ECED 4301 Assignment #6 Solution :

Prob. 10.49 Since $\mu_o = \mu_1 = \mu_2$,

$$\sin\theta_{t1} = \sin\theta_i\sqrt{\frac{\varepsilon_o}{\varepsilon_1}} = \frac{\sin 45^o}{\sqrt{4.5}} = 0.3333 \longrightarrow \underline{\underline{\theta_{t1} = 19.47^o}}$$

$$\sin\theta_{t2} = \sin\theta_{t1}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{1}{3}\sqrt{\frac{4.5}{2.25}} = 0.4714 \longrightarrow \underline{\underline{\theta_{t2} = 28.13^o}}$$

Prob. 10.50

$$\boldsymbol{E}_s = \frac{20(e^{jk_x x} - e^{-jk_x x})}{j2}\frac{(e^{jk_y y} + e^{-jk_y y})}{2}\boldsymbol{a}_z$$

$$= -j5\left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)}\right]\boldsymbol{a}_z$$

which consists of four plane waves.

$$\nabla\times\boldsymbol{E}_s = -j\omega\mu_o\boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = \frac{j}{\omega\mu_o}\nabla\times\boldsymbol{E}_s = \frac{j}{\omega\mu_o}\left(\frac{\partial E_z}{\partial y}\boldsymbol{a}_x - \frac{\partial E_z}{\partial x}\boldsymbol{a}_y\right)$$

$$\underline{\underline{\boldsymbol{H}_s = -\frac{j20}{\omega\mu_o}\left[k_y\sin(k_x x)\sin(k_y y)\boldsymbol{a}_x + k_x\cos(k_x x)\cos(k_y y)\boldsymbol{a}_y\right]}}$$

Prob.10.54

(a) $k = \sqrt{k_x^2 + k_x^2} = \sqrt{6^2 + 10^2} = 10$

$$\omega = kc = 3\times10^9 = 2\pi f \longrightarrow f = \underline{\underline{477.5\text{ MHz}}}$$

$$\lambda = 2\pi / k = \frac{2\pi}{10} = \underline{\underline{0.6283\text{ m}}}$$

(b) $\boldsymbol{H} = \dfrac{\boldsymbol{k}\times\boldsymbol{E}}{\omega\mu}, \qquad \boldsymbol{k} = 6\boldsymbol{a}_x + 8\boldsymbol{a}_z$

$$\boldsymbol{k}\times\boldsymbol{a}_y = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 6 & 0 & 8 \\ 0 & 1 & 0 \end{vmatrix} = -8\boldsymbol{a}_x + 6\boldsymbol{a}_z$$

$$\frac{E_o}{\omega\mu} = \frac{50}{3\times10^9\times4\pi\times10^{-7}} = \frac{50}{1200\pi} = 0.01326$$

$$\boldsymbol{H} = \underline{\underline{13.26\sin(\omega t - 6x - 8z)(-8\boldsymbol{a}_x + 6\boldsymbol{a}_z)\text{ mA/m}}}$$

Prob. 10.57 (a)

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{1}{\sqrt{8}} \longrightarrow \underline{\underline{\theta_i = \theta_r = 19.47^o}}$$

$$\sin\theta_t = \sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}(3) = 1 \longrightarrow \underline{\underline{\theta_t = 90^o}}$$

(b) $\beta_1 = \dfrac{\omega}{c}\sqrt{\varepsilon_{r1}} = \dfrac{10^9}{3\times10^8}\times 3 = 10 = k\sqrt{1+8} = 3k \longrightarrow \underline{\underline{k = 3.333}}$

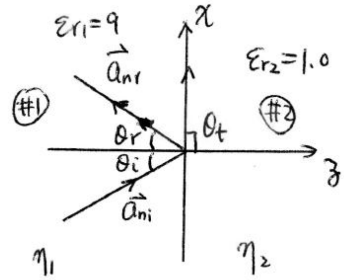
(c) $\lambda = 2\pi/\beta$, $\lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283}}$ m

$\beta_2 = \omega/c = 10/3$, $\lambda_2 = 2\pi/\beta_2 = 2\pi x 3/10 = \underline{\underline{1.885}}$ m

(d)
$$\begin{aligned} \boldsymbol{E}_i &= \eta_1 \boldsymbol{H}_x \times \boldsymbol{a}_k = 40\pi(0.2)\cos(\omega t - \boldsymbol{k}\bullet\boldsymbol{r})\boldsymbol{a}_y \times \frac{(\boldsymbol{a}_x + \sqrt{8}\boldsymbol{a}_z)}{3} \\ &= \underline{\underline{(23.6954\boldsymbol{a}_x - 8.3776\boldsymbol{a}_z)\cos(10^9 t - kx - k\sqrt{8}z)\ \text{V/m}}} \end{aligned}$$

Problem 10.57 (e)

(e)



$\theta_r=\theta_i$

$\theta_t=90°$

$$\eta_1=\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{r1}}}=\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\sqrt{\varepsilon_r}}=\eta_0\frac{1}{\sqrt{9}}=\frac{\eta_0}{3},\quad \eta_2=\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{r2}}}=\sqrt{\frac{\mu_0}{\varepsilon_0}}=\eta_0=120\pi\ (\Omega)$$

$$\tau_{//}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=\frac{2\eta_2\cos\theta_i}{\eta_2\underbrace{\cos 90°}_{=0}+\eta_1\cos\theta_i}=\frac{2\eta_2}{\eta_1}=6$$

$$\Gamma_{//}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=\frac{\eta_2\cos 90°-\eta_1\cos\theta_i}{\eta_2\underbrace{\cos 90°}_{=0}+\eta_1\cos\theta_i}=-1$$

$$\Rightarrow\ E_{t0}=\tau_{//}E_{i0}=6E_{i0},\quad E_{r0}=\Gamma_{//}E_{i0}=-E_{i0}$$

also: $K_1=\omega\sqrt{\mu_0\varepsilon_0\varepsilon_{r1}}=\frac{\omega}{c}\sqrt{\varepsilon_{r1}}=\frac{10^9}{3\times10^8}\cdot\sqrt{9}=10\quad(=\beta_1)$

