

(i)

$$D_1 = \begin{bmatrix} 1/16 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{41}E_{31}E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -7 & 0 & 0 & 1 \end{bmatrix} D_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -16/11 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{12}E_{32}E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 29/16 & 1 & 0 \\ 0 & 97/16 & 0 & 1 \end{bmatrix} D_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/784 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{13}E_{23}E_{43} = \begin{bmatrix} 1 & 0 & -14/16 & 0 \\ 0 & 1 & -29/11 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1793/176 & 1 \end{bmatrix}$$

(ii)

$$E_{41}E_{31}E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -10 & 0 & 0 & 1 \end{bmatrix} D_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 9 & 0 & 1 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 108 & 1 \end{bmatrix} D_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{117} \end{bmatrix}$$

$$E_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{21}E_{31}E_{41} = \begin{bmatrix} 1 & -1 & -10 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii) I did Gauss-Jordan to small scale. For example $n=2,3,\dots$ and find out the regularity.