**Prob. 11.2**

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi x 80x10^6 x 4\pi x 10^{-7} x 5.28x10^7}} = 7.744x10^{-6}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right] = \frac{\left[\dfrac{1}{0.8x10^{-3}}+\dfrac{1}{2.6x10^{-3}}\right]}{2\pi x 7.744x10^{-6} x 5.28x10^7} = \frac{10^3(1.25+0.3836)}{2569.09} = \underline{\underline{0.6359\ \Omega/\text{m}}}$$

$$L = \frac{\mu}{2\pi}\ln\frac{b}{a} = \frac{4\pi x10^{-7}}{2\pi}\ln\frac{2.6}{0.8} = \underline{\underline{2.357x10^{-7}\ \text{H/m}}}$$

$$G = \frac{2\pi\sigma}{\ln\frac{b}{a}} = \frac{2\pi x10^{-5}}{\ln\frac{2.6}{0.8}} = \underline{\underline{5.33x10^{-5}\ \text{S/m}}}$$

$$C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}} = \frac{2\pi x 3.5 x\frac{10^{-9}}{36\pi}}{\ln\frac{2.6}{0.8}} = \underline{\underline{1.65x10^{-10}\ \text{F/m}}}$$

**Prob.11.6**

$$Z_o = \sqrt{\frac{L}{C}}, \qquad \beta = \omega\sqrt{LC} = \frac{\omega}{u}, \qquad u = \frac{1}{\sqrt{LC}}$$

$$Z_o u = \frac{1}{C} \quad \longrightarrow \quad \frac{1}{C} = 50x2.8x10^8 = 1.4x10^{10}$$

$$C = \frac{10^{-9}}{5.6} = \underline{\underline{71.43 \text{ pF/m}}}$$

$$\frac{Z_o}{u} = L \quad \longrightarrow \quad L = \frac{50}{2.8x10^8} = \underline{\underline{1.786x10^{-7} \text{ H/m}}}$$

**Prob. 11.9**

(a) $R + j\omega L = 40 + j2\pi \times 10^7 \times 0.2 \times 10^{-6} = 41.93\angle 17.44^o$

$$G + j\omega C = 400 \times 10^{-6} + j2\pi \times 10^7 \times 0.5 \times 10^{-9} = 3.142 \times 10^{-2} \angle 89.27^o$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \underline{\underline{29.59 - j21.43\ \Omega}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega L)} = 0.685 + j0.921 = \alpha + j\beta$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{0.921} = \underline{\underline{6.823 \times 10^7\ \text{m/s}}}$$

(b) $\alpha = 0.685\ \text{Np/m} = 0.685 \times 8.686\ \text{dB/m} = 5.95\ \text{dB/m}$

$$\alpha l = 30 \rightarrow l = \frac{30}{5.95} = \underline{\underline{5.042\ \text{m}}}$$

**Prob. 11.22**

(a) $\Gamma = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{120-50}{170} = \underline{\underline{0.4118}}$

For resistive load, $s = \dfrac{Z_L}{Z_o} = \underline{\underline{2.4}}$

(b) $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$

$$\beta l = \frac{2\pi}{\lambda} . \frac{\lambda}{6} = 60^o$$

$$Z_{in} = 50\left[\frac{120 + j50\tan(60^o)}{50 + j120\tan(60^o)}\right] = \underline{\underline{34.63\angle -40.65^o\,\Omega}}$$

p.11.24

a) $\beta l = \frac{1}{4} \times 100 = 25 \text{ rad} = 1432.4^\circ = 352.4^\circ$

$$Z_{in} = 60 \left[ \frac{j40 + j60\tan 352.4^\circ}{60 - 40\tan 352.4^\circ} \right] = j\,29.375\,\Omega$$

$$V(z=0) = V_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g = \frac{j29.375\,(10\angle 0)}{j29.375 + 50 - j40} = 5.7\cancel{8}\angle 102^\circ$$

(b) $Z_{in} = Z_L = j40\,\Omega$

$$V(z=L) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = V_0^+ e^{-\gamma L} + V_0^- e^{\gamma L}$$

$\gamma = \alpha + j\beta$ lossless $\alpha = 0$

$\gamma = j\beta = j0.25$

$\gamma l = j\,(0.25)100 = j25$

$V(z=0) = 5.7\cancel{2}\angle 102 = V_0^+ + V_0^-$ (I)

$$I(z=0) = \frac{V(z=0)}{Z_{in}} = \frac{5.7\angle 102}{29.37\angle 90} = 0.19\angle 12$$

$$I_0 = \frac{V_0^+ - V_0^-}{Z_0}$$

$$\Rightarrow \frac{V_0^+ - V_0^-}{60} = 0.19\angle 12 \Rightarrow V_0^+ = 11.7\angle 12 + V_0^-$$

substituting in (I)

