Prob. 9.3

$$\Psi = \int B \cdot dS = BS$$

$$V_{emf} = -\frac{d\Psi}{dt} = -\frac{dB}{dt}S = 0.6x10x10^{-4} = 0.6 \text{ mV}$$

The emf is divided in the ratio of the resistances

$$v_1 = \frac{10}{15}x0.6 = \underline{\underline{0.4 \text{ mV}}}$$

$$v_2 = \frac{5}{15}x0.6 = \underline{\underline{0.2 \text{ mV}}}$$

Prob. 9.4

$$V_{emf} = -N\int \frac{\partial B}{\partial t} \bullet dS = N(10^3)(2)\int_{-0.1}^{0.1} \cos y dy \int_{-0.1}^{0.1} dx \sin 10^3 t$$

$$= 50(2)\sin 10^3 t(0.2)\sin y\Big|_{-0.1}^{0.1} \text{ kV} \cong \underline{\underline{4\sin 10^3 t \text{ kV}}}$$

Prob. 9.5

(a) $v = \int(\bar{u}\times\bar{B})\cdot d\bar{l}, \quad d\bar{l} = dy\bar{a}_y$

$\bar{u}\times\bar{B} = 2\bar{a}_x \times 0.1\bar{a}_z = -0.2\bar{a}_y$

y = x since the angle of the v-shaped conductor is 45°. Hence

y = x = ut. At t = 0, x = 0 = y

$v = -\int 0.2 \, dy = -0.2y, \quad y = ut = 2t$

$\underline{\underline{v = -0.4t \text{ V}}}$

(b) $v = \int(\bar{u}\times\bar{B})\cdot d\bar{l}, \quad d\bar{l} = dy\bar{a}_y$

$\bar{u}\times\bar{B} = 2\bar{a}_x \times 0.5x\bar{a}_z = -x\bar{a}_y$

But y = x and x = ut. When t = 0, x = 0 = y

$v = -\int x \, dy = -\int y \, dy = -y^2/2$

But x = y = ut = 2t

$\underline{\underline{v = -2t^2 \text{ V}}}$

Prob. 9.6

$$\bar{B} = \frac{\mu_o I}{2\pi y}(-\bar{a}_x)$$

$$\psi = \int \vec{B} \bullet d\vec{S} = \frac{\mu_o I}{2\pi} \int_{z=0}^{a} \int_{y=\rho}^{\rho+a} \frac{dzdy}{y} = \frac{\mu_o Ia}{2\pi} \ln \frac{\rho + a}{\rho}$$

$$V_{emf} = -\frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial \rho} \bullet \frac{\partial \rho}{\partial t} = -\frac{\mu_o Ia}{2\pi} u_o \frac{d}{d\rho}[\ln(\rho + a) - \ln \rho]$$

$$= -\frac{\mu_o Ia}{2\pi} u_o \left[\frac{1}{\rho + a} - \frac{1}{\rho}\right] = \underline{\underline{\frac{\mu_o a^2 I u_o}{2\pi\rho(\rho + a)}}}$$

where $\rho = \rho_o + u_o t$

Prob. 9.7

$$V_{emf} = \int_{\rho}^{\rho+a} 3\boldsymbol{a}_z \times \frac{\mu_o I}{2\pi\rho} \boldsymbol{a}_\phi \bullet d\rho \boldsymbol{a}_\rho = -\frac{3\mu_o I}{2\pi} \ln \frac{\rho + a}{\rho}$$

$$= -\frac{4\pi \times 10^{-7}}{2\pi} \times 15 \times 3 \ln \frac{60}{20} = -9.888 \mu V$$

Thus the induced emf = 9.888μV, point A at higher potential.

Prob. 9.8

$$V_{emf} = -\int \frac{\partial \vec{B}}{\partial t} \bullet dS + \int (\vec{u} \times \vec{B}) \bullet d\vec{l}$$

where $\vec{B} = B_o \cos \omega t \vec{a}_x, \vec{u} = u_o \cos \omega t \vec{a}_y, d\vec{l} = dz \vec{a}_z$

$$V_{emf} = \int_{z=0}^{l} \int_{y=-a}^{y} B_o \omega \sin \omega t dy dz - \int_{0}^{l} B_o u_o \cos^2 \omega t dz$$

$$= B_o \omega l(y+a)\sin\omega t - B_o u_o l\cos^2 \omega t$$

Alternatively,

$$\psi = \int \vec{B} \bullet d\vec{s} = \int_{z=0}^{l} \int_{y=-a}^{y} B_o \cos \omega t \vec{a}_x \bullet dy dz \vec{a}_x = B_o(y+a)l\cos wt$$

$$V_{emf} = -\frac{\partial \psi}{\partial t} = B_o(y + a) l \omega \sin \omega t - B_o \frac{dy}{dt} l \cos \omega t$$

