Prob. 9.30 Using Maxwell's equations,

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \qquad (\sigma = 0) \qquad \longrightarrow \qquad \boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

But

$$\nabla \times \boldsymbol{H} = -\frac{1}{r\sin\theta}\frac{\partial H_\theta}{\partial \phi}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)\boldsymbol{a}_\phi = \frac{12\sin\theta}{r}\beta \sin(2\pi x 10^8 t - \beta r)\boldsymbol{a}_\phi$$

$$\boldsymbol{E} = \frac{12\sin\theta}{\varepsilon_o}\beta \int \sin(2\pi x 10^8 t - \beta r) dt \boldsymbol{a}_\phi$$

$$= \underline{\underline{-\frac{12\sin\theta}{\omega \varepsilon_o r}\beta \cos(\omega t - \beta r)\boldsymbol{a}_\phi}}, \qquad \omega = 2\pi x 10^8$$

Prob. 9.31

$$\nabla \times \vec{E} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi)\vec{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 t e^{-\rho - t})\vec{a}_z$$

$$= (2-\rho)te^{-\rho-t}\vec{a}_z$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} \rightarrow \vec{B} = -\int \nabla \times \vec{E} dt = \int \frac{(\rho-2)t}{V} \frac{e^{-\rho-t}dt}{du}\vec{a}_z$$

Integrating by parts yields

$$\vec{B} = [-(\rho-2)te^{-\rho-t} + \int(\rho-2)e^{-\rho-t}dt]\vec{a}_z$$

$$= \underline{\underline{(2-\rho)(1+t)e^{-\rho-t}\vec{a}_z}} \text{ Wb/m}^2$$

$$\vec{J} = \nabla \times \vec{H} = \nabla \times \frac{\vec{B}}{\mu_o} = -\frac{1}{\mu_o}\frac{\partial B_z}{\partial \rho}\vec{a}_\phi$$

$$= -\frac{1}{\mu_o}(1+t)(-1-2+\rho)e^{-\rho-t}\vec{a}_\phi$$

$$\vec{J} = \underline{\underline{\frac{(1+t)(3-\rho)e^{-\rho-t}}{4\pi x 10^{-7}}\vec{a}_\phi}} \text{ A/m}^2$$

Prob. 9.32

With the given **A**, we need to prove that

$$\nabla^2 \boldsymbol{A} = \mu\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2}$$

$\nabla^2 A = \mu\varepsilon(j\omega)(j\omega)A = -\omega^2\mu\varepsilon A$

Let $\beta^2 = \omega^2\mu\varepsilon$, then $\nabla^2 A = -\beta^2 A$ is to be proved. We recognize that

$$A = \frac{\mu_o}{4\pi r}e^{j\omega t}e^{-j\beta r}\boldsymbol{a}_z$$

Assume $\varphi = \dfrac{e^{-j\beta r}}{r}$, $\quad A = \dfrac{\mu_o}{4\pi}e^{j\omega t}\varphi\boldsymbol{a}_z$

$$\nabla^2\varphi = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta\frac{\partial\varphi}{dr})\right] = \frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2)\left(\frac{-j\beta}{r}-\frac{1}{r^2}\right)e^{-j\beta r}\right]$$

$$= \frac{1}{r^2}\left(-\beta^2 r + j\beta - j\beta\right)e^{-j\beta r} = -\beta^2\frac{e^{-j\beta r}}{r} = -\beta^2\varphi$$

Therefore, $\nabla^2 A = -\beta^2 A$

We can find V using Lorentz gauge.

$$V = \frac{-1}{\mu_o\varepsilon_o}\int\nabla\bullet A dt = \frac{-1}{j\omega\mu_o\varepsilon_o}\nabla\bullet A$$

$$= \frac{-1}{j\omega\mu_o\varepsilon_o}\frac{\partial}{\partial r}\left(\frac{\mu_o}{4\pi r}e^{-j\beta r}e^{j\omega t}\right) = \frac{-1}{j\omega\varepsilon_o(4\pi)}\left(\frac{-j\beta}{r}-\frac{1}{r^2}\right)e^{-j\beta r}e^{j\omega t}\cos\theta$$

$$V = \underline{\underline{\frac{\cos\theta}{j4\pi\omega\varepsilon_o r}\left(j\beta+\frac{1}{r}\right)e^{j(\omega t-\beta r)}}}$$

Prob. 9.33

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$$

But $\boldsymbol{B} = \nabla x A$

$$\nabla\times\boldsymbol{E} = -\frac{\partial}{\partial t}(\nabla\times A) = \nabla\times\left(-\frac{\partial A}{\partial t}\right)$$

