

**Prob. 10.6 (a)**  $\frac{\sigma}{\omega \varepsilon} = \tan 2\theta_\eta = \tan 60^\circ = \underline{1.732}$

(b)  $|\eta| = 240 = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{2\varepsilon_r}} \rightarrow \varepsilon_r = \frac{\pi^2}{8} = \underline{1.234}$

(c)  $\varepsilon_c = \varepsilon(1 - j\frac{\sigma}{\omega \varepsilon}) = 1.234 \times \frac{10^{-9}}{36\pi} (1 - j1.732) = \underline{(1.091 - j1.89) \times 10^{-11}}$  F/m

(d)

$$\alpha = \frac{\omega}{c} \sqrt{\frac{\mu_r \varepsilon_r}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} - 1 \right]} = \frac{2\pi \times 10^6}{3 \times 10^8} \sqrt{\frac{1}{2} \frac{\pi^2}{8} \left[ \sqrt{1+3} - 1 \right]} = \underline{0.0164} \text{ Np/m}$$

### Prob. 10.7

$$\frac{\sigma}{\omega \varepsilon} = \frac{4}{2\pi \times 10^5 \times 80 \times 10^{-9} / 36\pi} = 9,000 \gg 1$$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\frac{2\pi \times 10^5}{2} \times 4\pi \times 10^{-7} \times 4} = 0.4\pi$$

(a)  $u = \omega / \beta = \frac{2\pi \times 10^5}{0.4\pi} = \underline{5 \times 10^5}$  m/s

(b)  $\lambda = 2\pi / \beta = \frac{2\pi}{0.4\pi} = \underline{5}$  m

(c)  $\delta = l / \alpha = \frac{l}{0.4\pi} = \underline{0.796}$  m

(d)  $\eta = |\eta| \angle \theta_\eta, \theta_\eta = 45^\circ$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2}} \approx \sqrt{\frac{\mu \omega \varepsilon}{\varepsilon \sigma}} = \sqrt{\frac{4\pi \times 10^{-7} \times 2\pi \times 10^5}{4}} = 0.4443$$

**Prob. 10.10 (a)**  $\gamma = \alpha + j\beta = \underline{0.05 + j2}$  /m

(b)  $\lambda = 2\pi / \beta = \underline{\pi} = \underline{3.142}$  m

$$(c) u = \omega / \beta = \frac{2 \times 10^8}{2} = \underline{10^8} \text{ m/s}$$

$$(d) \delta = l / \alpha = \frac{l}{0.05} = \underline{20} \text{ m}$$

**Prob. 10.11 (a)**  $\beta = \omega / c = \frac{2\pi \times 10^6}{3 \times 10^8} = \underline{0.02094}$  rad/m,

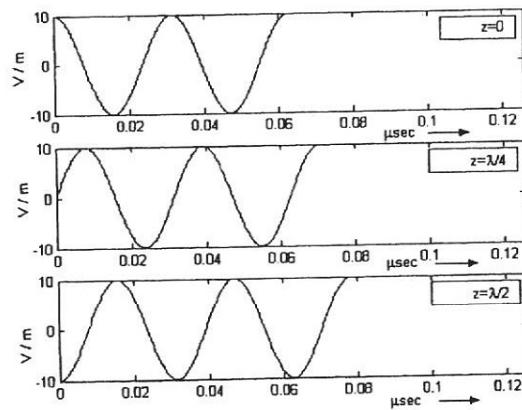
$$\lambda = 2\pi / \beta = \underline{300} \text{ m}$$

(b) When  $z = 0$ ,  $E_y = 10 \cos \omega t$

$$z = \lambda / 4, \quad E_y = 10 \cos(\omega t - \frac{2\pi}{\lambda} \frac{\lambda}{4}) = 10 \sin \omega t$$

$$z = \lambda / 2, \quad E_y = 10 \cos(\omega t - \pi) = -10 \cos \omega t$$

Thus E is sketched below.



