Information Arrival in Financial Markets

Job Market Paper

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This article introduces a new high-frequency analysis of six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange. I quantify in real time the information contained in the probability measure implied by option prices, using concepts developed in information theory. Here information is analogous to a reduction in uncertainty surrounding the future price of the underlying security. A simple nonparametric estimator allows us to measure the amount of information gained as an option approaches maturity. I then test for jumps in the expectation of said future price. I find the intraday flow of information in a large and important market is not continuous, and often increases in discrete intervals. This fact is used to identify events in which a large amount of information is revealed to investors.

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How does information arrive in financial markets? In this paper, I confront the basic characterization of the process by which investors learn about the future value of an asset. The topic is of importance to much of financial economics, yet continues to be one of the least explored. Indeed, this paper is the first to quantify in real time how information drives price discovery in option markets. In doing so, the paper offers three methodological contributions to the literature on measuring the information found in option prices, and documents two empirical facts not explained by existing theoretical models. First, I find the arrival of information drives jumps in investor expectations of the future price, and second, that this process is not constant over the life of an option.

This paper joins a growing literature of high-frequency analysis of investor expectations, of which Birru and Figlewski (2012) offer another example. Both the literature and this paper estimate the distribution of future returns as implied by observed option prices. Following Cox and Ross (1976) and Cox, Ross, and Rubinstein (1979), this estimation relies on a representative investor's ability to arbitrage an option and its underlying asset. In that way, all risk except the underlying uncertainty surrounding the asset's future price may be hedged away. The resulting implied distribution is known as the "risk-neutral density". Following Harrison and Kreps (1979), if this distribution is known, then options may be priced as if all investors are "risk-neutral". In other words, the price of an option is independent of the individual risk-preferences of an investor. Figlewski (2018) offers a recent review of the key ideas in this literature. Option prices therefore reflect investor beliefs over the probability the world will achieve some future state, and it is this information that is of vital interest to investors, researchers, and policymakers. This fact combined with the growth in derivative markets has inspired renewed interest in understanding how the beliefs of investors respond to new information.

In this paper, I provide a high-frequency analysis of the price discovery process in option markets. Using six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange, I characterize the intraday evolution of the density function implied by the price of options with the same maturity date. The analysis is done for the final 3 months of each option's life-cycle, as the density is shown to become more and more concentrated over time. This paper is the first to estimate the intraday dynamics of the risk-neutral density over the life-cycle, and offers the following three methodological contributions to the literature. First, I show how a simple nonparametric estimator can be used to approximate the implied density of future returns at high-frequencies. Second, I show how concepts developed in information theory can be used to quantify the amount of information contained in the estimated density. Third, I show how this novel approach permits a simple testing procedure for the presence of jumps in the evolution of the risk-neutral density, coinciding with the arrival of new information. The results of this testing procedure represent the paper's main contribution to the literature.

I find that information often arrives in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump, a result not anticipated by existing theoretical models. The testing procedure reveals both the frequency and magnitude of these jumps in investor expectations. I identify at least one jump for a majority of days, and find days without jumps contribute little to the total information gained over the life-cycle. I then document two empirical facts new to the literature: First, the majority of information accrues only in the final month. I show investors learn little about the future price of an asset for much of an option's life. Second, jumps contribute a majority of information early in the life-cycle. Only in the final month does information arrive often enough to contribute more to the total gained.

The paper builds on earlier work in many ways, but several features distinguish the findings from previous results. These include; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, 2. a fully non-parametric estimation technique, 3. a measure of information as a reduction in uncertainty, 4. a simple framework to test for jumps, and 5. the frequency and length of the sample of options data used. These features are discussed briefly in turn.

This paper targets the evolution of the risk-neutral density over time. In a departure from much of the earlier literature (Aït-sahalia and Lo, 1998), among others, the density is not estimated for a fixed maturity. Instead, for options with the same expiration date, the focus is on how the density becomes more concentrated as the maturity date approaches. The new perspective is shown to permit a simple testing procedure for the presence of jumps in the evolution of the density, and does not require the estimated density to be interpolated over time.

The paper employs an estimation procedure that is fully nonparametric. Inspired by the original procedure proposed by Breeden and Litzenberger (1978), the estimator places no restrictions on the shape of the density function or the dynamics of the underlying asset's price. In contrast to semiparametric procedures, such as those following Shimko (1993), the estimation procedure does not require the interpolation or the extrapolation of observed option prices, or the need for significant tradeoffs in measures of goodness-of-fit and smoothness.

This paper introduces concepts from information theory to quantify the uncertainty investors face about the future value of an underlying asset. The basic insight is that information can be measured as the reduction in uncertainty over time. The basic quantity of information theory is entropy. The concept is not new to economics, and both Sims (2003) and Frankel and Kamenica (2019) use entropy to model rational inattention and the information in decision problems respectively. Stutzer (2000) and Buchen and Kelly (1996) go so far as to use the concept of maximum entropy to infer the risk-neutral density from observed prices. However, this paper represents the first application of entropy to the problem of quantifying the information gained in the density function over time.

The paper employs a framework and hypothesis test for jumps in the risk-neutral density. Here the evolution of the entropy of the density reflects the arrival of new information. The procedure is equivalent to testing for jumps in a nonstationary time series, and follows Zivot and Andrews (1992). The test derives from the literature on testing for unit roots in economic time series, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984).

The choice of procedure is threefold. First the test is straightforward and transparent. It is fast and simple to implement, and the results are easily interpreted. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items that are of immediate interest. Third, the procedure permits a statistical test for each identified jump. The analysis therefore differs in this respect from other high-frequency event-studies, such as Goldberg and Grisse (2013) and Andersen, Bollerslev, Diebold and Vega (2003), among others, who examine the response of interest rates and exchange rates to economic news announcements.

Finally, the paper uses a new dataset of intraday quotes for all options written on the S&P 500 and traded on the Chicago Board of Exchange. The data is novel in two respects, namely the high frequency and long calendar span of the sample of SPX options analyzed. The intraday analysis covers six years, or nearly 1,500 trading days, beginning in January 2009 and ending in December 2014. In comparison to other high-frequency studies, Jiang and Tian (2005) and Birru and Figlewski (2012), who focus on forecasting realized variance and the change in the quantiles of the risk-neutral density during the fall of 2008, the data presented here represents a more complete picture of the intraday evolution of investor expectations.

The paper proceeds as follows. Background in section 2. Section 3 describes the high-frequency options data sample. Section 4 proposes the use of information theory to quantify the information in the estimated density. Section 5 characterizes the nonparametric estimator used to approximate the risk-neutral density. Section 6 describes the framework to test for jumps in the arrival of information. Section 7 discusses the results of the hypothesis tests. Section 8 [Under Revision] presents select case studies around events where a large jump in information was identified. Section 9 concludes.

II. Background

A. Options

A derivative is a financial instrument whose value depends on the price of an underlying asset. For example, the value of the index options used in this paper "derive" from the price of an underlying stock index, the S&P 500. Introduced in 1957, the S&P 500 was the first stock index weighted by market capitalization. Today over $9.9 trillion dollars is benchmarked to the index, with indexed assets totaling $3.4 trillion. The S&P 500 covers roughly 80% of total US market capitalization. Options written on the index trade worldwide, in both over-the-counter and exchange-traded markets. The largest and most liquid exchange-traded contract is the SPX, traded on the Chicago Board of Exchange (CBOE). The CBOE is the largest exchange for trading stock options. An options exchange offers standardized contracts and manages credit risk between counterparties, typically through a centralized clearing house. The CBOE began trading standardized contracts in 1973. Today, the notional value of SPX options is roughly \$5.5 trillion, with the number of open contracts exceeding 20 million. In 2019, the average daily volume exceeded 1.28 million contracts.

