

**Prob. 11.27**

$$z_L = \frac{Z_L}{Z_o} = \frac{210}{100} = 2.1 = s$$

$$\text{Or } \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{110}{310},$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.1$$

$$\text{But } s = \frac{V_{\max}}{V_{\min}} \rightarrow V_{\max} = s V_{\min}$$

Since the line is  $\frac{\lambda}{4}$  long,  $\frac{\lambda}{4} \rightarrow \frac{720^\circ}{4} = 120^\circ$

Hence the sending end will be  $V_{\min}$ , while the receiving end at  $V_{\max}$

$$V_{\min} = V_{\max} / s = 80 / 2.1 = 38.09$$

$$V_{\text{sending}} = \underline{\underline{38.09 < 90^\circ}}$$

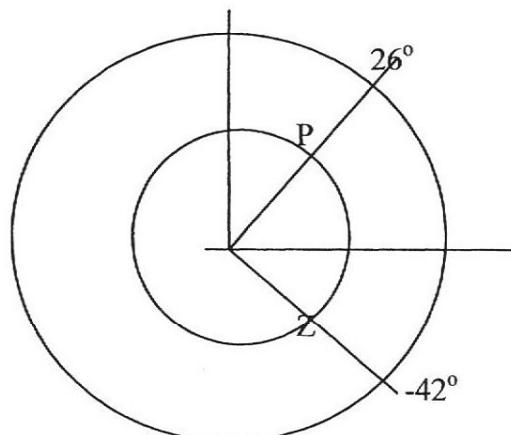
**Prob. 11.28**

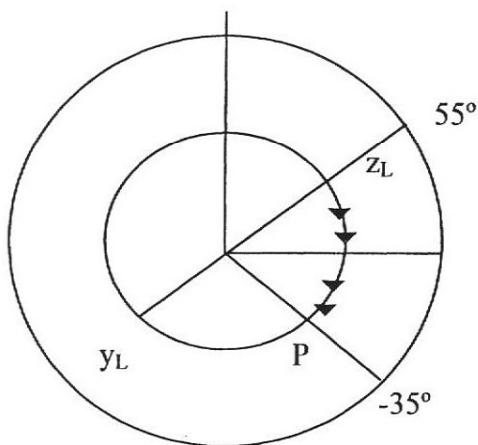
$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{50} = 2 + j3$$

$$z_{in} = \frac{Z_{in}}{Z_o} = \frac{50 + j110}{50} = 1 + j2.2$$

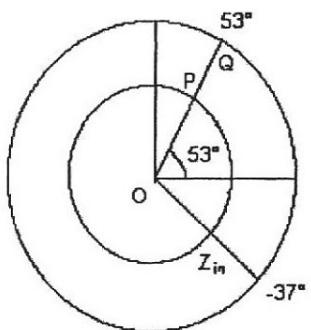
$$\theta = 26^\circ - -42^\circ = 68^\circ$$

$$\text{If } 720 \rightarrow \lambda, \quad 68^\circ \rightarrow d = \frac{\lambda}{720^\circ} 68^\circ = \underline{\underline{0.0944\lambda}}$$





At P,  $z_{in} = \underline{\underline{1.4 - j2.6}}$   
 (Exact value = 1.379-j2.448 )  
 (c)



$$\Gamma = 0.3 + j0.4 = 0.5 < 53.13^\circ$$

$$\frac{OP}{OQ} = 0.5, \quad z_{in} = \underline{\underline{1.667 - j1.333}}$$

#### Prob. 11.34

$$(a) \quad z_L = \frac{Z_L}{Z_o} = \frac{75 + j60}{50} = 1.5 + j1.2$$

$$|\Gamma| = \frac{OP}{OQ} = \frac{3.8\text{cm}}{8\text{cm}} = 0.475, \quad \theta_\Gamma = 42^\circ$$

$$\Gamma = \underline{0.475 \angle 42^\circ}$$

(Exact value =  $0.4688 \angle 41.76^\circ$ )

(b)  $s=2.8$

(Exact value = 2.765)

$$(c) \quad 0.2\lambda \rightarrow 0.2 \times 720^\circ = 144^\circ$$

$$z_{in} = 0.55 - j0.65$$

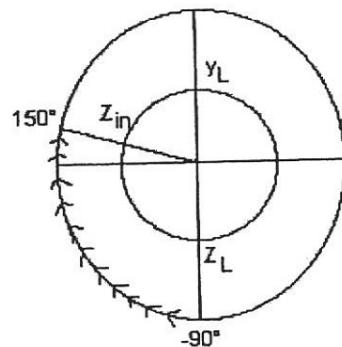
$$Z_{in} = Z_o z_{in} = 50(0.55 + j0.65) = \underline{\underline{27.5 + j32.5 \Omega}}$$