(c)

$$H = \frac{10}{120\pi} \cos(2\pi \times 10^6 t - 2\pi z / 300) \mathbf{a}_x = \underline{26.53 \cos(2\pi \times 10^6 t - 0.02094 z)} \mathbf{a}_x \text{ mA/m}$$

$$(c) u = c = \underline{\underline{3 \times 10^8 \text{ m/s}}}$$

$$(d) \text{ Let } E = E_o \cos(\beta z + 40,000t) \mathbf{a}_E$$

$$E_o = 25 \times 120\pi = 9.425 \times 10^3$$

$$\mathbf{a}_E \cdot \mathbf{x} \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_E \cdot \mathbf{x} \mathbf{a}_y = -\mathbf{a}_z \longrightarrow \mathbf{a}_E = -\mathbf{a}_x$$

$$E = \underline{\underline{-9.425 \cos(\beta z + 40,000t) \mathbf{a}_x \text{ kV/m}}}$$

### Prob. 10.17

(a) along +  $\mathbf{a}_x$

$$(b) u = \frac{\omega}{\beta} = \frac{1}{3} \times 10^8 \text{ m/s}$$

$$(c) \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{3} = \underline{\underline{2.0944 \text{ m}}}$$

### Prob. 10.18

$$(a) \nabla \times \mathbf{E} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z,t) & E_y(z,t) & 0 \end{vmatrix} = -\frac{\partial E_y}{\partial z} \mathbf{a}_x + \frac{\partial E_x}{\partial z} \mathbf{a}_y$$

$$= -6\beta \cos(\omega t - \beta z) \mathbf{a}_x + 8\beta \sin(\omega t - \beta z) \mathbf{a}_y$$

$$\text{But } \nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \longrightarrow \mathbf{H} = -\frac{1}{\mu} \int \nabla \times \mathbf{E} dt$$

$$\mathbf{H} = \underline{\underline{\frac{6\beta}{\mu\omega} \sin(\omega t - \beta z) \mathbf{a}_x + \frac{8\beta}{\mu\omega} \cos(\omega t - \beta z) \mathbf{a}_y}}$$

$$(b) \beta = \omega \sqrt{\mu\epsilon} = \frac{2\pi f}{c} \sqrt{4.5} = \frac{2\pi \times 40 \times 10^6}{3 \times 10^8} \sqrt{4.5} = \underline{\underline{1.777 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.777} = \underline{\underline{3.536 \text{ m}}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{4.5}} = \underline{\underline{177.72 \Omega}}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{4.5}} = \frac{3 \times 10^8}{\sqrt{4.5}} = \underline{\underline{1.4142 \times 10^8 \text{ m/s}}}$$

**Prob. 10.19**  $\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi \times 10^7}{3 \times 10^8}(10) = \underline{\underline{2.0943}} \text{ rad/m}$

$$\mathbf{H} = -\frac{1}{\mu} \int \nabla \times \mathbf{E} dt$$

$$\nabla \times \mathbf{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x) & E_z(x) \end{vmatrix} = -\frac{\partial E_z}{\partial x} \mathbf{a}_y + \frac{\partial E_y}{\partial x} \mathbf{a}_z = -10\beta \sin(\omega t - \beta x)(\mathbf{a}_y - \mathbf{a}_z)$$

$$\mathbf{H} = -\frac{10\beta}{\omega\mu} \cos(\omega t - \beta x)(\mathbf{a}_y - \mathbf{a}_z) = -\frac{10 \times 2\pi/3}{2\pi \times 10^7 \times 50 \times 4\pi \times 10^{-7}} \cos(\omega t - \beta x)(\mathbf{a}_y - \mathbf{a}_z)$$

$$\underline{\underline{\mathbf{H} = 5.305 \cos(2\pi \times 10^7 t - 2.0943x)(-\mathbf{a}_y + \mathbf{a}_z)}} \text{ mA/m}$$