Hence,

$$\boldsymbol{E} = -\frac{\partial A}{\partial t}$$

Prob. 9.34

(a)

$$z = 4\angle 30^o - 10\angle 50^o = 3.464 + 2j - 6.427 - j7.66 = -2.963 - j5.66$$

$$= 6.389\angle -117.64^o$$

$$z^{1/2} = \underline{\underline{2.5277\angle -58.82^o}}$$

(b)

$$\frac{1+j2}{6-j8-7\angle 15^o}=\frac{2.236\angle 63.43^o}{6-j8-7.761-j1.812}=\frac{2.236\angle 63.43^o}{9.841\angle 265.57^o}$$

$$=\underline{\underline{0.2272\angle -202.1^o}}$$

(c) $$z=\frac{(5\angle 53.13^o)^2}{12-j7-6-j10}=\frac{25\angle 106.26^o}{18.028\angle -70.56^o}$$

$$=\underline{\underline{1.387\angle 176.8^o}}$$

(d)

$$\frac{1.897\angle -100^o}{(5.76\angle 90^o)(9.434\angle -122^o)}=\underline{\underline{0.0349\angle -68^o}}$$

Prob. 9.35

(a) $\boldsymbol{A}=5\cos(2t+\pi/3-\pi/2)\boldsymbol{a}_x+3\cos(2t+30^o)\boldsymbol{a}_y=\mathrm{Re}(A_s e^{j\omega t}),\omega=2$

$$\underline{\underline{A_s=5e^{-j30^o}\boldsymbol{a}_x+3e^{j30^o}\boldsymbol{a}_y}}$$

(b) $\boldsymbol{B}=\frac{100}{\rho}\cos(\omega t-2\pi z-90^o)\boldsymbol{a}_\rho$

$$\underline{\underline{\boldsymbol{B}_s=\frac{100}{\rho}e^{-j(2\pi z+90^o)}\boldsymbol{a}_\rho}}$$

(c) $\boldsymbol{C}=\frac{\cos\theta}{r}\cos(\omega t-3r-90^o)\boldsymbol{a}_\theta$

$$\underline{\underline{\boldsymbol{C}_s=\frac{\cos\theta}{r}e^{-j(3r+90^o)}\boldsymbol{a}_z}}$$

(d) $\boldsymbol{D}_s=\underline{\underline{10\cos(k_1x)e^{-jk_2z}\boldsymbol{a}_y}}$

Prob. 9.36 (a) $(4-j3)=5e^{-j36.87^o}$

$$A_s=5e^{-j(\beta x+36.87^o)}\boldsymbol{a}_y$$

$$A=\mathrm{Re}\left[A_s e^{j\omega t}\right]=\underline{\underline{5\cos(\omega t-\beta x-36.87^o)\boldsymbol{a}_y}}$$

(b)

$$\boldsymbol{B} = \mathrm{Re}\left[\boldsymbol{B}_s e^{j\omega t}\right] = \mathrm{Re}\left[\frac{20}{\rho} e^{j(\omega t - 2z)} \boldsymbol{a}_\rho\right]$$

$$= \underline{\underline{\frac{20}{\rho}\cos(\omega t - 2z)\boldsymbol{a}_\rho}}$$

(c) $1 + j2 = 2.23e^{j63.43^o}$

$$\boldsymbol{C}_s = \frac{10}{r^2}(2.236)e^{j63.43^o} e^{-j\phi}\sin\theta \boldsymbol{a}_\phi$$

$$\boldsymbol{C} = \mathrm{Re}\left[C_s e^{j\omega t}\right] = \mathrm{Re}\left[\frac{22.36}{r^2} e^{j(\omega t - \phi + 63.43^o)} \sin\theta \boldsymbol{a}_\phi\right]$$

$$= \underline{\underline{\frac{22.36}{r^2}\cos(\omega t - \phi + 63.43^o)\sin\theta \boldsymbol{a}_\phi}}$$

Prob. 9.37

(a) $\nabla \cdot \boldsymbol{E}_s = 0$

$$\nabla \times \boldsymbol{E}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix} = -\frac{\partial E_{sx}}{\partial y}\boldsymbol{a}_z = j40e^{-j4y}\boldsymbol{a}_z$$

But $\nabla \times \boldsymbol{E}_s = -j\mu_o \omega \boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = -\frac{40}{\mu_o \omega} e^{-j4y}\boldsymbol{a}_z$

$\nabla \cdot \boldsymbol{H}_s = 0$

$$\nabla \times \boldsymbol{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_{sz}(y) \end{vmatrix} = \frac{\partial H_{sz}}{\partial y}\boldsymbol{a}_x = \frac{j160}{\mu_o \omega} e^{-j4y}\boldsymbol{a}_x$$

But $\nabla \times \boldsymbol{H}_s = j\omega\varepsilon_o \boldsymbol{E}_s \longrightarrow \boldsymbol{E}_s = \frac{160}{\mu_o \varepsilon_o \omega^2} e^{-j4y}\boldsymbol{a}_x = 10e^{-j4y}\boldsymbol{a}_x$

$$\omega^2 = \frac{16}{\mu_o \varepsilon_o} = \frac{16}{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}} = 16 \times 9 \times 10^{16}$$

$\omega = \underline{\underline{12 \times 10^8 \text{ rad/s}}}$

(b) $\boldsymbol{H}_s = -\frac{40}{4\pi x 10^{-7} x 12 x 10^8} e^{-j4y}\boldsymbol{a}_z = \underline{\underline{-26.53e^{-j4y}\boldsymbol{a}_z \text{ mA/m}}}$