The SPX contracts used here are European-style call options on the underlying S&P 500 Index. The buyer of a European call option has the right to buy the underlying asset on a predetermined date (T) and for a predetermined price (K). The predetermined date is known as the expiration or maturity date. The price is the exercise or strike price. A single SPX contract is for 100 times the index at the given strike price. Settlement for index options is always in cash. If the buyer chooses to exercise the call option, they receive $(X_T - K) \times 100$ from the seller where, X is the settlement value of the index. Strike prices for SPX options are defined by the exchange. Strike prices for options near the value of the underlying index are typically offered at \$5 intervals. For options far from the current value of the underlying index, strike prices may only be available at \$10, \$25, \$50, or \$100 intervals. Trading in SPX options ends on the business day before the day on which the final settlement value of the index is calculated.

SPX options trade on a March cycle. All US stock options trade on either a January, February, or March cycle. The SPX cycle consists of options expiring every 3 months; March, June, September, and December. The standard expiration date is the third Friday of the near expiration month. For example, the "June call" is the SPX call option expiring on the third Friday of June, and trading in the June call ends on the Thursday before the third Friday. When an option expires, trading in a new option with a maturity date set in the next expiration month in the cycle begins. Regular trading in SPX options occurs every business day from 8:30 am to 3:15 pm Central time.

B. Risk-Neutral Valuation

A Risk-neutral valuation is a general result in the pricing of derivatives. In theory, when valuing a derivative we may assume that investors are risk-neutral. This assumption means that the individual risk-preferences of investors do not enter into the pricing equation. Introduced by Cox and Ross (1976) and expanded on in Cox, Ross, and Rubinstein (1979), the concept arises from the relationship between a derivative and its underlying asset. Since both are affected by the same underlying source of uncertainty, a portfolio can be constructed to arbitrage any gain or loss in the asset with an equivalent loss or gain in the derivative. The resulting portfolio would have no risk, and its return would be the risk-free rate. The cost of constructing such a portfolio is then the price of the derivative. Harrison and Kreps (1979) show that in a market with no profitable arbitrage opportunities, the risk-neutral price is the correct price, even under risk aversion. In a complete market their result can be extended to include the risk-neutral price being unique.

The idea that information could be extracted from the observed price of an option was introduced by Black and Scholes (1973). By inverting the valuation formula, Black and Scholes were able to estimate the future volatility of an asset, as implied by the current price of its derivative. Merton (1973) extends the Black-Scholes model to continuous time. In continuous time, the evolution of the price of the underlying asset is modelled as a diffusion process. The Black-Scholes-Merton model assumes this process to be a geometric Brownian motion. This assumption implies the shape of the terminal distribution of the asset's price is lognormal. If known, this distribution may be used to price an option independent of the risk preferences of investors. This is the basic idea of risk-neutral valuation. If it is possible to arbitrage an asset and its derivative, the derivative may be priced as if it were riskless. The terminal distribution required for obtaining the riskless price is known as the "risk-neutral" density.

In practice, the ability to continuously rebalance such a portfolio is limited by transaction costs and financing requirements. The resulting arbitrage is then unlikely to be riskless. Consequently, theoretical models do not always perform well empirically. Black and Scholes (1972) discovered this fact early on, noting that for options on the same stock the volatility implied by their model is not constant across strike prices, thus inventing the well-known Black- Scholes "volatility smile". There now exists a large body of empirical evidence that implied volatility is not constant across strike prices and maturities. Taken together, the evidence strongly suggests the assumption of a lognormal terminal distribution implied by diffusion process that is a geometric Brownian motion does not hold in practice. A phenomenon known as the Black-Scholes-Merton anomaly. Figlewski (2018) offers a more intensive introduction to the key ideas, issues, and finding introduced here.

The Black-Scholes-Merton anomaly has led to a large literature that attempts to estimate a risk-neutral density which better fits the observed option prices, and it is to this literature that the nonparametric estimator I propose contributes. The discussion of the estimator is therefore limited to a class of nonparametric and semiparametric methods. This class of methods includes other direct implementations of Breeden and Litzenberger (1978), (Shimko (1993), Malz (1997), Bliss and Panigirtzoglous (2002), Weinberg (2001), and Dumas, Fleming, and Whaley (1998)), and other methods based on such concepts as maximum entropy (Buchen and Kelly (1996), and Stutzer (1996)), kernel regression (Ait-shalia and Lo (1998)), and the binomial tree method of Cox, Ross, Rubinstein (1979), (Rubinstein (1994), Rubinstein (1996), Jackwerth (1997,1999)). For an extensive review of methods for estimating the density from observed option prices see Jackwerth (1999), Jondeau and Rockinger (2000), and Bliss and Panigirtzoglou (2002) Coutant, Jondeau, and Rockinger (2001), and Datta, Londono, and Ross (2017)).

III. Data

Throughout the paper I use data for a sample of options written on the S&P 500 and traded on the Chicago Board of Exchange. The data is novel in two respects, principally the high frequency and long calendar span of the sample of SPX options analyzed. The data represent the most complete sample of high-frequency investor expectations to date. Here I describe the data in detail.

Daily 'Trade And Quote' or TAQ data were obtained from the Chicago Board of Exchange (CBOE). The raw files contain all trades and quotes for options written on the S&P 500, and traded on the CBOE. The raw data consists of daily files for each of what is typically 252 trading days per year. The sample covers six years, or approximately 1,500 trading days beginning January 2, 2009 and ending December 31, 2014. Regular trading hours occur from 8:30 am to 3:15 pm Central time each day. Quotes are updated throughout the day for all available strikes. Trades in SPX options are sparse relative to quotes, and are rarely observed simultaneously across strikes. To avoid introducing additional pricing-errors, quotes are used in the analysis which follows. Each quote consists of bid and ask price, size, and the corresponding strike price, together with a timestamp, underlying index price, and flags indicating the class of option and market condition, either open or pre-open. Very few quotes are observed pre-open.

The traditional SPX options chain is AM-settled on the third Friday of every month. Options trading on the March cycle consist of those expiring every 3 months; March, June, September, and December. Nontraditional SPX options are PM-settled, with varying expiration dates, including the last trading day of the month and weekly options. ‘Long-term Equity AnticiPation Securities’ or LEAPS are traditional SPX options with expiration dates up to five years in the future.

The sample consists of 24 nonoverlapping option chains, beginning with the options expiring March 21, 2009 and ending with those expiring December 20, 2014. Following convention, option chains are referred to by their expiration date. There are on average 62 trading days for each curve, with the March 2009 option having only 55 trading days observed. The sample consists of 83 5-minute intervals for each trading day, beginning at 8:25 am and ending at 3:15 pm. Intraday quotes are aggregated by strike to the corresponding 5-minute interval.

Only quotes for traditional SPX call options are used, those which are AM-settled on the third Friday of the near expiration month. Nontraditional SPX options and LEAPS are excluded from the sample. To create the 5-minute samples used in the analysis, the average of the best bid and offer were taken for the sample of options with quotes posted in the preceding 5-minute. The reported 9:00 am call price for a given strike is then the mid-price of the best bid and best offer observed from 8:55:00.000 am to 8:59:59.999 am. In the rare instance where quotes were not observed for a particular strike, the average of the best bid and offer price were carried over from the previous interval.

There are two additional notes regarding the sample. First, trading in traditional SPX options ends on the business day before the settlement date, and before trading in a new option in the following cycle begins. No quotes for the current option chain are observed on these days, and they are excluded from the sample. Second, far out of the money options are often illiquid, and I exclude call options written on strikes issued at intervals greater than $50.

The result is a sample of 1,479 days, each day containing 83 5-minute intervals. Each interval contains the average of the best bid and best offer price for call options written on strike prices at intervals less than $50. Table 1 reports daily summary statistics for the sample of options and the underlying index. Columns 1-3 report the average number, minimum and maximum observed strikes, call and underlying index prices observed for each day in the sample. Columns 4 and 5 report the average and standard deviation of the sample of strike, call and underlying index prices for each day. Table 1 reports summary statistics for the entire six-year sample, as well as these statistics for each year. Table A in the appendix reports the separate summary statistics for each option chain.

