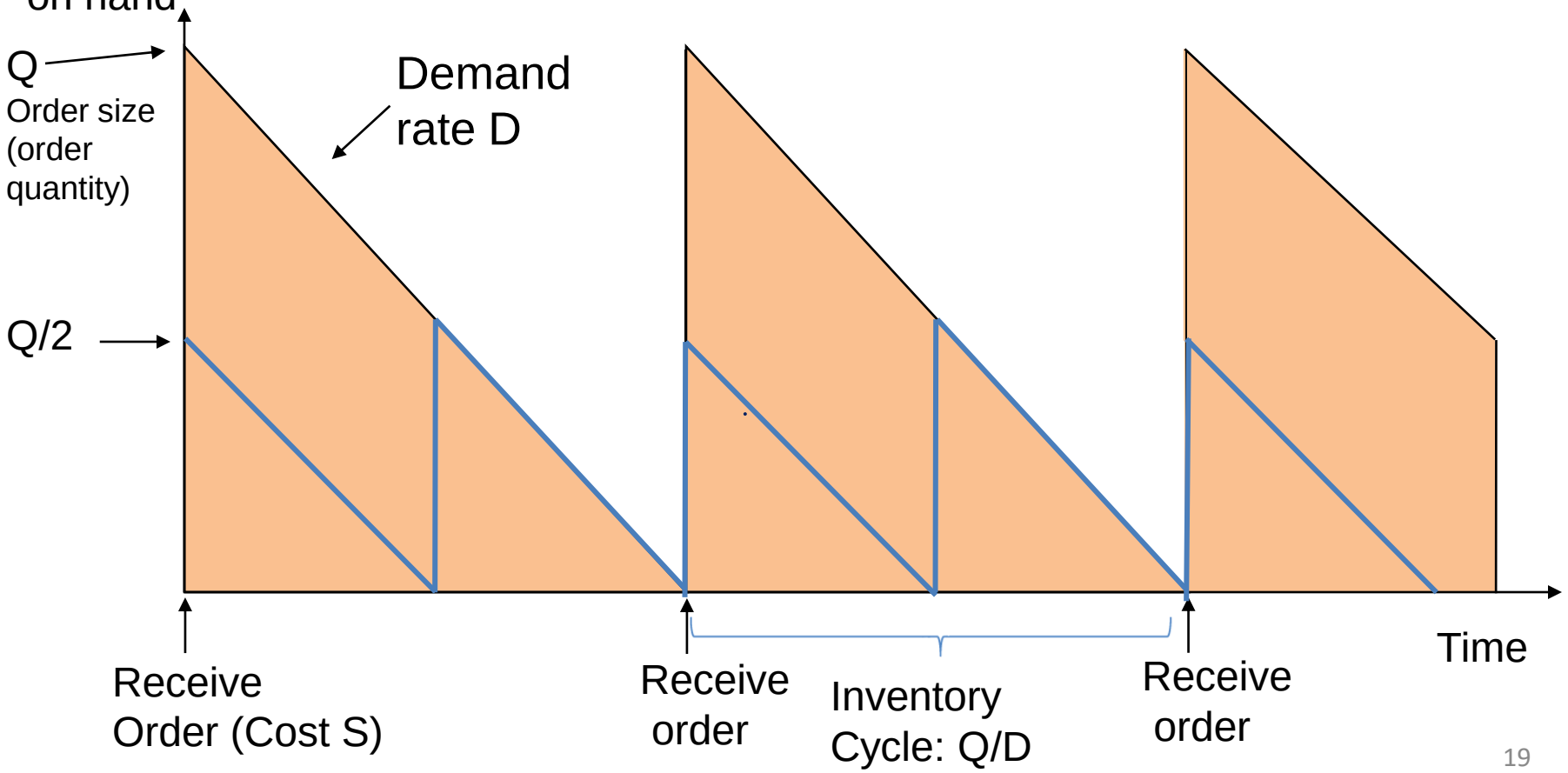
D	Annual Demand Rate
Q	Lot or batch size (of an order)
S	Set-up cost per lot/batch, or average cost of processing/placing an order
C	Unit cost ($/unit)
H	Annual holding and storage cost per unit of average inventory ($/unit/year)
i	Percent carrying cost (e.g., “interest” rate) (%/year)

