ECED 4301- ELECTROMAGNETIC WAVES AND PROPOGATION SOLUTION ASSIGNMENT #4

Prob. 10.4 (a) Let $u = \frac{\sigma}{\omega \varepsilon}$ = loss tangent

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+u^2}+1\right]}$$

$$10 = \frac{\omega}{c}\sqrt{\frac{5\times 2}{2}\left[\sqrt{1+u^2}+1\right]} = \frac{2\pi\times 5\times 10^6\sqrt{5}}{3\times 10^8}\sqrt{\left[\sqrt{1+u^2}+1\right]}$$

which leads to

$$u = \frac{\sigma}{\omega\varepsilon} = \underline{\underline{1823}}$$

(b) $\sigma = \omega\varepsilon u = 2\pi\times 5\times 10^6 \times 2\times 1823\times 2\times\frac{10^{-9}}{36\pi} = \underline{\underline{1.013}}$ S/m

(c) $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\frac{\sigma}{\omega} = 2\times\frac{10^{-9}}{36\pi} - j\frac{1.023}{2\pi\times 5\times 10^6} = \underline{\underline{1.768\times 10^{-11} - j3.224\times 10^{-8}}}$ F/m

d) $\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+u^2}-1}}{\sqrt{\sqrt{1+u^2}+1}} = \sqrt{\frac{1822}{1824}}$

$\beta = 10$

$\frac{\alpha}{\beta} = \underline{\underline{0.9995}}$ Np/m ; $\alpha = \underline{\underline{9.99}}$ Np/m

(e) $$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+u^2}} = \frac{120\pi\sqrt{\frac{5}{2}}}{\sqrt[4]{1+1823^2}} = 13.96$$

$$\tan 2\theta_\eta = u = 1823 \longrightarrow \theta_\eta = 44.98^\circ$$

$$\eta = \underline{\underline{13.96\angle 44.98^\circ\,\Omega}}$$

Prob. 10.6 (a) $\dfrac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60^o = \underline{\underline{1.732}}$

(b) $|\eta| = 240 = \dfrac{\dfrac{120\pi}{\sqrt{\varepsilon_r}}}{\sqrt[4]{1+3}} = \dfrac{120\pi}{\sqrt{2\varepsilon_r}} \longrightarrow \varepsilon_r = \dfrac{\pi^2}{8} = \underline{\underline{1.234}}$

(c) $\varepsilon_c = \varepsilon(1 - j\dfrac{\sigma}{\omega\varepsilon}) = 1.234\times\dfrac{10^{-9}}{36\pi}(1 - j1.732) = \underline{\underline{(1.091 - j1.89)\times 10^{-11}}}$ F/m

(d)

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi\times 10^6}{3\times 10^8}\sqrt{\frac{1}{2}\frac{\pi^2}{8}\left[\sqrt{1+3}-1\right]} = \underline{\underline{0.0164}} \text{ Np/m}$$

Prob. 10.10 (a) $\gamma = \alpha + j\beta = \underline{\underline{0.05 + j2}}$ /m

(b) $\lambda = 2\pi/\beta = \pi = \underline{\underline{3.142}}$ m

(c) $u = \omega/\beta = \dfrac{2\times 10^8}{2} = \underline{\underline{10^8}}$ m/s

(d) $\delta = 1/\alpha = \dfrac{1}{0.05} = \underline{\underline{20}}$ m

Prob. 10.19 $\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi\times10^7}{3\times10^8}(10) = \underline{\underline{2.0943}}$ rad/m

$$H = -\frac{1}{\mu}\int \nabla\times E dt$$

$$\nabla\times E = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x) & E_z(x) \end{vmatrix} = -\frac{\partial E_z}{\partial x}a_y + \frac{\partial E_y}{\partial x}a_z = -10\beta\sin(\omega t - \beta x)(a_y - a_z)$$

$$H = -\frac{10\beta}{\omega\mu}\cos(\omega t - \beta x)(a_y - a_z) = -\frac{10\times2\pi/3}{2\pi\times10^7\times50\times4\pi\times10^{-7}}\cos(\omega t - \beta x)(a_y - a_z)$$

$$H = \underline{\underline{5.305\cos(2\pi\times10^7 t - 2.0943x)(-a_y + a_z)}} \quad \text{mA/m}$$

It is a Linearly Polarized Wave.

