Tutorial

Problem (#1)

The electric field component of a uniform VLF electromagnetic field propagating vertically down in the z direction in the ocean ($\sigma = 4sm$, $\varepsilon_r = 81$, $\mu_r = 1$) is approximated given by

$$\vec{E}(z,t) = \vec{a}_x E_0 e^{-\alpha z} \cos(6\pi \times 10^3 t - \beta z) \ V/m$$

$z = 0$ is the interface between the air and ocean water.

(a) Find the attenuation constant α and Phase shift constant β

(b) Find the wavelength, phase velocity U_p, skin depth δ and intrinsic impedance η_c and compare them to their values in air.

(c) Write the instantaneous expression for $\vec{H}(z,t)$

(d) A submarine located at a depth of 100m has a receiver antenna capable of measuring electric fields with amplitudes of 1 $\mu V/m$. What is the minimum required electric field amplitude immediately below the ocean surface (i.e. E_0) in order to communicate with the submarine? What is the corresponding value for the amplitude of the magnetic field?

[Solution] $\varepsilon'' = 0$, $\varepsilon' = \varepsilon_0 \varepsilon_r = 81\varepsilon_0$, $\sigma = 4S$, $\mu = \mu_0 \mu_r = \mu_0$

(a) $\alpha = \omega \sqrt{\frac{\mu_0 \mu_r}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon'}\right)^2} - 1\right]} = 0.218 \ np/m$ ←

$\beta = \omega \sqrt{\frac{\mu_0 \mu_r}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon'}\right)^2} + 1\right]} = 0.218 \ rad/m$ ←

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.218} = 28.9 \ m$; $f = 3000 \ Hz$ ←

In air, $\lambda = \frac{c}{f} = 3 \times 10^8 / 3000 = 100 \ Km \Rightarrow$

$\Rightarrow$ Wavelength in ocean water is 3646 times smaller!

$$u_p = f\lambda = 3\times10^3 \times 28.9 = 8.66\times10^4 \ m/s \leftarrow$$

In air, $u_p = c = 3\times10^8$. Phase velocity in ocean water is 3464 times slower!

$$\delta = \frac{1}{\alpha} = 4.59 m \leftarrow$$

$$\eta_c = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r - j\frac{\sigma}{\omega}}} = 7.70\times10^{-2} e^{j45^\circ} (\Omega) \leftarrow$$

Compared to $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$ in air, the magnitude of η_c is about 4900 times smaller. In addition, η_c in the ocean has a phase of 45°

(c)

$$\vec{H}(z) = \frac{1}{\eta_c} \vec{a}_z \times \vec{E}$$

$$= \frac{1}{\eta_c} E_0 e^{-\alpha z} e^{-j\beta z} \vec{a}_y$$

$$\vec{H}(z,t) = Re[\vec{H}(z) e^{j\omega t}]$$

$$= \vec{a}_y \frac{E_0}{|\eta_c|} e^{-\alpha z} \cos(\omega t - \beta z - \phi_\eta) \leftarrow$$

$$= \vec{a}_y \, 13E_0 \, e^{-0.218z} \cos(6\pi\times10^3 t - 0.218z - \frac{\pi}{4}) \ A/m$$

(d) at $z_1 = 100m$.

$$E_0 e^{-\alpha z_1} = E_0 e^{-0.218\times100} \geq E_{min} = 1\mu V/m$$

$$\Rightarrow E_0 = 2.84 \ kV/m \leftarrow$$

$$\Rightarrow H_0 = \frac{E_0}{|\eta_c|} = 13E_0 = 36.9 \ kA/m. \leftarrow$$

Problem #2.

A survey conducted in US indicated that ~50% of the population is exposed to an averaged power densities of approximately 0.005 $\mu W/m^2$ due to VHF and UHF broadcast radiation. Find the corresponding amplitudes of the electric field and magnetic field. (Assume plane waves)

