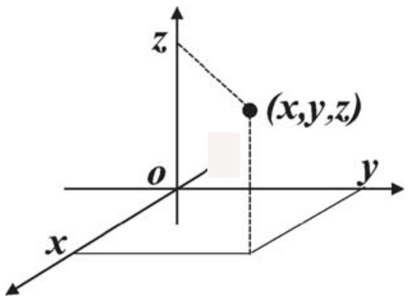
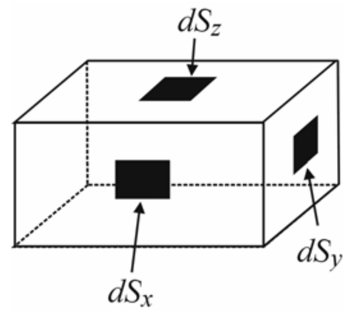
$\mu_o = 4\pi \times 10^{-7}\, H/m$ $\varepsilon_o = \frac{1}{36\pi} \times 10^{-9}\, F/m$

Cartesian Coordinates (x, y, z) :





$\boldsymbol{r} = x\,\mathbf{a}_x + y\,\mathbf{a}_y + z\,\mathbf{a}_z$ $dS_x = dy\,dz$

$d\boldsymbol{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$ $dS_y = dx\,dz$

$dv = dx\ dy\ dz$ $dS_z = dx\,dy$

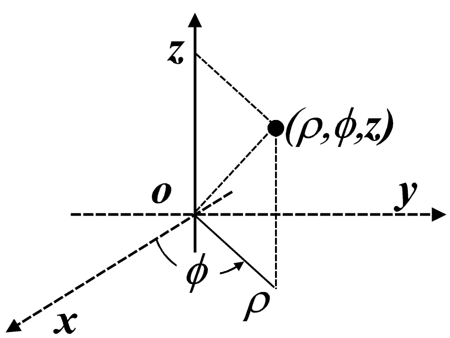
$\nabla f = \frac{\partial f}{\partial x}\mathbf{a}_x + \frac{\partial f}{\partial y}\mathbf{a}_y + \frac{\partial f}{\partial z}\mathbf{a}_z$

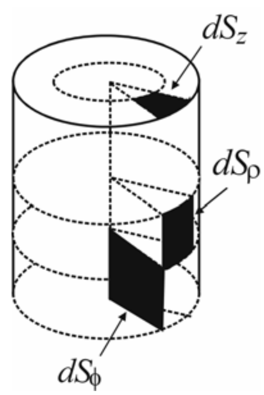
$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ (where $\mathbf{A} = A_x\mathbf{a}_x + A_x\mathbf{a}_y + A_z\mathbf{a}_z$)

$\nabla \times \mathbf{A} = [\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}]\mathbf{a}_x + [\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}]\mathbf{a}_y + [\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}]\mathbf{a}_z$

$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Cylindrical Coordinates (ρ, ϕ, z):





$\boldsymbol{r} = \rho\, cos\phi\,\mathbf{a}_x + \rho\, sin\phi\,\mathbf{a}_y + z\,\mathbf{a}_z$ $dS_\rho = \rho\, d\phi dz$

$d\boldsymbol{l} = d\rho\,\mathbf{a}_\rho + \rho\, d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z$ $dS_\phi = d\rho\, dz$

$dv = \rho\, d\rho\ d\phi\ dz$ $dS_z = \rho\, d\rho\, d\phi$

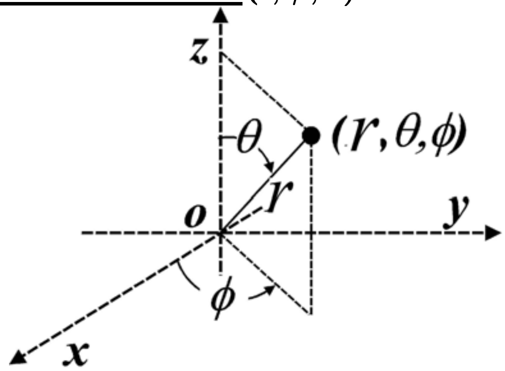
$\nabla f = \frac{\partial f}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\mathbf{a}_\phi + \frac{\partial f}{\partial z}\mathbf{a}_z$