The number of strikes observed each day is large. Looking at Table 1, there are between 117 and 187, and on average 146, strikes observed. Strike prices range from $150 to $2,250 and are traded at intervals of less than $50. The options sampled cover a large degree of moneyness, with the average ratio of the strike price to the current index price ranging from 0.3 to 1.4. Thus on a typical day, probabilities between a 70% decrease and 40% increase in the index over the following days are observed.

The average number of quotes observed each day is also large. For the options sampled, I observe on average approximately 718,000 quotes per day, ranging in price from $0.30 to $1,373, with an average mid-price of about $243 and average daily standard deviation of $219. The value of the underlying index ranges from $674 to $2,075, with an average price of around $1,380, and a small average daily standard deviation of just $0.18. Finally, the observed number of strikes and quotes increases consistently over the six years in the sample, as does the value of the underlying index. For simplicity and convenience, much of the analysis refers to options expiring in either the first or last year of the sample, 2009 and 2014. This is done to show the findings are not artifacts of any larger trends in the market. Figure 1 reports the daily level of both the underlying S&P500 Index and the CBOE Volatility Index or VIX Index for the sample period. For much of the sample, the value of the underlying index is increasing. Implied volatility as calculated by the CBOE is highest early in the sample, spikes in mid-2010 and late 2011, and declines steadily from the beginning of 2012. In Table 1 these trends are evident in the range of strike prices available and the average value of the underlying index.

[Insert Table 1 Here]

Table 1: Daily Summary Statistics

Year		Count	Minimum	Maximum	Average	Std. Dev.
All	*Strikes*	146	150	2,250	1,185.85	265.62
	Calls	717,850	0.0300	1,373.14	242.59	219.01
	Underlying	704,148	674.1500	2,075.37	1,380.50	0.18
2009	*Strikes*	117	200	1,650	880.37	74.42
	Calls	713,010	0.0300	663.64	145.91	40.37
	Underlying	713,317	674.15	1,114.16	941.21	112.36
2010	*Strikes*	125	350	1,500	996.84	22.67
	Calls	461,231	0.0300	892.70	189.69	35.84
	Underlying	459,336	1,022.24	1,242.87	1,134.38	51.55
2011	*Strikes*	138	150	1,600	1,087.30	62.90
	Calls	646,719	0.0300	1,131.37	232.68	41.98
	Underlying	613,337	1,099.65	1,363.65	1,268.69	62.43
2012	*Strikes*	150	350	1,800	1,157.43	53.06
	Calls	527,438	0.0300	1,105.25	255.45	26.21
	Underlying	511,441	1,203.72	1,464.25	1,373.27	51.94
2013	*Strikes*	160	350	2,100	1,389.30	101.63
	Calls	792,863	0.0300	1,212.67	277.69	40.67
	Underlying	790,280	1,403.03	1,810.83	1,634.21	98.41
2014	*Strikes*	187	650	2,250	1,600.05	41.70
	Calls	1,170,790	0.0300	1,373.14	352.24	42.29
	Underlying	1,165,436	1,741.25	2,075.37	1,923.74	76.28

Table 1 reports standard summary statistics for the sample of call options used. Column 1 reports the average number of options, bids, and quotes observed each day. Columns 2 through 4 report their minimum, maximum, and average values for the sample or year listed. Column 5 reports the standard deviation of the average.

TABLE A: Daily Summary Statistics

Maturity		Count	Minimum	Maximum	Average	Std. Dev.	Option Ex Date		Count	Minimum	Maximum	Average	Std. Dev.
2009-03-21	*Strikes*	110	200	1,600.00	857.65	82.57	2012-03-17	*Strikes*	154	400	1,550.00	1,080.10	14.59
	Calls	730,566	0.03	630.69	89.90	14.44		*Calls*	311,975	0.03	1,001.60	270.89	33.50
	Underlying	724,788	674	934.39	807.95	66.88		*Underlying*	293,866	1,204	1,402.61	1,323.45	44.76
2009-06-20	*Strikes*	118	300	1,600.00	802.55	25.08	2012-06-16	*Strikes*	157	450	1,750.00	1,156.40	27.44
	Calls	948,751	0.03	645.55	155.58	26.68		*Calls*	613,567	0.03	962.40	251.98	14.47
	Underlying	950,198	786	947.71	880.67	44.19		*Underlying*	583,803	1,278	1,419.18	1,360.77	40.63
2009-09-19	*Strikes*	114	450	1,650.00	899.55	41.57	2012-09-22	*Strikes*	131	400	1,600.00	1,169.96	6.13
	Calls	666,463	0.03	617.52	149.95	31.37		*Calls*	423,603	0.03	1,064.22	245.27	26.85
	Underlying	669,623	879	1,068.76	976.62	53.17		*Underlying*	412,665	1,314	1,464.25	1,387.58	39.40
2009-12-19	*Strikes*	125	450	1,550.00	958.43	16.40	2012-12-22	*Strikes*	157	350	1,800.00	1,222.06	15.78
	Calls	508,810	0.03	663.64	180.09	21.49		*Calls*	768,118	0.03	1,105.25	254.77	19.29
	Underlying	510,326	1,025	1,114.16	1,080.27	24.07		*Underlying*	758,723	1,353	1,462.10	1,420.25	26.64
2010-03-20	*Strikes*	134	400	1,500.00	974.39	12.82	2013-03-16	*Strikes*	156	350	1,800.00	1,217.09	9.21
	Calls	326,984	0.03	765.84	199.05	21.50		*Calls*	640,450	0.03	1,212.67	303.08	28.06
	Underlying	328,884	1,057	1,166.21	1,116.48	27.72		*Underlying*	630,545	1,403	1,563.25	1,498.68	35.88
2010-06-19	*Strikes*	124	550	1,500.00	1,024.83	15.83	2013-06-22	*Strikes*	149	900	1,850.00	1,397.21	14.06
	Calls	592,298	0.03	662.63	175.34	25.00		*Calls*	911,785	0.03	766.75	236.32	30.46
	Underlying	579,694	1,051	1,217.16	1,146.51	50.43		*Underlying*	908,343	1,542	1,669.34	1,603.54	39.20
2010-09-18	*Strikes*	116	600	1,400.00	987.32	14.19	2013-09-21	*Strikes*	171	650	2,000.00	1,415.36	13.62
	Calls	603,780	0.03	525.39	156.42	26.13		*Calls*	775,906	0.03	1,074.66	295.04	32.17
	Underlying	605,497	1,022	1,128.25	1,086.60	27.73		*Underlying*	774,767	1,573	1,725.28	1,667.12	33.00
2010-12-18	*Strikes*	127	350	1,500.00	999.74	7.97	2013-12-21	*Strikes*	163	850	2,100.00	1,505.10	6.25
	Calls	315,365	0.03	892.70	228.38	21.24		*Calls*	813,782	0.03	959.51	283.22	33.18
	Underlying	316,956	1,124	1,242.87	1,187.25	31.12		*Underlying*	814,691	1,656	1,810.83	1,753.01	44.79
2011-03-19	*Strikes*	132	350	1,600.00	1,077.37	6.22	2014-03-22	*Strikes*	187	650	2,150.00	1,536.28	5.75
	Calls	398,175	0.03	990.37	256.34	19.85		*Calls*	1,004,186	0.03	1,226.51	331.89	32.44
	Underlying	385,724	1,247	1,343.01	1,294.92	25.64		*Underlying*	994,344	1,741	1,878.04	1,832.26	32.15
2011-06-18	*Strikes*	123	550	1,550.00	1,154.30	7.48	2014-06-21	*Strikes*	179	900	2,150.00	1,595.54	9.14
	Calls	545,121	0.03	810.19	212.51	14.07		*Calls*	1,036,519	0.03	1,058.53	320.98	33.35
	Underlying	524,984	1,266	1,363.65	1,322.02	24.30		*Underlying*	1,052,149	1,816	1,959.47	1,889.78	34.05
2011-09-17	*Strikes*	132	550	1,550.00	1,124.09	28.17	2014-09-20	*Strikes*	187	700	2,200.00	1,617.38	2.62
	Calls	861,006	0.03	799.34	188.95	29.96		*Calls*	666,667	0.03	1,312.05	378.46	21.67
	Underlying	817,286	1,119	1,353.83	1,245.15	72.85		*Underlying*	645,371	1,908	2,011.64	1,972.74	24.53
2011-12-17	*Strikes*	164	150	1,550.00	993.27	6.88	2014-12-20	*Strikes*	196	700	2,250.00	1,648.19	14.01
	Calls	778,565	0.03	1,131.37	273.30	32.09		*Calls*	1,959,596	0.03	1,373.14	377.08	42.93
	Underlying	732,673	1,100	1,285.92	1,213.10	43.48		*Underlying*	1,942,217	1,862	2,075.37	1,996.52	58.27

Table A reports standard summary statistics for the sample of call options used. Column 1 reports the average number of options, bids, and quotes observed each day. Columns 2 through 4 report their minimum, maximum, and average values for the sample or year listed. Column 5 reports the standard deviation of the average.

