Fourier Transform for Maths Dummy

How can we obtain Fourier Series?

- What if I use a series of cosine function with different cycle to convolve the signal?
- Sine waves are orthogonal to each other….

$$x_T(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_1 t + \varphi_n)$$

$$= c_0 + c_1 \cos(\omega_1 t + \varphi_1) + c_2 \cos(2\omega_1 t + \varphi_2) + \cdots + c_n \cos(n\omega_1 t + \varphi_n) + \cdots$$

http://madebyevan.com/dft/

This tells how features are selected

$$\int_0^{2\pi} \cos(mx)\cos(nx)\,dx = 0; (m \neq n)$$

$$\int_0^{2\pi} \sin(nx)\sin(nx)\,dx = \pi;$$

$$\int_0^{2\pi} \cos(nx)\cos(nx)\,dx = \pi;$$

Convolution Theorem

- The discrete *convolution* of two functions *f* (*x*,*y*) and *h* (*x*,*y*) of size *M* x *N* is defined as

$$f(x,y) \star h(x,y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n).$$

- This is equivalent to the *correlation* of *f* (*x*,*y*) with *h*(*x*,*y*) flipped about the origin.
- Convolution theorem:

$$f(x,y) \star h(x,y) \Leftrightarrow F(u,v)\, H(u,v)$$

$$f(x,y)\, h(x,y) \Leftrightarrow F(u,v) \star H(u,v)$$

- Where “ $\Leftrightarrow$ ” indicates a Fourier transform pair.

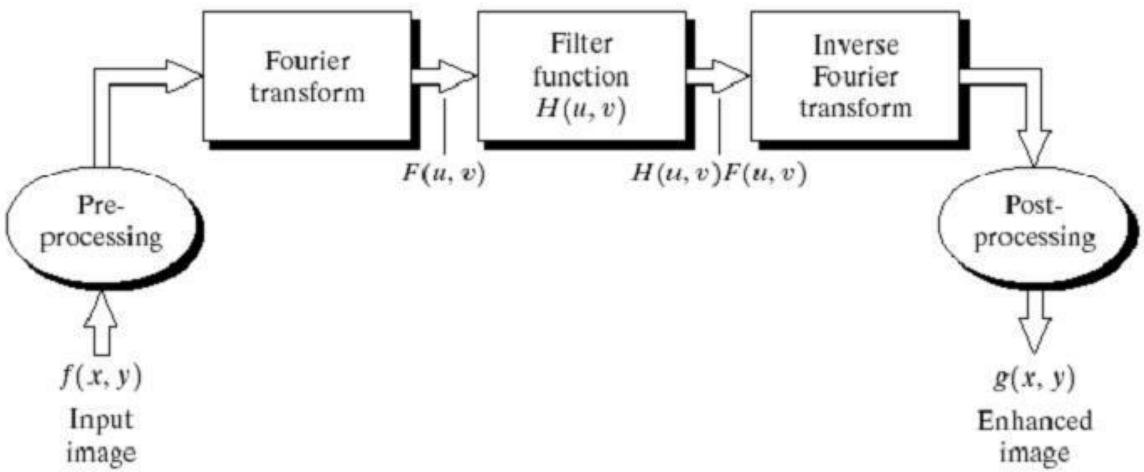
Frequency Domain Filtering

Filter image $f(x, y)$ **with mask** $h(x, y)$

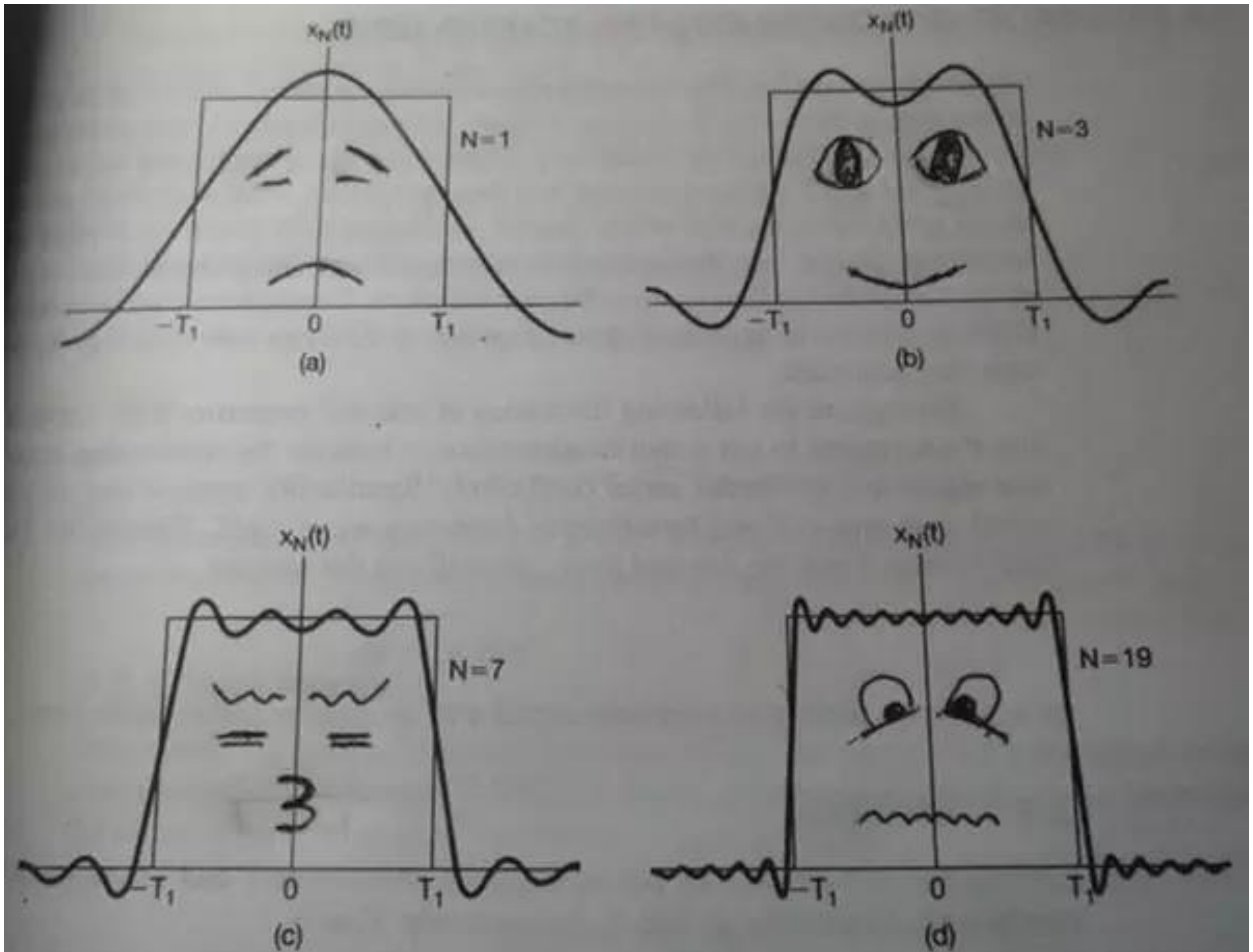
(1) Fourier transform the image $f(x, y)$ to obtain its frequency rep. $F(u, v)$.
(2) Fourier transform the mask $h(x, y)$ to obtain its frequency rep. $H(u, v)$
(3) multiply $F(u, v)$ and $H(u, v)$ pointwise to obtain $F'(u, v)$
(4) apply the inverse Fourier transform to $F'(u, v)$ to obtain the filtered image $f'(x, y)$.

Algorithm 3: Filtering image $f(x, y)$ with mask $h(x, y)$ using the Fourier transform

