Technology Choice, Energy Efficiency, and Second-Best Climate Policy

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Abstract: I study the effectiveness of subsidies as an alternative to carbon taxes to reduce carbon emissions in a quantitative climate-economy model. An energy firm uses brown and green energy inputs to produce energy. A firm-household then combines energy with capital and labor to produce final goods. The short-run elasticity between energy and other inputs is low. However, higher energy prices encourage higher energy efficiency, leading to a higher elasticity in the long run. The key weakness of green energy subsidies, as an alternative to carbon taxes, is that they cannot promote higher energy efficiency. Thus, in the baseline model, the optimal green subsidies result in a modest 1.0% decrease in emissions by the end of the century compared to the business-as-usual scenario. However, if the government can subsidize green energy usage and energy-saving technical change simultaneously, the optimal subsidies could be nearly as effective in reducing emissions as the first-best brown energy taxes. Under this approach, cumulative carbon emissions are reduced by 12.2% by the end of the century.

Keywords: Integrated Assessment Models, Climate Change, Subsidy, Second-Best Policies

JEL Classification Codes: H23, O44, Q43, Q54, Q58

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1. Introduction

Economists generally regard carbon taxes as the first-best solution for reducing carbon emissions and addressing climate change. Since the social cost of one ton of carbon emission is higher than its private cost, a Pigouvian tax should be imposed to correct its negative externality. However, like most taxes, carbon taxes and similar policies designed to increase energy costs are massively unpopular among the public.[1] A survey by the Energy Policy Institute and the Associated Press-NORC Center revealed that 42% of respondents would not accept even a $1 monthly increase in their electricity bill to fight climate change. In contrast, government subsidies for renewable energy sources, such as solar and wind power, are more popular and frequently implemented. A notable example is the Inflation Reduction Act, which allocates billions in renewable energy tax credits. Therefore, assessing the effectiveness of green energy subsidies as a viable alternative to the carbon tax approach is crucial.

In this paper, I build a macro-climate model with an upstream energy sector and a downstream final goods sector. A perfectly competitive firm in the energy sector uses brown and green energy inputs, which are imperfect substitutes, to produce energy. Brown energy, while cheaper in terms of private cost than green energy, has a higher social cost due to its carbon emissions, which lead to future climate damage. The energy is then sold to a representative firm-household, which combines energy with capital and labor to produce final goods, which can be used either for consumption or investment. The energy intensity of the final goods production process is endogenous and influenced by the energy price. This is because the firm-household can make technology choices on a technology frontier and, therefore, change the energy efficiency of its production process over time. Adopting new technologies with energy efficiencies different from existing ones would incur a technology adjustment cost. Under such a setting, the elasticity of substitution between energy and other inputs is low in the short run

[1] The reasons for opposing carbon taxes include (1) concerns about tax burdens (2) concerns about foreign free-riding, and (3) the distributional consequences of carbon taxes (e.g. Beiser-McGrath and Bernauer 2019, Känzig 2023).

and higher in the long run. A benevolent government has access to different policy instruments, including brown taxes, green subsidies, and energy efficiency subsidies, under different policy scenarios and tries to correct the negative externality of brown energy usage. I solve this Ramsey problem globally using the Value Function Iteration augmented with Gaussian Processes Regression (Scheidegger and Bilionis, 2019).

The model has two key mechanisms for reducing carbon emissions: (1) reducing the share of brown energy inputs in energy production by substituting them with green inputs and (2) reducing overall energy usage in final goods production, mainly by improving energy efficiency. Taxes on brown energy would reduce carbon emissions by utilizing both mechanisms. On the other hand, if the government only subsidizes green energy in the energy sector, the carbon emission reductions under the optimal policy path would be very limited. The limited impact stems from their key weakness, namely, their tendency to lower energy costs, which inadvertently encourages firms in the downstream sector to use more energy, mainly by adopting less energy-efficient technology in their production. According to the baseline model, brown taxes can reduce cumulative emissions by 13.6% by the end of this century compared to emissions under the business-as-usual (BAU), whereas optimal green subsidies achieve a 1.0% reduction in cumulative emissions. The optimal taxes on brown energy increase household welfare by 0.16% in terms of equivalent consumption. In contrast, optimal green energy subsidies increase household welfare by only 0.002%.

To illustrate, consider the various decisions within our economy aimed at reducing energy usage by enhancing energy efficiency, often at a cost. These decisions include opting for a more expensive, energy-saving air conditioner, choosing a car with higher fuel economy but lower horsepower, utilizing trains instead of trucks for transporting goods, and allocating our limited research and development (R&D) resources towards improving energy efficiency rather than other research projects. All these choices can be regarded as technology choices on the technology frontier. The efforts that can improve the energy efficiency of the economy become economically viable for agents only when energy costs are sufficiently high. Green energy subsidies are not able to provide incentives for higher energy efficiency. As a result, they have become an inefficient tool for the government to use to reduce carbon emissions and should not be implemented on a scale that can reduce carbon emissions substantially.

Therefore, the government needs an additional lever to increase the economy's energy efficiency. Technology subsidies that reward energy-efficient technology choices, that is, subsidies that reward higher energy efficiency, are effective policy instruments for achieving this goal. Since the long-run substitution between them is largely driven by changes in the firm's technology choice, the government can reduce the firm's energy usage by subsidizing energy-saving technology. When energy and other inputs are perfect complements in the short run, subsidies on energy efficiency and green subsidies can jointly implement the solution to the social optimum. In the baseline model, even though the assumption of perfect complementarity is not imposed, these two types of subsidies can still achieve a 12.3% reduction in cumulative emissions, approaching the level of carbon abatement achieved through the optimal imposition of taxes on brown energy. With respect to welfare, green subsidies and technology subsidies jointly increase welfare by 0.14% in terms of equivalent consumption, which parallels the welfare gain brought by taxes on brown energy.[2]

In summary, relying solely on green subsidies is costly and ineffective in reducing carbon emissions and brings very limited welfare gains relative to the BAU. However, combining green and energy efficiency subsidies can lead to an almost socially optimal outcome. This key finding remains robust across various scenarios, including different degrees of elasticity of substituting in the energy sector and varying technology adjustment costs for energy efficiency.

In the baseline model, the production cost of green energy is constant and exogenously given. As an extension to the baseline model, I introduce learning-by-doing in green energy production: more green energy usage leads to cheaper green energy. This setting allows green

[2] In fact, IRA does provide tax credit for energy efficiency. For example, homeowners can claim at most $1,200 for energy property costs and various types of residence energy efficiency improvements.

subsidies to stimulate directed technical change towards green energy (e.g., Acemoglu et al. 2012, Kalkuhl et al., 2012, Fried 2018, Hassler et al., 2021, Lemoine 2024). Still, the key insights from the baseline model are still valid: even if green subsidies can lead to cheaper green energy sources, they still cannot address the issue of energy efficiency. So, subsidies on energy efficiency are still warranted when brown taxes are not feasible for the government.

This paper is built upon and contributes to the development of the integrated assessment models (Nordhaus 1977, Nordhaus 1993, Golosov et al. 2014, Barrage 2020, Hassler et al. 2021, Barrage and Nordhaus 2023). Within this literature, this paper is closest to works on subsidies as second-best climate policies (Popp 2006, Gerlagh 2008, Kalkuhl et al. 2013, van der Ploeg and Withagen 2014, Cruz and Rossi-Hansberg 2021, Nordhaus 2017, Lemoine 2017, Airaudo et al. 2023). Usually, subsidies on green technologies, either on their usage or R&D, only have modest impacts on carbon emissions. Highly relevant to this paper, Hassler et al. (2020) demonstrate that a lower price of green energy can increase carbon emissions when brown and green energy are gross complements. Casey et al. (2023) argue that green subsidies' impact on brown energy usage is determined by the elasticity of substitution between clean and dirty energy as well as the price elasticity of demand for energy services. They also analytically derive the conditions for green subsidies to reduce carbon emissions and find that the welfare gain brought by implementing green subsidies alone is modest at its best. Bistline et al. (2023) and Cruz and Rossi-Hansberg (2024) also find the welfare gain from green subsidies is very limited. This paper contributes to this literature by explicitly modeling firms' energy efficiency choices and distinguishing the short- and long-run elasticities between energy and other inputs. More importantly, I further show that subsidies targeted at long-run energy efficiency can address the energy efficiency problem caused by green subsidies, thereby making subsidy-based climate policy schemes almost as effective as first-best carbon taxes.

This paper also relates to studies that examine the relationship between technological change and the elasticities between energy and conventional inputs. The low short-run and high long-run elasticities of energy use are well documented (Pindyck, 1979). Akteson and Kehoe

(1999) propose a model with capitals with different fixed energy-capital proportions. Because of the existence of capital adjustment cost, the elasticity of energy usage is low in the short run but high in the long run. Hassler et al. (2021) emphasize the near-zero short-run elasticity between energy and capital/labor inputs. Yet, they conclude that this elasticity is higher in the long run because a higher energy price can induce energy-saving technological change. Casey (2022) develops a model with directed technical change in economic growth and energy efficiency and highlights the importance of final-use energy efficiency for climate policies. Airaudo et al. (2023) propose a model that also features an energy sector and directed technical change in energy efficiency to evaluate the macro impacts of green transition in a small open economy induced by pre-determined brown taxes, green subsidies, or green infrastructure investments. This paper introduces the time-varying elasticity of substitution based on the "appropriate technology" literature that describes the technological choice of a firm to select the efficiencies of different inputs. Prominent examples of this literature are Jones (2005), Caselli and Coleman (2006), Growiec (2008, 2013), and León-Ledesma and Satchi (2019). Hinkelmann (2023) builds a quantitative model featuring the time-varying elasticity of substitution between fossil fuel and electricity in the energy sector following León-Ledesma and Satchi (2019) and finds that a sizable carbon tax is required to achieve net zero emissions by 2050. The connection between directed technical change (e.g., Acemoglu et al. 2012, Fried 2018, Hassler et al., 2021, Casey 2023, Lemoine 2024) and technological choice is also discussed.

This paper also connects to microeconomics studies on second-best environmental policies. Baumol and Oates (1988) demonstrate that subsidies for clean alternatives to pollution can increase pollution if production is scaled up. Sinn (2008) discusses the possibility of the so-called "green paradox": subsidies on renewable energy encourage the owner of finite oil stock to extract more oil, which increases carbon emissions. Fischer and Newell (2008) and Gugler et al. (2021) also conclude that green subsidies are less effective than brown taxes. Newell et al. (2019) further highlight clean energy subsidies can lead to inefficiently high electricity production. Belfiori and Rezai (2024) show that any sequence of explicit carbon prices can be achieved implicitly through a combination of conventional taxes.

The remainder of this paper is structured as follows. Section II lays out a static model that highlights the key ideas of this paper. Section III presents the full dynamic model and quantitatively evaluates the effectiveness of different policy tools, including taxes and subsidies. Section IV introduces learning-by-doing in the green sector as an extension of the baseline model. Section V concludes.

II. A Static Model

This section lays out a static, decentralized model that highlights the key mechanisms in the full model. I compare the decentralized economy to the social planning problem, show that the solution to the social planner (SP)'s problem can achieved in the decentralized economy by either (1) implementing brown taxes or (2) implementing green subsidies as well as subsidies on energy efficiency. However, green energy subsidies alone cannot implement the social optimum.

Decentralized Economy

A perfectly competitive energy firm has access to a constant elasticity of substitution (CES) technology that uses brown energy inputs e_b and green energy inputs e_g to produce the final energy e with a Constant Elasticity of Substitution (CES) production function:

$$e = \left(\lambda(e_b)^{\frac{\epsilon_e - 1}{\epsilon_e}} + (1-\lambda)(e_g)^{\frac{\epsilon_e - 1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e - 1}}, \tag{1}$$

where ϵ_e is the elasticity of substitution between green and brown energy. We think brown energy inputs e_b as abundant "coal." Their usage causes carbon emissions. Green energy inputs e_g, such as solar or wind energy, are expensive but clean. Both brown and green energy inputs are produced with constant marginal costs p_b and p_g, respectively, in terms of final output. The produced energy e is then sold to the firm-household at price p_e. This energy firm is also potentially subject to tax or subsidy imposed on both types of energy sources, τ_b and τ_g, which are rebated lump-sum (T_e). The profit maximization problem of this energy firm is as follows:

$$\max_{e_b, e_g} p_e e - p_b e_b - p_g e_g - \tau_b e_b - \tau_g e_g + T_e$$

s.t.