[Solution]: Set the $\vec{a}_z$ as the propagation direction

$$\Rightarrow \quad \vec{E} = E_0 e^{-j\beta z} \vec{a}_x$$

$$\vec{H} = \frac{E_0}{\eta} e^{-j\beta z} \vec{a}_y \quad \left(= \frac{1}{\eta} \vec{a}_z \times \vec{E}\right)$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \ (\Omega)$$

$$\Rightarrow \quad \vec{P}_{av} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*)$$

$$= \frac{1}{2} Re\left[E_0 e^{-j\beta z} \vec{a}_x \times \frac{E_0^*}{\eta^*} (e^{-j\beta z})^* \vec{a}_y \right]$$

$$= \frac{1}{2} \frac{|E_0|^2}{|\eta|} \vec{a}_z$$

$$\Rightarrow \quad \frac{1}{2} \frac{|E_0|^2}{\eta} = 0.005$$

$$\Rightarrow \quad E_0 = \sqrt{2 \times 0.005 \times 120\pi} = 194 \ mV/m$$

$$H_0 = \frac{E_0}{\eta} = \frac{E_0}{120\pi} = 515 \ \mu A/m$$

problem #3:

Consider a planar interface between air and muscle tissue. If a plane wave is normally incident onto the boundary, find the percentage of incident power absorbed by the muscle tissue at (a) 915 MHz and (b) 2.45 GHz. The muscle tissue has $\sigma = 0.889$ S/m, $\varepsilon_r = 71.7$.

[Solution]. $\mu_{r2} = 1$, $\mu_2 = \mu_r\mu_0 = \mu_0$, $\varepsilon_{r2} = 71.7$, $\sigma_2 = 0.889$

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ (\Omega)$

(a) at 915 MHz,

$\omega = 2\pi \times 915\,\text{MHz}$ $\Rightarrow \eta_{2c} = \sqrt{\dfrac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2} - j\frac{\sigma_2}{\omega}}} = 48.7e^{j15.8^\circ}\ (\Omega)$

$$\Rightarrow \Gamma = \frac{\eta_{2c} - \eta_1}{\eta_{2c} + \eta_1} = \frac{48.7e^{j15.8^\circ} - 120\pi}{48.7e^{j15.8^\circ} + 120\pi}$$

$$= 0.779e^{j176^\circ}$$

Incident power density: $(P_{av})_{inc.} = \frac{1}{2}\frac{|E_{i0}|^2}{\eta_0}$

Reflected power density: $(P_{av})_{ref} = \frac{1}{2}\frac{|E_{r0}|^2}{\eta_1}$

The power absorbed in %

$$= \frac{(P_{av})_{in} - (P_{av})_{ref}}{(P_{av})_{in}} = 1 - \frac{(P_{av})_{ref}}{(P_{av})_{inc}}$$

$$= 1 - \left|\frac{E_{r0}}{E_{i0}}\right|^2 = 1 - |\Gamma|^2 = 39.3\% \leftarrow$$

(b) at 2.45 GHz.

$\omega = 2\pi \times 2.45\times10^9$, $\Rightarrow \eta_{2c} = \sqrt{\dfrac{\mu_0}{\varepsilon_0\varepsilon_r - j\frac{\sigma_2}{\omega}}} = 53.5e^{j9.52^\circ}$

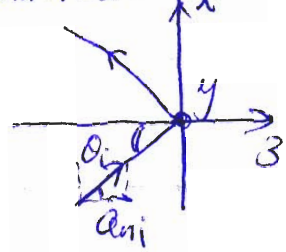
$\Rightarrow \Gamma = \dfrac{\eta_{2c} - \eta_0}{\eta_{2c} + \eta_0} = 0.755e^{j177^\circ}$

The power absorbed in % $= 1 - |\Gamma|^2 = 43\% \leftarrow$

Problem #4.

Consider a 100-MHz wave with amplitude of 6 V/m obliquely incident from air onto a slab of lossless, non-magnetic material with $\varepsilon_r = 9.00$. The angle is 60.0° and the wave is a perpendicular polarization. Find the incident, reflected and transmitted fields.

$\vec{a}_{ni} = \sin\theta_i \vec{a}_x + \cos\theta_i \vec{a}_z$



[Solution].

In air, $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ (\Omega)$

$\lambda_1 = \frac{c}{f} = \frac{3\times10^8}{100\times10^6} = 3\ m.$

$\beta_1 = k_1 = \frac{2\pi}{\lambda_1} = \frac{2}{3}\pi\ (rad/m)$

$\theta_i = 60° \Rightarrow \sin\theta_i = 0.866,\ \cos\theta_i = 0.5.$

$$\Rightarrow \vec{E}_i = 6e^{-jk\vec{a}_n\cdot\vec{r}}\vec{a}_y$$

$$= 6e^{-j\frac{2}{3}\pi(0.866\vec{a}_x+0.5\vec{a}_z)\cdot(x\vec{a}_x+y\vec{a}_y+z\vec{a}_z)}\vec{a}_y$$

$$= 6e^{-j(1.814x+1.047z)}\vec{a}_y \quad (V/m)$$

$$\vec{H}_i = \frac{1}{\eta_1}\vec{a}_n\times\vec{E}_i = \frac{6}{120\pi}e^{-j(1.814x+1.047z)}(-0.5\vec{a}_x+0.866\vec{a}_z) \quad (A/m)$$