Frequency domain filtering operation



傅里叶级数(Fourier Series)的频谱



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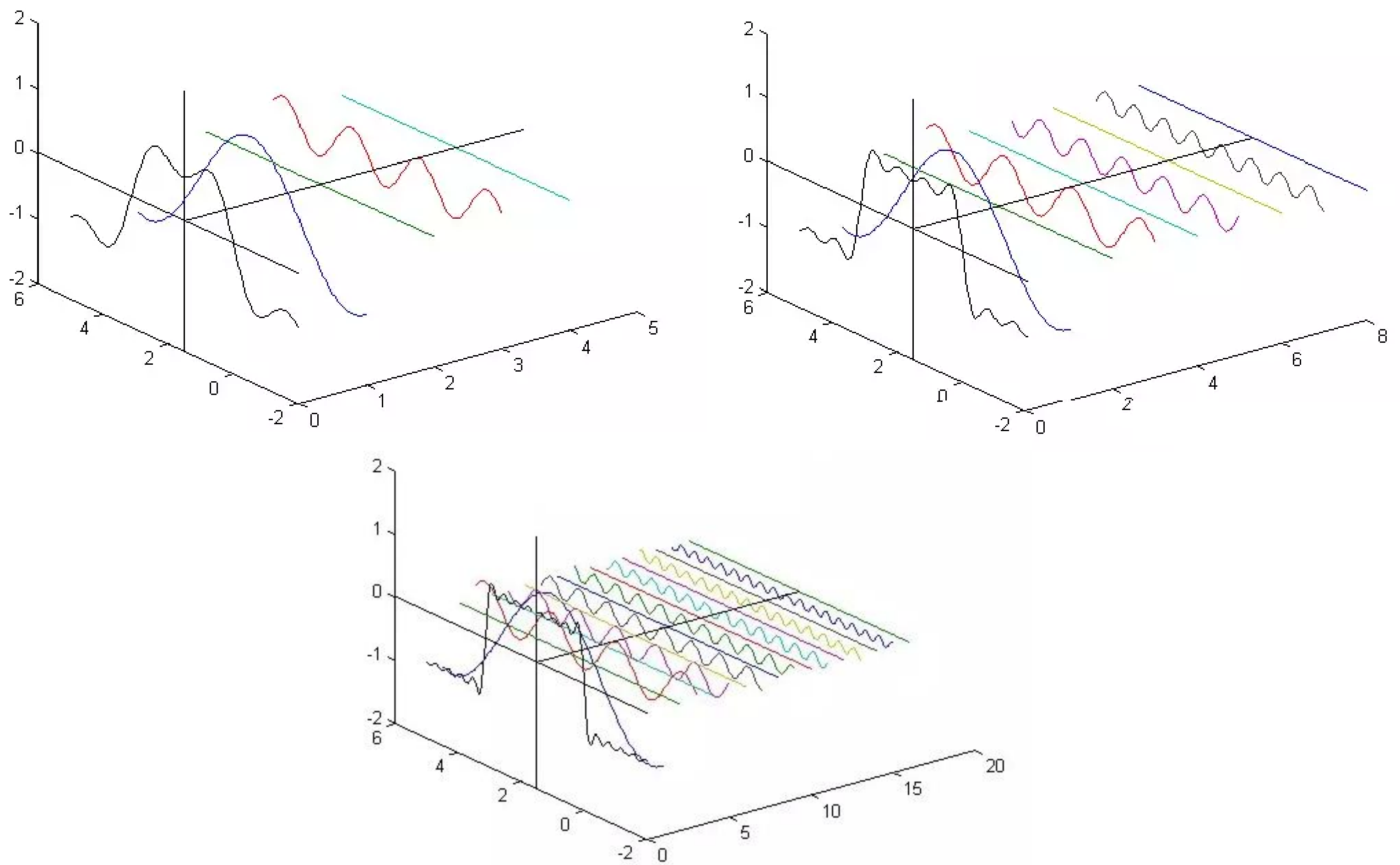

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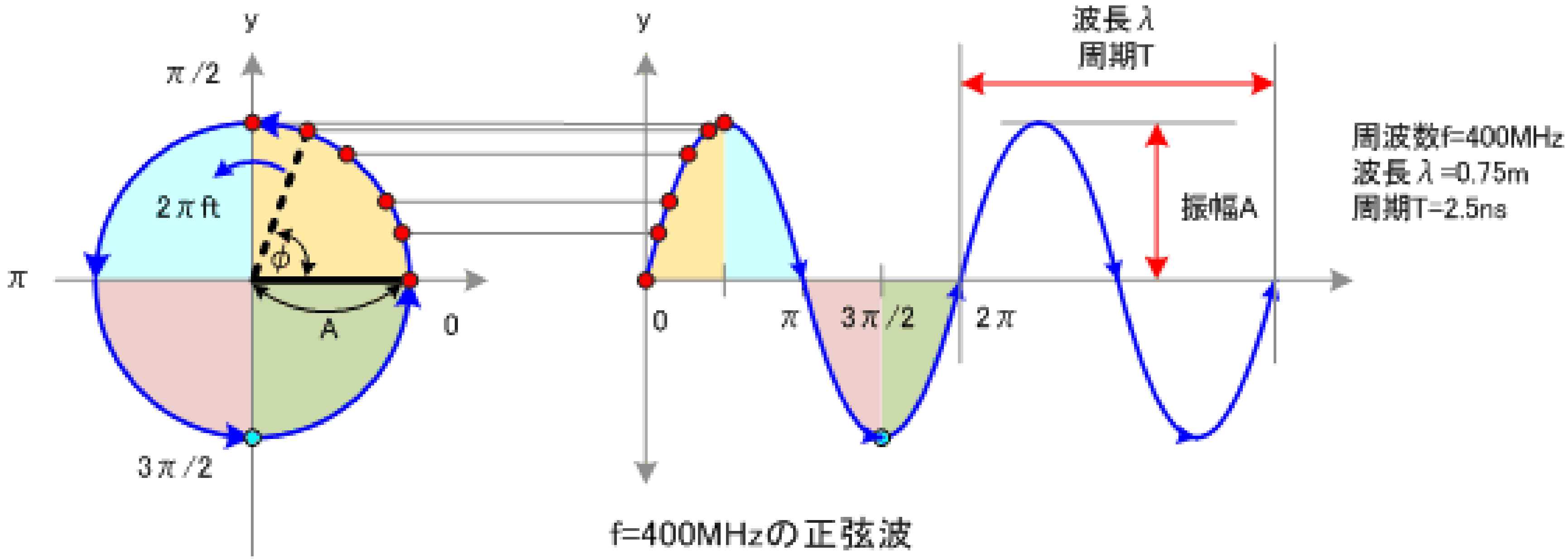

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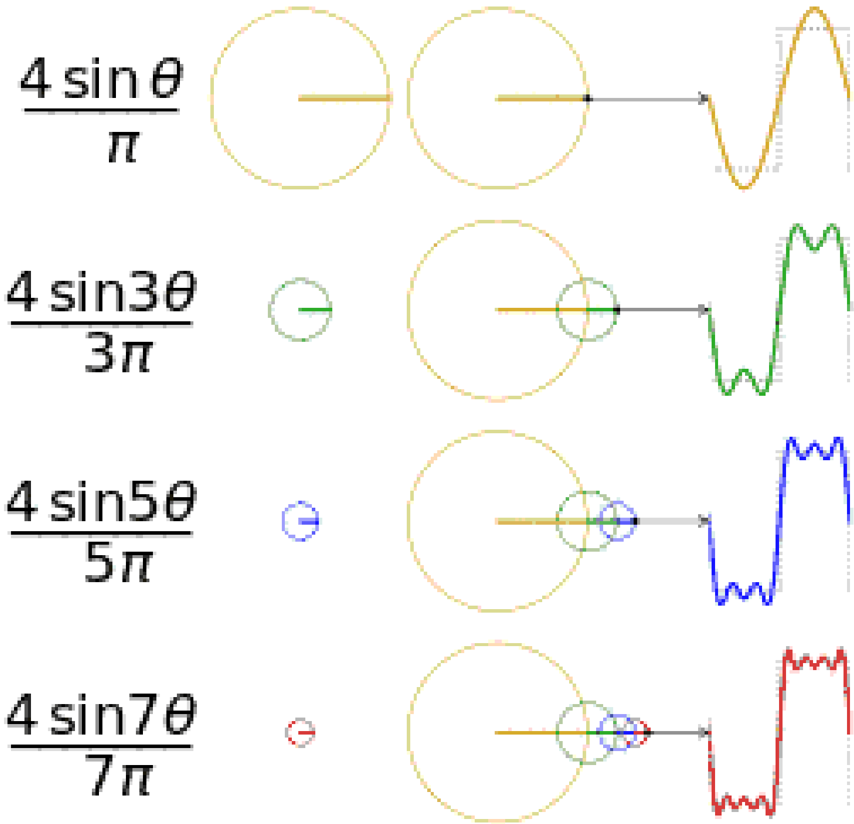

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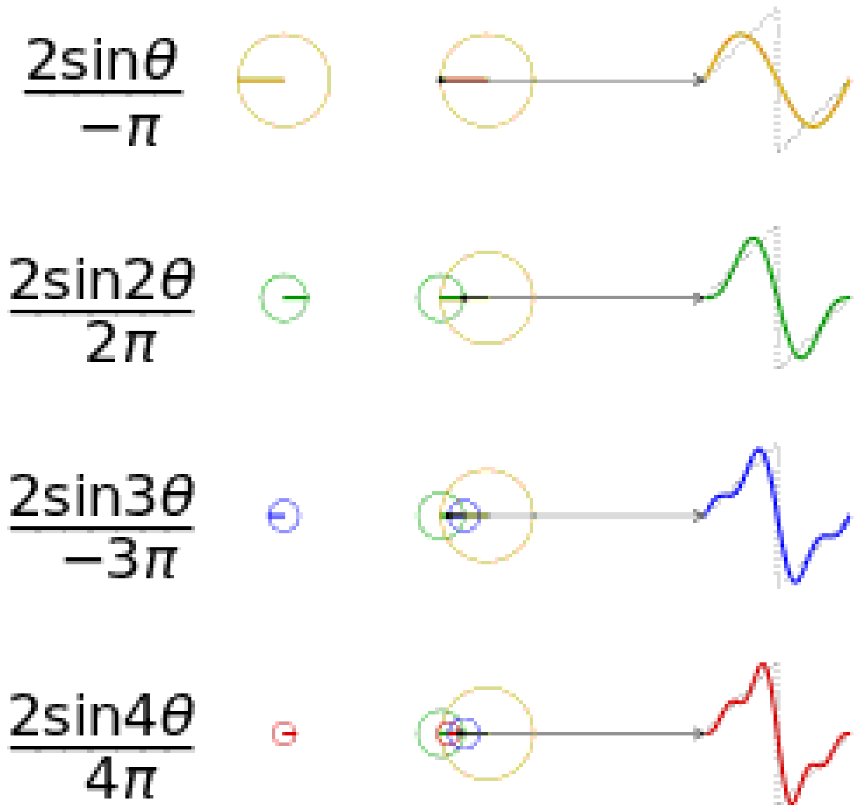

