**ECED 4301- ELECTROMAGNETIC WAVES AND PROPOGATION SOLUTION ASSIGNMENT #2**

**Prob. 9.3**

$$\Psi = \int \boldsymbol{B} \cdot dS = BS$$

$$V_{emf} = -\frac{d\Psi}{dt} = -\frac{dB}{dt} S = 0.6x10x10^{-4} = 0.6 \text{ mV}$$

The emf is divided in the ratio of the resistances

$$v_1 = \frac{10}{15} x0.6 = \underline{\underline{0.4 \text{ mV}}}$$

$$v_2 = \frac{5}{15} x0.6 = \underline{\underline{0.2 \text{ mV}}}$$

**Prob. 9.8**

$$V_{emf} = -\int \frac{\partial \vec{B}}{\partial t} \bullet dS + \int (\vec{u} \times \vec{B}) \bullet d\vec{l}$$

where $\vec{B} = B_o \cos \omega t \vec{a}_x, \vec{u} = u_o \cos \omega t \vec{a}_y, d\vec{l} = dz \vec{a}_z$

$$V_{emf} = \int_{z=0}^{l} \int_{y=-a}^{y} B_o \omega \sin \omega t dy dz - \int_0^l B_o u_o \cos^2 \omega t dz$$

$$= B_o \omega l(y+a) \sin \omega t - B_o u_o l \cos^2 \omega t$$

Alternatively,

$$\psi = \int \vec{B} \bullet d\vec{s} = \int_{z=0}^{l} \int_{y=-a}^{y} B_o \cos \omega t \vec{a}_x \bullet dy dz \vec{a}_x = B_o(y+a) l \cos wt$$

$$V_{emf} = -\frac{\partial \psi}{\partial t} = B_o (y+a) l \omega \sin \omega t - B_o \frac{dy}{dt} l \cos \omega t$$

But $\frac{dy}{dt} = u = u_o \cos \omega t \rightarrow y = \frac{u_o}{\omega} \sin \omega t$

$$V_{emf} = B_o \omega l(y+a) \sin \omega t - B_o u_o l \cos^2 \omega t$$

$= B_o u_o l \sin^2 \omega t + B_o \omega a l \sin\omega t - B_o u_o l \cos^2 \omega t$

$= -B_o u_o l \cos 2\omega t + B_o \omega a l \sin\omega t$

$= 6 \times 10^{-3} \times 5[10 \times 10 \sin 10t - 2\cos 20t]$

$V_{emf} = \underline{\underline{3\sin 10t - 0.06\cos 20t \text{ V}}}$

**Prob. 9.11**

$d\psi = 0.64 - 0.45 = 0.19,\ dt = 0.02$

$$V_{emf} = N\frac{d\psi}{dt} = 10\left(\frac{0.19}{0.02}\right) = 95V$$

$$I = \frac{V_{emf}}{R} = \left(\frac{95}{15}\right) = \underline{\underline{6.33 \text{ A}}}$$

Using Lenz's law, the direction of the induced current is <u>counterclockwise.</u>

**Prob. 9.22**

$$\nabla \times H = J_d$$

$$\boldsymbol{J}_d = \nabla \times \boldsymbol{H} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z(x,t) \end{vmatrix} = -\frac{\partial H_z}{\partial x}\boldsymbol{a}_y$$

$$\boldsymbol{J}_d = \underline{\underline{20\cos(10^8 t - 2x)\boldsymbol{a}_y \text{ A/m}^2}}$$

But $\boldsymbol{J}_d = \varepsilon \dfrac{\partial \boldsymbol{E}}{\partial t} \longrightarrow \boldsymbol{E} = \dfrac{1}{\varepsilon}\displaystyle\int \boldsymbol{J}_d dt = \dfrac{1}{\varepsilon}\dfrac{20}{10^8}\sin(10^8 t - 2x)\boldsymbol{a}_y$

Also,

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x) & 0 \end{vmatrix} = \frac{\partial E_y}{\partial x}\boldsymbol{a}_z = -\frac{40}{10^8 \varepsilon}\cos(10^8 t - 2x)\boldsymbol{a}_z$$

$$-\mu \frac{\partial \boldsymbol{H}}{\partial t} = -10\mu_o x 10^8 \cos(10^8 t - 2x)\boldsymbol{a}_z$$