(1)

The first-order conditions (FOCs) and the zero-profit condition yield the equilibrium conditions:

$$\frac{e_b}{e_g} = \frac{\lambda}{1-\lambda} \left(\frac{p_b + \tau_b}{p_g + \tau_g} \right)^{-\epsilon_e} \tag{2}$$

$$p_e = \left(\lambda^{\epsilon_e} (p_b + \tau_b)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e} (p_g + \tau_g)^{1-\epsilon_e} \right)^{\frac{1}{1-\epsilon_e}} \tag{3}$$

A firm-household purchases energy e from the energy firm at price p_e. It uses non-energy endowed inputs h and energy e to produce a final good y. The efficiencies of energy and non-energy inputs are denoted to B_e and B_h, respectively. The production function follows the Leontief form:

$$y = \min\{B_h h, B_e e\}.$$

For a given relative energy efficiency ratio B_e/B_h, non-energy inputs h and energy e are perfect complements. Following Jones (2005) and León-Ledesma and Satchi (2019), I assume that the firm-household also can choose its non-energy-inputs-efficiency B_h and energy efficiency B_e on a log-linear technology menu:

$$\nu \log B_h + (1-\nu) \log B_e = 0 \ (0 < \nu < 1).$$

Here, we define the technology choice θ as

$$\theta \equiv \frac{B_e}{B_h}.$$

Then, we have

$$B_h = B_e^{\frac{\nu-1}{\nu}}$$

$$\frac{B_h}{B_e} = B_e^{-\frac{1}{\nu}}$$

$$\theta^{-1} = B_e^{-\frac{1}{\nu}}$$

$$\theta^{\nu} = B_e$$

$$\theta^{\nu-1} = B_h$$

The production function can then be re-written as

$$y = \min\{\theta^{\nu-1}h, \theta^{\nu}e\}$$

Since a higher relative energy efficiency θ implies a higher B_e and a lower B_h, it implies the agent is using less energy to produce one unit of output. Conversely, a lower θ leads to a less energy-efficient production process. Here, I follow Jones (2005) and León-Ledesma and Satchi (2019) to interpret the trade-off between efficiencies as a choice on the "technology frontier." In Appendix A, I demonstrate that change in θ also can be interpreted as the outcome of directed technical change (Acemoglu et al., 2012; Hassler et al., 2022).

The firm's technology choice on θ highlights the difference between the ex-ante elasticity of substitution between energy and other inputs with a given θ and the ex-post substitution with a chosen θ. For any given θ, energy and other inputs are perfect complements. Yet, the ex-post elasticity of substitution after the agent chooses θ is 1 as the technology frontier is log-linear.[3] (León-Ledesma and Satchi, 2019) To see that, if we assume an interior solution in which capital is fully utilized following Hassler et al. (2021), then we have

$$\theta e = h \tag{4}$$

$$y = \theta^{\nu-1}h \tag{5}$$

[3] Similarly, directed technical change implies a long-run unitary elasticity between energy and other inputs if and only if energy efficiency and capital efficiency have a log-linear relationship. For further discussions, I refer the interested readers to León-Ledesma and Satchi (2019) and Hassler et al. (2021).

Then, the production function yields an alternative Cobb-Douglas form:

$$y = \theta^{\nu-1} h = \left(\frac{h}{e}\right)^{\nu-1} h = h^{\nu} e^{1-\nu}.$$

Intuitively, energy cannot be substituted with other inputs to produce one unit of goods without a corresponding technology adjustment. For example, the electricity (energy) needed to run the air conditioner to keep a house warm cannot be just substituted with more air conditioners (other inputs). Suppose the agent wants to reduce her energy usage to more energy-efficient technologies. In that case, she needs to use energy-efficient air conditioners and make her house more heat-isolating by installing insulation materials. In this model, these technology choices are modeled as the adjustment in θ. In section III, I introduce frictions to the adjustment of θ. Allowing for different ex-ante and ex-post elasticities then leads to differences in short-run and long-run elasticities between energy and other inputs. This setting is consistent with the empirical finding by Hassler et al. (2021) that “energy-saving technical change” leads to a significant difference in the short and long-run price elasticity of energy.

The usage of brown energy inputs warms the planet and leads to damage that is proportionate to the final outputs. The damage ratio is given by the climate damage function $D(e_B)$. The firm-household does not internalize this climate damage and take e_b as given. After incurring climate damage and paying the final energy firm, the remaining final goods are consumed by the firm's household. In addition, a technology subsidy as a function of θ, $\mathrm{T}_\theta(\theta) = \tau_\theta \theta < 0$ can be given to the firm-household to encourage the adoption of more energy-efficient technology choices on the technology menu.[4] Like the green and brown tax/subsidies, this technology choice subsidy is financed by a lump-sum tax ($T_y < 0$). Therefore, the firm-household’s maximization problem is

$$\max_{\theta,e} D(e_b) \min\{\theta^{\nu-1} h, \theta^{\nu} e\} - p_e e - \tau_\theta \theta + T_y$$

[4] Since θ_t also can be modelled as a product of directed technical change, subsidies on θ_t can be interpreted as subsidies on directed technical change.

Here, I still focus on the interior solution. Plugging (4) and (5) back into the optimization problem, we have

$$\max_{\theta} D(e_b)\theta^{\nu-1}h - p_e\frac{h}{\theta} - \tau_\theta\theta + T_y$$

This problem yields the first-order condition:

$$D(e_b)(\nu-1)\theta^{\nu-2}h + p_e\frac{h}{\theta^2} - \tau_\theta = 0 \tag{6}$$

We shall notice that, if $\tau_\theta = 0$,

$$\theta = \left(\frac{p_e}{D(e_b)(1-\nu)}\right)^{\frac{1}{\nu}}$$

That is, a lower energy price p_e leads to a lower θ. The economic intuition here is that less expensive energy encourages the firm to choose a less energy-efficient technology on the technology menu. From the energy production function (1), we know that, for a given brown/green share, $\frac{e_b}{e_g}$, lower e always leads to lower e_b. Therefore, brown energy usage can lead to less brown energy usage, too.

The Social Planner's Problem and Green Subsidies

In this subsection, I state the planning problem, in which a social planner (SP) has access to the production technology of energy e and final goods y and internalizes the damage caused by brown energy inputs $D(e_b)$. The SP's maximization problem is

$$\max_{e_b,e_g} D(e_b)\min\{\theta^{\nu-1}h, \theta^{\nu}e\} - p_b e_b - p_g e_g$$

s.t.

$$e = \left(\lambda(e_b)^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)(e_g)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}} \tag{1*}$$

If we still assume that capital is fully utilized, the FOCs of the problem yield the following

$$\frac{e_b}{e_g} = \frac{\lambda}{1-\lambda}\left(\frac{p_b+\kappa}{p_g}\right)^{-\epsilon_e} \tag{2*}$$

$$\mu_e = (\lambda^{\epsilon_e}(p_b+\kappa)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_g)^{1-\epsilon_e})^{\frac{1}{1-\epsilon_e}} \tag{3*}$$

$$\theta e = h \tag{4*}$$

$$y = \theta^{\nu} e \tag{5*}$$

$$D(e_B)(\nu-1)\theta^{\nu-2}h = -\mu_e \frac{h}{\theta^2} \tag{6*}$$

$$\kappa = D'(e_b)y \tag{7*}$$

Here, κ is the Social Cost of Carbon (SCC), which characterizes the marginal climate damage of brown energy usage.

By comparing the decentralized equilibrium (1) – (6) and the solution for the SP (1*)-(6*), we can see that, in the decentralized competitive equilibrium, the share between green and brown energy inputs e_b/e_g and the price of energy p_e are determined by the after-tax private costs of two types of energy sources:

$$\frac{e_b}{e_g} = \frac{\lambda}{1-\lambda}\left(\frac{p_b+\tau_b}{p_g+\tau_g}\right)^{-\epsilon_e} \tag{2}$$

$$p_e = (\lambda^{\epsilon_e}(p_b+\tau_b)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_g+\tau_g)^{1-\epsilon_e})^{\frac{1}{1-\epsilon_e}} \tag{3}$$

In contrast, in the solution to SP, they are determined by their respective social costs, with the social costs of brown energy in addition to its brown cost summarized by the SCC $\kappa = D'(e_b)y$. In the decentralized equilibrium conditions, τ_b appears in both equations (2) and (3), identical to how the SCC κ is present in both equations (2*) and (3*) of the SP problem solution. Therefore, a brown tax $\tau_b = \kappa = D'(e_b)y$ alone implements the solution to the SP. As the classical Pigou logic states, a tax should equalize the private cost to the social cost when there is a negative externality. Such a brown tax can produce two separate effects. First, it encourages the substitution away from brown energy towards green energy by making brown energy relatively more expensive as it changes the after-tax relative price to be $(p_b+\tau_b)/p_g$ (*the substitution effect*) in (2). At the same time, it incentivizes the firm-household to use less energy

e by choosing a higher θ as it makes the price energy p_e higher as $\frac{dp_e}{d\tau_b} > 0$ and $\frac{\partial\theta}{\partial p_e} > 0$. Less energy usage leads to less brown energy usage (*the scale effect*).

On the other hand, a green subsidy $-\tau_g < 0$ cannot implement the social optimum. It is true that such a subsidy can replicate the substitution effect of the socially optimal brown tax by setting τ_g^* such that

$$\frac{p_b}{p_g + \tau_g^*} = \frac{p_b + \kappa}{p_g} \Rightarrow \tau_g^* = p_b \frac{p_g}{p_b + \kappa} - p_g = p_g \left(\frac{p_b}{p_b + \kappa} - 1 \right) < 0.$$

On the other hand, green subsidies alone, in general, cannot mimic the scale effect of brown tax because $\frac{dp_e}{d(-\tau_g)} < 0$. While a brown tax increases the energy price p_e and incentivizes the adoption of a higher θ, a green subsidy decreases the energy price p_e and leads to a level of θ that is different from that according to the solution to SP. Because of the negative externality of brown energy inputs, there is a wedge between the private and social costs of energy. A brown tax can close this wedge, while a green subsidy further widens it and leads to higher energy production than the social optimum. In fact, when green and brown energy inputs are gross complements ($\epsilon_y < 1$), green energy subsidies actually increase brown energy usage. [5] To see that, by manipulating the energy production function (1), we have

$$e_b = \frac{\lambda}{1 - \lambda} \left(\frac{p_b}{p_e(\tau_g)} \right)^{-\epsilon_e} e\left(p_e(\tau_g)\right)$$

Then, the change of brown energy usage induced by green energy subsidies relative to the business-as-usual with no green energy subsidies, $\hat{e}_b \equiv \frac{e_b}{e_{b,BA}}$, is

$$\hat{e}_b = \left(\frac{p_e(\tau_g)}{p_{\text{BAU}}} \right)^{-\epsilon_e} \frac{e\left(p_e(\tau_g)\right)}{e_{\text{BAU}}} = C\left(p_e(\tau_g)\right)^{-\epsilon_e} e\left(p_e(\tau_g)\right),$$

with

[5] Casey et al. (2023) derive the conditions under which the scale effect can outweigh the substitution effect, leading to the conclusion that green energy ought to be subjected to taxation when green and brown inputs are gross complements. The following analysis largely follows their argument. For the analysis of this topic in a model general setting, see Casey et al. (2023).

$$C \equiv \left(\frac{1}{p_{\text{BAU}}}\right)^{-\epsilon_e} \frac{1}{e_{\text{BAU}}} > 0$$

Then,

$$\frac{d\hat{e}_b}{d\tau_g} = C\left(p_e(\tau_g)\right)^{-\epsilon_e - 1}(-\epsilon_e)\frac{dp_e(\tau_g)}{\tau_g} e\left(p_e(\tau_g)\right) + \left(p_e(\tau_g)\right)^{-\epsilon_e} \frac{de\left(p_e(\tau_g)\right)}{dp_e(\tau_g)} \frac{dp_e(\tau_g)}{d\tau_g}$$

$$\Rightarrow$$

$$\frac{d\hat{e}_b}{d(-\tau_g)} < 0 \Leftrightarrow -\epsilon_e + \frac{de\left(p_e(\tau_g)\right)}{dp_e(\tau_g)} \frac{p_e(\tau_g)}{e\left(p_e(\tau_g)\right)} > 0 \Leftrightarrow \epsilon_e > 1,$$

as the ex-post price elasticity of substitution is -1. Only when green and brown energy are gross substitutes standalone subsidies on green energy inputs can reduce brown energy usage. Otherwise, the scale effect would dominate the substitution effect, and emissions would increase as a response to subsidies imposed on green energy inputs.