Usually, H = i*C

Cycle Stocks: Tradeoff between fixed costs and holding costs

What if we change the order size to Q/2?

decrease average inventory (holding cost)
but increase number of orders (setup cost)



Example: “The South Face”

- Some facts about The South Face retail shop

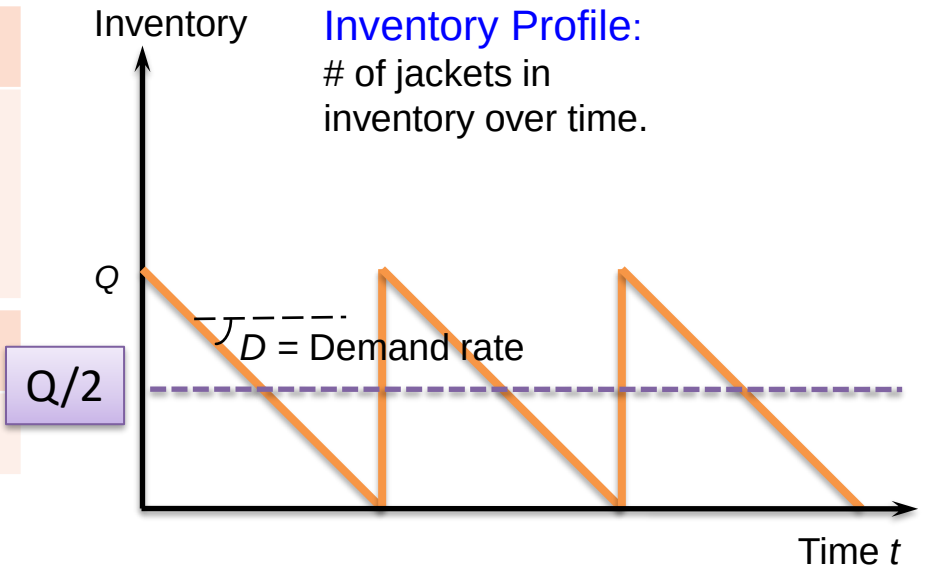
D	Annual Demand Rate	1200 jackets/year
S	Set-up cost per lot/batch, or average cost of processing/placing an order	$2,000
C	Unit cost	$200 per jacket
i	Percent carrying cost (e.g., interest rate)	25% (per year)

Thus, H = iC = (0.25)*($200) = $50 per unit-year

- What order size (Q) would you recommend for The South Face?

Cost Calculation

D	Annual Demand
Q	Lot or batch size (Number of jackets per replenishment order)
S	Order or setup cost
H	Annual Holding cost



