ECED 4301 Assignment #5 solution

Prob. 10.31

(a) $$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} = \underline{\underline{\frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \cos^2(\omega t - \beta z)\boldsymbol{a}_z}}$$

(b) $$\boldsymbol{P}_{ave} = \frac{1}{T}\int_0^T \boldsymbol{E} \times \boldsymbol{H} dt = \underline{\underline{\frac{V_o I_o}{4\pi\rho^2 \ln(b/a)} \boldsymbol{a}_z}}$$

Prob.10.34

(a)

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = 10^{-2}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+10^{-4}} - 1\right]} = \frac{2\pi\times 2\times 10^9}{3\times 10^8}\sqrt{\frac{2.5}{2}}\sqrt{0.00005} = 0.3311$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+10^{-4}} + 1\right]} = \frac{2\pi\times 2\times 10^9}{3\times 10^8}\sqrt{\frac{2.5}{2}}\sqrt{2.00005} = 66.23$$

(In this case, $\beta = \omega\sqrt{\mu\varepsilon}$.)

$$(1-0.2)E_o = E_o e^{-\alpha z} \longrightarrow 0.8E_o = E_o e^{-\alpha z}$$

$$e^{\alpha z} = 1.25 \longrightarrow z = \frac{1}{\alpha}\ln 1.25 = \underline{\underline{0.674\text{ m}}}$$

(b) $\beta z = 180^o = \pi \longrightarrow z = \frac{\pi}{\beta} = \underline{\underline{0.04743\text{ m}}}$

(c) $P = P_o e^{-2\alpha z} \longrightarrow 0.9P_o = P_o e^{-2\alpha z}$

$$e^{2\alpha z} = 1/0.9 = 1.111 \longrightarrow z = \frac{1}{2\alpha}\ln 1.111 = \underline{\underline{0.159\text{ m}}}$$

Prob. 10.35

(a) $\boldsymbol{u} = \omega/\beta \longrightarrow \omega = u\beta = \frac{\beta c}{\sqrt{4.5}} = \frac{2\times 3\times 10^8}{\sqrt{4.5}} = \underline{\underline{2.828\times 10^8}}$ rac

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7\Omega$$

$$\boldsymbol{H} = \boldsymbol{a}_k \times \frac{\boldsymbol{E}}{\eta} = \frac{\boldsymbol{a}_z}{\eta} \times \frac{40}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\rho = \underline{\underline{\frac{0.225}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\phi}}\ \text{A/m}$$

(b) $\mathcal{P} = \boldsymbol{E}\times\boldsymbol{H} = \underline{\underline{\frac{9}{\rho^2}\sin^2(\omega t - 2z)\boldsymbol{a}_z\ \text{W/m}^2}}$

(c) $\mathcal{P}_{ave} = \frac{4.5}{\rho^2}\boldsymbol{a}_z,\ d\mathbf{S} = \rho d\phi d\rho \boldsymbol{a}_z$

$$P_{ave} = \int \mathbf{P}_{ave} \bullet dS = 4.5 \int_{2mm}^{3mm} \frac{d\rho}{\rho}\int_0^{2\pi} d\phi = 4.5\ln(3/2)(2\pi) = \underline{\underline{11.46\text{ W}}}$$

Prob. 10.40

(a) $$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{4}} = 60\pi, \qquad \eta_1 = \eta_o = 120\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{180\pi} = -\frac{1}{3}, \qquad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{120\pi}{180\pi} = \frac{2}{3}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \underline{\underline{2}}$$

(b) $$\lambda_1 = c/f = \frac{3\times 10^8}{10^8} = \underline{\underline{3\text{ m}}}$$

$$\lambda_2 = u/f = \frac{\frac{c}{\sqrt{\varepsilon_r}}}{f} = \frac{3}{\sqrt{4}} = \underline{\underline{1.5\text{ m}}}$$

(c) $$P_i = \frac{E_{oi}^2}{2\eta_1}, \qquad P_r = \frac{E_{or}^2}{2\eta_1} = \frac{\Gamma^2 E_{oi}^2}{2\eta_1}$$

$$\frac{P_r}{P_i} = \Gamma^2 = \frac{1}{9} = 0.1111 \text{ or } \underline{\underline{11.11\%}}$$

Prob. 10.41 $\quad \eta_1 = \eta_o = 120\pi, \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad (1)$$

But $\quad E_{ro} = \eta_o H_{ro} \qquad (2)$

Combining (1) and (2),

$$E_{ro} = \eta_o H_{ro} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{io} \qquad \longrightarrow \qquad \eta_o = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \frac{E_{io}}{H_{ro}}$$

But $\dfrac{E_{io}}{H_{ro}} = \dfrac{3.6}{1.2\times10^{-3}} = 3000$

$$\eta_o = 3000\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \qquad \longrightarrow \qquad 377 = 3000\left(\frac{\eta_2 - 377}{\eta_2 + 377}\right)$$

Thus, $\eta_2 = 485.37$. Since $\eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}}$,

$$\mu_2 = \varepsilon_o \varepsilon_r \eta_2{}^2 = \frac{10^{-9}}{36\pi} \times 12.5 \times (485.37)^2 = \underline{\underline{2.604\times10^{-5}}} \text{ H/m}$$

Prob. 10.47 (a) $\omega = \beta c = 3 \times 3 \times 10^8 = \underline{\underline{9 \times 10^8 \text{ rad/s}}}$

(b) $\lambda = 2\pi / \beta = 2\pi / 3 = \underline{\underline{2.094 \text{ m}}}$

(c) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{4}{9 \times 10^8 \times 80 \times 10^{-9} / 36\pi} = 2\pi = \underline{\underline{6.288}}$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 6.288 \quad \longrightarrow \quad \theta_\eta = 40.47^\circ$$

$$|\eta_2| = \frac{\sqrt{\mu_2/\varepsilon_2}}{\sqrt[4]{1+\left(\dfrac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{377/\sqrt{80}}{\sqrt[4]{1+4\pi^2}} = 16.71$$

$$\eta_2 = \underline{\underline{16.71\angle 40.47^\circ \ \Omega}}$$

d) $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{16.71\angle 40.47^\circ - 377}{16.71\angle 40.47^\circ + 377} = 0.935\angle 179.7^\circ$

$$E_{or} = \Gamma E_{oi} = 9.35\angle 179.7^\circ$$

$$\boldsymbol{E}_r = \underline{\underline{9.35\sin(\omega t - 3z + 179.7)\boldsymbol{a}_x \text{ V/m}}}$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{\mu_{r2}\varepsilon_{r2}}{2}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2} - 1\right]} = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} - 1\right]} = 43.94 \text{ Np/m}$$

$$\beta_2 = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} + 1\right]} = 51.48 \text{ rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\times 16.71\angle 40.47^\circ}{16.71\angle 40.47^\circ + 377} = 0.0857\angle 38.89^\circ$$

$$E_{ot} = \tau E_o = 0.857\angle 38.89^o$$

$$\underline{E_t = 0.857e^{43.94z}\sin(9\times 10^8 t + 51.48z + 38.89^o)\ \text{V/m}}$$