Figure 1: Underlying Index Value and Volatility

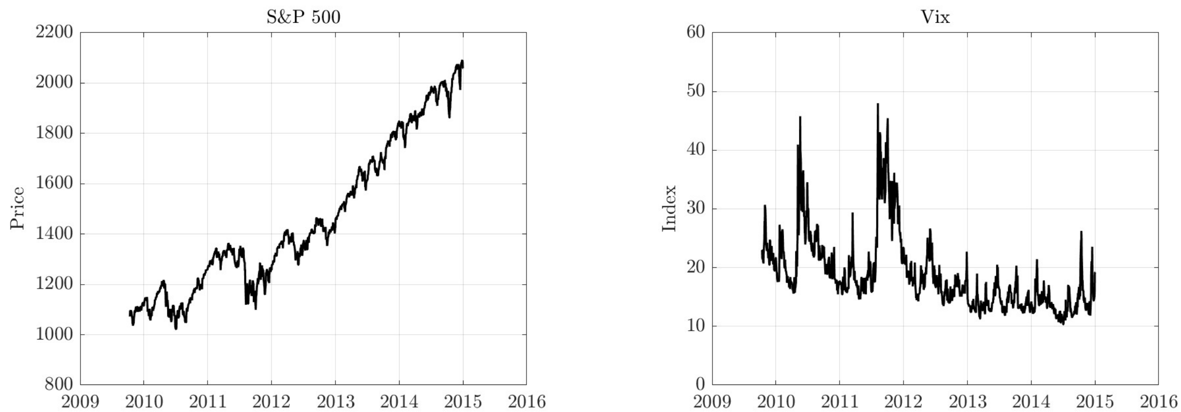

Figure 1 reports the daily level of both the underlying S&P500 Index and the CBOE Volatility Index or VIX Index for the sample period. Source: Fred and Chicago Board Options Exchange.

IV. Information

A. In Option Prices

Black and Scholes (1973) introduced the idea that information could be extracted from the price of traded options. In general, option prices reflect investor beliefs over the probability the underlying asset will take a particular value. Consider two options with adjacent strikes. Intuitively, any difference in their price must reflect the likelihood that the value of the underlying asset will fall between them. This holds across all strikes. Therefore the observed difference in option prices across strikes, or state-prices, contains information about how likely investors believe different states are. Breeden and Litzenberger (1978) show how to obtain the entire risk-neutral density from these differences. Here the information contained in option prices is extracted as a probability distribution over the future value of the underlying index. This information allows for a more complete view of investor beliefs; both how they evolve over time, and how they change in response to new information or events. To extract this information, option prices are combined across strikes to mimic a security which pays \$1 if an individual state is realized, and \$0 if not. The price of such security would then be proportional to the probability of a state occurring. This idea was first introduced in time-state preference model of Arrow (1964) and Debreu (1959). The result is known as an 'Arrow-Debreu' security, a type of elementary claim whose value is contingent on a state being realized.

Interpreting the information extracted from option prices requires summarizing the characteristics of the implied probability distribution of random process. There much of early the literature focused on using the risk-neutral density to estimate the implied volatility of the underlying process, often for the purpose of forecasting realized volatility (Canina and Figlewski (1993), Weinberg (2001), and Jiang and Tian (2005), among others). Later literature has expanded the focus to examining the response of higher-order moments, principally skewness and kurtosis, to key events, see for example Birru and Figlewski (2012). The approach has known limitations, including the sensitivity of the estimated risk-neutral density to the choice of estimation methods. As a result, quantile moments are often used and are found to be a more robust (Datta, Londono, and Ross (2014), Flamouris and Giamouridis (2002) and Campa, Chang, and Refalo (2002), Birru and Figlewski (2012)).

A growing literature studies the information contained in option prices in this manner, using the moments of the risk-neutral density to examine a wide range of markets; including but not limited to international stock indexes, (Kang and Kim (2006), Shiratsuka (2001), Glatzer and Scheicher (2005), Äijö (2008), Kim and Kim (2003), and Syrdal (2002)), exchange rates, (Weinberg (2001), Bahra (1997), Campa, Chang, and Refalo (2002), and Londono and Zhou (2012)), government bonds, (Neuhaus (1995), Shiratsuka (2001), and Cheng (2010)), inflation options, (Kitsul and Wright (2012)), and oil commodities, (Datta, Londono, and Ross (2014), Melick and Thomas (1997), Flamouris and Giamouridis (2002), and Askari and Krichene (2008)).

This paper contributes to the literature analyzing the information contained in the price of index options, but departs in the methodology employed. Similar to Weinberg (2001), Jiang and Tian (2005), Kang, Kim, and Yoon (2010), and Birru and Figlewski (2012), the focus here is for options written on the S&P 500. Two features however distinguish the analysis from earlier work. This include a departure from using both a fixed horizon risk-neutral density and the moments of the estimated density function. Instead, I propose the use of information theoretic concepts to quantify the amount of information investors gain about the likely value of the underlying index at maturity over time. It is to these concepts I now turn.

B. Information Theory

The basic insight of information theory is to measure information as a reduction in uncertainty, and the basic building block is entropy. Entropy quantifies the amount of uncertainty in a random variable, here the outcome of a stochastic process (Shannon 1948). Given a random variable X, with probability density function $p(X)$, entropy is simply $-E[log(p(X))]$. It should be noted that the concept applies whether X is a discreet or continuous random variable. That is, whether p(X) is a density with respect to a Lebesgue measure on $\mathcal{R}^k$, or a discreet measure on a countable set of points. By convention, $\text{p log p} = 0$ for any value X such that $\text{p} = 0$. The base of the logarithm determines only a scaling factor for the amount of information. Base 2 is often used as intuitively the outcome of a fair coin toss contains one *"bit"* of information. Here I provided a brief introduction to the basic ideas of information theory, for a complete technical introduction see Cover and Thomas (1991).

The amount of information or entropy in an event is calculated using the probability of that event. The more deterministic or certain an event is, the less surprising or informative its realization is. The 'surprisal' or 'self-information' of a discreet event x is defined as $\text{h(x)} = -\text{lo}g_2(p(\text{x}))$, and is 0 when $\text{p(x)} = 1$. Here the intuitive is clear, since no information is learned from a certain outcome. The classic example is that of a fair coin toss, where the self-information of a single toss is $\text{lo}g_2(1/2) = 1$. Compare this event to the outcome of a fair dice, and the intuition and idea behind entropy becomes even clearer, $-\log_2(1/6) \approx 2.58$. In a sentence; the more certain an event is, the less informative its outcome, and the lower its entropy.

Calculating the information in a random variable is the same as calculating the information of the probability distribution of the events of a random variable. In other words, the amount of information in a random variable is the surprisal of each event weighted by the probability of those events. In practice, estimating the entropy of a random variable is equivalent to estimating the information or surprisal for the probability distribution of events. Entropy then is the average amount of information of an event drawn from the probability distribution of a random variable.

