Density Based Fuzzy Thresholding for Image Segmentation

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Abstract. In this paper, we introduce an image segmentation framework which applies automatic threshoding selection using fuzzy set theory and fuzzy density model. With the use of different types of fuzzy membership function, the proposed segmentation method in the framework is applicable for images of unimodal, bimodal and multimodal histograms. The advantages of the method are as follows: (1) the threshoding value is automatically retrieved thus requires no prior knowledge of the image; (2) it is not based on the minimization of a criterion function therefore is suitable for image intensity values distributed gradually, for example, medical images; (3) it overcomes the problem of local minima in the conventional methods. The experimental results have demonstrated desired performance and effectiveness of the proposed approach.

Keywords: Segmentation, Histogram thresholding, Fuzzy density.

1 Introduction

Image segmentation is an indispensable preprocessing task in most image processing, recognition and analysis applications. As the most intuitive and least computation-intensive approach, segmentation methods using global threshoding separate objects and background pixels into non-overlapping regions [1]. The key of applying this type of method is to find an appropriate threshold. Gray level of the pixels under or higher than this value are assigned respectively into two different groups [2].

Most previous works on various thresholding techniques are good at particular kinds of images. Otsu's method [3] automatically perform histogram shape-based image thresholding. Some previous works are based on information theorem which suggests that entropy is a measure of the uncertainty of an event [4], [5]. In these methods, rather than maximizing the inter-class variance, the inter-class entropy is maximized in order to find the optimal thresholds.

Fuzzy set theory is suitable to be applied on image thresholding to partition the image into meaningful regions. The nature of the fuzziness in image arises from the

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uncertainty present and provides a new tool for image segmentation [6] – [12]. Fuzzy clustering is an important application of the theory and becomes popular in the recent decades [14] – [16]. To apply fuzzy set theory on quick image segmentation, several researchers have investigated fuzzy based thresholding techniques. Li et al. [6] combined the fuzzy set theory and information theorem to develop a criterion of maximum fuzzy entropy to obtain the threshold. Chaira and Ray [7] [8] applied four types of methods, i.e. fuzzy divergence proposed in [7], linear and quadratic indices of fuzziness, fuzzy compactness and fuzzy similarity. This type of method minimizes the fuzzy divergence or the separation between the actual and the ideal thresholded image. In [9] [10], Tobias et.al introduced a method based on criterion of similarity between gray levels instead of using a criterion function to be minimized with the use of Zadeh's S-function. Lopes et.al [11] further developed this method to make it fully automatic through a statistical approach with image equalization. Further improvement by Prasad et.al [12] uses $\pi-$function instead of S-function to produce more accurate and reliable results compared to the algorithm proposed in [10], the πfunction is chosen as one standard deviation of the arithmetic mean to locate the intensities of the misclassification regions. Huang et.al [13] further improved Tobias' algorithm by fixing boundary value on medical images.

In this paper, we propose an automatic image thresholding method based on fuzzy set theory, i.e. fuzzy density model. The framework with the use of this method applies different types of fuzzy membership function to be suitable for images of unimodal, bimodal and multimodal histograms.

2 Fuzziness and Fuzzy Density in Image

2.1 Fuzzy Set Theory

On the basis of the principles of uncertainty, ambiguity and vagueness, Zadeh introduced the fuzzy set theory in 1965 [17]. A fuzzy set is a class of objects with a continuum of grades of membership.

Let us assume *X* be a space of points, this is also called the universe. In *X*, its elements are denoted as x, that is, $x_i : X = \{x_1, x_2, \dots, x_n\}, 0.0 \le x_i \le 1.0$. A fuzzy set *A* in *X* is formally defined as

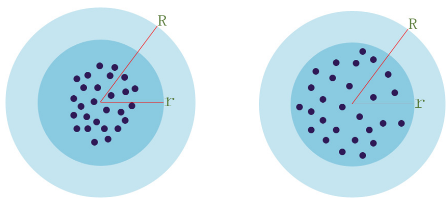
$$A = \{(x_i, \mu_A(x_i))\}, x_i \in X, i = 1,2 \cdots, n \tag{1}$$

where $\mu_A(.)$ is termed as the characteristic or membership function of the elements in the set.

