Prob. 9.32

With the given **A**, we need to prove that

$$\nabla^2 A = \mu\varepsilon \frac{\partial^2 A}{\partial t^2}$$

$$\nabla^2 A = \mu\varepsilon (j\omega)(j\omega) A = -\omega^2 \mu\varepsilon A$$

Let $\beta^2 = \omega^2 \mu\varepsilon$, then $\nabla^2 A = -\beta^2 A$ is to be proved. We recognize that

$$A = \frac{\mu_o}{4\pi r} e^{j\omega t} e^{-j\beta r} \mathbf{a}_z$$

Assume $\varphi = \dfrac{e^{-j\beta r}}{r}$, $\quad A = \dfrac{\mu_o}{4\pi} e^{j\omega t} \varphi \mathbf{a}_z$

$$\nabla^2 \varphi = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(r^2 \sin\theta \frac{\partial \varphi}{dr})\right] = \frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2)\left(\frac{-j\beta}{r} - \frac{1}{r^2}\right)e^{-j\beta r}\right]$$

$$= \frac{1}{r^2}\left(-\beta^2 r + j\beta - j\beta\right)e^{-j\beta r} = -\beta^2 \frac{e^{-j\beta r}}{r} = -\beta^2 \varphi$$

Therefore, $\quad \nabla^2 A = -\beta^2 A$

We can find V using Lorentz gauge.

$$V = \frac{-1}{\mu_o \varepsilon_o} \int \nabla \bullet A dt = \frac{-1}{j\omega \mu_o \varepsilon_o} \nabla \bullet A$$

$$= \frac{-1}{j\omega \mu_o \varepsilon_o} \frac{\partial}{\partial r}\left(\frac{\mu_o}{4\pi r} e^{-j\beta r} e^{j\omega t}\right) = \frac{-1}{j\omega \varepsilon_o (4\pi)}\left(\frac{-j\beta}{r} - \frac{1}{r^2}\right) e^{-j\beta r} e^{j\omega t} \cos\theta$$

$$\underline{\underline{V = \frac{\cos\theta}{j4\pi\omega\varepsilon_o r}\left(j\beta + \frac{1}{r}\right) e^{j(\omega t - \beta r)}}}$$

Prob. 9.34

(a)

$$z = 4\angle 30^o - 10\angle 50^o = 3.464 + 2j - 6.427 - j7.66 = -2.963 - j5.66$$

$$= 6.389\angle -117.64^o$$

$$z^{1/2} = \underline{\underline{2.5277\angle -58.82^o}}$$

(b)

$$\frac{1+j2}{6-j8-7\angle 15^o} = \frac{2.236\angle 63.43^o}{6-j8-7.761-j1.812} = \frac{2.236\angle 63.43^o}{9.841\angle 265.57^o}$$

$$= \underline{\underline{0.2272\angle -202.1^o}}$$

(c) $$z = \frac{(5\angle 53.13^o)^2}{12-j7-6-j10} = \frac{25\angle 106.26^o}{18.028\angle -70.56^o}$$

$$= \underline{\underline{1.387\angle 176.8^o}}$$

(d)

$$\frac{1.897\angle -100^o}{(5.76\angle 90^o)(9.434\angle -122^o)} = \underline{\underline{0.0349\angle -68^o}}$$

Prob. 9.35

(a) $A = 5\cos(2t + \pi/3 - \pi/2)\boldsymbol{a}_x + 3\cos(2t + 30^o)\boldsymbol{a}_y = \mathrm{Re}(A_s e^{j\omega t}), \omega = 2$

$$\underline{\underline{A_s = 5e^{-j30^o}\boldsymbol{a}_x + 3e^{j30^o}\boldsymbol{a}_y}}$$

(b) $\boldsymbol{B} = \dfrac{100}{\rho}\cos(\omega t - 2\pi z - 90^o)\boldsymbol{a}_\rho$

$$\boldsymbol{B}_s = \underline{\underline{\frac{100}{\rho}e^{-j(2\pi z + 90^o)}\boldsymbol{a}_\rho}}$$

(c) $\boldsymbol{C} = \dfrac{\cos\theta}{r}\cos(\omega t - 3r - 90^o)\boldsymbol{a}_\theta$

$$\boldsymbol{C}_s = \underline{\underline{\frac{\cos\theta}{r}e^{-j(3r + 90^o)}\boldsymbol{a}_z}}$$