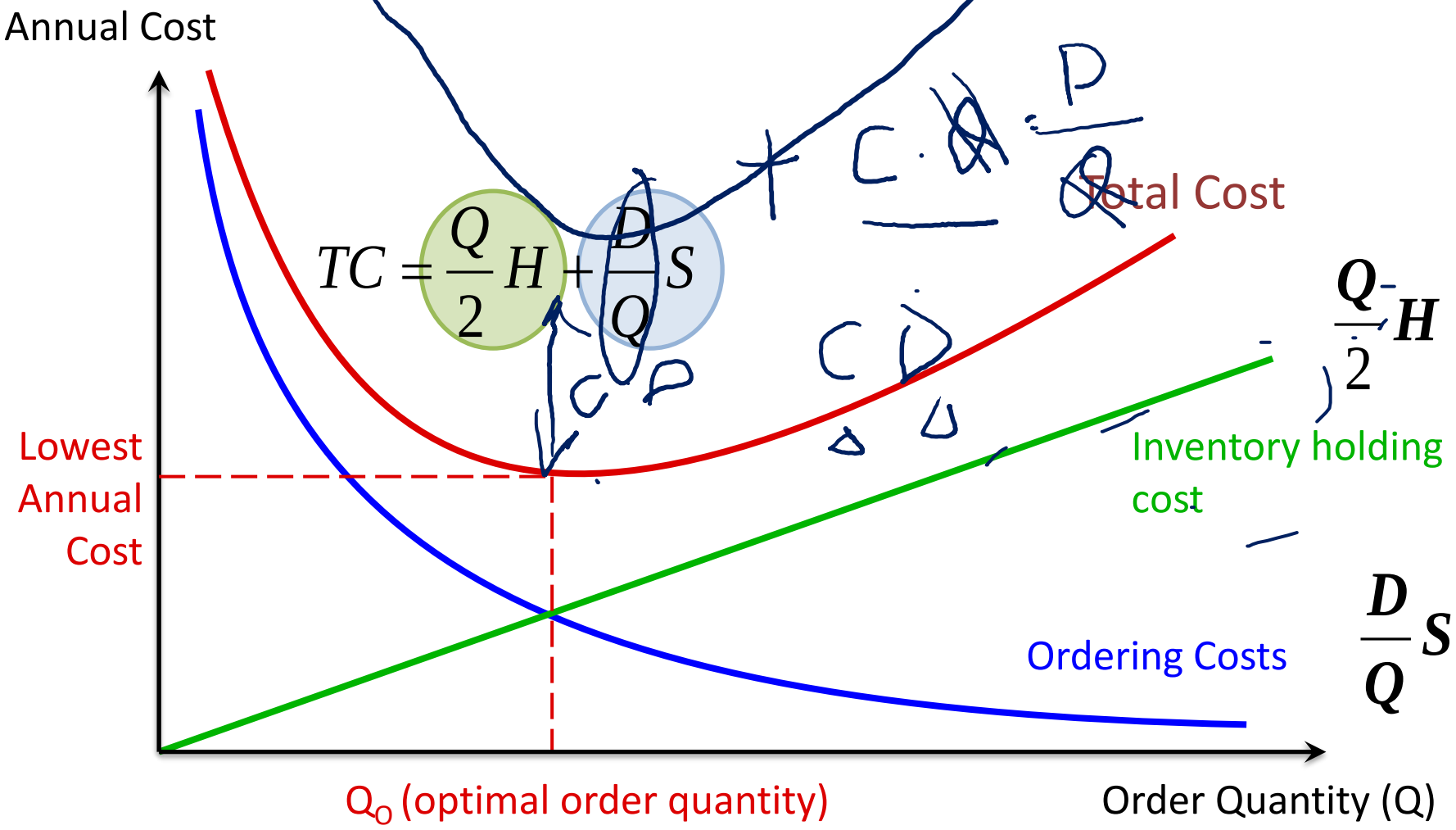
Number of orders per year	D/Q
Average inventory	Q/2
Annual Setup Cost	(D/Q) * S
Annual Holding Cost	(Q/2) * H
Annual Total Cost	Annual Setup Cost + Holding Cost

$$\frac{D}{Q}S + \frac{Q}{2}H$$

The South Face: Costs

Per Order/Batch Q	Batches per Year D/Q	Annual Setup Cost	Annual Holding Cost	Annual Total Cost
50	24.0	48000	1250	49250
100	12.0	24000	2500	26500
150	8.0	16000	3750	19750
200	6.0	12000	5000	17000
250	4.8	9600	6250	15850
260	4.6	9231	6500	15731
270	4.4	8889	6750	15639
280	4.3	8571	7000	15571
290	4.1	8276	7250	15526
300	4.0	8000	7500	15500
310	3.9	7742	7750	15492
320	3.8	7500	8000	15500
330	3.6	7273	8250	15523
340	3.5	7059	8500	15559
350	3.4	6857	8750	15607
400	3.0	6000	10000	16000
500	2.4	4800	12500	17300
600	2.0	4000	15000	19000
700	1.7	3429	17500	20929

Goal: Minimize Total Cost



Economic Order Quantity

$$\text{Total Cost} = TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

D	Annual Demand Rate
S	Order or Setup Cost
H	Annual Holding Cost

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

$$TC_{OPT} = \sqrt{2SDH}$$

Economic Order Quantity
The South Face

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

D	1200 jackets/year
S	$2,000
H	$50 per unit-year

- What is the optimal order quantity?

- How many times would you place orders per year, i.e., ***frequency*** of ordering?

- What is the time duration between successive orders (this is also called the ***cycle time*** or ***reorder interval***)?

EOQ and Sensitivity Analysis

- What happens to the cost and optimal quantity as the parameters change?