In a fuzzy set, the membership function functions can be viewed as mappings of diverse human choices to an interval [0,1]. Thus, a fuzzy set is a more generalized set where the membership values lie between 0 and 1.There are a numerous membership functions described in the literature as monotonic and non-monotonic families [18].

2.2 Fuzzy Density Model

The mass density or density of a material is defined as its mass per unit volume. In [19], Wang introduced the neighborhood counting, a general methodology for devising similarity functions (NCM) used in the framework of *k*NN algorithm, to find the nearest neighbors. To introduce this concept, here we first consider an example. Fig. 1a and Fig. 1b show a 2D data space along with some data points respectively. To simplify, let us assume these points are all on average distribution, that is, at a same radius of r (we use circle for its boundary here, see Fig. 1a and Fig. 1b), the centroids of these points are at the same coordinate but with different distance to its centroid. In this case, although these points both have the same centroid, but in fuzzy density model we say Fig. 1a has a high density in its fuzzy region. We could describe this character with the help of fuzzy membership function, that is, a closer distance to the centroid would get a higher fuzzy value so its fuzzy average would also be higher. It is intuitively sensible that the higher this value, the more similar these points are. When the radius become larger, that is, R > r (see Fig. 1a), there are no effect that we assume no more point is at the region between r and R. It is obvious that a small region would have a high value of fuzzy density under the same situation. To quantify this intuition, we could use the Euclidean distance (see Fig. 1).



(a) (b)

Fig. 1. An example of fuzzy density model. (a) has a higher fuzzy density in the same region at the radius of *r* than it in (b).

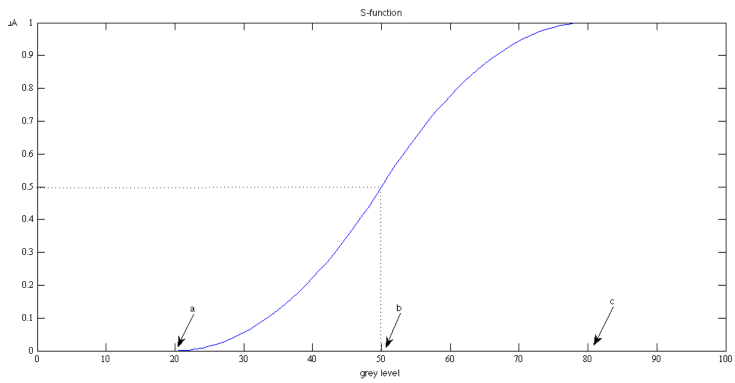


Fig. 2. S-membership function. *a, b, c* are the three control points and μ_a is membership value.

Using the notion of fuzzy density model, we now state the above intuition formally. Let U be a data space, and *fdm*(*r*,*p*) be the function to calculate the fuzzy density, where r denotes the region and p is the points within this region of boundary. The higher the *fdm*(*r*,*p*), the more similar these points are. When taking image into account, in a simple manner, the gray scale is divided into two parts by a selected value *X*. *X* is the boundary to both parts and we could get two regions at the same time, i.e. [*Min*,*X*], [*X*,*Max*], where *Min*, *Max* denotes the minimum and maximum gray scale respectively. Pixels under this partition are these points in p. We could use gray level to express the distance of pixels. As a whole, we calculate *fdm*(*r*,*p*) at the selected value of *X* to choose a proper threshold.

