Q1:

A $100 + j150$–Ω load is connected to a 75-Ω lossless line. Find:

(a) Γ

(b) s

(c) The load admittance Y_L

(d) Z_{in} at 0.4λ from the load

(e) The locations of V_{max} and V_{min} with respect to the load if the line is 0.6λ long

(f) Z_{in} at the generator.

Q2

With an unknown load connected to a slotted air line, $s = 2$ is recorded by a standing wave indicator and minima are found at 11 cm, 19 cm, . . . on the scale. When the load is replaced by a short circuit, the minima are at 16 cm, 24 cm, ... If $Z_o = 50\ \Omega$, calculate λ, f, and Z_L.

[Solution #1]

(a) We can use the Smith chart to solve this problem. The normalized load impedance is

$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{75} = 1.33 + j2$$

We locate this at point P on the Smith chart of Figure 11.16. At P, we obtain

$$|\Gamma_L| = \frac{OP}{OQ} = \frac{6\text{ cm}}{9.1\text{ cm}} = 0.659$$

$$\theta_L = \text{angle } POS = 40°$$

(b) Draw the constant $|\Gamma|$ circle passing through P and obtain

$$s = 4.82$$

Check:

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.659}{1 - 0.659} = 4.865$$

(c) To obtain Y_L, extend PO to POP' and note point P' where the constant $|\Gamma|$-circle meets POP'. At P', obtain

$$y_L = 0.228 - j0.35$$

The load admittance is

$$Y_L = Y_o y_L = \frac{1}{75}(0.228 - j0.35) = 3.04 - j4.67\text{ m℧}$$

Check:

$$Y_L = \frac{1}{Z_L} = \frac{1}{100 + j150} = 3.07 - j4.62\text{ m℧}$$

$(720° = 4\pi)$

(d) 0.4λ corresponds to an angular movement of $0.4 \times 720° = 288°$ on the constant s-circle. From P, we move 288° toward the generator (clockwise) on the $|\Gamma|$-circle to reach point R. At R,

