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**Title: MEASURING SOLAR DIFFERENTIAL ROTATION WITH AN ITERATIVE PHASE CORRELATION METHOD**

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**Stats Reviewer's Comments:**

*1. There is a vast literature on the image registration problem in the computer vision literature, some of it involving phase registration. Can the authors describe how their method relates to published work such as:*

*- Kuglin, C.D., Hines, D.C.: The phase correlation image alignment method. In: Proceeding of IEEE International Conference on Cybernetics and Society, 163-165 (1975)*

*- Foroosh, H., Zerubia, J.B., Marc, B.: Extension of phase correlation to subpixel registration. IEEE Trans. Image Process. 11(3), 188-200 (2002)*

*- Takita, K., et al.: High-accuracy subpixel image registration based on phase-only correlation. IEICE Trans. Fundam. E86A(8), 1925-1934 (2003)*

*- Foroosh, H., Balci, M.: Subpixel registration directly from the phase difference. EURASIP J. Appl. Signal Process. 2006, 1-11 (2006)*

*- Y. Douini et al., An image registration algorithm based on phase correlation and the classical Lucas-Kanade technique, Signal, Image and Video Processing volume 11, 1321-1328 (2017)*

The Iterative Phase Correlation method (IPC) is one of the subpixel extensions of the classical phase correlation method (Kuglin 1975). The method is mainly based on the upscaling of the phase correlation landscape, adaptable band-pass filtering of the cross-power spectrum and it’s iterative character, which iteratively refines the resulting image shift by computing weighted centroids on a small region of interest (ROI) around the phase correlation peak in each iteration, followed by shifting the ROI towards the current centroid, converging to a high precision subpixel solution. Foroosh (2002) used another approach to extend the method to subpixel accuracy, which is analytical, with no iterative refinement. Takita (2003) used least squares 2D regression to fit the neighborhood around the maximum peak and then based the resulting image shift on the fit. Foroosh (2006) inferred the image shift directly from the frequency domain, counting the number of phase-difference sawtooth cycles, while calculating the subpixel part of the image shift from the fraction of the last (incomplete) cycle. Douini (2017) was mainly concerned about the computational complexity of the algorithm while registering images that are not only shifted, but also rotated and scaled, and thus used a combination of different algorithms for each task. The Lucas-Kanade algorithm mentioned in the title was used only for rotation/scale estimation, while the subpixel shift estimation (which is the only important part in the differential rotation measurement) was done again by 2D least squares fitting and filtering.

Since the authors also think the context of other image registration algorithms is important, a shortened version of the response was added to a new chapter called “Discussion”.

*2. Figure 1d shows a difference between the mean (centroid) and mode of the statistic used for image registration. The authors choose to use the mean: would the science result differ significantly if the mode was used?*

If only the mode was used, there would be no possible improvement during the iterative refinement and the method would therefore default to the classic phase correlation algorithm with pixel accuracy only. Since the science result depends heavily on the subpixel component of the image shift, it would differ significantly if the mode was used instead of the mean.

**Reviewer's Comments:**

*1. The technique determines shifts in the x- and y-direction. I suggest to use a different naming convention throughout the manuscript: Both shifts represent horizontal flows on the solar surface and not a radial flow. The "horizontal" shift could be named "east-west" shift, while the "vertical" shift could be called "north-south" shift or something similar.*

The authors fully agree. The naming of the horizontal and vertical image shifts and flow speeds has been updated to west-east and north-south all throughout the manuscript text, tables and figures.

*2. The authors state that they apply their technique to full-disk continuum intensity data obtained with the HMI instrument onboard SDO. The wavelength is given as 1700 A. There seems to be a mistake: the HMI continuum intensity wavelength is Fe I 6173 A, while 1700 A is a wavelength of the AIA instrument on SDO. Please clarify.*

The given wavelength of 1700 A is a mistake, as has been correctly pointed out. The wavelength was corrected in the manuscript to the correct HMI continuum intensity wavelength of 6173 A.

