ECED 4301 Assignment #7 Solution

Prob. 11.2

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi x 80x10^6 x 4\pi x10^{-7} x5.28x10^7}} = 7.744x10^{-6}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right] = \frac{\left[\frac{1}{0.8x10^{-3}}+\frac{1}{2.6x10^{-3}}\right]}{2\pi x7.744x10^{-6}x5.28x10^7} = \frac{10^3(1.25+0.3836)}{2569.09} = \underline{\underline{0.6359\ \Omega}}$$

$$L = \frac{\mu}{2\pi}\ln\frac{b}{a} = \frac{4\pi x10^{-7}}{2\pi}\ln\frac{2.6}{0.8} = \underline{\underline{2.357x10^{-7}\ \text{H/m}}}$$

$$G = \frac{2\pi\sigma}{\ln\frac{b}{a}} = \frac{2\pi x10^{-5}}{\ln\frac{2.6}{0.8}} = \underline{\underline{5.33x10^{-5}\ \text{S/m}}}$$

$$C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}} = \frac{2\pi x3.5x\frac{10^{-9}}{36\pi}}{\ln\frac{2.6}{0.8}} = \underline{\underline{1.65x10^{-10}\ \text{F/m}}}$$

Prob. 11.7

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma Z_o = R + j\omega L = (0.01 + j4)(50 + j0) = 0.5 + j200$$

$$R = \underline{\underline{0.5\ \Omega/\text{m}}}$$

$$L = \frac{200}{\omega} = \frac{200}{2\pi x 800x10^6} = \underline{\underline{3.979x10^{-8}\ \text{H/m}}}$$

$$\frac{\gamma}{Z_o} = G + j\omega C = \frac{(0.01 + j4)}{50}$$

$$G = \frac{0.01}{50} = \underline{\underline{2x10^{-4}\ \text{S/m}}}$$

$$C = \frac{4}{50\omega} = \frac{4}{50x2\pi x800x10^6} = \underline{\underline{1.591x10^{-11}\ \text{F/m}}}$$

Prob. 11.9

(a) $R + j\omega L = 40 + j2\pi \times 10^7 \times 0.2 \times 10^{-6} = 41.93\angle 17.44^o$

$$G + j\omega C = 400 \times 10^{-6} + j2\pi \times 10^7 \times 0.5 \times 10^{-9} = 3.142 \times 10^{-2} \angle 89.27^o$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \underline{\underline{29.59 - j21.43\ \Omega}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega L)} = 0.685 + j0.921 = \alpha + j\beta$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{0.921} = \underline{\underline{6.823 \times 10^7\ \text{m/s}}}$$

(b) $\alpha = 0.685\ \text{Np/m} = 0.685 \times 8.686\ \text{dB/m} = 5.95\ \text{dB/m}$

$$\alpha l = 30 \rightarrow l = \frac{30}{5.95} = \underline{\underline{5.042\ \text{m}}}$$

Prob. 11.14

(a) $\alpha = \underline{\underline{0.0025\text{ Np/m}}}, \quad \beta = \underline{\underline{2\text{ rad/m}}},$

$$u = \frac{\omega}{\beta} = \frac{10^8}{2} = \underline{\underline{5 \times 10^7\ \text{m/s}}}$$

(b) $\Gamma = \frac{V_o^-}{V_o^+} = \frac{60}{120} = \frac{1}{2}$

But $\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \rightarrow \frac{1}{2} = \frac{300 - Z_o}{300 + Z_o} \rightarrow \underline{\underline{Z_o = 100\Omega}}$

$$I(l') = \frac{120}{Z_o} e^{0.0025l'} \cos\left(10^8 + 2l'\right) - \frac{60}{Z_o} e^{-0.0025l'} \cos\left(10^8 t - 2l'\right)$$

$$= \underline{\underline{1.2e^{0.0025l'} \cos\left(10^8 + 2l'\right) - 0.6e^{-0.0025l'} \cos\left(10^8 t - 2l'\right) A}}$$

