Master of Science in Analytics

MSCA 31006 Time Series Analysis and Forecasting

Assignment #1 - Introduction

Due Date – Beginning of Session #2

Total Points: 6%

Student: Duo Zhou

(18 points) Question 1:

Suppose E(X) = 2, Var(X) = 9, E(Y) = 0, Var(Y) = 4, and Corr(X,Y) = 0.25.

Find:

(a) Var(X + Y)

*Cov[X,Y]=Corr[X,Y]*Sqrt(Var[X]*Var[Y])=0.25*Sqrt(9*4)=1.5*

Var[X+Y]=Var[X]+Var[Y]+2Cov[X,Y]=9+4+2×1.5=16

(b) Cov(X, X + Y):

Cov[X,X+Y]=Cov[X,X]+Cov[X,Y]=Var[X]+Cov[X,Y]=9+1.5=10.5

(c) Corr(X + Y, X − Y):

Var[X-Y]= Var[X]+Var[Y]-2Cov[X,Y]=9+4-2×1.5=10

Cov[X+Y,X-Y]= Cov[X,X]+Cov[X,Y]- Cov[X,Y]- Cov[Y,Y]=Var[X]-Var[Y]=9-4=5

*Corr[X+Y,X-Y] = Cov[X+Y,X-Y]/Sqrt(Var[X+Y]*Var[X-Y])=5/Sqrt(16*10)=0.3953*

(6 points) Question 2:

If X and Y are dependent but Var(X) = Var(Y), find Cov(X + Y, X − Y).

Cov[X+Y,X−Y]=Cov[X,X]+Cov[X,−Y]+Cov[Y,X]+Cov[Y,−Y]=Var[X]−Cov[X,Y]+Cov[X,Y]−Var[Y]
since Var[X]=Var[Y]
Cov[X+Y,X−Y]= Var[X]−Cov[X,Y]+Cov[X,Y]−Var[X]=0

(18 points) Question 3:

Suppose $Y_t = 5 + 2t + X_t$, where {X_t } is a zero-mean stationary series with autocovariance function γ_k.

(a) Find the mean function for $\{Y_t\}$.

$E[Y_t]=E[5+2t+X_t]=5+2E[t]+E[X_t]=5+2t+0=2t+5$

(b) Find the autocovariance function for $\{Y_t\}$.

$Cov[5+2t+X_t,5+2(t-k)+X_{t-k}]$

$=Cov[5+2t, 5+2(t-k)]+ Cov[5+2t, X_{t-k}]+ Cov[X_t, 5+2(t-k)]+ Cov[X_t, X_{t-k}]$

Since $Cov[5+2t, 5+2(t-k)]=Cov[5+2t, X_{t-k}]= Cov[X_t, 5+2(t-k)]=0$

$Cov[5+2t+X_t,5+2(t-k)+X_{t-k}]=Cov[X_t,X_{t-k}]= \gamma_k$

(b) Is $\{Y_t\}$ stationary? Why or why not?

A stationary time series is one whose properties (i.e., probability laws) do not depend on the time at which the series is observed.
Based on the result from a), the mean function for $\{Y_t\}$ is 2t+5, which is not independent of time t. $\{Y_t\}$ is not stationary.