Technology Choice, Energy Efficiency, and Second-Best Climate Policy

Danchen Zhao*

Sep 01, 2024

Abstract

I study the effectiveness of subsidies as an alternative to carbon taxes in reducing carbon emissions in a quantitative climate-economy model. An energy firm uses brown and green energy inputs to produce energy. A representative firm-household then uses energy, capital, and labor to produce final goods. The short-run elasticity between energy and other inputs is low. However, higher energy prices encourage higher energy efficiency, leading to a higher elasticity in the long run. The key weakness of green energy subsidies, as an alternative to carbon taxes, is that they cannot promote higher energy efficiency. Thus, in the baseline model, the optimal green subsidies result in a modest 1.0% decrease in emissions by the end of the century relative to cumulative emissions in the business-as-usual scenario. However, if the government subsidizes green energy usage and energy-saving technical change simultaneously, the optimal subsidies are nearly as effective in reducing emissions as the first-best taxes on carbon emissions. Under this approach, cumulative carbon emissions are reduced by 12.2% by the end of the century.

Keywords: Integrated Assessment Models, Climate Change, Subsidy, Second-Best Policies

*Zhao: Department of Economics, University of Notre Dame, 1399 N Notre Dame Ave, Notre Dame, IN 46617 (dzhao@nd.edu). I would like to express my sincere gratitude to my advisor, Nelson Mark, and committee members Toan Phan, Robert Johnson, and Zach Stangebye. I also thank Jeff Campbell, Eric Sims, César Sosa-Padilla, Matthias Hoelzlein, Benjamin Pugsley, Wendong Zhang (discussant), Christoph Boehringer (discussant), and participants at ND-Econ Macro Workshop, 2024 CES North America Annual Conference, 2024 CEF, and EAERE 2024 for their helpful comments. All errors are my own.

1 Introduction

Economists generally regard carbon taxes as the first-best solution for reducing carbon emissions and addressing climate change. Since the social cost of one ton of carbon emissions is higher than the private cost, a Pigouvian tax should be imposed to correct its negative externality. However, like most taxes, carbon taxes and similar policies designed to increase energy costs are massively unpopular among the public.[1] A survey by the Energy Policy Institute and the Associated Press-NORC Center revealed that 42% of respondents would not accept even a $1 monthly increase in their electricity bill to fight climate change. In contrast, government subsidies for renewable energy sources, such as solar and wind power, are more popular and frequently implemented. A notable example of such a subsidy is the Inflation Reduction Act (IRA), which allocates billions of dollars in renewable energy tax credits. Given the potential real-world constraints on the implementation of climate policy, it is important to investigate the effectiveness of green energy subsidies as a viable alternative to the carbon tax.

To study this issue, I build a macro-climate model with an upstream energy sector and a downstream final goods sector. A perfectly competitive firm in the energy sector uses brown and green energy inputs, which are imperfect substitutes, to produce energy. Brown energy has a lower private cost than green energy, but it has a higher social cost because of its carbon emissions and the resulting future climate damage. The energy is then sold to a representative firm-household, which combines energy with capital and labor to produce final goods. The final goods can be used for either consumption or investment. The energy intensity of the final goods production process is endogenous and influenced by the energy price because the firm-household can make technology choices on a technology frontier and, therefore, change the energy efficiency of its production process over time. If the firm-household adopts a new technology with energy efficiency different from its existing production process,

[1] Reasons for opposing carbon taxes include (1) concerns about the tax burden, (2) concerns about foreign free-riding, and (3) the distributional consequences of carbon taxes (e.g., Beiser-McGrath and Bernauer 2019, Känzig 2023).

it will incur a technology adjustment cost. Under such a setting, the elasticity of substitution between energy and other inputs is low in the short run and higher in the long run. Depending on the policy scenario, a benevolent government has access to alternative policy instruments, including brown taxes, green subsidies, and energy efficiency subsidies, which it can implement to try to correct the negative externality of brown energy usage. I solve the government's constrained-efficient optimization problem and evaluate the effectiveness of different policy instruments by comparing the outcomes under different policy scenarios. For the accuracy of the solution and to handle the high dimensionality of the models, this constrained-efficient optimization problem is solved globally using value function iteration augmented with Gaussian processes regression (Scheidegger and Bilionis, 2019).

The economy has two mechanisms for reducing carbon emissions: (1) reducing the share of brown energy inputs in energy production by substituting them with green inputs and (2) reducing overall energy usage in final goods production, mainly by improving energy efficiency. That is, carbon dioxide emissions of an economy can be decomposed as

$$\mathrm{CO}_2 = GDP \frac{Energy}{GDP} \frac{\mathrm{CO}_2}{Energy}.$$

The aggregate emissions can be reduced by either having a lower energy-to-GDP ratio or a lower emission-to-energy ratio, holding GDP constant. Taxes on brown energy are effective for reducing carbon emissions because they can utilize both mechanisms. By contrast, if the government only subsidizes green energy in the energy sector, the carbon emission reductions under the optimal policy path would be very limited because green subsidies tend to lower energy costs. Cheaper energy cannot incentivize firms in the downstream sector to adopt more energy-efficient technology, and the energy-efficiency carbon mitigation mechanism is not utilized. According to the baseline model, brown taxes can reduce cumulative emissions by 13.6% by the end of this century compared to cumulative emissions under business-as-usual (BAU), whereas the optimal green subsidies achieve a 1.0% reduction in cumulative emissions. The optimal taxes on brown energy increase household welfare by 0.16% in terms of equivalent consumption. In contrast, optimal green energy subsidies increase household welfare by only 0.002%.

To illustrate, consider the various decisions within our economy aimed at saving energy by enhancing energy efficiency, often at a cost. These decisions include purchasing a more expensive, energy-saving air conditioner, choosing a car with higher fuel economy but lower horsepower, utilizing trains instead of trucks for transporting goods, and allocating more research and development (R&D) resources towards improving energy efficiency rather than other research projects. All these choices can be regarded as choices on the technology frontier of our economy. These efforts to improve the energy efficiency of the economy become economically viable for agents only when energy costs are sufficiently high. Green energy subsidies fail to provide incentives for these efforts.[2] As a result, if only green subsidies are feasible for government implementation, they are an inefficient tool and should not be implemented on a scale that can deliver substantial carbon emission reduction.

The government thus needs a second lever as a complement to green subsidies to increase the economy's energy efficiency. Technology subsidies that reward energy-efficient technology choices, that is, subsidies that reward higher energy efficiency, can be effective policy instruments for achieving this goal. Since energy saving is largely driven by changes in the firm-household's technology choice over time, the government can reduce the firm's energy usage by subsidizing energy-saving technology.[3] When energy and other inputs are perfect complements in the short run, the joint implementation of energy efficiency and green subsidies delivers the social optimum. In the baseline model, even without imposing the assumption of perfect complementarity, these two types of subsidies can still achieve a

[2]Under the most extreme scenario in which renewable energy is provided by the government for free, even with great brown energy usage reductions, everyone will use energy recklessly on activities like Bitcoin mining.

[3]In fact, IRA does provide tax credits for energy efficiency. For example, homeowners can claim up to $1,200 for energy property costs and various types of residential energy efficiency improvements. Similarly, the European Union has the so-called "Energy Efficiency First (EE1st)" guiding principle in the areas of sustainability, climate neutrality, and green growth.

12.3% reduction in cumulative emissions, approaching the level of carbon abatement achieved through the optimal imposition of taxes on brown energy. With respect to welfare, the joint implementation of energy efficiency and green subsidies increases welfare by 0.14% in terms of equivalent consumption, which parallels the welfare gain brought by taxes on brown energy.

In summary, relying solely on green subsidies is costly and ineffective in reducing carbon emissions and brings very limited welfare gains relative to the BAU. However, combining green and energy efficiency subsidies can lead to an outcome close to the social optimum. This key finding remains robust across various scenarios, including different degrees of elasticity of substitution in the energy sector and varying technology adjustment costs for energy efficiency.

In the baseline model, the production cost of green energy is constant and exogenously given. As an extension to the baseline model, I introduce learning-by-doing to green energy production: more green energy usage leads to cheaper green energy. This setting allows green subsidies to stimulate directed technical change toward green energy (e.g., Acemoglu et al., 2012; Kalkuhl et al., 2012; Fried, 2018; Hassler et al., 2021; Lemoine, 2024). Still, the key insights from the baseline model remain valid: even if green subsidies can lead to cheaper green energy, they still cannot address the issue of energy efficiency. So, subsidies for energy efficiency are still warranted when brown taxes are not feasible.

This paper is built upon and contributes to the development of integrated assessment models (Nordhaus 1977, Nordhaus 1993, Golosov et al. 2014, Barrage 2020, Hassler et al. 2021, Barrage and Nordhaus 2023). Within this literature, this paper is closest to works on subsidies as second-best climate policies (Popp, 2006; Gerlagh, 2008; Kalkuhl et al., 2013; van der Ploeg and Withagen, 2014; Cruz and Rossi-Hansberg, 2021; Nordhaus, 2017; Lemoine, 2017; Airaudo et al., 2023). Usually, subsidies to promote either the usage or R&D on green energy are shown to have only modest impacts on carbon emissions by the literature. Highly relevant to this paper, Hassler et al. (2020) demonstrate that a lower green energy price can increase carbon emissions when brown and green energy are gross complements. Casey et al. (2023) argue that green subsidies' impact on brown energy usage is determined by the elasticity of substitution between clean and dirty energy as well as by the price elasticity of demand for energy services. They also analytically derive the conditions under which green subsidies reduce carbon emissions and find that the welfare gain brought by green subsidies alone is modest at its best. Bistline et al. (2023) and Cruz and Rossi-Hansberg (2024) also find that the welfare gain from green subsidies is very limited. This paper contributes to this literature by explicitly modeling firms' energy efficiency choices and distinguishing the short- and long-run elasticities between energy and other inputs. More importantly, I further show that subsidies targeting long-run energy efficiency can address the energy efficiency problem caused by green subsidies, thereby making subsidy-based climate policy schemes almost as effective as first-best carbon taxes.

This paper also relates to studies that examine the relationship between technical change and the elasticities between energy and other inputs. The low short-run and high long-run elasticities of energy use are well documented (Akteson and Kehoe, 1999). Akteson and Kehoe (1999) propose a model featuring capitals with different fixed energy-capital proportions. Due to the existence of capital adjustment costs, the elasticity of energy usage is low in the short run but high in the long run. Hassler et al. (2021) emphasize the near-zero short-run elasticity between energy and capital/labor inputs. Yet, they conclude that this elasticity is higher in the long run because a higher energy price can induce energy-saving technical change. Casey (2022) develops a model with the directed technical change in economic growth and energy efficiency and highlights the importance of final-use energy efficiency for climate policies. Airaudo et al. (2023) propose a model that also features an energy sector and directed technical changes in energy efficiency to evaluate the macro impacts of green transitions in a small open economy induced by pre-determined brown taxes, green subsidies, or green infrastructure investments. This paper introduces the time-varying elasticity of substitution based on the "appropriate technology" literature that describes the technology choice of a firm to select the efficiencies of different inputs on a technology frontier. Prominent examples of this literature are Jones (2005), Caselli and Coleman (2006), Growiec (2008, 2013), and León-Ledesma and Satchi (2019). Hinkelmann (2023) builds a quantitative model featuring the time-varying elasticity of substitution between fossil fuels and electricity in the energy sector following León-Ledesma and Satchi (2019) and finds that sizable carbon taxes are required to achieve net zero emissions by 2050. The connection between directed technical change (e.g., Acemoglu et al. 2012, Fried 2018, Hassler et al., 2021, Casey 2023, Lemoine 2024) and technical choice is also discussed.

This paper also connects to microeconomics studies on second-best environmental policies. Baumol and Oates (1988) demonstrate that subsidies for clean alternatives to polluting inputs may increase pollution if the final production is scaled up. Sinn (2008) discusses the possibility of the so-called "green paradox": subsidies on renewable energy encourage the owner of finite oil stock to extract more oil, which increases carbon emissions. Fischer and Newell (2008) and Gugler et al. (2021) also conclude that green subsidies are less effective than brown taxes. Newell et al. (2019) further highlight that clean energy subsidies can lead to inefficiently high electricity production. Belfiori and Rezai (2024) show that any sequence of explicit carbon prices can be achieved implicitly through a combination of conventional taxes.

The remainder of this paper is structured as follows. Section 2 lays out a static model that highlights the key ideas of this paper. Section 3 presents the full dynamic model and quantitatively evaluates the effectiveness of different policy tools, including taxes and subsidies. Section 4 introduces learning-by-doing in the green sector as an extension of the baseline model. Section 5 concludes.

2 The Static Model

This section lays out a static, decentralized model that highlights the key mechanisms in the full model. I compare the decentralized economy to the social planner (SP) problem and show that the solution to the SP problem can be achieved in the decentralized economy by either (1) implementing brown taxes or (2) implementing green subsidies along with subsidies on energy efficiency. However, green energy subsidies alone cannot implement the social optimum.

2.1 Decentralized Economy

A perfectly competitive energy firm has access to a constant elasticity of substitution (CES) technology that uses brown energy inputs e_b and green energy inputs e_g to produce the final energy e:

$$e = \left(\lambda \left(e_b\right)^{\frac{\epsilon_e - 1}{\epsilon_e}} + (1 - \lambda)\left(e_g\right)^{\frac{\epsilon_e - 1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e - 1}} \tag{1}$$

where ϵ_e is the elasticity of substitution between green and brown energy. We may think of brown energy inputs e_b as abundant "coal." Their usage generates carbon emissions. Green energy inputs e_g, such as solar or wind energy, are expensive but emission-free (clean). Both brown and green energy inputs are produced with constant marginal costs $cost_b$ and $cost_g$, respectively, in terms of final output. The produced energy e is then sold to the firm-household at price p_e. This energy firm is also potentially subject to a tax or a subsidy imposed on both types of energy sources, τ_b and τ_g, which are rebated lump-sum (T_e). The profit maximization problem of this energy firm is as follows:

$$\max_{e_b, e_g} p_e e - cost_b e_b - cost_g e_g - \tau_b e_b - \tau_g e_g + T_e$$

$$\text{s.t.}$$

$$(1)$$

The first-order conditions (FOCs) and the zero-profit condition yield the equilibrium conditions:

$$\frac{e_b}{e_g} = \frac{\lambda}{1-\lambda}\left(\frac{cost_b+\tau_b}{cost_g+\tau_g}\right)^{-\epsilon_e} \tag{2}$$

$$p_e = \left(\lambda^{\epsilon_e}\left(cost_b+\tau_b\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_g+\tau_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{3}$$

A firm-household purchases energy e from the energy firm at price p_e. It uses endowed non-energy inputs h and energy input e to produce a final good y. The efficiencies of energy and nonenergy inputs are denoted as B_e and B_h, respectively. The production function follows the Leontief form:

$$y = \min\left\{B_h h, B_e e\right\}$$

For a given relative energy efficiency ratio B_e/B_h, non-energy inputs h and energy e are perfect complements. Following Jones (2005) and León-Ledesma and Satchi (2019), I assume that the firm-household also can choose its non-energy-input-efficiency B_h and energy efficiency B_e on a log-linear technology menu:

$$\nu \log B_h + (1-\nu)\log B_e = 0 \ (0 < \nu < 1)$$

Here, we define the technology choice θ as

$$\theta \equiv \frac{B_e}{B_h}$$

Then, we have

$$B_h = B_e^{\frac{\nu-1}{\nu}}$$
$$\frac{B_h}{B_e} = B_e^{-\frac{1}{\nu}}$$
$$\theta^{-1} = B_e^{-\frac{1}{\nu}}$$
$$\theta^{\nu} = B_e$$
$$\theta^{\nu-1} = B_h$$

The production function can then be re-written as

$$y = \min\left\{\theta^{\nu-1}h, \theta^{\nu}e\right\}$$

Higher relative energy efficiency θ implies a higher B_e and a lower B_h, which in turn implies that the agent is using less energy to produce one unit of output. Conversely, a lower θ leads to a less energy-efficient production process. Here, I follow Jones (2005) and León-Ledesma and Satchi (2019) and interpret the trade-off between efficiencies as a choice on a "technology frontier." In Appendix A, in a more general setting, I demonstrate that change in θ can also be interpreted as the long-run outcome of directed technical change (Acemoglu et al., 2012; Hassler et al., 2022).