Prob. 11.23

$$I_l = \frac{V_L}{Z_L}, \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{50e^{j30^o} - 50}{50e^{j30^o} + 50}$$
$$\approx j0.2679$$

From eq.(11.30),

$$V_o^+ = \frac{1}{2}(V_L + Z_o . \frac{V_L}{Z_L})e^{\gamma l} = \frac{V_L}{2Z_L}(Z_L + Z_o)e^{\gamma l}$$

$$V_o^- = \frac{V_L}{2Z_L}(Z_L - Z_o)e^{-\gamma l}$$

Substituting these in eq.(11.25),

$$I_s = \frac{V_L}{2Z_L Z_o}\left[(Z_L + Z_o)e^{\gamma l}e^{-\gamma z} - (Z_L - Z_o)e^{-\gamma l}e^{\gamma z}\right]$$
$$= \frac{V_L / \ Z_o}{1+\Gamma}\left[e^{-\gamma(z-l)} - \Gamma e^{\gamma(z-l)}\right]$$

But $l - z = \dfrac{\lambda}{8} \quad or \quad z - l = -\dfrac{\lambda}{8}$

$$I_s = \frac{10\angle 25^o}{1.035\angle 15^o}\left(\frac{1}{50}\right)\left(e^{j\pi/4} - j0.2679e^{-j\pi/4}\right)$$
$$= \underline{\underline{0.1414\angle 55^o\, A}}$$

Prob. 11.29

Using the Smith chart, $z_L = \dfrac{60 - j40}{75} = 0.8 - j0.533$

$$l = \frac{3}{4}\lambda \quad \longrightarrow \quad \frac{3}{4}x720^o = 540^o$$

At C, $Z_{in} = 75(0.8654 + j0.5769) = 65 + j43 \ \Omega$

$$z_{in} = \frac{65 + j43}{100} = 0.65 + j0.43$$

$$\frac{\lambda}{2} \longrightarrow \frac{720^o}{2} = 360^o$$

At B, $Z_{in} = 65 + j43$

$$z_{in} = \frac{65 + j43}{50} = 1.2981 + 0.8654$$

$$\frac{\lambda}{4} \longrightarrow \frac{720^o}{4} = 180^o$$

At A,

$$Z_{in} = 50(0.53 - j0.35) = \underline{\underline{26.7 - j17.8 \ \Omega}}$$

Prob. 11.34

(a) $$z_L = \frac{Z_L}{Z_o} = \frac{75 + j60}{50} = 1.5 + j1.2$$

$$|\Gamma| = \frac{OP}{OQ} = \frac{3.8\text{cm}}{8\text{cm}} = 0.475, \qquad \theta_\Gamma = 42^o$$

$$\Gamma = \underline{\underline{0.475\angle 42^0}}$$

(Exact value = $0.4688 \angle 41.76^o$)

(b) s=2.8

(Exact value = 2.765)

(c) $$0.2\lambda \rightarrow 0.2x720^o = 144^o$$

$$z_{in} = 0.55 - j0.65$$

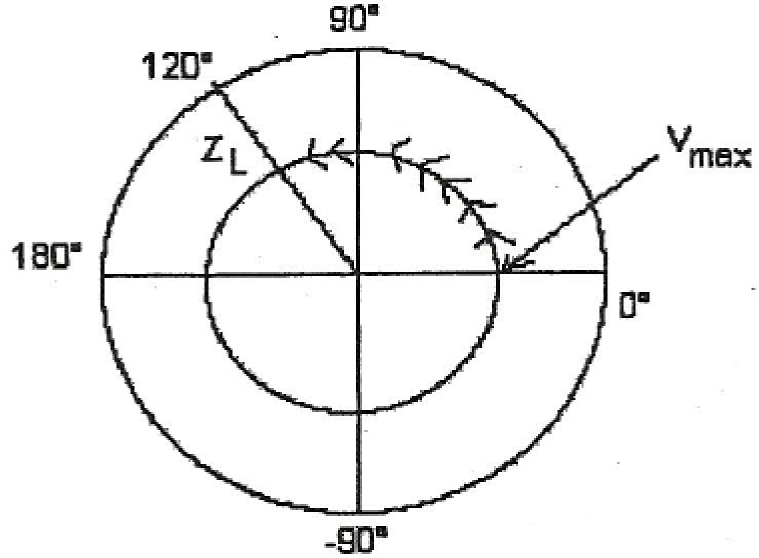
$$Z_{in} = Z_o z_{in} = 50(0.55 + j0.65) = \underline{\underline{27.5 + j32.5\ \Omega}}$$