**Prob. 10.20**

$$0.4E_o = E_o e^{-\alpha z} \longrightarrow \frac{1}{0.4} = e^{2\alpha}$$

$$\text{Or } \alpha = \frac{1}{2} \ln \frac{1}{0.4} = 0.4581 \longrightarrow \delta = 1/\alpha = \underline{\underline{2.183}} \text{ m}$$

$$\lambda = 2\pi/\beta = 2\pi/1.6$$

$$u = f\lambda = 10^7 \times \frac{2\pi}{1.6} = \underline{\underline{3.927 \times 10^7}} \text{ m/s}$$

**Prob. 10.21**

$$(a) \quad \mathbf{E} = \operatorname{Re}[E_s e^{j\omega t}] = (5\mathbf{a}_x + 12\mathbf{a}_y)e^{-0.2z} \cos(\omega t - 3.4z)$$

$$\text{At } z = 4\text{m}, t = T/8, \quad \text{so } t = \frac{2\pi}{T} \frac{T}{8} = \frac{\pi}{4}$$

$$\mathbf{E} = (5\mathbf{a}_x + 12\mathbf{a}_y)e^{-0.8} \cos(\pi/4 - 13.6)$$

$$|E| = 13e^{-0.8} |\cos(\pi/4 - 13.6)| = \underline{\underline{5.662}} \text{ V/m}$$

(b) loss =  $\alpha \Delta z = 0.2(3) = 0.6 \text{ Np}$ . Since  $1 \text{ Np} = 8.686 \text{ dB}$ ,

$$\text{loss} = 0.6 \times 8.686 = \underline{\underline{5.212}} \text{ dB}$$

$$(c) \text{ Let } x = \sqrt{I + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$$

$$\frac{\alpha}{\beta} = \left(\frac{x-I}{x+I}\right)^{1/2} = 0.2 / 3.4 = \frac{I}{17}$$

$$\frac{x-I}{x+I} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega \sqrt{\mu \epsilon / 2} \sqrt{x-I} = \frac{\omega}{c} \sqrt{\epsilon_r / 2} \sqrt{x-I}$$

$$\sqrt{\frac{\epsilon_r}{2}} = \frac{\alpha c}{\omega \sqrt{x-1}} = \frac{0.2 \times 3 \times 10^8}{10^8 \sqrt{0.00694}} = 7.2 \quad \longrightarrow \quad \epsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\mu_o} \cdot \frac{1}{\sqrt{\epsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68 \times 1.00694}} = 36.896$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega \epsilon} = \sqrt{x^2 - 1} = 0.118 \quad \longrightarrow \quad \theta_\eta = 3.365^\circ$$

$$\eta = 36.896 \angle 3.365^\circ \Omega$$

$$H_s = \mathbf{a}_k \times \frac{\mathbf{E}_s}{\eta} = \frac{\mathbf{a}_z}{\eta} \times (5\mathbf{a}_x + 12\mathbf{a}_y) e^{-\gamma z} = \frac{(5\mathbf{a}_y - 12\mathbf{a}_x)}{|\eta|} e^{-j3.365^\circ} e^{-\gamma z}$$

$$H = (-325.24\mathbf{a}_x + 135.5\mathbf{a}_y) e^{-0.2z} \cos(\omega t - 3.4z - 3.365^\circ) \text{ mA}$$

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 5 & 12 & 0 \\ -325.24 & 135.5 & 0 \end{vmatrix} \times 10^{-3} e^{-0.4z} \cos(\omega t - 3.4z) \cos(\omega t - 3.4z - 3.365^\circ)$$

$$\mathbf{P} = 4.58 e^{-0.4z} \cos(\omega t - 3.4z) \cos(\omega t - 3.4z - 3.365^\circ) \mathbf{a}_z$$

At z=4, t=T/4,

$$\mathbf{P} = 4.58 e^{-1.6} \cos(\pi/4 - 13.6) \cos(\pi/4 - 13.6 - 0.0587) \mathbf{a}_z = \underline{\underline{0.8545 \mathbf{a}_z \text{ W/m}^2}}$$

**Prob. 10.22** For a good conductor,  $\frac{\sigma}{\omega \epsilon} \gg I$ , say  $\frac{\sigma}{\omega \epsilon} > 100$