In the slab. $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi\ (\Omega)$

$$\beta_2 = k_2 = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = 2\pi\ (rad/m)$$

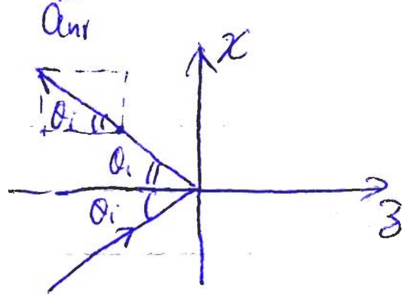
$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{\frac{\omega}{\beta_2}}{\frac{\omega}{\beta_1}} = \frac{\beta_1}{\beta_2} = \frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$$

$$\Rightarrow \theta_t = \sin^{-1}(\tfrac{1}{3}\sin\theta_i) = \sin^{-1}(\tfrac{1}{3}\sin 60°) = 16.8°$$

$$\Gamma_\perp = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = -0.613$$

$$\tau_\perp = \frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = 0.387$$

$$\vec{a}_{nr} = \sin\theta_i \vec{a}_x - \cos\theta_i \vec{a}_z$$



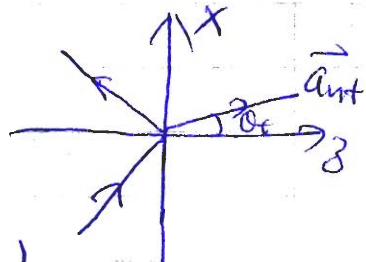
$$\vec{E}_r = \Gamma E_{io} e^{-j\beta_1 \vec{a}_{nr} \cdot \vec{r}}$$

$$= -3.68 e^{-j(1.814x - 1.047z)} \vec{a}_y$$

$$\vec{H}_r = \frac{1}{\eta_1} \vec{a}_{nr} \times \vec{E}_r$$

$$= -9.76 e^{-j(1.814x - 1.047z)} (-0.5\vec{a}_x + 0.866\vec{a}_z) \text{ (mA/m)}$$

$$\vec{a}_{nt} = \sin\theta_t \vec{a}_x + \cos\theta_t \vec{a}_z$$



$$\vec{E}_t = \tau E_{io} e^{-j\beta_2 \vec{a}_{nt} \cdot \vec{r}}$$

$$= 2.32 e^{-j(1.82x + 6.02z)} \vec{a}_y \text{ (V/m)}$$

$$\vec{H}_t = \frac{1}{\eta_2} \vec{a}_{nt} \times \vec{E}_t$$

$$= 18.5 e^{-j(1.82x + 6.02z)} (-0.96\vec{a}_x + 0.29\vec{a}_z) \text{ (mA/m)}$$

Problem #5 :

Consider a high-speed microstrip transmission line of 20 cm in length. It is used to connect a 1-V amplitude, 1-GHz, 50-Ω sinusoidal voltage source to a load of 1 kΩ. Based on measurements at 1GHz, the line has $R = 5\,\Omega/\text{cm}$, $L = 5\,\text{nH/cm}$, $C = 0.4\,\text{pF/cm}$ and $G = 0$.

(a) Find the propagation constant γ and characteristic impedance Z_0

(b) Find the voltage at the source and the load ends of the line

(c) Find the time-averaged power delivered to the line by the source and that delivered to the load. What is the power dissipated along the line?