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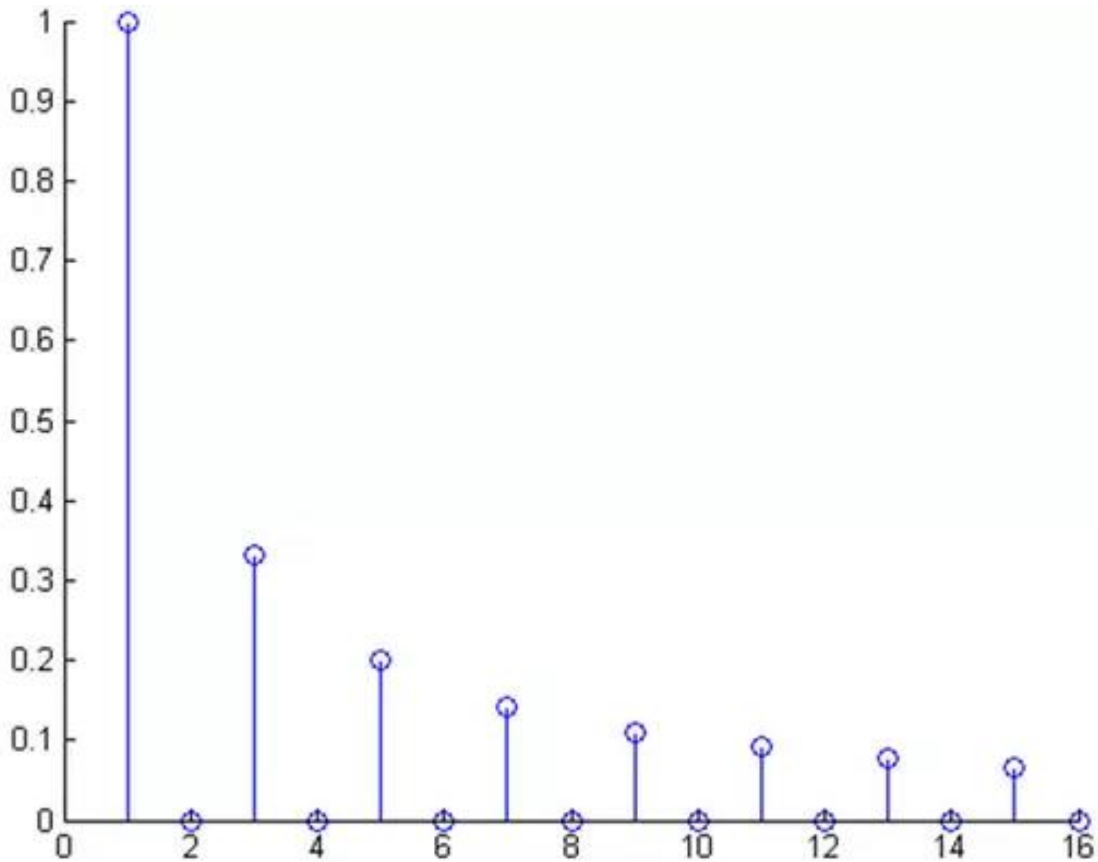

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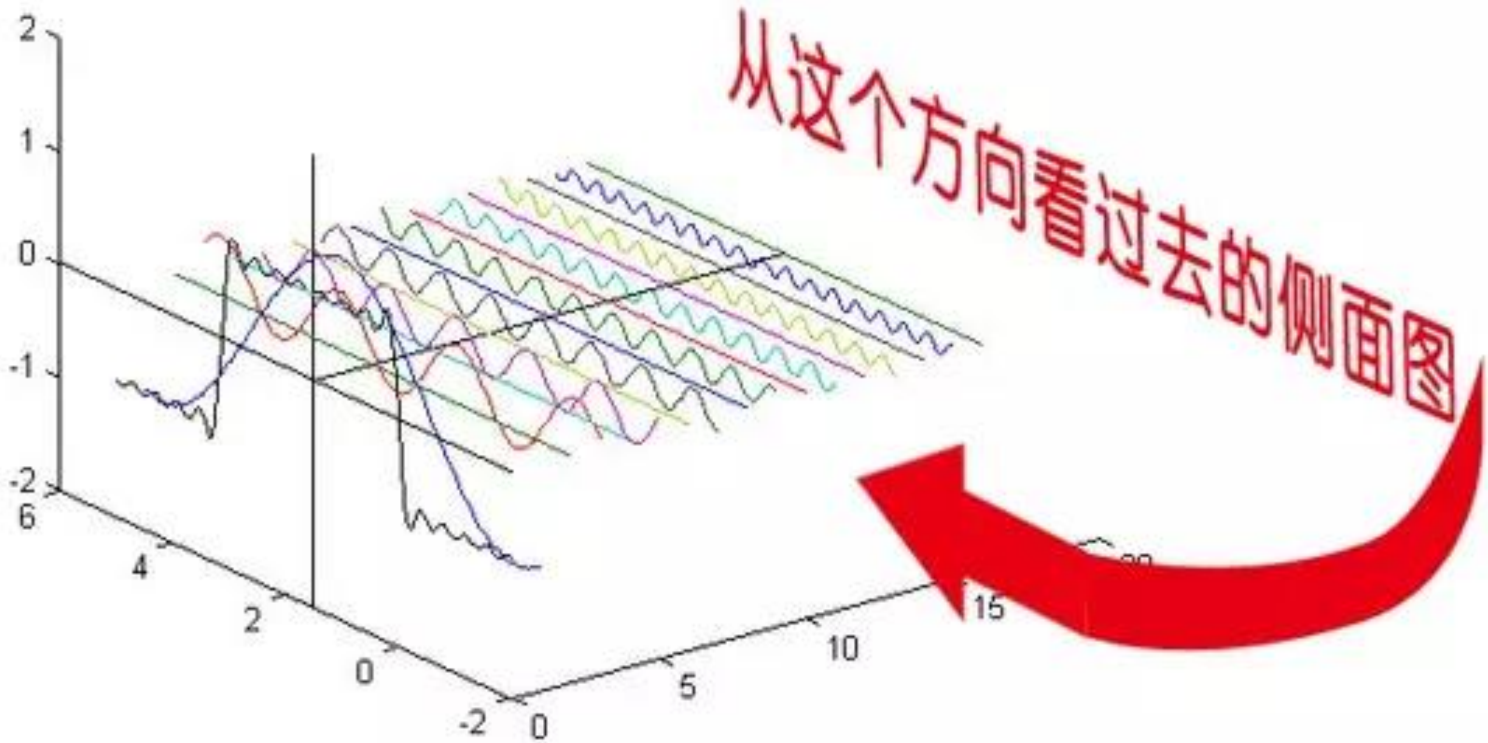