(b)  $R_{ac} = \frac{l}{\sigma 2\pi a \delta}$ . At 100 MHz,  $\delta = 6.6 \times 10^{-3}$  mm for copper (see Table 10.2).

$$R_{ac} = \frac{600}{5.8 \times 10^7 \times 2\pi \times (1.2 \times 10^{-3}) \times 6.6 \times 10^{-3}} = \underline{\underline{0.2079 \Omega}}$$

(c)  $\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = 1 \longrightarrow \delta = a/2 = \frac{66.1 \times 10^{-3}}{\sqrt{f}}$

$$\sqrt{f} = \frac{66.1 \times 2 \times 10^{-3}}{a} = \frac{66.1 \times 2}{1.2} \longrightarrow f = \underline{\underline{12.137 \text{ kHz}}}$$

### Prob. 10.25

(a)  $\tan \theta = \frac{\sigma}{\omega \epsilon} = \frac{3.5 \times 10^7}{2\pi \times 150 \times 10^6 \times \frac{10^{-9}}{36\pi}} = \frac{3.5 \times 18 \times 10^9}{15} \gg 1$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\pi f \mu \sigma} = \sqrt{150 \pi \times 10^6 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7} = 143,965.86$$

$$\gamma = \alpha + j\beta = \underline{\underline{1.44(1+j)x10^5 /m}}$$

(b)  $\delta = 1/\alpha = \underline{\underline{6.946 \times 10^{-6} \text{ m}}}$

(c)  $u = \frac{\omega}{\beta} = \frac{150 \times 2\pi \times 10^6}{1.44 \times 10^5} = \underline{\underline{6547 \text{ m/s}}}$

### Prob. 10.26

$$\omega = 10^6 \pi = 2\pi f \longrightarrow f = 0.5 \times 10^6$$

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\sqrt{\pi \times 0.5 \times 10^6 \times 3.5 \times 10^7 \times 4\pi \times 10^{-7}}} = \underline{\underline{0.1203 \text{ mm}}}$$

$$R_{ac} = \frac{l}{\sigma \delta w}$$

since  $\delta$  is very small,  $w = 2\pi \rho_{outer}$

$$R_{ac} = \frac{l}{\sigma 2\pi \rho_{outer} \delta} = \frac{40}{3.5 \times 10^7 \times 2\pi \times 12 \times 10^{-6} \times 0.1203} = \underline{\underline{0.126 \Omega}}$$

$$\mathcal{P}_{ave} = \frac{1}{2} \operatorname{Re}(E_s \times H_s^*)$$

as required.

### Prob. 10.31

$$(a) \quad P = E \times H = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \cos^2(\omega t - \beta z) \mathbf{a}_z$$

$$(b) \quad P_{ave} = \frac{1}{T} \int_0^T E \times H dt = \frac{V_o I_o}{4\pi\rho^2 \ln(b/a)} \mathbf{a}_z$$

### Prob. 10.32

$$(a) \quad H_s = \frac{j30\beta I_o dl}{120\pi r} \sin \theta e^{-j\beta r} \mathbf{a}_H$$

$$\text{where } \mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \quad \longrightarrow \quad \mathbf{a}_\theta \times \mathbf{a}_H = \mathbf{a}_r \quad \longrightarrow \quad \mathbf{a}_H = \mathbf{a}_\phi$$

$$H_s = \frac{j\beta I_o dl}{4\pi r} \sin \theta e^{-j\beta r} \mathbf{a}_\phi$$

$$(b) \quad P_{ave} = \frac{1}{2} \operatorname{Re}[E_s \times H_s^*] = \frac{1}{2} \operatorname{Re}\left[\frac{30\beta^2 I_o^2 dl^2 \sin^2 \theta}{4\pi r^2} \mathbf{a}_r\right] = \frac{15\beta^2 I_o^2 dl^2 \sin^2 \theta}{4\pi r^2} \mathbf{a}_r$$