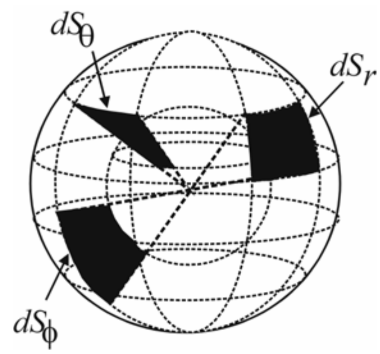
$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$ (where $\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$)

$\nabla \times \mathbf{A} = [\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}]\mathbf{a}_\rho + [\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}]\mathbf{a}_\phi + \frac{1}{\rho}[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}]\mathbf{a}_z$

$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial f}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates $(\boldsymbol{r}, \phi, \theta)$**:**





$$\boldsymbol{r} = r\,sin\theta\,cos\phi\,\mathbf{a}_x + r\,sin\theta\,sin\phi\,\mathbf{a}_y + r\,cos\theta\,\mathbf{a}_z \qquad dS_r = r^2 sin\theta\,d\theta\,d\phi$$

$$d\boldsymbol{l} = \mathbf{a}_r dr + \mathbf{a}_{\boldsymbol{\theta}} r d\theta + \mathbf{a}_{\boldsymbol{\phi}} r sin\theta\,d\phi \qquad dS_\theta = r sin\theta\,dr\,d\phi$$

$$dv = r^2 sin\theta\ dr\ d\phi\ d\theta \qquad dS_\phi = r\,d\theta\,dr$$

$$\nabla f = \frac{\partial f}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{a}_{\boldsymbol{\theta}} + \frac{1}{r\,sin\theta}\frac{\partial f}{\partial \phi}\mathbf{a}_{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\,sin\theta}\frac{\partial}{\partial \theta}(A_\theta sin\theta) + \frac{1}{r\,sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

(where $\mathbf{A} = A_r \mathbf{a}_r + A_\theta \mathbf{a}_{\boldsymbol{\theta}} + A_\phi \mathbf{a}_\phi$)

$$\nabla \times \mathbf{A} = \frac{1}{r sin\theta}[\frac{\partial}{\partial \theta}(A_\phi sin\theta) - \frac{\partial A_\theta}{\partial \phi}]\mathbf{a}_r + \frac{1}{r}[\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)]\mathbf{a}_{\boldsymbol{\theta}}$$

$$+ \frac{1}{r}[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}]\mathbf{a}_{\boldsymbol{\phi}}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Conversion Between the Different Coordinates:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} cos\phi & -\,sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$x = \rho\,cos\phi \qquad \rho = (x^2 + y^2)^{\frac{1}{2}}$$

$$y = \rho\,sin\phi \qquad \phi = tan^{-1}\frac{y}{x}$$

$$z = z \qquad z = z$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} cos\phi\,sin\theta & cos\phi\,cos\theta & -\,sin\phi \\ sin\phi\,sin\theta & sin\phi\,cos\theta & cos\phi \\ cos\theta & -\,sin\theta & 0 \end{bmatrix}\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$x = r\,sin\theta\,cos\phi \qquad r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$y = r\,sin\theta\,sin\phi \qquad \theta = tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$$

$$z = r\,cos\theta \qquad \phi = tan^{-1}\frac{y}{x}$$

Vector Identities:

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = (\mathbf{B} \times \mathbf{C}) \cdot \mathbf{A} = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

$$\nabla(\psi V) = \psi\,\nabla V + V\,\nabla\psi$$

$$\nabla \cdot (V\mathbf{A}) = \nabla V \cdot \mathbf{A} + V\nabla \cdot \mathbf{A}$$

$$\nabla \times (V\mathbf{A}) = \nabla V \times \mathbf{A} + V\nabla \times \mathbf{A}$$

$\nabla\cdot(\mathbf{A}\times\mathbf{B})=\mathbf{B}\cdot(\nabla\times\mathbf{A})-\mathbf{A}\cdot(\nabla\times\mathbf{B})$
$\nabla(\mathbf{A}\cdot\mathbf{B})=(\mathbf{A}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{A})$
$\nabla\cdot\nabla=\nabla^2$
$\nabla\cdot(\nabla\times\mathbf{A})=0$
$\nabla\times(\nabla V)=0$
$\nabla\times\nabla\times\mathbf{A}=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A}$
$\int_V(\nabla\cdot\mathbf{A})dv=\oint_s\mathbf{A}\cdot d\boldsymbol{s}$
(Gauss' Theorem)
$\int_S(\nabla\times\mathbf{A})\cdot d\boldsymbol{s}=\oint_C\mathbf{A}\cdot d\boldsymbol{l}$
(Stokes' Theorem)