[Solution]. $\omega = 2\pi f = 2\pi \times 10^9$ (rad/s)

(a) $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = 28.3 e^{j85.5°}$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = 112.5 e^{-j4.522°} (\Omega)$$

(b) the reflection coefficient at the load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1000 - 112.5 e^{-j4.522°}}{1000 + 112.5 e^{-j4.522}} = 0.798 e^{j1.03°}$$

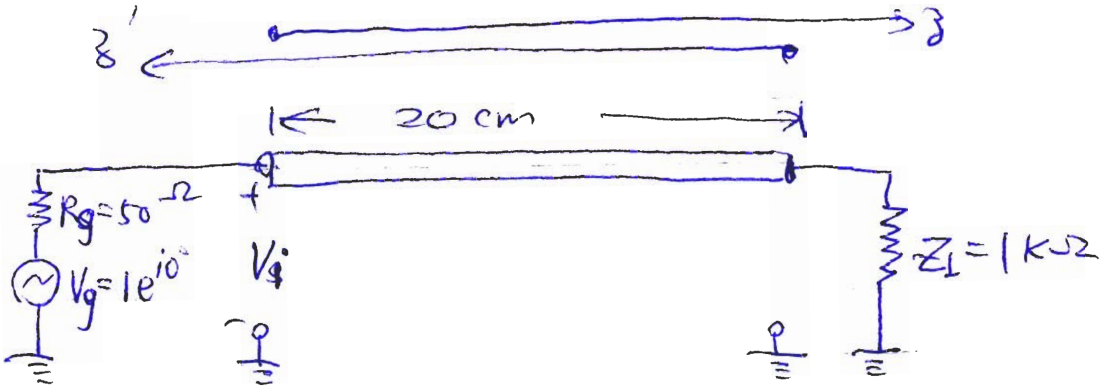
The reflection coefficient at any location z' away from Z_L is

$$\Gamma(z') = \Gamma_L e^{-2\gamma z'} = 0.798 e^{j1.03°} e^{-2 \times 28.3 e^{j85.5°} z'}$$

$$Z(z') = Z_0 \frac{1 + \Gamma(z')}{1 - \Gamma(z')}$$

$$Z_{in} = Z(z' = 0.2) = Z_0 \frac{1 + \Gamma(z' = 0.2)}{1 - \Gamma(z' = 0.2)}$$

$$= 131.2 e^{+j30.79°} (\Omega) = 112.75 + j\,67.174\ (\Omega)$$



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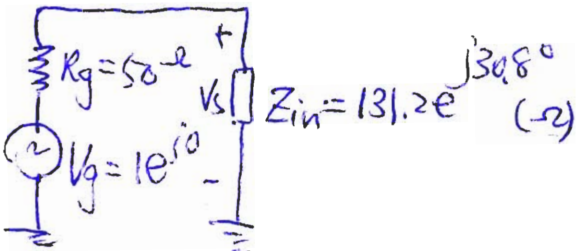


Based on voltage division,

$$V_i = \frac{Z_{in}}{Z_{in} + Z_s} V_g$$
$$= 0.745 e^{j8.361°} \text{ (V)}$$

$$V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$
$$= V^+ e^{-\gamma z} [1 + \Gamma(z)]$$

$$\Rightarrow \quad V^+ = \frac{V(z)}{e^{-\gamma z}[1 + \Gamma(z)]}$$

at $z' = 0.2$ m, or $z = 0$

$$V(z=0) = V(z'=0.2) = V_i$$

$$\Gamma(z=0) = \Gamma(z'=0.2)$$
$$= \Gamma_L e^{-2\gamma \times 0.2} = 0.3273 e^{j75.05}$$

$$\Rightarrow \quad V^+ = \frac{0.745 e^{j8.361°}}{(1 + 0.327 e^{j75.05°})} = 0.6598 e^{-j7.895°}$$

Voltage at the load

$$V_L = V(z=0.2) = V^+ e^{-\gamma z}(1 + \Gamma_L) = 0.760 e^{j29.57} \text{ (V)}$$

(c) time-averaged power delivered to the line by the source is given by

$$P_{in} = \frac{1}{2} \left| \frac{V_g}{Z_{in}} \right|^2 \cdot R_{in} = \frac{1}{2} \left(\frac{0.745}{131.2} \right)^2 \times 112.75 = 1.82 \text{ mW}$$