Prob. 10.21

(a) $E = \text{Re}[E_s e^{j\omega t}] = (5a_x + 12a_y)e^{-0.2z}\cos(\omega t - 3.4z)$

At $z = 4$m, $t = T/8$, $\omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$$E = (5a_x + 12a_y)e^{-0.8}\cos(\pi/4 - 13.6)$$

$$|E| = 13e^{-0.8}|\cos(\pi/4 - 13.6)| = \underline{\underline{5.662}} \text{ V/m}$$

(b) loss $= \alpha\Delta z = 0.2(3) = 0.6$ Np. Since 1 Np = 8.686 dB,

loss = 0.6 x 8.686 = $\underline{5.212 \text{ dB}}$

(c) Let $x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2/3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega\sqrt{\mu\varepsilon/2}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r/2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2x3x10^8}{10^8\sqrt{0.00694}} = 7.2 \quad \longrightarrow \quad \varepsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o}}.\frac{1}{\sqrt{\varepsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68\times1.00694}} = 36.896$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2-1} = 0.118 \quad \longrightarrow \quad \theta_\eta = 3.365^o$$

$$\eta = 36.896\angle 3.365^o\ \Omega$$

$$\boldsymbol{H}_s = \boldsymbol{a}_k \times \frac{\boldsymbol{E}_s}{\eta} = \frac{\boldsymbol{a}_z}{\eta}\times(5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-\gamma z} = \frac{(5\boldsymbol{a}_y - 12\boldsymbol{a}_x)}{|\eta|}e^{-j3.365^o}e^{-\gamma z}$$

$$H = (-325.24\boldsymbol{a}_x + 135.5\boldsymbol{a}_y)e^{-0.2z}\cos(\omega t - 3.4z - 3.365^o)\ \text{mA}$$

$$\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 5 & 12 & 0 \\ -325.24 & 135.5 & 0 \end{vmatrix} \times 10^{-3}e^{-0.4z}\cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)$$

$$\boldsymbol{P} = 4.58e^{-0.4z}\cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)\boldsymbol{a}_z$$

At z=4, t=T/4,

$$\mathbf{P} = 4.58e^{-1.6}\cos(\pi/4 - 13.6)\cos(\pi/4 - 13.6 - 0.0587)\mathbf{a}_z = \underline{\underline{0.8545\mathbf{a}_z \text{ W/m}^2}}$$

Prob. 10.26

$\omega = 10^6\pi = 2\pi f \longrightarrow f = 0.5\times10^6$

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\sqrt{\pi\times0.5\times10^6\times3.5\times10^7\times4\pi\times10^{-7}}} = \underline{\underline{0.1203}} \text{ mm}$$

$$R_{ac} = \frac{l}{\sigma\delta w}$$

since δ is very small, $w = 2\pi\rho_{outer}$

$$R_{ac} = \frac{l}{\sigma 2\pi\rho_{outer}\delta} = \frac{40}{3.5\times10^7\times2\pi\times12\times10^{-6}\times0.1203} = \underline{\underline{0.126\Omega}}$$

Question **A radar sends a signal of 2.4GHz towards an incoming missile at an elevation of 30 degrees. The echo signal received by the radar is found to have a frequency shift of 53.6 KHz and the time delay of 0.001 second. Find (a) the speed of the missile and (b) the height of the missile (above the ground).

Ans: - (a) F= 2.4 GHz

$$F' = \frac{F}{1 - \frac{u}{c}\cos\theta}$$

$$F'' = \frac{F'}{1 - \frac{u}{c}\cos\theta}$$

$$F'' = \frac{F'}{\left(1 - \frac{u}{c}\cos\theta\right)^2} \quad \text{----------} \ (1)$$

$u\ cos\theta > 0$ as the missile moves towards the radar

F" = F + 53.6 KHz , $\theta = 30°$

C= 3 x 10^8 m/s

Now, solve for u from equation (1) by putting the above values

$$\left(1 - \frac{u}{c}\cos\theta\right)^2 = \frac{F}{F''}$$

Solving the above equation for u, the value u = 3868.18 m/s as u<<c

Where c = speed of light

So, u= 3868.18 m/s

(b) In time delay Δt = 0.001 ms, the signal travelled 2d with speed c

$$c = \frac{2d}{\Delta t}$$

Hence, $d = \frac{c\Delta t}{2} = \frac{(3 \times 10^8) \times (0.001)}{2} = 150000 \text{ m} = 150 \text{ km}$

As, $\sin\theta = \frac{h}{d}$ so, h = dsinθ = $150000\ sin\ 30°$ = 75000 m = 75 km

h = 75 km