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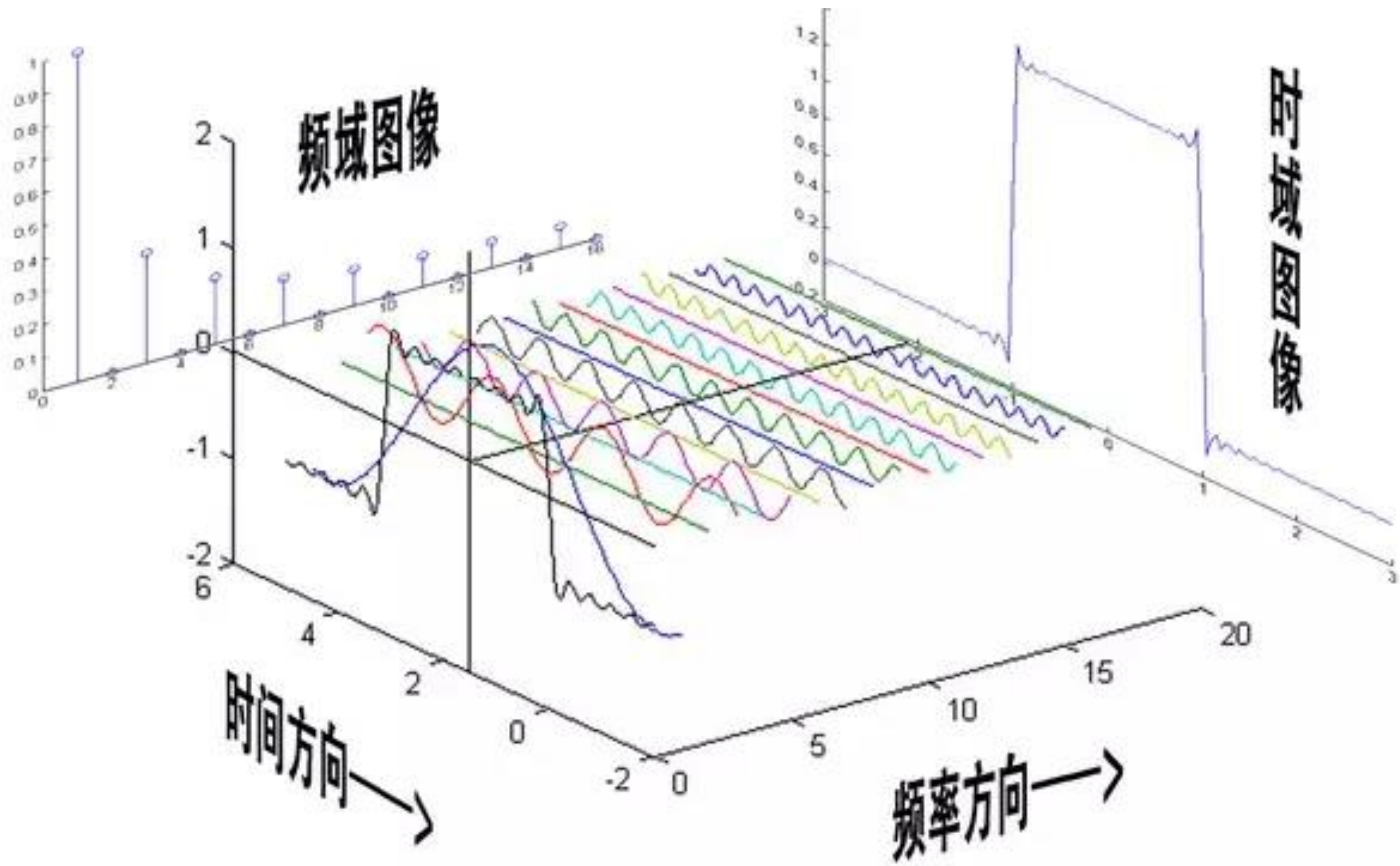
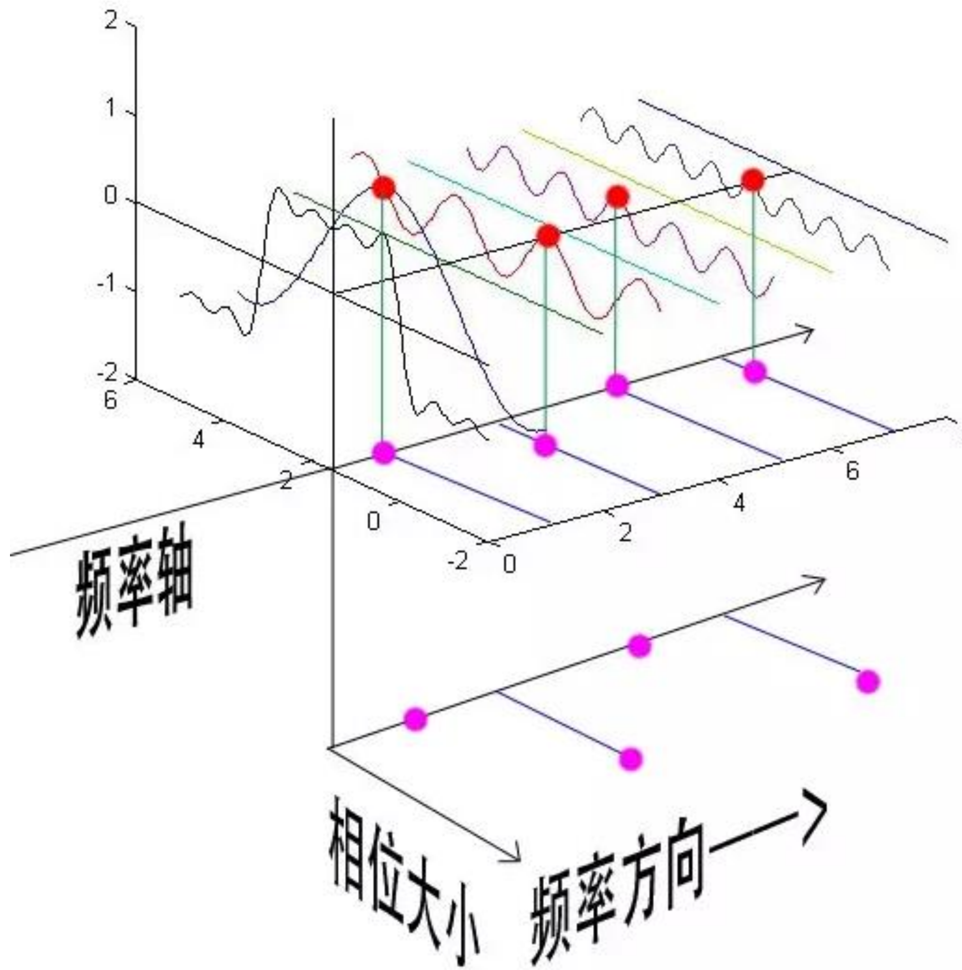


傅里叶级数(Fourier Series)的频谱

傅里叶级数（Fourier Series）的相位谱



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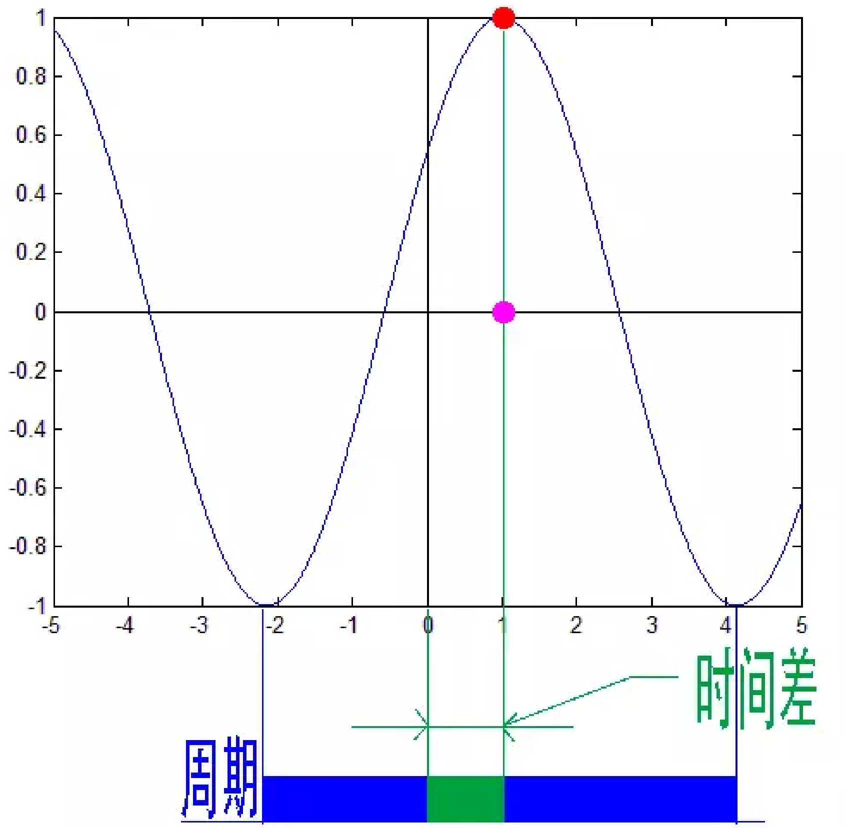

