Tutorial #6

Q1: An EM wave travels in free space with electric field component $\vec{E}_s = 100 e^{j(0.866y+0.5z)} \vec{a}_x$ (V/m)

Determine

(a) ω and λ

(b) The magnetic field component

(c) The time average power density in the wave

Q2: An uniform plane wave in air with

$$\vec{E} = 8\cos(\omega t - 4x - 3z)\, \vec{a}_y \text{ V/m}$$

is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1.0$, $\varepsilon_r = 2.5$, $\sigma = 0$, find

(a) Polarization of the wave

(b) the angle of incidence

(c) the reflected $\vec{E}$ field.

(d) the transmitted $\vec{H}$ field

Q1 [Solution]

(a) Comparing the given $\vec{E}$ with

$$\vec{E}_s = \vec{E}_o e^{-jk\vec{a}_n \cdot \vec{R}} = 100 e^{j(0.866y+0.5z)} \vec{a}_x \quad (\text{V/m})$$

$$\Rightarrow \quad k\vec{a}_n = k_x \vec{a}_x + k_y \vec{a}_y + k_z \vec{a}_z = -0.866\vec{a}_y - 0.5\vec{a}_z$$

$$\Rightarrow \quad k_x = 0, \quad k_y = -0.866, \quad k_z = -0.5$$

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{(0.866)^2 + (0.5)^2} = 1.0$$

$$= \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{C_o}$$

$$\Rightarrow \omega = k C_o = 3\times 10^8 \text{ (rad/s)}$$

$$\lambda = \frac{2\pi}{k} = 2\pi = 6.283 \text{ (m)}$$

(b) $$\vec{a}_n = \frac{k\vec{a}_n}{k} = -0.866\vec{a}_y - 0.5\vec{a}_z$$

$$\vec{H}_s = \frac{1}{\eta}\vec{a}_n \times \vec{E} = (2.3\vec{a}_z - 1.33\vec{a}_y) e^{j(0.866y+0.5z)} \text{ (mA/m)}$$

(c) $P_{ave} = \frac{1}{2} Re [E_s \times H_s^*]$

$$= \frac{-(100)^2}{2 \times (120\pi)} (+0.866 \vec{a}_y + 0.5 \vec{a}_z)$$

$$= -11.49 \vec{a}_y - 6.631 \vec{a}_z \quad (W/m^2)$$

Q2 [Solution]

(a) $\vec{E}_i = 8 e^{-j(4x+3z)} \vec{a}_y \quad (V/m)$

$\Rightarrow \quad \beta_1 \vec{a}_{ni} = 4\vec{a}_x + 3\vec{a}_z$

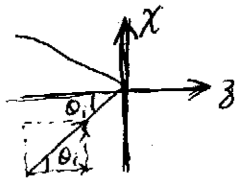
$\Rightarrow \quad \beta_1 = \sqrt{4^2+3^2} = 5 = \frac{\omega}{c} \Rightarrow \omega = 5c = 15 \times 10^8 \text{ (rad/s)}$

$\vec{a}_{ni} = \frac{\beta_1 \vec{a}_{ni}}{\beta_1} = \frac{4}{5} \vec{a}_x + \frac{3}{5} \vec{a}_z$

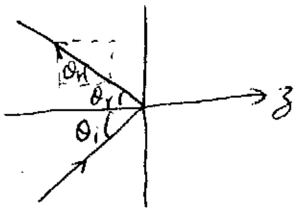
$\Rightarrow$ The wave is perpendicular polarization

(b)

$$\tan\theta_i = \frac{\vec{a}_{ni-x}}{\vec{a}_{ni-z}} = \frac{4}{3} \Rightarrow \theta_i = 53.13°$$



(c)



$\theta_r = \theta_i = 53.13°$, $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$, $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = 238.4 (\Omega)$

$\vec{a}_{nr} = -\cos\theta_r \vec{a}_z + \sin\theta_r \vec{a}_x$

$= -\frac{3}{5} \vec{a}_z + \frac{4}{5} \vec{a}_x$

$\beta_1 = 5$ (rad/m)

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{\beta_1}{\beta_2} = \frac{\sqrt{\mu_0 \varepsilon_0 \varepsilon_{r1}}}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_{r2}}} = \frac{1}{\sqrt{2.5}}$$

$$\Rightarrow \theta_t = \sin^{-1}\left(\frac{\sin\theta_i}{\sqrt{2.5}}\right) = 30.39°$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = -0.389$$

$E_{ro} = \Gamma_\perp E_{io} = -3.112 \quad (V/m)$

$\Rightarrow \vec{E}_r = -3.112 \vec{a}_y e^{-j\beta_1 \vec{a}_{nr} \cdot \vec{R}} = -3.112 \vec{a}_y e^{-j(4x-3z)} \quad (V/m)$

$\vec{\mathcal{E}}_r = -3.112 \cos(15 \times 10^8 t - 4x + 3z) \vec{a}_y \quad (V/m)$

(d)

$$\beta_2 = \omega\sqrt{\mu_2 \varepsilon_2} = \frac{\omega}{c}\sqrt{\mu_{r2}\varepsilon_{r2}} = \frac{15\times10^8}{3\times10^8}\sqrt{1\times2.5} = 7.906$$

$$\beta_2 \vec{a}_{nt} = 7.906\,(\cos\theta_t\,\vec{a}_z + \sin\theta_t\,\vec{a}_x) = 4\vec{a}_x + 6.819\,\vec{a}_z$$

$$\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = 0.611.$$

$$E_{t0} = \tau_\perp E_{i0} = 0.611\times 8 = 4.888 \Rightarrow \dot{\vec{E}}_t = 4.888\,e^{-j(4x+6.819z)}$$

$$\vec{E}_t = 4.888\cos(15\times10^8 t - 4x - 6.819z)\,\vec{a}_y \quad (V/m)$$

$$\Rightarrow \dot{\vec{H}}_t = \frac{1}{\eta_2}\vec{a}_{nt}\times\dot{\vec{E}}_t = (-17.69\,\vec{a}_x + 10.37\,\vec{a}_z)\,e^{-j(4x+6.819z)}$$

(mA/m)

$$\vec{H}_t = (-17.69\,\vec{a}_x + 10.37\,\vec{a}_z)\cos(15\times10^8 t - 4x - 6.819z)$$

(mA/m)