The Energy Efficiency Subsidies and the Social Optimum

The analysis above indicates that if we want to achieve the social optimum through subsidies, the scale effect of taxes on brown goods must be generated using alternative policy measures. A subsidy on technology choice θ is an ideal candidate. Rewarding the firm-household with higher θ incentivizes the agent to choose a more energy-efficient technology. By comparing (3) and (6) to (3*) and (6*), we see that a technology subsidy

$$\begin{aligned}
\tau_\theta^* = &-\frac{h(\mu_e - p_e)}{\theta^2} \\
&= -\frac{h}{\theta^2}\Big(\left(\lambda^{\epsilon_e}(p_b+\kappa)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_g)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \\
&- \left(\lambda^{\epsilon_e}(p_b)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_g+\tau_g)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}\Big) < 0
\end{aligned}$$

together with a green subsidy

$$\tau_g^* = p_g\left(\frac{p_b}{p_b+\kappa} - 1\right) < 0$$

exactly implements the solution to the social planning problem. Specifically, the technology subsidy on θ can implement the socially optimum energy usage level e because this level is entirely determined by θ as $\theta e = h$. So, by (6), if $\tau_\theta^* = \frac{h(\mu_e - p_e)}{\theta^2}$, we have

$$D(e_b)(\nu-1)\theta^{\nu-2}k + \frac{hp_e}{\theta^2} + \frac{h(\mu_e - p_e)}{\theta^2} = 0$$

$$\Rightarrow$$

$$D(e_b)(\nu-1)\theta^{\nu-2}h + \mu_e \frac{h}{\theta^2} \; = 0$$

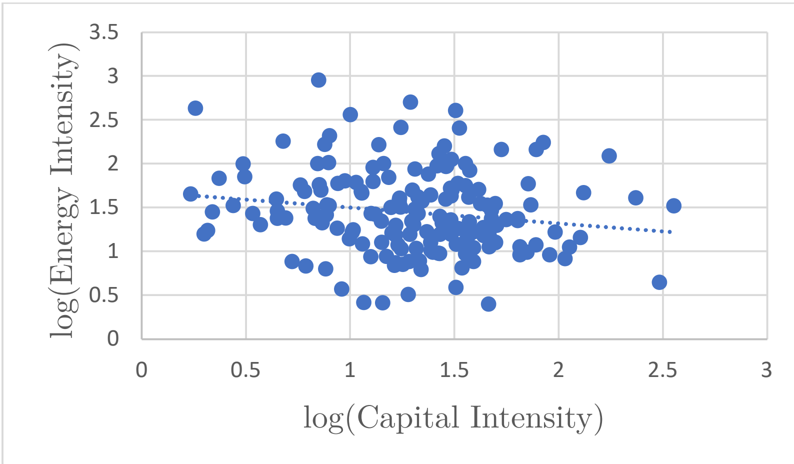
which is the same as (6)* in solution to SP. The intuition here is that the government can use τ_θ to implement a relative energy efficiency level θ that is consistent with the social cost of energy. With such a subsidy in place, the private marginal benefit of adjustment in θ is consistent with its social marginal benefit. As the optimal green subsidy already implements the optimal brown-green share, both the green and brown share e_b/e_g and the overall energy usage e are equivalent to those according to the solution to SP when an optimal technology subsidy and an optimal green subsidy are placed. We conclude that, when the short-run elasticity between energy and capital is zero, a technology subsidy and a green subsidy can implement the social optimum together. The effectiveness of this policy scheme is evaluated in a more generalized setting in the next section.

The Empirical Evidence for the Endogenous Energy Efficiency

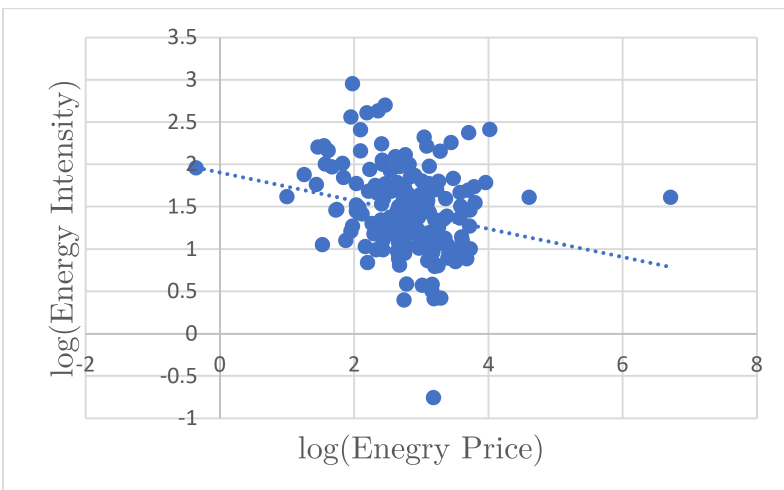
In this subsection, I provide some indirect evidence for the existence of technology choice, which determines energy efficiency.[6] From Panel A of Figure 1, we can observe a negative relationship between the capital and energy intensity across the country level. Even though it is a correlation instead of causation, this relationship can still be seen as empirical evidence for the trade-off

[6] I also refer the readers to Knittel (2011), which shows the trade-offs faced when choosing between passenger vehieles' fuel economy, weight, and engine power characteristics.

between energy and non-energy input efficiencies. Panel B also confirms the model's prediction, showing that an increase in energy prices leads to a decrease in energy intensity.



Panel A: The Relationship between Capital Intensity and Energy Intensity



Panel B: The Relationship between Energy Price and Energy Intensity

Notes: Panel A depicts the cross-country relationship between the log energy price (x-axis) and log energy intensity (y-axis). Panel B depicts the relationship between the log capital intensity (x-axis) and log energy intensity (y-axis). The energy price is proxied by price of electricity (US cents per kWh). The energy intensity is proxied by energy intensity level of primary energy (MJ/$2017 PPP GDP in 2015). The energy capital is proxied by Energy intensity level of primary energy ($2017 PPP Capital Stock/$2017 PPP GDP in 2015). This figure reveals the negative correlation between energy intensity and energy price as well as the negative correlation between energy intensity and capital intensity.

Figure 1: Energy Price, Capital Intensity, and Energy Intensity

The Non-Zero Ex-Ante Elasticity between Energy and Other Inputs

In the analysis above, I assume that energy and other inputs are perfect complements with a given technology choice θ. In this subsection, I relax this assumption by assuming that they are imperfect complements and the firm-household has access to a CES production function:

$$y = \left((\theta^{\nu-1} h)^{\frac{\epsilon_y - 1}{\epsilon_y}} + (\theta^{\nu} e)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}.$$

The firm-household then becomes

$$\max_{\theta, e} \left((\theta^{\nu-1} h)^{\frac{\epsilon_y - 1}{\epsilon_y}} + (\theta^{\nu} e)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - p_e e - \tau_\theta \theta$$

The FOCs are

$$y^{\frac{1}{\epsilon_y}} (\theta^{\nu} e)^{-\frac{1}{\epsilon_y}} \theta^{\nu} - p_e = 0$$

$$y^{\frac{1}{\epsilon_y}} (\theta^{\nu} e)^{-\frac{1}{\epsilon_y}} (\nu) \theta^{\nu-1} + y^{\frac{1}{\epsilon_y}} (\theta^{\nu-1} h)^{-\frac{1}{\epsilon_y}} (\nu - 1) \theta^{\nu-2} - \tau_\theta = 0$$

We then have

$$y^{\frac{1}{\epsilon_y}} (e)^{-\frac{1}{\epsilon_y}} (\theta^{\nu})^{1-\frac{1}{\epsilon_y}} - p_e = 0 \Rightarrow$$

$$(e)^{-\frac{1}{\epsilon_y}} = p_e (\theta^{\nu})^{\frac{1-\epsilon_y}{\epsilon_y}} y^{-\frac{1}{\epsilon_y}} \Rightarrow$$

$$\frac{e}{y} = p_e^{-\epsilon_y} (\theta^{\nu})^{\epsilon_y - 1}.$$

If $\epsilon_y = 0$, then

$$\frac{e}{y} = (\theta^{\nu})^{-1}.$$

The energy usage is entirely determined by the energy efficiency θ. On the other hand, if $\epsilon_y > 0$, then with any given θ, a lower energy price would still lead to higher energy usage as

$$\frac{\partial \left(\frac{e}{y}\right)}{\partial p_e} \frac{p_e}{\left(\frac{e}{y}\right)} = -\epsilon_y.$$

Even if the government can use energy efficiency subsidies τ_θ to implement a socially optimal θ, energy is still overused if a wedge between its private and social costs exists. Nevertheless, in the next section, I show that, even though they are not able to implement the social optimum exactly, green energy subsidies and energy efficiency subsidies can still implement an equilibrium sufficiently close to it.

III. A Dynamic Model

We now consider a dynamic model to assess the effectiveness of subsidies versus brown taxes over time. In addition to the time dimensionality, this model also extends the static model in several ways. It relaxes the assumption of the Leontief production function by allowing some degree of ex-ante elasticity of substitution between energy and other inputs. Technology adjustment costs and endogenous capital accumulation are also introduced. The government's Ramsey problems are solved under different policy scenarios. The equilibriums under different policy scenarios are then compared.

The Energy Firm

The setting for the energy sector is identical to the one in the static model. A perfectly competitive energy firm uses CES technology to produce the final energy e with brown and green energy inputs $e_{b,t}$ and $e_{g,t}$. Both brown and green energy inputs are produced with constant marginal costs $p_{b,t}$ and $p_{g,t}$, respectively, in terms of final output. In the baseline model, $p_{b,t}$ and $p_{g,t}$ are assumed to be constant over time. The energy e_t is then sold to the firm-household at price $p_{e,t}$. This firm is also potentially subject to taxes or subsidies imposed on both types of energy inputs $\tau_{b,t}$ and $\tau_{g,t}$, which are rebated lump sum ($T_{e,t}$). The profit maximization problem of this energy firm is

$$\max_{e_{b,t},e_{g,t}} p_{e,t}e_t - p_{b,t}e_{b,t} - p_{g,t}e_{g,t} - \tau_{b,t}e_{b,t} - \tau_{g,t}e_{g,t} + T_{e,t}$$

s.t.

$$e_t = \left(\lambda(e_{b,t})^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)(e_{g,t})^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}} \tag{7}$$

The Firm-Household

A representative firm-household has an infinite life horizon with an annual period length. Its preferences given by

$$W = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\alpha}}{1-\alpha}.$$

In this expression, $1/\alpha$ is the intertemporal elasticity of consumption, β is the discount factor, and c_t is consumption. This agent, with labor-augmenting total factor productivity (TFP) A_t, combines capital k_t, endowed labor l, and energy e_t to produce final goods using a CES production technology with $\epsilon_y < 1$:

$$y_t = \left((\theta_t^{\nu-1}A_t^{1-\sigma}k_t^{\sigma}l^{1-\sigma})^{\frac{\epsilon_y-1}{\epsilon_y}} + (\theta_t^{\nu}e_t)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} \tag{8}$$

Labor l is supplied inelastically. The law of motion for labor-augmenting TFP A_t is $A_{t+1} = \exp(g_A)\, A_t$. The energy e_t is purchased from the energy firm at the price $p_{e,t}$. Similar to the static model, the firm can choose θ_t, the relative efficiency between energy and other inputs. A higher θ implies a higher relative energy efficiency, and vice versa.[7]

If the agent's technology choice in this period, θ_t, is different from the technology choice in the last period, θ_{t-1}, the technology adjustment cost is $\left(1-\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\right) y_t$ where $0 \le \Psi(.) \le 1$, $\Psi(1) = 0$, $\Psi'(1) = 0$, and $\Psi''(.) > 0$. Under such a setting, the ex-ante elasticity of substitution

[7] In other words, the firm can choose its energy efficiency $B_e \equiv \theta_t^{\nu}$ and capital-labor efficiency $B_k \equiv \theta_t^{\nu-1}$ on a log-linear technology frontier: $\nu \log B_k + (1-\nu)\log B_e = 0$ $(0 < \nu < 1)$.

between the Cobb-Douglas composite $A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}$ and energy e_t with a given θ_t is ϵ_y while the ex-post elasticity is 1. Because of the technology adjustment cost, the short-run elasticity of substitution between the Cobb-Douglas composite $A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}$ and energy e_t is mainly determined by the ex-ante elasticity ϵ_y. On the other hand, the long-run elasticity equals one, with a long-run energy share of $1-\nu$ (Leon-Ledesma and Satchi, 2019). In other words, the production function is Cobb-Douglas in the long run, while the short-run production process is characterized by complementarity between energy and other inputs. Allowing for different short- and long-run elasticities is consistent with the empirical finding of Hassler et al. (2021). Technology choice subsidies, as a function of θ, $\mathrm{T}_{\theta,t}(\theta_t) = \tau_{\theta,t}\theta_t < 0$, are financed by lump-sum taxes $(T_{y,t})$ and given to the firm-household.