(d) Since  $\theta_\Gamma = 42^\circ$ ,  $V_{min}$  occurs at

$$\frac{42}{720} \lambda = \underline{\underline{0.05833\lambda}}$$

(e) same as in (d), i.e..  $\underline{\underline{0.05833\lambda}}$

### Prob. 11.35



If  $\lambda \rightarrow 720^\circ$ , then  $\frac{\lambda}{6} \rightarrow 120^\circ$

$$z_{in} = \underline{\underline{0.35 + j0.24}}$$

**Prob. 11.36**

$$(a) \quad Z_{in} = \frac{Z_o}{Z_o} = \frac{100 - j120}{80} = 1.25 - j1.5$$

$$\lambda = \frac{u}{f} = \frac{0.8 \times 3 \times 10^8}{12 \times 10^6} = 20 \text{ m}$$

$$l_1 = 22 \text{ m} = \frac{22\lambda}{20} = 1.1\lambda \rightarrow 720^\circ + 72^\circ$$

$$l_2 = 28 \text{ m} = \frac{28\lambda}{20} = 1.4\lambda \rightarrow 720^\circ + 72^\circ + 216^\circ$$

To locate P(the load), we move 2 revolutions plus  $72^\circ$  toward the load. At P,

$$|\Gamma_L| = \frac{OP}{OQ} = \frac{5.1 \text{ cm}}{9.2 \text{ cm}} = 0.5543$$

$$\theta_R = 72^\circ - 47^\circ = 25^\circ$$

$$\Gamma_L = \underline{0.5543 \angle 25^\circ}$$

(Exact value =  $0.5624 \angle 25.15^\circ$ )

$$Z_{in,\max} = sZ_o = 3.7(80) = \underline{296 \Omega}$$

(Exact value =  $285.59 \Omega$ )

$$Z_{in,\min} = \frac{Z_o}{s} = \frac{80}{3.7} = \underline{21.622 \Omega}$$

(Exact value =  $22.41 \Omega$ )

$$(b) \text{ Also, at P, } Z_L = 2.3 + j1.55$$

$$Z_L = 80(2.3 + j1.55) = \underline{184 + j124 \Omega}$$

(Exact value =  $183.45 + j128.25 \Omega$ )

$$\text{At S, } s = \underline{\underline{3.7}}$$

To Locate  $Z'_{in}$ , we move  $216^\circ$  from  $Z_{in}$  toward the generator

At  $Z'_{in}$ ,

$$z'_{in} = 0.48 + j0.76$$

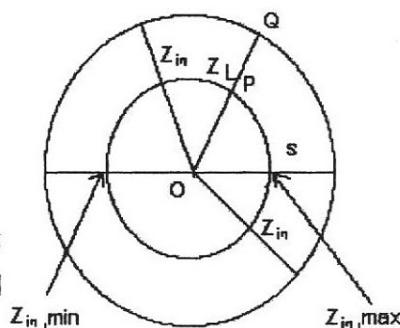
$$Z'_{in} = 80(0.48 + j0.76) = \underline{38.4 + j60.8 \Omega}$$

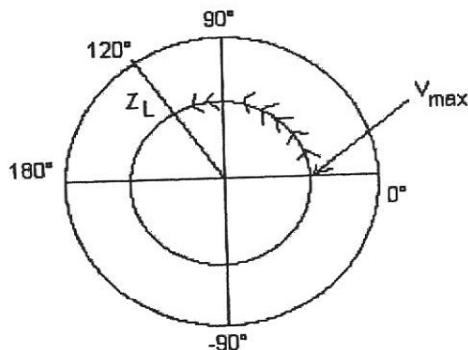
(Exact =  $37.56 + j61.304 \Omega$ )

(c) Between  $Z_L$  and  $Z'_{in}$ , we move 2 revolutions a the movement, we pass through  $Z_{in,\max}$  3 times and

