Tutorial #1.

Q1: Point P & Q are located at (0, 2, 4) and (-3, 1, 5). Calculate

(a) The position vector P

(b) The distance vector from P to Q

(c) A vector parallel to PQ with magnitude of 10

Q2. $\vec{Q} = 2\vec{a}_x - \vec{a}_y + 2\vec{a}_z$, $\vec{R} = 2\vec{a}_x - 3\vec{a}_y + \vec{a}_z$

Find $\sin(\theta_{QR})$

Q3: Given P(-2, 6, 3).

Express P in cylindrical and spherical coordinates. ($\vec{A}$ is to be evaluated at P)

Q1. [Solution]

(a) $\vec{r}_P = 0\vec{a}_x + 2\vec{a}_y + 4\vec{a}_z = 2\vec{a}_y + 4\vec{a}_z$

(b) $\vec{R}_{PQ} = \vec{r}_Q - \vec{r}_P = (-3,1,5)-(0,2,4) = (-3,-1,1)$

$= -3\vec{a}_x - 1\vec{a}_y + \vec{a}_z = -3\vec{a}_x - \vec{a}_y + \vec{a}_z$

(c) $\vec{A} = A\vec{a}_A = 10\vec{a}_A$

$\because\ \vec{a}_A \parallel \vec{R}_{PQ} \Rightarrow \vec{a}_A = \pm\frac{\vec{R}_{PQ}}{|\vec{R}_{PQ}|} = \pm\frac{(-3,-1,1)}{\sqrt{3^2+1^2+1^2}} = \pm\frac{(-3,-1,1)}{3.317}$

$\Rightarrow\ \vec{A} = 10\vec{a}_A = 10\times(\pm1)\times\frac{(-3,-1,1)}{3.317}$

$= \pm(-9.045\vec{a}_x - 3.015\vec{a}_y + 3.015\vec{a}_z)$

Q2 [Solution]

$$\sin(\theta_{QR}) = \frac{|\vec{Q}\times\vec{R}|}{|\vec{Q}||\vec{R}|} = \frac{\left|\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}\right|}{|(2,-1,2)|\,|(2,-3,1)|}$$

$$= \frac{|(5,2,-4)|}{|(2,-1,2)|\,|(2,-3,1)|} = \frac{\sqrt{45}}{3\sqrt{14}} = 0.5976$$

Q3 [Solution]:

at P, $x=-2,\ y=6,\ z=3$.

$\Rightarrow\ \rho = \sqrt{x^2+y^2} = \sqrt{4+36} = 6.32$

$\phi = \tan^{-1}\frac{y}{x} = \tan\frac{6}{-2} = 108.43°$

$z = 3.$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{4+36+9} = 7$

$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\frac{\sqrt{40}}{3} = 64.62°$

$\Rightarrow\ P(x=-2, y=6, z=3) = P(\rho=6.32, \phi=108.43°, z=3)$

$= P(r=7, \theta=64.62°, \phi=108.43°)$