$$z_{in} = 0.3 + j0.63$$

Hence

$$Z_{in} = Z_o z_{in} = 75\,(0.3 + j0.63)$$
$$= 22.5 + j47.25\ \Omega$$

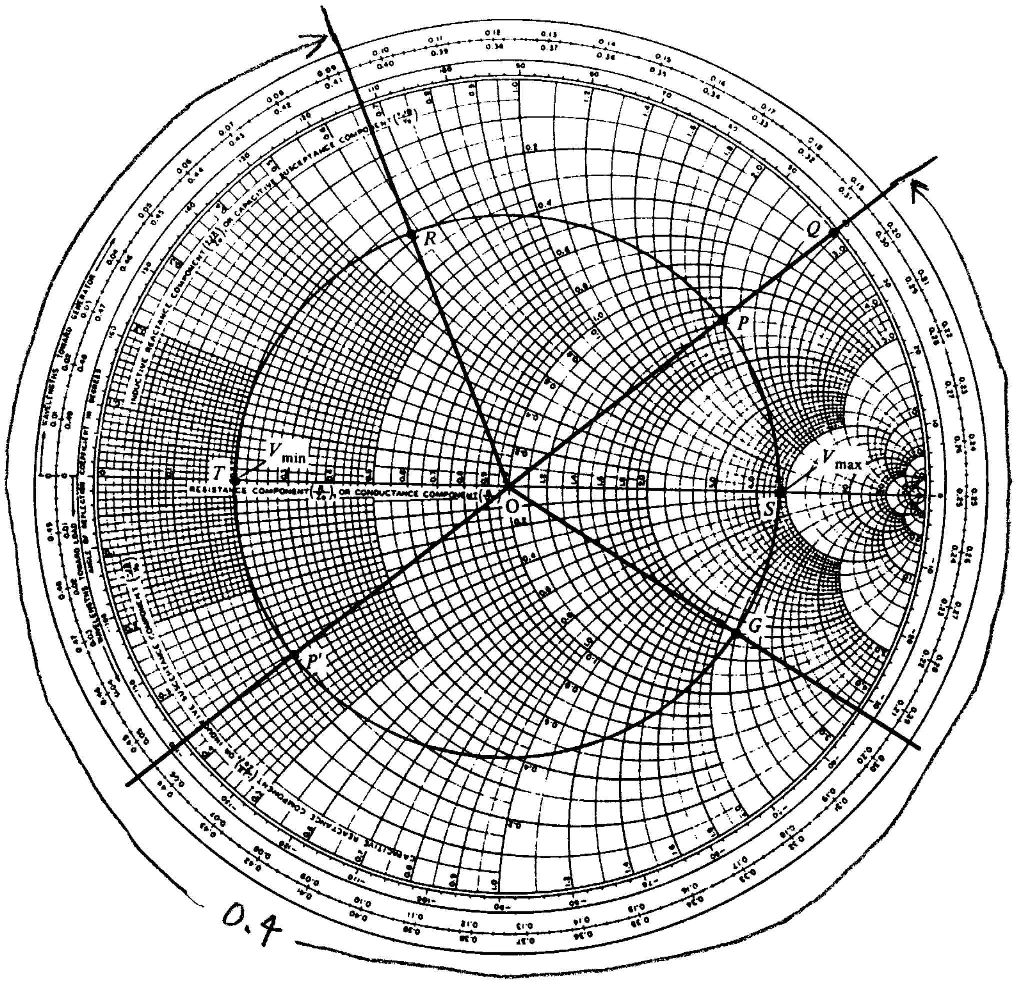


Figure 11.16 For Example 11.5.

Hence,

$$\Gamma_L = 0.659 \angle 40^\circ$$

Check:

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100 + j150 - 75}{100 + j150 + 75}$$

$$= 0.659 \angle 40^\circ$$

Check:

$$\beta\ell = \frac{2\pi}{\lambda}(0.4\lambda) = 360^\circ\,(0.4) = 144^\circ$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell}\right]$$

$$= \frac{75\,(100 + j150 + j75 \tan 144^\circ)}{[75 + j(100 + j150) \tan 144^\circ]}$$

$$= 54.41 \angle 65.25^\circ$$

or

$$Z_{in} = 21.9 + j47.6\ \mho$$

(e) 0.6λ corresponds to an angular movement of

$$0.6 \times 720^\circ = 432^\circ = 1 \text{ revolution} + 72^\circ$$

$(720^\circ = 4\pi)$

Thus, we start from P (load end), move along the $|\Gamma|$-circle 432°, or one revolution plus 72°, and reach the generator at point G. Note that to reach G from P, we have passed through point T (location of V_{min}) once and point S (location of V_{max}) twice. Thus, from the load,

$$\text{1st } V_{max} \text{ is located at } \frac{40°}{720°}\lambda = 0.055\lambda$$

$$\text{2nd } V_{max} \text{ is located at } 0.0555\lambda + \frac{\lambda}{2} = 0.555\lambda$$

and the only V_{min} is located at $0.055\lambda + \lambda/4 = 0.3055\lambda$

(f) At G (generator end),

$$z_{in} = 1.8 - j2.2$$

$$Z_{in} = 75(1.8 - j2.2) = 135 - j165\ \Omega.$$

This can be checked by using eq. (11.34), where $\beta\ell = \dfrac{2\pi}{\lambda}(0.6\lambda) = 216°$.

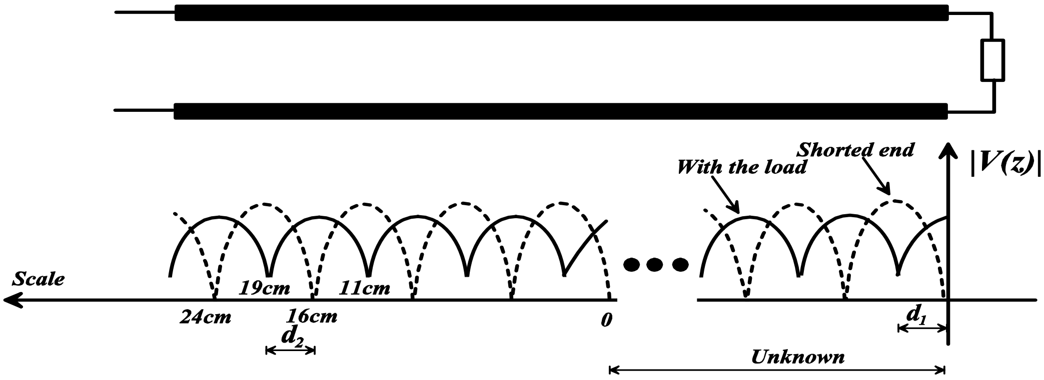
We can see how much time and effort is saved using the Smith chart. ■

[Solution #2]

Consider the standing wave patterns as in Figure 11.23(a). From this, we observe that

$$\frac{\lambda}{2} = 19 - 11 = 8 \text{ cm} \qquad \text{or} \qquad \lambda = 16 \text{ cm}$$

$$f = \frac{u}{\lambda} = \frac{3 \times 10^8}{16 \times 10^{-2}} = 1.875 \text{ GHz}$$



From the above figure, we can see that the distance of $|V|_{\min}$ with the load, d_1, is equal to d_2. That is,