Other formulas:

$\int x^\alpha\,dx=\frac{1}{\alpha+1}x^{\alpha+1}+C \quad (\alpha\neq-1)$
$\int x^{-1}\,dx=ln(x)+C$
$\int sin(ax)\,dx=-\frac{1}{a}cos(ax)+C$
$\int cos(ax)\,dx=\frac{1}{a}sin(ax)+C$
$\int e^{ax}dx=\frac{1}{a}e^{ax}+C$
$\int\frac{1}{(x^2\pm a^2)^{\frac{1}{2}}}\,dx=ln(x+\sqrt{x^2\pm a^2})+C$
$\int\frac{1}{(x^2\pm a^2)^{\frac{3}{2}}}\,dx=\frac{x}{\pm a^2(x^2\pm a^2)^{\frac{1}{2}}}+C$
$\int\frac{x^2dx}{(x^2\pm a^2)^{\frac{1}{2}}}\,dx=\frac{x}{2}\sqrt{x^2\pm a^2}\mp\frac{a^2}{2}ln(x+\sqrt{x^2\pm a^2})+C$
$\int\frac{x^2dx}{(x^2\pm a^2)^{\frac{3}{2}}}\,dx=-\frac{x}{(x^2\pm a^2)^{\frac{1}{2}}}+ln(x+\sqrt{x^2\pm a^2})+C$

$\int ln(1+x^2)dx=xln(1+x^2)-2x+2\,arctan(x)$

$\int\frac{1}{1+x^2}dx=arctan(x)$

$sin^2x=\frac{1-cos(2x)}{2}$
$cos^2x=\frac{1+cos(2x)}{2}$
The general solution of $\frac{d^2y(x)}{dx^2}=d$ is:
$y(x)=\frac{d}{2}x^2+c_1x+c_2$
c_1 and c_2 are the constants to be determined.

The general solution of $\frac{d^2y(x)}{dx^2}+cy(x)=d$ is:
$y(x)=c_1cos(\sqrt{c}\,x)+c_2sin(\sqrt{c}\,x)+\frac{d}{c}$
where $c(>0)$, and d are constants.
c_1 and c_2 are the constants to be determined.

The general solution of $\frac{d^2y(x)}{dx^2}+b\frac{dy(x)}{dx}+cy(x)=d$ is:
$y(x)=c_1e^{r_1x}+c_2e^{r_2x}+\frac{d}{c}$

where $b,\ c$ and d are constants.
c_1 and c_2 are the constants to be determined.
$r_{1,2}=\frac{-b\pm\sqrt{b^2-4c}}{2}$.

Elcctromagnetic Formulas and Equations:

$\mu_o=4\pi\times10^{-7}\,H/m$

$\varepsilon_o=\frac{1}{36\pi}\times10^{-9}\,F/m\;F/m$

$\mathbf{F}=\mathbf{F}_e+\mathbf{F}_m=q(\mathbf{E}+\mathbf{u}\times\mathbf{B})$

$\mathbf{D}=\varepsilon_o\varepsilon_r\mathbf{E}=\varepsilon\mathbf{E}$

$\mathbf{B}=\mu_o\mu_r\mathbf{H}=\mu\mathbf{H}$

$I=\int_s\mathbf{J}\cdot d\mathbf{s}=\int_s\mathbf{J}\cdot\mathbf{a}_n ds$

$\mathbf{J}=\sigma\mathbf{E}$

$J_{1n}=J_{2n}\qquad(\text{A/m}^2)$

$\frac{J_{1t}}{J_{2t}}=\frac{\sigma_1}{\sigma_2}$

$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$

$\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}+\mathbf{J}$

$\nabla\cdot\mathbf{D}=\rho$

$\nabla\cdot\mathbf{B}=0$

$\mathbf{E}_{1t}=\mathbf{E}_{2t}$

$\mathrm{D}_{1n}-\mathrm{D}_{2n}=\rho_s$

$\boldsymbol{a}_{n2}\times(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$