Although there are many other membership functions, in this work, we use three different types of formula to calculate the membership function according to the distance in its fuzzy density region.

a). Zadeh's S-membership function [10]

Such a function is defined as

$$\mu_S(x) = S(x,a,b,c) = \begin{cases} 0\,, x < a \\ 2[(x-a)/(c-a)]^2, a < x \le b \\ 1-2[(x-c)/(c-a)]^2\,, b < x < c \\ 1, x > c \end{cases} \quad (2)$$

where a, b, and c, are the three parameters as shown in Fig.2. b denotes the crossover point and could be any value between a and c. Here we define b by b = (a+c)/2 with $\mu_A(b) = 0.5$. For a = Xmin, c = Xmax, the membership function plot is shown in Fig.2 for a normalized set.

b). Gamma membership function [7]

The general formula for the probability density function of the Gamma distribution is:

$$\mathrm{f(x)} = \frac{\left(\left(\frac{x-\mu}{\beta}\right)^{\gamma-1}\right)\exp\left(-\left(\frac{x-\mu}{\beta}\right)\right)}{\Gamma(\gamma)}, x \ge \mu, \gamma, \beta > 0 \quad (3)$$

where γ is the shape parameter, μ is the location parameter, $\beta$is the scale parameter and Γ is the Gamma function.

when $\mu \ne 0, \beta = 1, \gamma = 1$, the Gamma distribution takes the form

$$\mu(x) = \exp(-c \cdot |x-\mu|) \quad (4)$$

It may be pointed out that in the membership function, the constant '*c*' has been taken to ensure membership value of the gray level feasible in the range [0,1] and would explain how to chose in the following section.

c). Gaussian membership function [18]

$$\mu(x) = \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right), m = \text{mean} \quad (5)$$

3 Threshold Selection Method

We introduce the key idea in section 2, that is, by combining fuzzy set theory we could use fuzzy density model to calculate the character of image which can is illustrated in Fig.3 as below.

The histogram consists by two groups of pixels, dark part and light part. The target is to split the image histogram into two crisp subsets, namely, object subset O and background subset B, $O \cup B = A$. Firstly, we get two initial fuzzy subsets, denoted by L and R, are associated with initial histogram intervals located at the beginning and the end regions of the histogram, i.e. *[Xmin,Xl], [Xr,Xmax]*, and we assume there are both enough initial points that the two parts contain. For bright objects $R \subset O, L \subset B$, for dark objects $L \subset O, R \subset B$. The place between *[Xl,Xr]* is called fuzzy region. In order to get a proper threshold value, we continuously choose gray level in *[Xl,Xr]*, Xi denotes. Every time we calculate the fuzzy density of *[Xl,Xi],[Xi,Xr]* respectively, then a comparison between them is made as described in Equation (6):

$$\begin{cases} IF\big(fdm(Xmin, Xi) > fdm(Xi, Xmax)\big), THEN\ Xi \subset L \\ IF\big(fdm(Xmin, Xi) < fdm(Xi, Xmax)\big), THEN\ Xi \subset R \end{cases} \tag{6}$$

Note that in the proposed framework the fuzzy density model is used, where a fuzzy set with a high value of fuzzy density indicates its elements is much closer, or similar. And with its region get large, its value would become low normally (see Fig.3).

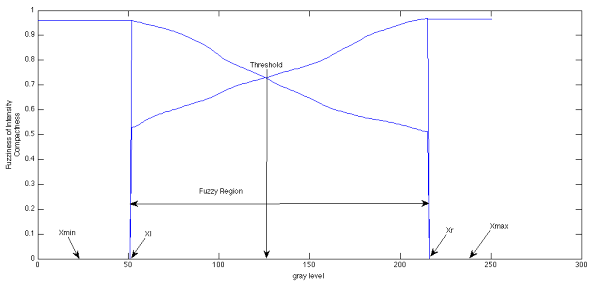


Fig. 3. Base idea of the fuzzy density model thresholding method. *Xmin, Xmax* denotes the minimum and maximum gray level that has pixels in the histogram of the image respectively.