In this paper, I propose the use of entropy to quantify the amount of information in the risk-neutral density implied by option prices. I employ two measures of information, the traditional Shannon (1948) entropy, and the generalized Rényi (1961) entropy. Shannon entropy is the expected or average amount of information for an event drawn from its distribution. Rényi entropy is equivalent to the L_2 norm of the distribution. Shannon entropy can be shown to generalize to Rényi entropy. Both concepts measure how broadly distributed the outcome of a random variable is. Intuitively, the higher the entropy, the broader the distribution. For example, the uniform distribution is the 'maximum entropy distribution' on any given interval $[a, b]$.

Two final points of clarification, I break somewhat with convention in the analysis that follows by reporting $-H(x)$ not $H(x)$, and using the natural logarithm in place of log base 2 for both Shannon and Rényi entropies. The units are then known as *"nats"*. This is done only for convenience, and does not affect the results. The negative rescaling simply allows for both entropies to be displayed on a single positive axis, and for the magnitude and direction of the gains in information, trend, and jumps to be more readily compared. For the remainder of the paper, when uncertainty declines the reported measures entropy will be shown to increase! Finally, entropy is not variance, which is the measure of variation in a random variable. No, here entropy is defined as a measure of uncertainty in the distribution of probabilities, and is not equivalent to implied volatility.

Figure 2: Level of Information

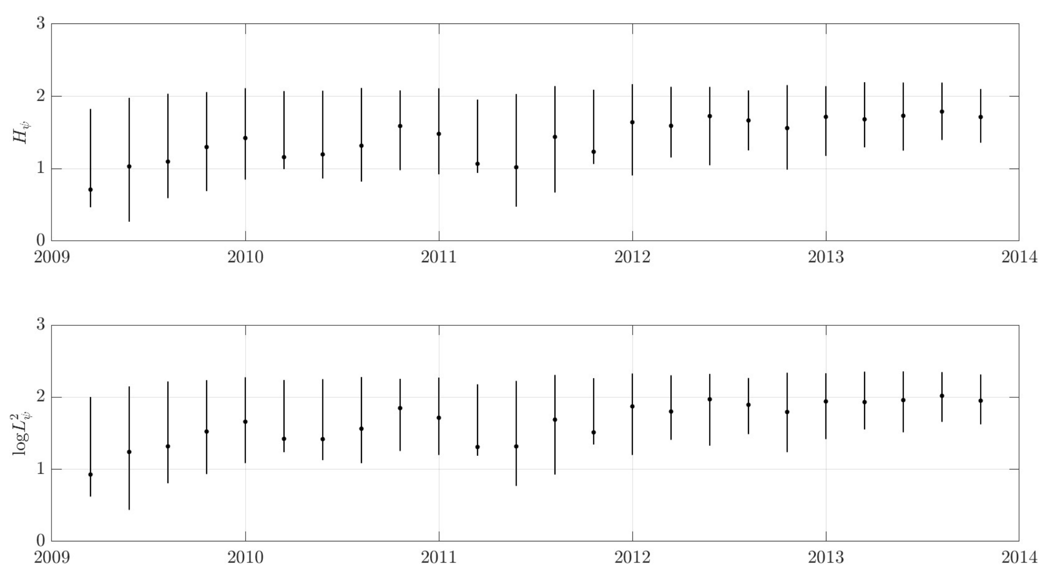

Figure 2 reports the level of Shannon, H_ψ, and Rényi, $logL^2_\psi$, entropy for each of 24 option chains over their 3-month life-cycles. Each bar represents the initial 3-month (lower bound), 2-months (center dot), and final 1-day (upper bound) level of information over time.

Table 2: Entropy Summary Statistics

		Level Shannon			Standard Deviation			Level Rényi			Standard Deviation		
Maturity	Days	3-Month	1-Month	1-Day	3-Month	2-Month	1-Month	3-Month	1-Month	1-Day	3-Month	2-Month	1-Month
2009-03-21	53	0.47	0.71	1.83	0.021	0.018	0.025	0.62	0.93	2.00	0.022	0.020	0.023
2009-06-20	62	0.27	1.03	1.98	0.017	0.014	0.022	0.44	1.24	2.15	0.016	0.014	0.019
2009-09-19	62	0.59	1.10	2.04	0.017	0.015	0.020	0.81	1.32	2.22	0.015	0.014	0.015
2009-12-19	63	0.69	1.30	2.06	0.019	0.020	0.018	0.94	1.52	2.24	0.017	0.018	0.016
2010-03-20	60	0.85	1.42	2.11	0.018	0.019	0.016	1.09	1.66	2.28	0.015	0.017	0.012
2010-06-19	62	0.99	1.16	2.07	0.026	0.023	0.031	1.24	1.42	2.24	0.025	0.022	0.030
2010-09-18	62	0.86	1.20	2.08	0.019	0.019	0.020	1.13	1.42	2.25	0.019	0.019	0.019
2010-12-18	63	0.82	1.32	2.12	0.027	0.030	0.022	1.09	1.56	2.28	0.022	0.025	0.016
2011-03-19	61	0.98	1.59	2.08	0.030	0.032	0.027	1.25	1.85	2.25	0.024	0.024	0.023
2011-06-18	62	0.92	1.48	2.11	0.026	0.029	0.019	1.20	1.72	2.27	0.022	0.025	0.017
2011-09-17	62	0.94	1.07	1.96	0.032	0.029	0.039	1.19	1.31	2.18	0.029	0.028	0.033
2011-12-17	63	0.48	1.02	2.03	0.039	0.043	0.029	0.77	1.32	2.23	0.030	0.035	0.020
2012-03-17	60	0.67	1.44	2.14	0.021	0.022	0.019	0.93	1.69	2.31	0.016	0.017	0.014
2012-06-16	62	1.07	1.23	2.09	0.030	0.030	0.030	1.34	1.51	2.26	0.022	0.021	0.023
2012-09-22	67	0.91	1.64	2.17	0.025	0.025	0.023	1.20	1.87	2.33	0.022	0.021	0.023
2012-12-22	61	1.16	1.59	2.13	0.024	0.026	0.021	1.41	1.80	2.30	0.020	0.023	0.016
2013-03-16	55	1.05	1.73	2.13	0.021	0.018	0.027	1.33	1.97	2.32	0.016	0.013	0.020
2013-06-22	67	1.25	1.67	2.08	0.017	0.016	0.020	1.49	1.89	2.27	0.014	0.013	0.016
2013-09-21	62	0.99	1.56	2.16	0.020	0.019	0.020	1.24	1.80	2.34	0.014	0.014	0.015
2013-12-21	63	1.18	1.72	2.14	0.019	0.016	0.026	1.42	1.94	2.33	0.015	0.012	0.020
2014-03-22	60	1.30	1.68	2.20	0.021	0.018	0.027	1.55	1.93	2.35	0.015	0.014	0.015
2014-06-21	62	1.25	1.73	2.19	0.018	0.018	0.020	1.51	1.96	2.36	0.014	0.014	0.013
2014-09-20	62	1.40	1.79	2.19	0.019	0.017	0.021	1.66	2.02	2.35	0.014	0.014	0.014
2014-12-20	63	1.36	1.72	2.10	0.022	0.020	0.027	1.62	1.95	2.32	0.018	0.017	0.021
Average	62	0.94	1.41	2.09				1.19	1.65	2.27			

Table 2 reports the level of both measures of information, Shannon and Rényi entropy, at three points in each option's life-cycle, 3-months, 2-months, and 1-day from maturity, as well as the standard deviation of each measure of information over that respective month.

V. Estimation

A. Semiparametric

Two features distinguish the following estimation from earlier work; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, and 2. a fully nonparametric estimation technique. These features are discussed briefly now.

Nonparametric estimation places no restrictions on the shape of the density function, and makes no assumptions on the dynamics of an underlying asset's price. Applying the original Breeden and Litzenberger (1978) method requires only the assumption that markets are competitive, and that the call price function is twice differentiable. Yet in much of the literature, semiparametric methods are preferred. The reasoning for this is twofold. First, semiparametric techniques are easier to estimate and require fewer assumptions than parametric methods. Second, semiparametric methods are thought to produce estimates of the density that are smoother and more interpretable than nonparametric techniques.