Thus there are:

$$\underline{\underline{3 Z_{in,\max} \text{ and } 2 Z_{in,\min}}}$$



**Prob. 11.37**

$$(a) \quad \frac{\lambda}{2} = 120 \text{ cm} \rightarrow \lambda = 2.4 \text{ m}$$

$$u = f\lambda \rightarrow f = \frac{u}{\lambda} = \frac{3 \times 10^8}{2.4} = \underline{\underline{125 \text{ MHz}}}$$

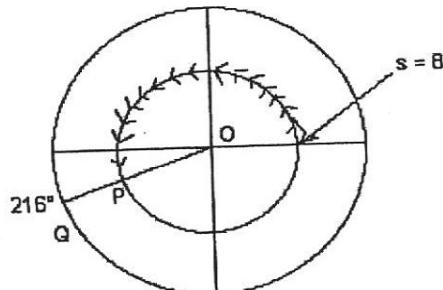
$$(b) \quad 40 \text{ cm} = \frac{40\lambda}{240} = \frac{\lambda}{6} \rightarrow \frac{720^\circ}{6} = 120^\circ$$

$$\begin{aligned} Z_L &= Z_o z_L = 150(0.48 + j0.48) \\ &= \underline{\underline{72 + j72 \Omega}} \end{aligned}$$

(Exact value =  $73.308 + j70.324 \Omega$ )

$$(c) \quad |\Gamma| = \frac{s-1}{s+1} = \frac{1.6}{3.9} = 0.444,$$

$$\Gamma = \underline{\underline{0.444 \angle 120^\circ}}$$

**Prob. 11.38**

$$0.3\lambda \rightarrow 720^\circ \times 0.3 = 216^\circ$$

$$\text{At } P, z_L = 0.15 - j0.32$$

$$Z_L = Z_o z_L = \underline{\underline{15 - j32\Omega}}$$

(Exact value =  $13.7969 - j31.9316 \Omega$ )

$$|\Gamma| = \frac{OP}{OQ} = \frac{7.2 \text{ cm}}{9.3 \text{ cm}} = 0.7742$$

$$\Gamma = \underline{\underline{0.7742 \angle 216^\circ}}$$

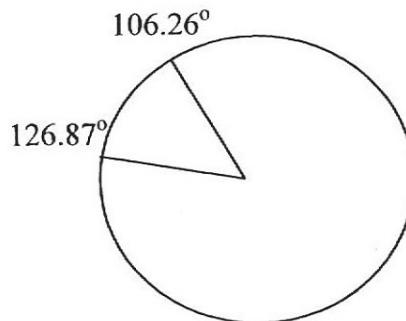
(Exact value =  $0.7778 \angle 216^\circ$ )

### Prob. 11.39

(a)

$$z_L = \frac{Z_L}{Z_o} = \frac{j60}{80} = j0.75, \quad z_{in} = \frac{Z_{in}}{Z_o} = \frac{j40}{80} = j0.5$$

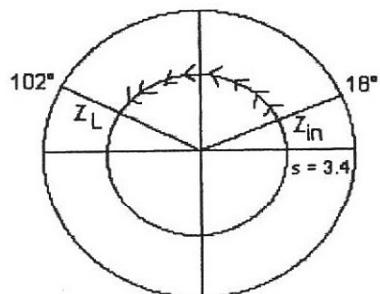
The two loads fall on the  $r=0$  circle, the outermost resistance circle. The shortest distance between them is



$$\frac{360^\circ - (126.87^\circ - 106.26^\circ)\lambda}{720^\circ} = \underline{\underline{0.4714\lambda}}$$

$$(b) \underline{\underline{s = \infty}}, \quad \Gamma_L = \underline{\underline{1 \angle 106.26^\circ}}$$

### Prob. 11.40



$$l = 0.2\lambda \rightarrow 720^\circ \times 0.2 = 144^\circ$$

$$Z_{in} = \frac{V_s}{I_s} = \frac{2+j}{10 \times 10^{-3}} = 200 + j100$$

$$Z_{in} = \frac{Z_{in}}{Z_L} = 2.667 + j1.33$$

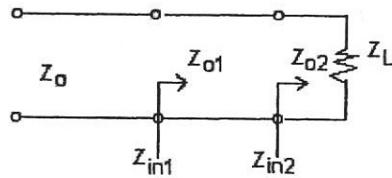
$$Z_L = 0.3 + j0.12$$

$$Z_L = 75(0.3 + j0.12) = \underline{\underline{22.5 + j9\Omega}}$$

(Exact value =  $22.4233 + j10.4251 \Omega$ )