As ….	"Cost"	EOQ	Frequency
S ↑	↑	↑	↓
H↑	↑	↓	↑
D↑	↑	↑	↑

$$TC_{OPT} = \sqrt{2SDH} \qquad Q_{OPT} = \sqrt{\frac{2SD}{H}} \qquad \frac{D}{Q_{OPT}} = \sqrt{\frac{HD}{2S}}$$

Managerial Implications of EOQ

- Cost curve is almost “flat” near the optimal point
  - Use the EOQ formula, but do not worry about making minor adjustments to get a number that is “more realistic” for your organization
- The flatness of the cost curve implies that the EOQ figure is “robust”
  - Estimating holding cost is usually difficult
  - The EOQ formula guarantees that the “optimal” order quantity is not very sensitive to errors in estimation .

Managerial Challenges
How to estimate costs?

Ordering/Setup Costs (Fixed Costs): S

- Estimate costs incurred during the start of each new order
- Do not count “sunk” costs and fixed overhead

Unit Cost: C

- Estimate variable cost incurred in the production of each additional unit

Holding or Carrying Costs (as % of unit cost): i

- Estimate opportunity cost of working capital
- Estimate cost of storage, handling, etc

Managerial Challenges
How to reduce the EOQ inventory?

- Reduce the set-up cost
  - Re-evaluate sources of fixed costs, and find ways to reduce, spread-out, or eliminate these costs

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

Bulk Orders?

D	Annual Demand Rate
Q	Lot or batch size (of an order)
S	Set-up cost per lot/batch, or average cost of processing/placing an order
C	Unit cost ($/unit)
H	Annual holding and storage cost per unit of average inventory ($/unit/year)
i	Percent carrying cost (e.g., “interest” rate) (%/year)

- Which parameter changes if your supplier is offering discount for large orders?

EOQ with Quantity Discounts

- Unit cost structure: Cj(Q)
  - $11/unit: order < 10,000 units
  - $10/unit: 10,000 <= order < 80,000 units
  - $9.5/unit: 80,000 <= order
- Demand (D): 8,000 units/year
- Setup cost (S): $12,000/order
- Holding cost (H): $0.3/unit/year