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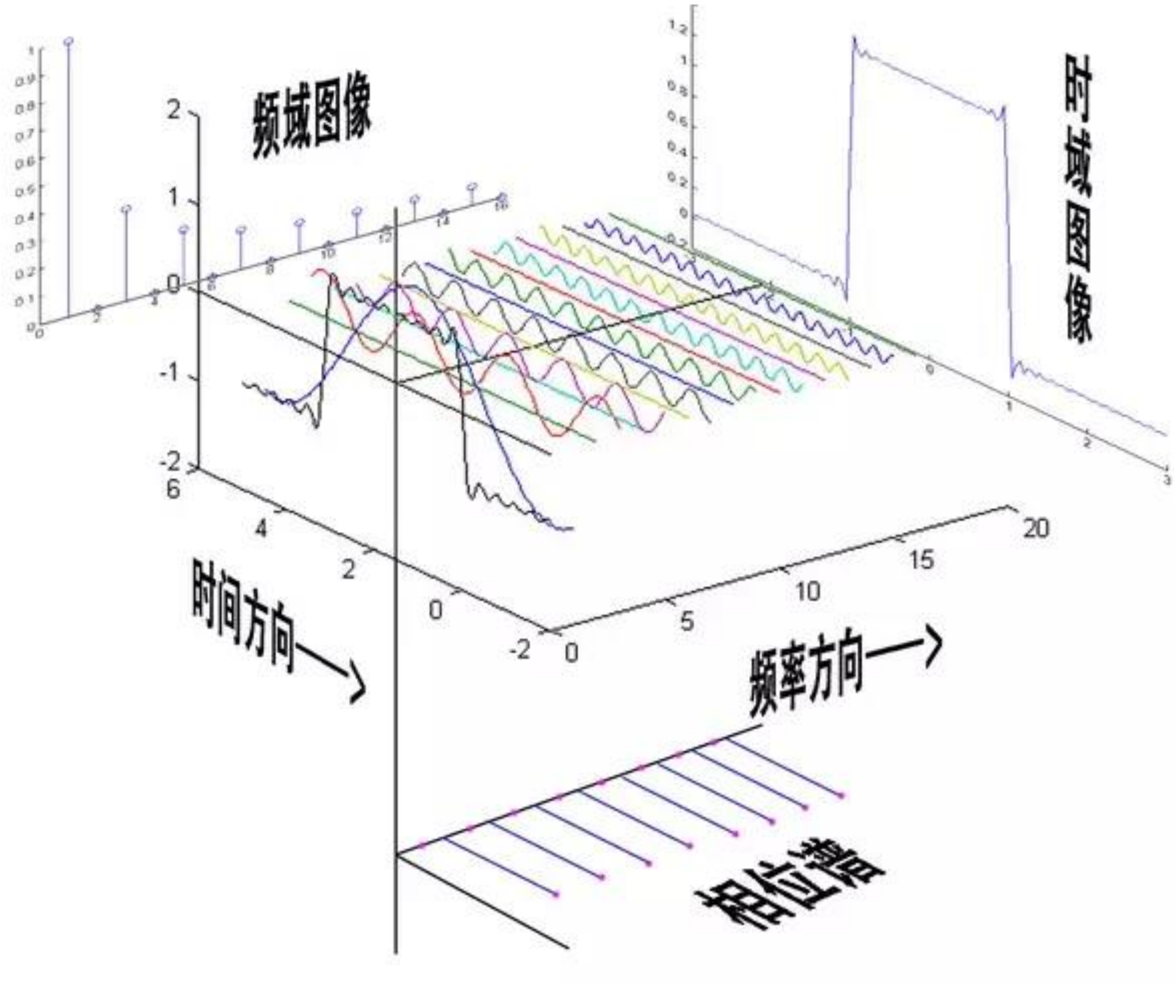
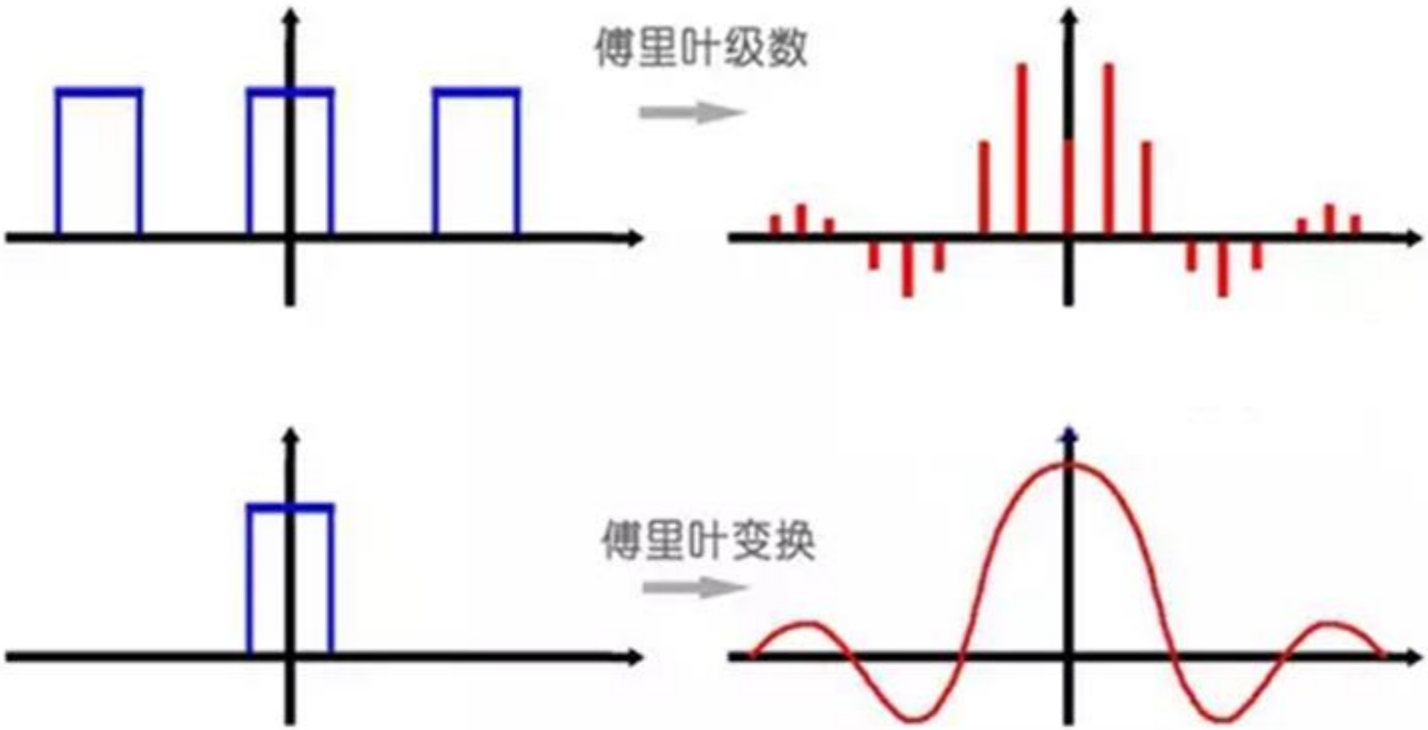


傅里叶变换（Fourier Transformation）



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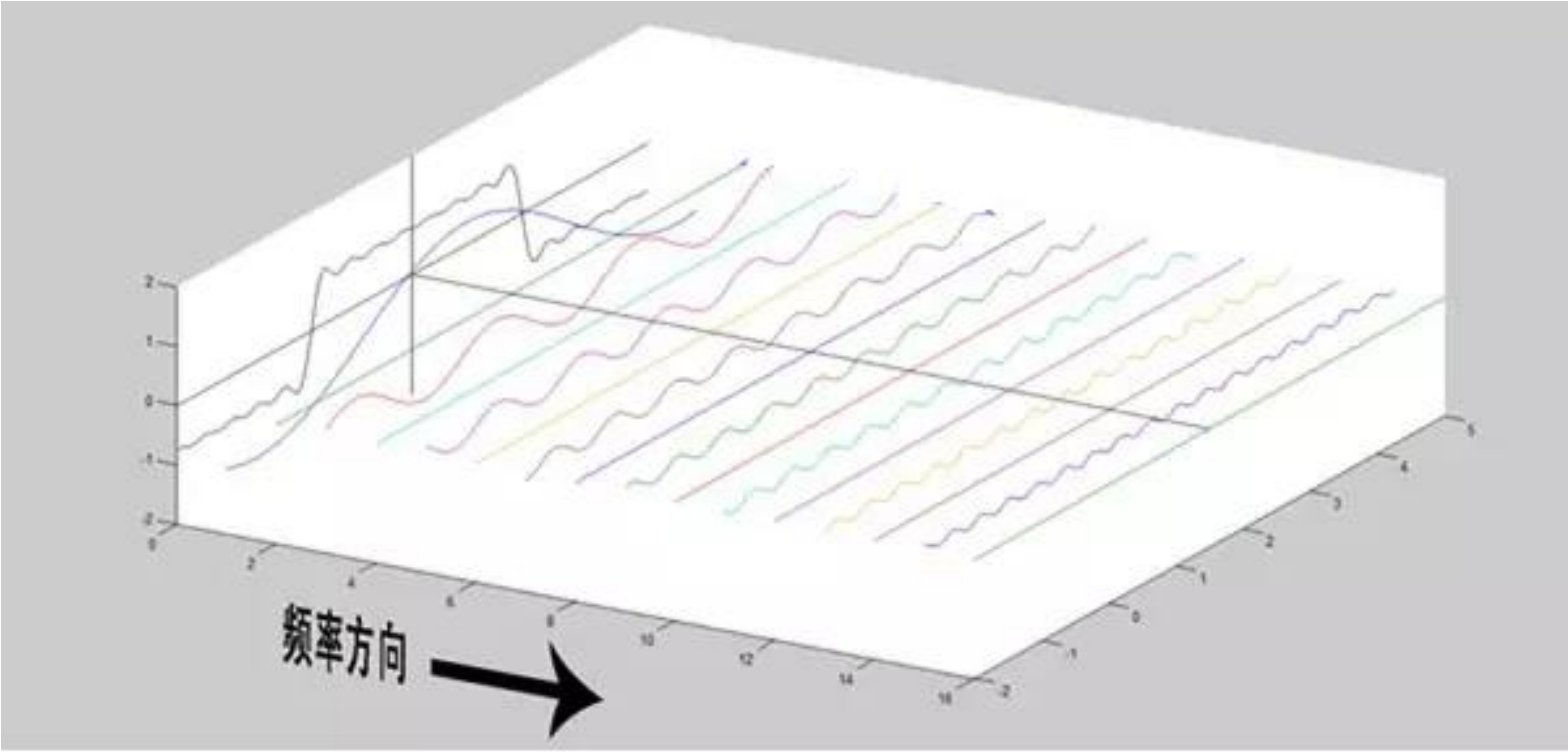

