Homework for Linear Algebra
September 24, 2024

Chengyu Zhang

Exercise 1. Assume $A = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_n \end{bmatrix}$, $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_m \end{bmatrix}$, $C = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_n \end{bmatrix}$ and $E = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_m \end{bmatrix}$

1. $A\mathbf{b}_2$
2. $\mathbf{a}_1 B$
3. $\mathbf{a}_3\mathbf{b}_5$
4. $\mathbf{c}_1 D\mathbf{e}_1$

Exercise 2.

$(A+B)^2 = \begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix}^2 = \begin{bmatrix} 10 & 4 \\ 6 & 6 \end{bmatrix}$, $A^2+2AB+B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 2 \\ 3 & 0 \end{bmatrix}$

$$(A+B)(A+B) = A^2 + AB + BA + B^2$$

Exercise 3. Because when we do a row swapping operation ,we actually won't change which column each entry in.
When we do a scaling or addition operation , it follows the distributive law : $(a_1 + a_2) * c = a_1 * c + a_2 * c$. So the order is arbitrary.

Exercise 4. Assume $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}$
If $AB = I$, for $\mathbf{b}_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, we have

$$\begin{cases} 2 * x_1 + 3 * x_2 = 1 \\ 1 * x_1 + 2 * x_2 = 0 \\ 7 * x_1 + 100 * x_2 = 0 \end{cases}$$

No solution.

Exercise 5. For $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$, $A\mathbf{x}$ equals the first column of A. So if for all the x with one entry 1 and the rest 0 , we have $A\mathbf{x} = B\mathbf{x}$, we have A = B.