The firm's choice of θ highlights the difference between the ex-ante elasticity of substitution between energy and other inputs with a given θ and the ex-post substitution with a chosen θ. Without adjustments in θ, energy and other inputs are perfect complements. Yet, the ex-post elasticity of substitution after the agent chooses the optimal θ is higher, and the production function is isomorphic to the Cobb-Douglas production function as long as the technology frontier is log-linear. [4] (León-Ledesma and Satchi, 2019) To see that, if we assume an interior solution in which all other inputs are fully utilized following Hassler et al. (2021), then we have

[4] Similarly, directed technical change implies a long-run unitary elasticity between energy and other inputs if and only if energy efficiency and capital efficiency have a log-linear relationship, as I demonstrate in Appendix A. For further discussion, I refer interested readers to León-Ledesma and Satchi (2019) and Hassler et al. (2021).

$$\theta e = h \tag{4}$$

$$y = \theta^{\nu-1} h \tag{5}$$

Then, the production function yields an alternative Cobb-Douglas form:

$$y = \left(\frac{h}{e}\right)^{\nu-1} h = h^{\nu} e^{1-\nu}$$

Also,

$$\frac{e}{y} = \frac{e}{h^{\nu} e^{1-\nu}} = \left(\frac{e}{h}\right)^{\nu} = \theta^{-\nu}$$

and

$$\frac{h}{y} = \frac{h}{h^{\nu} e^{1-\nu}} = \left(\frac{e}{h}\right)^{1-\nu} = \theta^{\nu-1}$$

That is, the energy intensity and non-energy-inputs intenisty of the firm is determined by θ.

Intuitively, energy required to produce one unit of goods cannot usually be substituted with other inputs without a corresponding technology adjustment. For example, the electricity (energy) needed to run the air conditioner to keep a house cool cannot be just substituted with more air conditioners (other inputs). Suppose one wants to reduce energy usage. In that case, one needs to buy a more energy-efficient air conditioner or make the house more heat-isolating by installing insulation materials in the walls. More generally speaking, one needs to change the technology used in her house.

In this paper, these technology choices are modeled as an adjustment in θ. In section 3, I introduce frictions to the adjustment of θ. Allowing for different ex-ante and ex-post elasticities then leads to differences in short- and long-run elasticities between energy and other inputs. This setting is consistent with the empirical finding by Hassler et al. (2021) that "energy-saving technical change" leads to a significant difference in the short- and long-run price elasticity of energy.

Using brown energy inputs warms the planet and leads to damage proportional to the final outputs. The damage ratio is given by the climate damage function $D(e_b)$. The firm-household does not internalize this climate damage and takes e_b as given. After incurring climate damage and paying the energy firm, the remaining final goods are consumed by the firm-household. In addition, a technology subsidy as a function of θ, $\mathrm{T}_\theta(\theta) = \tau_\theta \theta < 0$, can be given to the firm-household to encourage a higher θ through the adoption of a more energy-efficient technology on the technology menu.[5] Similar to the green and brown taxes/subsidies, this technology choice subsidy is financed lump-sum tax ($T_y < 0$). The output y net of climate damage, energy price, taxes, and subsidies are consumed as consumption goods c. Therefore, the firm-household's maximization problem is

$$\max_{\theta,e} c$$

$$c = D(e_b) \min\left\{\theta^{\nu-1} h, \theta^{\nu} e\right\} - p_e e - \tau_\theta \theta + T_y$$

Here, I continue to focus on the interior solution. Plugging (4) and (5) back into the optimization problem, we have

$$\max_{\theta} D(e_b)\, \theta^{\nu-1} h - p_e \frac{h}{\theta} - \tau_\theta \theta + T_y$$

This problem yields the first-order condition:

$$D(e_b)(\nu - 1)\theta^{\nu-2} h + p_e \frac{h}{\theta^2} - \tau_\theta = 0 \tag{6}$$

We shall notice that, if $\tau_\theta = 0$,

$$\theta = \left(\frac{p_e}{D(e_b)(1-\nu)}\right)^{\frac{1}{\nu}}$$

That is, a lower energy price p_e leads to a lower θ. The economic intuition here is that less expensive energy encourages the firm to choose a less energy-efficient technology on

[5] Since θ_t also can be modeled as a product of directed technical change, subsidies on θ_t can be interpreted as subsidies on directed technical change.

the technology frontier. From the energy production function (1), we know that, for a given brown/green ratio, $\frac{e_b}{e_g}$, a lower e always leads to a lower e_b. That is, a higher energy efficiency can lead to reduced brown energy usage.

2.2 The Social Planner's Problem, Brown Taxes, and Green Subsidies

In this subsection, I state the planning problem, in which the social planner (SP) has access to the production technology for energy e and final good y and internalizes the damage caused by brown energy inputs $D(e_b)$. This planner wants to maximize the consumption level of the firm-household. Her maximization problem is

$$\max c$$

$$s.t.$$

$$c = D(e_b) \min\left\{\theta^{\nu-1}h, \theta^{\nu}e\right\} - cost_b e_b - cost_g e_g$$

$$e = \left(\lambda (e_b)^{\frac{\epsilon_e - 1}{\epsilon_e}} + (1-\lambda)(e_g)^{\frac{\epsilon_e - 1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e - 1}}$$

If we still assume that capital is fully utilized, the FOCs of the problem yield the following conditions

$$e=\left(\lambda\left(e_b\right)^{\frac{\epsilon_e-1}{\epsilon_e}}+(1-\lambda)\left(e_g\right)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}} \tag{1*}$$

$$\frac{e_b}{e_g}=\frac{\lambda}{1-\lambda}\left(\frac{cost_b+\kappa}{cost_g}\right)^{-\epsilon_e} \tag{2*}$$

$$\mu_e=\left(\lambda^{\epsilon_e}\left(cost_b+\kappa\right)^{1-\epsilon_e}+(1-\lambda)^{\epsilon_e}\left(cost_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{3*}$$

$$\theta e=h \tag{4*}$$

$$y=\theta^{\nu-1}h \tag{5*}$$

$$D\left(e_B\right)(\nu-1)\theta^{\nu-2}h=-\mu_e\frac{h}{\theta^2} \tag{6*}$$

$$\kappa=D'\left(e_b\right)y \tag{7*}$$

Here, κ is the social cost of carbon (SCC), which characterizes the marginal climate damage of brown energy usage.

By comparing the decentralized equilibrium (1) - (6) and the solution for SP (1^*) - (6^*), we can see that, in the decentralized competitive equilibrium, the share between green and brown energy inputs e_b/e_g and the price of energy p_e, which in turn determines energy demand e, are determined by the after-tax private costs of the two types of energy sources:

$$\frac{e_b}{e_g}=\frac{\lambda}{1-\lambda}\left(\frac{cost_b+\tau_b}{cost_g+\tau_g}\right)^{-\epsilon_e} \tag{2}$$

$$p_e=\left(\lambda^{\epsilon_e}\left(cost_b+\tau_b\right)^{1-\epsilon_e}+(1-\lambda)^{\epsilon_e}\left(cost_g+\tau_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{3}$$

In contrast, in the solution to SP, they are determined by their respective social costs, with the climate cost of brown energy in addition to its production cost being summarized by the SCC $\kappa=D'\left(e_b\right)y$. In the decentralized equilibrium conditions, τ_b appears in both equations (2) and (3), identical to the presence of SCC κ in both equations (2^*) and (3^*) of SP's problem solution. Therefore, a brown tax $\tau_b^*=\kappa=D'\left(e_b\right)y$ alone implements the solution to the SP. As the classical Pigou logic states, a tax should equalize the private cost and the social cost when there is a negative externality. Such a brown tax produces two separate effects. First, it encourages the substitution away from brown energy towards green energy by making brown energy relatively more expensive as it changes the after-tax relative price to $(cost_b + \tau_b)/cost_g$ (the substitution effect) in (2). At the same time, it incentivizes the firm-household to use less energy e by choosing a higher θ as it increases price energy p_e for $\frac{dp_e}{d\tau_b} > 0$ and $\frac{\partial \theta}{\partial p_e} > 0$. Less energy usage leads to less brown energy usage (the scale effect).

By contrast, a green subsidy $\tau_g < 0$ cannot implement the social optimum. It is true that such a subsidy can replicate the substitution effect of the socially optimal brown tax by setting τ_g^* such that

$$\frac{cost_b}{cost_g + \tau_g^*} = \frac{cost_b + \kappa}{cost_g} \Rightarrow \tau_g^* = cost_b \frac{cost_g}{cost_b + \kappa} - cost_g = cost_g \left(\frac{cost_b}{cost_b + \kappa} - 1 \right) < 0$$

Nevertheless, green subsidies alone, in general, cannot mimic the scale effect of a brown tax because $\frac{dp_e}{d(-\tau_g)} < 0$. While a brown tax increases the energy price p_e and incentivizes the adoption of a higher θ, a green subsidy decreases the energy price p_e and leads to a level of θ that is different from that in SP. The negative externality of brown energy inputs creates a wedge between the private and social energy costs. A brown tax can close this wedge, while a green subsidy further widens it and leads to higher energy production than that in the social optimum. In fact, when green and brown energy inputs are strong complements ($\epsilon_y << 1$), green energy subsidies actually increase brown energy usage. To see this, the demand function for e_b is

$$e_b = \frac{\lambda}{1-\lambda} \left(\frac{cost_b}{p_e(\tau_g)} \right)^{-\epsilon_e} e(p_e(\tau_g))$$

Then, the change in brown energy usage induced by green energy subsidies relative to the BAU with no green energy subsidies, $\hat{e}_b \equiv \frac{e_b}{e_{b,BAU}}$, is

$$\hat{e}_b = \left(\frac{p_e(\tau_g)}{p_{\text{BAU}}}\right)^{-\epsilon_e} \frac{e(p_e(\tau_g))}{e_{\text{BAU}}} = C(p_e(\tau_g))^{-\epsilon_e} e(p_e(\tau_g))$$

with

$$C \equiv \left(\frac{1}{p_{\text{BAU}}}\right)^{-\epsilon_e} \frac{1}{e_{\text{BAU}}} > 0$$

Then,

$$\frac{d\hat{e}_b}{d\tau_g} = C(p_e(\tau_g))^{-\epsilon_e-1}(-\epsilon_e)\frac{dp_e(\tau_g)}{d\tau_g} e(p_e(\tau_g)) + C(p_e(\tau_g))^{-\epsilon_e} \frac{de(p_e(\tau_g))}{dp_e(\tau_g)} \frac{dp_e(\tau_g)}{d\tau_g}$$

$$\Rightarrow$$

$$\frac{d\hat{e}_b}{d(-\tau_g)} < 0 \Leftrightarrow -\epsilon_e + \frac{de(p_e(\tau_g))}{dp_e(\tau_g)} \frac{p_e(\tau_g)}{e(p_e(\tau_g))} < 0 \Leftrightarrow \frac{de}{dp_e}\frac{p_e}{e} < \epsilon_e$$

Only when the elasticity of substitution between green and brown inputs is greater than the price elasticity of energy can standalone subsidies on green energy inputs reduce brown energy usage. Otherwise, the scale effect dominates the substitution effect, and emissions would increase as a response to subsidies imposed on green energy inputs.[6]

2.3 Implementing the Social Optimum through Green and Energy Efficiency Subsidies

The analysis above indicates that if we want to achieve the social optimum through subsidies, the scale effect of taxes on brown goods must be generated using alternative policy instruments. A subsidy on technology choice θ is an ideal candidate. Rewarding the firm-household with higher θ incentivizes the agent to choose a more energy-efficient technology.

[6]Casey et al. (2023) derive the conditions under which the scale effect can outweigh the substitution effect, leading to the conclusion that green energy ought to be subject to taxation when green and brown inputs are gross complements. The following analysis largely follows their argument. For the analysis of this topic in a more general setting, see Casey et al. (2023).

By comparing (3) and (6) to (3*) and (6*), we see that a technology subsidy

$$\begin{aligned}
\tau_\theta^* &= -\frac{h\left(\mu_e - p_e\right)}{\theta^2} \\
&= -\frac{h}{\theta^2}\Big(\left(\lambda^{\epsilon_e}\left(cost_b + \kappa\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \\
&\quad - \left(\lambda^{\epsilon_e}\left(cost_b\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_g + \tau_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}\Big) < 0
\end{aligned}$$

together with a green subsidy

$$\tau_g^* = cost_g\left(\frac{cost_b}{cost_b + \kappa} - 1\right) < 0$$

exactly implements the solution to SP. Specifically, the technology subsidy on θ can implement the socially optimum energy usage level e because this level is entirely determined by θ as $\theta e = h$. So, by (6), by setting $\tau_\theta^* = \frac{h(\mu_e - p_e)}{\theta^2}$, we have

$$D\left(e_b\right)(\nu - 1)\theta^{\nu-2}h + \frac{hp_e}{\theta^2} + \frac{h\left(\mu_e - p_e\right)}{\theta^2} = 0$$

$$\Rightarrow$$

$$D\left(e_b\right)(\nu - 1)\theta^{\nu-2}h + \mu_e\frac{h}{\theta^2} = 0$$

which is the same as (6*) in solution to SP. The intuition here is that the government can use τ_θ to implement a relative energy efficiency level θ that is consistent with the social cost of energy. With such a subsidy in place, the private marginal benefit of adjustment in θ is consistent with its social marginal benefit. As the optimal green subsidy already implements the optimal brown-green share, both the green and brown share e_b/e_g and the overall energy usage e are equivalent to those according to the solution to SP when an optimal technology subsidy and an optimal green subsidy are placed. We conclude that when the short-run elasticity between energy and capital is zero, a technology subsidy and a green subsidy can jointly implement the social optimum. The effectiveness of this policy scheme is evaluated in a more generalized setting in the next section.

2.4 Empirical Evidence for the Endogenous Energy Efficiency

In this subsection, I shed some light on the existence of technology choice, with this choice, in turn, determines energy efficiency in the long run. One important feature of the model is that there is a negative correlation between energy intensity and non-energy-inputs intensity of an economy as

$$\frac{e}{y} = \frac{e}{h^{\nu}e^{1-\nu}} = \left(\frac{e}{h}\right)^{\nu} = \theta^{-\nu}$$

and

$$\frac{h}{y} = \frac{h}{h^{\nu}e^{1-\nu}} = \left(\frac{e}{h}\right)^{1-\nu} = \theta^{\nu-1}$$

.

Also, a higher energy price always leads to lower energy intensity as

$$\theta = \left(\frac{p_e}{D\left(e_b\right)\left(1-\nu\right)}\right)^{\frac{1}{\nu}}.$$

To test whether these model predictions are consistent with the empirical evidence, I use the energy intensity level of primary energy (measured in MJ per \$2017 PPP GDP in 2015) as a proxy for energy intensity, the ratio of 2017 PPP capital stock to \$2017 PPP as a proxy for capital intensity, and the price of electricity as a proxy for energy prices.

From Panel A of Figure 1, we can observe a negative correlation between the capital and energy intensity across the country. Countries with higher energy usage per unit of production tend to have lower capital per unit of production, and verse versa. This relationship can still be seen as indirect empirical evidence for the trade-off between energy and non-energy input efficiencies, as indicated by the log-linear technology menu. I also refer the readers to Knittel (2011), which shows the trade-offs faced when choosing between passenger vehicles' fuel economy, weight, and engine power characteristics.

Panel B shows that an increase in energy prices leads to a decrease in energy intensity. It is consistent with the prediction of the model: a higher energy price can encourage firms and households to adopt more energy-efficient technology to, therefore, reduce energy usage. On the other hand, energy economists have identified that, within a country, energy use does not change much when price prices change, especially within a year. (Labandeira et al., 2017, Gao et al. 2021, Hassel et al. 2022) The different price elasticities of energy identified from cross-country data and cross-time data suggest low short-run and high long-run elasticities of energy use (Atekeson and Kehoe, 1999).