*3. The authors should acknowledge the source of their data. The obvious source of SDO data is the JSOC (jsoc.stanford.edu).*

The source of the used data (JSOC) has been added to Section 1.

*4. The authors state that the advantages and limitations of their technique are well known (discussed in a previous paper). The authors might want to briefly describe those that are relevant to this study.*

A discussion about the main relevant advantages and limitations of the IPC technique was added to Section 2.3.

*5. The authors use areas of 64x64 pixel shifted by 1 time step of 45s (pairs of consecutive images). Have the authors tried larger spatial sizes and/or longer time steps? The rotation rate is about 0.25 pixel at the equator and less than 0.1 pixel at 60 deg latitude. The authors rely on accurately deriving subpixel shifts of the order of 0.1 to 0.01 pixels. By using longer time steps and appropriately sized areas, the authors could improve the results. This is especially important for the meridional flow, the large-scale flow in the north-south direction, which is called vertical flow by the authors. It is well-established that the meridional flow is about two orders of magnitude smaller than the rotation rate. This would require an accuracy of 0.001 to 0.0001 pixels when using a temporal shift of 1 time step of 45s. The authors should therefore repeat the analysis with a longer time step. Given the claimed accuracy of their technique and the magnitude of solar flows, suitable choices would range from 100 to 1,000 time steps (1.25 hrs to 12.5 hrs).*

The authors have indeed tried larger spatial sizes and longer time steps. However, in the larger time step case, there is a very important limitation. Since the HMI images mainly show the granulation on the Sun’s surface, the time difference between the two pictures is limited by the lifetime of these granules. If the image shift measurements were done with the suggested time step of 1.25 hrs to 12.5 hrs, there would be absolutely no possible overlap/correlation between the two images in each pair, and thus the differential rotation rate measurement would not be possible. A typical granule has a lifetime of 10-15 minutes, which limits the number of 45s time steps to around 5 (which equals to around 4 minutes), to still keep good overlap/correlation between the images. The authors thus chose the lowest possible time step to maximize the signal-to-noise ratio.

The choice of an optimal time step is an important matter and was partly neglected in the original version of the manuscript, so a new subsection called “Time step” was added to the manuscript, along with two new Figures showing the time step dependency. To further demonstrate the rapid changes in the overlap/correlation, the authors created a short video of an upscaled 100x100px region of roughly 30 min of continuous SDO/HMI imagery - see at <https://www.youtube.com/watch?v=2iR5Ax75Ax4>.

Furthermore, in the larger image size case, authors initially chose very small areas of 64x64 pixels to maximize the spatial resolution of the flow maps and the final measurement and not cause averaging over larger areas due to larger image sizes. However, it is true, that larger images contain more information and therefore are more probable to produce results with higher precision. These two phenomena (spatial resolution and spatial accuracy) thus go against each other. The authors have changed the area size used for the main result (Figure 5, Table 1) to 256x256 pixels, which seems like a good compromise between the two mentioned factors.

*6. The pixel shifts of 0.01 at midlatitudes of 25 deg and 0.025 at 50 deg imply an equatorward flow of about 80 m/s and 200 m/s respectively (without taking geometry into account). The meridional flow measured by other techniques is mainly poleward in each hemisphere with a maximum amplitude of about 25 m/s at mid-latitudes. A 3- to 8-times larger equatorward flow would not have escaped notice.*

Most other techniques observe poleward flows at the surface and return equatorward flows beneath. However, there are also techniques which show the possibility of a circulation with a shallow equatorward return flow (e.g., Zhao et al. 2013; Miesch et al. 2006; Kapyla et al. 2012; Guerrero et al. 2013). Furthermore, this result could be connected with the well established migration of active regions, which initially emerge at mid-latitudes and appear at progressively lower latitudes as the cycle progresses, thus exhibiting equatorward migration.

Both image shift and flow speed profiles display equatorward flow, which eliminates the possibility of this result being caused by wrong geometric equations. The image shift and flow speed profiles are roughly symmetric around the origin, which also eliminates other systematic effects, like not-so-accurate center pixel coordinates from the FITS image header (these would cause the whole profile to be shifted up or down by an equal amount at each latitude in each picture pair and thus would not be able to cause the observed latitudinal variation). The authors leave the interpretation of this result to the broader scientific community.

