Prob. 9.3

$$\Psi = \int \mathbf{B} \cdot d\mathbf{S} = BS$$

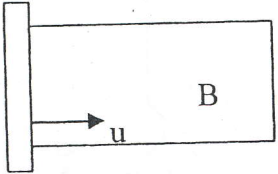
$$V_{emf} = -\frac{d\Psi}{dt} = -\frac{dB}{dt}S = 0.6x10x10^{-4} = 0.6 \text{ mV}$$

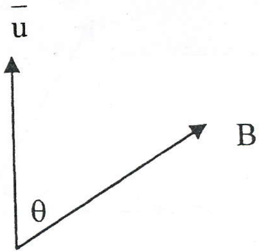
The emf is divided in the ratio of the resistances

$$v_1 = \frac{10}{15}x0.6 = \underline{\underline{0.4 \text{ mV}}}$$

$$v_2 = \frac{5}{15}x0.6 = \underline{\underline{0.2 \text{ mV}}}$$

Prob. 9.9





$$V_{emf} = \int (\bar{u} \times \bar{B}) \cdot d\bar{l} = uBl\cos\theta$$

$$= \left(\frac{120 \times 10^3}{3600} m/s \right) (4.3 \times 10^{-5})(1.6) \cos 65^o$$

$$= 2.293 \cos 65^o = \underline{\underline{0.97}} \text{ mV}$$

Prob. 9.12

$$V = \int (\vec{u} \times \vec{B}) \bullet d\vec{l}\ \text{, where } \vec{u} = \rho\omega\vec{a}_\phi,\ \vec{B} = B_o\vec{a}_z$$

$$V = \int_{\rho_1}^{\rho_2} \rho\omega B_o d\rho = \frac{\omega B_o}{2}(\rho^2{}_2 - \rho^2{}_1)$$

$$V = \frac{60 \times 15}{2} \bullet 10^{-3}(100-4) \bullet 10^{-4} = \underline{\underline{4.32 \text{ mV}}}$$

Prob. 9.23

$$J_d = \frac{\partial D}{\partial t} = \varepsilon_o \frac{\partial E}{\partial t} = \frac{50\varepsilon_o}{\rho}(-10^8)\sin(10^8 t - kz)\boldsymbol{a}_\rho = \underline{\underline{-\frac{4.421x10^{-2}}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho \text{ A/m}^2}}$$

$$\nabla \times \boldsymbol{E} = -\mu_o \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = \frac{\partial E_\rho}{\partial z}\boldsymbol{a}_\phi = \frac{50k}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\phi$$

$$\boldsymbol{H} = -\frac{1}{\mu_o}\int \nabla \times \boldsymbol{E} dt = \frac{1}{4\pi x10^{-7}}\frac{50k}{10^8 \rho}\cos(10^8 t - kz)\boldsymbol{a}_\phi$$

$$\underline{\underline{\boldsymbol{H} = \frac{2.5k}{2\pi\rho}\cos(10^8 t - kz)\boldsymbol{a}_\phi \text{ A/m}}}$$

$$\nabla \times \boldsymbol{H} = -\frac{\partial H_\phi}{\partial z}\boldsymbol{a}_\rho = -\frac{2.5k^2}{2\pi\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_d \longrightarrow -\frac{4.421x10^{-2}}{\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho = \frac{-2.5k^2}{2\pi\rho}\sin(10^8 t - kz)\boldsymbol{a}_\rho$$

$$k^2 = \frac{2\pi}{2.5}x4.421x10^{-2} \longrightarrow \underline{\underline{k = 0.333}}$$

Prob. 9.26

$$\nabla \bullet \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x(y,z,t) & 0 & 0 \end{vmatrix} = \frac{\partial E_x}{\partial z}\boldsymbol{a}_y - \frac{\partial E_x}{\partial y}\boldsymbol{a}_z$$

$$= \frac{\beta\omega\mu b}{\pi} H_o \sin(\pi y/b)\cos(\omega t - \beta z)\boldsymbol{a}_y + \omega\mu H_o \cos(\pi y/b)\sin(\omega t - \beta z)\boldsymbol{a}_z$$

$$\nabla \times \boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t} \quad \longrightarrow \quad \boldsymbol{H} = -\frac{1}{\mu}\int \nabla \times \boldsymbol{E} dt$$

$$\boldsymbol{H} = -\frac{\beta b}{\pi} H_o \sin(\pi y/b)\sin(\omega t - \beta z)\boldsymbol{a}_y + H_o \cos(\pi y/b)\cos(\omega t - \beta z)\boldsymbol{a}_z$$

which is the given H field.

$$\nabla \times \boldsymbol{H} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\boldsymbol{a}_x$$

$$= [-\frac{\pi}{b} H_o \sin(\pi y/b)\cos(\omega t - \beta z) + \frac{\beta^2 b}{\pi} H_o \sin(\pi y/b)\cos(\omega t - \beta z)]\boldsymbol{a}_x$$

$$\nabla \times \boldsymbol{H} = \varepsilon\frac{\partial \boldsymbol{E}}{\partial t} \quad \longrightarrow \quad \boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

$$\boldsymbol{E} = [-\frac{\pi}{\omega b\varepsilon} H_o \sin(\pi y/b)\sin(\omega t - \beta z) + \frac{\beta^2 b}{\pi\omega\varepsilon} H_o \sin(\pi y/b)\sin(\omega t - \beta z)]\boldsymbol{a}_x$$

Setting this equal to the given E,

$$\frac{\omega\mu b}{\pi} H_o = \frac{\pi}{\omega b\varepsilon} H_o - \frac{\beta^2 b}{\pi\omega\varepsilon} H_o \quad \longrightarrow \quad \beta^2 = -\frac{\pi^2}{b^2} - \omega^2 \mu\varepsilon$$

$$\beta = \sqrt{\omega^2\mu\varepsilon + \frac{\pi^2}{b^2}}$$

Prob. 9.31

$$\nabla \times \vec{E} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi)\vec{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 t e^{-\rho-t})\vec{a}_z$$

$$= (2-\rho)te^{-\rho-t}\vec{a}_z$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} \to \vec{B} = -\int \nabla \times \vec{E} dt = \int \frac{(\rho-2)t}{V}\frac{e^{-\rho-t}dt}{du}\vec{a}_z$$

Integrating by parts yields

$$\vec{B} = [-(\rho-2)te^{-\rho-t} + \int(\rho-2)e^{-\rho-t}dt]\vec{a}_z$$

$$= \underline{\underline{(2-\rho)(1+t)e^{-\rho-t}\vec{a}_z}} \text{ Wb/m}^2$$

$$\vec{J} = \nabla \times \vec{H} = \nabla \times \frac{\vec{B}}{\mu_o} = -\frac{1}{\mu_o}\frac{\partial B_z}{\partial \rho}\vec{a}_\phi$$

$$= -\frac{1}{\mu_o}(1+t)(-1-2+\rho)e^{-\rho-t}\vec{a}_\phi$$

$$\vec{J} = \underline{\underline{\frac{(1+t)(3-\rho)e^{-\rho-t}}{4\pi x 10^{-7}}\vec{a}_\phi}} \text{ A/m}^2$$