Tutorial #3

Problem 1.

The $z=0$ plane separate two media. Medium 1 lies in $z>0$ with $\varepsilon_r = \mu_r = 1$ while medium 2 lies in $z<0$ with $\varepsilon_r = 81$ and $\mu_r = 3$. ($\vec{J} = 0$ at the interface)

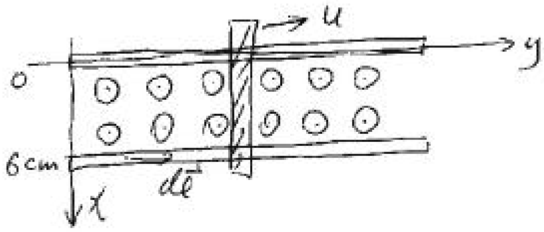
(1) Suppose $\vec{E}_1 = 8\cos(\omega t - 4x - 3z)\,\vec{a}_y$ (V/m)
find $\vec{E}_2$ at $z=0$

(2) Suppose $\vec{H}_1 = 4\cos(\omega t - 4x - 3z)\,\vec{a}_x + 5\cos(\omega t - 4x - 3z)\,\vec{a}_y + 6\cos(\omega t - 4x - 3z)\,\vec{a}_z$ (mA/m)

find $\vec{H}_2$ at $z=0$

Problem 2

A conducting bar can slide freely over two conducting rails as shown in the figure. Calculate the voltage in the bar.



$\vec{u} = 20\,\vec{a}_y$ m/s

$\vec{B} = 4\cos(10^6 t - y)\,\vec{a}_z$ (mWb/m²)

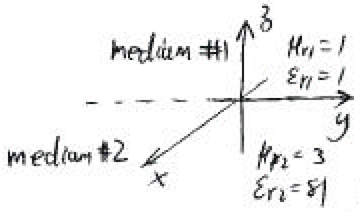
Problem 3

In a medium by $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$, and

$$\vec{E} = 20\sin(10^8 t - \beta z)\,\vec{a}_y \text{ V/m}$$

Find β and $\vec{H}$.

P1. [Solution]



(1) $\vec{E}_1 = \vec{E}_{1t} = 8\cos(\omega t - 4x)$ (V/m) at $z=0$

$\Rightarrow \vec{E}_2 = \vec{E}_{2t} = \vec{E}_{1t} = 8\cos(\omega t - 4x)$ (V/m)

(2) $\vec{J} = 0$, $\vec{H}_{2t} = \vec{H}_{1t} = 4\cos(\omega t - 4x)\vec{a}_x + 5\cos(\omega t - 4x)\vec{a}_y$ at $z=0$

$$\vec{B}_{1n} = \vec{B}_{2n} \Rightarrow \mu_{r1}\vec{H}_{1n} = \mu_{r2}\vec{H}_{2n}$$

$$\Rightarrow \vec{H}_{2n} = \frac{\mu_{r1}}{\mu_{r2}}\vec{H}_{1n} = \frac{1}{3}\cdot 6\cos(\omega t - 4x)\vec{a}_z = 2\cos(\omega t - 4x)\vec{a}_z \quad \text{at } z=0$$

$$\Rightarrow \vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n} = (4\vec{a}_x + 5\vec{a}_y + 2\vec{a}_z)\cos(\omega t - 4x) \text{ mA/m} \quad \text{at } z=0$$

P2 [Solution]

$$\Phi = \int \vec{B}\cdot d\vec{S} = \int \vec{B}\cdot\vec{a}_y\,dx\,dy$$

$$= \int_{y=0}^{y=ut}\int_{x=0}^{x=0.06} 4\cos(10^6 t - y)\,\vec{a}_y\cdot\vec{a}_y\,dx\,dy$$

$$= -0.24\sin(10^6 t - 20t) + 0.24\sin(10^6 t) \text{ mWb/m}$$

$$V_{emf} = -\frac{d\Phi}{dt} = 0.24(10^6 - 20)\cos(10^6 t - 20t) - 0.24\times 10^6\cos 10^6 t$$

$$\approx 240[\cos(10^6 t - 20t) - \cos(10^6 t)] \text{ V}$$

P3 [Solution]

$$-\mu\frac{\partial\vec{H}}{\partial t} = \nabla\times\vec{E} \Rightarrow \vec{H} = -\frac{1}{\mu}\int(\nabla\times\vec{E})dt = -\frac{20\beta}{\mu 10^8}\sin(10^8 t - \beta z)\vec{a}_x$$

$$\varepsilon\frac{\partial\vec{E}}{\partial t} = \nabla\times\vec{H} \Rightarrow \vec{E} = \frac{1}{\varepsilon}\int(\nabla\times\vec{H})dt = \frac{20\beta^2}{\mu\varepsilon\times 10^{16}}\sin(10^8 t - \beta z)\vec{a}_y$$

$$= 20\sin(10^6 t - \beta z)$$

$$\Rightarrow \frac{20\beta^2}{\mu\varepsilon 10^{16}} = 20 \Rightarrow \beta = \pm 10^8\sqrt{\mu\varepsilon} = \pm\frac{2}{3}, \Rightarrow \vec{H} = \pm\frac{1}{3\pi}\sin(10^8 t \pm \frac{2}{3}z)\vec{a}_x \text{ (A/m)}$$