Since the key of the proposed classification method is the comparison of fuzzy density value, normalization to the L or R region is needed. We take the trivial method proposed in [9] by first computing the initial subsets L or R to get a normalization factor α according to formula listed below and then normalize the value at every round, take L region that needed be normalized as example:

$$\alpha = \frac{fdm(R)}{fdm(L)} \tag{7}$$

A way to calculate the fdm(r,p) is by making a little change to the index of fuzziness [18]. The index of fuzziness is defined as:

$$I(A) = \frac{2}{n^k} . d(A, \bar{A}) \tag{8}$$

where $d(A, \bar{A})$ denotes the distance between image A and its nearest ordinary image $\bar{A}$.

There are two types of indices of fuzziness:(a) linear and (b) quadratic. These are defined as:

1, If k = 1, d becomes the Hamming distance, the linear index of fuzziness may be rewritten as:

$$I(A) = \frac{2}{n}\left[\sum_{i=1}^{n} |\mu_A(x_i) - \mu_{\bar{A}}(x_i)|\right] \tag{9}$$

2, If k =2, d becomes the Euclidean distance, then the quadratic index of fuzziness is defined as:

$$I(A) = \frac{2}{n^{1/2}}\left[\sum_{i=1}^{n} |\mu_A(x_i) - \mu_{\bar{A}}(x_i)|^2\right]^{1/2} \tag{10}$$

We take the centroid of pixels in each region to be the origin of $\bar{A}$. So the closer points to centroid would get a higher value in each membership function thus making more contribution to its fuzzy density value.

This is the basic idea of the approach. The concept presented above sounds attractive but has some limitations concerning the initialization of the seed subsets thus it sometimes needs manually operations to select initial boundary. More general, it runs not well in low contrast images. The work proposed in [11] overcomes a similar problem in [10] by a procedure with statistical parameters P1 and P2. In fact, the initial subsets are defined automatically and they are large enough to accommodate a minimum number of pixels defined at the beginning of the process. The minimum number of pixels of each set i.e. object or background depends on the shape of the histogram and it is a function of the number of pixels in the gray level intervals [0,127] and [128,255].

$$MinPix_{Bseed(Wseed)} = P_1 \sum_{i=0\ (128)}^{127\ (255)} h(x_i) \tag{11}$$

where $P1 \in [0,1]$ and $h(x_i)$ denotes number of occurrences at gray level x_i. If $P1$ is too high, the fuzzy region between the initial intervals will be small and the values are gray levels for thresholds are limited, on the other hand if the $P1$ is too low the initial subsets are not representative and the method does not converge. For low contrast images, popular histogram equalization is performed to bring minimum number of pixels into the region with poor number of pixels. If the number of pixels belonging to either side of the histogram from the intensity 128 is smaller than $P_{min} = P2 * MN$, where $P_2 \in [0.1]$ and M * N are the total number of pixels in the image, then histogram equalization is recommended. Finally the $P1$ and $P2$ are estimated as 39.64% with standard deviation 13.37% and 20% with standard deviation 14.30% respectively.

Now the whole approach can be summarized in the following algorithm:

```
  Let B,W describe individually the Background and Object.
Assume that Object is the white region.
  Input: An image
  Output: Thresholding Value
  Preliminary Step:
  1, Determine if image needs to equalization through P2
  2, Calculate the initial region boundary Xl,Xr through P1
  3, Get the normalization factor α
  Basic Step:
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for i =  (Xl + 1) to (Xr - 1)
     compute LEFT = fdm([Xmin,Xi]);
     compute RIGHT = fdm([Xi,Xmax]);
     if LEFT * α is larger than RIGHT
            Set Xi belongs to B
     else
            Set Xi belongs to W
     end
end
```

4 Experimental Results and Analysis

In order to validate the proposed algorithm, various types of images with the use of different membership functions discussed in the Section 2 are listed. Two other results based on minimal criterion methods, i.e. Fuzzy divergence [7], Otsu's method [3], are also presented.