(d) $\boldsymbol{D}_s = \underline{\underline{10\cos(k_1 x)e^{-jk_2 z}\boldsymbol{a}_y}}$

Prob. 9.36 (a) $(4 - j3) = 5e^{-j36.87^o}$

$$A_s = 5e^{-j(\beta x + 36.87^o)}\boldsymbol{a}_y$$

$$A = \mathrm{Re}\left[A_s e^{j\omega t}\right] = \underline{\underline{5\cos(\omega t - \beta x - 36.87^o)\boldsymbol{a}_y}}$$

(b)

$$\boldsymbol{B} = \text{Re}\left[\boldsymbol{B}_s e^{j\omega t}\right] = \text{Re}\left[\frac{20}{\rho} e^{j(\omega t - 2z)} \boldsymbol{a}_\rho\right]$$

$$= \underline{\underline{\frac{20}{\rho}\cos(\omega t - 2z)\boldsymbol{a}_\rho}}$$

(c) $1 + j2 = 2.23e^{j63.43^o}$

$$\boldsymbol{C}_s = \frac{10}{r^2}(2.236)e^{j63.43^o} e^{-j\phi} \sin\theta \boldsymbol{a}_\phi$$

$$\boldsymbol{C} = \text{Re}\left[C_s e^{j\omega t}\right] = \text{Re}\left[\frac{22.36}{r^2} e^{j(\omega t - \phi + 63.43^o)} \sin\theta \boldsymbol{a}_\phi\right]$$

$$= \underline{\underline{\frac{22.36}{r^2}\cos(\omega t - \phi + 63.43^o)\sin\theta \boldsymbol{a}_\phi}}$$

Prob. 9.37

(a) $\nabla \cdot \boldsymbol{E}_s = 0$

$$\nabla \times \boldsymbol{E}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix} = -\frac{\partial E_{sx}}{\partial y}\boldsymbol{a}_z = j40e^{-j4y}\boldsymbol{a}_z$$

But $\nabla \times \boldsymbol{E}_s = -j\mu_o \omega \boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = -\frac{40}{\mu_o \omega} e^{-j4y}\boldsymbol{a}_z$

$\nabla \cdot \boldsymbol{H}_s = 0$

$$\nabla \times \boldsymbol{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_{sz}(y) \end{vmatrix} = \frac{\partial H_{sz}}{\partial y}\boldsymbol{a}_x = \frac{j160}{\mu_o \omega} e^{-j4y}\boldsymbol{a}_x$$

But $\nabla \times \boldsymbol{H}_s = j\omega \varepsilon_o \boldsymbol{E}_s \longrightarrow \boldsymbol{E}_s = \frac{160}{\mu_o \varepsilon_o \omega^2} e^{-j4y}\boldsymbol{a}_x = 10e^{-j4y}\boldsymbol{a}_x$

$$\omega^2 = \frac{16}{\mu_o \varepsilon_o} = \frac{16}{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}} = 16 \times 9 \times 10^{16}$$

$\omega = \underline{\underline{12 \times 10^8 \text{ rad/s}}}$

(b) $\boldsymbol{H}_s = -\frac{40}{4\pi x 10^{-7} x 12 x 10^8} e^{-j4y}\boldsymbol{a}_z = \underline{\underline{-26.53e^{-j4y}\boldsymbol{a}_z \text{ mA/m}}}$

P. E. 10.1 (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\times10^8} = \underline{\underline{31.42\ \text{ns}}},$$

$$\lambda = uT = 3x10^8 x31.42x10^{-9} = \underline{\underline{9.425\ \text{m}}}$$

$$k = \beta = 2\pi / \lambda = \underline{\underline{0.6667\ \text{rad/m}}}$$

(b) $t_1 = T/8 = \underline{\underline{3.927\ \text{ns}}}$

(c)

$$\boldsymbol{H}(t = t_1) = 0.1\cos(2\times10^8 \frac{\pi}{8\times10^8} - 2x/3)\boldsymbol{a}_y = 0.1\cos(2x/3 - \pi/4)\boldsymbol{a}_y$$

as sketched below.