MSBA7003 Quantitative Analysis Methods

Assignment 1 (Due September 18 at 23:55 am. Please submit your solutions with the template.)

Q1.

The joint probability distribution between two random variables X and Y is given below.

Joint Probabilities	Y = 1	Y = 2	Y = 3
X = 0	0.1	0.2	0.3
X = 1	0.2	0.1	0.1

Which of the following statement(s) is(are) true?

(A) X = 0 and Y = 1 are unconditionally independent.

(B) The event of X = 0 is less likely to occur given Y = 2 than given Y = 1.

(C) Pr(X = 1 | Y > 1) = 2/7.

(D) Suppose we randomly and independently draw two pairs of (X,Y): (X1,Y1) and (X2,Y2). We only see the values of Y. Then Pr(X1 + X2 = 0 | Y1 = 2 and Y2 = 3) = 1/2.

(E) None of the above.

Solution: We check each statement as follows:

(A) **Not True.** Pr(X = 0) = 0.6, Pr(Y = 1) = 0.3 and Pr(X = 0 & Y = 1) = 0.1. Hence, Pr(X = 0 & Y = 1) $\neq$ Pr(X = 0)$\times$Pr(Y = 1).

(B) **Not True.** Pr(X = 0 | Y = 2) = Pr(X = 0 & Y = 2) / Pr(Y = 2) = 0.2 / (0.2 + 0.1) = 2/3, while Pr(X = 0 | Y = 1) = Pr(X = 0 & Y = 1)/Pr(Y = 1) = 0.1 / (0.2 + 0.1) = 1/3.

(C) **True.** Pr(X = 1|Y > 1) = Pr(X = 0 &Y > 1)/Pr(Y > 1) = [Pr(X = 1 & Y = 2) + Pr(X = 1 & Y = 3)]/[Pr(Y = 2) + Pr(Y = 3)] = 2/7

(D) **True.** Pr(X1 + X2 = 0 | Y1 = 2 and Y2 = 3) = Pr(X1 = 0 & X2=0 & Y1=2 & Y2=3)/Pr(Y1=2 & Y2 =3) = Pr(X1 = 0 & Y1 =2) Pr(X2 = 0 & Y2 =3)/ Pr(Y1=2 & Y2 =3) = (0.2$\times$0.3)/(0.3$\times$ 0.4)=1/2.

Q2.

Suppose you can see a lawn out of your window. The lawn may be wet or dry in the afternoon. The state depends on the whether it rained earlier today and the previous operation of the sprinkler on the lawn. We are given the conditional probability of the afternoon lawn being wet given the previous operation of the sprinkler and whether it rained earlier today as below:

Previous operation of the sprinkler		On B1	On B1	Off B0	Off B0
Whether it rained earlier today		Yes C1	No C0	Yes C1	No C0
Lawn state	Wet A1	0.99	0.9	0.8	0.0
	Dry A0	0.01	0.1	0.2	1.0

We also know that the operation of the sprinkler is affected by the weather probabilistically in the following way in terms of conditional probabilities:

Whether it rained earlier today		Yes C1	No C0
Sprinkler	On B1	0.1	0.4
	Off B0	0.9	0.6

In addition, our prior belief is that it rains with a probability of 0.2 every day. Pr(C1) = 0.2

Which of the following statement(s) is(are) true?

(A) If you know that the sprinkler was on earlier today, the probability that the lawn will be wet in the afternoon is 0.2*0.99 + 0.8*0.9 = 0.918.

(B) If you know that it rained in the morning, the probability that the lawn will be wet in the afternoon is 0.1*0.99 + 0.9*0.8 = 0.819.

(C) If you see in the afternoon that the lawn is wet, the probability that it rained before is (0.99 + 0.8)/(0.99 + 0.9 + 0.8 + 0) = 0.6654.

(D) If you see in the afternoon that the lawn is wet, the probability that the sprinkler was on before is (0.8*0.1*0.99 + 0.2*0.4*0.9)/(0.8*0.1*0.99 + 0.2*0.4*0.9 + 0.8*0.9*0.8 + 0.2*0.6*0) = 0.2.

(E) None of the above.

Solution:

To simplify notation, let's denote the following events:
A1: The lawn will be wet in the afternoon.
B1: The sprinkler was on earlier today.
C1: It rained earlier today.