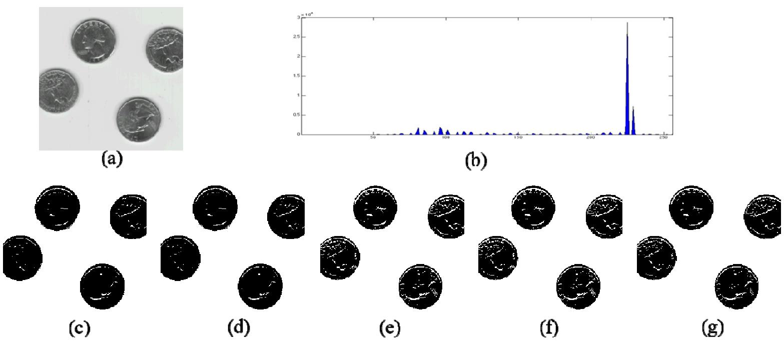

Fig. 4. (a) Input image 'coin', (b) its histogram and thresholded using (c) Fuzzy Divergence[7], (d) Otsu's method[3], (e) S-membership function, (f) Gamma membership function, (g) Gaussian membership function. Image copied from [13].

As we mentioned before, our method is good at segmenting images with ambiguous objects or gradually distributed intensity. Fig. 4 and Fig. 5 are two unimodal images (See their histograms). In Fig. 4, the image 'coin' with size 252 x 252 particularly demonstrates this kind of advantage. It obtains more details of information located with the center of coin stamp (See Fig. 4(e), (f), (g)) while the two other methods don't (See Fig.4 (c), (d)). And in Fig. 5, the image 'block' with size 200 x 200 has four block regions directly shown in different gray level that needs to be fully partitioned. By choosing Gaussian membership function in our framework (See Fig. 5(e)), it successfully obtains the right result while the two other methods fail (See Fig. 5(c), (d)). Because the proposed membership function is originally distributed in values and could easily be selected by testing the most suitable one for given kinds of images, under such particular condition which need more details around objects or images with intensities distributed in gray level, the proposed approach is more flexible and useful.

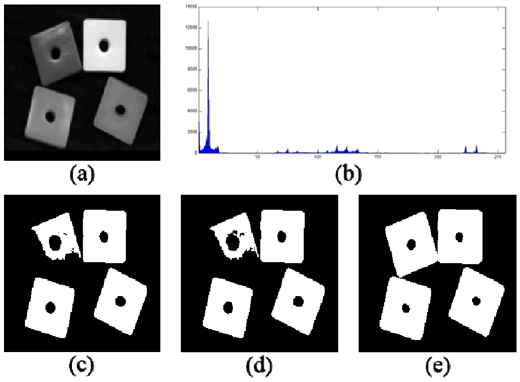

Fig. 5. (a) Input image 'block', (b) its histogram and thresholded using (c) Fuzzy Divergence[7], (d) Otsu's method[3], (e) Gaussian membership function.

Fig. 6 shows a typical bimodal image (See its histogram) 'blood' of size 272 x 265. All the five results are satisfactory that we can accept for correctly dividing the object. In details, we could notice that Fig. 6(c) and Fig. 6(d) get a similar partition because they both find the minimum value of criterion function, however, our approach gets more details around the region between background and object region. Fig. 6(b), (c), (d) get different contours in the middle of blood clot, different weights of its membership function are in accordance with its intensity which gradually change at that place. It can be seen that the threshold using Gamma membership function, S-membership function and Gaussian membership function obtains outer contour, inner contour, and region between contour and outer contours, respectively. It is flexible to choose the more suitable one by switching membership function.

When comes to the object and background of the input image has some overlapping regions in the histogram. In Fig. 7, the typical multimodal image 'lena' of size 512 x 512 shows a result. Our approach also works well comparing to these methods of finding an absolute minimum of the histogram (See Fig. 7(e)).

