Prob.1.24

(a) $\cos\theta_{AB} = \dfrac{\boldsymbol{A}\cdot\boldsymbol{B}}{AB} = \dfrac{-14}{\sqrt{83}\sqrt{6}} = -0.6273 \quad \longrightarrow \quad \theta_{AB} = \underline{\underline{128.86^{o}}}$

(b) $\boldsymbol{A}_{//} = (\boldsymbol{A}\cdot\boldsymbol{a}_B)\boldsymbol{a}_B = \dfrac{(\boldsymbol{A}\bullet\boldsymbol{B})\boldsymbol{B}}{B^2} = \dfrac{-14(1,-2,1)}{6} = \underline{\underline{-2.333\boldsymbol{a}_x + 4.667\boldsymbol{a}_y - 2.333\boldsymbol{a}_z}}$

(c) $\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{//} = (3,5,-7) - (-2.333, 4.667, -2.333) = \underline{\underline{5.333\boldsymbol{a}_x + 0.333\boldsymbol{a}_y - 4.667\boldsymbol{a}_z}}$

Prob. 2.10 (a)

$$\begin{bmatrix} H_\rho \\ H_\phi \\ H_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}$$

$$H_\rho = 3\cos\phi + 2\sin\phi, \quad H_\phi = -3\sin\phi + 2\cos\phi, \quad H_z = -4$$

$$\boldsymbol{H} = \underline{\underline{(3\cos\phi + 2\sin\phi)\boldsymbol{a}_\rho + (-3\sin\phi + 2\cos\phi)\boldsymbol{a}_\phi - 4\boldsymbol{a}_z}}$$

(b) At P, $\rho = 2, \quad \phi = 60^\circ, \quad z = -1$

$$\boldsymbol{H} = (3\cos 60^\circ + 2\sin 60^\circ)\boldsymbol{a}_\rho + (-3\sin 60^\circ + 2\cos 60^\circ)\boldsymbol{a}_\phi - 4\boldsymbol{a}_z$$
$$= \underline{\underline{3.232\boldsymbol{a}_\rho - 1.598\boldsymbol{a}_\phi - 4\boldsymbol{a}_z}}$$

Prob 2.25

(a)

At P, $\rho = 2,\ \phi = 30°,\ \ z = -1$

$$\bar{H} = 10\sin 30\,\bar{a}_\rho + 2\cos 30°\,\bar{a}_\phi + 4\bar{a}_z.$$

$$= 5\bar{a}_\rho + 1.732\bar{a}_\phi + 4\bar{a}_z.$$

$$\bar{a}_H = \frac{(5,\ 1.732,\ 4)}{\sqrt{5^2 + 1.732^2 + 4^2}} = \underline{\underline{0.7538\bar{a}_\rho + 0.2611\bar{a}_\phi + 0.603\bar{a}_z.}}$$

(b) $H_x = H_\rho \cos\phi - H_\phi \sin\phi = 5\rho\sin\phi\cos\phi + \rho z\cos\phi\sin\phi$

or P at $\rho = 2,\ \ \phi = 30,\ \ z = -1;$

$$\bar{H}_x = H_x\,\bar{a}_x = (10\sin 30°\cos 30° - 2\sin 30°\cos 30°)\bar{a}_x = 8\sin 30°\cos 30°\,\mathbf{a}_x$$

$$= \underline{\underline{3.4641\bar{a}_x}}$$

(c) Normal to $\rho = 2$ is $\mathbf{H}_n = H_\rho\,\mathbf{a}_\rho = 10\sin\phi\,\mathbf{a}_\rho$;

(d) Tangential to $\phi = 30°$.

$$\mathbf{H}_t = H_\rho\,\mathbf{a}_\rho + H_z\mathbf{a}_z = \underline{\underline{5\mathbf{a}_\rho + 4\mathbf{a}_z}}$$

Prob. 3.9

(a)

$$\nabla U = \frac{\partial U}{\partial x}\bar{a}_x + \frac{\partial U}{\partial y}\bar{a}_y + \frac{\partial U}{\partial z}\bar{a}_z$$

$$= \underline{\underline{4z^2\,\bar{a}_x + 3z\,\bar{a}_y + (8xz + 3y)\bar{a}_z}}$$

(b)

$$\nabla W = \frac{\partial W}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial W}{\partial \phi}\bar{a}_\phi + \frac{\partial W}{\partial z}\bar{a}_z$$

$$= \underline{\underline{2(z^2+1)\cos\phi\,\bar{a}_\rho - 2(z^2+1)\sin\phi\,\bar{a}_\phi + 4\rho z\cos\phi\,\bar{a}_z}}$$

(c)

$$\nabla H = \frac{\partial H}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial H}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial H}{\partial \phi}\bar{a}_\phi$$

$$= 2r\cos\theta\cos\phi\,\bar{a}_r - r\sin\theta\cos\phi\,\bar{a}_\theta - r\cot\theta\sin\phi\,\bar{a}_\phi$$

Prob 3.12

$$\nabla T = 2x\bar{a}_x + 2y\bar{a}_y - \bar{a}_z$$

At $(1,1,2)$, $\nabla T = (2,2,-1)$. The mosquito should move in the direction

$$\underline{\underline{2\bar{a}_x + 2\bar{a}_y - \bar{a}_z}}$$

Prob 3.20

Transform $\bar{F}$ into cylindrical system.

