Prob. 10.21

(a) $\quad \boldsymbol{E} = \text{Re}[\boldsymbol{E}_s e^{j\omega t}] = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-0.2z}\cos(\omega t - 3.4z)$

At $z = 4$m, $t = T/8$, $\quad \omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$$\boldsymbol{E} = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-0.8}\cos(\pi/4 - 13.6)$$

$$|E| = 13e^{-0.8}\,|\cos(\pi/4 - 13.6)| = \underline{\underline{5.662}}\ \text{V/m}$$

(b) loss $= \alpha\Delta z = 0.2(3) = 0.6$ Np. Since 1 Np = 8.686 dB,

$$\text{loss} = 0.6 \times 8.686 = \underline{\underline{5.212\ \text{dB}}}$$

(c) Let $\quad x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2/3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega\sqrt{\mu\varepsilon/2}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r/2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2x3x10^8}{10^8\sqrt{0.00694}} = 7.2 \quad \longrightarrow \quad \varepsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o}}.\frac{1}{\sqrt{\varepsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68 \times 1.00694}} = 36.896$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2 - 1} = 0.118 \longrightarrow \theta_\eta = 3.365^o$$

$$\eta = 36.896\angle 3.365^o\ \Omega$$

$$\boldsymbol{H}_s = \boldsymbol{a}_k \times \frac{\boldsymbol{E}_s}{\eta} = \frac{\boldsymbol{a}_z}{\eta} \times (5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-\gamma z} = \frac{(5\boldsymbol{a}_y - 12\boldsymbol{a}_x)}{|\eta|} e^{-j3.365^o} e^{-\gamma z}$$

$$H = (-325.24a_x + 135.5a_y)e^{-0.2z}\cos(\omega t - 3.4z - 3.365^o)\ \text{mA}$$

$$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 5 & 12 & 0 \\ -325.24 & 135.5 & 0 \end{vmatrix} \times 10^{-3} e^{-0.4z}\cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)$$

$$\boldsymbol{P} = 4.58e^{-0.4z}\cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)\boldsymbol{a}_z$$

At z=4, t=T/4,

$$\boldsymbol{P} = 4.58e^{-1.6}\cos(\pi/4 - 13.6)\cos(\pi/4 - 13.6 - 0.0587)\boldsymbol{a}_z = \underline{\underline{0.8545\boldsymbol{a}_z\ \text{W/m}^2}}$$

Prob. 10.23

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi f}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1.0049} - 1\right]} = \frac{2\pi \times 6\times 10^6}{3\times 10^8}\sqrt{\frac{4}{2}\times 2.447\times 10^{-3}}$$

$$\alpha = 8.791x10^{-3}$$

$$\delta = 1/\alpha = \underline{\underline{113.75\ \text{m}}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} = \frac{4\pi}{100}\sqrt{\frac{4}{2}}\left[\sqrt{1.0049} + 1\right] = 0.2515$$

$$u = \omega/\beta = \frac{2\pi\times 6\times 10^6}{0.2515} = \underline{\underline{1.5\times 10^8}}\ \text{m/s}$$

Prob. 10.25

(a) $\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{3.5\times10^{7}}{2\pi\times150\times10^{6}\times\dfrac{10^{-9}}{36\pi}} = \dfrac{3.5\times18\times10^{9}}{15} >> 1$

$\alpha = \beta = \sqrt{\dfrac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma} = \sqrt{150\pi\times10^{6}\times4\pi\times10^{-7}\times3.5\times10^{7}} = 143,965.86$

$\gamma = \alpha + j\beta = \underline{\underline{1.44(1+j)x10^{5}\ /\text{m}}}$

(b) $\delta = 1/\alpha = \underline{\underline{6.946\times10^{-6}\ \text{m}}}$

(c) $u = \dfrac{\omega}{\beta} = \dfrac{150\times2\pi\times10^{6}}{1.44\times10^{5}} = \underline{\underline{6547\ \text{m/s}}}$

Prob. 10.32

(a) $\boldsymbol{H}_s = \dfrac{j30\beta I_o dl}{120\pi r}\sin\theta e^{-j\beta r}\boldsymbol{a}_H$

where $\boldsymbol{a}_E\times\boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_\theta\times\boldsymbol{a}_H = \boldsymbol{a}_r \longrightarrow \boldsymbol{a}_H = \boldsymbol{a}_\phi$

$\boldsymbol{H}_s = \underline{\underline{\dfrac{j\beta I_o dl}{4\pi r}\sin\theta e^{-j\beta r}\boldsymbol{a}_\phi}}$

(b) $\boldsymbol{P}_{ave} = \dfrac{1}{2}\text{Re}[\boldsymbol{E}_s\times\boldsymbol{H}_s^*] = \dfrac{1}{2}\text{Re}[\dfrac{30\beta^2 I_o^2 dl^2\sin^2\theta}{4\pi r^2}\boldsymbol{a}_r] = \underline{\underline{\dfrac{15\beta^2 I_o^2 dl^2\sin^2\theta}{4\pi r^2}}}\boldsymbol{a}_r$

Prob. 10.33

(a) $P_{ave} = \frac{1}{2}\mathrm{Re}(E_s H_s^*) = \frac{1}{2}\mathrm{Re}(\frac{|E_s|}{|\eta|}) = \frac{8^2}{2|\eta|}e^{-0.2z}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

Let $\mathrm{x} = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = 0.1/0.3 = 1/3$$

$$\frac{x-1}{x+1} = \frac{1}{9} \longrightarrow x = 5/4$$

$$\frac{5}{4} = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \longrightarrow \frac{\sigma}{\omega\varepsilon} = 3/4$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi/\sqrt{81}}{\sqrt{\frac{5}{4}}} = 37.4657$$

$$P_{ave} = \frac{64}{2(37.4657)}e^{-0.2z} = \underline{\underline{0.8541e^{-0.2z}\ \mathrm{W/m^2}}}$$

(b) $20dB = 10\log\frac{P_1}{P_2} \longrightarrow \frac{P_1}{P_2} = 100$

$$\frac{P_2}{P_1} = e^{-0.2z} = \frac{1}{100} \longrightarrow e^{0.2z} = 100$$

$$z = 5\log 100 = \underline{\underline{23\ \mathrm{m}}}$$