2.5 What if the Ex-Ante Elasticity between Energy and Other Inputs is not Zero

In the analysis above, I assume that energy and other inputs are perfect complements with a given technology choice θ. In this subsection, I relax this assumption by assuming that they are imperfect complements and that the firm-household has access to a Constant Elasticity of Substitution (CES) production function:

$$y = \left(\left(\theta^{\nu-1} h\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta^{\nu} e\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}$$

Here, the ex-ante elasticity of substitution is ϵ_y while the ex-post elasticity is still unitary. The firm-household's problem then becomes

$$\max_{\theta, e} \left(\left(\theta^{\nu-1} h\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta^{\nu} e\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - p_e e - \tau_\theta \theta$$

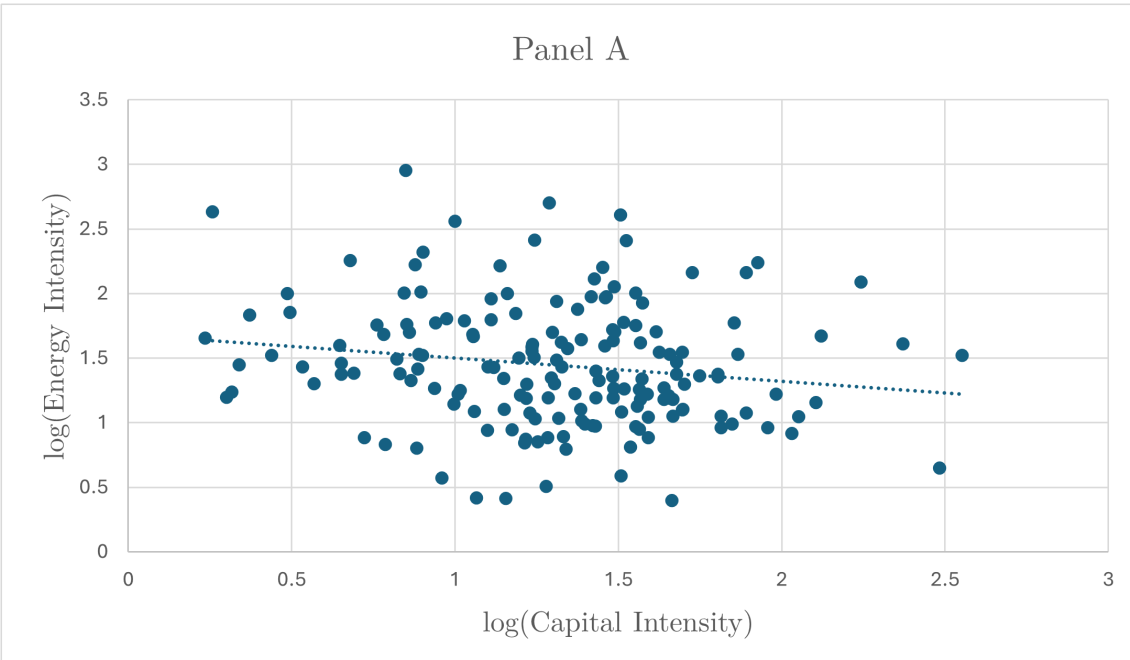
The FOCs are

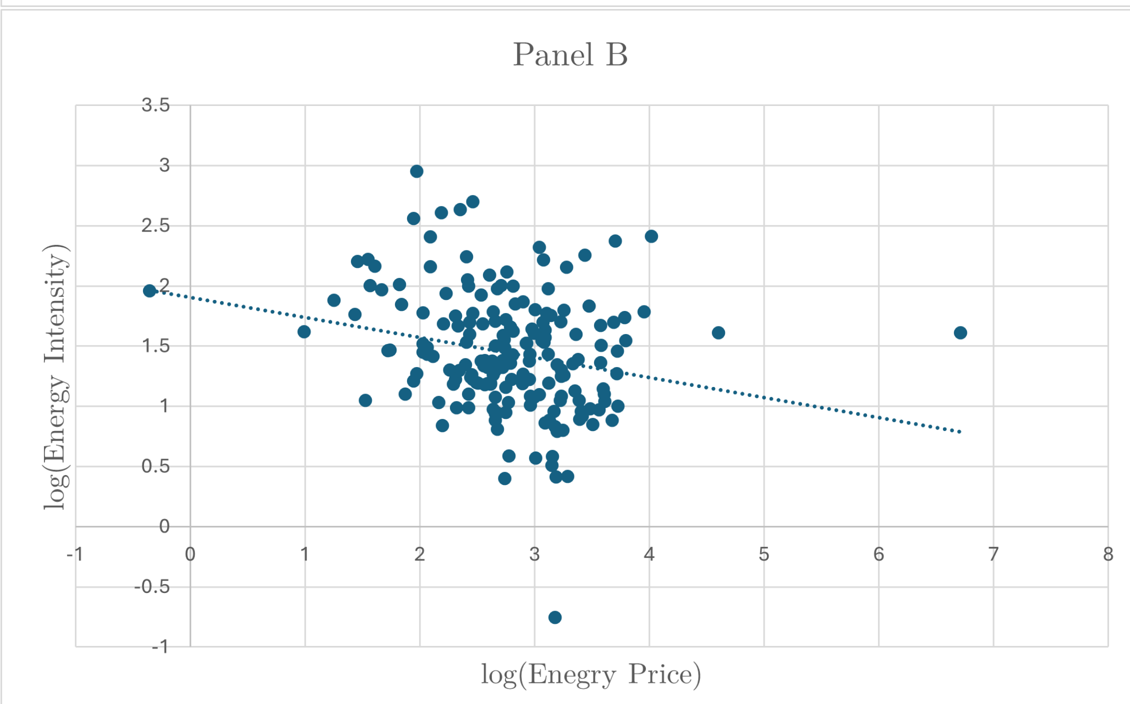
$$y^{\frac{1}{\epsilon_y}} \left(\theta^{\nu} e\right)^{-\frac{1}{\epsilon_y}} \theta^{\nu} - p_e = 0$$

$$y^{\frac{1}{\epsilon_y}} \left(\theta^{\nu} e\right)^{-\frac{1}{\epsilon_y}} (\nu) \theta^{\nu-1} + y^{\frac{1}{\epsilon_y}} \left(\theta^{\nu-1} h\right)^{-\frac{1}{\epsilon_y}} (\nu - 1) \theta^{\nu-2} - \tau_\theta = 0$$

We then have

Figure 1: Energy Price, Capital Intensity, and Energy Intensity: The first panel depicts the cross-country relationship between the log energy price (x-axis) and log energy intensity (y-axis). The second panel depicts the relationship between the log capital intensity (x-axis) and log energy intensity (y-axis). The energy price is proxied by price of electricity (US cents per kWh). The energy intensity is proxied by energy intensity level of primary energy (MJ/$2017 PPP GDP in 2015). The capital intensity is proxied by Energy by 2017 PPP Capital Stock/$2017 PPP GDP in 2015. This figure reveals the negative correlation between energy intensity and energy price, as well as the negative correlation between energy intensity and capital intensity.





$$y^{\frac{1}{\epsilon_y}} e^{-\frac{1}{\epsilon_y}} \left(\theta^{\nu}\right)^{1-\frac{1}{\epsilon_y}} - p_e = 0 \Rightarrow$$

$$e^{-\frac{1}{\epsilon_y}} = p_e \left(\theta^{\nu}\right)^{\frac{1-\epsilon_y}{\epsilon_y}} y^{-\frac{1}{\epsilon_y}} \Rightarrow$$

$$e = p_e^{-\epsilon_y} \left(\theta^{\nu}\right)^{\epsilon_y - 1} y$$

If ex-ante elasticity of substitution $\epsilon_y = 0$, then

$$e = \theta^{-\nu} y$$

The energy usage for a given level of output is entirely determined by the energy efficiency θ. However, if $\epsilon_y > 0$, then even if θ is fixed at the optimal level θ^*, a lower energy price would still lead to higher energy usage as

$$\frac{\partial e}{\partial p_e} \frac{p_e}{e} = -\epsilon_y$$

Even if the government can use energy efficiency subsidies τ_θ to implement a socially optimal θ, energy is still overused if a wedge exists between its private and social costs. In the next section, I show that, even though these two subsidies are not able to exactly implement the social optimum, green energy subsidies and energy efficiency subsidies can still implement an equilibrium very close to it.

3 A Dynamic Model

I now consider a dynamic model to assess the effectiveness of subsidies versus brown taxes over time. In addition to incorporating dynamics across time, this model also extends the static model in several ways. Here, I relax the assumption of the Leontief production function by allowing some degree of ex-ante elasticity of substitution between energy and other inputs. Technology adjustment costs and endogenous capital accumulation are also introduced. The government's constrained-efficient optimization problems are solved under different policy scenarios: business-as-usual, brown taxes, green subsidies, and green subsidies paired with energy efficiency subsidies. The equilibrium outcomes under different policy scenarios are then compared.

3.1 The Energy Firm

The setting for the energy sector is identical to that in the static model. A perfectly competitive energy firm uses CES technology to produce the final energy e_t with brown and green energy inputs $e_{b,t}$ and $e_{g,t}$. Both brown and green energy inputs are produced with constant marginal costs $cost_{b,t}$ and $cost_{g,t}$, respectively, in terms of final output. In the baseline model, $cost_{b,t}$ and $cost_{g,t}$ are assumed to be constant over time. Energy e_t is then sold to the firm-household at price $p_{e,t}$. This firm is also potentially subject to taxes or subsidies imposed on either type of energy inputs $\tau_{b,t}$ and $\tau_{g,t}$, which are rebated lump sum ($T_{e,t}$). The profit maximization problem of this energy firm is

$$\max_{e_{b,t},e_{g,t},e_t} p_{e,t}e_t - cost_{b,t}e_{b,t} - cost_{g,t}e_{g,t} - \tau_{b,t}e_{b,t} - \tau_{g,t}e_{g,t} + T_{e,t}$$

s.t.

$$e_t = \left(\lambda \left(e_{b,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)\left(e_{g,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}} \tag{7}$$

3.2 The Firm-Household

A representative firm-household has an infinite life horizon with a year-long period length. Its preference is given by

$$W = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\alpha}}{1-\alpha}$$

In this expression, $1/\alpha$ is the intertemporal elasticity of consumption, β is the discount factor, and c_t is consumption. This agent, with labor-augmenting total factor productivity

(TFP) A_t, combines capital k_t, endowed labor l, and energy e_t to produce final goods using a CES production technology with $\epsilon_y < 1$:

$$y_t = \left(\left(\theta_t^{\nu-1} A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} e_t \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \tag{8}$$

Labor l is supplied inelastically. The law of motion for labor-augmenting TFP A_t is $A_{t+1} = \exp(g_A) A_t$. Energy e_t is purchased from the energy firm at the price $p_{e,t}$. Similar to the static model, the firm can choose θ_t, the relative efficiency between energy and other inputs. A higher θ implies a higher relative energy efficiency, and vice versa.[7]

If the agent's technology choice in this period, θ_t, is different from the technology choice in the last period, θ_{t-1}, the technology adjustment cost is $\left(1 - \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\right) y_t$ where $0 \leq \Psi() \leq .1$, $\Psi(1) = 0, \Psi'(1) = 0$, and $\Psi''() > .0$. Under such a setting, the ex-ante elasticity of substitution between the non-energy-input composite $A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}$ and energy e_t with a given θ_t is ϵ_y while the ex-post elasticity is 1. Because of the technology adjustment cost, the short-run elasticity of substitution between the non-energy-input composite $A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}$ and energy e_t is mainly determined by the ex-ante elasticity ϵ_y. Nevertheless, the long-run elasticity equals one, with a long-run energy share (of income) being $1 - \nu$ (Leon-Ledesma and Satchi, 2019). In other words, the production function is Cobb-Douglas in the long run, while the short-run production process is characterized by complementarity between energy and other inputs. Allowing for different short and long-run elasticities is consistent with the empirical findings of Hassler et al. (2021). Technology choice subsidies, as a function of $\theta, \mathrm{T}_{\theta,t}(\theta_t) = \tau_{\theta,t}\theta_t < 0$, are financed by lump-sum taxes $(T_{y,t})$ and given to the firm-household.

Cumulative carbon emissions M_t, generated by brown energy usage with exogenously given emission intensity η_t, are

[7] In other words, the firm can choose its energy efficiency $B_e \equiv \theta_t^{\nu}$ and capital-labor efficiency $B_k \equiv \theta_t^{\nu-1}$ on a loglinear technology frontier: $\nu \log B_k + (1-\nu) \log B_e = 0$ $(0 < \nu < 1)$.

$$M_t = M_0 + \sum_{s=0}^{t} \eta_s e_{b,s} = M_{t-1} + \eta_{t-1} e_{b,t-1}$$

The cumulative emissions warm the planet and damage a share of final outputs given by the function $D(M_t)$. Emission intensity η_t declines at the rate g_η :

$$\eta_{t+1} = \exp(g_\eta)\, \eta_t$$

The agent takes the path of M_t as given and does not internalize the effects of her decision on M_t. After incurring climate damage, the technology adjustment cost, and the energy cost, the remaining final outputs can either be consumed as c_t or be invested into capital k_{t+1}, which depreciates at the rate δ. The resource constraint for the agent is

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t & \\ = D(M_t)\, \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) & \left(\left(\theta_t^{\nu-1} A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} e_t\right)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \\ & - p_{e,t} e_t + \tau_{\theta,t}\theta_t + T_{y,t} \end{aligned} \quad (9)$$

The agent's optimization problem is then characterized by the Bellman equation:

$$\begin{gathered} \mathcal{V}(\theta_{t-1}, k_t, M_t, A_t, \eta_t) = \\ \max_{\theta_t, k_{t+1}, e_t, c_t} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta \mathcal{V}(\theta_t, k_{t+1}, M_{t+1}, A_{t+1}, \eta_{t+1}) \\ \text{s.t.} \\ (8) \text{ and } (9) \end{gathered}$$

Following Leon-Ledesma and Satchi (2019), I assume the technology adjustment cost is a symmetric exponential function with γ as the technology adjustment scale factor. For a given adjustment in θ_t relative to θ_{t-1}, a higher γ implies a higher adjustment cost [8]:

$$\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) = \exp\left(-\frac{1}{2}\gamma * \left(\frac{\theta_t}{\theta_{t-1}} - 1\right)^2\right)$$

The functional form of the damage function is based on the recent advancement in climate science that indicates the degree of global warming H_t is proportional to the cumulative emissions with the transient climate response to emissions χ (IPCC, 2013):

$$H_t = \chi * M_t$$

Following Barrage and Nordhaus (2023), the functional form of damage function is

$$D\left(M_t\right) = 1.0 - d\left(H_t\right)^2 = 1.0 - d\left(\chi * M_t\right)^2$$

Here, the damage scale factor d determines the output loss with a given degree of warming.

3.3 The Decentralized Equilibrium

The Recursive Competitive Equilibrium is defined as follows: I denote s as the vector of state variables $(\theta_{t-1}, k_t, M_t, A_t, \eta_t)$. A recursive competitive equilibrium is defined by a perceived law of motion for the cumulative carbon emissions $\widehat{M}(s)$, tax/subsidy rules $\hat{\tau}_b(s), \hat{\tau}_g(s), \hat{\tau}_\theta(s)$, and decision rules $k'(s), \theta(s), e(s), e_b(s), e_g(s)$ with associated value function $V(s)$ such that:

1. $\{k'(s), \theta(s), e(s)\}$ and $V(s)$ solve the firm-household's recursive optimization problem, taking as given perceived $\widehat{M}(s)$, energy price $p_e(s)$, and tax/subsidy rules $\hat{\tau}_b(s), \hat{\tau}_g(s), \hat{\tau}_\theta(s)$;
2. $\{e_b(s), e_g(s), e(s)\}$ solves the energy firm's static optimization problem with the given $p_e(s)$;
3. The profit of the energy firm is zero;

[8] The results presented in this paper still hold under the alternative quadratic adjustment cost function.