But $\frac{dy}{dt} = u = u_o \cos \omega t \rightarrow y = \frac{u_o}{\omega} \sin \omega t$

$$V_{emf} = B_o \omega l(y+a)\sin\omega t - B_o u_o l\cos^2 \omega t$$

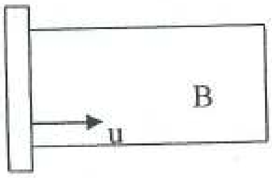
$= B_o u_o l \sin^2\omega t + B_o \omega a l \sin\omega t - B_o u_o l \cos^2\omega t$

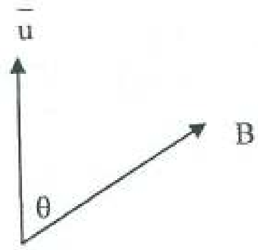
$= -B_o u_o l \cos 2\omega t + B_o \omega a l \sin\omega t$

$= 6 \times 10^{-3} \times 5[10 \times 10\sin 10t - 2\cos 20t]$

$V_{emf} = \underline{\underline{3\sin 10t - 0.06\cos 20t \text{ V}}}$

Prob. 9.9



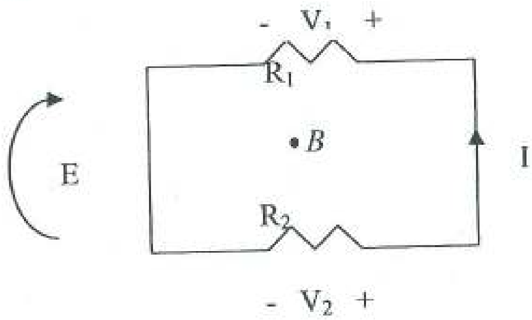


$$V_{emf} = \int (\bar{u} \times \bar{B}) \cdot d\bar{l} = uBl\cos\theta$$

$$= \left(\frac{120 \times 10^3}{3600} m/s\right)(4.3 \times 10^{-5})(1.6)\cos 65^o$$

$$= 2.293\cos 65^o = \underline{\underline{0.97}} \text{ mV}$$

Prob. 9.10



$$\oint \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\int \vec{B} \bullet dS$$

$$= I(R_1 + R_2)$$

$$\frac{dB}{dt} \bullet S = I(R_1 + R_2) \qquad (1)$$

Also, $\oint \vec{E} \bullet d\vec{l} = V_1 - V_2 = -\frac{dB}{dt} \bullet S \qquad (2)$

Hence, $V_1 = IR_1 = -\frac{SR_1}{R_1 + R_2}\frac{dB}{dt}$

$$V_2 = -IR_2 = \frac{SR_2}{R_1 + R_2}\frac{dB}{dt}$$

$$V_1 = \frac{10\times10^{-4}\times10}{15}\times0.2\times150\pi\sin150\pi t = \underline{\underline{0.0628\sin150\pi t}}\ \text{V}$$

$$V_2 = \frac{-10\times10^{-4}\times5}{15}\times0.2\times150\pi\sin150\pi t = \underline{\underline{-0.0314\sin150\pi t}}\ \text{V}$$

Prob. 9.11

$d\psi = 0.64 - 0.45 = 0.19$, $dt = 0.02$

$$V_{emf} = N\frac{d\psi}{dt} = 10\left(\frac{0.19}{0.02}\right) = 95V$$

$$I = \frac{V_{emf}}{R} = \left(\frac{95}{15}\right) = \underline{\underline{6.33\ \text{A}}}$$

Using Lenz's law, the direction of the induced current is counterclockwise.