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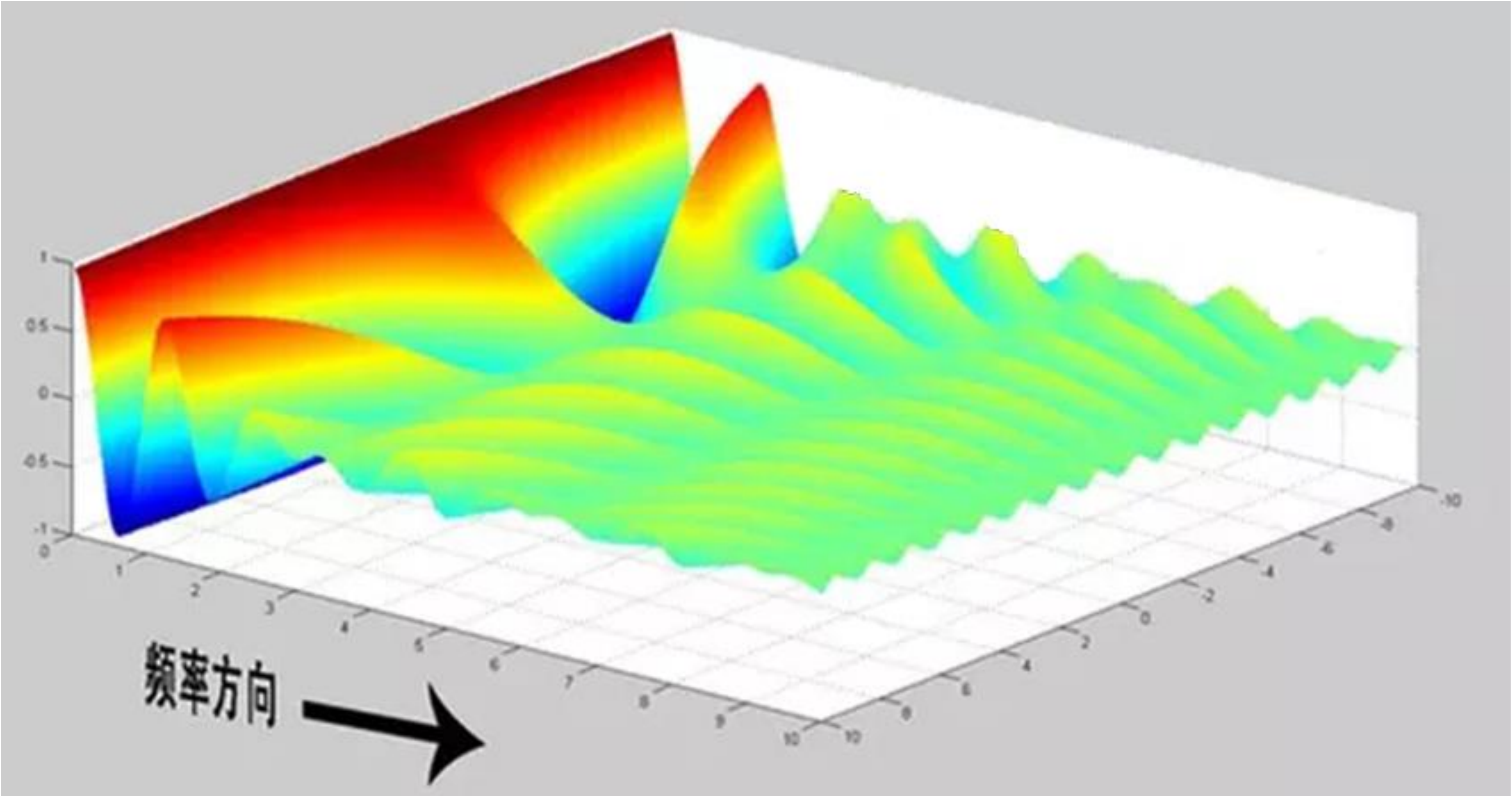
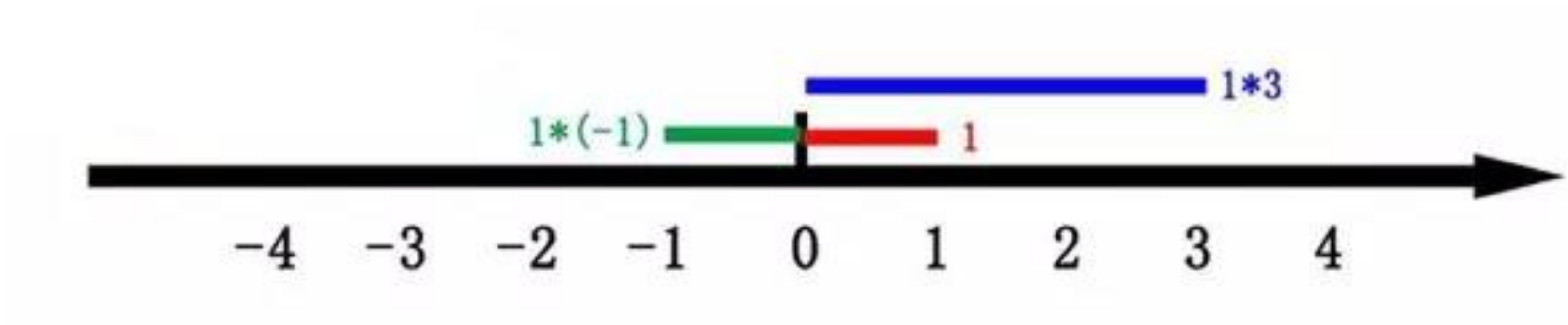


欧拉公式（Euler's Formula）



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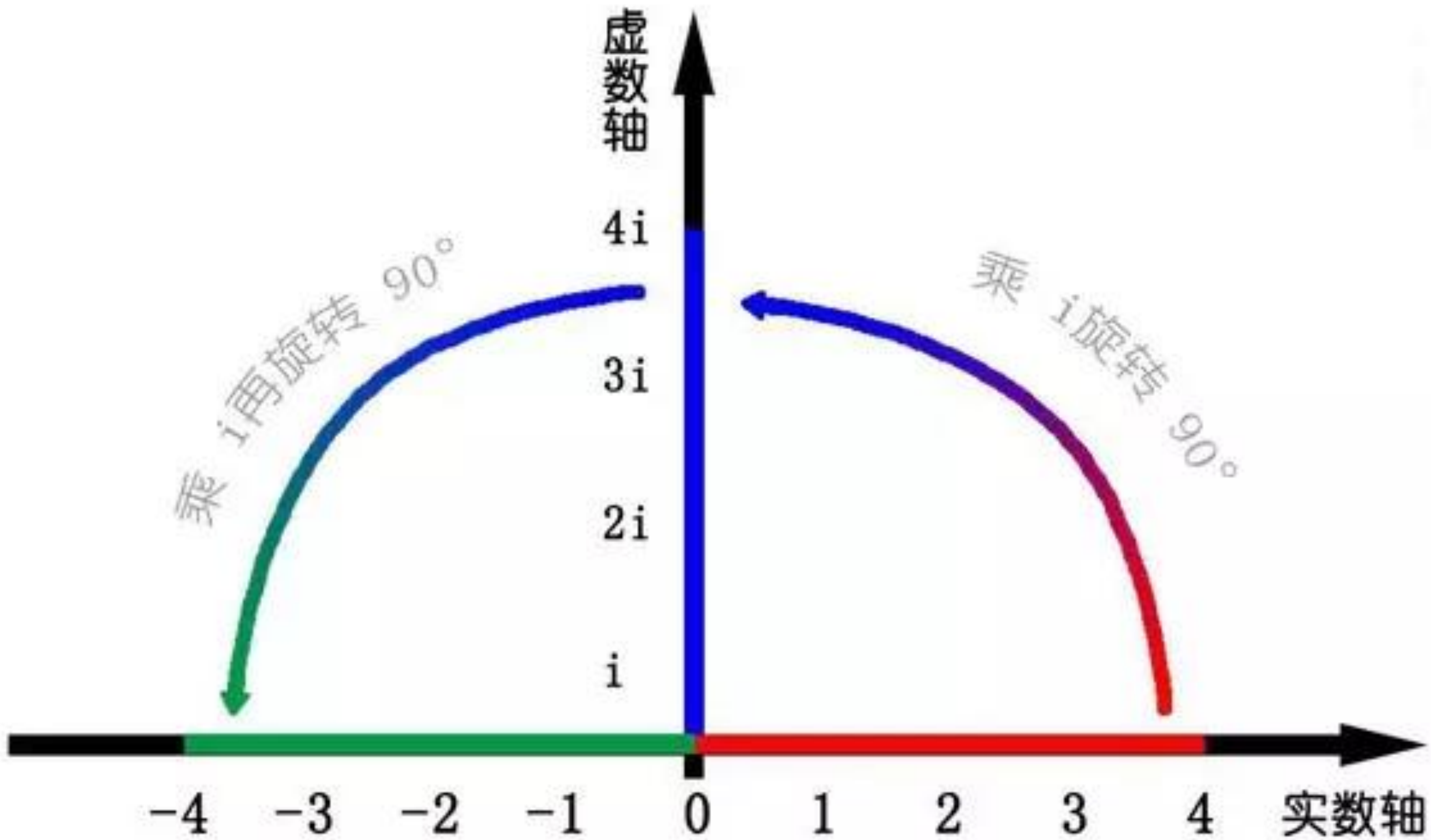
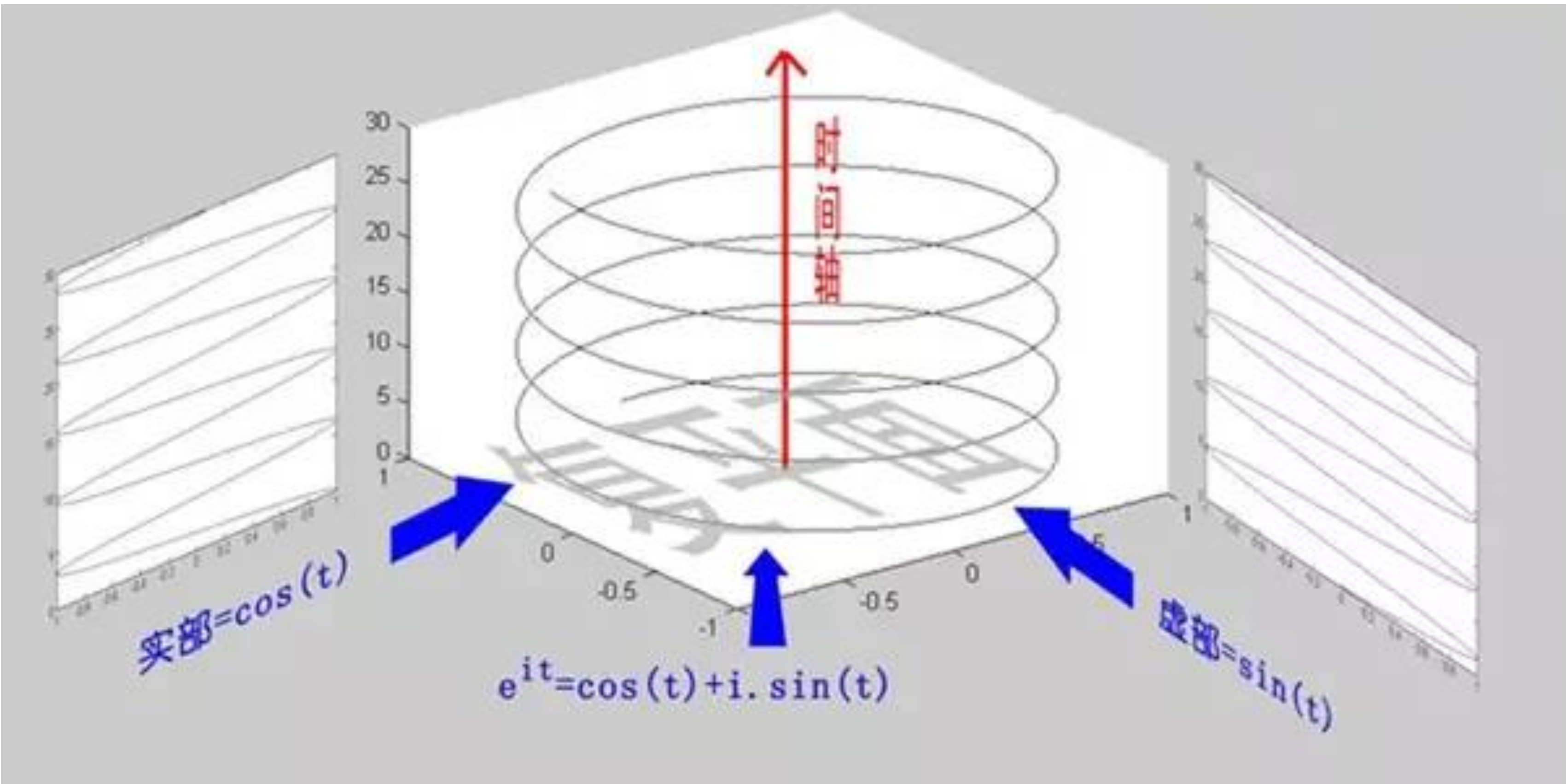