4. The perceived law of motion for cumulative carbon emissions is consistent with the actual law of motion $\widehat{M}'(s) = M + \eta e_B(s)$; and

5. The perceived tax/subsidy rules are consistent with the actual rules: $\hat{\tau}_i(s) = \tau_i(s)$.

Solving the maximization problem of the energy firm and the firm-household yields a set of endogenous variables $\{\theta_{t+1}, k_{t+1}, c_t, e_t, e_{b,t}, e_{g,t}, y_t, p_{e,t}\}$ such that both (7) - (9) and the following equations are satisfied.

$$p_{e,t} = \left(\lambda^{\epsilon_e} \left(cost_{b,t} + \tau_{b,t}\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e} \left(cost_{g,t} + \tau_{g,t}\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{10}$$

$$\frac{e_{b,t}}{e_{g,t}} = \left(\frac{\lambda}{1-\lambda} \frac{cost_{b,t} + \tau_{b,t}}{cost_{g,t} + \tau_{g,t}}\right)^{-\epsilon_e} \tag{11}$$

$$\left(1 - d\left(\chi M_t\right)^2\right) * \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) y_t^{\frac{1}{\epsilon_y}} * \left(\theta_t^{\nu} e_t\right)^{-\frac{1}{\epsilon_y}} \theta_t^{\nu} = p_{e,t} \tag{12}$$

$$\begin{aligned} & c_t^{-\alpha} \Bigg\{ \left(1 - d\left(\chi M_t\right)^2\right) \Psi'\left(\frac{\theta_t}{\theta_{t-1}}\right) \frac{1}{\theta_{t-1}} y_t \\ & + \left(1 - d\left(\chi M_t\right)^2\right) \Psi\left(\frac{\theta_t}{\theta_{t-1}}\right) y^{\frac{1}{\epsilon_y}} \left(\begin{aligned} \left(\theta_t^{\nu-1} A_{t+1}^{1-\sigma} k_{t+1}^{\sigma} l^{1-\sigma}\right)^{-\frac{1}{\epsilon_y}} (\nu-1)\theta_t^{\nu-2} A_{t+1}^{1-\sigma} k_{t+1}^{\sigma} l^{1-\sigma} \\ + \left(\theta_t^{\nu} e_t\right)^{\frac{1}{\epsilon_y}} \nu \theta_t^{\nu-1} e_t + \tau_{\theta,t} \end{aligned} \right) \Bigg\} \\ & + \beta c_{t+1}^{-\alpha} \left(\left(1 - d\left(\chi M_{t+1}\right)^2\right) \Psi'\left(\frac{\theta_{t+1}}{\theta_t}\right) \left(-\frac{\theta_{t+1}}{\theta_t^2}\right) y_{t+1} \right) = 0 \end{aligned} \tag{13}$$

$$\begin{aligned} & - c_t^{-\alpha} + \beta c_{t+1}^{-\alpha} * \Bigg(\bigg(\left(1 - d\left(\chi M_{t+1}\right)^2\right) \Psi\left(\frac{\theta_{t+1}}{\theta_t}\right) y_{t+1}^{\frac{1}{\epsilon_y}} \left(\theta_{t+1}^{\nu-1} * A_{t+1}^{1-\sigma} k_{t+1}^{\sigma} l^{1-\sigma}\right)^{-\frac{1}{\epsilon_y}} \theta_{t+1}^{\nu-1} * \sigma \\ & * A_{t+1}^{1-\sigma} * k_{t+1}^{\sigma-1} l^{1-\sigma} + (1-\delta) \bigg) \Bigg) = 0 \end{aligned} \tag{14}$$

Here, equation (10) is the standard CES price index for energy. Equation (11) describes how the share of brown and green energy sources is influenced by their respective prices after taxes and subsidies. Equation (12) indicates that the marginal cost of energy equals its marginal benefit to the firm-household. Lastly, equations (13) and (14) are the Euler equations governing technology choice θ and capital k, respectively.

3.4 The Constrained-Efficient Optimization Problem

Since the focus of this paper is on the second-best policies, I formulate the optimization of a benevolent government as the solution to a constrained-efficient optimization problem. The government internalizes the negative externality of brown energy and aims to maximize the welfare of the firm-household. Through the tax or subsidy instruments $\tau_{b,t}$, $\tau_{g,t}$, and $\tau_{\theta,t}$, the government can choose allocations subject to the resource and implementability constraints (7)-(14). It also faces bounds on policy instruments: $\tau_{i,LB}$ and $\tau_{i,UB}$. These bounds determine which policy instruments are feasible for the government. For example, if the upper bounds $\tau_{b,UB}$ and $\tau_{b,LB}$ are both zero, the government can neither tax nor subsidize the brown input. lower bounds This setting yields the constrained-efficient optimization problem characterized by the Bellman equation as follows:

$$\mathcal{V}\left(M_t,\theta_{t-1},k_t,A_t,\eta_t\right) =$$

$$\max_{\substack{\tau_{b,t},\tau_{g,t},\tau_{\theta,t},M_{t+1},\theta_t,k_{t+1},\\ e_t,e_{b,t},e_{g,t},y_t,c_t,p_{e,t}}} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta\mathcal{V}\left(M_{t+1},\theta_t,k_{t+1},A_{t+1},\eta_{t+1}\right)$$

$$\text{s.t.}$$

$$\text{(7)-(14)}$$

$$M_{t+1} = M_t + \eta_t e_{b,t}$$

$$\tau_{i,LB} \leq \tau_i(s) \leq \tau_{i,UB}$$

The constrained-efficient equilibrium is defined by the policy functions $k_{t+1}(s), M_{t+1}(s)$ with decision rules $\theta_t(s), e_t(s), e_{b,t}(s), e_{g,t}(s), c_t(s), \tau_{i,t}(s)$, policy constraints $\tau_{i,LB}$ and $\tau_{i,UB}$ with $\tau_{i,LB} \leq \tau_i(s) \leq \tau_{i,UB}$, the value function $\mathcal{V}(s)$, and the conjectured functions characterizing the decision rule of the future planner $\theta'(s), e'(s), c'(s)$ such that the following conditions hold:

1. Planner's optimization: $\mathcal{V}(s)$ and $k_{t+1}(s), M_{t+1}(s), \theta_t(s), e_t(s), e_{b,t}(s), e_{g,t}(s), \tau_{i,t}(s)$ solve the Bellman equation defined above; and

Table 1: Policy Scenarios and the Corresponding Tax/Subsidy Constraints

Policy Scenarios	Policy Instrument Constraints
Brown taxes only	$\tau_{b,t} \geq 0, \tau_{g,t} = 0, \tau_{\theta,t} = 0$
Green subsidies only	$\tau_{b,t} = 0, \tau_{g,t} \leq 0, \tau_{\theta,t} = 0$
Technology choice subsidies + green subsidies	$\tau_{b,t} = 0, \tau_{g,t} \leq 0, \tau_{\theta,t} \leq 0$
The business-as-usual (BAU)	$\tau_{b,t} = 0, \tau_{g,t} = 0, \tau_{\theta,t} = 0$

Notes: This table lists the constraints the government faces under different policy scenarios.

2. The conjectured decision rules are consistent with the actual decision rules.

I study four policy scenarios in which different policy instruments are feasible for the government: brown taxes only, green energy subsidies only, technology choice subsidies together with green subsidies, and BAU. The corresponding constraints of these scenarios are summarized in Table 1.

As in the static model, setting carbon taxes equal to the marginal utility damage of brown energy (i.e., SCC) usage,

$$\tau_{b,t}^{*} = SCC_t = \eta_t \kappa_t$$
$$\kappa_t = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} \left(\left(-D'\left(M_{t+1}\right)\Psi\left(\frac{\theta_{t+1}}{\theta_t}\right) y_{t+1}\right) + \kappa_{t+1}\right),$$

implements the social optimum.

Also, when the ex-ante elasticity between energy and other inputs $\epsilon_y = 0$, if subsidies on technology choice $\tau_{\theta,t}$ and $\tau_{g,t}$ are both feasible for the government, then

$$\tau_{\theta,t}^{*} = \frac{(\mu_t - p_t)}{c_t^{-\alpha}} \left(-\frac{1}{\theta_t^2} k_t^{\sigma} l^{1-\sigma}\right)$$
$$\tau_{g,t}^{*} = cost_{g,t}\left(\frac{cost_{b,t}}{cost_{b,t} + \tau_{b,t}^{*}} - 1\right)$$

with

$$\mu_t = \left(\lambda^{\epsilon_e}\left(cost_b + SCC_t\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}$$
$$p_t = \left(\lambda^{\epsilon_e}\left(cost_b\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_{g,t} + \tau_g^{*}\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}$$

can implement the social optimum. The formula for optimal subsidies has an interpretation identical to that of its static counterpart. To achieve the social optimum, the government needs to use two subsidies: one on green energy inputs to increase its share in the energy sector and one on technology choice to promote higher energy efficiency.

However, as I argued in section 2.5, technology choice subsidies and green subsidies cannot implement the social optimum if the ex-ante elasticity of substitution between other inputs and energy, ϵ_y, is not zero. Nevertheless, as demonstrated later in the quantitative results in sections 3.7 and 3.8, technology choice subsidies, together with green subsidies, can bring the magnitude of emission reduction to a level close to that under brown taxes.

3.5 Solution Method

To solve the model numerically, the government's optimization problem is normalized by TFP and labor. I define $\tilde{k}_t = \frac{k_t}{A_t l}, \tilde{c}_t = \frac{c_t}{A_t l}, \tilde{y}_t = \frac{y_t}{A_t l}, \tilde{e}_t = \frac{e_t}{A_t l}, \tilde{e}_{b,t} = \frac{e_{b,t}}{A_t l}, \tilde{e}_{g,t} = \frac{e_{g,t}}{A_t l}, \tilde{\tau}_{\theta,t} = \frac{\tau_{\theta,t}}{A_t l}, \tilde{T}_{\theta,t} = \frac{T_{\theta,t}}{A_t l}, \tilde{T}_{e,t} = \frac{T_{e,t}}{A_t l}, \tilde{\eta}_t = A_t * l * \eta_t$, and $\tilde{V}_t = \frac{V_t}{A_t^{1-\alpha} l}$. Here, $\tilde{\eta}_t$ captures the carbon intensity of brown energy per unit of effective labor $\tilde{e}_{b,t}$. The model after normalization is described in detail in Appendix C. The recursive constrained-efficient optimization problem of the government is then rewritten as

$$V\left(M_{t,}\theta_{t-1}, \tilde{k}_t, \tilde{\eta}_t\right) = \max_{\substack{\tau_{b,t},\tau_{g,t},\tilde{\tau}_{\theta,t},M_{t+1},\theta_{t+1},k_{t+1},\\ \tilde{c}_t,\tilde{e}_t,\tilde{e}_{b,t},\tilde{e}_{g,t},\tilde{y}_l,p_{e,t}}} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta e^{(1-\alpha)g} V\left(M_{t+1,}, \theta_t, \tilde{k}_{t+1}, \tilde{\eta}_{t+1}\right)$$

s.t.

$$(7^*) - (14^*)$$

$$\tilde{\eta}_{t+1} = \tilde{\eta}_t \exp\left(g_A + g_\eta\right)$$

$$M_{t+1} = M_t + \tilde{\eta}_t * \tilde{e}_{b,t}$$

$$\tau_{i,UB} < \tau_i(s) < \tau_{i,LB}$$

For notional convenience, I will omit the tilde sign for the rest of the paper. Any lower-case variables denote normalized variables unless otherwise noted. The normalized model is solved using backward value function iteration. At each iteration, with a given value function $V(M_{t+1}, \theta_t, k_{t+1}, \eta_{t+1})$ as well as future decision rules $\theta_{t+1}(M_{t+1}, \theta_t, k_{t+1}, \eta_{t+1})$, $e_{t+1}(M_{t+1}, \theta_t, k_{t+1}, \eta_{t+1})$, and $c_{t+1}(M_{t+1}, \theta_t, k_{t+1}, \eta_{t+1})$, the maximization problem above is solved by non-linear programming on a finite number of grid points. Using the results as the training inputs, I then train Gaussian processes to approximate the value function $V(M_t, \theta_t, k_t, \eta_t)$ as well as decision rules for the firm-household $\theta_t(M_{t,}\theta_{t-1}, k_t, \eta_t)$, $e_t(M_{t,}, \theta_{t-1}, k_t, \eta_t)$, $c_t(M_{t,}\theta_{t-1}, k_t, \eta_t)$, which are used as inputs for the next value function iterations. This procedure is repeated until the value function converges.

3.6 Calibration

I follow Nordhaus (2017) and set $\alpha = 1.45$ and $\beta = 0.985$. I set $\sigma = 0.35$ and $\nu = 0.94$ to match the long-run capital and energy share of income. I assume that TFP A_t, grows at a rate of 2.36% annually. The carbon intensity of brown energy η_t also declines at the same speed such that the carbon intensity of brown energy usage per unit of effective labor is constant over time.[9] The ex-ante elasticity of substitution between the non-energy-inputs composite and the energy, ϵ_y, is 0.05. This value is consistent with the empirical finding of Hassler et al. (2021), which suggests a very low short-run elasticity between energy and other inputs. The transient climate response to emissions χ, which links the cumulative emissions and the degree of warming, is 1.6 (Rudik, 2020). I set the damage parameter d to 0.003476 so that 3 degrees of warming leads to a 3.13% output loss (Barrage and Nordhaus, 2023). The technology adjustment cost parameter γ is set to 2.498, which implies that a 0.01% increase in the energy price decreases the annual energy usage by a 0.005% when θ is in the steady state of the baseline economy. This setting is generally consistent with the empirical estimation of the annual price elasticity of energy, which suggests a value of

[9] I examine the robustness of my conclusion to this assumption in Appendix D.

Table 2: Calibrated Parameters

Parameter	Value	Source/Target
The Energy Firm:		
ϵ_e	1.85	Papageorgiou et al. 2017
$cost_b/cost_g$	74/600	Hassler et al. 2021
λ	0.644	Golosov et al. 2014
The Firm-Household:		
α	1.45	Nordhaus 2017
β	0.985	Nordhaus 2017
ϵ_y	0.05	
$1-\nu$	0.06	Long-run energy share of income
σ	0.35	Long-run capital share of income
δ	0.06	
γ	2.49	Calibrated
χ	1.6	Rudik 2020
d	0.003467	Barrage and Nordhaus 2023
g_A	0.023	Calibrated

Notes: Lists model parameters that are calibrated or taken from the literature.

approximately -0.2 (Labandeira et al., 2017, Gao et al. 2021); I also run a sensitivity analysis on this parameter and find that my results are robust to a wide range of values. The relative brown and green energy prices are based on the coal and green energy prices in Hassler et al. (2017). Their absolute per-unit prices and the initial carbon emission intensity are set to match BAU emissions to the 46 billion global carbon dioxide emissions in 2015. The share parameter for green energy $1-\lambda$ follows Golosov et al. (2014). The value of the elasticity of substitution between green and brown energy, ϵ_e, is set to 1.8 based on the empirical study by Papageogiou et al. (2017), so that green and brown energy sources are substitutable inputs. I also examine the results when $\epsilon_e = 0.95$ (Stern, 2007; Golosov et al., 2014). The initial technology choice θ_{-1} is set according to its steady state when climate change damage is zero. Without climate change or government interventions, the firm-household would never adjust its technology choice θ_t, and the technology choice stays at θ_{-1} forever. The initial capital stock k_0 is calibrated to match the 26% global gross capital formation rate in 2015. Table 2 lists some of the key model parameters that are calibrated or taken from the literature.