Prob. 9.12

$$V = \int(\vec{u}\times\vec{B})\bullet d\vec{l}\text{, where } \vec{u} = \rho\omega\vec{a}_\phi,\ \vec{B} = B_o\vec{a}_z$$

$$V = \int_{\rho_1}^{\rho_2}\rho\omega B_o d\rho = \frac{\omega B_o}{2}(\rho^2{}_2 - \rho^2{}_1)$$

$$V = \frac{60\times15}{2}\bullet10^{-3}(100-4)\bullet10^{-4} = \underline{\underline{4.32\ \text{mV}}}$$

Prob. 9.13

$$J_{ds} = j\omega D_s \rightarrow |J_{ds}|_{\max} = \omega\varepsilon E_s = \omega\varepsilon\frac{V_s}{d}$$

$$= \frac{10^{-9}}{36\pi}\times\frac{2\pi\times20\times10^6\times50}{0.2\times10^{-3}}$$

$$= \underline{\underline{277.8}}\ \text{A/m}^2$$

$$I_{ds} = J_{ds}\bullet S = \frac{1000}{3.6}\times2.8\times10^{-4} = \underline{\underline{77.78}}\ \text{mA}$$

Prob. 9.14 $\quad \dfrac{J_c}{J_d} = \dfrac{\sigma E}{\omega\varepsilon E} = \dfrac{\sigma}{\omega\varepsilon}$

(a) $$\frac{\sigma}{\omega\varepsilon} = \frac{2x10^{-3}}{2\pi x10^9 x81x\dfrac{10^{-9}}{36\pi}} = \underline{\underline{0.444x10^{-3}}}$$

$$-\mu\frac{\partial \boldsymbol{H}}{\partial t} = -10\mu_o x10^8\cos(10^8 t - 2x)\boldsymbol{a}_z$$

$$\nabla\times\boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t} \longrightarrow -\frac{40}{10^8\varepsilon} = -10\mu x10^8$$

$$\frac{4}{\mu_o\varepsilon_r\varepsilon_o} = 10^{16} \longrightarrow \varepsilon_r = \frac{4}{4\pi x10^{-7}x\dfrac{10^{-9}}{36\pi}x10^{16}} = \underline{\underline{36}}$$

$$\boldsymbol{E} = \frac{20}{36x\dfrac{10^{-9}}{36\pi}x10^8}\sin(10^8 t - 2x)\boldsymbol{a}_y = \underline{\underline{628.32\sin(10^8 t - 2x)\boldsymbol{a}_y \text{ V/m}}}$$

Prob. 9.23

$$\boldsymbol{J}_d = \frac{\partial \boldsymbol{D}}{\partial t} = \varepsilon_o\frac{\partial \boldsymbol{E}}{\partial t} = \frac{50\varepsilon_o}{\rho}(-10^8)\sin(10^8 t - kz)\boldsymbol{a}_\rho = \underline{\underline{-\frac{4.421x10^{-2}}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho \text{ A/m}^2}}$$

$$\nabla\times\boldsymbol{E} = -\mu_o\frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla\times\boldsymbol{E} = \frac{\partial E_\rho}{\partial z}\boldsymbol{a}_\phi = \frac{50k}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\phi$$

$$\boldsymbol{H} = -\frac{1}{\mu_o}\int\nabla\times\boldsymbol{E}dt = \frac{1}{4\pi x10^{-7}}\frac{50k}{10^8\rho}\cos(10^8 t - kz)\boldsymbol{a}_\phi$$

$$\boldsymbol{H} = \underline{\underline{\frac{2.5k}{2\pi\rho}\cos(10^8 t - kz)\boldsymbol{a}_\phi \text{ A/m}}}$$

$$\nabla\times\boldsymbol{H} = -\frac{\partial H_\phi}{\partial z}\boldsymbol{a}_\rho = -\frac{2.5k^2}{2\pi\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{J}_d \longrightarrow -\frac{4.421x10^{-2}}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho = \frac{-2.5k^2}{2\pi\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho$$

$$k^2 = \frac{2\pi}{2.5}x4.421x10^{-2} \longrightarrow \underline{\underline{k = 0.333}}$$

Prob. 9.27 (a) $\nabla \bullet A = 0$

$$\nabla \times A = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z(x,t) \end{vmatrix} = -\frac{\partial E_z(x,t)}{\partial x}\boldsymbol{a}_y \neq 0$$

Yes, A is a possible EM field.

(b) $\nabla \bullet B = 0$

$$\nabla \times \boldsymbol{B} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[10\cos(\omega t - 2\rho)\right]\boldsymbol{a}_z \neq 0$$

Yes, B is a possible EM field.