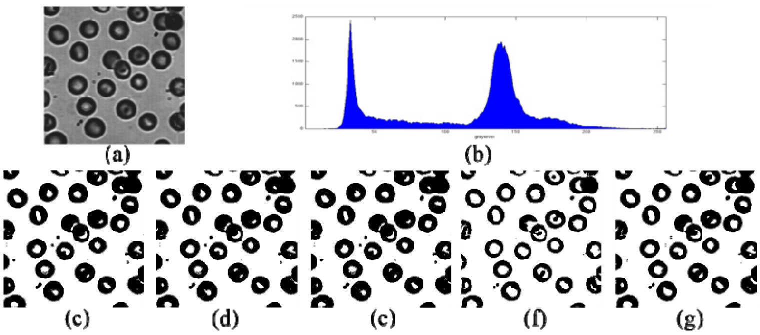

Fig. 6. (a) Input image 'blood', (b) its histogram and thresholded using (c) Fuzzy Divergence[7], (d) Otsu's method[3], (e) S-membership function, (f) Gamma membership function, (g) Gaussian membership function.

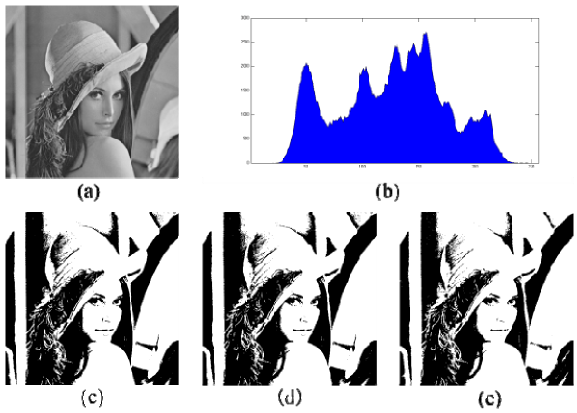

Fig. 7. (a) Input image ‘lena’, (b) its histogram and thresholded using (c) Fuzzy Divergence[7], (d) Otsu’s method[3], (e) Gamma membership function

For a particular series of medical image, a desired threshold can be obtained from our proposed method, while the other two do not (See Fig. 8 (c), (d), (e)), such as getting the sacroiliac bone region. Fig. 8 shows the experimental results by using S-membership function and manually setting the boundary value of $a = X_{min} + 10$ and $c = X_{max} - 10$ each time.

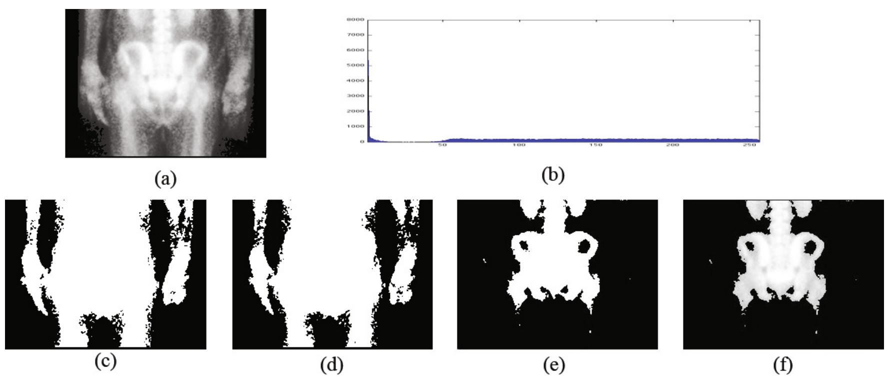

Fig. 8. (a) Input image ‘sacroiliac’, (b) its histogram and thresholded using (c) Fuzzy Divergence[7], (d) Otsu’s method[3], (e) S-membership function, (f) the final result from thresholding and input images.

5 Conclusion

Based on the fuzzy set theory, a new procedure of automatic histogram threshold for image segmentation is presented in this paper. The concept of fuzzy density model is developed to choose the proper threshold for object extraction. The threshold level is obtained by its combined method and in such case yields optimal segmentation of the objects from the background. It is fully automatic that no prior knowledge of the image is required and not based on the minimization of a criterion function thus good at coping some kinds of images with gray level distributed gradually in illumination. Comparative experimental results suggest that the proposed method is suitable some medical image segmentation tasks. It is also applicable for real-time applications owing to the its less computation-intensiveness.

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