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \longrightarrow -\frac{40}{10^8 \varepsilon} = -10\mu x 10^8$$

$$\frac{4}{\mu_o \varepsilon_r \varepsilon_o} = 10^{16} \longrightarrow \varepsilon_r = \frac{4}{4\pi x 10^{-7} x \frac{10^{-9}}{36\pi} x 10^{16}} = \underline{\underline{36}}$$

$$\boldsymbol{E} = \frac{20}{36x\frac{10^{-9}}{36\pi}x10^8}\sin(10^8 t - 2x)\boldsymbol{a}_y = \underline{\underline{628.32\sin(10^8 t - 2x)\boldsymbol{a}_y \text{ V/m}}}$$

**Prob. 9.26**

$$\nabla \bullet \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x(y,z,t) & 0 & 0 \end{vmatrix} = \frac{\partial E_x}{\partial z}\boldsymbol{a}_y - \frac{\partial E_x}{\partial y}\boldsymbol{a}_z$$

$$= \frac{\beta\omega\mu b}{\pi} H_o \sin(\pi y/b)\cos(\omega t - \beta z)\boldsymbol{a}_y + \omega\mu H_o \cos(\pi y/b)\sin(\omega t - \beta z)\boldsymbol{a}_z$$

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \quad \longrightarrow \quad \boldsymbol{H} = -\frac{1}{\mu}\int \nabla \times \boldsymbol{E} dt$$

$$\boldsymbol{H} = -\frac{\beta b}{\pi} H_o \sin(\pi y/b)\sin(\omega t - \beta z)\boldsymbol{a}_y + H_o \cos(\pi y/b)\cos(\omega t - \beta z)\boldsymbol{a}_z$$

which is the given H field.

$$\nabla \times \boldsymbol{H} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & H_y & H_z \end{vmatrix} = \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right)\boldsymbol{a}_x$$

$$= [-\frac{\pi}{b} H_o \sin(\pi y/b)\cos(\omega t - \beta z) + \frac{\beta^2 b}{\pi} H_o \sin(\pi y/b)\cos(\omega t - \beta z)]\boldsymbol{a}_x$$

$$\nabla \times \boldsymbol{H} = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \quad \longrightarrow \quad \boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

$$\boldsymbol{E} = [-\frac{\pi}{\omega b \varepsilon} H_o \sin(\pi y/b)\sin(\omega t - \beta z) + \frac{\beta^2 b}{\pi\omega\varepsilon} H_o \sin(\pi y/b)\sin(\omega t - \beta z)]\boldsymbol{a}_x$$

Setting this equal to the given E,

$$\frac{\omega\mu b}{\pi} H_o = \frac{\pi}{\omega b \varepsilon} H_o \overset{-}{\cancel{+}} \frac{\beta^2 b}{\pi\omega\varepsilon} H_o \quad \longrightarrow \quad \beta^2 = -\frac{\pi^2}{b^2} \overset{-}{\cancel{+}} \omega^2 \mu\varepsilon$$

$$\underline{\underline{\beta = \sqrt{\omega^2 \mu\varepsilon + \frac{\pi^2}{b^2}}}}$$

**Prob. 9.30** Using Maxwell's equations,

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \qquad (\sigma = 0) \qquad \longrightarrow \qquad \boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

But

$$\nabla \times \boldsymbol{H} = -\frac{1}{r\sin\theta}\frac{\partial H_\theta}{\partial \phi}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)\boldsymbol{a}_\phi = \frac{12\sin\theta}{r}\beta \sin(2\pi x10^8 t - \beta r)\boldsymbol{a}_\phi$$

$$\boldsymbol{E} = \frac{12\sin\theta}{\varepsilon_o}\beta \int \sin(2\pi x10^8 t - \beta r) dt\boldsymbol{a}_\phi$$

$$= -\frac{12\sin\theta}{\omega \varepsilon_o r}\beta \cos(\omega t - \beta r)\boldsymbol{a}_\phi, \qquad \omega = 2\pi x10^8$$