Table 3: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy

Policy Scenarios	Cumulative Emissions in 2100 (billion Ton)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1406	0.0
Brown (Carbon) taxes	1214	0.16
Green subsidies	1391	0.002
Technology choice subsidies + green subsidies	1233	0.14

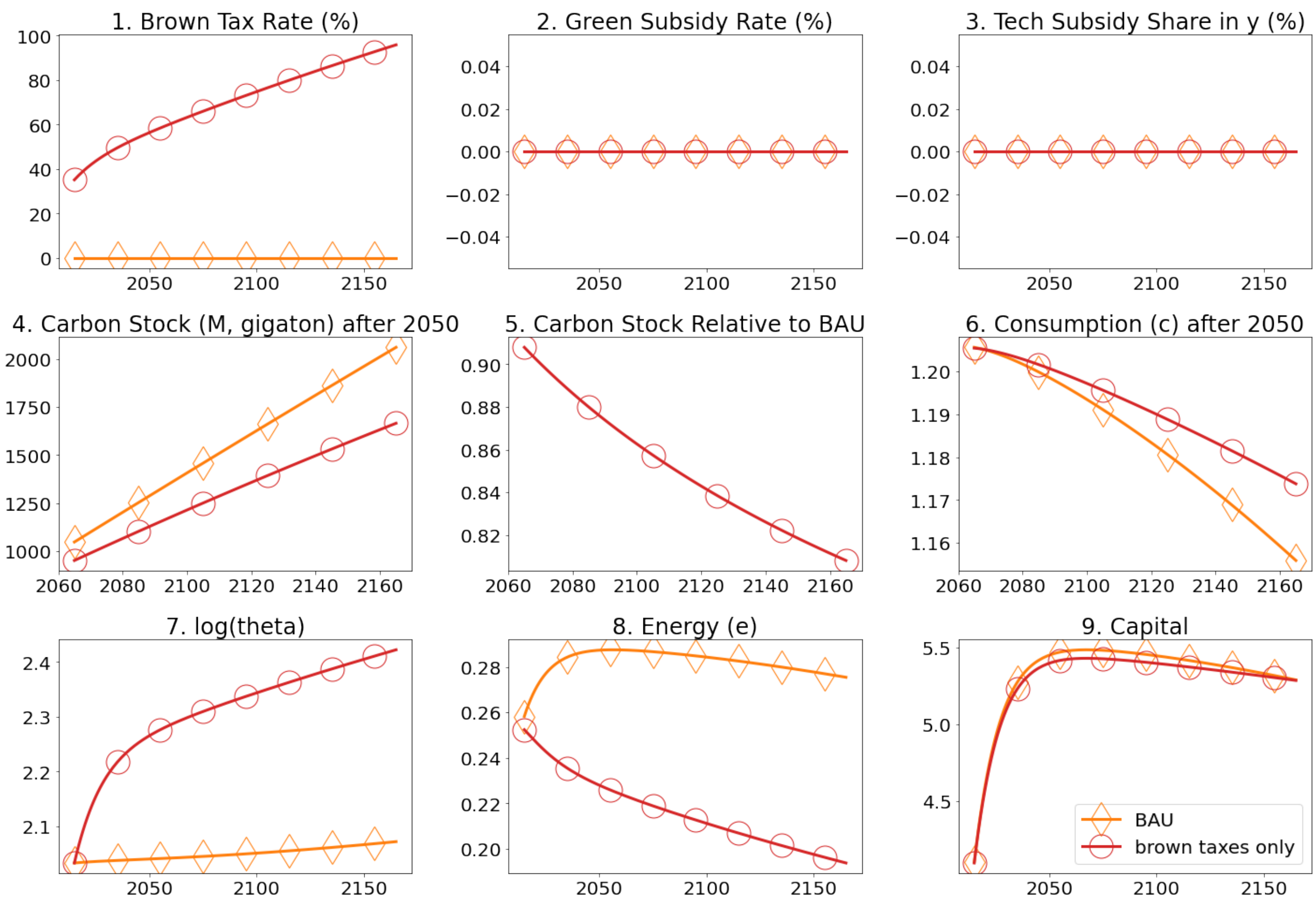
Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios.

3.7 Baseline Quantitative Results a High Elasticity of Substitution between Brown and Green

This subsection evaluates the baseline quantitative results of the model given a high elasticity of substitution between green and brown energy ($\epsilon_e = 1.85$) in a simulation spanning 150 years starting in 2015 under the four policy scenarios listed in Table 1: BAU, brown (carbon) taxes, green subsidies, and green plus energy efficiency subsidies. Table 3 summarizes the cumulative carbon emissions in 2100 and the welfare gain relative to BAU.

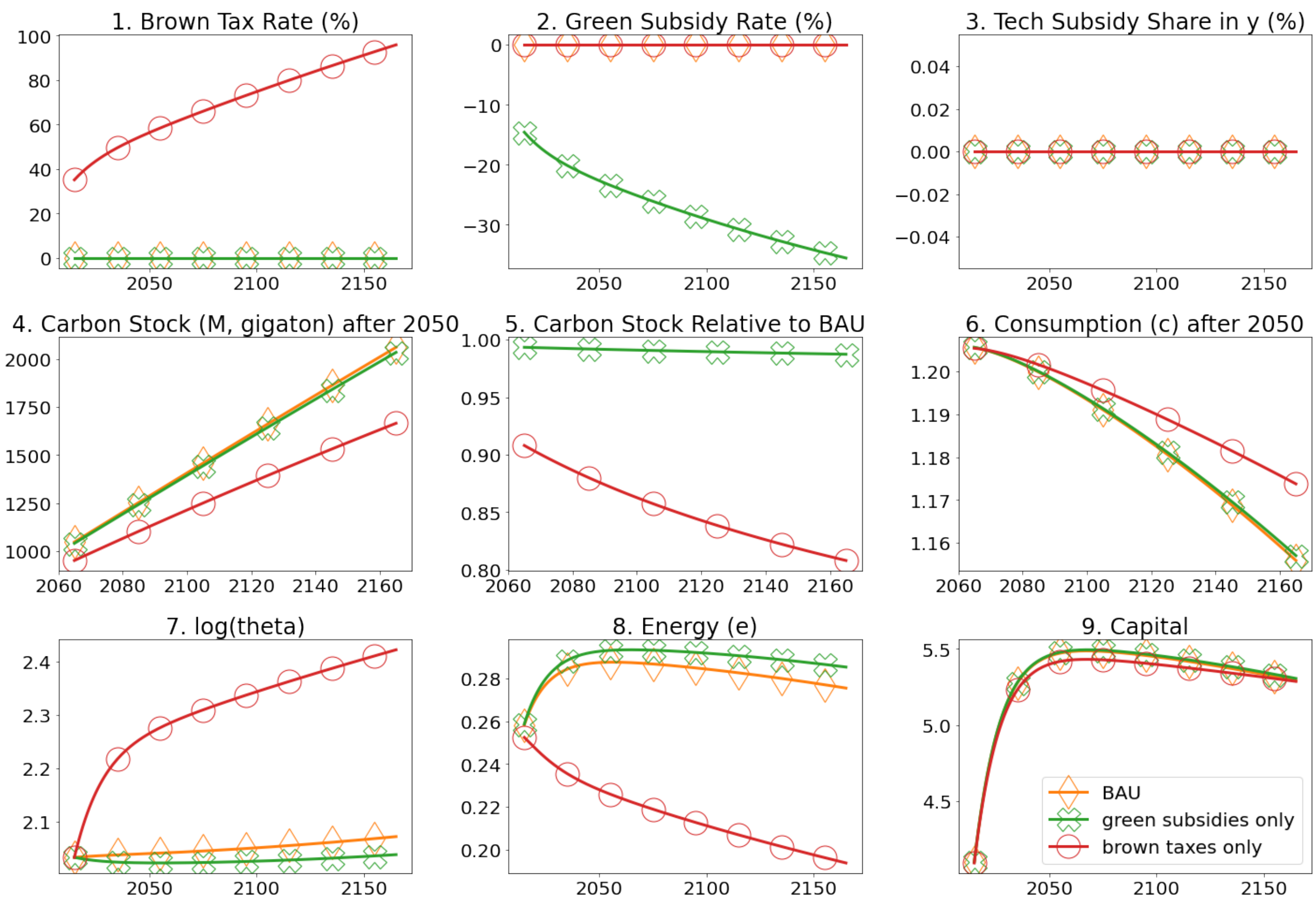
Figure 2 shows the path of several endogenous variables under BAU and brown taxes. Under the BAU scenario (depicted by yellow lines with diamond markers in the figure), the government does not implement any policy instruments. Cumulative carbon emissions reach 1,406 gigatons by the end of the century (as shown in subplot 4), which is equivalent to 2.25 degrees Celsius of warming. Without any policy interventions from the government, technology choice θ_t remains relatively unchanged over time, even though energy demand escalates due to capital accumulation. If the government can impose carbon taxes $\tau_{b,t}$ (red lines with circle markers), the optimal tax rate $\tau_{b,t}/cost_b$ would be 35% in 2015, and it would rise to 75% by the end of the century (subplot 1). Implementing the optimal carbon taxes could lead to a 13.6% reduction in cumulative emissions, bringing them down to 1214 gigatons in 2100. Fewer emissions correspond to a smaller increase in global temperatures, which in this scenario is limited to 1.92 degrees Celsius. Brown taxes lead to higher consumption levels in the long run. If the optimal taxes are imposed on brown energy, the welfare gain in terms of equivalent consumption is 0.16%. In line with theoretical expectations, the introduction of brown taxes notably enhances energy efficiency (θ_t), as illustrated in subplot 7. Thanks to higher energy efficiency, compared to energy consumption in the BAU scenario, this policy results in a significant 24% decrease in energy consumed by 2050, despite the higher capital stock, as shown in subplot 8.

Figure 2: Paths of Endogenous Variables under Brown (Carbon) Tax and BAU. This figure demonstrates the paths of endogenous variables under BAU and brown taxes policy scenarios in Table 1 when brown-green elasticity is high. It shows that optimal brown taxes can effective reduce carbon emissions, improve energy efficiency, and bring long-run consumption gains.



The endogenous variables under green subsidies are shown in Figure 3, along with those under BAU and brown taxes. In the scenario in which only green subsidies are implemented (illustrated by green lines with cross markers), the absence of technology subsidies results in a green subsidy rate $\tau_{g,t}/cost_g$ of -14.5% in 2015, as shown in subplot 2. Green subsidies, without the support of technology subsidies, only reduce cumulative emissions by merely 1% by the century's end compared to emissions in the BAU scenario, highlighted in subplot 4. The green-subsidies-only policy scenario leads to a limited welfare gain of 0.002%. The limited effectiveness of green energy subsidies on their own is attributed to the fact that, without technology subsidies, green energy subsidies fail to improve the energy efficiency of the downstream sector. The negative scale effect of green subsidies on energy efficiency, while not substantially large, is observable in the changes to energy efficiency and energy usage patterns, as depicted in subplots 7 and 8.

Figure 3: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, and Green Subsidies. This figure demonstrates the paths of endogenous variables under BAU, brown taxes, and green subsidies policy scenarios in Table 1 when brown-green elasticity is high. It shows that the emission reductions and welfare gain brought by green subsidies alone are very modest as green energy subsidies fail to improve the energy efficiency of the economy.



The blue lines with star markers represent the green subsidy plus technology choice subsidy policy scenario in Figure 4. When the optimal technology subsidies $\tau_{\theta,t}$ are also imposed, the optimal green subsidy rate reduces to 14.2% per unit in 2015 (subplot 2). Just as in the static model, a modest level of technology subsidies is warranted to ensure the energy efficiency of the economy if both technology subsidies and green subsidies are feasible. In 2015, the optimal technology subsidy $\tau_{\theta,t}$ costs approximately 0.4% of total output. This rate rises to approximately 0.6% by the end of the century (subplot 3). When both energy efficiency subsidies and green subsidies are feasible, cumulative emissions are only slightly higher than those under the first-best carbon tax scenario. These two types of subsidies together reduce cumulative emissions by 12.3%, leading to a cumulative emission of 1,233 tons, or a 1.93 degree of warming, by the end of the century. In subplot 5, which illustrates the consumption levels across different policy scenarios, we clearly see that the consumption levels under the green subsidies plus technology choice subsidies policy align closely with those under the first-best brown taxes. When green subsidies are paired with technology subsidies, the consumption-equivalent welfare gain is 0.14%. The patterns for the technology choice θ and energy e are similar to those under the first-best carbon taxes. (subplots 7 and 8). Subplot 9 demonstrates that the patterns of capital accumulation remain consistent across the various policy scenarios.

Figure 4: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, and green + energy Efficiency Subsidies. This figure demonstrates the paths of endogenous variables under BAU, Brown Taxes, Green + Energy Efficiency Subsidies policy scenarios in Table 1 when brown-green elasticity is high. It shows that the emission and energy usage reduction and consumption level improvement relative to BAU can match those under first-best brown taxes scenerio.

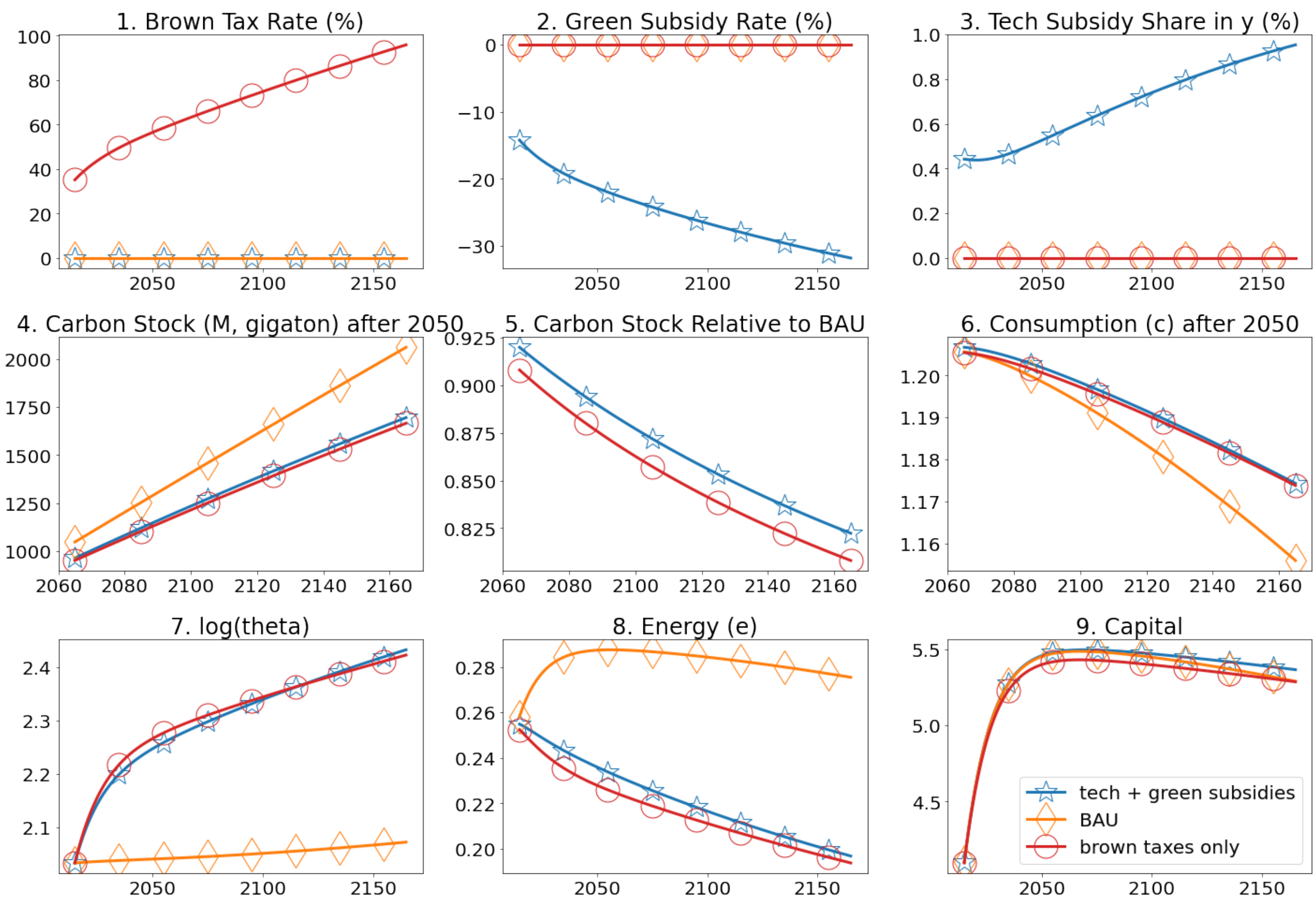


Table 4: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when $\epsilon_e = 0.95$