欧拉公式（**Euler's Formula**）

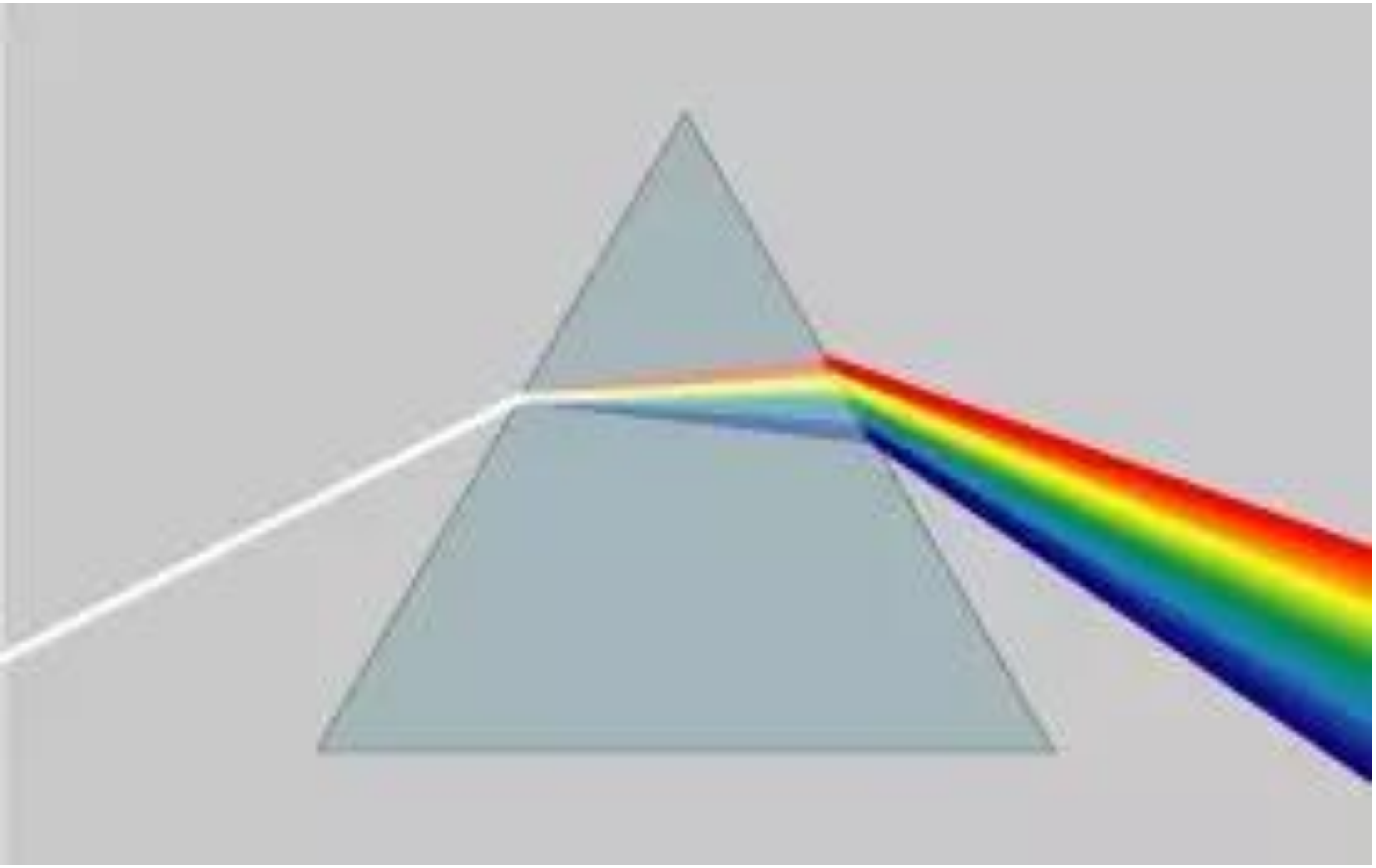
$$e^{ix} = \cos x + i \sin x$$

特殊形式：$e^{i\pi} + 1 = 0$

欧拉公式（Euler's Formula）



指数形式的傅里叶变换



指数形式的傅里叶变换

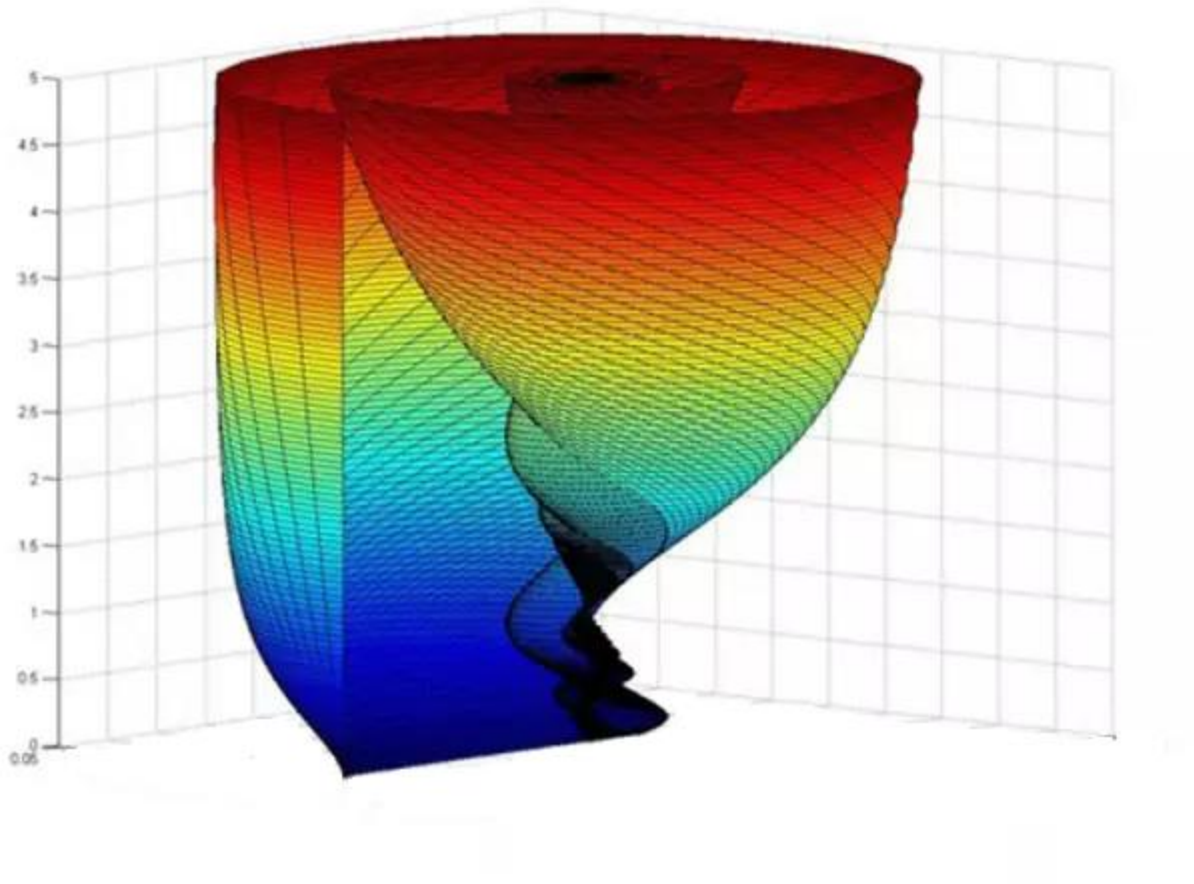
$$e^{ix} = \cos x + i \sin x$$

另一种形式：

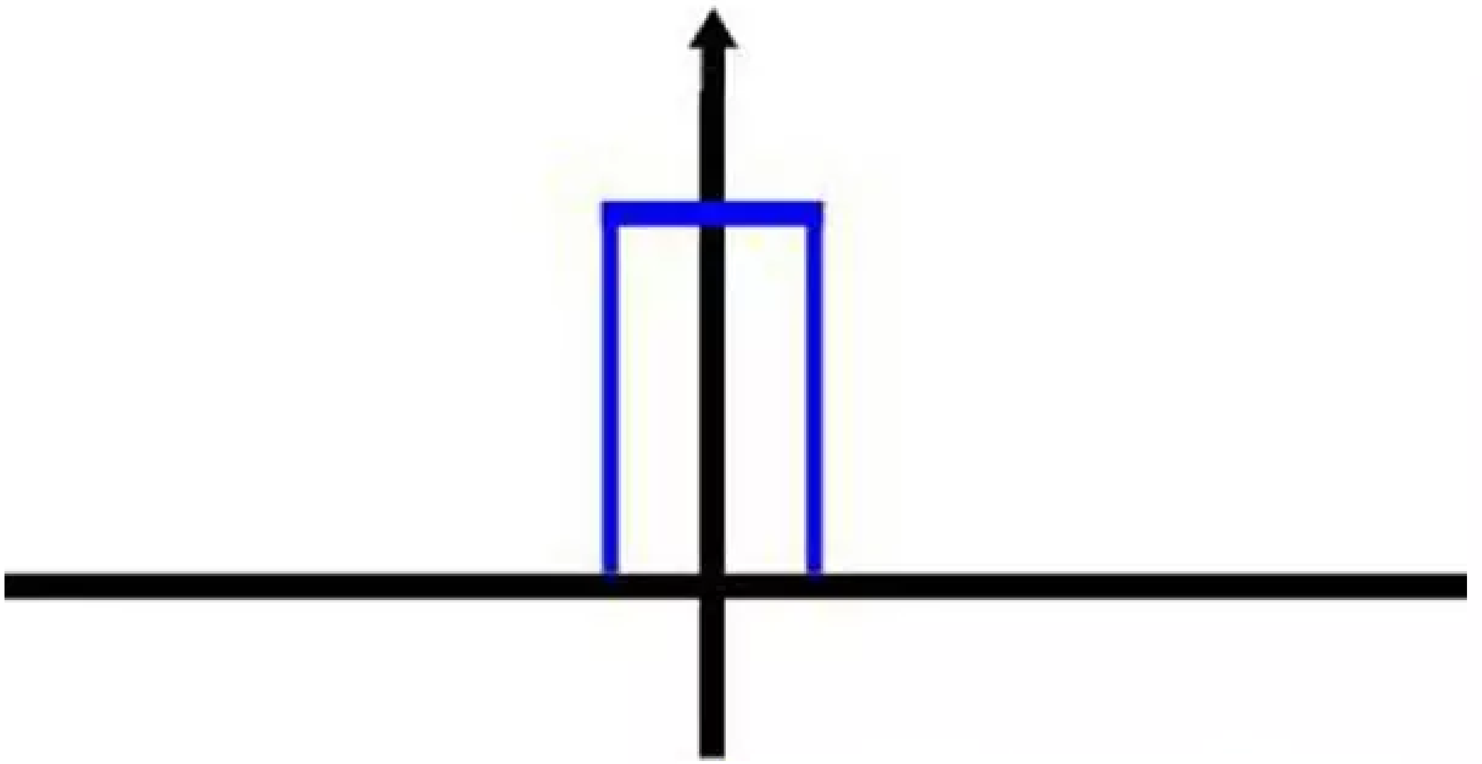
