

ECED 4301 Assignment# 3 Solution

Prob. 9.32

With the given A , we need to prove that

$$\nabla^2 A = \mu\epsilon \frac{\partial^2 A}{\partial t^2}$$

$$\nabla^2 A = \mu\epsilon(j\omega)(j\omega)A = -\omega^2 \mu\epsilon A$$

Let $\beta^2 = \omega^2 \mu\epsilon$, then $\nabla^2 A = -\beta^2 A$ is to be proved. We recognize that

$$A = \frac{\mu_o}{4\pi r} e^{j\omega t} e^{-j\beta r} \mathbf{a}_z$$

Assume $\phi = \frac{e^{-j\beta r}}{r}$, $A = \frac{\mu_o}{4\pi} e^{j\omega t} \phi \mathbf{a}_z$

$$\begin{aligned} \nabla^2 \phi &= \frac{1}{r^2 \sin \theta} \left[\frac{\partial}{\partial r} (r^2 \sin \theta \frac{\partial \phi}{\partial r}) \right] = \frac{1}{r^2} \left[\frac{\partial}{\partial r} (r^2) \left(\frac{-j\beta}{r} - \frac{1}{r^2} \right) e^{-j\beta r} \right] \\ &= \frac{1}{r^2} (-\beta^2 r + j\beta - j\beta) e^{-j\beta r} = -\beta^2 \frac{e^{-j\beta r}}{r} = -\beta^2 \phi \end{aligned}$$

Therefore, $\nabla^2 A = -\beta^2 A$

We can find V using Lorentz gauge.

$$\begin{aligned} V &= \frac{-1}{\mu_o \epsilon_o} \int \nabla \cdot A dt = \frac{-1}{j\omega \mu_o \epsilon_o} \nabla \cdot A \\ &= \frac{-1}{j\omega \mu_o \epsilon_o} \frac{\partial}{\partial r} \left(\frac{\mu_o}{4\pi r} e^{-j\beta r} e^{j\omega t} \right) = \frac{-1}{j\omega \epsilon_o (4\pi)} \left(\frac{-j\beta}{r} - \frac{1}{r^2} \right) e^{-j\beta r} e^{j\omega t} \cos \theta \\ V &= \underline{\underline{\frac{\cos \theta}{j4\pi \omega \epsilon_o r} \left(j\beta + \frac{1}{r} \right) e^{j(\omega t - \beta r)}}} \end{aligned}$$

Prob. 9.34

(a)

$$\begin{aligned}
 z &= 4\angle 30^\circ - 10\angle 50^\circ = 3.464 + 2j - 6.427 - j7.66 = -2.963 - j5.66 \\
 &= 6.389\angle -117.64^\circ \\
 z^{1/2} &= \underline{\underline{2.5277\angle -58.82^\circ}}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{1+j2}{6-j8-7\angle 15^\circ} &= \frac{2.236\angle 63.43^\circ}{6-j8-7.761-j1.812} = \frac{2.236\angle 63.43^\circ}{9.841\angle 265.57^\circ} \\
 &= \underline{\underline{0.2272\angle -202.1^\circ}}
 \end{aligned}$$

(c) $z = \frac{(5\angle 53.13^\circ)^2}{12-j7-6-j10} = \frac{25\angle 106.26^\circ}{18.028\angle -70.56^\circ}$

$$= \underline{\underline{1.387\angle 176.8^\circ}}$$

(d)

$$\frac{1.897\angle -100^\circ}{(5.76\angle 90^\circ)(9.434\angle -122^\circ)} = \underline{\underline{0.0349\angle -68^\circ}}$$

Prob. 9.35

(a) $A = 5 \cos(2t + \pi/3 - \pi/2) \mathbf{a}_x + 3 \cos(2t + 30^\circ) \mathbf{a}_y = \text{Re}(A_s e^{j\omega t}), \omega = 2$

$$\underline{\underline{A_s = 5e^{-j30^\circ} \mathbf{a}_x + 3e^{j30^\circ} \mathbf{a}_y}}$$

(b) $B = \frac{100}{\rho} \cos(\omega t - 2\pi z - 90^\circ) \mathbf{a}_\rho$

$$\underline{\underline{B_s = \frac{100}{\rho} e^{-j(2\pi z + 90^\circ)} \mathbf{a}_\rho}}$$