Similarly, their mutually exclusive and collectively exhaustive (MECE) events can be denoted as:
A0: The lawn will not be wet in the afternoon.
B0: The sprinkler was off earlier today.
C0: It did not rain earlier today.

(A) Not True. $Pr(A1 \middle| B1) = Pr(A1|B1, C1) * Pr(C1|B1) + Pr(A1|B1, C0) * Pr(C0|B1)$, wherein $Pr(C1|B1) = 0.1 * 0.2/(0.1 * 0.2 + 0.4 * 0.8)\ = 0.0588$, and $Pr(C0|B1) = 1 - 0.0588 = 0.9412$. Hence, $Pr(A1 \middle| B1) = 0.99 * 0.0588\ +\ 0.9 * 0.9412\ =\ 0.9053$.

Another method to calculate:

$$Pr(A1|B1) = \frac{\Pr(A1B1)}{\Pr(B1)} = \frac{\Pr(A1B1C1)+\Pr(A1B1C0)}{\sum_{i=0,1}\Pr(B1,Ci)} =$$

$$\frac{\Pr(A1|B1,C1)\Pr(B1|C1)\Pr(C1)+\Pr(A1|B1,C0)\Pr(B1|C0)\Pr(C0)}{\sum_{i=0,1}\Pr(B1|,Ci)\Pr(Ci)} =$$

$$\frac{0.99*0.1*0.2+0.9*0.4*0.8}{0.1*0.2+0.4*0.8} = 0.9053.$$

(B) True. $Pr(A1|C1) = Pr(A1|C1,B1)Pr(B1|C1) + Pr(A1|C1,B1)\,Pr(B1|C1) = 0.99 * 0.1 + 0.8 * 0.9 = 0.819.$

(C) Not True. $Pr(C1 \mid A1) = \frac{Pr(A1C1)}{Pr(A1)} = \frac{\Pr(A1B1C1)+\Pr(A1B0C1)}{\sum_{i,j=0,1}\Pr(A1,Bi,Cj)} =$

$$\frac{\Pr(A1|B1,C1)\Pr(B1|C1)\Pr(C1)+\Pr(A1|B0,C1)\Pr(B0|C1)\Pr(C1)}{\sum_{i,j=0,1}\Pr(A1|Bi,Cj)\Pr(Bi|Cj)\Pr(Cj)} =$$

$$\frac{0.99*0.1*0.2+0.8*0.9*0.2}{0.99*0.1*0.2+0.9*0.4*0.8+0.8*0.9*0.2+0*0.6*0.8} = 0.3625.$$

(D) Not Ture. $Pr(B1|A1) = \frac{\Pr(A1B1)}{\Pr(A1)} = \frac{\Pr(A1B1C1)+\Pr(A1B1C0)}{\sum_{i,j=0,1}\Pr(A1,Bi,Cj)} =$

$$\frac{\Pr(A1|B1,C1)\Pr(B1|C1)\Pr(C1)+\Pr(A1|B1,C0)\Pr(B1|C0)\Pr(C0)}{\sum_{i,j=0,1}\Pr(A1|Bi,Cj)\Pr(Bi|Cj)\Pr(Cj)} =$$

$$\frac{0.99*0.1*0.2+0.9*0.4*0.8}{0.99*0.1*0.2+0.9*0.4*0.8+0.8*0.9*0.2+0*0.6*0.8} = 0.6813.$$

Q3.

ABC Inc. is considering launching a new product and there are three options: product X, product Y, and do nothing. Product X requires an initial investment of $15 million and product Y requires $5 million. The gross profit (before subtracting the initial investment) that can be generated by each product depends on the market condition. If the market is strong, product X can generate a gross profit of $100 million and product Y can generate $20 million; if the market is weak, product X will lead to a loss of $80 million and product Y will cause a loss of $9 million (before subtracting the initial investment). The company's prior belief of a strong market is 60%.

To get a better understanding of the market before deciding the choice of the new product, the manager hired a consulting firm to conduct market research. According to historical data, this consulting firm can successfully predict a strong market in 65% cases and can correctly predict a weak market in 55% cases.

Based on this information, which of the following statement(s) is(are) true?

