ECED 4301 WAVES AND PROPOGATIONS ASSIGNMENT # 1 SOLUTION

·ob.1.24

) $\cos\theta_{AB} = \dfrac{A \cdot B}{AB} = \dfrac{-14}{\sqrt{83}\sqrt{6}} = -0.6273 \longrightarrow \theta_{AB} = \underline{\underline{128.86^o}}$

) $A_{//} = (A \cdot a_B)a_B = \dfrac{(A \bullet B)B}{B^2} = \dfrac{-14(1,-2,1)}{6} = \underline{\underline{-2.333a_x + 4.667a_y - 2.333a_z}}$

) $A_{\perp} = A - A_{//} = (3,5,-7) - (-2.333, 4.667, -2.333) = \underline{\underline{5.333a_x + 0.333a_y - 4.667a_z}}$

Prob. 2.10 (a)

$$\begin{bmatrix} H_\rho \\ H_\phi \\ H_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}$$

$H_\rho = 3\cos\phi + 2\sin\phi, \quad H_\phi = -3\sin\phi + 2\cos\phi, \quad H_z = -4$

$$\underline{\underline{\mathbf{H} = (3\cos\phi + 2\sin\phi)\mathbf{a}_\rho + (-3\sin\phi + 2\cos\phi)\mathbf{a}_\phi - 4\mathbf{a}_z}}$$

b) At P, $\rho = 2, \quad \phi = 60^\circ, \quad z = -1$

$$\mathbf{H} = (3\cos 60^\circ + 2\sin 60^\circ)\mathbf{a}_\rho + (-3\sin 60^\circ + 2\cos 60^\circ)\mathbf{a}_\phi - 4\mathbf{a}_z$$
$$= \underline{\underline{3.232\mathbf{a}_\rho - 1.598\mathbf{a}_\phi - 4\mathbf{a}_z}}$$

rob 2.25

)

At P, $\rho = 2,\ \phi = 30°,\ \ z = -1$

$$\bar{H} = 10\sin 30\,\bar{a}_\rho + 2\cos 30°\,\bar{a}_\phi + 4\,\bar{a}_z.$$

$$= 5\bar{a}_\rho + 1.732\,\bar{a}_\phi + 4\,\bar{a}_z.$$

$$\bar{a}_H = \frac{(5, 1.732,\ 4)}{\sqrt{5^2 + 1.732^2 + 4^2}} = \underline{\underline{0.7538\,\bar{a}_\rho + 0.2611\bar{a}_\phi + 0.603\bar{a}_z.}}$$

) $H_x = H_\rho \cos\phi - H_\phi \sin\phi = 5\rho\sin\phi\cos\phi + \rho z\cos\phi\sin\phi$

or P at $\rho = 2,\ \phi = 30,\ \ z = -1;$

$$\bar{H}_x = H_x\,\bar{a}_x = (10\sin 30°\cos 30° - 2\sin 30°\cos 30°)\bar{a}_x = 8\sin 30°\cos 30°\,\mathbf{a}_x$$

$$= \underline{\underline{3.4641\bar{a}_x}}$$

Normal to $\rho = 2$ is $\mathbf{H}_n = H_\rho\,\mathbf{a}_\rho = 10\sin\phi\,\mathbf{a}_\rho;$

i.e. $\mathbf{H}_n = \underline{\underline{5\ \mathbf{a}_\rho}}.$

Tangential to $\phi = 30°$.

$$= H_\rho\,\mathbf{a}_\rho + H_z\mathbf{a}_z = \underline{\underline{5\mathbf{a}_\rho + 4\mathbf{a}_z}}$$

Prob. 3.9

(a)

$$\nabla U = \frac{\partial U}{\partial x}\bar{a}_x + \frac{\partial U}{\partial y}\bar{a}_y + \frac{\partial U}{\partial z}\bar{a}_z$$

$$= \underline{\underline{4z^2\,\bar{a}_x + 3z\,\bar{a}_y + (8xz + 3y)\,\bar{a}_z}}$$

(b)

$$\nabla W = \frac{\partial W}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial W}{\partial \phi}\bar{a}_\phi + \frac{\partial W}{\partial z}\bar{a}_z$$

$$= \underline{\underline{2(z^2+1)\cos\phi\,\bar{a}_\rho - 2(z^2+1)\sin\phi\,\bar{a}_\phi + 4\rho z\cos\phi\,\bar{a}_z}}$$

(c)

$$\nabla H = \frac{\partial H}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial H}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial H}{\partial \phi}\bar{a}_\phi$$

$$= \underline{\underline{2r\cos\theta\cos\phi\,\bar{a}_r - r\sin\theta\cos\phi\,\bar{a}_\theta - r\cot\theta\sin\phi\,\bar{a}_\phi}}$$

Prob 3.12

$$\nabla T = 2x\bar{a}_x + 2y\bar{a}_y - \bar{a}_z$$

At $(1,1,2)$, $\nabla T = (2,2,-1)$. The mosquito should move in the direction of

$$\underline{\underline{2\bar{a}_x + 2\bar{a}_y - \bar{a}_z}}$$

Prob 3.20

Transform $\bar{F}$ into cylindrical system.