Semiparametric methods require the interpolation and extrapolation of observed option prices. Since prices are available for only a finite number of discreet strikes, the call function must first be interpolated across strike-prices. Popular methods, such as those following Shimko (1993), begin by converting prices to implied volatilities and interpolating the volatility 'smile'. The interpolated smile is then converted back into prices. This process produces a continuous and twice-differentiable call price function, but introduces additional pricing errors. Furthermore, substantial effort must be exerted to choose a curve fitting method that minimize the added pricing errors while also producing an estimate of the density function that is sufficiently smooth (Jondeau and Rockinger (2000) and Campa, Chang, and Refalo (2002)).

Semiparametric methods may also require the interpolated function to be extrapolated beyond the observed prices to ensure the estimated probabilities form a sufficient density, that is, sum to one. If so, another choice must be made of how to extrapolate either the interpolated call price function or volatility smile, and additional effort must be exerted to minimize these additional errors (Datta, Londono, and Ross, 2014). This is often required because prices are not observed for a sufficient range of strike prices, or the observed prices are not sufficiently liquid. Finally, options have fixed maturities, and do not mature every day. Therefore to generate a fixed-horizon density, as is common in the literature, requires interpolating the estimated functions not only across strikes, but across days. This may introduce further errors into the call price function, which requires additional tradeoffs between smoothness and goodness of fit.

The testing procedure employed here does not rely on a fixed-horizon estimate of the density function. In a departure from the literature, the target of the estimation is the evolution of the risk-neutral density over an option's life cycle, and not the day-to-day variation of the density for a fixed maturity. This is done for two reasons. First, targeting the dynamics removes the need for further interpolation of the call price function, and second, the approach permits a simple testing procedure for the presence of jumps in the evolution of the density. As a result, the nonparametric estimation does not require the interpolation or the extrapolation of prices across either strikes or time.

B. Nonparametric

Recall that the buyer of a call option has the right to buy the underlying asset on a predetermined date (T) and for a predetermined price (K). The date is known as the expiration or maturity date, and the price as the exercise or strike price. A European call option cannot be excised before its maturity date. The payoff function of European call is then:

$$\max(0, X_T - K) \tag{4.1}$$

If the settlement price of the underlying index is less than a given strike, $X_T \leq K$, then the option will not be exercised and its value at time T is 0. Alternatively, if the price of the index is greater than a given strike, $X_T > K$, then the option is worth $X_T - K$ at time T. At time $t < T$, or τ, the value of a European call option is then:

$$C(K, X_t, t) = e^{-r\tau} E^*[\, \max(X_T - K, 0)\, | X_t, t\ ] \tag{4.2}$$

$$= e^{-r\tau} \int_{\tau} \max(X_T - K, 0)\, dQ(X_T) \tag{4.2}$$

$$= e^{-r\tau} \int_0^{\infty} \max(X_T - K, 0)\, q(X_T) dX_T$$

Given payoff function (4.1), the price of an option is equivalent to the expected value of its payoff under the risk-neutral measure, E^*, where $Q(X_T)$ is the risk-neutral probability, and $e^{-r\tau}$ is the discount factor. Here Q is the time T or terminal distribution of returns of the underlying asset, conditional on the observed price of its derivative at time $t < T$. Intuitively, Q is the aggregate "belief" of the market in the distribution of the future returns of the asset at time t. Here q is the target of our nonparametric estimation.

Breeden and Litzenberger (1978) first showed that there exists a unique relationship between option prices and the risk-neutral density. Given the price function of a European call option $C(K, X_t)$ or C for simplicity, the second derivative with respect to its strike price K is the discounted probability density function:

$$\frac{\partial C}{\partial K} = -e^{-r\tau} \int_K^{\infty} q(X_T) d\, X_T \tag{4.3}$$

$$\left.\frac{\partial^2 C}{\partial K^2}\right|_{K = X_T} = e^{-r\tau} q(X_T) \tag{4.4}$$

The result suggests the second derivative of the call price function can be used to estimate the risk-neutral density from observed option prices. However, as Breeden and Litzenberger (1978) noted, the estimation is very unstable, as small errors in observed prices are exacerbated by numerically differencing twice. Typically these errors are small and do not represent a serious mispricing of the option or any arbitrage opportunities, and are usually a result of observing nonsynchronous prices. This problem however is not unique to lower frequency studies. Even at high-frequencies, I do not observe prices for all options simultaneously. The estimation therefore faces two challenges: 1. A finite number of observed strikes, often at non-regular intervals, and 2. nonsynchronous pricing errors.

C. Butterfly Option

The intuition behind the original procedure is straightforward. Consider any two options with adjacent strikes, the difference in their price must reflect the probability of the final price of the underlying asset falling between them. This fact holds across strikes, and price differences must reflect the probability of the different outcomes. This fact can be used to extract the targeted probability measure, Q.

Arrow (1964) and Debreu (1959) first formalized the intuition of how option prices may be used to mimic such state-contingent claims. The result is an elementary security whose return depends on the 'state' of the world at a particular time T in the future. Their key insight was that the price of such a claim would reflect investors' assessment of the probability of that state occurring. Breeden and Litzenberger (1978) later showed how traded options can be combined across strikes to mimic such an Arrow-Debreu security. The goal is to construct a security which pays \$1 at time T if the underlying asset takes a value, X_T, and \$0 otherwise, using options with the same expiration date and underlying asset.

Recall that given the price of an asset X_t, a single European call option with strike price K pays $\max(0, X_T - K)$ at maturity $T > t$, and at any given time, t, we can observe M call prices for C_i, for $i = 1, \dots, M$ options written on X across, K_i, for $i = 1, .., M$ strike prices. These options can be combined to mimic an Arrow-Debreu security as follows:

First, suppose I buy $\frac{1}{K_i - K_{i-1}}$ call options with strike price K_{i-1} and simultaneously sell the same number of call options with strike price K_i. The cost (4.5) of such a trade would be $\frac{C_{i-1} - C_i}{K_i - K_{i-1}}$, and at maturity I would receive a payout of (4.6). Such a trade is commonly known as a bull spread.

$$price_{bull} = \frac{C_{i-1} - C_i}{K_i - K_{i-1}} \tag{4.5}$$

$$\begin{aligned} payout_{bull} &= \frac{\max(0, X_T - K_{i-1}) - \max(0, X_T - K_i)}{K_i - K_{i-1}} \\ &= \begin{cases} 1 \text{ if } X_T \geq K_i \\ 0 \text{ if } X_T \leq K_{i-1} \\ \text{Linear if } K_{i-1} \leq X_T \leq K_i \end{cases} \end{aligned} \tag{4.6}$$

At the same time, suppose I sell another $\frac{1}{K_{i+1}-K_i}$ call options with strike price K_i and buy the same number of options with strike price K_{i+1}. The cost (4.7) of the trade would be $\frac{C_i-C_{i+1}}{K_{i+1}-K_i}$, and at maturity I would again receive a payout of (4.8). Such a trade is known as a bear spread.

$$price_{bear} = \frac{C_i - C_{i+1}}{K_{i+1} - K_i} \tag{4.7}$$

$$\begin{aligned} payout_{bear} &= \frac{\max(0, X_T - K_i) - \max(0, X_T - K_{i+1})}{K_{i+1} - K_i} \\ &= \begin{cases} -1 \text{ if } X_T \geq K_{i+1} \\ 0 \text{ if } X_T \leq K_i \\ \text{Linear if } K_i \leq X_T \leq K_{i+1} \end{cases} \end{aligned} \tag{4.8}$$

The combination of these two trades is known as butterfly option or spread, and it has the following properties. First, the payout of the combined option (4.9) is \$0 for both $X_T \leq K_{i-1}$ and for $X_T \geq K_{i+1}$, since in the first instance neither option is exercised, and in the second, the bull option pays \$1 and the bear option pays $-$\$1. Second, the spread is linearly increasing in $K_{i-1} \leq X_T \leq K_i$, as only the first option is exercised. Likewise the option is linearly decreasing in $K_i \leq X_T \leq K_{i+1}$, as both options are exercised. Finally, the option will have a payout equal to \$1 for any $X_T = K_i$, and the cost or price, p_i, of setting up the butterfly is (4.10).