$5.7\angle 102 = 11.7\angle 12 + V_0^- + V_0^- \Rightarrow V_0^- = 6.5\angle 166^\circ$ (please check the answers)

$V_0^+ = 5.72\angle 102 - 6.5\angle 166 = 6.5\angle 38.1$

$V(Z=L=100) = (6.5\angle 38.1)e^{-j25\,rad} + (6.5\angle 166)e^{j25\,rad}$

$= \ldots$

c) $\beta l' = \frac{1}{4} \times 4 = 1\,rad = 57.3^\circ$

$Z_{in} = 60\left[\frac{j40 + j60\tan 57.3}{60 - 40\tan 57.3}\right] = -j\,3486.75\,\Omega$

$l = 4 \Rightarrow Z = 100 - 4 = 96$

$\gamma L = j\,2.25 \times 96 = j\,24 = j\,1375^\circ$

$V(Z=L=96) = V_0^+ e^{-\gamma Z} + V_0^- e^{\gamma Z}$

$= (6.5\angle 38.1)e^{-1375j} + (6.5\angle 166)e^{1375j}$

$= 13\angle 102^\circ$ (check the answers)

d) 3m from the source is the same as 97m from the load, i.e.

$l' = 100 - 3 = 97m \Rightarrow \beta l' = \frac{1}{4} \times 97 = 24.25\,rad = 309.42^\circ$

$Z_{in} = 60\left[\frac{j40 + j60\tan 309.42}{60 - 40\tan 309.42}\right] = -j18.2\,\Omega$

$V(Z) = V_0^+ e^{-\gamma Z} + V_0^- e^{\gamma Z}$

$\gamma Z = j\left(\frac{1}{4}\right) \times 3 = j41^\circ$

$V(Z=3) = (6.5\angle 38.1)e^{-j41} + (6.5\angle 166)e^{+j41}$

$= 6.3\angle -83$

**Prob. 11.26**

$$V_1 = V_s(z=0) = V_o^+ + V_o^- \quad (1)$$

$$V_2 = V_s(z=l) = V_o^+ e^{-\gamma l} + V_o^- e^{\gamma l} \quad (2)$$

$$I_1 = I_s(z=0) = \frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o} \quad (3)$$

$$I_2 = -I_s(z=l) = -\frac{V_o^+}{Z_o} e^{-\gamma l} + \frac{V_o^-}{Z_o} e^{\gamma l} \quad (4)$$

$$(1) + (3) \rightarrow V_o^+ = \frac{1}{2}(V_1 + Z_o I_1)$$

$$(1) - (3) \rightarrow V_o^- = \frac{1}{2}(V_1 - Z_o I_1)$$

Substituting $V_o^+$ and $V_o^-$ in (2) gives

$$V_2 = \frac{1}{2}(V_1 + Z_o I_1)e^{-\gamma l} + \frac{1}{2}(V_1 - Z_o I_1)e^{\gamma l}$$

$$= \frac{1}{2}(e^{\gamma l} + e^{-\gamma l})V_1 + \frac{1}{2}Z_o(e^{-\gamma l} - e^{\gamma l})I_1$$

$$V_2 = \cosh \gamma l V_1 + Z_o \sinh \gamma l I_1 \quad (5)$$

Substituting $V_o^+$ and $V_o^-$ in (4),

$$I_2 = -\frac{1}{2Z_o}(V_1 + Z_o I_1)e^{-\gamma l} + \frac{1}{2Z_o}(V_1 - Z_o I_1)e^{\gamma l}$$

$$= \frac{1}{2Z_o}(e^{\gamma l} - e^{-\gamma l})V_1 + \frac{1}{2}(e^{\gamma l} + e^{-\gamma l})I_1$$

$$I_2 = -\frac{1}{Z_o}\sinh \gamma l V_1 - \cosh \gamma l I_1 \quad (6)$$

From (5) and (6)

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & +Z_o \sinh \gamma l \\ -\frac{1}{Z_o}\sinh \gamma l & -\cosh \gamma l \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

But

$$\begin{bmatrix} \cosh \gamma l & +Z_o \sinh \gamma l \\ -\frac{1}{Z_o}\sinh \gamma l & -\cosh \gamma l \end{bmatrix}^{-1} = \begin{bmatrix} \cosh \gamma l & Z_o \sinh \gamma l \\ \frac{1}{Z_o}\sinh \gamma l & \cosh \gamma l \end{bmatrix}$$

Thus

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_o \sinh \gamma l \\ \frac{1}{Z_o}\sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$