Cumulative carbon emissions M_t, generated by brown energy usage with exogenously given emission intensity η_t, are

$$M_t = M_0 + \sum_{s=0}^{t} \eta_s e_{b,s} = M_{t-1} + \eta_{t-1} e_{b,t-1}.$$

The cumulative emissions warm the planet and, therefore, damage a share of final outputs given by the function $D(M_t)$. Emission intensity η_t declines at the rate g_η:

$$\eta_{t+1} = \exp(g_\eta)\, \eta_t.$$

The agent takes the path of M_t as given and does not internalize the effects of her decision on M_t. After incurring climate damage, the technology adjustment cost, and the energy cost, the remaining final outputs can either be c_t or be invested into capital k_{t+1} which depreciates at the rate δ. The resource constraint for the agent is

$$\begin{aligned} &c_t + k_{t+1} - (1-\delta)k_t \\ &\quad = D(M_t)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\left(\left(\theta_t^{\nu-1} * A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} + \left(\theta_t^{\nu} e_t\right)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} - p_{e,t}e_t \qquad (9) \\ &\quad + \tau_{\theta,t}\theta_t + T_{y,t}. \end{aligned}$$

The agent's optimization problem is then characterized by the Bellman equation:

$$\mathcal{V}(\theta_{t-1}, k_t, M_t, A_t, \eta_t)$$

$$= \max_{\theta_t, k_{t+1}, e_t, c_t} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta \mathcal{V}(\theta_t, k_{t+1}, M_{t+1}, A_{t+1}, \eta_{t+1})$$

s.t.

(8) and (9)

Following Leon-Ledesma and Satchi (2019), I assume the technology adjustment cost is a symmetric exponential function[8] with γ as the technology adjustment scale factor. The parameter γ determines the speed of any adjustments of θ_t relative to θ_{t-1}:

$$\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) = \exp\left(-\frac{1}{2}\gamma * \left(\frac{\theta_t}{\theta_{t-1}} - 1\right)^2\right).$$

The functional form of the damage function is based on the recent advancement in climate science that indicates the degree of global warming H_t is proportional to the cumulative emissions with the transient climate response to emissions χ:

$$H_t = \chi * M_t$$

Following Barrage and Nordhaus (2023), the functional form of damage function is

$$D(M_t) = 1.0 - d(H_t)^2 = 1.0 - d(\chi * M_t)^2.$$

Here, the damage scale factor d determines the output loss with a given degree of warming.

The Decentralized Equilibrium

The Recursive Competitive Equilibrium is defined as follows: I denote s as the vector of state variables $\left(\bar{\theta}, k, M, A, \eta\right)$. A recursive competitive equilibrium is defined by a perceived law of

[8] The results presented in the results subsections still holds under the alternative quadratic adjustment cost function.

motion for the cumulative carbon emissions $\widehat{M}(s)$, tax/subsidy rules $\hat{\tau}_b(s)$, $\hat{\tau}_g(s), \hat{\tau}_\theta(s)$, and decision rules $k'(s)$, $\theta(s)$, $e(s)$, $e_b(s)$, $e_g(s)$ with associated value function $V(s)$ such that:

1. $\{k'(s),\ \theta(s)\ ,\ e(s)\}$ and $V(s)$ solve the firm-household's recursive optimization problem, taking as given perceived $\widehat{M}(s)$ and tax/subsidy rules $\hat{\tau}_b(s)$, $\hat{\tau}_g(s), \hat{\tau}_\theta(s)$;
2. $\{e_b(s),\ e_g(s)\}$ solves the energy firm's static optimization problem with the given $p_e(s)$;
3. The profit of the energy firm is zero;
4. The perceived law of motion for cumulative carbon emissions is consistent with the actual law of motion $\widehat{M}'(s) = M + \eta e_B(s)$; and
5. The perceived tax/subsidy rules are consistent with the actual rules: $\hat{\tau}_i(s) = \tau_i(s)$.

Solving the maximization problem of the energy firm and the firm-household yields a set of endogenous variables $\{\theta_{t+1}, k_{t+1}, c_t, e_t, e_{b,t}, e_{g,t}, y_t, p_{e,t}\}$ such that both (7) - (9) and the following equations are satisfied.

$$p_{e,t} = (\lambda^{\epsilon_e}(p_{b,t} + \tau_{b,t})^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_{g,t} + \tau_{g,t})^{1-\epsilon_e})^{\frac{1}{1-\epsilon_e}} \tag{10}$$

$$\frac{e_{b,t}}{e_{g,t}} = \left(\frac{\lambda}{1-\lambda}\frac{p_{b,t} + \tau_{b,t}}{p_{g,t} + \tau_{g,t}}\right)^{-\epsilon_e} \tag{11}$$

$$(1 - d(\chi M_t)^2) * \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) y_t^{\frac{1}{\epsilon_y}} * (\theta_t^\nu e_t)^{-\frac{1}{\epsilon_y}} \theta_t^\nu = p_{e,t} \tag{12}$$

$$\begin{aligned} &c_t^{-\alpha}\ \Bigg\{(1 - d(\chi M_t)^2)\Psi'\left(\frac{\theta_t}{\theta_{t-1}}\right)\frac{1}{\theta_{t-1}} y_t \\ &+ (1 - d(\chi M_t)^2)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) y^{\frac{1}{\epsilon_y}} \begin{pmatrix} (\theta_t^{\nu-1} A_{t+1}^{1-\sigma} k_{t+1}^\sigma l^{1-\sigma})^{-\frac{1}{\epsilon_y}}(\nu - 1)\theta_t^{\nu-2} A_{t+1}^{1-\sigma} k_{t+1}^\sigma l^{1-\sigma} \\ +(\theta_t^\nu e_t)^{-\frac{1}{\epsilon_y}} \nu \theta_t^{\nu-1} e_t + \tau_{\theta,t} \end{pmatrix}\Bigg\} \\ &+ \beta c_{t+1}^{-\alpha}\left((1 - d(\chi M_{t+1})^2)\Psi'\left(\frac{\theta_{t+1}}{\theta_t}\right)\left(-\frac{\theta_{t+1}}{\theta_t^2}\right) y_{t+1}\right) = 0 \end{aligned} \tag{13}$$

$$
\begin{aligned}
-c_t^{-\alpha}+\beta c_{t+1}^{-\alpha}*\Bigg(\Bigg(&(1-d(\chi M_{t+1})^2)\Psi\left(\frac{\theta_{t+1}}{\theta_t}\right)y_{t+1}^{\frac{1}{\epsilon_y}}(\theta_{t+1}^{\nu-1}*A_{t+1}^{1-\sigma}k_{t+1}^{\sigma}l^{1-\sigma})^{-\frac{1}{\epsilon_y}}\theta_{t+1}^{\nu-1}*\sigma\\
&*A_{t+1}^{1-\sigma}*k_{t+1}^{\sigma-1}l^{1-\sigma}+(1-\delta)\Bigg)\Bigg)=0
\end{aligned}
\tag{14}
$$

Here, equation (10) is the standard CES price index for energy. Equation (11) describes how the share of brown and green energy sources is influenced by their respective prices after taxes and subsidies. Equation (12) indicates that the marginal cost of energy equals its marginal benefits to the firm-household. Lastly, equations (13) and (14) are the Euler equations governing technology choice θ and capital k, respectively.

The Ramsey Problem

I formulate the optimization of a benevolent government as the solution to a Ramsey problem. The government internalizes the negative externality of brown energy and aims to maximize the welfare of the firm-household. Through the tax and/or subsidy instruments $\tau_{b,t}$, $\tau_{g,t}$, and $\tau_{\theta,t}$, the government can choose allocations subject to the resources and implementability constraints (7)–(14) and constraints on policy instruments. This setting yields the Ramsey problem characterized by the Bellman equation as follows:

Table 1: Policy Scenarios and the Corresponding Tax/Subsidy Constraints

Policy Scenarios	Policy Instrument Constraints
Brown taxes	$\tau_{b,t} > 0, \tau_{g,t} = 0, \tau_{\theta,t} = 0$
Green subsidies	$\tau_{b,t} = 0, \tau_{g,t} < 0, \tau_{\theta,t} = 0$
Technology choice subsidies + green subsidies	$\tau_{b,t} = 0, \tau_{g,t} < 0, \tau_{\theta,t} < 0$
The business-as-usual (BAU)	$\tau_{b,t} = 0, \tau_{g,t} = 0, \tau_{\theta,t} = 0$

Notes: This table lists the constraints faced by the government under different policy scenarios.

$$\mathcal{V}(M_{t,}\ \theta_{t-1}, k_t, A_t, \eta_t) =$$

$$\max_{\substack{\tau_{b,t},\tau_{g,t},\tau_{\theta,t},\\ M_{t+1},\theta_t,k_{t+1},\\ c_t,e_t,e_{b,t},e_{g,t},y_t,p_{e,t}}} \quad \frac{c_t^{1-\alpha}}{1-\alpha} + \beta\mathcal{V}(M_{t+1,}\ \theta_t, k_{t+1}, A_{t+1}, \eta_{t+1})$$

s.t.

(7) – (14)

$$M_{t+1} = M_t + \eta_t e_{b,t}$$

$$\tau_{i,LB} \le \tau_i(s) \le \tau_{i,UB}$$

The constrained-efficient equilibrium is defined by the policy functions $k_{t+1}(s)$, $M_{t+1}(s)$ with decision rules $\theta_t(s)$, $e_t(s)$, $e_{b,t}(s)$, $e_{g,t}(s)$, $c_t(s), \tau_{i,t}(s)$, policy constraints $\tau_{i,LB}$ and $\tau_{i,UB}$ with $\tau_{i,LB} \le \tau_i(s) \le \tau_{i,UB}$, the value function $\mathcal{V}(s)$, and the conjectured functions characterizing the decision rule of the future planner $\theta'(s)$, $e'(s)$, $c'(s)$ such that the following condition holds:

1. Planner's optimization: $\mathcal{V}(s)$ with $k_{t+1}(s)$, $M_{t+1}(s)$, $\theta_t(s)$, $e_t(s)$, $e_{b,t}(s)$, $e_{g,t}(s)$, $\tau_{i,t}(s)$ solve the Bellman equation defined above; and
2. The conjectured decision rules are consistent with the actual decision rules.

In this section, I study four policy scenarios in which different policy instruments are feasible to the government: carbon taxes alone, green energy subsidies alone, technology choice subsidies together with green subsidies, and the business-as-usual (BAU). The corresponding constraints of these scenarios are summarized in Table 1.

As in the static model, carbon taxes which equal the marginal utility damage of brown energy (i.e., SCC) usage,

$$\kappa_t = \beta \left(c_{t+1}^{-\alpha} \left(-\eta_t * D'(M_{t+1}) \Psi \left(\frac{\theta_{t+1}}{\theta_t} \right) y_{t+1} \right) + \kappa_{t+1} \right)$$

$$\tau_{b,t} = \frac{\eta_t \kappa_t}{c_t^{-\alpha}}$$

implements the social optimum. As I argued in the section above, technology choice subsidies and green subsidies cannot implement the social optimum if the ex-ante elasticity of substitution between the Cobb-Douglas composite and energy, ϵ_y, is not zero. Nevertheless, with a low short-run elasticity ϵ_y, this intensive margin is relatively small. As a result, as demonstrated later in the quantitative results, technology choice subsidies, together with green subsidies, can bring the magnitude of emission reduction to a level close to that under brown taxes.

Solution Method

For the convenience of numerical implementation, the government's optimization problem is normalized by TFP and labor. I define $\tilde{k}_t = \frac{k_t}{A_t l}$, $\tilde{c}_t = \frac{c_t}{A_t l}$, $\tilde{y}_t = \frac{y_t}{A_t l}$, $\tilde{e}_t = \frac{e_t}{A_t l}$, $\tilde{e}_{b,t} = \frac{e_{b,t}}{A_t l}$, $\tilde{e}_{g,t} = \frac{e_{g,t}}{A_t l}$, $\tilde{\tau}_{\theta,t} = \frac{\tau_{\theta,t}}{A_t l}$, $\tilde{T}_{\theta,t} = \frac{T_{\theta,t}}{A_t l}$, $\tilde{T}_{e,t} = \frac{T_{e,t}}{A_t l}$, $\tilde{\eta}_t = A_t * l * \eta_t$, and $V_t = \frac{\mathcal{V}_t}{A_t^{1-\alpha}}$. Here, $\tilde{\eta}_t$ captures the carbon intensity of brown energy per unit of effective labor $\tilde{e}_{b,t}$. The model after normalization is described in detail in Appendix B. The Ramsey problem of the government is rewritten as

$$V\Big(M_t, \theta_{t-1}, \tilde{k}_t, \tilde{\eta}_t\Big) = \max_{\substack{\tau_{b,t}, \tau_{g,t}, \tilde{\tau}_{\theta,t}, \\ M_{t+1}, \theta_{t+1}, \tilde{k}_{t+1}, \tilde{c}_t, \tilde{e}_t, \tilde{e}_{b,t}, \tilde{e}_{g,t}, \tilde{y}_t, p_{e,t}}} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta e^{(1-\alpha)g} V\Big(M_{t+1}, \theta_t, \tilde{k}_{t+1}, \tilde{\eta}_{t+1}\Big)$$

s.t.