$$
payout_{butterfly} = \begin{cases} 0 & \text{if } X_T \leq K_{i-1} \\ \text{Increasing} & \text{if } K_{i-1} \leq X_T \leq K_i \\ 1 & \text{if } X_T = K_i \\ \text{Decreasing} & \text{if } K_i \leq X_T \leq K_{i+1} \\ 0 & \text{if } X_T \geq K_{i+1} \end{cases} \tag{4.9}
$$

$$
p_i = \frac{C_{i-1} - C_i}{K_i - K_{i-1}} - \frac{C_i - C_{i+1}}{K_{i+1} - K_i} \tag{4.10}
$$

The result is an approximate Arrow-Debreu security, and the price, p_i, of constructing such an option centered at K_i can be shown to be proportional to the risk-neutral probability of that state occurring (Ross 1976). Here no interpolation is required, and it is possible to construct the spread using only the available options. In summary, an individual butterfly spread is a near binary option, whose price has point mass p_i in K_i.

To approximate the entire density from these simple functions, notice that it is always possible to construct a series of butterfly options across any range of strikes I and J, that is, one option centered at each K_i, for $i = I, \ldots, J$ strike prices. The resulting sum of their payouts has the following properties; \$0 for $X_T \leq K_{I-1}, X_T \geq K_{J+1}$, \$1 for $X_T = K_I, K_{I+1}, \ldots, K_J$, and piecewise linear in between. Since at maturity X_T is a single point, the payout for the sum of butterflies must be smaller than $\mathbb{1}_{[K_{i-1}, K_{J+1}]}$ and larger than $\mathbb{1}_{[K_i, K_J]}$, making it an increasing sequence of simple or step functions. Recall that any non-negative measurable function can be shown to be the pointwise limit of a sequence of non-negative monotonically increasing simple functions.

$$\sup|K_i - K_{i-1}| \to 0 \tag{4.11}$$

In other words, it can be shown that as (4.11) tends to zero, the payoff function of the butterfly option tends to a Dirac delta function with its mass at $X_T = K_i$. In the limit, the constructed butterfly is equivalent to an Arrow-Debreu security paying \$1 if $X_T = K_i$ and \$0 for all other states, $X_T \neq K_i$. Assuming the target measure has a continuous cumulative distribution function, a measure constructed from functions with point mass p_i in K_i can be shown to be a very good approximation of the risk-neutral density.

For the sample of SPX options employed, the difference between available K_i is typically \$5, and here I only use options spaced at intervals of \$25 or less. If we assume condition (4.11) is fulfilled, it is then possible to work with the measures based p_i, and its modifications. Note that for convenience, I choose to work with the logarithmic scale of this measure, such that $\log(25) < 1.4$. After applying the log-transform, the result is a measure with point mass p_i in $log(K_i)$.

D. Convolution

In the previous section, the price of a combination of butterfly options was shown to be a point mass approximation of the risk-neutral density. The goal then is to convert this point mass approximation to a continuous density function. This is complicated by the likely presence of nonsynchronous pricing errors in the data.

If the observed prices C_i contain even small pricing errors, equation (4.10) will be highly unstable. This is intuitive since the butterfly spread divides the difference of prices C_i by the difference of strikes K_i, which are already assumed to be small. In practice then, I observe $\widehat{p_i}$, the estimated cost of the butterfly spread including pricing errors, and not p_i. To recover p_i, I consult the literature on inverse problems in econometrics and choose to approximate the measure as the convolution of $\widehat{p_i}$ and a smoothing function φ. Convolutions also play a central role in the identification and estimation of measurement error models (Schennach 2016, 2019). Beyond econometrics, they have broad applications in statistics, physics, engineering, acoustics, image processing, and probability theory. In statistics, a convolution is a weighted moving average. In kernel density estimation, a distribution can be approximated from its sample points by a convolution with a kernel (Diggle 1995).

$$\widehat{p_i} = p_i + \varepsilon_i \tag{4.12}$$

$$\begin{aligned} \psi(x) &= \sum\nolimits_i \widehat{p_i}\,\varphi(x - \log K_i) \\ &= \sum\nolimits_i p_i \varphi(x - \log K_i) + \sum\nolimits_i \varepsilon_i \varphi(x - \log K_i) \end{aligned} \tag{4.13}$$

Following Carrasco, Florens, and Renault (2014), a convolution is employed to reduce the noise in $\widehat{p_i}$ and dampen the estimation error of p_i. Consider any non-negative function φ such that $\int \varphi(x)d\,x = 1$, the convolution of φ and $\widehat{p_i}$ is the function ψ as defined in (4.13), where ψ has the following two components. First, a systematic component, $\sum_i p_i \varphi(x - \log K_i)$, here the density of a convolution of the constructed measure p_i with smoothing function φ. Second, an idiosyncratic component, $\sum_i \varepsilon_i \varphi(x - \log K_i)$, which can be shown to converge to zero under reasonable conditions; that is, φ is three times differentiable, square integrable, and the pricing errors have only short-range dependence and are sufficiently tame. Stated simply, given a choice of smoothing function φ, if the pricing errors are not too large and not too persistent then ψ will be a good approximation of the continuous risk-neutral measure.

A few final thoughts. First, here the choice of smoothing function is the density of a $N(0{,}6.25 \times 10^{-4})$, and φ is therefore three times differentiable and square integrable. Second, the target of the remaining investigation is the information contained in the convolution ψ of the approximate risk-neutral measure with this smoothing function. Third, there is no guarantee that this ψ will be positive. In practice however, the contribution of the negative portion of $\psi < 0$ to the total variation of the measure is typically small, less than 10^{-4} with few exceptions. Finally, the convolution can be shown to preserve all higher order moments, here the cumulants, and therefore the nonlinearities of the estimated density.

E. Initial Estimates

[Under Revision]

Figures 3 and 4 report the results of the nonparametric estimation procedure. Figure 3 reports the evolution of the intraday measures of information for each option chain covering six years, 2009-2014. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index $\tau = (T - n)/T$ normalizes each option's life-cycle.

Figure 4 reports the intraday evolution of the approximate risk-neutral measure ψ as the convolution of $\widehat{p_\imath}$, the estimated cost of the sum of butterfly options, and φ, a chosen smoothing function. The estimated densities ψ for the SPX options expiring in June 2014 are shown in color for a single day, May 30th, 2014, and for 83 5-minute intervals beginning at 8:25 am and ending 3:15 pm Central time. The moving average of $\widehat{p_\imath}$, the price of the sum of butterfly option, is shown in grey. Here $\widehat{p_\imath}$ is an estimate of a measure with point mass p_i in $log(K_i)$. Table 2 reports the level of both measures of information, Shannon (4.14) and Rényi (4.15) entropy, at three points in each option's life, 3-months, 2-months, and 1-day from maturity, as well as the standard deviation of each measure of information over that respective month.

$$H_\psi = \int \psi(x) \log \psi\,(x) \tag{4.14}$$

$$logL_\psi^2 = \log \int \big(\psi(x)\big)^2 \tag{4.15}$$

[Insert Figures 3 and 4 Here]