### Prob. 10.33

$$(a) \quad P_{ave} = \frac{1}{2} \operatorname{Re}(E_s H_s^*) = \frac{1}{2} \operatorname{Re}\left(\frac{|E_s|}{|\eta|}\right) = \frac{8^2}{2|\eta|} e^{-0.2z}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega\varepsilon} \right)^2} - 1 \right]}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega\varepsilon} \right)^2} + 1 \right]}$$

$$\text{Let } x = \sqrt{1 + \left( \frac{\sigma}{\omega\varepsilon} \right)^2}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = 0.1/0.3 = 1/3$$

$$\frac{x-1}{x+1} = \frac{1}{9} \quad \longrightarrow \quad x = 5/4$$

$$\frac{5}{4} = \sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} \quad \longrightarrow \quad \frac{\sigma}{\omega \epsilon} = 3/4$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt[4]{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2}} = \frac{120\pi / \sqrt{81}}{\sqrt[4]{\frac{5}{4}}} = 37.4657$$

$$P_{ave} = \frac{64}{2(37.4657)} e^{-0.2z} = \underline{\underline{0.8541e^{-0.2z} \text{ W/m}^2}}$$

$$(b) \quad 20dB = 10 \log \frac{P_1}{P_2} \quad \longrightarrow \quad \frac{P_1}{P_2} = 100$$

$$\frac{P_2}{P_1} = e^{-0.2z} = \frac{1}{100} \quad \longrightarrow \quad e^{0.2z} = 100$$

$$z = 5 \log 100 = \underline{\underline{23 \text{ m}}}$$

### Prob.10.34

(a)

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right]}$$

$$\tan \theta = \frac{\sigma}{\omega \epsilon} = 10^{-2}$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[ \sqrt{1 + 10^{-4}} - 1 \right]} = \frac{2\pi \times 2 \times 10^9}{3 \times 10^8} \sqrt{\frac{2.5}{2}} \sqrt{0.00005} = 0.3311$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[ \sqrt{1 + 10^{-4}} + 1 \right]} = \frac{2\pi \times 2 \times 10^9}{3 \times 10^8} \sqrt{\frac{2.5}{2}} \sqrt{2.00005} = 66.23$$

(In this case,  $\beta = \omega \sqrt{\mu \epsilon}$ .)

$$(1 - 0.2)E_o = E_o e^{-\alpha z} \quad \longrightarrow \quad 0.8E_o = E_o e^{-\alpha z}$$

$$e^{\alpha z} = 1.25 \quad \longrightarrow \quad z = \frac{1}{\alpha} \ln 1.25 = \underline{\underline{0.674 \text{ m}}}$$

$$(b) \quad \beta z = 180^\circ = \pi \quad \longrightarrow \quad z = \frac{\pi}{\beta} = \underline{0.04743 \text{ m}}$$

$$(c) \quad P = P_o e^{-2\alpha z} \quad \longrightarrow \quad 0.9 P_o = P_o e^{-2\alpha z}$$

$$e^{2\alpha z} = 1/0.9 = 1.111 \quad \longrightarrow \quad z = \frac{1}{2\alpha} \ln 1.111 = \underline{0.159 \text{ m}}$$

**Prob. 10.35**

$$(a) \quad u = \omega / \beta \quad \longrightarrow \quad \omega = u\beta = \frac{\beta c}{\sqrt{4.5}} = \frac{2 \times 3 \times 10^8}{\sqrt{4.5}} = \underline{2.828 \times 10^8 \text{ rad/s}}$$