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^2 \\ y^2 \\ z^2-1 \end{bmatrix}$$

$$F_\rho = x^2\cos\phi + y^2\sin\phi = \rho^2\cos^3\phi + \rho^2\sin^3\phi, F_z = z^2-1$$

$$F_\phi = -x^2\sin\phi + y^2\cos\phi = -\rho^2\cos^2\phi\sin\phi + \rho^2\sin^2\phi\cos\phi$$

$$\begin{aligned}\nabla \bullet \bar{F} &= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^3\cos^3\phi + \rho^3\sin^3\phi) + 2z - \rho\cos^3\phi - 2\rho\cos\phi\sin^2\phi \\ &\quad + 2\rho\sin\phi\cos^2\phi - \rho\sin^3\phi \\ &= 2\rho\cos^3\phi + 2\rho\sin^3\phi - 2\rho\cos\phi\sin^2\phi + 2\rho\cos^2\phi\sin\phi + 2z\end{aligned}$$

$$\int \bar{F} \bullet d\bar{S} = \int \nabla \bullet \bar{F}\, dv$$

Due to the fact that we are integrating $\sin\phi$ and $\cos\phi$ over terms involving $\cos\phi$ *and* $\sin\phi$ will vanish. Hence, ence,

$$\int \bar{F}\, d\bar{S} = \iiint 2z\ \rho\, d\rho\, d\phi\, dz = 2\int_0^{2\pi} d\phi \int_0^2 z\,dz \int_0^2 \rho\, d\rho$$

$$= 2(2\pi)\left(\frac{z^2}{2}\Big|_0^2\right)\left(\frac{\rho^2}{2}\Big|_0^2\right) = 16\pi$$

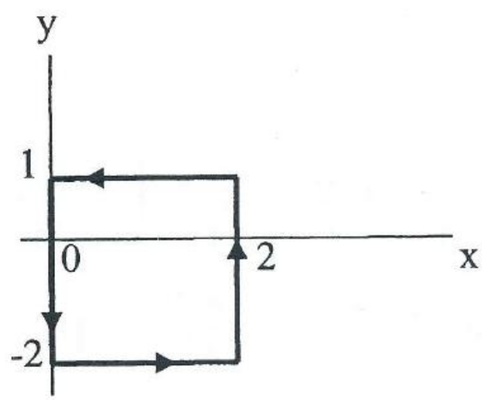
$$= \underline{\underline{50.26}}$$

Prob.3.32

$$\oint \mathbf{A} \bullet dl = \int_{\rho=2}^{1} \rho \sin\phi d\rho \Big|_{\phi=0} + \int_{\phi=0}^{\pi/2} \rho^2 \rho d\phi d\rho \Big|_{\rho=1} + \int_{\rho=1}^{2} \rho \sin\phi \Big|_{\phi=90^\circ} + \int_{\phi=\pi/2}^{0} \rho^3 d\phi \Big|_{\rho=2}$$

$$= \frac{\pi}{2} + \frac{1}{2}(4-1) + 8\left(-\frac{\pi}{2}\right) = \underline{\underline{-9.4956}}$$

Prob. 3.34



$$\oint_L \mathbf{F} \cdot dl = \int_0^2 3y^2 z dx \Big|_{y=-2, z=1} + \int_{-2}^{1} 6x^2 y dy \Big|_{x=2, z=1}$$

$$+ \int_2^0 3y^2 z dx \Big|_{y=1, z=1} + \int_1^{-2} 6x^2 y dy \Big|_{x=0, z=1}$$

$$= 3(4)(1)(2) + 6(4)\frac{y^2}{2}\Big|_{-2}^{1} + 3(1)(1)(-2) + 0 = \underline{\underline{-18}}$$

$$\nabla \mathrm{x}\mathbf{F} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3y^2z & 6x^2y & 9xz^2 \end{vmatrix} = (12xy - 6yz)\boldsymbol{a}_z + \ldots$$

$$\int_S (\nabla \mathrm{x} F)\cdot dS = \iint (12xy - 6yz)\,dx\,dy \Big|_{z=1} = 12\int_0^2 x\,dx \int_{-2}^{1} y\,dy - 6\int_{-2}^{1} y\,dy \int_0^2 dx$$

$$= 3x^2\Big|^2 y^2\Big|^1 - 3y^2\Big|^1 (2) = 3(4)((1-4) - 6(1-4) = -18$$