$$s = \underline{\underline{3.4}}$$

### Prob. 11.41



$$(a) \text{ From Eq. (11.43), } Z_{in2} = \frac{Z_{o2}^2}{Z_L}$$

$$Z_{in1} = \frac{Z_{o1}^2}{Z_{in2}} = Z_o, \text{ i.e. } Z_{in2} = \frac{Z_{o1}^2}{Z_o} = \frac{Z_{o2}^2}{Z_L}$$

$$Z_{o1} = Z_{o2} \sqrt{\frac{Z_o}{Z_L}} = 30 \sqrt{\frac{50}{75}} = \underline{\underline{24.5\Omega}}$$

$$(b) \text{ Also, } \frac{Z_o}{Z_{o1}} = \left( \frac{Z_{o2}}{Z_L} \right) \rightarrow Z_{o2} = \frac{Z_o Z_L}{Z_{o1}} \quad (1)$$

$$\text{Also, } \frac{Z_{o1}}{Z_{o2}} = \left( \frac{Z_{o2}}{Z_L} \right)^2 \rightarrow (Z_{o2})^3 = Z_{o1} Z_L^2 \quad (2)$$

$$\text{From (1) and (2), } (Z_{o2})^3 = Z_{o1} Z_L^2 = \frac{Z_o^3 Z_L^3}{Z_{o1}^3} \quad (3)$$

$$\text{or } Z_{o1} = \sqrt[4]{Z_o^3 Z_L} = \sqrt[4]{(50)^3 (75)} = \underline{\underline{55.33\Omega}}$$

$$\text{From (3), } Z_{o2} = \sqrt[3]{Z_{o1} Z_L^2} = \sqrt[3]{(55.33)(75)^2} = \underline{\underline{67.74\Omega}}$$

### Prob. 11.42

$$\frac{\lambda}{4} \rightarrow 180^\circ, \quad z_L = \frac{74}{50} = 1.48, \quad \frac{1}{z_L} = 0.6756$$

This acts as the load to the left line. But there are two such loads in parallel due to

the two lines on the right. Thus

$$Z_L' = 50 \frac{\left(\frac{1}{Z_L}\right)}{2} = 25(0.6756) = 16.892$$

$$z_L' = \frac{16.892}{50} = 0.3378, \quad z_{in} = \frac{1}{z_L'} = 2.96$$

$$Z_{in} = 50(2.96) = \underline{\underline{148\Omega}}$$

### Prob. 11.43

From the previous problem,  $Z_{in} = 148\Omega$

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{120}{80 + 148} = 0.5263A$$

$$P_{ave} = \frac{1}{2} |I_{in}|^2 R_{in} = \frac{1}{2} (0.5263)^2 (148) = 20.5W$$

Since the lines are lossless, the average power delivered to either antenna is 10.25W

### Prob. 11.44

$$(a) \quad \beta l = \frac{2\pi}{4} \cdot \frac{\lambda}{4} = \frac{\pi}{2}, \quad \tan \beta l = \infty$$

$$Z_{in} = Z_o \left( \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right) = Z_o \frac{\left( \frac{Z_L}{\tan \beta l} + jZ_o \right)}{\left( \frac{Z_o}{\tan \beta l} + jZ_L \right)}$$

As  $\tan \beta l \rightarrow \infty$ ,

$$Z_{in} = \frac{Z_o^2}{Z_L} = \frac{(50)^2}{200} = \underline{\underline{12.5\Omega}}$$

(b) If  $Z_L = 0$ ,

$$Z_{in} = \frac{Z_o^2}{0} = \underline{\underline{\infty}} \quad (\text{open})$$

$$(c) \quad Z_L = 25 // \infty = \frac{25 \times \infty}{25 + \infty} = \frac{25}{1 + \frac{25}{\infty}} = 25\Omega$$

$$Z_{in} = \frac{(50)^2}{12.5} = \underline{\underline{200\Omega}}$$

**Prob. 11.45**

$$l_1 = \frac{\lambda}{4} \rightarrow Z_{in1} = \frac{Z_o^2}{Z_L} \quad or \quad y_{in1} = \frac{Z_L}{Z_o^2}$$

$$y_{in1} = \frac{200 + j150}{(100)^2} = \underline{\underline{20 + j15 \text{ mS}}}$$