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7 \Omega$$

$$\mathbf{H} = \mathbf{a}_k \times \frac{\mathbf{E}}{\eta} = \frac{\mathbf{a}_z}{\eta} \times \frac{40}{\rho} \sin(\omega t - 2z) \mathbf{a}_\rho = \frac{0.225}{\rho} \sin(\omega t - 2z) \mathbf{a}_\phi \quad \text{A/m}$$

$$(b) \quad \mathcal{P} = \mathbf{E} \times \mathbf{H} = \frac{9}{\rho^2} \sin^2(\omega t - 2z) \mathbf{a}_z \quad \text{W/m}^2$$

$$(c) \quad \mathcal{P}_{\text{ave}} = \frac{4.5}{\rho^2} \mathbf{a}_z, \quad dS = \rho d\phi d\rho \mathbf{a}_z$$

$$P_{\text{ave}} = \int \mathbf{P}_{\text{ave}} \bullet dS = 4.5 \int_{2\pi}^{3\pi} \frac{d\rho}{\rho} \int_0^{2\pi} d\phi = 4.5 \ln(3/2)(2\pi) = \underline{11.46 \text{ W}}$$

$$\text{Prob. 10.36 (a)} \quad P_{i,\text{ave}} = \frac{E_{io}^2}{2\eta_1}, \quad P_{r,\text{ave}} = \frac{E_{ro}^2}{2\eta_1}, \quad P_{t,\text{ave}} = \frac{E_{to}^2}{2\eta_2}$$

$$R = \frac{P_{r,\text{ave}}}{P_{i,\text{ave}}} = \frac{E_{ro}^2}{E_{io}^2} = \Gamma^2 = \left( \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right)^2$$

$$R = \left( \frac{\sqrt{\frac{\mu_o}{\epsilon_2}} - \sqrt{\frac{\mu_o}{\epsilon_1}}}{\sqrt{\frac{\mu_o}{\epsilon_2}} + \sqrt{\frac{\mu_o}{\epsilon_1}}} \right)^2 = \left( \frac{\sqrt{\mu_o \epsilon_1} - \sqrt{\mu_o \epsilon_2}}{\sqrt{\mu_o \epsilon_1} + \sqrt{\mu_o \epsilon_2}} \right)^2$$

$$\text{Since } n_1 = c\sqrt{\mu_1 \epsilon_1} = c\sqrt{\mu_o \epsilon_1}, \quad n_2 = c\sqrt{\mu_o \epsilon_2},$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{120\pi}{\sqrt{4}} = 60\pi, \quad \eta_2 = \eta_o = 120\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 60\pi}{180\pi} = \frac{1}{3}, \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2 \times 120\pi}{180\pi} = \frac{4}{3}$$

$$E_{io} = 10, \quad E_{ro} = \Gamma E_{io} = 10/3, \quad E_{to} = \tau E_{io} = 40/3$$

$$P_i = P_r = \frac{E_{io}^2}{2\eta_1} \mathbf{a}_z + \frac{E_{ro}^2}{2\eta_1} (-\mathbf{a}_z) = \frac{100}{2 \times 60\pi} \mathbf{a}_z - \frac{100}{92 \times 60\pi} \mathbf{a}_z = \underline{\underline{0.2358 \mathbf{a}_z \text{ W/m}^2}}$$

$$P_t = P_o = \frac{E_{to}^2}{2\eta_2} \mathbf{a}_z = \frac{1600}{9 \times 2 \times 120\pi} \mathbf{a}_z = \underline{\underline{0.2358 \mathbf{a}_z \text{ W/m}^2}}$$

**Prob. 10.40**

(a)  $\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{4}} = 60\pi, \quad \eta_1 = \eta_o = 120\pi$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{180\pi} = -\frac{1}{3}, \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{120\pi}{180\pi} = \frac{2}{3}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = 2$$

(b)  $\lambda_1 = c/f = \frac{3 \times 10^8}{10^8} = \underline{\underline{3 \text{ m}}}$

$$\lambda_2 = u/f = \frac{\frac{c}{\sqrt{\epsilon_r}}}{f} = \frac{3}{\sqrt{4}} = \underline{\underline{1.5 \text{ m}}}$$

(c)  $P_i = \frac{E_{oi}^2}{2\eta_1}, \quad P_r = \frac{E_{or}^2}{2\eta_1} = \frac{\Gamma^2 E_{oi}^2}{2\eta_1}$

$$\frac{P_r}{P_i} = \Gamma^2 = \frac{1}{9} = 0.1111 \text{ or } \underline{\underline{11.11\%}}$$