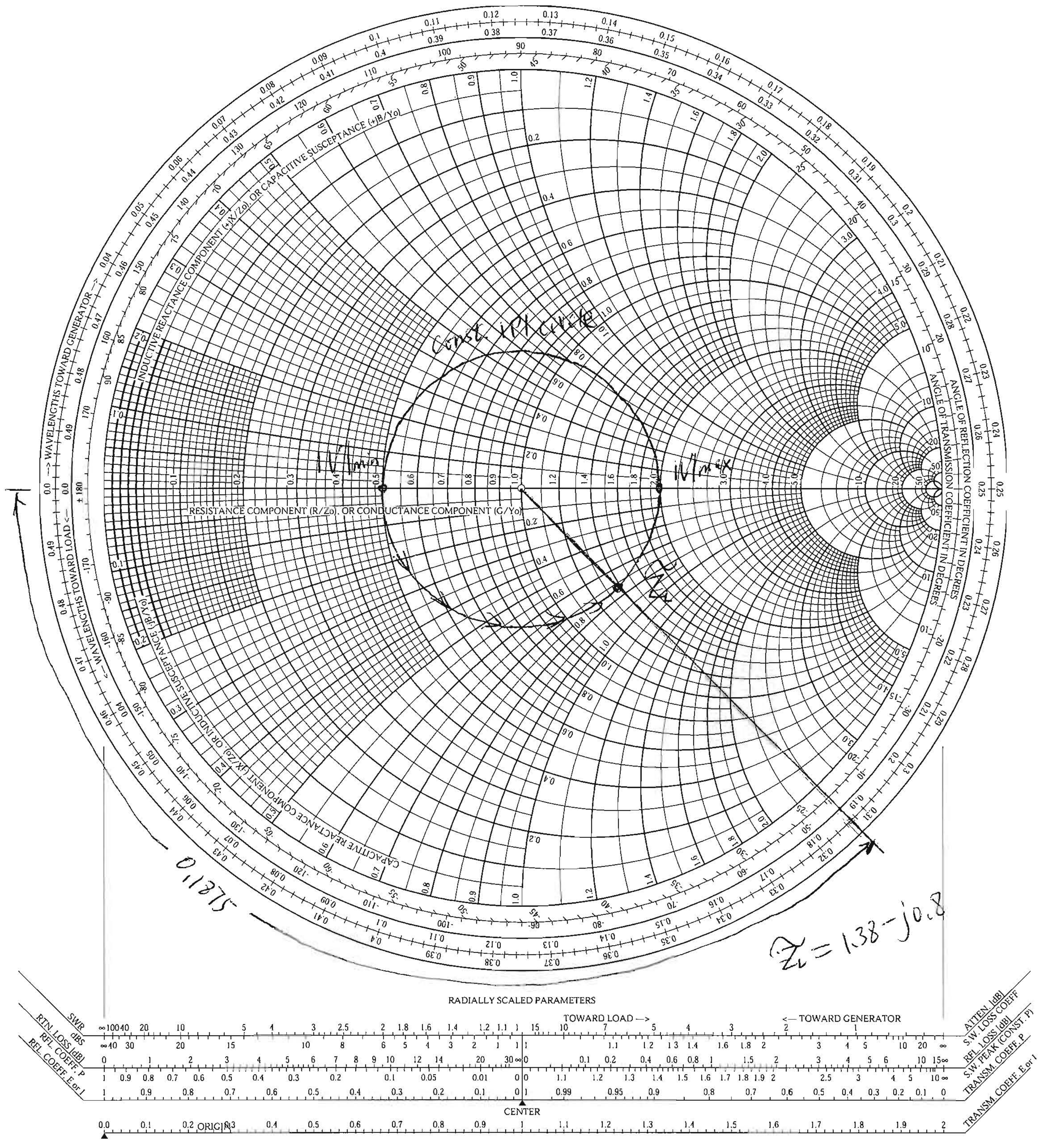
$$d_1 = d_2 = 19cm - 16cm = 3cm.$$

Since $s=2$ (*i.e. VSWR=2*), we can locate point P: $z=r=s=2.0$ (which is also the $|V|_{\max}$ point). Then draw the constant-$|\Gamma|$ circle and find the $|V|_{\min}$ point.

By moving from the $|V|_{\min}$ point towards the load by a distance of *3cm/16cm=0.1875*, we can read:

$$\tilde{Z}_L = 1.38 - j0.80$$

$$Z_L = 50(1.38 - j0.80) = 69 - j40 \ (\Omega)$$



Const. |Γ| circle
|V|min
|V|max
z_L
0.1875
$z_L = 1.38 - j0.8$
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN