Tutorial #4

Problem #1.

Evaluate complex numbers $Z_1 = \dfrac{j(3-j4)^*}{(-1+6j)(2+j)^2}$

and $Z_2 = \left(\dfrac{1+j}{4-j8}\right)^{1/2}$

Problem #2

Given that $\vec{A} = 10\cos(10^8 t - 10x + 60^\circ)\vec{a}_z$ and $\vec{B} = \frac{20}{j}\vec{a}_x + 10e^{j\frac{2\pi x}{3}}\vec{a}_y$, express $\vec{A}$ in phasor form and $\vec{B}$ in instantaneous form.

Problem #3.

Given in the free space that:

$$\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\,\vec{a}_\phi \ \ \text{V/m}$$

$$\vec{H} = \frac{H_o}{\rho}\cos(10^6 t + \beta z)\,\vec{a}_\rho \ \ \text{A/m}$$

in the cylindrical Coordinates.

Express these in phasor form and then determine the constants H_o and β.

[Solution #1].

$$Z_1 = \frac{j(3-j4)^*}{(-1+j6)(2+j)^2} = \frac{j(3+j4)}{(-1+j6)(2^2+4j+j^2)} = \frac{-4+j3}{(-1+j6)(3+4j)}$$

$$= \frac{(-4+j3)}{-3-4j+j18-24} = \frac{-4+j3}{-27+j14}\cdot\frac{(-27-j14)}{(-27-j14)} = \frac{150-j25}{27^2+14^2}$$

$$= 0.1622 - j0.027 = 0.1644\angle -9.46^\circ$$

$$Z_2 = \left(\frac{1+j}{4-j8}\right)^{1/2} = \left(\frac{\sqrt{2}\,e^{j45^\circ}}{4\sqrt{5}\,e^{-j63.4^\circ}}\right)^{1/2} = \left[\frac{\sqrt{2}}{4\sqrt{5}}\,e^{j(45^\circ+63.4^\circ)}\right]^{1/2}$$

$$= 0.3976\,e^{j(45^\circ+63.4^\circ)/2} = 0.3976\angle 54.2^\circ$$

[Solution #2]

$$\dot{\vec{A}} = 10\, e^{j(60^\circ - 4x)}\, \vec{a}_z$$

$$\vec{B}(t) = Re[\,\dot{\vec{B}} e^{j\omega t}\,] = Re\left[\left(20 e^{-j90^\circ} \vec{a}_x + 10 e^{j\frac{2\pi x}{3}} \vec{a}_y\right) e^{j\omega t}\right]$$

$$= 20 \cos(\omega t - 90^\circ)\, \vec{a}_x + 10 \cos(\omega t + \tfrac{2\pi x}{3})\, \vec{a}_y$$

$$= 20 \sin(\omega t)\, \vec{a}_x + 10 \cos(\omega t + \tfrac{2}{3}\pi x)\, \vec{a}_y$$

[Solution #3]

$$\dot{\vec{E}} = \frac{50}{\rho} e^{+j\beta z}\, \vec{a}_\phi$$

$$\dot{\vec{H}} = \frac{H_0}{\rho} e^{+j\beta z}\, \vec{a}_\rho$$

In free space, $\rho = 0$, $\sigma = 0$, $\varepsilon = \varepsilon_0$. $\mu = \mu_0 \Rightarrow \vec{D} = \varepsilon_0 \vec{E}$

$\vec{B} = \mu_0 \vec{H}$

, $\dot{\vec{E}}$ and $\dot{\vec{H}}$ into

$$\begin{cases} \nabla \times \dot{\vec{E}} = -j\omega \mu_0 \dot{\vec{H}} \\ \nabla \times \dot{\vec{H}} = j\omega \varepsilon_0 \dot{\vec{E}} \end{cases}$$

$$\Rightarrow \begin{cases} -j\beta \frac{50}{\rho} e^{j\beta z} \vec{a}_\rho = -j\omega\mu_0 \frac{H_0}{\rho} e^{j\beta z} \vec{a}_\rho \\ \frac{jH_0\beta}{\rho} e^{j\beta z} \vec{a}_\phi = j\omega\varepsilon_0 \frac{50}{\rho} e^{j\beta z} \vec{a}_\phi \end{cases}$$

$$\Rightarrow \begin{cases} \frac{H_0}{\beta} = \frac{50}{\omega\mu_0} \\ H_0\beta = 50\omega\varepsilon_0 \end{cases} \Rightarrow \begin{cases} H_0 = \pm 50\sqrt{\frac{\varepsilon_0}{\mu_0}} = \pm 0.1326 \\ \beta = \pm\omega\sqrt{\mu_0\varepsilon_0} = \pm 3.33 \times 10^{-3} \end{cases}$$