**Prob. 10.41**  $\eta_1 = \eta_o = 120\pi, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (1)$$

But  $E_{ro} = \eta_o H_{ro}$  (2)

Combining (1) and (2),

**Prob. 10.43 (a)** Medium 1 is free space. Given that  $\beta = 1$ ,

$$\beta = 1 = \frac{\omega}{c} \longrightarrow \omega = c = \underline{3 \times 10^8 \text{ rad/s}}$$

$$(b) \quad \eta_1 = \eta_o, \quad \eta_2 = \eta_o \sqrt{\frac{\mu_r}{\epsilon_r}} = \eta_o \sqrt{\frac{3}{12}} = \eta_o / 2$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3, \quad \tau = I + \Gamma = 2/3$$

$$s = \frac{I + |\Gamma|}{I - |\Gamma|} = \frac{I + 1/3}{I - 1/3} = 2$$

(c) Let  $H_r = H_{or} \cos(\omega t + z) \mathbf{a}_z$ , where

$$\mathbf{E}_r = -\frac{1}{3}(30) \cos(\omega t + z) \mathbf{a}_y = -10 \cos(\omega t + z) \mathbf{a}_y, \quad H_{or} = \frac{10}{\eta_o} = \frac{10}{120\pi}$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow -\mathbf{a}_y \times \mathbf{a}_H = -\mathbf{a}_z \longrightarrow \mathbf{a}_H = -\mathbf{a}_x$$

$$H_r = -\frac{10}{120\pi} \cos(3 \times 10^8 t + z) \mathbf{a}_x \text{ A/m} = \underline{-26.53 \cos(3 \times 10^8 t + z) \mathbf{a}_x \text{ mA/m}}$$

$$(d) \quad P_t = \frac{E_{ot}^2}{2\eta_2} \mathbf{a}_z, \quad E_{ot} = \tau E_{oi} = \frac{2}{3}(30) = 20, \quad \eta_2 = 60\pi$$

$$P_t = \frac{20^2}{120\pi} (\mathbf{a}_z) = \underline{1.061 \mathbf{a}_z \text{ W/m}^2}$$

**Prob. 10.44**

(a) In air,  $\beta_1 = 1, \lambda_1 = 2\pi/\beta_1 = 2\pi = \underline{6.283 \text{ m}}$

$$\omega = \beta_1 c = \underline{3 \times 10^8 \text{ rad/s}}$$

In the dielectric medium,  $\omega$  is the same.

$$\omega = \underline{3 \times 10^8 \text{ rad/s}}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\epsilon_{r2}} = \beta_1 \sqrt{\epsilon_{r2}} = \sqrt{3}$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_z \times \mathbf{a}_H = \mathbf{a}_y \longrightarrow \mathbf{a}_H = \mathbf{a}_x$$

$$\underline{\underline{H_t = 35.37 \cos(\omega t - 16\pi y) \mathbf{a}_x \text{ mA/m}}}$$

**Prob. 10.47 (a)**  $\omega = \beta c = 3 \times 3 \times 10^8 = \underline{\underline{9 \times 10^8 \text{ rad/s}}}$

(b)  $\lambda = 2\pi / \beta = 2\pi / 3 = \underline{\underline{2.094 \text{ m}}}$

(c)  $\frac{\sigma}{\omega \epsilon} = \frac{4}{9 \times 10^8 \times 80 \times 10^{-9} / 36\pi} = 2\pi = \underline{\underline{6.288}}$

$$\tan 2\theta_n = \frac{\sigma}{\omega \epsilon} = 6.288 \longrightarrow \theta_n = 40.47^\circ$$

$$|\eta_2| = \frac{\sqrt{\mu_2 / \epsilon_2}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega \epsilon_2}\right)^2}} = \frac{377 / \sqrt{80}}{\sqrt[4]{1 + 4\pi^2}} = 16.71$$