(c) $\nabla \bullet \boldsymbol{C} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(3\rho^3 \cot\phi \sin\omega t\right) - \frac{\sin\phi \sin\omega t}{\rho^2} \neq 0$

$$\nabla \times \boldsymbol{C} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\cos\phi \sin\omega t\right)\boldsymbol{a}_z - 3\rho^2\frac{\partial}{\partial \phi}(\cot\phi \sin\omega t)\boldsymbol{a}_z \neq 0$$

No, C cannot be an EM field.

(d) $\nabla \bullet D = \frac{1}{r^2 \sin\theta}\sin(\omega t - 5r)\frac{\partial}{\partial \theta}(\sin^2\theta) \neq 0$

$$\nabla \times \boldsymbol{D} = -\frac{\partial D_\theta}{\partial \phi}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial r}(rD_\theta)\boldsymbol{a}_\phi = \frac{1}{r}\sin\theta(-5)\sin(\omega t - 5r)\boldsymbol{a}_\phi \neq 0$$

No, D cannot be an EM field.

Prob. 9.28 From Maxwell's equations,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (2)$$

Dotting both sides of (2) with $\vec{E}$ gives:

$$\vec{E} \bullet (\nabla \times \vec{H}) = \vec{E} \bullet \vec{J} + \vec{E} \bullet \frac{\partial \vec{D}}{\partial t} \qquad (3)$$

But for any arbitrary vectors $\vec{A}$ and $\vec{B}$,

$$\nabla \bullet (\vec{A} \times \vec{B}) = \vec{B} \bullet (\nabla \times \vec{A}) - \vec{A} \bullet (\nabla \times \vec{B})$$

Applying this on the left-hand side of (3) by letting $\vec{A} \equiv \vec{H}$ and $\vec{B} \equiv \vec{E}$, we get

Prob. 9.30 Using Maxwell's equations,

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \qquad (\sigma = 0) \qquad \longrightarrow \qquad \boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

But

$$\nabla \times \boldsymbol{H} = -\frac{1}{r\sin\theta}\frac{\partial H_\theta}{\partial \phi}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)\boldsymbol{a}_\phi = \frac{12\sin\theta}{r}\beta\sin(2\pi x10^8 t - \beta r)\boldsymbol{a}_\phi$$

$$\boldsymbol{E} = \frac{12\sin\theta}{\varepsilon_o}\beta\int \sin(2\pi x10^8 t - \beta r)dt\boldsymbol{a}_\phi$$

$$= \underline{\underline{-\frac{12\sin\theta}{\omega\varepsilon_o r}\beta\cos(\omega t - \beta r)\boldsymbol{a}_\phi}}, \qquad \omega = 2\pi x10^8$$

Prob. 9.31

$$\nabla \times \vec{E} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi)\vec{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 te^{-\rho-t})\vec{a}_z$$

$$= (2-\rho)te^{-\rho-t}\vec{a}_z$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} \to \vec{B} = -\int \nabla \times \vec{E} dt = \int \frac{(\rho-2)t}{V}\frac{e^{-\rho-t}dt}{du}\vec{a}_z$$

Integrating by parts yields

$$\vec{B} = [-(\rho-2)te^{-\rho-t} + \int(\rho-2)e^{-\rho-t}dt]\vec{a}_z$$

$$= \underline{\underline{(2-\rho)(1+t)e^{-\rho-t}\vec{a}_z}} \text{ Wb/m}^2$$

$$\vec{J} = \nabla \times \vec{H} = \nabla \times \frac{\vec{B}}{\mu_o} = -\frac{1}{\mu_o}\frac{\partial B_z}{\partial \rho}\vec{a}_\phi$$

$$= -\frac{1}{\mu_o}(1+t)(-1-2+\rho)e^{-\rho-t}\vec{a}_\phi$$

$$\vec{J} = \underline{\underline{\frac{(1+t)(3-\rho)e^{-\rho-t}}{4\pi x10^{-7}}\vec{a}_\phi}} \text{ A/m}^2$$

Prob. 9.32

With the given **A**, we need to prove that

$$\nabla^2 \boldsymbol{A} = \mu\varepsilon\frac{\partial^2 \boldsymbol{A}}{\partial t^2}$$