$$l_2 = \frac{\lambda}{8} \rightarrow Z_{in2} = Z_o \lim_{Z_L \rightarrow 0} \left( \frac{Z_L + jZ_o \tan \frac{\pi}{4}}{Z_o + jZ_L \tan \frac{\pi}{4}} \right) = jZ_o$$

$$y_{in2} = \frac{1}{jZ_o} = \frac{1}{j100} = \underline{\underline{-j10 \text{ mS}}}$$

$$l_3 = \frac{7\lambda}{8} \rightarrow Z_{in3} = Z_o \frac{\left( Z_i + jZ_o \tan \frac{7\pi}{4} \right)}{\left( Z_o + jZ_i \tan \frac{7\pi}{4} \right)} = \frac{Z_o(Z_i - jZ_o)}{(Z_o - jZ_i)}$$

But

$$y_i = y_{in1} + y_{in2} = 20 + j5 \text{ mS}$$

$$z_i = \frac{1}{y_i} = \frac{1000}{20 + j5} = 47.06 - j11.76$$

$$y_{in3} = \frac{Z_o - jZ_{in}}{Z_o(Z_{in} - jZ_o)} = \frac{100 - j47.06 - 11.76}{100(47.06 - j11.76 - j100)} \\ = \underline{\underline{6.408 + j5.189 \text{ mS}}}$$

If the shorted section were open,

$$y_{in1} = \underline{\underline{20 + j15 \text{ mS}}}$$

$$y_{in2} = \frac{1}{Z_{in2}} = \frac{j \tan \pi/4}{Z_o} = \frac{j}{100} = \underline{\underline{j10 \text{ mS}}}$$

$$l_3 = \frac{7\lambda}{8} \rightarrow Z_{in3} = Z_o \frac{\left( Z_i + jZ_o \tan \frac{7\pi}{4} \right)}{\left( Z_o + jZ_i \tan \frac{7\pi}{4} \right)} = \frac{Z_o(Z_i - jZ_o)}{(Z_o - jZ_i)}$$

$$y_i = y_{in1} + y_{in2} = 20 + j15 + j10 = 20 + j25 \text{ mS}$$

$$Z_i = \frac{1}{y_i} = \frac{1000}{20 + j25} = 19.51 - j24.39 \Omega$$

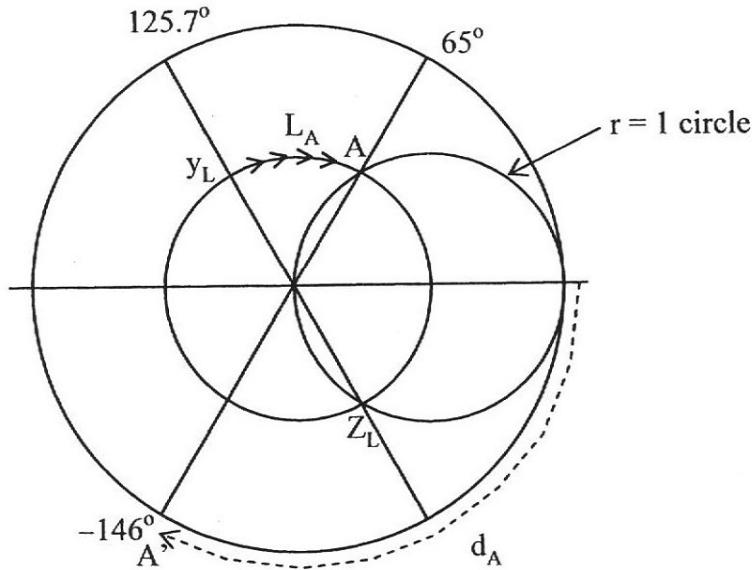
$$y_{in3} = \frac{Z_o - jZ_i}{Z_o(Z_i - jZ_o)} = \frac{75.61 - j19.51}{100(19.51 - j124.39)}$$

$$= \underline{\underline{2.461 + j5.691 \text{ mS}}}$$

**Prob. 11.46**

$$z_L = \frac{Z_L}{Z_o} = \frac{60 - j50}{50} = 1.2 - j1$$

$$y_L = \frac{1}{z_L}$$



At A,  $y = 1 + j0.92$ ,  $y_s = -j0.92$

$$Y_s = Y_o y_s = \frac{-j0.92}{50} = \underline{\underline{-j18.4 \text{ mS}}}$$

Stub length = 0.1307λ

Stub position = 0.0843λ