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^2 \\ y^2 \\ z^2-1 \end{bmatrix}$$

$$F_\rho = x^2\cos\phi + y^2\sin\phi = \rho^2\cos^3\phi + \rho^2\sin^3\phi, F_z = z^2 - 1$$

$$F_\phi = -x^2\sin\phi + y^2\cos\phi = -\rho^2\cos^2\phi\sin\phi + \rho^2\sin^2\phi\cos\phi$$

$$\nabla \bullet \bar{F} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^3\cos^3\phi + \rho^3\sin^3\phi) + 2z - \rho\cos^3\phi - 2\rho\cos\phi\sin^2\phi$$
$$+ 2\rho\sin\phi\cos^2\phi - \rho\sin^3\phi$$
$$= 2\rho\cos^3\phi + 2\rho\sin^3\phi - 2\rho\cos\phi\sin^2\phi + 2\rho\cos^2\phi\sin\phi + 2z$$

$$\int \bar{F} \bullet d\bar{S} = \int \nabla \bullet \bar{F}\, dv$$

Due to the fact that we are integrating $\sin\phi$ and $\cos\phi$ over $0 < \phi < 2\pi$, terms involving $\cos\phi$ *and* $\sin\phi$ will vanish. Hence,

$$\int \bar{F}\, d\bar{S} = \iiint 2z\ \rho d\rho d\phi dz = 2\int_0^{2\pi} d\phi \int_0^2 z dz \int_0^2 \rho d\rho$$

$$= 2(2\pi)\left(\frac{z^2}{2}\Big|_0^2\right)\left(\frac{\rho^2}{2}\Big|_0^2\right) = 16\pi$$

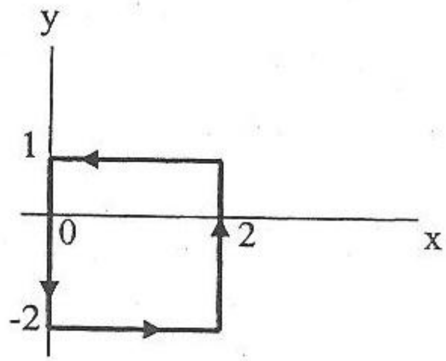
$$= \underline{\underline{50.26}}$$

Prob.3.32

$$\oint \mathbf{A} \bullet dl = \int_{\rho=2}^{1} \rho\sin\phi d\rho\Big|_{\phi=0} + \int_{\phi=0}^{\pi/2} \rho^2 \rho d\phi d\rho\Big|_{\rho=1} + \int_{\rho=1}^{2} \rho\sin\phi\Big|_{\phi=90^\circ} + \int_{\phi=\pi/2}^{0} \rho^3 d\phi\Big|_{\rho=2}$$

$$= \frac{\pi}{2} + \frac{1}{2}(4-1) + 8\left(-\frac{\pi}{2}\right) = \underline{\underline{-9.4956}}$$

Prob. 3.34



$$\oint_L \mathbf{F}\cdot d\mathbf{l} = \int_0^2 3y^2 z\,dx\Big|_{y=-2,z=1} + \int_{-2}^{1} 6x^2 y\,dy\Big|_{x=2,z=1}$$

$$+\int_2^0 3y^2 z\,dx\Big|_{y=1,z=1} + \int_1^{-2} 6x^2 y\,dy\Big|_{x=0,z=1}$$

$$= 3(4)(1)(2) + 6(4)\frac{y^2}{2}\Big|_{-2}^{1} + 3(1)(1)(-2) + 0 = \underline{\underline{-18}}$$

$$\nabla \mathrm{x}\mathbf{F} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y^2 z & 6x^2 y & 9xz^2 \end{vmatrix} = (12xy - 6yz)\mathbf{a}_z + \ldots$$

$$\int_S (\nabla \mathrm{x} F)\cdot dS = \iint (12xy - 6yz)dxdy\Big|_{z=1} = 12\int_0^2 xdx \int_{-2}^{1} ydy - 6\int_{-2}^{1} ydy \int_0^2 dx$$

$$= 3x^2\Big|_0^2 y^2\Big|_{-2}^{1} - 3y^2\Big|_{-2}^{1}(2) = 3(4)((1-4) - 6(1-4) = \underline{\underline{-18}}$$