Since the authors also think this matter is important and was not previously discussed properly in the article, this response was also added to the new “Discussion” section.

*7. The authors should test whether the results are the same for longer temporal shifts (see comment above).*

This is not possible, because the HMI images contain mostly information about granules, which are relatively short-lived (see the response to comment 5). However, for slightly larger time steps (e.g. 90s, 135s, 180s), the measured shifts are displaying linear behaviour (see the newly added Figure 2.), which means that for these smaller time steps, the results are indeed very similar.

*8. The first sentence needs to be rephrased. Studies from 2000 and 2015 have shown faster rotation rates in the southern hemisphere compared to the northern one, which agrees with the presented results. However, that does not mean that the previous studies predicted the rotation rate in 2020.*

The study from 2000 was omitted, as it is indeed 20 years older than the data used for the measurement. Another (quite recent) study was referenced here instead, showing similar results in north-south asymmetry as those obtained by the iterative phase correlation method (Lamb 2017).

*9. I suggest to change the name of the "C" parameter in equation (13) to "B" to match equation (12) and use a different letter for the linear parameter in equation (13), such as "b", for example. This would reduce the potential confusion in Table 1.*

The least-squares coefficients of the polynomial fits were renamed to lowercase “a, b, c, d” and were put in a separate table (Table 2.) for more clarity.

*10. The authors include two rotation rates from other studies for comparison. Lamb (2017) tracked small magnetic features in SDO/HMI magnetograms. Howard et al. (1983) determined the solar rotation rate from MWO Dopplergrams. The three curves agree reasonably well in the southern hemisphere. However, the measured rotation rate (black) is much slower than the other two (red, blue) in the northern hemisphere. The curves of the previous studies (red, blue) thus agree better with each other than the results of this study (black) with either of them. The authors should discuss this discrepancy.*

The discrepancy was caused by a bug in the visualization/interpolation code used to create the plots, which the authors failed to detect earlier but have now discovered, based on this comment. With this bug in the software fixed, the curves agree very well with each other – see the updated Figure 5.

*11. Since the authors use small patches of 64 pixels, their rotation rate might be close to the rate determined from granular or supergranular patterns. The authors might want to search whether the rotation rates of these patterns have been reported by other studies.*

As mentioned in the response to comment 5, the area size was increased to 256x256px, which yielded similar results, just somewhat smoother. Some of the supergranular cross-correlation studies suggest that the rotation rate of the supergranular patterns could be up to 3% faster than the surrounding photoshperic plasma, however, faster rates are mostly reported for longer time lags (~ 8-24hrs), which is much longer than the time lag used in this article (45s). Furthermore, studies of supergranular patters mostly focus on the supergranules only, while the presented method does not select and focus on any particular regions or features and thus more likely produces more average/general results.

*12. The authors overstate the usefulness of their technique compared to other correlation techniques. While it is true that their technique does not rely on the presence of contrasty (magnetic) features, this is also true for any analysis of Dopplergrams or intensity images with a correlation technique or with a helioseismology technique, for example.  
Feature tracking is used to determine the rotation rate of magnetic features, such as sunspots. Different magnetic tracers lead to different rotation rates due to their differences in age, evolution, and other characteristics and their study helps to improve the understanding of magnetic activity.  
However, a technique, such as used by Howard et al. (1983), does not depend on magnetic features and can derive the rotation rate as a function of latitude throughout a solar cycle. In addition, small magnetic features are present everywhere at all times and can be used in a correlation analysis to derive the rotation rate as a function of latitude any time during a solar cycle.  
The advantage of the presented technique is the ability to determine shifts with subgrid accuracy, which allows the authors to use short time steps and small spatial scales.*

The sentence was modified with “mainly in the domain of high spatial and temporal resolution” at the end, which reflects the mentioned remarks.