Policy Scenarios	Cumulative Emissions in 2100 (billion tons)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1561	0.0
Brown taxes	1230	0.3
Technology choice subsidies + green subsidies	1249	0.28

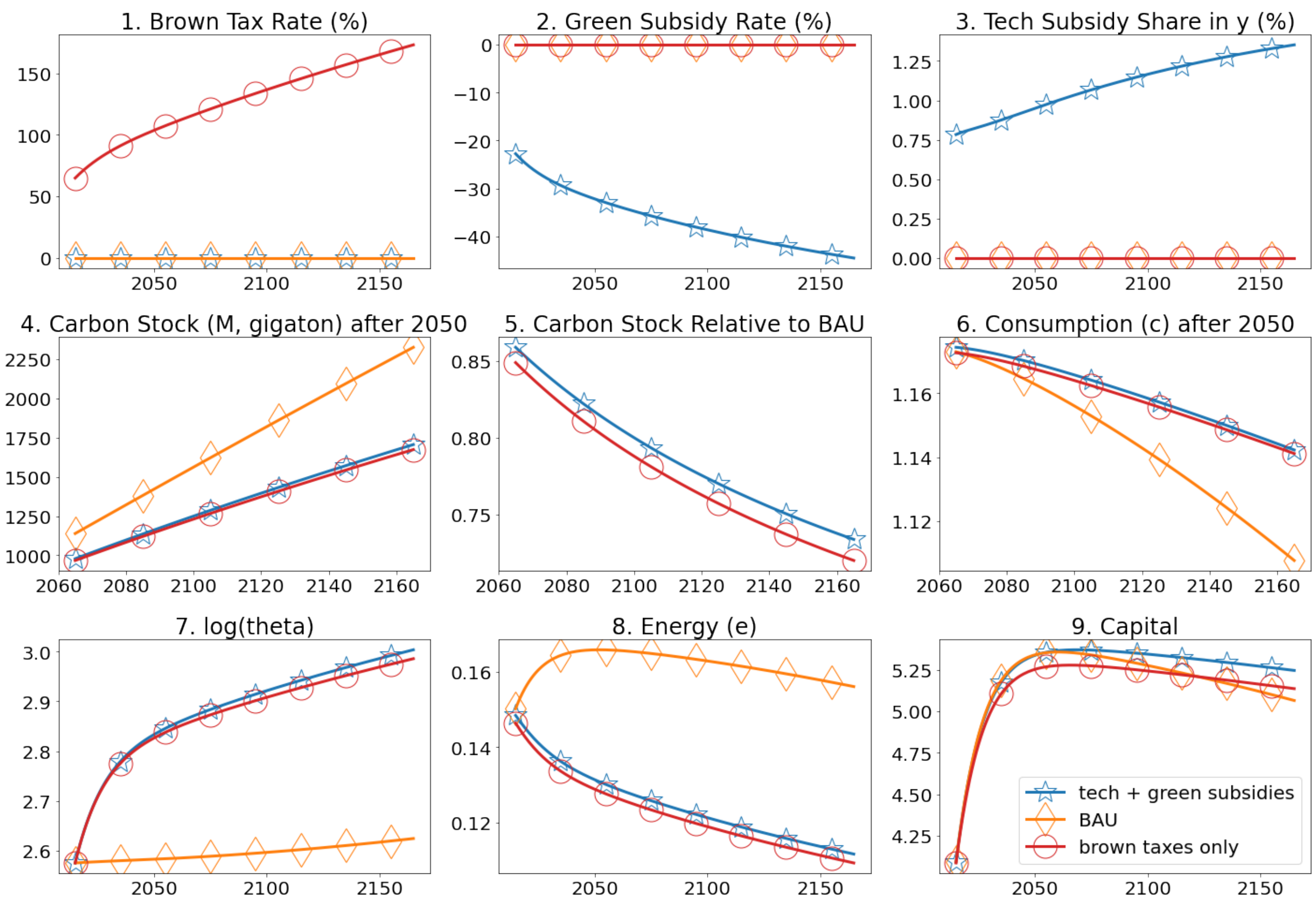
Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios when $\epsilon_e = 0.95$.

3.8 Quantitative Results with an Alternative Low Elasticity of Substitution between Brown and Green

This sub-section re-evaluates the baseline quantitative results of the model in the context of a low elasticity of substitution between green and brown energy ($\epsilon_e = 0.95$). The re-evaluated results are exhibited in Figure 5. Table 4 lists the cumulative carbon emissions in 2100 and the welfare gains relative to BAU under the different policy scenarios.[10] Figure 3 shows the path of endogenous variables under these different policy scenarios. Although green and brown energy inputs are now complementary goods, the emission reductions and welfare gains brought by the paired green and energy efficiency subsidies remain similar to those brought by brown taxes.

[10]The green-subsidies-only scenario is not examined here because the optimal green energy subsidies are negative when green and brown energy are complementary goods, as shown in the static model and Casey et al. (2023).

Figure 5: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidies, and Green + Energy Efficiency Subsidies when ϵ_e=0.95. It shows that the emission reductions and welfare gain brought by green subsidies alone are very modest as green energy subsidies fail to improve the energy efficiency of the economy ubder an alternative ϵ_e.



Under the BAU scenario, the government still does not impose any taxes or subsidies (subplots 1, 2, and 3). The cumulative emissions by the end of the century are 1,561 gigatons of carbon (subplot 4). Still, without policy interventions, θ is broadly stable over time. If brown taxes are accessible to the government, the optimal tax rate is \$176 per ton of carbon dioxide. Subplot 4 reveals that the optimal implementation of brown taxes can reduce cumulative emissions by the end of the century by 21.2% to 1,230 gigatons. From subplot 6, we can learn that the energy efficiency of the final goods production process improves significantly over time. The welfare gains under different policy scenarios, compared to those under the BAU, are evaluated. If the optimal taxes are imposed on brown energy inputs, the welfare gain in terms of equivalent consumption is 0.30%.

When both green subsidies and technology choice subsidies are accessible, then, in 2015, the optimal green subsidy rate is \$139.6, and the technology subsidy costs approximately 0.75% of GDP; in 2050, these two numbers rise to \$231 and 1%, respectively. The combination of these two policy instruments reduces cumulative carbon emissions almost as effectively as brown taxes: subplot 4 shows that the cumulative emissions in 2100 are 1,249 gigatons. The path of θ_t is also similar to that when the optimal brown taxes are imposed. When green subsidies are paired with technology subsidies, the consumption-equivalent welfare gain is 0.28%.

3.9 The Role of Technology Adjustment Cost

In this subsection, I examine how the optimal paths under different policy scenarios are affected by the technology adjustment cost parameter γ. In Appendix F, Figure F.1 shows the optimal endogenous paths when $\gamma = 10.0$, and Figure F.2 shows these paths when $\gamma = 20.0$. From these figures, we see that a higher adjustment cost γ leads to a slower increase in energy efficiency θ and ultimately brings higher cumulative carbon emissions. Nevertheless, regardless of the value of γ, the combination of energy efficiency subsidies and green subsidies remains an ideal alternative to the first-best brown taxes. When $\gamma = 10.0$, the brown tax leads to a 12.3% reduction in cumulative emissions, whereas a combination of subsidies achieves an 11.4% reduction. When $\psi = 20.0$, brown taxes can reduce cumulative emissions by 11.1% compared to a 10.2% reduction achieved through the subsidy combination.

4 Extension: Learning-by-Doing

I extend the baseline model in this section by allowing directed technical change in the green energy sector. Here, I assume that the cost of green energy inputs $cost_{g,t}$ decreases as green energy usage $e_{g,t}$ increases the knowledge stock S_t about green energy (learning-by-doing). That is,

$$cost_{g,t} = cost_{g,\text{ min}} + \left(\frac{\Omega}{S_t}\right)^{\gamma_g}$$

$$S_{t+1} = S_t + e_{g,t}$$

In the expressions above, $cost_{g,\text{ min}}$ is the long-run price of green energy, $\Omega > 0$ is a scaling factor, and $\gamma_g > 0$ is the learning exponent. I set $\gamma_g = 0.3$ and $\Omega = 0.02$ following Kalkuhl et al (2012). $cost_{g,\min}$ is set to be one tenth of $cost_{g,0}$. The positive learning externality from green energy usage is not internalized by the firm-household. The government's constrained-efficient optimization problem then becomes the following:

$$V\left(M_{t,}, \theta_{t-1}, k_t, S_t\right) = \max_{\substack{\tau_{b,t},\tau_{g,t},\tau_{\theta,t},M_{t+1},\theta_{t+1},k_{t+1}, \\ c_t,e_t,e_{b,t},e_{g,t},y_t,p_{e,t},cost_{g,t},S_{t+1}}} \frac{c_t^{1-\alpha}}{1-\alpha} + \beta e^{(1-\alpha)g} V\left(M_{t+1}, \theta_t, k_{t+1}, \eta_{t+1}, S_{t+1}\right)$$

s.t.

$$(7^{*}) - (14^{*})$$

$$\eta_{t+1} = \eta_t \exp\left(g_A + g_\eta\right)$$

$$M_{t+1} = M_t + \eta_t e_{b,t}$$

$$cost_{g,t} = p_{cost,\ \min} + \left(\frac{\Omega}{S_t}\right)^{\gamma_g}$$

$$S_{t+1} = S_t + e_{g,t}$$

$$\tau_{i,UB} < \tau_i(s) < \tau_{i,LB}$$

In contrast to the baseline model without learning-by-doing, brown taxes alone cannot correct the positive externality of green energy usage because they can not directly address the positive externality of learning. So, in this section, I also explore the social planner (SP) policy scheme in which the government faces no implementability constraints to evaluate how close each type of policy scheme can bring us to the social optimum.

Figure 6: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy with Learning-by-Doing

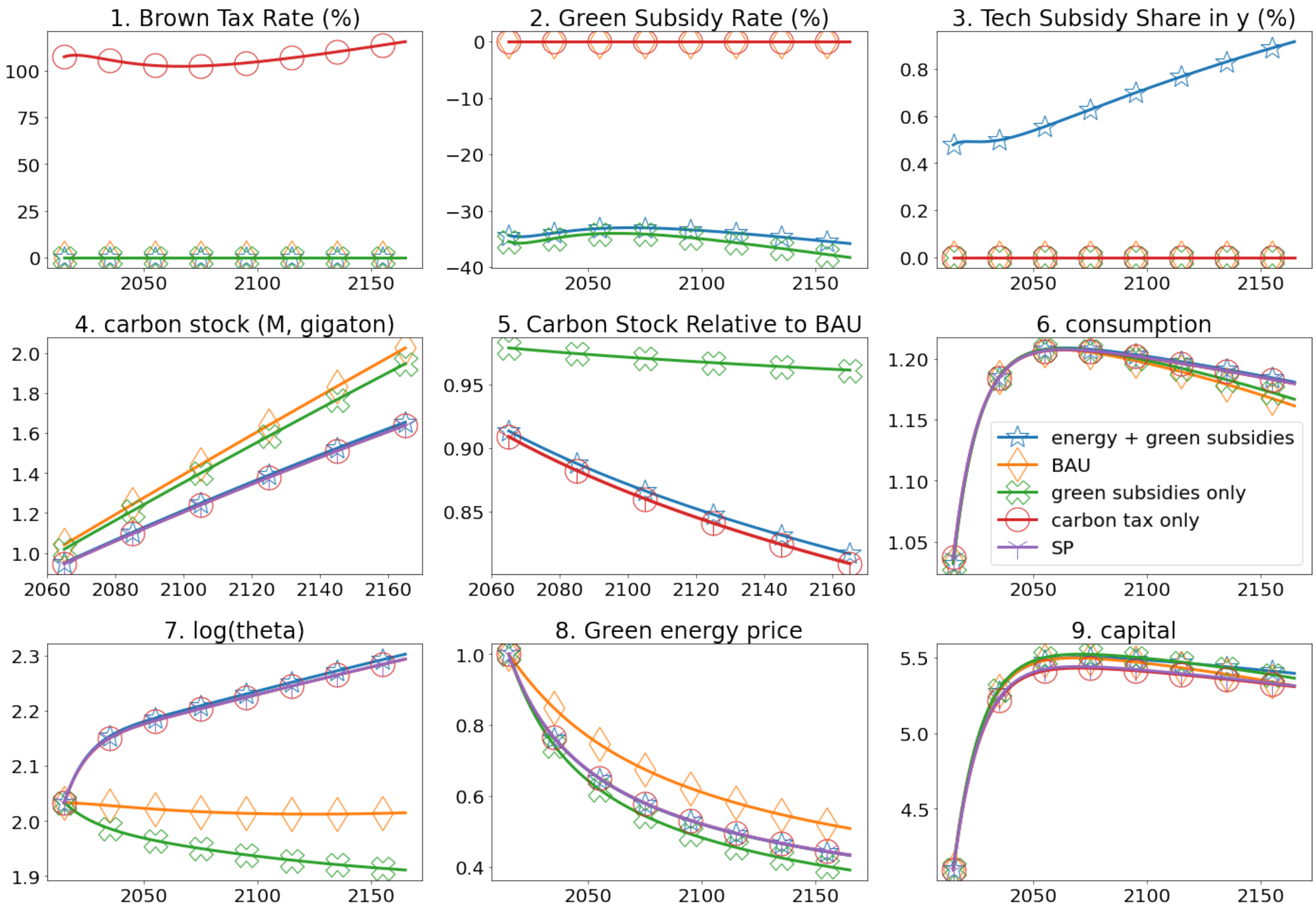


Table 5: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy with Learning-by-Doing

Policy Scenarios	Cumulative Emissions in 2100 (billion Ton)	Welfare Gain (% in terms of CEV)
The business-as-usual (BAU)	1392	0.0
SP	1205	0.15
Brown taxes	1204	0.14
Green subsidies	1352	0.05
Technology choice subsidies + green subsidies	1213	0.14

Notes: This table lists the cumulative carbon emissions in 2100 and the welfare gain relative to BAU under different policy scenarios.

I solve the model after setting ϵ_e to 1.85 and ψ to 2.498, as in the baseline model. The results are summarized in Figure 6 and Table 5. Under the BAU scenario, the cumulative carbon emissions by the end of the century are 1392 gigatons. Compared to the no-learning scenario, the cumulative emissions are lower even in the absence of policy interventions. The unit price of green energy decreases by 39%. Similar to the no-learning case, θ is, in general, stable over time. In contrast, under the SP scenario, the cumulative emissions are 1205 gigatons in 2100. The cost of green energy then is halved, and the relative energy efficiency θ increases by approximately 20%. In terms of equivalent consumption, the SP policy brings a 0.15% gain in welfare compared to the BAU policy.

The optimal standalone brown tax rate in 2015 is higher than 100% because brown taxes also need to be imposed to indirectly promote higher productivity in the green sector. This approach leads to cumulative emissions of 1204 billion tons in 2100 and significantly increases θ while reducing energy usage, e. The welfare gain under standalone brown taxes compared to that under BAU is 0.14%.

Under the green-subsidies-only scenario, the optimal green subsidy rate in 2015 is approximately 35%. We should note that such a policy leads to the cheapest green energy price because the low energy efficiency generated by green subsidies leads to energy usage being higher than that under the social optimum. The emission reduction brought by green subsidies alone is still limited: in 2100, the cumulative emissions are 1352 billion tons. Moreover, the energy efficiencies of final goods production are decreasing over time. Green energy subsidies alone can only give the household a welfare gain of 0.05%

Under the green-and-technology-choice-subsidies scenario, the optimal rate is still approximately 35%, and the subsidies on energy efficiency cost approximately 0.5% of total output. The cumulative emissions in 2100 are then 1213 billion tons. The welfare gain is 0.14%. We should notice that both brown taxes and two types of subsidies can implement energy efficiency levels that are close to those under the SP. In summary, both brown taxes and paired subsidies are shown to be good-performing second-best policies with directed technical change in the energy sector.