(7*) – (14*)

$$\tilde{\eta}_{t+1} = \tilde{\eta}_t \exp(g_A + g_\eta)$$

$$M_{t+1} = M_t + \tilde{\eta}_t * \tilde{e}_{b,t}$$

$$\tau_{i,UB} < \tau_i(s) < \tau_{i,LB}$$

Table 2: Calibrated Parameters

Parameter	Value	Source
The Energy Firm:		
ϵ_e	1.85	Papageorgiou et al. 2017
p_b/p_g	74/600	Hassler et al. 2021
λ	0.644	Golosov et al. 2014
The Firm-Household:		
α	1.45	Nordhaus 2017
β	0.985	Nordhaus 2017
ϵ_y	0.05	
$1-\nu$	0.06	Golosov et al. 2014
σ	0.35	Golosov et al. 2014
δ	0.06	Casey et al. 2023
γ	2.49	Calibrated
χ	1.6	Rudik 2020
d	0.003467	Barrage and Nordhaus 2023
g_A	0.023	Calibrated

Notes: Lists model parameters that are calibrated or taken from the literature.

For notional convenience, I will omit the tilde sign for the rest of the paper. Any lower-case variables denote normalized variables unless otherwise noted. The renormalized model is solved using backward value function iteration. At each iteration, with a given value function $V(M_{t+1,}\theta_t, k_{t+1}, \eta_{t+1})$ as well as the future decision rules $\theta_{t+1}(M_{t+1,}\theta_t, k_{t+1}, \eta_{t+1})$, $e_{t+1}(M_{t+1,}\theta_t, k_{t+1}, \eta_{t+1})$, and $c_{t+1}(M_{t+1,}\theta_t, k_{t+1}, \eta_{t+1})$, the maximization problem above is solved by non-linear programming on a finite number of grid points. Using the results as the training inputs, I then train Gaussian processes to approximate the value function $V(M_t, \theta_t, k_t, \eta_t)$ as well as decision rules for the firm-household $\theta_t(M_{t,}\theta_{t-1}, k_t, \eta_t)$, $e_t(M_{t,}\theta_{t-1}, k_t, \eta_t)$, $c_t(M_{t,}\theta_{t-1}, k_t, \eta_t)$, which are used as inputs for the next value function iterations. This procedure is repeated until the value function converges.

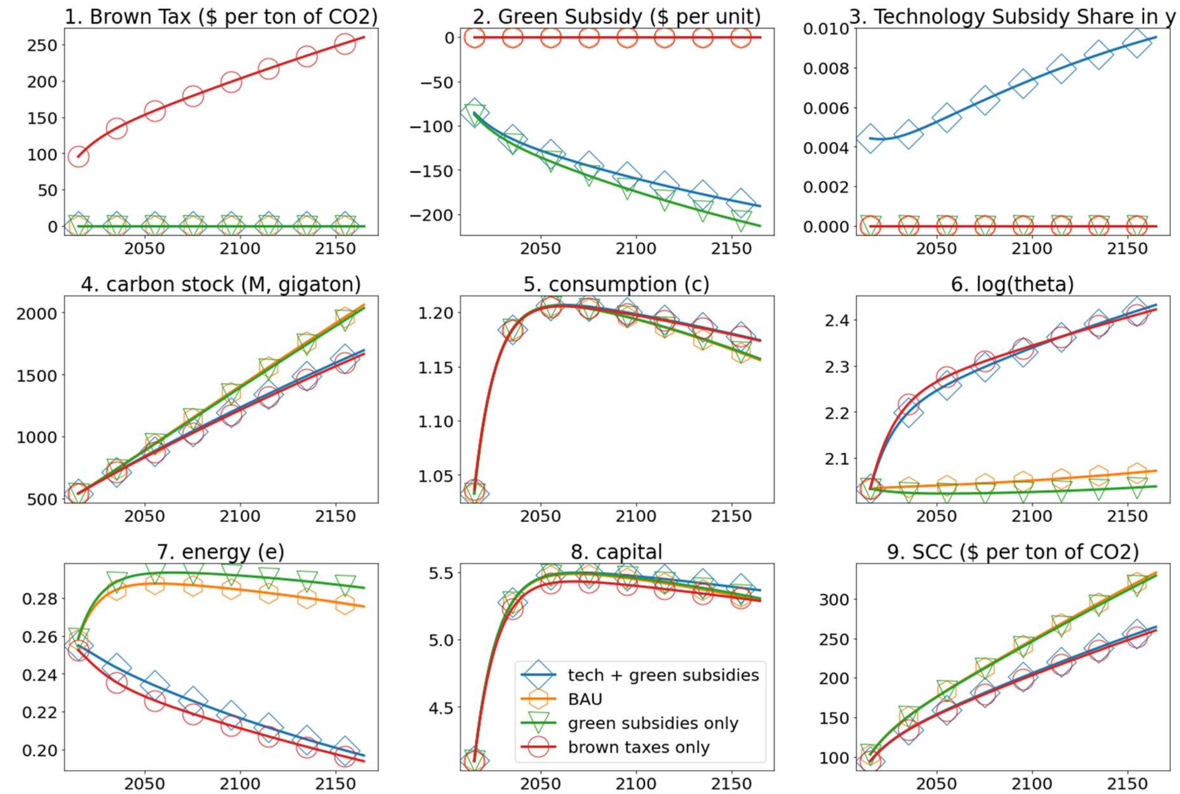
Calibration

I follow Nordhaus (2017) and set $\alpha = 1.45$ and $\beta = 0.985$. The long-run capital share of income σ is 0.35, while the long-run energy share $1 - \nu$ is 0.06. I assume that the growth rate of TFP A_t grows at 2.36% annually. The declining rate of the carbon intensity of brown energy g_η is also at 2.36% to keep the carbon intensity of brown energy usage per unit of effective labor is constant over time. The ex-ante elasticity of substitution between the Cobb-Douglas composite and the energy, ϵ_y, set to be 0.05. This value is consistent with the empirical finding of Hassler et al. (2021), which suggests a very low short-run elasticity between energy and other inputs. The transient climate response to emissions χ, which links the cumulative emissions and the degree of warming, is 1.6 (Rudik, 2020). I set the damage parameter d to 0.003476 such that a 3-degree of warming leads to a 3.13% output loss (Barrage and Nordhaus, 2023). The technology adjustment cost parameter γ is set to be 2.498, which implies that 0.01% increase in the energy price decreases the annual energy usage by 0.005% on the steady state. This is generally consistent with the empirical estimation of the short-run price elasticity of energy, which suggests that a value of around -0.2 (Labandeira et al., 2017, Gao et al. 2021). I also run a sensitivity analysis on this parameter and find that my results are robust to a wide range of values. The relaive brown and green energy prices are based on the coal and green energy prices in Hassler et al. (2017), while their absolute per-unit prices are set to match BAU emissions to the 46 trillion global carbon dixodie emissions in 2015. The share parameter for green energy $1 - \lambda$ follows Golosov et al. (2014). The value of the elasticity of substitution between green and brown energy, ϵ_e, is set to be 1.8 based on the empirical study by Papageogiou et al. (2017), such that green and brown energy sources are substitutable inputs. I also examine the results when $\epsilon_e = 0.95$ (Golosov et al., 2014). The initial carbon emission intensity ζ_0 is normalized to match the global carbon emissions in 2015. The initial technology choice θ_{-1} is according to its steady state when the climate change damage is zero. Without climate change or government interventions, the firm-household would never adjust its technology choice θ_t, and the technology stays at θ_{-1} forever. The initial capital stock k_0 is to match the 26% global gross capital formation rate in 2015. Table 2 lists some of the key model parameters that are calibrated or taken from the literature.

Baseline Quantitative Results with High Elasticity of Substitution between Brown and Green

This subsection evaluates the baseline quantitative results of the model when the elasticity of substitution between green and brown energy is high ($\epsilon_e = 1.85$) in a simulation spanning 150 years starting in 2015 under four policy scenarios in Table 1: BAU, carbon taxes, green subsidies, and green plus energy efficiency subsidies. Figure 1 shows the path of several endogenous variables under these different policy scenarios. Table 3 summarizes the cumulative carbon emissions in 2100 and the welfare gain relative to BAU.

Under the BAU scenario (depicted by yellow lines with hexagon markers in the figure), no policy instruments are implemented by the government. Cumulative carbon emissions reach 1,406 gigatons by the end of the century (as shown in subplot 4). That is equivalent to 2.25 degrees Celsius of warming. Without any policy interventions from the government, technology choice θ_t remains relatively unchanged over time, even as energy demand escalates due to increasing capital accumulation. If the government can impose carbon taxes $\tau_{b,t}$ (red lines with round markers), the optimal tax rate would be \$98.2 per ton of carbon dioxide emission in 2015, and it would rise above \$202 by the end of the century (subplot 1). Implementing optimal carbon taxes could lead to a 13.6% reduction in cumulative emissions, bringing them down to 1214 gigatons in 2100. This reduction corresponds to a smaller increase in global temperatures, limited to 1.92 degrees Celsius. Brown taxes lead to higher consumption levels in the long run. If optimal taxes are imposed on brown energy, the welfare gain in terms of equivalent consumption is 0.16%. In line with theoretical expectations, the introduction of brown taxes notably enhances energy efficiency (θ_t), as illustrated in subplot 6. Compared to the BAU scenario, this policy results in a significant 24% decrease in energy consumption by 2050, as shown in subplot 7.



Notes: This figure demonstrates the paths of endogenous variables under four different policy scenarios in Table 1 in which different policy instruments are feasible to the government when brown-green elasticity is high.

Figure 2: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy

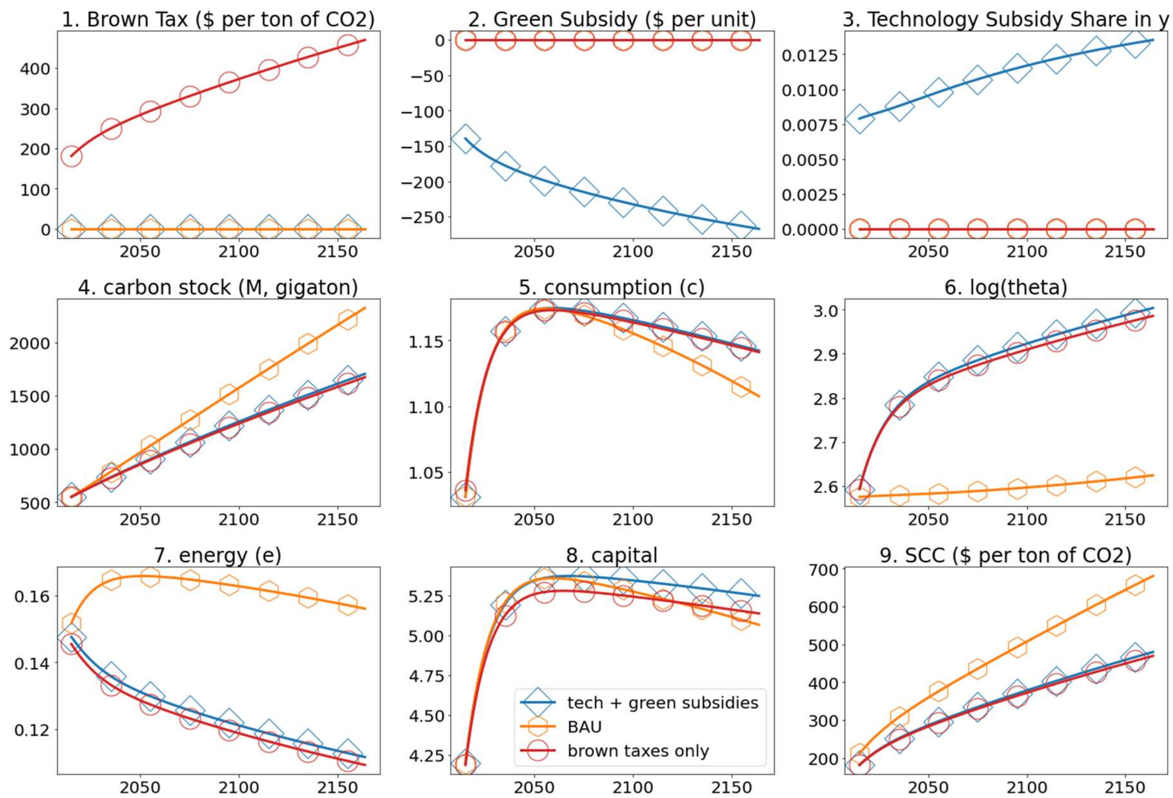
Table 3: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy

Policy Scenarios	Cumulative Emissions in 2100 (Trillion Ton)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1406	0.0
Brown taxes	1214	0.16
Green subsidies	1391	0.002
Technology choice subsidies + green subsidies	1233	0.14

Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios.