Prob. 9.38

$$A = 4\cos(\omega t - 90^o)\boldsymbol{a}_x + 3\cos\omega t\boldsymbol{a}_y = \text{Re}\left[4e^{j(\omega t-90^o)}\boldsymbol{a}_x + 3e^{j\omega t}\boldsymbol{a}_y\right] = \text{Re}\left[\boldsymbol{A}_s e^{j\omega t}\right]$$

$$\boldsymbol{A}_s = 4e^{-j90^o}\boldsymbol{a}_x + 3\boldsymbol{a}_y = \underline{\underline{-j4\boldsymbol{a}_x + 3\boldsymbol{a}_y}}$$

$$\boldsymbol{B}_s = 10ze^{j90^o}e^{-jz}\boldsymbol{a}_x$$

$$\boldsymbol{B} = \text{Re}\left[\boldsymbol{B}_s e^{j\omega t}\right] = 10z\cos(\omega t - z + 90^o)\boldsymbol{a}_x = \underline{\underline{-10z\sin(\omega t - z)\boldsymbol{a}_x}}$$

Prob. 9.39 We begin with Maxwell's equations:

$$\nabla \bullet \boldsymbol{D} = \rho_v / \varepsilon = 0, \qquad \nabla \bullet \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

We write these in phasor form and in terms of E_s and H_s only.

$$\nabla \bullet \boldsymbol{E}_s = 0 \qquad (1)$$

$$\nabla \bullet \boldsymbol{H}_s = 0 \qquad (2)$$

$$\nabla \times \boldsymbol{E}_s = -j\omega\mu\boldsymbol{H}_s \qquad (3)$$

$$\nabla \times \boldsymbol{H}_s = (\sigma + j\omega\varepsilon)\boldsymbol{E}_s \qquad (4)$$

Taking the curl of (3),

$$\nabla \times \nabla \times \boldsymbol{E}_s = -j\omega\mu\nabla \times \boldsymbol{H}_s$$

$$\nabla(\nabla \bullet \boldsymbol{E}_s) - \nabla^2\boldsymbol{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\boldsymbol{E}_s$$

$$\nabla^2\boldsymbol{E}_s + (\omega^2\mu\varepsilon - j\omega\mu\sigma)\boldsymbol{E}_s = 0 \quad \longrightarrow \quad \underline{\underline{\nabla^2\boldsymbol{E}_s + \gamma^2\boldsymbol{E}_s = 0}}$$

Similarly, by taking the curl of (4),

$$\nabla \times \nabla \times \boldsymbol{H}_s = (\sigma + j\omega\varepsilon)\nabla \times \boldsymbol{E}_s$$

$$\nabla(\nabla \bullet \boldsymbol{H}_s) - \nabla^2\boldsymbol{H}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\boldsymbol{H}_s$$

$$\nabla^2\boldsymbol{H}_s + (\omega^2\mu\varepsilon - j\omega\mu\sigma)\boldsymbol{H}_s = 0 \quad \longrightarrow \quad \underline{\underline{\nabla^2\boldsymbol{H}_s + \gamma^2\boldsymbol{H}_s = 0}}$$

Prob. 10.1 (a) Wave propagates <u>along $+\mathbf{a}_x$</u>.

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi x 10^6} = \underline{\underline{1\mu s}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{\underline{1.047\,\text{m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi x 10^6}{6} = \underline{\underline{1.047 x 10^6\,\text{m/s}}}$$

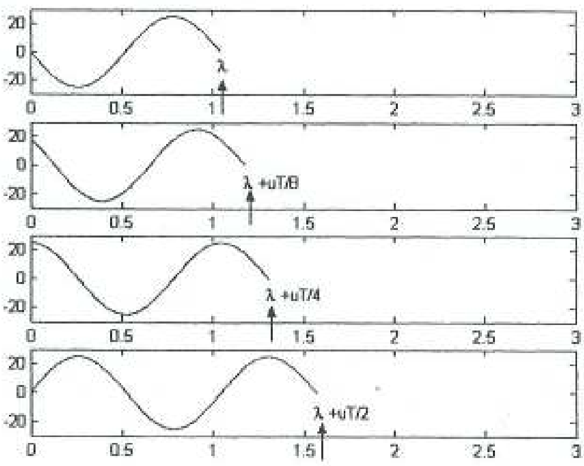
(c) At t=0, $E_z = 25\sin(-6x) = -25\sin 6x$

At t=T/8, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{8} - 6x) = 25\sin(\frac{\pi}{4} - 6x)$

At t=T/4, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{4} - 6x) = 25\sin(-6x + 90^o) = 25\cos 6x$

At t=T/2, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{2} - 6x) = 25\sin(-6x + \pi) = 25\sin 6x$

These are sketched below.



Prob. 10.2 If

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = -\omega^2\mu\varepsilon + j\omega\mu\sigma \quad \text{and}\, \gamma = \alpha + j\beta\,, \text{ then}$$