Prob.10.34

(a)

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = 10^{-2}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+10^{-4}}-1\right]} = \frac{2\pi\times2\times10^9}{3\times10^8}\sqrt{\frac{2.5}{2}}\sqrt{0.00005} = 0.3311$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+10^{-4}}+1\right]} = \frac{2\pi\times2\times10^9}{3\times10^8}\sqrt{\frac{2.5}{2}}\sqrt{2.00005} = 66.23$$

(In this case, $\beta \simeq \omega\sqrt{\mu\varepsilon}$.)

$$(1-0.2)E_o = E_o e^{-\alpha z} \longrightarrow 0.8E_o = E_o e^{-\alpha z}$$

$$e^{\alpha z} = 1.25 \longrightarrow z = \frac{1}{\alpha}\ln 1.25 = \underline{\underline{0.674 \text{ m}}}$$

(b) $\beta z = 180^{o} = \pi \longrightarrow z = \dfrac{\pi}{\beta} = \underline{\underline{0.04743 \text{ m}}}$

(c) $P = P_o e^{-2\alpha z} \longrightarrow 0.9P_o = P_o e^{-2\alpha z}$

$$e^{2\alpha z} = 1/0.9 = 1.111 \longrightarrow z = \frac{1}{2\alpha}\ln 1.111 = \underline{\underline{0.159 \text{ m}}}$$

Prob. 10.35

(a) $u = \omega / \beta \longrightarrow \omega = u\beta = \dfrac{\beta c}{\sqrt{4.5}} = \dfrac{2\times 3\times 10^{8}}{\sqrt{4.5}} = \underline{\underline{2.828\times 10^{8}}}$ rad/s

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7\Omega$$

$$\boldsymbol{H} = \boldsymbol{a}_k \times \frac{\boldsymbol{E}}{\eta} = \frac{\boldsymbol{a}_z}{\eta} \times \frac{40}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\rho = \underline{\underline{\frac{0.225}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\phi}} \text{ A/m}$$

(b) $\mathscr{P} = \boldsymbol{E}\times\boldsymbol{H} = \underline{\underline{\dfrac{9}{\rho^2}\sin^2(\omega t - 2z)\boldsymbol{a}_z \text{ W/m}^2}}$

$P_{av} = \dfrac{|\vec{P}|}{2}\,\overline{a_z}$

$\mathscr{P}_{ave} = \dfrac{1}{2}\mathrm{Re}\{E_s \times H_s^*\}$

(c) $\mathscr{P}_{ave} = \dfrac{4.5}{\rho^2}\boldsymbol{a}_z$, $\mathrm{d}\mathbf{S} = \rho\,\mathrm{d}\phi\,\mathrm{d}\rho\,\boldsymbol{a}_z$

$$P_{ave} = \int \mathbf{P}_{ave} \bullet dS = 4.5 \int_{2\text{mm}}^{3\text{mm}} \frac{\mathrm{d}\rho}{\rho}\int_0^{2\pi}\mathrm{d}\phi = 4.5\ln(3/2)(2\pi) = \underline{\underline{11.46 \text{ W}}}$$

Prob. 10.40

(a) $$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{4}} = 60\pi, \qquad \eta_1 = \eta_o = 120\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{180\pi} = -\frac{1}{3}, \qquad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{120\pi}{180\pi} = \frac{2}{3}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \underline{\underline{2}}$$

(b) $$\lambda_1 = c/f = \frac{3\times10^8}{10^8} = \underline{\underline{3\text{ m}}}$$

$$\lambda_2 = u/f = \frac{\frac{c}{\sqrt{\varepsilon_r}}}{f} = \frac{3}{\sqrt{4}} = \underline{\underline{1.5\text{ m}}}$$

(c) $$P_i = \frac{E_{oi}^2}{2\eta_1}, \qquad P_r = \frac{E_{or}^2}{2\eta_1} = \frac{\Gamma^2 E_{oi}^2}{2\eta_1}$$

$$\frac{P_r}{P_i} = \Gamma^2 = \frac{1}{9} = 0.1111 \text{ or } \underline{\underline{11.11\%}}$$

Prob. 10.41 $\eta_1 = \eta_o = 120\pi, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad (1)$$

But $$E_{ro} = \eta_o H_{ro} \qquad (2)$$

Combining (1) and (2),

$$E_{ro} = \eta_o H_{ro} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) E_{io} \longrightarrow \eta_o = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) \frac{E_{io}}{H_{ro}}$$

But $\dfrac{E_{io}}{H_{ro}} = \dfrac{3.6}{1.2 \times 10^{-3}} = 3000$

$$\eta_o = 3000 \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) \longrightarrow 377 = 3000 \left(\frac{\eta_2 - 377}{\eta_2 + 377} \right)$$

Thus, $\eta_2 = 485.37$. Since $\eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}}$,

$$\mu_2 = \varepsilon_o \varepsilon_r \eta_2^{\,2} = \frac{10^{-9}}{36\pi} \times 12.5 \times (485.37)^2 = \underline{\underline{2.604 \times 10^{-5}}} \text{ H/m}$$