time average power delivered to the load

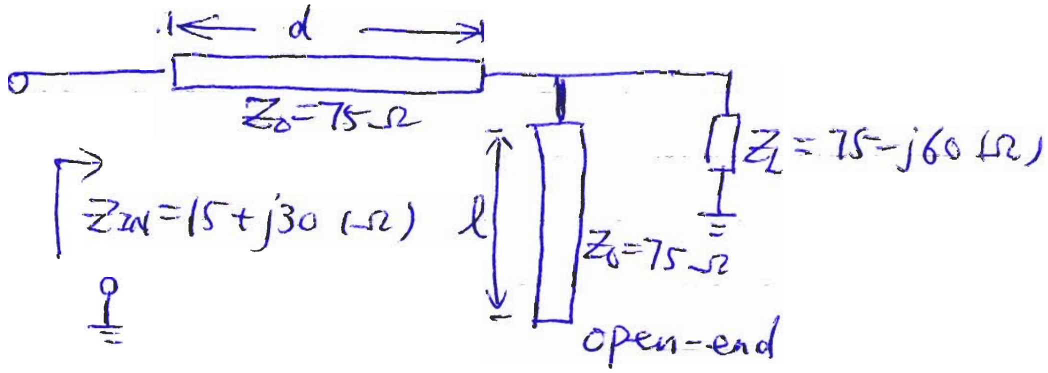
$$P_L = \frac{1}{2} \frac{|V_L|^2}{R_2} = \frac{1}{2} \frac{|0.760|^2}{1000} = 0.289\,mW \leftarrow$$

$\Rightarrow$ power dissipated in the line

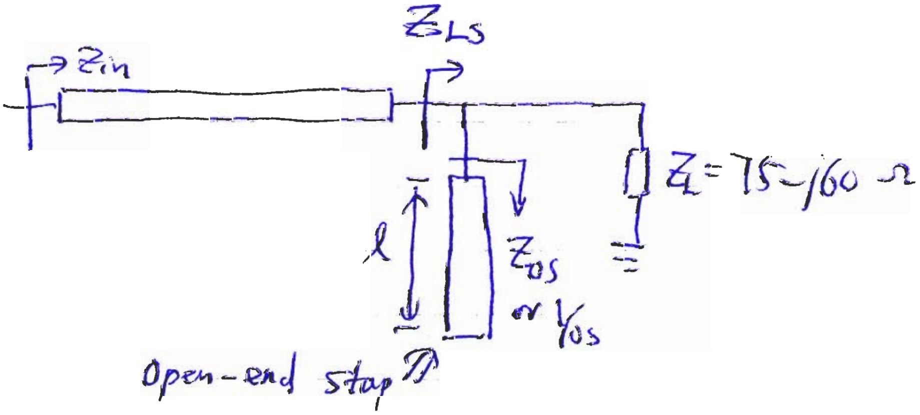
$$P_{lost} = P_{in} - P_L = 1.53\,mW \leftarrow$$

Problem #6

A microstrip transmission line matching network is shown below and is to transform the load $Z_L = 75 - j60\ (\Omega)$ to an input impedance of $Z_{in} = 15 + j30\ (\Omega)$. Find the lengths d and l. Assume that the line has $Z_0 = 75\ (\Omega)$.



[Solution]



For open-end stub, $$Z_{OS} = \left.\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right|_{Z_L \to \infty} = \frac{Z_0}{j\tan\beta l}$$

$$Z_{LS} = Z_{OS} \,//\, Z_L = \frac{Z_{OS} \cdot Z_L}{Z_{OS} + Z_L} = \frac{\frac{Z_0}{j\tan\beta l} Z_L}{\frac{Z_0}{j\tan\beta l} + Z_L}$$

$$= \frac{Z_0 Z_L}{Z_0 + Z_L j\tan\beta l}$$

$$Z_{in} = \frac{Z_{LS} + jZ_0 \tan(\beta d)}{Z_0 + jZ_{LS} \tan(\beta d)} = \frac{\frac{Z_0 Z_L}{Z_0 + jZ_L \tan\beta l} + jZ_0 \tan(\beta d)}{Z_0 + j\frac{Z_0 Z_L}{Z_0 + jZ_L \tan\beta l} \tan(\beta d)}$$

$$= \frac{Z_0 Z_L - Z_0 Z_L \tan(\beta l)\tan(\beta d) + jZ_0^2 \tan(\beta d)}{Z_0^2 + j(Z_0 Z_L \tan\beta l + Z_0 Z_L \tan\beta d)}$$

$$= 15 + j30 \qquad (*)$$

$\Rightarrow$ Solve for ℓ and d with $\beta = \frac{2\pi}{\lambda}$

$$\Rightarrow \begin{cases} \dfrac{d}{\lambda} = 0{,}149 \\ \dfrac{\ell}{\lambda} = 0{,}126 \end{cases}$$

Note: solutions are obtained by letting real part of LHS = real part of RHS
imaginary part of LHS = imaginary part of RHS

in equation (*)