(d) Since $\theta_\Gamma = 42^o$, V_{min} occurs at

$$\frac{42}{720}\lambda = \underline{\underline{0.05833\lambda}}$$

(e) same as in (d), i.e.. $\underline{\underline{0.05833\lambda}}$

Prob. 11.37



(a) $\dfrac{\lambda}{2} = 120cm \rightarrow \lambda = 2.4m$

$$u = f\lambda \rightarrow f = \frac{u}{\lambda} = \frac{3\times10^8}{2.4} = \underline{\underline{125MHz}}$$

(b) $40cm \quad = \dfrac{40\lambda}{240} = \dfrac{\lambda}{6} \rightarrow \dfrac{720°}{6} = 120°$

$$Z_L = Z_o z_L = 150(0.48 + j0.48$$

$$= \underline{\underline{72 + j72}}\ \Omega$$

(Exact value = 73.308+j70.324 Ω)

(c) $|\Gamma| = \dfrac{s-1}{s+1} = \dfrac{1.6}{3.9} = 0.444,$

$$\Gamma = \underline{\underline{0.444\angle 120°}}$$

Prob.11.48

$$z_L = \frac{Z_L}{Z_o} = \frac{120 + j220}{50} = 2.4 + j4.4$$

We follow Example 11.7. At A, y_s=-j3 and at B, admittance is

$$Y_s = Y_o y_s = \frac{\pm j3}{50} = \pm j0.06 \text{ S}$$

The distance between the load and the stub is deter

$$l_A = \frac{180 - (17.2 - 10)}{720}\lambda = \underline{\underline{0.24\lambda}}$$

(Exact value = 0.2308λ)

For B,

$$l_B = \frac{180 + 10 + 17}{720}\lambda = \underline{\underline{0.2875\lambda}}$$

The length of the stub line is determined as follows.

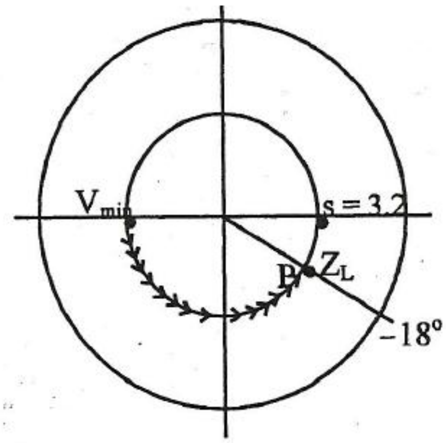
$$d_A = \frac{19}{720}\lambda = \underline{\underline{0.0264\lambda}}$$

Exact value = 0.0515 λ)

$$d_B = \frac{360 - 19}{720}\lambda = \underline{\underline{0.4736\lambda}}$$

Exact value = 0.4485λ)

Prob. 11.51



$$\frac{\lambda}{2} = 32 - 12 = 20cm \rightarrow \lambda = 40\,\text{cm}$$

$$f = \frac{u}{\lambda} = \frac{3\times 10^8}{40\times 10^{-2}} = \underline{\underline{0.75\,\text{GHz}}}$$

$$l = 21 - 12 = 9cm = \frac{9\lambda}{40} \rightarrow \frac{9}{40}\times 720^o = 162^o$$

At P, $z_L = 2.6 - j1.2$

$$Z_L = z_L Z_o = 50(2.6 - j1.2) = \underline{\underline{130 - j60\Omega}}$$

(Exact value = $130.49 - j58.219\ \Omega$)