In the scenario where only green subsidies are implemented (illustrated by green lines with triangle markers), the absence of technology subsidies results in an optimal green subsidy rate of approximately \$89.9 per unit of green energy in 2015, as shown in subplot 2. Green subsidies, without the support of technology subsidies, only reduce cumulative emissions by merely 1% by the century's end compared to the BAU scenario, highlighted in subplot 4. The green-subsidies-only policy scenario leads to a limited welfare gain of 0.002%. The limited effectiveness of green energy subsidies on their own is attributed to the fact that, without technology subsidies, green energy subsidies fail to improve the energy efficiency of the downstream sector. The negative scale effect of green subsidies on energy efficiency, while not substantially large, is observable in the changes to energy efficiency and energy usage patterns, as depicted in subplots 6 and 7.

The blue lines with square markers represent the green subsidy plus technology choice subsidy policy scenario. When optimal technology subsidies $\tau_{\theta,t}$ are also imposed, the optimal rate increases to \$87.6 per unit in 2015 (subplot 2). Just as in the static model, a modest level of technology subsidies is warranted to ensure the energy efficiency of the economy if both technology subsidies and green subsidies can be imposed. In 2015, the optimal technology subsidy $\tau_{\theta,t}$ costs approximately 0.4% of total output. This rate rises to approximately 0.6% by the end of the century (subplot 3). When both energy efficiency subsidies and green subsidies are feasible, cumulative emissions are only slightly higher than those under the first-best carbon tax scenario. These two types of subsidies together reduce the cumulative emissions by 12.3%, leading to a cumulative emission of 1,233 tons, or a 1.93 degree of warming, by the end of the century. From subplot 5, which illustrates consumption levels across different policy scenarios, we clearly see that the consumption levels under the green subsidies plus technology choice subsidies policy align closely with those under the first-best brown taxes. When green subsidies are paired with technology subsidies, this consumption-equivalent welfare gain is 0.14%. The patterns for the technology choice θ and energy e are similar to those under the first-best carbon taxes. (subplots 6 and 7). Subplot 8 demonstrates that the patterns of capital accumulation remain consistent across the various policy scenarios. Lastly, the Social Cost of Carbon (SCC) patterns align with those for cumulative carbon emissions, highlighting that higher emissions lead to increased costs of carbon emission damage.



Notes: This figure demonstrates the paths of endogenous variables under policy scenarios in Table 1 in which different policy instruments are feasible to the government when brown-green elasticity is low.

Figure 3: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when $\epsilon_e = 0.95$

Table 4: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when $\epsilon_e = 0.95$

Policy Scenarios	Cumulative Emissions in 2100 (Trillion Ton)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1561	0.0
Brown taxes	1230	0.3
Technology choice subsidies + green subsidies	1249	0.28

Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios when $\epsilon_e = 0.95$.

Quantitative Results with Alternative Low Elasticity of Substitution between Brown and Green[9]

This sub-section evaluates the baseline quantitative results of the model when the elasticity of substitution between green and brown energy is low ($\epsilon_e = 0.95$), which are exhibited in Figure 3. Table 4 lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios. Figure 3 shows the path of endogenous variables under these different policy scenarios. Although green and brown energy inputs are complementary goods now, the emission reductions and welfare gains brought by the paired green and energy efficiency subsidies remain parallel to those brought by brown taxes.

Under the BAU scenario, the government still cannot impose any taxes or subsidies (subplots 1, 2, and 3). The cumulative emissions by the end of the century are 1,561 gigatons of carbon (subplot 4). Still, without policy interventions, θ is broadly stable over time. If brown taxes are accessible to the government, the optimal tax rate is \$176 per ton of carbon dioxide. Subplot 4 reveals that the optimal implemented brown taxes can reduce cumulative emissions by the end of the century by 21.2% to 1,230 gigatons. From subplot 6, we can learn that the energy efficiency of the final goods production process is improved significantly over time. The welfare gains under different policy scenarios, compared to those under the BAU, are evaluated. If optimal taxes are imposed on brown energy inputs, the welfare gain in terms of equivalent consumption is 0.30%.

When both green subsidies and technology choice subsidies are accessible, then, in 2015, the optimal green subsidy rate is \$139.6, and the technology subsidy costs approximately 0.75% of GDP; in 2050, these two numbers rise to \$231 and 1%, respectively. The combination of these two policy instruments reduces cumulative carbon emissions almost as effectively as brown taxes: subplot 4 shows that the cumulative emissions in 2100 are 1,249 gigatons. The path of

[9] The green-subsidies-only scenario is not examined here because the optimal green energy subsidies are zero when green and brown energy are complementary goods. (Casey et al., 2023)

θ_t is also similar to that when the optimal brown taxes are imposed. When green subsidies are paired with technology subsidies, this consumption-equivalent welfare gain is 0.28%.

The Role of Technology Adjustment Cost ψ

In the subsection, I examine how the optimal paths under different policy scenarios are affected by the technology adjustment cost parameter γ. In Appendix C, Figure A.1 shows the optimal endogenous paths when $\gamma = 10.0$, and Figure A.2 shows these paths when $\gamma = 20.0$. From these figures, we see that a higher adjustment cost γ leads to a slower increase in energy efficiency θ and ultimately brings higher cumulative carbon emissions. Nevertheless, regardless of the value of ψ, the combination of energy efficiency subsidies and green subsidies remains an ideal alternative to the first-best brown taxes. When $\psi = 10.0$, the brown tax leads to a 12.3% reduction in cumulative emissions, whereas a combination of subsidies achieves an 11.4% reduction. When $\psi = 20.0$, brown taxes can reduce cumulative emissions by 11.1% compared to a 10.2% reduction achieved through the subsidy combination.

IV. Extensions: Learning-by-Doing

In this section, I extend the baseline model by allowing learning-by-doing in the green energy sector. Here, I assume that the cost of green energy inputs $p_{g,t}$ decreases as green energy usage $e_{g,t}$ increases the knowledge stock S_t about green energy (*Learning-by-Doing*). That is,

$$p_{g,t} = p_{g,min} + \left(\frac{\Omega}{S_t}\right)^{\gamma_g},$$

$$S_{t+1} = S_t + e_{g,t}.$$

In the expressions above, $p_{g,min}$ is the long-run price of green energy, $\Omega > 0$ is a scaling factor, and $\gamma_g > 0$ is the learning exponent. I set $\gamma_g = 0.3$ and $\Omega = 0.02$ following Kalkuhl et al (2012).

$p_{g,min}$ is set to \$60. The positive learning externality from green energy usage is not internalized by the firm-household. The government Ramsey problem then becomes the following:

$$\mathcal{V}(M_t,\theta_{t-1},k_t,\eta_t,S_t) = \max_{\substack{\tau_{b,t},\tau_{g,t},\tau_{\theta,t},\\ M_{t+1},\theta_{t+1},k_{t+1},\\ c_t,e_t,e_{b,t},e_{g,t},y_t,p_{e,t}\\ p_{g,t},S_{t+1}}} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta e^{(1-\alpha)g}\mathcal{V}(M_{t+1},\theta_t,k_{t+1},\eta_{t+1},S_{t+1})$$

s.t.

(7)* – (14)*

$$\zeta_{t+1} = \zeta_t \exp(g_A + g_\eta)$$

$$M_{t+1} = M_t + \eta_t e_{b,t}$$

$$p_{g,t} = p_{g,min} + \left(\frac{\Omega}{S_t}\right)^{\gamma_g},$$

$$S_{t+1} = S_t + e_{g,t}.$$

$$\tau_{i,UB} < \tau_i(s) < \tau_{i,LB}$$

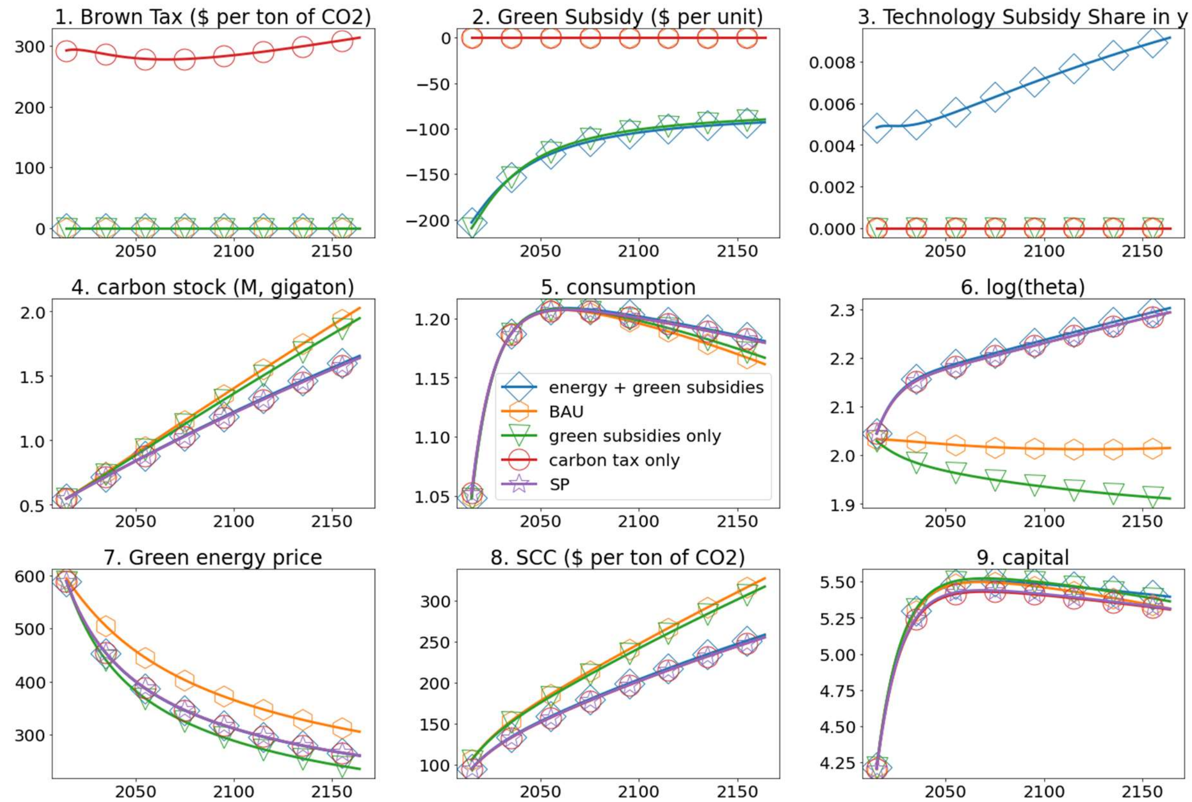
In contrast to the baseline model without learning-by-doing, brown taxes alone cannot correct the positive externality of green energy usage because it cannot address the positive externality of learning. So, in this section, I also explore the social planner (SP) policy scheme in which the government faces no implementability constraints to evaluate how close each type of policy scheme can bring us to the social optimum.

I solve the model after setting ϵ_e to be 1.85 and ψ to be 2.498, as in the baseline model. The results are summarized in Figure 4 and Table 5. Under the BAU scenario, the cumulative carbon emissions by the end of the century are 1392 gigatons. The unit price of green energy decreases to \$366. Similar to the no-learning case, θ is, in general, stable across time. In contrast, under the SP scenario, the cumulative emissions are 1205 gigatons in 2100. The cost of green energy then is \$300, and the relative energy efficiency θ increases by approximately 20%. The relative energy efficiency θ also increases by approximately 20%. In terms of equivalent consumption, the SP policy brings a 0.15% gain in welfare compared to the BAU policy.

The optimal standalone brown tax rate in 2015 is \$291.16 because brown taxes also need to be imposed to indirectly promote higher productivity in the green sector. It leads to cumulative emissions of 1204 trillion tons in 2100 and significantly increases θ and reduces energy usage, e. The welfare gain compared under standalone brown taxes to BAU is 0.14%.

Under the green-subsidies-only scenario, the optimal green subsidy rate in 2015 is around \$209. We should notice that such a policy leads to the cheapest green energy price because the low energy efficiency generated by green subsidies leads to higher energy usage than that according to the social optimum. The emission reduction brought by green subsidies alone is still very limited: in 2100, the cumulative emissions are 1352 trillion tons. Moreover, the energy efficiencies of the final goods production ARE decreasing over time. Green energy subsidies alone can only give the household a welfare gain of 0.05%

Under the green-and-technology-choice-subsidies scenario, the optimal rate is still around \$209, and the subsidies on energy efficiency cost around 0.5% of total output. The cumulative emissions in 2100 are then 1213 trillion tons. The welfare gain is 0.14%. We should notice that both brown taxes and two types of subsidies can implement energy efficiency levels that are close to those under the SP. In summary, both brown taxes and paired subsidies are shown to be good-performing second-best policies with directed technical change in the energy sector.