$$\eta_2 = \underline{\underline{16.71 \angle 40.47^\circ \Omega}}$$

(d)  $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{16.71 \angle 40.47^\circ - 377}{16.71 \angle 40.47^\circ + 377} = 0.935 \angle 179.7^\circ$

$$E_{or} = \Gamma E_{oi} = 9.35 \angle 179.7^\circ$$

$$\underline{\underline{E_r = 9.35 \sin(\omega t - 3z + 179.7) \mathbf{a}_x \text{ V/m}}}$$

$$\alpha_2 = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \left[ \sqrt{1 + \left( \frac{\sigma_2}{\omega \epsilon_2} \right)^2} - 1 \right]} = \frac{9 \times 10^8}{3 \times 10^8} \sqrt{\frac{80}{2} \left[ \sqrt{1 + 4\pi^2} - 1 \right]} = 43.94 \text{ Np/m}$$

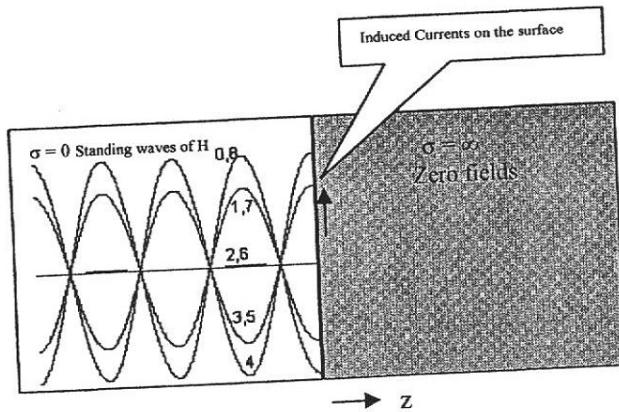
$$\beta_2 = \frac{9 \times 10^8}{3 \times 10^8} \sqrt{\frac{80}{2} \left[ \sqrt{1 + 4\pi^2} + 1 \right]} = 51.48 \text{ rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 16.71 \angle 40.47^\circ}{16.71 \angle 40.47^\circ + 377} = 0.0857 \angle 38.89^\circ$$

$$E_{ot} = \tau E_o = 0.857 \angle 38.89^\circ$$

$$\underline{E_t = 0.857 e^{j3.94z} \sin(9 \times 10^8 t + 51.48z + 38.89^\circ) \text{ V/m}}$$

### Prob. 10.48



Curve 0 is at  $t = 0$ ; curve 1 is at  $t = T/8$ ; curve 2 is at  $t = T/4$ ; curve 3 is at  $t = 3T/8$ , etc.

**Prob. 10.49** Since  $\mu_o = \mu_1 = \mu_2$ ,

$$\sin \theta_{t1} = \sin \theta_i \sqrt{\frac{\epsilon_o}{\epsilon_1}} = \frac{\sin 45^\circ}{\sqrt{4.5}} = 0.3333 \quad \longrightarrow \quad \underline{\theta_{t1} = 19.47^\circ}$$

$$\sin \theta_{t2} = \sin \theta_{t1} \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{3} \sqrt{\frac{4.5}{2.25}} = 0.4714 \quad \longrightarrow \quad \underline{\theta_{t2} = 28.13^\circ}$$

### Prob. 10.50

$$\begin{aligned} \mathbf{E}_s &= \frac{20(e^{jk_x x} - e^{-jk_x x})}{j2} \frac{(e^{jk_y y} + e^{-jk_y y})}{2} \mathbf{a}_z \\ &= -j5 \left[ e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)} \right] \mathbf{a}_z \end{aligned}$$

which consists of four plane waves.

$$\nabla \times \mathbf{E}_s = -j\omega\mu_o \mathbf{H}_s \quad \longrightarrow \quad \mathbf{H}_s = \frac{j}{\omega\mu_o} \nabla \times \mathbf{E}_s = \frac{j}{\omega\mu_o} \left( \frac{\partial E_z}{\partial y} \mathbf{a}_x - \frac{\partial E_z}{\partial x} \mathbf{a}_y \right)$$