Prob. 10.49 Since $\mu_o = \mu_1 = \mu_2$,

$$\sin\theta_{t1} = \sin\theta_i \sqrt{\frac{\varepsilon_o}{\varepsilon_1}} = \frac{\sin 45^o}{\sqrt{4.5}} = 0.3333 \longrightarrow \underline{\underline{\theta_{t1} = 19.47^o}}$$

$$\sin\theta_{t2} = \sin\theta_{t1} \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{1}{3}\sqrt{\frac{4.5}{2.25}} = 0.4714 \longrightarrow \underline{\underline{\theta_{t2} = 28.13^o}}$$

Prob. 10.57 (a)

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{1}{\sqrt{8}} \longrightarrow \underline{\underline{\theta_i = \theta_r = 19.47^\circ}}$$

$$\sin\theta_t = \sin\theta_i \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}(3) = 1 \quad \longrightarrow \quad \underline{\underline{\theta_t = 90^o}}$$

(b) $$\beta_1 = \frac{\omega}{c}\sqrt{\varepsilon_{r1}} = \frac{10^9}{3\times 10^8}\times 3 = 10 = k\sqrt{1+8} = 3k \quad \longrightarrow \quad \underline{\underline{k = 3.333}}$$

(c) $$\lambda = 2\pi/\beta, \qquad \lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283}}\text{ m}$$

$$\beta_2 = \omega/c = 10/3, \qquad \lambda_2 = 2\pi/\beta_2 = 2\pi x3/10 = \underline{\underline{1.885}}\text{ m}$$

(d)
$$\begin{aligned} \boldsymbol{E}_i &= \eta_1 \boldsymbol{H}_x \times \boldsymbol{a}_k = 40\pi(0.2)\cos(\omega t - \boldsymbol{k}\bullet\boldsymbol{r})\boldsymbol{a}_y \times \frac{(\boldsymbol{a}_x + \sqrt{8}\boldsymbol{a}_z)}{3} \\ &= \underline{\underline{(23.6954\boldsymbol{a}_x - 8.3776\boldsymbol{a}_z)\cos(10^9 t - kx - k\sqrt{8}z)\ \text{V/m}}} \end{aligned}$$

(e) $$\tau_{//} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i+\theta_t)\cos(\theta_t-\theta_i)} = \frac{2\cos 19.47^o \sin 90^o}{\sin 19.47^o \cos 19.47^o} = 6$$

$$\Gamma_{//} = -\frac{\cot 19.47^o}{\cot 19.47^o} = -1$$

Let $\boldsymbol{E}_t = -E_{io}(\cos\theta_t \boldsymbol{a}_x - \sin\theta_t \boldsymbol{a}_z)\cos(10^9 t - \beta_2 x\sin\theta_t - \beta_2 z\cos\theta_t)$

where

$$\boldsymbol{E}_t = -E_{io}(\cos\theta_i \boldsymbol{a}_x - \sin\theta_i \boldsymbol{a}_z)\cos(10^9 t - \beta_1 x\sin\theta_i - \beta_1 z\cos\theta_i)$$

$$\sin\theta_t = 1, \qquad \cos\theta_t = 0, \qquad \beta_2 \sin\theta_t = 10/3$$

$$E_{to}\sin\theta_t = \tau_{\|}E_{io} = 6(24\pi)(3)(1) = 1357.2$$

Hence,

$$\underline{\underline{\boldsymbol{E}_t = 1357\cos(10^9 t - 3.333x)\boldsymbol{a}_z}}\ \text{V/m}$$

$E_t = \tau_{\|} |E_i|\, \bar{a}_z \cos(10^9 t - kx)$

$E_t = 6 \times \sqrt{23.69^2 + 8.37^2} \cos(10^9 t -$

$= 150.76 \cos(10^9 t - 3.333x)\bar{a}$

Since $\Gamma = -1, \quad \theta_r = \theta_i$

$+ 8.37 a_z$

$$\underline{\underline{\boldsymbol{E}_r = (\cancel{213.3}\boldsymbol{a}_x + \cancel{75.4}\boldsymbol{a}_z)\cos(10^9 t - kx + k\sqrt{8}z)}}\ \text{V/m}$$

$23.69 a_x$

(f) $\tan\theta_{B//} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\dfrac{\varepsilon_o}{9\varepsilon_o}} = 1/3 \longrightarrow \underline{\underline{\theta_{B//} = 18.43^o}}$

Prob. 10.58

$$\beta_1 = \sqrt{3^2+4^2} = 5 = \omega / c \longrightarrow \underline{\underline{\omega = \beta_1 c = 15\times 10^8 \text{ rad/s}}}$$

Let $\quad E_r = (E_{ox}, E_{oy}, E_{oz})\sin(\omega t + 3x + 4y)$. In order for

$$\nabla \bullet E_r = 0, \qquad 3E_{ox} + 4E_{oy} = 0 \qquad (1)$$

Also, at y=0, $E_{1\tan} = E_{2\tan} = 0$

$$E_{1\tan} = 0, \qquad 8a_x + 5a_z + E_{ox}a_x + E_{oz}a_z = 0$$

Equating components, $\quad E_{ox} = -8, \qquad E_{oz} = -5$

From (1), $\quad 4E_{oy} = -3E_{ox} = 24 \qquad E_{oy} = 6$

Hence,

$$\underline{\underline{E_r = (-8\boldsymbol{a}_x + 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\sin(15\times 10^8 t + 3x + 4y) \quad \text{V/m}}}$$