5 Conclusion

I propose a model that features the long-run energy efficiency decision of a firm-household to address the effectiveness of subsidies as an alternative to carbon taxes to correct the negative externality of carbon emissions. Compared to the first-best carbon taxes, green energy subsidies reduce the price of energy and deter the adoption of energy-saving technologies. In the baseline model in which green and brown energy sources are substitute goods, the optimal carbon tax can reduce cumulative emissions by 13.6% by the end of the century. By contrast, optimal green subsidies alone can mitigate the cumulative emissions by only 1.0%. Nevertheless, green subsidies jointly implemented with technology subsidies can approximate the impact of carbon taxes. With both types of subsidies in place, cumulative emissions can be reduced by 12.3%. The model is also evaluated under alternative values of elasticity of substitution between energy sources and the technology adjustment cost. The role of learning-by-doing is also addressed. In summary, subsidies are shown to be an ideal substitute for carbon pricing if both the substitution from green energy toward brown energy and the substitution of energy toward other inputs are encouraged.

Finally, I offer a few directions for future research. Because I assume a representative household in the model, the distributional consequences of taxes and subsidies are not addressed, even though carbon taxes may be regressive. The fiscal implications of taxes and subsidies are also overlooked in this paper. In the static model, we can see that, even though subsidies can replicate the solution to the social planner's problem, implementation of such subsidies places more substantial information demands on the government compared to the needs for brown taxes' implementation. To implement the optimal brown taxes, the government only needs to know the social cost of carbon, which is defined as the marginal damage of carbon emissions. In contrast, the implementation of optimal subsidies requires almost perfect knowledge of the production cost of brown and green energy and the production functions of both sectors. This weakness of subsidies is not surprising. In essence, brown taxes are a type of price regulation. Subsidies instead can be seen as a type of quantity regulation as the government aims to implement an optimal quantity of carbon emissions by encouraging substitutes for fossil fuels. As Hassler et al. (2016) argue, the quantity regulation may require more information than the price regulation. These information demands of subsidies pose practical challenges. In this paper, price-induced energy efficiency changes are interpreted as adjustments of the technology choice along a given technology menu. In the real world, such changes can be caused by different mechanisms, including (1) directed technical change, as highlighted by Hassler et al. (2021), (2) the production process chosen by the firm, as discussed by Hawkins-Pierot and Wagner (2023), and (3) the relative weight of energy-intensive industry in the whole economy, as illustrated in Hart (2008) and Bachmann et al. (2022). That is, higher energy prices can encourage scientists to put more effort into developing energy-efficient technology and firms to adopt existing energy-efficient technology, thereby shrinking the energy-intensive sector and growing the energy-saving industry. Understanding and distinguishing the influences of these three factors is crucial for crafting effective and practical policy measures.

References

[1] Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. "The environment and directed technical change." *American Economic Review* 102, no. 1 (2012): 131-166.

[2] Airaudo, Florencia S., Evi Pappa, and Hernán D. Seoane. *The green metamorphosis of a small open economy.* Centre for Economic Policy Research, 2023.

[3] Bachmann, Ruediger, David Baqaee, Christian Bayer, Moritz Kuhn, Andreas Löschel, Benjamin Moll, Andreas Peichl, Karen Pittel, and Moritz Schularick. *What if? The economic effects for Germany of a stop of energy imports from Russia.* No. 028. ECONtribute Policy Brief, 2022.

[4] Barrage, Lint. "Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy." *The Review of Economic Studies* 87, no. 1 (2020): 1-39.

[5] Baumol, William J., and Wallace E. Oates. *The theory of environmental policy.* Cambridge University Press, 1988.

[6] Beiser-McGrath, Liam F., and Thomas Bernauer. "Could revenue recycling make effective carbon taxation politically feasible?." *Science Advances* 5, no. 9 (2019): eaax3323.

[7] Belfiori, Elisa, and Armon Rezai. "Implicit carbon prices: Making do with the taxes we have." *Journal of Environmental Economics and Management* (2024): 102950.

[8] Caselli, Francesco, and Wilbur John Coleman. "The world technology frontier." *American Economic Review* 96, no. 3 (2006): 499-522.

[9] Casey, Gregory, Woongchan Jeon, and Christian Traeger. "The Macroeconomics of Clean Energy Subsidies." (2023).

[10] Casey, Gregory. "Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation." *Review of Economic Studies*, 2023. https://doi.org/10.1093/restud/rdad001.

[11] Cruz, José-Luis, and Esteban Rossi-Hansberg. *The economic geography of global warming.* No. w28466. National Bureau of Economic Research, 2021.

[12] Fischer, Carolyn, and Richard G. Newell. "Environmental and technology policies for climate mitigation." *Journal of Environmental Economics and Management* 55, no. 2 (2008): 142-162.

[13] Fried, Stephie. "Climate policy and innovation: A quantitative macroeconomic analysis." *American Economic Journal: Macroeconomics* 10, no. 1 (2018): 90-118.

[14] Gerlagh, Reyer. "A climate-change policy induced shift from innovations in carbon-energy production to carbon-energy savings." *Energy Economics* 30, no. 2 (2008): 425-448.

[15] Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. "Optimal taxes on fossil fuel in general equilibrium." *Econometrica* 82, no. 1 (2014): 41-88.

[16] Growiec, Jakub. "A microfoundation for normalized CES production functions with factor-augmenting technical change." *Journal of Economic Dynamics and Control* 37, no. 11 (2013): 2336-2350.

[17] Growiec, Jakub. "A new class of production functions and an argument against purely labor-augmenting technical change." *International Journal of Economic Theory* 4, no. 4 (2008): 483-502.

[18] Gugler, Klaus, Adhurim Haxhimusa, and Mario Liebensteiner. "Effectiveness of climate policies: Carbon pricing vs. subsidizing renewables." *Journal of Environmental Economics and Management* 106 (2021): 102405.

[19] Hassler, John, Per Krusell, and Conny Olovsson. "Directed technical change as a response to natural resource scarcity." *Journal of Political Economy* 129, no. 11 (2021): 3039-3072.

[20] Hassler, John, Per Krusell, and Conny Olovsson. "Presidential address 2020 suboptimal climate policy." *Journal of the European Economic Association* 19, no. 6 (2021): 2895-2928.

[21] Hassler, John, Per Krusell, and Jonas Nycander. "Climate policy." *Economic Policy* 31, no. 87 (2016): 503-558.

[22] Hassler, John, Per Krusell, Conny Olovsson, and Michael Reiter. "On the effectiveness of climate policies." *IIES WP* 53 (2020): 54.

[23] Hawkins-Pierot, Jonathan T., Consumer Financial Protection Bureau, and Katherine RH Wagner. "Technology Lock-In and Costs of Delayed Climate Policy." (2023).

[24] Hinkelmann, Stefan. (Be-)Coming Clean: A Model of the U.S. Energy Transition. \bibitem{ipcc2013} IPCC Climate Change, \textit{The Physical Science Basis}, Intergovernmental Panel on Climate Change, 2013.

[25] Jones, Charles I. "The shape of production functions and the direction of technical change." *The Quarterly Journal of Economics* 120, no. 2 (2005): 517-549.

[26] Kalkuhl, Matthias, Ottmar Edenhofer, and Kai Lessmann. "Renewable energy subsidies: Second-best policy or fatal aberration for mitigation?." *Resource and Energy Economics* 35, no. 3 (2013): 217-234.

[27] Känzig, Diego R., and Maximilian Konradt. *Climate Policy and the Economy: Evidence from Europe's Carbon Pricing Initiatives*. No. w31260. National Bureau of Economic Research, 2023.

[28] Knittel, Christopher R. "Automobiles on steroids: Product Attribute Trade-Offs and Technological Progress in the Automobile Sector." *American Economic Review* 101, no.

7 (2011): 3368-3399.

[29] Labandeira, Xavier, José M. Labeaga, and Xiral López-Otero. "A meta-analysis on the price elasticity of energy demand." *Energy Policy* 102 (2017): 549-568.

[30] Lemoine, Derek. "Innovation-led transitions in energy supply." *American Economic Journal: Macroeconomics* 16, no. 1 (2024): 29-65.

[31] León-Ledesma, Miguel A., and Mathan Satchi. "Appropriate technology and balanced growth." *The Review of Economic Studies* 86, no. 2 (2019): 807-835.

[32] Newell, Richard G., William A. Pizer, and Daniel Raimi. "US federal government subsidies for clean energy: Design choices and implications." *Energy Economics* 80 (2019): 831-841.

[33] Nordhaus, William D. "Revisiting the social cost of carbon." *Proceedings of the National Academy of Sciences* 114, no. 7 (2017): 1518-1523.

[34] Nordhaus, William D. "Economic growth and climate: the carbon dioxide problem." *The American Economic Review* 67, no. 1 (1977): 341-346.

[35] Nordhaus, William D. "Optimal greenhouse-gas reductions and tax policy in the 'DICE' model." *The American Economic Review* 83, no. 2 (1993): 313-317.

[36] Nordhaus, William D., and Zili Yang. "A regional dynamic general-equilibrium model of alternative climate-change strategies." *The American Economic Review* (1996): 741-765.

[37] Pindyck, Robert S. "Interfuel substitution and the industrial demand for energy: an international comparison." *The Review of Economics and Statistics* (1979): 169-179.

[38] Popp, David. "R&D subsidies and climate policy: is there a "free lunch"?." *Climatic Change* 77, no. 3-4 (2006): 311-341.

[39] Rudik, Ivan. "Optimal climate policy when damages are unknown." *American Economic Journal: Economic Policy* 12, no. 2 (2020): 340-373.

[40] Scheidegger, Simon, and Ilias Bilionis. "Machine learning for high-dimensional dynamic stochastic economies." *Journal of Computational Science* 33 (2019): 68-82.

[41] Stern, D. I. (2012). "Interfuel Substitution: A Meta-Analysis," *Journal of Economic Surveys*, 26, 307-331.

[42] Sinn, Hans-Werner. "The green paradox." In *CESifo Forum*, vol. 10, no. 3, pp. 10-13. München: ifo Institut für Wirtschaftsforschung an der Universität München, 2009.

[43] Van Der Ploeg, Frederick, and Cees Withagen. "Growth, renewables, and the optimal carbon tax." *International Economic Review* 55, no. 1 (2014): 283-311.

[44] Gao, Jiti, Bin Peng, and Russell Smyth. "On income and price elasticities for energy demand: A panel data study." *Energy Economics* 96 (2021): 105168.

Appendix A: Directed Technical Change and Technology Choice on the Technology Menu

In this appendix, I show that the baseline model featuring the technology choice of the firm-household also can be interpreted as the directed technical change between capita-labor-augmenting technology $B_{K,t}$ and the energy-augmenting technology $B_{E,t}$. That is, the production technology of the firm-household is

$$Y_t = \left(\left(B_{K,t} K_t^{\sigma} L^{1-\sigma} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(B_{E,t} E_t \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}$$

Here, we define

$$\theta_t \equiv \frac{B_{E,t}}{B_{K,t}}$$

and

$$X_t \equiv B_{K,t} \theta_t^{1-\nu}$$

Then,

$$B_{K,t} = X_t \theta_t^{\nu - 1}$$

and

$$B_{E,t} = X_t \theta_t^{\nu}$$

The production function then becomes

$$Y_t = X_t \left(\left(\theta_t^{\nu - 1} K_t^{\sigma} L^{1-\sigma} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} E_t \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}$$

We assume that the firm-household gets access to a unit mass of R&D resources that can be either allocated to the improvement of $B_{E,t}$ (n_t) or that of $B_{K,t}$ $(1 - n_t)$ with the speed g :

$$B_{E,t} = B_{E,t-1} \left(1 + n_t g \right)$$

$$B_{K,t} = B_{K,t-1} \left(1 + \left(1 - n_t \right) g \right)$$

Then,

$$\frac{\theta_t}{\theta_{t-1}} = \frac{B_{E,t}}{B_{K,t}} / \frac{B_{E,t-1}}{B_{K,t-1}} = \frac{1+n_t g}{1+(1-n_t)\,g}$$

That is, the firm-household effectively chooses θ_t on a log-linear technology menu. In contrast to the baseline model, the model does not feature a technology adjustment cost. Instead, the friction that leads to the difference between the short-run and long-run price elasticity of energy is caused by the limited R&D resources as $\frac{1}{1+g} \leq \frac{\theta_t}{\theta_{t-1}} \leq 1+g$. In the end, we can detrend the model by defining $y_t = \frac{Y_t}{X_t^m L}, k_t = \frac{K_t}{X_t^m L}, e_t = \frac{E_t}{X_t^m L}$, and $\tilde{\theta}_t = \frac{\theta_t}{X_t^n}$ with $m = \frac{1}{1-\sigma}\frac{1}{\nu}$ and $n = -\frac{1}{\nu}$. The normalized production function is then

$$y_t = \left(\left(\tilde{\theta}_t^{\nu-1} k_t^{\sigma}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\tilde{\theta}_t^{\nu} e_t\right)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}}$$

Appendix B: The Normalized Model

In this appendix, I lay out the model after the normalization. I define $\tilde{k}_t = \frac{k_t}{A_t l}, \tilde{c}_t = \frac{c_t}{A_t}, \tilde{y}_t = \frac{y_t}{A_t l}, \tilde{e}_t = \frac{e_t}{A_t t}, \tilde{e}_{b,t} = \frac{e_{b,t}}{A_t l}, \tilde{e}_{g,t} = \frac{e_{g,t}}{A_t l}, \tilde{\tau}_{\theta,t} = \frac{\tau_{\theta,t}}{A_t l}, \tilde{T}_{\theta,t} = \frac{T_{\theta,t}}{A_t l}, \tilde{T}_{e,t} = \frac{T_{e,t}}{A_t l}, \tilde{\eta}_t = A_t * l * \eta_t, \tilde{\mathcal{V}}_t = \frac{\mathcal{V}_t}{A_t^{1-\alpha} l}$, and $\tilde{\beta} = e^{(1-\alpha)g}\beta$. Here, $\tilde{\eta}_t$ captures the carbon intensity of brown energy per unit of effective labor $\tilde{e}_{b,t}$.