$\mathbf{B}_{1n}=\mathbf{B}_{2n}$

$\mathbf{R}=\mathbf{r}\text{-}\mathbf{r}'=(x-x')\mathbf{a}_x+(y-y')\mathbf{a}_y+(z-z')\mathbf{a}_z$

$R=|\mathbf{R}|$

$\mathbf{a}_{\mathbf{R}}=\frac{\mathbf{R}}{|\mathbf{R}|}=\mathbf{a}_{\mathbf{r}\text{-}\mathbf{r}'}=\frac{\mathbf{r}\text{-}\mathbf{r}'}{|\mathbf{r}\text{-}\mathbf{r}'|}$

$\nabla^2V-\mu_o\mu_r\varepsilon_o\varepsilon_r\frac{\partial^2V}{\partial t^2}=-\frac{\rho}{\varepsilon_o\varepsilon_r}$

$V(\mathbf{r})=\frac{1}{4\pi\varepsilon_o\varepsilon_r}\int_{v'}\frac{\rho(t-\frac{R}{c})}{R}dv'$

$\nabla^2\mathbf{A}-\mu_o\mu_r\varepsilon_o\varepsilon_r\frac{\partial^2\mathbf{A}}{\partial t^2}=-\mu_o\mu_r\mathbf{J}$

$\mathbf{A}(\mathbf{r})=\frac{\mu_o\mu_r}{4\pi}\int_{v'}\frac{\mathbf{J}(t-\frac{R}{c})}{R}dv'$

$\mathbf{E}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}$

$\mathbf{B}=\nabla\times\mathbf{A}$

$F[\frac{d^nf(t)}{dt^n}]=(j\omega)^n\,F[f(t)]$

$F[f(t-t_0)]=e^{-j\omega t_0}F[f(t)]$

$\mathbf{E}=\mathbf{E}_0\,e^{-j\,\mathbf{k}\cdot\mathbf{r}}=-\eta\,\boldsymbol{a}_n\times\mathbf{H}$

$\mathbf{H}=\mathbf{H}_0\,e^{-j\,\mathbf{k}\cdot\mathbf{r}}=\frac{1}{\eta}\,\boldsymbol{a}_n\times\mathbf{E}$

$\eta=\sqrt{\frac{\mu}{\varepsilon}}=\sqrt{\frac{\mu_o\mu_r}{\varepsilon_o\varepsilon_r}}$

$\mathbf{k}=k\,\boldsymbol{a}_n$

$k=\omega\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}\ ,\ \ \omega=2\pi f$

$f'=\frac{f}{1-\frac{u}{c}cos\theta}$

$\varepsilon_c=\varepsilon_o\varepsilon_r-j(\varepsilon''+\frac{\sigma}{\omega})=\varepsilon'-j(\varepsilon''+\frac{\sigma}{\omega}),\qquad tan\delta_c=\frac{(\varepsilon''+\frac{\sigma}{\omega})}{\varepsilon_o\varepsilon_r}=\frac{(\varepsilon''+\frac{\sigma}{\omega})}{\varepsilon'}$

$\gamma=jk_c=j\omega\sqrt{\mu_o\mu_r\varepsilon_c}=\alpha+j\beta$

$$\alpha=\omega\sqrt{\frac{\mu_o\mu_r\varepsilon_o\varepsilon_r}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon_o\varepsilon_r}+\frac{\varepsilon''}{\varepsilon_o\varepsilon_r})^2}-1]}=\omega\sqrt{\frac{\mu_o\mu_r\varepsilon'}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon'}+\frac{\varepsilon''}{\varepsilon'})^2}-1]}$$

$$\beta=\omega\sqrt{\frac{\mu_o\mu_r\varepsilon_o\varepsilon_r}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon_o\varepsilon_r}+\frac{\varepsilon''}{\varepsilon_o\varepsilon_r})^2}+1]}=\omega\sqrt{\frac{\mu_o\mu_r\varepsilon'}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon'}+\frac{\varepsilon''}{\varepsilon'})^2}+1]}$$

$u_p=\frac{\omega}{\beta}$

$u_g=1/\frac{d\beta}{d\omega}$

$\oint_s\boldsymbol{p}\cdot d\boldsymbol{s}=-\frac{\partial}{\partial t}\int_v(\frac{1}{2}\mu|\mathbf{H}|^2)dv-\frac{\partial}{\partial t}\int_v(\frac{1}{2}\varepsilon|\mathbf{E}|^2)dv-\int_v\sigma|\mathbf{E}|^2dv$