(c) $C = \frac{\cos \theta}{r} \cos(\omega t - 3r - 90^\circ) \mathbf{a}_\theta$

$$\underline{\underline{C_s = \frac{\cos \theta}{r} e^{-j(3r + 90^\circ)} \mathbf{a}_\theta}}$$

(d) $\underline{\underline{D_s = 10 \cos(k_1 x) e^{-jk_2 z} \mathbf{a}_y}}$

Prob. 9.36 (a) $(4 - j3) = 5e^{-j36.87^\circ}$

$$A_s = 5e^{-j(\beta x + 36.87^\circ)} \mathbf{a}_y$$

$$A = \text{Re}[A_s e^{j\omega t}] = \underline{\underline{5 \cos(\omega t - \beta x - 36.87^\circ) \mathbf{a}_y}}$$

(b)

$$B = \operatorname{Re} [B_s e^{j\omega t}] = \operatorname{Re} \left[\frac{20}{\rho} e^{j(\omega t - 2z)} a_\rho \right]$$

$$= \underline{\underline{\frac{20}{\rho} \cos(\omega t - 2z) a_\rho}}$$

$$(c) \quad 1 + j2 = 2.23 e^{j63.43^\circ}$$

$$C_s = \frac{10}{r^2} (2.236) e^{j63.43^\circ} e^{-j\phi} \sin \theta a_\phi$$

$$C = \operatorname{Re} [C_s e^{j\omega t}] = \operatorname{Re} \left[\frac{22.36}{r^2} e^{j(\omega t - \phi + 63.43^\circ)} \sin \theta a_\phi \right]$$

$$= \underline{\underline{\frac{22.36}{r^2} \cos(\omega t - \phi + 63.43^\circ) \sin \theta a_\phi}}$$

Prob. 9.37

(a) $\nabla \cdot \mathbf{E}_s = 0$

$$\nabla \times \mathbf{E}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix} = -\frac{\partial E_{sx}}{\partial y} \mathbf{a}_z = j40e^{-j4y} \mathbf{a}_z$$

But $\nabla \times \mathbf{E}_s = -j\mu_o\omega\mathbf{H}_s \longrightarrow \mathbf{H}_s = -\frac{40}{\mu_o\omega}e^{-j4y}\mathbf{a}_z$

$$\nabla \cdot \mathbf{H}_s = 0$$

$$\nabla \times \mathbf{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_{sz}(y) \end{vmatrix} = \frac{\partial H_{sz}}{\partial y} \mathbf{a}_x = \frac{j160}{\mu_o\omega}e^{-j4y}\mathbf{a}_x$$

But $\nabla \times \mathbf{H}_s = j\omega\epsilon_o\mathbf{E}_s \longrightarrow \mathbf{E}_s = \frac{160}{\mu_o\epsilon_o\omega^2}e^{-j4y}\mathbf{a}_x = 10e^{-j4y}\mathbf{a}_x$

$$\omega^2 = \frac{16}{\mu_o\epsilon_o} = \frac{16}{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}} = 16 \times 9 \times 10^{16}$$

$$\omega = \underline{\underline{12 \times 10^8 \text{ rad/s}}}$$

(b) $\mathbf{H}_s = -\frac{40}{4\pi \times 10^{-7} \times 12 \times 10^8}e^{-j4y}\mathbf{a}_z = \underline{\underline{-26.53e^{-j4y}\mathbf{a}_z \text{ mA/m}}}$

P. E. 10.1 (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2 \times 10^8} = \underline{31.42 \text{ ns}},$$

$$\lambda = uT = 3 \times 10^8 \times 31.42 \times 10^{-9} = \underline{9.425 \text{ m}}$$

$$k = \beta = 2\pi / \lambda = \underline{0.6667 \text{ rad/m}}$$

(b) $t_1 = T/8 = \underline{3.927 \text{ ns}}$

(c)

$$H(t = t_1) = 0.1 \cos(2 \times 10^8 \frac{\pi}{8 \times 10^8} - 2x/3) \mathbf{a}_y = 0.1 \cos(2x/3 - \pi/4) \mathbf{a}_y$$

as sketched below.