Notes: This figure demonstrates the paths of endogenous variables under four policy scenarios in Table 1 and the SP scenario in which different policy instruments are feasible to the government.

Figure 4: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy with Learning-by-Doing

Table 5: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy with Learning-by-Doing

Policy Scenarios	Cumulative Emissions in 2100 (Trillion Ton)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1392	0.0
SP	1205	0.15
Brown taxes	1204	0.14
Green subsidies	1352	0.05
Technology choice subsidies + green subsidies	1213	0.14

Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios.

V. Conclusion

I propose a model that features the long-run energy efficiency decision of a firm-household to address the effectiveness of subsidies as an alternative to carbon taxes to correct the negative externality of carbon emissions. Compared to the first-best carbon taxes, green energy subsidies reduce the price of energy and deter the adoption of energy-saving technologies. In the baseline model in which green and brown energy sources are substitute goods, the optimal carbon tax can reduce cumulative emissions by 13.6% by the end of the century. By contrast, optimal green subsidies alone can mitigate the cumulative emissions by only 1.0%. Nevertheless, green subsidies and technology subsidies can jointly approximate the impact of carbon taxes. With both types of subsidies in place, cumulative emissions can be reduced by 12.3%. The model is also evaluated under alternative values of elasticity of substitution between energy sources and technology adjustment cost. The role of learning-by-doing is also addressed. In summary, subsidies are shown to be an ideal substitute for carbon pricing if both substitution from green energy toward brown energy and substitution of energy toward other inputs are encouraged.

In the end of this paper, I offer a few directions for future research. Because I assume a representative household in the model, the distributional consequences of taxes and subsidies are not addressed, even though carbon taxes may be regressive. The fiscal implications of taxes and subsidies are also overlooked in this paper. From the static model, we can see that, even though subsidies can replicate the solution to the social planner's problem, the information demands on the government to implement these subsidies are more substantial compared to brown taxes and pose practical challenges. To implement the optimal brown taxes, the government only needs to know the Social Cost of Carbon, which is defined as the marginal cost of carbon emissions. In contrast, the implementation of optimal subsidies requires almost perfect knowledge of the production cost of brown and green energy and the production functions of both sectors. This result is not surprising. In essence, brown taxes are a type of price regulation. Subsidies instead can be seen as a type of quantity regulation as the government aims to implement an optimal quantity of carbon emissions by encouraging substitutes for fossil fuels. As

Hassler et al. (2016) argue, the quantity regulation may require more information than the price regulation to address carbon emissions. In this paper, price-induced energy efficiency changes are interpreted as adjustments of the technology choice along a given technology menu. In the real world, such changes can be caused by different mechanisms, including (1) directed technical change, as highlighted by Hassler et al. (2021), (2) the production process chosen by the firm, as discussed by Hawkins-Pierot and Wagner (2023), and (3) the relative weight of energy-intensive industry in the whole economy, as illustrated in Hart (2008) and Bachmann et al. (2022). That is, higher energy prices can encourage scientists to put more effort into developing energy-efficient technology, firms to adopt existing energy-efficient technology, shrinking the energy-intensive sector, and growing the energy-saving industry. Understanding and distinguishing the influences of these three factors is crucial for crafting effective and practical policy measures.

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. "The environment and directed technical change." American economic review 102, no. 1 (2012): 131-166.

Airaudo, Florencia S., Evi Pappa, and Hernán D. Seoane. The green metamorphosis of a small open economy. Centre for Economic Policy Research, 2023.

Bachmann, Ruediger, David Baqaee, Christian Bayer, Moritz Kuhn, Andreas Löschel, Benjamin Moll, Andreas Peichl, Karen Pittel, and Moritz Schularick. What if? The economic effects for Germany of a stop of energy imports from Russia. No. 028. ECONtribute Policy Brief, 2022.

Barrage, Lint. "Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy." The Review of Economic Studies 87, no. 1 (2020): 1-39.

Baumol, William J., and Wallace E. Oates. The theory of environmental policy. Cambridge university press, 1988.

Beiser-McGrath, Liam F., and Thomas Bernauer. "Could revenue recycling make effective carbon taxation politically feasible?." Science advances 5, no. 9 (2019): eaax3323.

Belfiori, Elisa, and Armon Rezai. "Implicit carbon prices: Making do with the taxes we have." Journal of Environmental Economics and Management (2024): 102950.

Caselli, Francesco, and Wilbur John Coleman. "The world technology frontier." American Economic Review 96, no. 3 (2006): 499-522.

Casey, Gregory, Woongchan Jeon, and Christian Traeger. "The Macroeconomics of Clean Energy Subsidies." (2023).

Casey, Gregory. "Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation." Review of Economic Studies, 2023. https://doi.org/10.1093/restud/rdad001.

Cruz, José-Luis, and Esteban Rossi-Hansberg. The economic geography of global warming. No. w28466. National Bureau of Economic Research, 2021.

Fischer, Carolyn, and Richard G. Newell. "Environmental and technology policies for climate mitigation." Journal of environmental economics and management 55, no. 2 (2008): 142-162.

Fried, Stephie. "Climate policy and innovation: A quantitative macroeconomic analysis." American Economic Journal: Macroeconomics 10, no. 1 (2018): 90-118.

Gerlagh, Reyer. "A climate-change policy induced shift from innovations in carbon-energy production to carbon-energy savings." Energy Economics 30, no. 2 (2008): 425-448.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. “Optimal taxes on fossil fuel in general equilibrium.” Econometrica 82, no. 1 (2014): 41-88.

Growiec, Jakub. "A microfoundation for normalized CES production functions with factor-augmenting technical change." Journal of Economic Dynamics and Control 37, no. 11 (2013): 2336-2350.

Growiec, Jakub. "A new class of production functions and an argument against purely labor-augmenting technical change." International Journal of Economic Theory 4, no. 4 (2008): 483-502.

Gugler, Klaus, Adhurim Haxhimusa, and Mario Liebensteiner. “Effectiveness of climate policies: Carbon pricing vs. subsidizing renewables.” Journal of Environmental Economics and Management 106 (2021): 102405.

Hassler, John, Per Krusell, and Conny Olovsson. "Directed technical change as a response to natural resource scarcity." Journal of Political Economy 129, no. 11 (2021): 3039-3072.

Hassler, John, Per Krusell, and Conny Olovsson. “Presidential address 2020 suboptimal climate policy.” Journal of the European Economic Association 19, no. 6 (2021): 2895-2928.

Hassler, John, Per Krusell, and Jonas Nycander. "Climate policy." Economic Policy 31, no. 87 (2016): 503-558.

Hassler, John, Per Krusell, Conny Olovsson, and Michael Reiter. “On the effectiveness of climate policies.” IIES WP 53 (2020): 54.

Hawkins-Pierot, Jonathan T., Consumer Financial Protection Bureau, and Katherine RH Wagner. “Technology Lock-In and Costs of Delayed Climate Policy.” (2023).

Hinkelmann, Stefan. (Be-)Coming Clean: A Model of the U.S. Energy Transition.

Jones, Charles I. "The shape of production functions and the direction of technical change." The Quarterly Journal of Economics 120, no. 2 (2005): 517-549.

Kalkuhl, Matthias, Ottmar Edenhofer, and Kai Lessmann. “Renewable energy subsidies: Second-best policy or fatal aberration for mitigation?.” Resource and Energy Economics 35, no. 3 (2013): 217-234.

Känzig, Diego R., and Maximilian Konradt. Climate Policy and the Economy: Evidence from Europe's Carbon Pricing Initiatives. No. w31260. National Bureau of Economic Research, 2023.

Knittel, Christopher R. "Automobiles on steroids: Product Attribute Trade-Offs and Technological Progress in the Automobile Sector." American Economic Review 101, no. 7 (2011): 3368-3399.

Labandeira, Xavier, José M. Labeaga, and Xiral López-Otero. "A meta-analysis on the price elasticity of energy demand." Energy policy 102 (2017): 549-568.

Lemoine, Derek. "Innovation-led transitions in energy supply." American Economic Journal: Macroeconomics 16, no. 1 (2024): 29-65.

León-Ledesma, Miguel A., and Mathan Satchi. “Appropriate technology and balanced growth.” The Review of Economic Studies 86, no. 2 (2019): 807-835.

Newell, Richard G., William A. Pizer, and Daniel Raimi. “US federal government subsidies for clean energy: Design choices and implications.” Energy Economics 80 (2019): 831-841.

Nordhaus, William D. "Revisiting the social cost of carbon." Proceedings of the National Academy of Sciences 114, no. 7 (2017): 1518-1523.

Nordhaus, William D. “Economic growth and climate: the carbon dioxide problem.” The American Economic Review 67, no. 1 (1977): 341-346.

Nordhaus, William D. “Optimal greenhouse-gas reductions and tax policy in the ‘DICE’ model.” The American Economic Review 83, no. 2 (1993): 313-317.

Nordhaus, William D., and Zili Yang. “A regional dynamic general-equilibrium model of alternative climate-change strategies.” The American Economic Review (1996): 741-765.

Pindyck, Robert S. "Interfuel substitution and the industrial demand for energy: an international comparison." The Review of Economics and Statistics (1979): 169-179.

Popp, David. “R&D subsidies and climate policy: is there a “free lunch”?.” Climatic Change 77, no. 3-4 (2006): 311-341.

Rudik, Ivan. “Optimal climate policy when damages are unknown.” American Economic Journal: Economic Policy 12, no. 2 (2020): 340-373.

Scheidegger, Simon, and Ilias Bilionis. "Machine learning for high-dimensional dynamic stochastic economies." Journal of Computational Science 33 (2019): 68-82.

Sinn, Hans-Werner. “The green paradox.” In CESifo forum, vol. 10, no. 3, pp. 10-13. München: ifo Institut für Wirtschaftsforschung an der Universität München, 2009.

Van Der Ploeg, Frederick, and Cees Withagen. “Growth, renewables, and the optimal carbon tax.” International Economic Review 55, no. 1 (2014): 283-311.

Gao, Jiti, Bin Peng, and Russell Smyth. "On income and price elasticities for energy demand: A panel data study." Energy Economics 96 (2021): 105168.

Online Appendix A: Directed Technical Change and Technology Choice on the Technology Menu

In the appendix, I show that the baseline model featuring the technology choice of the firm-household also can be interpreted as the directed technical change between capita-labor-augmenting technology $B_{K,t}$ and the energy-augmenting technology $B_{E,t}$. That is, the production technology of the firm-household is

$$Y_t = \left(\left(B_{K,t} K_t^{\sigma} L^{1-\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(B_{E,t} E_t\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}.$$

Here, we define

$$\theta_t \equiv \frac{B_{E,t}}{B_{K,t}}$$

and

$$X_t \equiv B_{K,t} \theta_t^{1-\nu}.$$

Then,

$$B_{K,t} = X_t \theta_t^{\nu - 1}$$

and

$$B_{E,t} = X_t \theta_t^{\nu}$$

The production function then becomes

$$Y_t = X_t \left(\left(\theta_t^{\nu-1} K_t^{\sigma} L^{1-\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} E_t\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}.$$

We assume that the firm-household gets access to a unit mass of R&D resources that can be either allocated to the improvement of $B_{E,t}$ (n_t) or that of $B_{K,t}$ ($1 - n_t$) with the efficiency g:

$$B_{E,t} = B_{E,t-1}(1 + n_t g)$$

$$B_{K,t} = B_{K,t-1}(1 + (1 - n_t)g)$$

Then,

$$\frac{\theta_t}{\theta_{t-1}} = \frac{B_{E,t}}{B_{K,t}} \Bigg/ \frac{B_{E,t-1}}{B_{K,t-1}} = \frac{1 + n_t g}{1 + (1 - n_t)g}.$$

That is, the firm-household effectively chooses θ_t on a log-linear technology menu. In contrast to the baseline model, the model does not feature a technology adjustment cost. Instead, the friction that leads to the difference between the short-run and long-run price elasticity of energy is generated by the finite R&D resources as $\frac{1}{1+g} \leq \frac{\theta_t}{\theta_{t-1}} \leq 1 + g$. In the end, we can detrend the model by defining $y_t = \frac{Y_t}{X_t^m L}$, $k_t = \frac{K_t}{X_t^m L}$, $e_t = \frac{E_t}{X_t^m L}$, and $\tilde{\theta}_t = \frac{\theta_t}{X_t^n}$ with $m = \frac{1}{1-\sigma}\frac{1}{\nu}$ and $n = -\frac{1}{\nu}$. The normalized production function is then

$$y_t = \left(\left(\tilde{\theta}_t^{\nu-1} k_t^{\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\tilde{\theta}_t^{\nu} e_t\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}.$$

Online Appendix B: The Normalized Model

In this appendix, I lay out the model after the normalization. I define $\tilde{k}_t = \frac{k_t}{A_t l}$, $\tilde{c}_t = \frac{c_t}{A_t l}$, $\tilde{y}_t = \frac{y_t}{A_t l}$, $\tilde{e}_t = \frac{e_t}{A_t l}$, $\tilde{e}_{b,t} = \frac{e_{b,t}}{A_t l}$, $\tilde{e}_{g,t} = \frac{e_{g,t}}{A_t l}$, $\tilde{\tau}_{\theta,t} = \frac{\tau_{\theta,t}}{A_t l}$, $\tilde{T}_{\theta,t} = \frac{T_{\theta,t}}{A_t l}$, $\tilde{T}_{e,t} = \frac{T_{e,t}}{A_t l}$, $\zeta_t = A_t * l * \eta_t$, and $\tilde{\mathcal{V}}_t = \frac{\mathcal{V}_t}{A_t^{1-\alpha}}$. Here, ζ_t captures the carbon intensity of brown energy per unit of effective labor $\tilde{e}_{b,t}$.

The energy firm's optimization problem then becomes

$$\max_{e_{b,t}, e_{g,t}} p_{e,t}\tilde{e}_t - p_{b,t}\tilde{e}_{b,t} - p_{g,t}\tilde{e}_{g,t} - \tau_{b,t}\tilde{e}_{b,t} - \tau_{g,t}\tilde{e}_{g,t} + \tilde{T}_{e,t}$$

s.t.

$$\tilde{e}_t = \left(\lambda(\tilde{e}_{b,t})^{\frac{\epsilon_e - 1}{\epsilon_e}} + (1-\lambda)(\tilde{e}_{g,t})^{\frac{\epsilon_e - 1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e - 1}} \tag{7*}$$

The firm-household's problem is then

$$\tilde{\mathcal{V}}(\theta_{t-1}, k_t, M_t, A_t, \eta_t)$$

$$= \max_{\theta_t, \tilde{k}_{t+1}, M_{t+1}, c_t} \frac{\tilde{c}_t^{1-\alpha}}{1-\alpha} + e^{(1-\alpha)g}\tilde{\mathcal{V}}(\theta_t, k_{t+1}, M_{t+1}, A_{t+1}, \eta_{t+1})$$

s.t.

$$M_{t+1} = M_t + \eta_t \tilde{e}_{b,t}$$

$$y_t = \left(\left(\theta_t^{\nu-1}\tilde{k}_t^{\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu}\tilde{e}_t\right)^{\frac{\epsilon_y - 1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \tag{8*}$$

$$\begin{aligned}
&\tilde{c}_t + e^g\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t \\
&\quad = D(M_t)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\left(\left(\theta_t^{\nu-1} * \tilde{k}_t^{\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu}\tilde{e}_t\right)^{\frac{\epsilon_y - 1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - p_{e,t}\tilde{e}_t + \tilde{\tau}_{\theta,t}\theta_t \\
&\quad + \tilde{T}_{y,t}.
\end{aligned} \tag{9*}$$

$$p_{e,t} = \left(\lambda^{\epsilon_e}(p_{b,t} + \tau_{b,t})^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}(p_{g,t} + \tau_{g,t})^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{10*}$$

Their FOCs then becomes

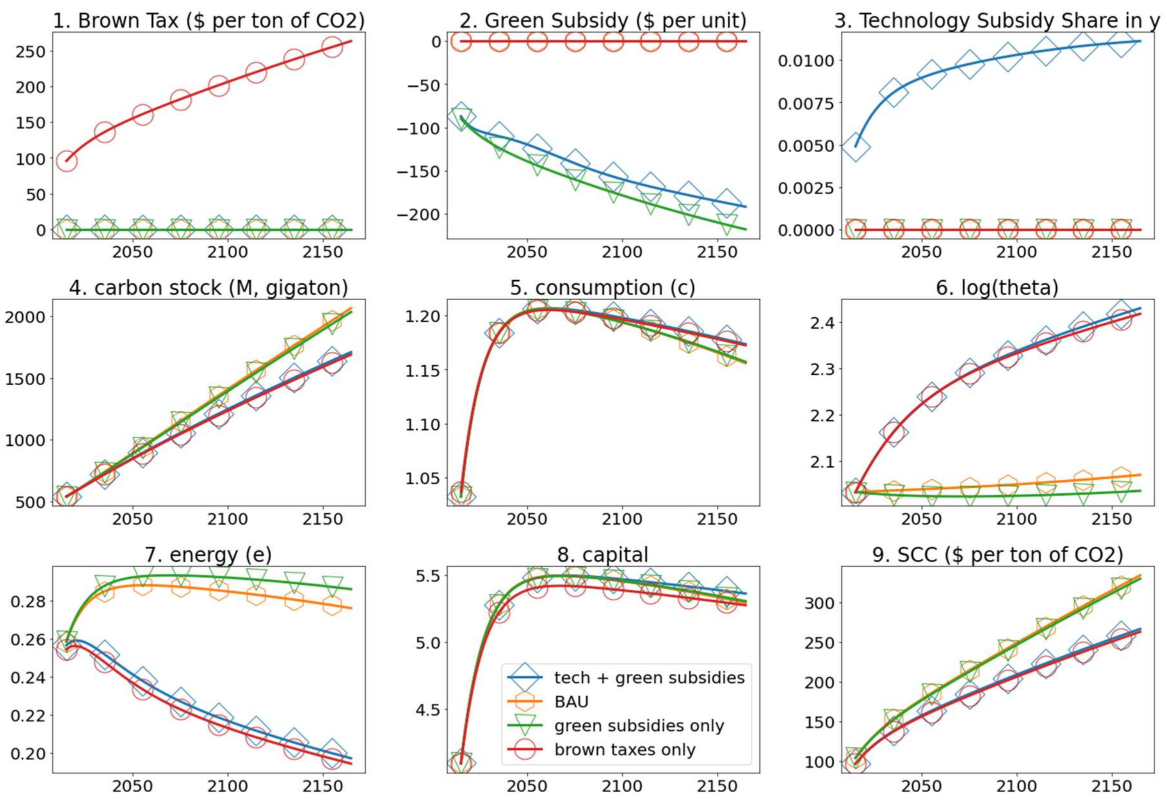
$$\frac{e_{b,t}}{e_{g,t}} = \left(\frac{\lambda}{1-\lambda}\frac{p_{b,t}+\tau_{b,t}}{p_{g,t}+\tau_{g,t}}\right)^{-\epsilon_e} \tag{11*}$$

$$(1-d(\chi M_t)^2) * \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) y_t^{\frac{1}{\epsilon_y}} * (\theta_t^{\nu} e_t)^{-\frac{1}{\epsilon_y}} \theta_t^{\nu} = p_{e,t} \tag{12*}$$

$$\begin{aligned}
&\tilde{c}_t^{-\alpha}\ \Bigg\{(1-d(\chi M_t)^2)\Psi'\left(\frac{\theta_t}{\theta_{t-1}}\right)\frac{1}{\theta_{t-1}}\tilde{y}_t \\
&\qquad + (1-d(\chi M_t)^2)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\tilde{y}^{\frac{1}{\epsilon_y}}\begin{pmatrix}\left(\theta_t^{\nu-1}\tilde{k}_{t+1}^{\sigma}\right)^{-\frac{1}{\epsilon_y}}(\nu-1)\theta_t^{\nu-2}\tilde{k}_{t+1}^{\sigma} \\ +(\theta_t^{\nu}\tilde{e}_t)^{-\frac{1}{\epsilon_y}}\nu\theta_t^{\nu-1}\tilde{e}_t + \tilde{\tau}_{\theta,t}\end{pmatrix}\Bigg\} \\
&\qquad + e^{(1-\alpha)g}\tilde{c}_{t+1}^{-\alpha}\left((1-d(\chi M_{t+1})^2)\Psi'\left(\frac{\theta_{t+1}}{\theta_t}\right)\left(-\frac{\theta_{t+1}}{\theta_t^2}\right)\tilde{y}_{t+1}\right) = 0
\end{aligned} \tag{13*}$$

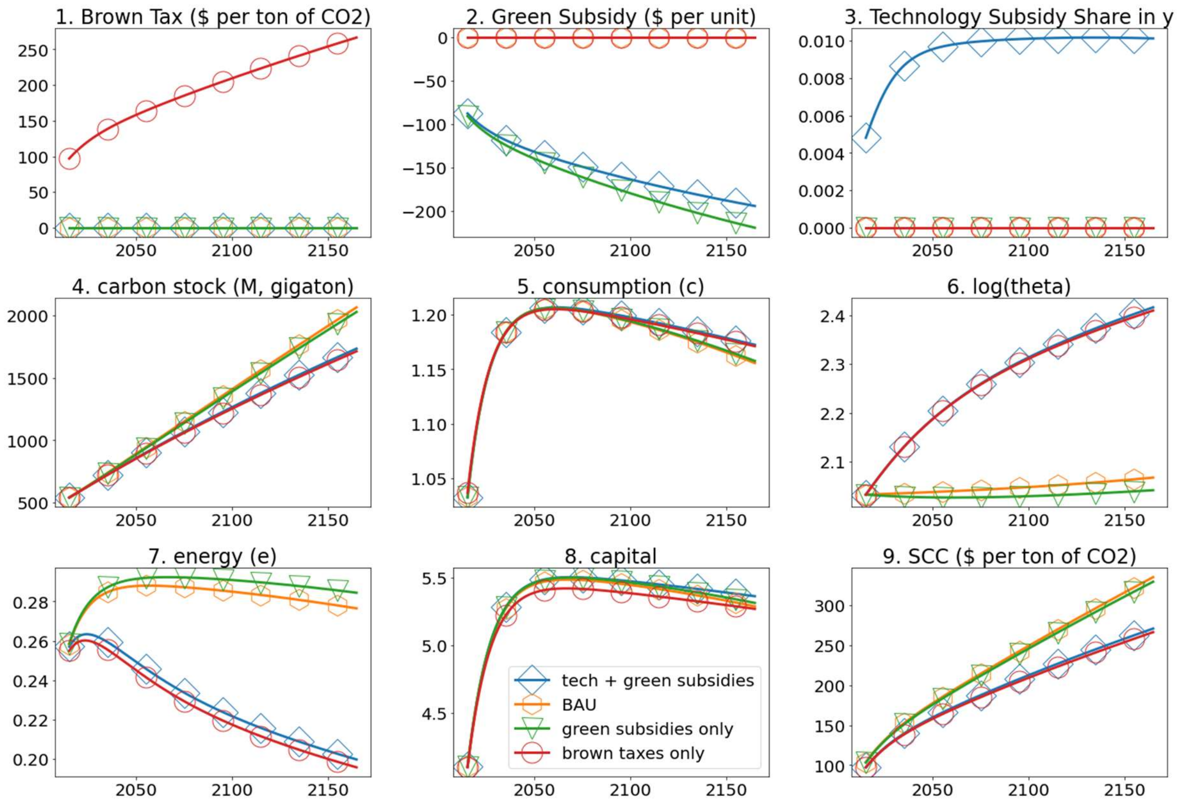
$$\begin{aligned}
&-e^{g}\tilde{c}_t^{-\alpha} + e^{(1-\alpha)g}\tilde{c}_{t+1}^{-\alpha} \\
&\qquad * \left(\left((1-d(\chi M_{t+1})^2)\Psi\left(\frac{\theta_{t+1}}{\theta_t}\right)\tilde{y}_{t+1}^{\frac{1}{\epsilon_y}}\left(\theta_{t+1}^{\nu-1} * \tilde{k}_{t+1}^{\sigma}\right)^{-\frac{1}{\epsilon_y}}\theta_{t+1}^{\nu-1}\tilde{k}_{t+1}^{\sigma-1}\right.\right. \\
&\qquad \left.\left. + (1-\delta)\right)\right) = 0
\end{aligned} \tag{14*}$$

Online Appendix C: Figures



Notes: This figure deconstruesthe paths of endogenous variables under four different policy scenarios in Table 1 in which different policy instruments are feasible to the government with a middle level of technology adjustment cost.

Figure C.1: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when the Adjustment Cost Parameter $\gamma = 10.0$



Notes: This figure demonstrates the paths of endogenous variables under four different policy scenarios in Table 1 in which different policy instruments are feasible to the government with a high level of technology adjustment cost.

Figure C.2: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when the Adjustment Cost Parameter $\gamma = 20.0$