$n = c/u_p \quad (c = 3 \times 10^8 m/s$ is the speed of light in vacuum)

$\Gamma_\perp = \frac{\eta_2/cos\theta_t - \eta_1/cos\theta_i}{\eta_2/cos\theta_t + \eta_1/cos\theta_i}$ $\qquad \tau_\perp = \frac{2(\eta_2/cos\theta_t)}{(\eta_2/cos\theta_t)+(\eta_1/cos\theta_i)}$

$\Gamma_{//} = \frac{\eta_2 cos\theta_t - \eta_1 cos\theta_i}{\eta_2 cos\theta_t + \eta_1 cos\theta_i}$ $\qquad \tau_{//} = \frac{2\eta_2 cos\theta_i}{\eta_2 cos\theta_t + \eta_1 cos\theta_i}$

For two-wire line:

$C = \frac{\pi\varepsilon_o\varepsilon_r}{cosh^{-1}(D/2a)} \quad (F/m)$ $\qquad L = \frac{\mu_o\mu_r}{\pi} cos^{-1}(D/2a) \quad (H/m)$

$G = \frac{\pi\sigma}{cos^{-1}(D/2a)} \quad (S/m)$ $\qquad R = 2(\frac{R_s}{2\pi a}) = \frac{1}{\pi a}\sqrt{\frac{\pi f \mu_o \mu_{rc}}{\sigma_c}} \quad (\Omega/m)$

For coaxial line:

$C = \frac{2\pi\varepsilon_o\varepsilon_r}{ln(b/a)} \quad (F/m)$ $\qquad L = \frac{\mu_o\mu_r}{2\pi} ln(b/a) \quad (H/m)$

$G = \frac{2\pi\sigma}{ln(b/a)} \quad (S/m)$ $\qquad R = \frac{R_s}{2\pi}(\frac{1}{a} + \frac{1}{b})$

$= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_o \mu_{rc}}{\sigma_c}}(\frac{1}{a} + \frac{1}{b})$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$\alpha = x_1 y_2 + x_2 y_1; \qquad \beta = j\omega\sqrt{LC}(x_1 y_1 - x_2 y_2);$$

$$x_1 = \sqrt{\frac{1}{2}(\sqrt{1 + (\frac{R}{\omega L})^2} + 1}; \quad x_2 = \sqrt{\frac{1}{2}(\sqrt{1 + (\frac{R}{\omega L})^2} - 1};$$

$$y_1 = \sqrt{\frac{1}{2}(\sqrt{1 + (\frac{G}{\omega C})^2} + 1}; \quad y_2 = \sqrt{\frac{1}{2}(\sqrt{1 + (\frac{G}{\omega C})^2} - 1};$$

$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$ $\qquad I(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} + \frac{-V_o^-}{Z_o} e^{\gamma z}$

$= V_o^+ e^{-\gamma z}(1 + \Gamma)$ $\qquad = \frac{V_o^+}{Z_o} e^{-\gamma z}(1 - \Gamma)$

$z = l - z'$

$\Gamma(z') = \Gamma_L e^{-2\gamma z'}$ $\qquad \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$

$Z(z') = Z_o \frac{Z_L + Z_o tanh(\gamma z')}{Z_o + Z_L tanh(\gamma z')} = Z_o \frac{1+\Gamma(z')}{1-\Gamma(z')}$

For lossless line:

$Z(z') = Z_o \frac{Z_L + jZ_o tan(\beta z')}{Z_o + jZ_L tan(\beta z')}$

$$sin(0^o) = cos(90^o) = 0; \quad sin(30^o) = cos(60^o) = \frac{1}{2};$$

$$sin(45^o) = cos(45^o) = \frac{\sqrt{2}}{2}; \quad sin(90^o) = cos(0^o) = 1;$$

$$tan(0^o) = cot(90^o) = 0; \quad tan(30^o) = cot(60^o) = \frac{\sqrt{3}}{3};$$

$$tan(60^o) = cot(30^o) = \sqrt{3}; \quad tan(90^o) = cot(0^o) = \infty;$$