$K_2=\omega\sqrt{\mu_0\varepsilon_0\varepsilon_{r2}}=\frac{\omega}{c}=\frac{10^9}{3\times10^8}=\frac{10}{3}\quad(=\beta_2)$

From the figure: $\vec{a}_{ni}=\cos\theta_i\,\vec{a}_z+\sin\theta_i\,\vec{a}_x=\frac{\sqrt{8}}{3}\vec{a}_z+\frac{1}{3}\vec{a}_x$;

note: $\cos\theta_i=\sqrt{1-\sin^2\theta_i}=\sqrt{1-(\frac{1}{3})^2}=\frac{\sqrt{8}}{3}$

$\vec{a}_{nr}=-\cos\theta_r\,\vec{a}_z+\sin\theta_r\,\vec{a}_x=-\frac{\sqrt{8}}{3}\vec{a}_z+\frac{1}{3}\vec{a}_x$

$\vec{a}_{nt}=\cos\theta_t\,\vec{a}_z+\sin\theta_t\,\vec{a}_x=\vec{a}_x$

From the question. $\vec{H}_i=\vec{H}_x=0.2\vec{a}_y e^{-j(kx+kz\sqrt{8})}=\frac{E_{i0}}{\eta_1}\vec{a}_y e^{-jk(x+\sqrt{8}z)}$

$$\Rightarrow\quad E_{i0}=0.2\eta_1=0.2\times\frac{\eta_0}{3}=0.2\times\frac{120\pi}{3}=8\pi\ \ (V/m)$$

$$\Rightarrow\quad E_{r0}=-E_{i0}=-8\pi,\quad E_{t0}=6E_{i0}=48\pi\ \ (V/m)$$

$$\Rightarrow\quad \vec{H}_r=-\vec{a}_y\frac{E_{r0}}{\eta_1}e^{-jk_1\vec{a}_{nr}\cdot\vec{r}}=+\frac{8\pi}{\eta_1}\vec{a}_y e^{-j10(-\frac{\sqrt{8}}{3}z+\frac{1}{3}x)}$$

$$\vec{E}_r=-\eta_1\vec{a}_{nr}\times\vec{H}_r=-\cancel{\eta_1}\times(-\tfrac{\sqrt{8}}{3}\vec{a}_z+\tfrac{1}{3}\vec{a}_x)\times(+\tfrac{8\pi}{\cancel{\eta_1}}\vec{a}_y)e^{-j10(-\frac{\sqrt{8}}{3}z+\frac{1}{3}x)}$$

$$=-\frac{8\pi}{3}(\sqrt{8}\vec{a}_x+\vec{a}_z)e^{-j10(-\frac{\sqrt{8}}{3}z+\frac{1}{3}x)}$$

$$\Rightarrow\quad \underline{\underline{\vec{E}_r=-\frac{8\pi}{3}(\sqrt{8}\vec{a}_x+\vec{a}_z)\cos(10^9t+\sqrt{8}kz-kx)}},\quad k=\frac{10}{3}=3.333\ (rad/s)$$

$$\Rightarrow\quad \vec{H}_t=\vec{a}_y\frac{E_{t0}}{\eta_2}e^{-jk_2\vec{a}_{nt}\cdot\vec{r}}=\vec{a}_y\frac{48\pi}{\eta_2}e^{-j\frac{10}{3}\vec{a}_x\cdot\vec{r}}$$

$$=\vec{a}_y\frac{48\pi}{\eta_2}e^{-j\frac{10}{3}x}$$

$$\vec{E}_t=-\eta_2\vec{a}_{nt}\times\vec{H}_t=-\cancel{\eta_2}\,\vec{a}_x\times\vec{a}_y\frac{48\pi}{\cancel{\eta_2}}e^{-j\frac{10}{3}x}=-48\pi\vec{a}_z e^{-j\frac{10}{3}x}$$

$$\Rightarrow\quad \underline{\underline{\vec{E}_t=-48\pi\vec{a}_z\cos(10^8t-\tfrac{10}{3}x)}}\quad (V/m)$$

$$=-48\pi\vec{a}_z\cos(10^8t-kx)$$

(f) $\tan\theta_{B//} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\dfrac{\varepsilon_o}{9\varepsilon_o}} = 1/3 \longrightarrow \underline{\underline{\theta_{B//} = 18.43^o}}$

Prob. 10.59 Since both media are nonmagnetic,

$$\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{2.6\varepsilon_o}{\varepsilon_o}} = 1.612 \longrightarrow \theta_{B//} = 58.19^o$$

But

$$\cos\theta_t = \frac{\eta_1}{\eta_2}\cos\theta_{B//} = \frac{\eta_o}{\eta_o/\sqrt{2.6}}\cos\theta_{B//} = \sqrt{2.6}\cos 58.19^o \longrightarrow \underline{\underline{\theta_t = 31.8^o}}$$