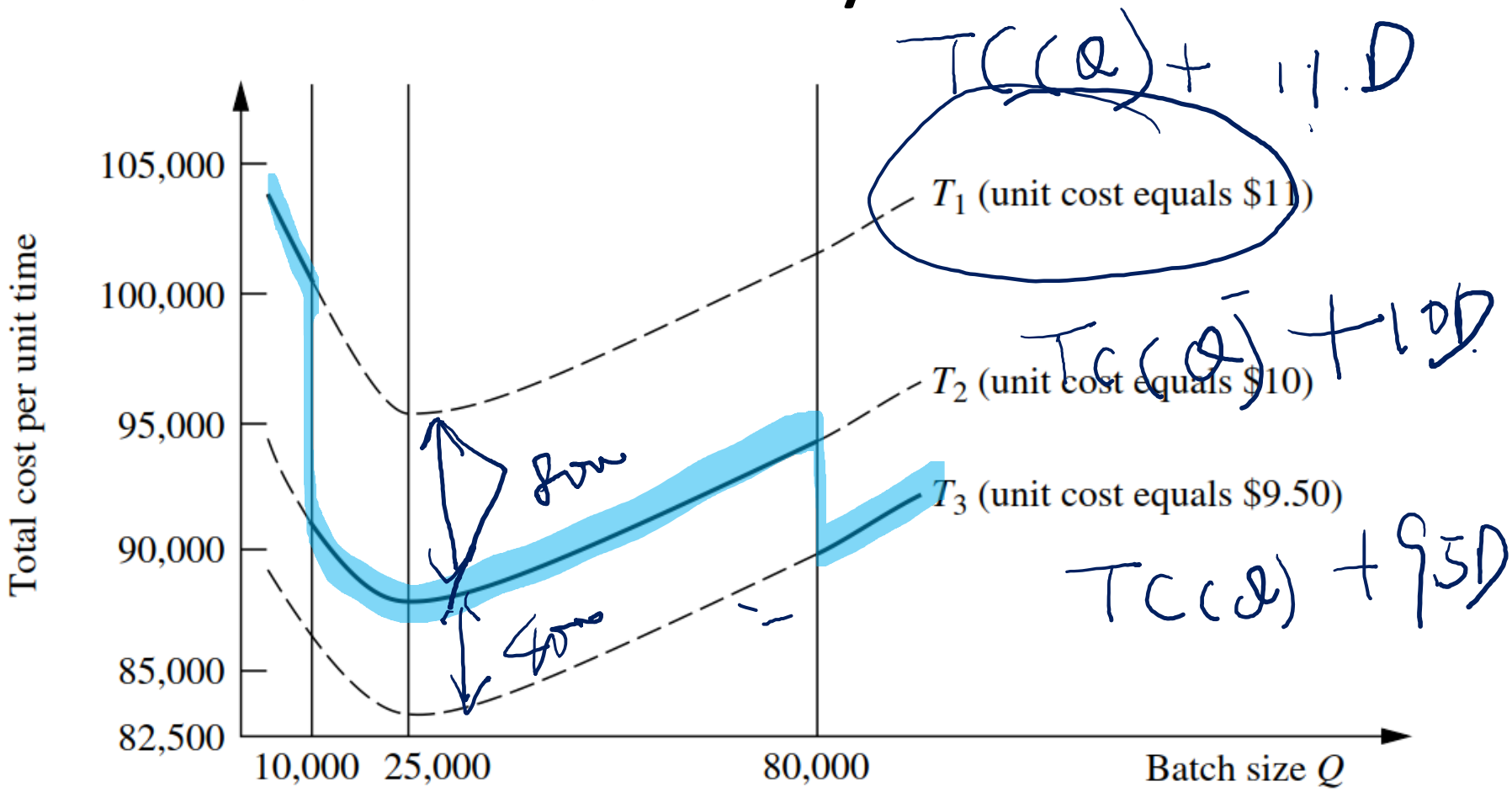
$$Q_{OPT} =$$

EOQ with Quantity Discounts

$$\text{Total Cost} = TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S + Dc_j$$

- Cost structure now includes procurement cost
- How does it differ from basic EOQ cost structure?
- How does the total cost curve look now?

EOQ with Quantity Discounts



- TC(25,298)=$87,589
- TC(80,000)=$89,200

EOQ with Quantity Discounts

- Unit cost structure: Cj(Q)
  - $11/unit: order < 10,000 units
  - $10/unit: 10,000 <= order < 80,000 units
  - $9/unit: 80,000 <= order
- TC(25,298)=$87,589
- TC(80,000)=$85,200

Practice Problem

The demand for spring water at the Plano Walmart is 600 liters per week. The setup cost for placing an order to replenish inventory is $25. The water loses its freshness while stored at the store. To account for this, Walmart charges an annual holding cost of $2.6/liter. (You may assume 1 year=52 weeks =365 days).

(1) Determine how often the Plano Walmart should order for water. (What are we asking for?)

Practice Problem

(1) What is the order cycle?

Let's match the time units in years. (default: 1 year = 52 weeks = 365 days)

H=$2.6/liter, D=600*52 =31200 liters/year, S=$25

$$Q_{OPT} = \sqrt{\frac{2SD}{H}} = \sqrt{\frac{2*25*600*52}{2.6}} = 774.6 \text{ liter}$$

Optimal Order Cycle: Q/D = 774.6/31200=0.0248 year = 9.06 days

Practice Problem

(2) Now suppose the order is delivered by the supplier which charges Walmart $0.10/liter for the cost of transportation from the Ozark mountains to Plano. This transportation cost increases the cost of water to $1.25/liter. Suppose the other parameters, including the $25 order cost and $2.6/liter holding cost will not change. How does the transportation cost impact the order cycle?

The order cycle will (*increase,* *decrease,* *not change*) when there is an additional transportation cost.

Practice Problem

(2) Not Change. Because the transportation cost does not depend on how often you make the order. It should be calculated in the unit cost C. However, in this problem, H is explicitly given and C does not impact the optimal quantity.

Practice Problem: QMH (1)

Queen Marry Hospital consumes 100 boxes of bandages per week. The price of bandages is $70 per box. The hospital operates 52 weeks per year. The cost of processing an order is $60, and the cost of holding one box for a year is 15% of the value of the material. What is the EOQ and the corresponding total cost?

D	100*52 per year
S	$60
C	$70
i	15%
H	i*C = (0.15)(70)=10.50

$$TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

$$EOQ = \sqrt{\frac{2SD}{H}} = 244$$

$$TC(244) = \frac{244}{2}H + \frac{D}{244}S = 2459.7$$