(A) If the consulting firm is not hired, it is better not to introduce any product.

(B) If there is not historical data about the consulting firm (i.e., we don't know the probabilities of accurate predictions), $38 million should be paid at most to hire the company.

(C) If the consulting firm gave an unfavorable report, ABC should introduce product X.

(D) The expect value of the consulting firm's report is about $3.12 million.

(E) None of the above.

Solution: Before hiring the consulting firm, ABC Inc's decision tree is as follows:

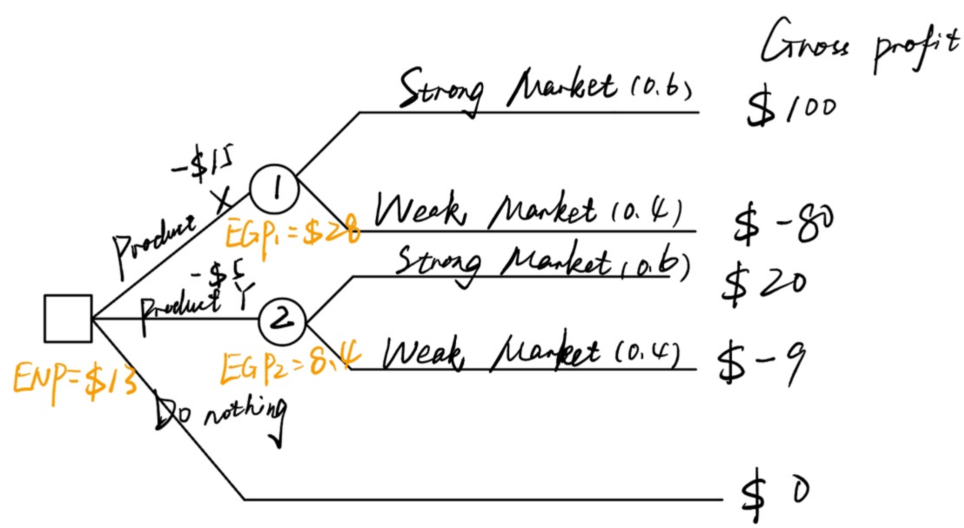

Fig.1. ABC Inc's decision tree

If the consulting firm is not hired, launching product X can get the best net profit, and EMV = $13, **A is not true.** Considering the best payoff of each market, EVwPI = 0.6*(100-15)+0.4*0 = 51, so EVPI = EVwPI-EMV = $38, **B is true.**

After hiring the consulting firm and have known the probability of accurate predictions, we can obtain:

Pr (Consulting Firm: Positive| Market: Strong) = 0.65

Pr (Consulting Firm: Negative| Market: Weak) = 0.55

Then we can calcite the updated probability of a strong or weak market.

Joint Prob.	Consulting Firm :Strong	Consulting Firm :Weak	Marginal
Market: Strong	0.65*0.6=0.39	0.35*0.6=0.21	0.6
Market: Weak	0.45*0.4=0.18	0.55*0.4=0.22	0.4
Marginal	0.57	0.43	

- If Consulting Firm: Weak, we get:

P(Market: Strong|Consult Firm: Weak) = 0.21/0.43=0.4884 and

P(Market: Weak|Consult Firm: Weak) = 0.5116.

EMV of launching product X is 100*0.4884-80*0.5116 -15 = -7.088,

EMV of launching product Y is 20*0.4884-9*0.5116 -5 = 0.1636.

Therefore, given the negative results from the consulting firm, ABC should introduce product Y(EMV = 0.1636), so **C is not true.**

- If Consulting Firm: Strong, we get:

P(Market: Strong|Consult Firm: Strong) = 0.39/0.57=0.6842 and

P(Market: Weak|Consult Firm: Stong) = 0.3158.

EMV of launching product X is 100*0. 6842-80*0. 3158 -15 = 28.156,

EMV of launching product Y is 20*0. 6842-9*0. 3158 -5 = 5.8418.

Therefore, given the positive results from the consulting firm, ABC should introduce product X (EMV = 28.156)**.**

Finally, we can calucute the value of consulting firm's report which is EVPI = 0.57*(28.156 - 13) + 0.43*(0.1636 - 13) = 3.119, **D is true.**