$$e^{it} = \cos(t) + i \sin(t)$$
$$e^{-it} = \cos(t) - i \sin(t)$$

$$\cos(t) = \frac{e^{it} + e^{-it}}{2}$$

指数形式的傅里叶变换



总结



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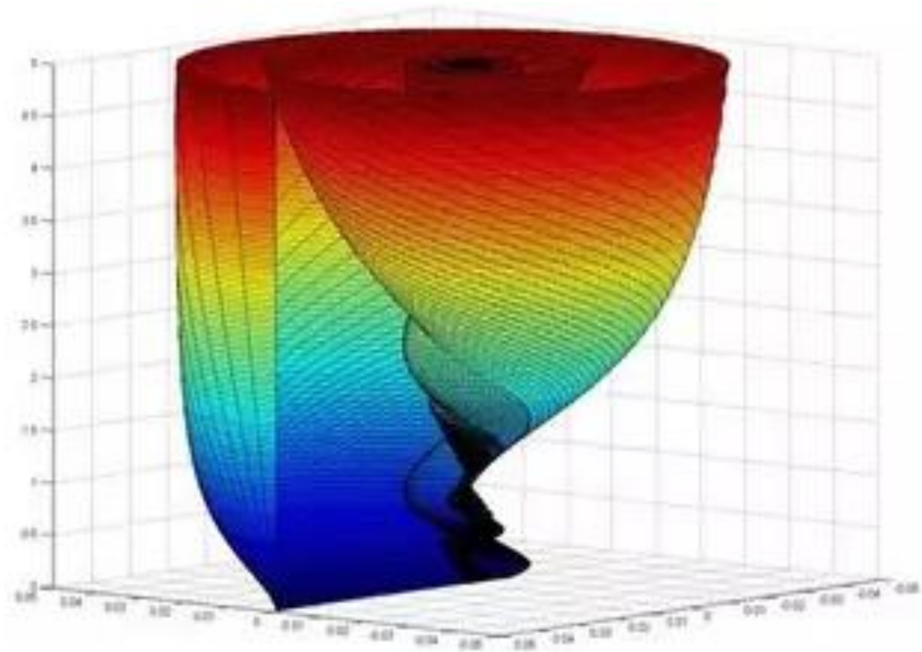
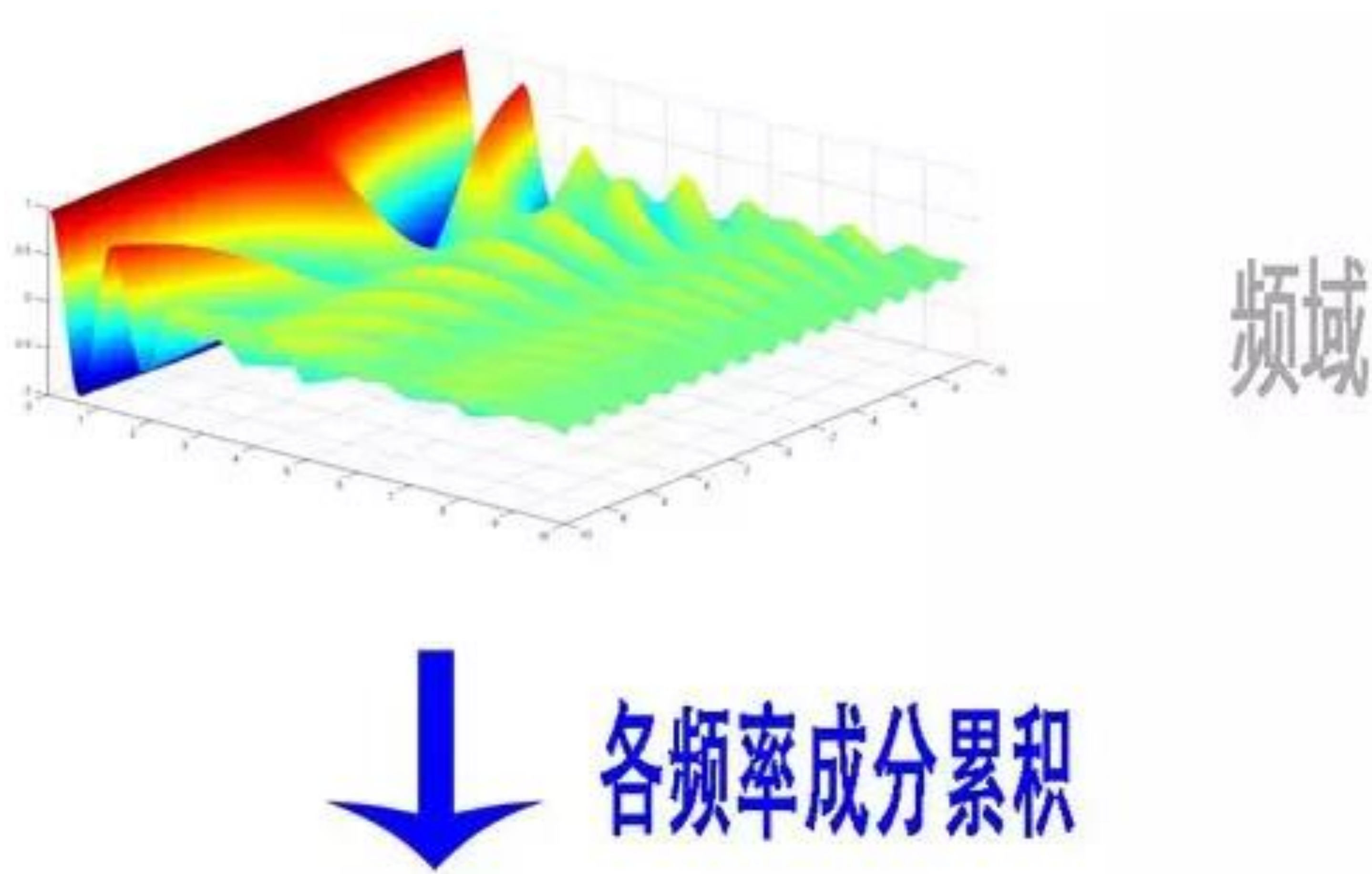


复频域

投影到实数空间

总结



总结