The energy firm's optimization problem then becomes

$$\max_{e_{b,t}, e_{g,t}} p_{e,t}\tilde{e}_t - cost_{b,t}\tilde{e}_{b,t} - cost_{g,t}\tilde{e}_{g,t} - \tau_{b,t}\tilde{e}_{b,t} - \tau_{g,t}\tilde{e}_{g,t} + \tilde{T}_{e,t}$$

s.t.

$$\tilde{e}_t = \left(\lambda\left(\tilde{e}_{b,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)\left(\tilde{e}_{g,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}} \tag{7*}$$

The firm-household's problem is then

$$\begin{aligned} &\tilde{\mathcal{V}}\left(\theta_{t-1}, k_t, M_t, A_t, \eta_t\right) \\ = &\max_{\theta_t, \tilde{k}_{t+1}, M_{t+1}, c_t} \frac{\tilde{c}_t^{1-\alpha}}{1-\alpha} + \tilde{\beta}\tilde{\mathcal{V}}\left(\theta_t, k_{t+1}, M_{t+1}, A_{t+1}, \eta_{t+1}\right) \end{aligned}$$

s.t.

$$M_{t+1} = M_t + \eta_t \tilde{e}_{b,t}$$

$$y_t = \left(\left(\theta_t^{\nu-1} \tilde{k}_t^{\sigma} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} \tilde{e}_t \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \tag{8*}$$

$$\tilde{c}_t + e^g \tilde{k}_{t+1} - (1-\delta) \tilde{k}_t \tag{9*}$$

$$= D(M_t) \Psi \left(\frac{\theta_t}{\theta_{t-1}} \right) \left(\left(\theta_t^{\nu-1} * \tilde{k}_t^{\sigma} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\theta_t^{\nu} \tilde{e}_t \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - p_{e,t} \tilde{e}_t + \tilde{\tau}_{\theta,t} \theta_t \tag{10*}$$

$$+ \tilde{T}_{y,t}$$

The implementability constraints faced by the government then become

$$\frac{e_{b,t}}{e_{g,t}} = \left(\frac{\lambda}{1-\lambda} \frac{cost_{b,t} + \tau_{b,t}}{cost_{g,t} + \tau_{g,t}} \right)^{-\epsilon_e} \tag{11*}$$

$$\left(1 - d(\chi M_t)^2\right) * \Psi \left(\frac{\theta_t}{\theta_{t-1}} \right) y_t^{\frac{1}{\epsilon_y}} * \left(\theta_t^{\nu} e_t \right)^{-\frac{1}{\epsilon_y}} \theta_t^{\nu} = p_{e,t} \tag{12*}$$

$$\tilde{c}_t^{-\alpha} \Bigg\{ \left(1 - d(\chi M_t)^2\right) \Psi' \left(\frac{\theta_t}{\theta_{t-1}} \right) \frac{1}{\theta_{t-1}} \tilde{y}_t$$

$$+ \left(1 - d(\chi M_t)^2\right) \Psi \left(\frac{\theta_t}{\theta_{t-1}} \right) \tilde{y}^{\frac{1}{\epsilon_y}} \begin{pmatrix} \left(\theta_t^{\nu-1} \tilde{k}_{t+1}^{\sigma} \right)^{-\frac{1}{\epsilon_y}} (\nu - 1) \theta_t^{\nu-2} \tilde{k}_{t+1}^{\sigma} \\ + \left(\theta_t^{\nu} \tilde{e}_t \right)^{-\frac{1}{\epsilon_y}} \nu \theta_t^{\nu-1} \tilde{e}_t + \tilde{\tau}_{\theta,t} \end{pmatrix} \Bigg\} \tag{13*}$$

$$- e^g \tilde{c}_t^{-\alpha} + \tilde{\beta} \tilde{c}_{t+1}^{-\alpha}$$

$$+ \left(\left(\left(1 - d(\chi M_{t+1})^2\right) \Psi \left(\frac{\theta_{t+1}}{\theta_t} \right) \tilde{y}_{t+1}^{\frac{1}{\epsilon_{t+1}}} \left(\theta_{t+1}^{\nu-1} * \tilde{k}_{t+1}^{\sigma} \right)^{-\frac{1}{\epsilon_y}} \theta_{t+1}^{\nu-1} \tilde{k}_{t+1}^{\sigma-1} \right.\right.$$

$$\left. * \left(1 - d(\chi M_{t+1})^2\right) \Psi' \left(\frac{\theta_{t+1}}{\theta_t} \right) \left(- \frac{\theta_{t+1}}{\theta_t^2} \right) \tilde{y}_{t+1} \right) = 0 \tag{14*}$$

$$+ (1 - \delta))) = 0$$

Appendix C: Implementation of the Social Optimum when $\epsilon_y = 0$

If $\epsilon_y = 0$, we have

$$y_t = \min \left\{ \theta_t^{\nu-1} A_t^{1-\sigma} k_t^{\sigma} l^{1-\sigma}, \theta_t^{\nu} e_t \right\}$$

Per Appendix B, after proper normalization, we have

$$y_t = \min\left\{\theta_t^{\nu-1}k_t^{\sigma}, \theta_t^{\nu}e_t\right\}$$
$$c_t + k_{t+1} - (1-\delta)k_t + cost_b e_{b,t} + cost_g e_{g,t} = D\left(M_t\right)\Psi\left(\frac{\theta_{t+1}}{\theta_t}\right)y_t$$
$$e_t = \left(\lambda\left(e_{b,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)\left(e_{g,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}}$$
$$M_{t+1} = M_t + \eta_t e_{b,t}$$

The social planner's problem is

$$\begin{aligned}&\max\sum_{t=0}^{\infty}\beta^t\frac{\left(D\left(M_t\right)\Psi\left(\frac{\theta_{t+1}}{\theta_t}\right)\theta_t^{\nu-1}k_t^{\sigma} - k_{t+1} + (1-\delta)k_t - cost_{b,t}e_{b,t} - cost_{g,t}e_{g,t}\right)^{1-\alpha}}{1-\alpha}\\ &\quad -\lambda_t\left(\frac{1}{\theta_t}k_t^{\sigma} - \left(\lambda\left(e_{b,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}} + (1-\lambda)\left(e_{g,t}\right)^{\frac{\epsilon_e-1}{\epsilon_e}}\right)^{\frac{\epsilon_e}{\epsilon_e-1}}\right) + \tilde{\kappa}_t\left(M_{t+1} - M_t - \eta_t e_{b,t}\right)\end{aligned}$$

The FOCs are

$$-c_t^{-\alpha} + \beta\left(\left(D\left(M_{t+1}\right)\Psi\left(\frac{\theta_{t+2}}{\theta_{t+1}}\right)\theta_{t+1}^{\nu-1}\sigma k_{t+1}^{\sigma-1} + (1-\delta)k_{t+1}\right)c_{t+1}^{-\alpha} - \lambda_{t+1}\frac{1}{\theta_{t+1}}\sigma k_{t+1}^{\sigma-1}\right) = 0 \tag{C.1}$$

$$\begin{aligned}&\left(D\left(M_t\right)\Psi'\left(\frac{\theta_t}{\theta_{t-1}}\right)\theta_t^{\nu-1}k_t^{\sigma} + D\left(M_t\right)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\frac{1}{\theta_{t-1}}(\nu-1)\theta_t^{\nu-2}k_t^{\sigma}\right)c_t^{-\alpha} - \lambda_t\left(-\frac{1}{\theta_t^2}k_t^{\sigma}\right) \qquad \text{(C.2)}\\ &+\beta\left(D\left(M_{t+1}\right)\Psi'\left(\frac{\theta_{t+2}}{\theta_{t+1}}\right)\left(-\frac{1}{\theta_{t+1}^2}\right)\theta_{t+1}^{\nu-1}\sigma k_{t+1}^{\sigma-1}\right)c_{t+1}^{-\alpha} = 0\end{aligned}$$

$$-cost_b c_t^{-\alpha} - \lambda_t\left(-\lambda\left(e_{b,t}\right)^{-\frac{1}{\epsilon_e}}e_t^{\frac{1}{\epsilon_e}}\right) - \tilde{\kappa}_t\eta_t = 0 \tag{C.3}$$

$$-cost_g c_t^{-\alpha} - \lambda_t\left(-(1-\lambda)\left(e_{g,t}\right)^{-\frac{1}{\epsilon_e}}e_t^{\frac{1}{\epsilon_e}}\right) = 0 \tag{C.4}$$

$$\tilde{\kappa}_t - \beta\left(D'\left(M_{t+1}\right)y_{t+1}c_{t+1}^{-\alpha} + \tilde{\kappa}_{t+1}\right) = 0 \tag{C.5}$$

The market equilibrium conditions are instead characterized by

$$-c_t^{-\alpha} + \beta\left(\left(D\left(M_{t+1}\right)\Psi\left(\frac{\theta_{t+2}}{\theta_{t+1}}\right)\theta_{t+1}^{\nu-1}\sigma k_{t+1}^{\sigma-1} + (1-\delta)k_{t+1}\right)c_{t+1}^{-\alpha} - \lambda_{t+1}\frac{1}{\theta_{t+1}}\sigma k_{t+1}^{\sigma-1}\right) = 0 \tag{C.6}$$

$$\left(D\left(M_t\right)\Psi'\left(\frac{\theta_t}{\theta_{t-1}}\right)\theta_t^{\nu-1}k_t^{\sigma} + D\left(M_t\right)\Psi\left(\frac{\theta_t}{\theta_{t-1}}\right)\frac{1}{\theta_{t-1}}(\nu-1)\theta_t^{\nu-2}k_t^{\sigma} + \tau_{\theta,t}\right)c_t^{-\alpha} \tag{C.7}$$

$$-\lambda_t\left(-\frac{1}{\theta_t^2}k_t^{\sigma}\right) + \beta\left(D\left(M_{t+1}\right)\Psi'\left(\frac{\theta_{t+2}}{\theta_{t+1}}\right)\left(-\frac{1}{\theta_{t+1}^2}\right)\theta_{t+1}^{\nu-1}\sigma k_{t+1}^{\sigma-1}\right)c_{t+1}^{-\alpha} = 0$$

$$-\left(cost_b + \tau_{b,t}\right)c_t^{-\alpha} - \lambda_t\left(-\lambda\left(e_{b,t}\right)^{-\frac{1}{\epsilon_e}}e_t^{\frac{1}{\epsilon_e}}\right) = 0 \tag{C.8}$$

$$-\left(cost_g + \tau_{g,t}\right)c_t^{-\alpha} - \lambda_t\left(-(1-\lambda)\left(e_{g,t}\right)^{-\frac{1}{\epsilon_e}}e_t^{\frac{1}{\epsilon_e}}\right) = 0 \tag{C.9}$$

By comparing C.1-5 and C.6-9, we can learn that either

$$\tau_{b,t}^* = \kappa_t\eta_t$$

or

$$\tau_g^* = \frac{(\mu_t - p_t)}{c_t^{-\alpha}}\left(-\frac{1}{\theta_t^2}k_t^{\sigma}\right)$$
$$\tau_\theta^* = cost_g\left(\frac{cost_b}{cost_b + \tau_{b,t}^* - 1}\right)$$

with

$$\kappa_t c_t^{-\alpha} = \tilde{\kappa}_t$$
$$\mu_t = \lambda_t^* = \left(\lambda^{\epsilon_e}\left(cost_b + SCC_t\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_g\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}$$
$$p_t = \lambda_t = \left(\lambda^{\epsilon_e}\left(cost_b\right)^{1-\epsilon_e} + (1-\lambda)^{\epsilon_e}\left(cost_{g,t} + \tau_g^*\right)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}}$$

Appendix D: The Robustness Test on the Carbon Intensity Trends

In this appendix, I examine how my results are affected by changes in the trend of per-effective labor carbon intensity. In the baseline model, $\eta_t = A_t * l * \eta_t$ is constant over time. Here, I extend the baseline model by assuming that this parameter is time-varying and follows the law of motion.

Table D.1: End-of-the-Century Cumulative Emissions and Welfare Gain Relative to BAU under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy

Policy Scenarios	M_t in 2100 when $\bar{\eta} = \eta_0 \times 0.5$	M_t in 2100 when $\bar{\eta} = \eta_0 \times 2.0$
The business-as-usual (BAU)	856	1758
Brown taxes	830	1327
Green subsidies	855	1742
Technology choice subsidies + green subsidies	832	1342

$$\eta_{t+1} = \eta_t \left(\frac{\bar{\eta}}{\eta_t} \right)^{\gamma_\eta}$$

where $\bar{\eta}$ is the steady state of η_t and γ_η denotes the convergence rate. γ_η is set to 0.1 . I examine two cases for the steady state $\bar{\eta}$: $\bar{\eta} = \eta_0 * 0.5$ and $\bar{\eta} = \eta_0 * 2.0$. The first scenario results in a declining per-effective labor intensity over time, while the second shows an increase. From Table D.1, we can see that introducing trends to η_t would not change my conclusion: the combined subsidies on green energy and energy efficiency still can match the welfare gain brought by carbon taxes.

Appendix E: Robustness Tests on a Range of Elasticities of Substitution between Green and Brown Energy Inputs

In this appendix, I test whether the combined subsidies can be an effective alternative to brown taxes on a range of brown-green elasticities of substitution, ϵ_e, in the baseline model. From Table E.1, we can learn that my conclusion is robust to variation of ϵ_e, regardless of whether the green and brown inputs are complementary goods or substitute goods.

Table E.1: End-of-the-Century Cumulative Emissions under Alternative Values of ϵ_e and Policy Scenarios

Policy Scenarios	$\epsilon_e = 0.985$	$\epsilon_e = 0.997$	$\epsilon_e = 1.01$	$\epsilon_e = 1.023$	$\epsilon_e = 1.037$
BAU	1536	1579	1580	1581	1581
Brown taxes	1214	1233	1231	1237	1239
Green subsidies	1232	1251	1255	1255	1257
Technology choice subsidies + green subsidies	1544	1587	1584	1580	1578
Policy Scenarios	$\epsilon_e = 1.05$	$\epsilon_e = 1.06$	$\epsilon_e = 1.08$	$\epsilon_e = 1.095$	$\epsilon_e = 1.11$
BAU	1582	1583	1583	1584	1585
Brown taxes	1241	1244	1246	1249	1251
Green subsidies	1258	1261	1262	1265	1267
Technology choice subsidies + green subsidies	1576	1574	1572	1570	1569

Appendix F: Figures

Figure F.1: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when the Adjustment Cost Parameter $\gamma = 10.0$

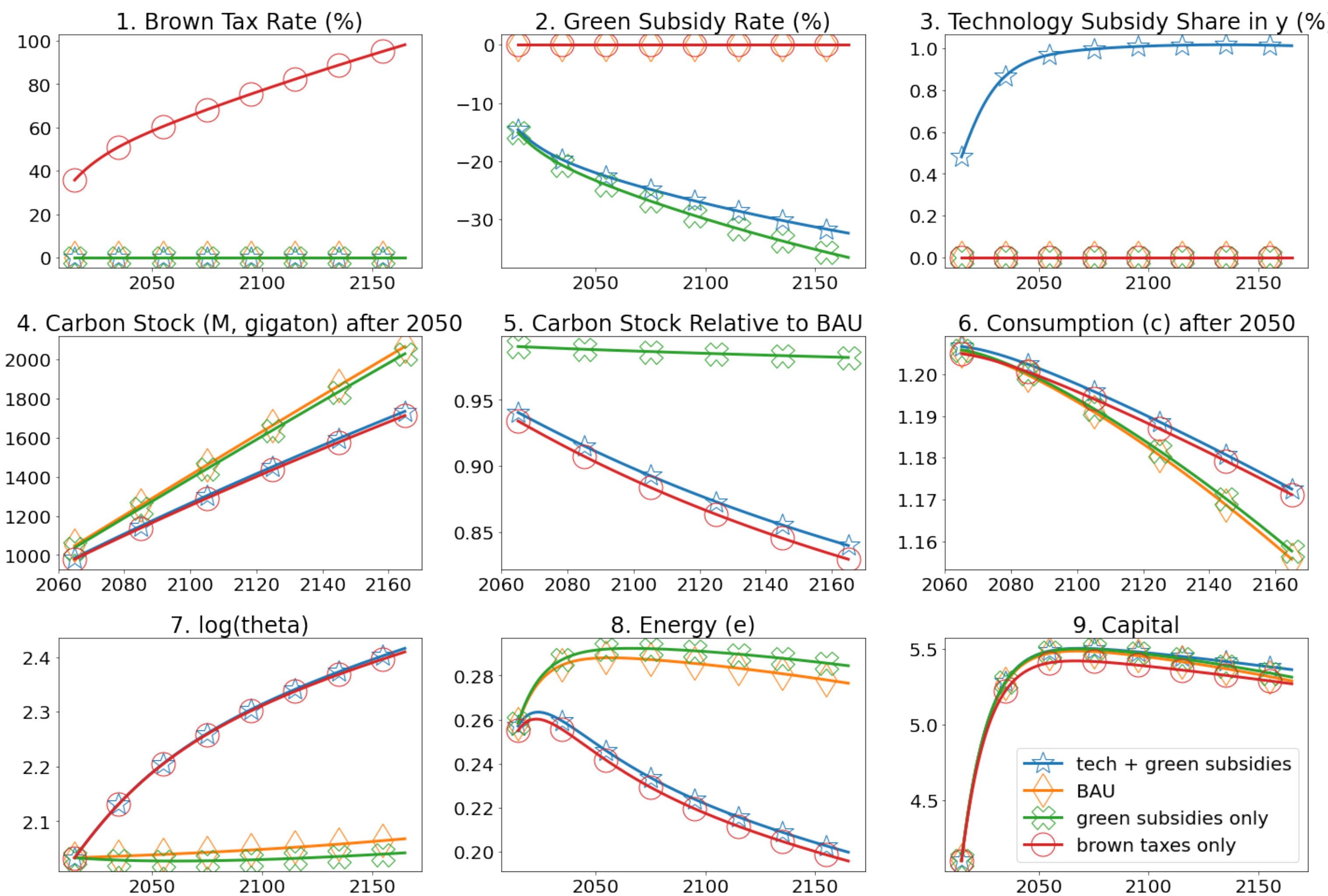


Figure F.2: Paths of Endogenous Variables under Carbon Tax (Brown Taxes), BAU, Green Subsidy, and Green + Energy Efficiency Subsidy when the Adjustment Cost Parameter $\gamma = 20.0$