Figure 3

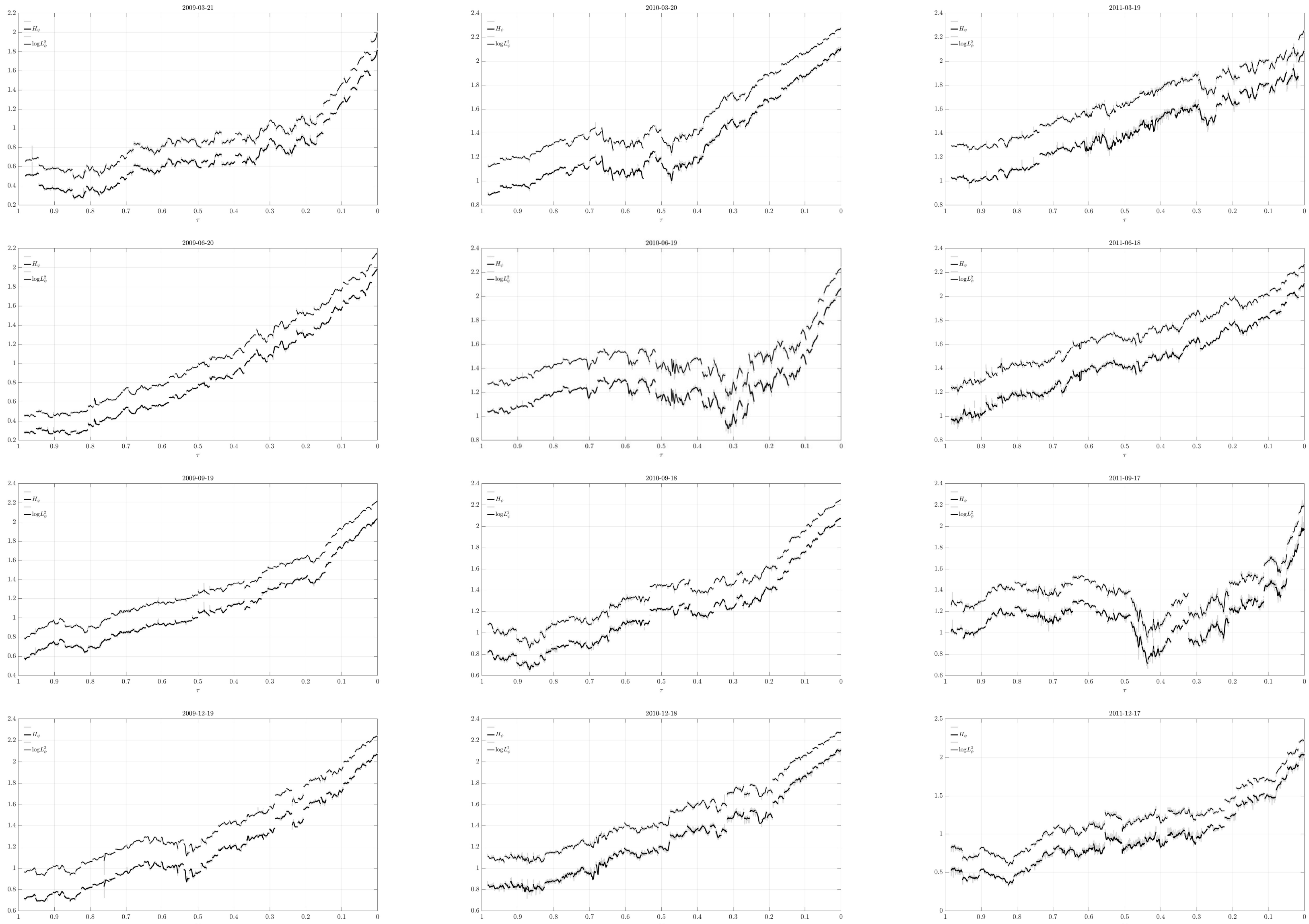

Figure 3 reports the evolution of the intraday measures of information for each option chain covering three years, 2009-2011. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index $\tau = (T - n)/T$ normalizes each option's life-cycle.

Figure 3

Figure 3 reports the evolution of the intraday measures of information for each option chain covering three years, 2012-2014. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index $\tau = (T-n)/T$ normalizes each option's life-cycle.

Figure 4

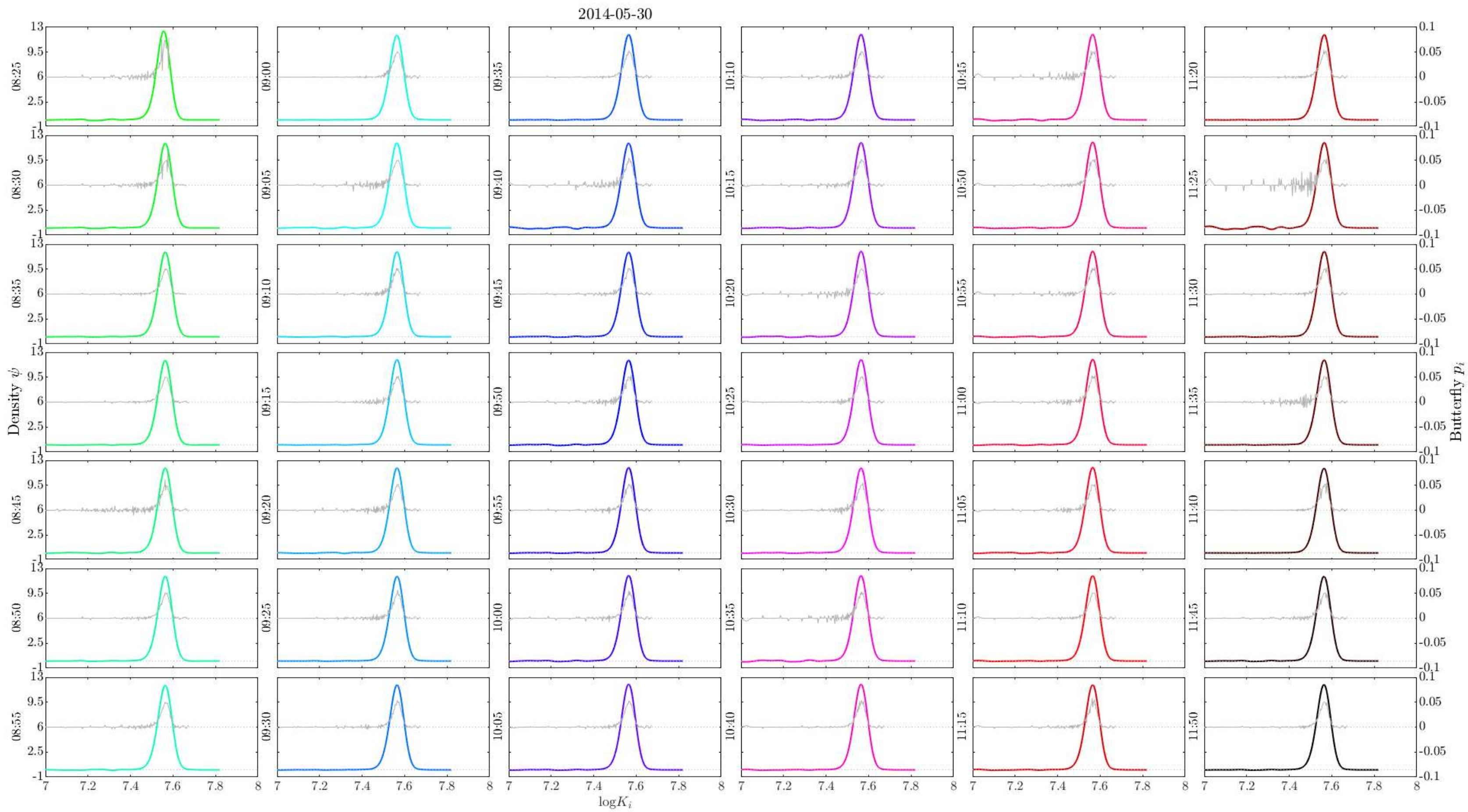

Figure 4 reports the intraday evolution of the approximate risk-neutral measure ψ as the convolution of $\widehat{p_i}$, the estimated cost of the sum of butterfly options, and φ, a chosen smoothing function. The estimated densities ψ for the SPX options expiring in June 2014 are shown in color for a single day, May 30th, 2014, and for 42 5-minute intervals beginning at 8:25 am and ending 11:50 am Central time. The moving average of $\widehat{p_i}$, the price of the sum of butterfly option, is shown in grey. Here $\widehat{p_i}$ is an estimate of a measure with point mass p_i in $log(K_i)$.

Figure 4

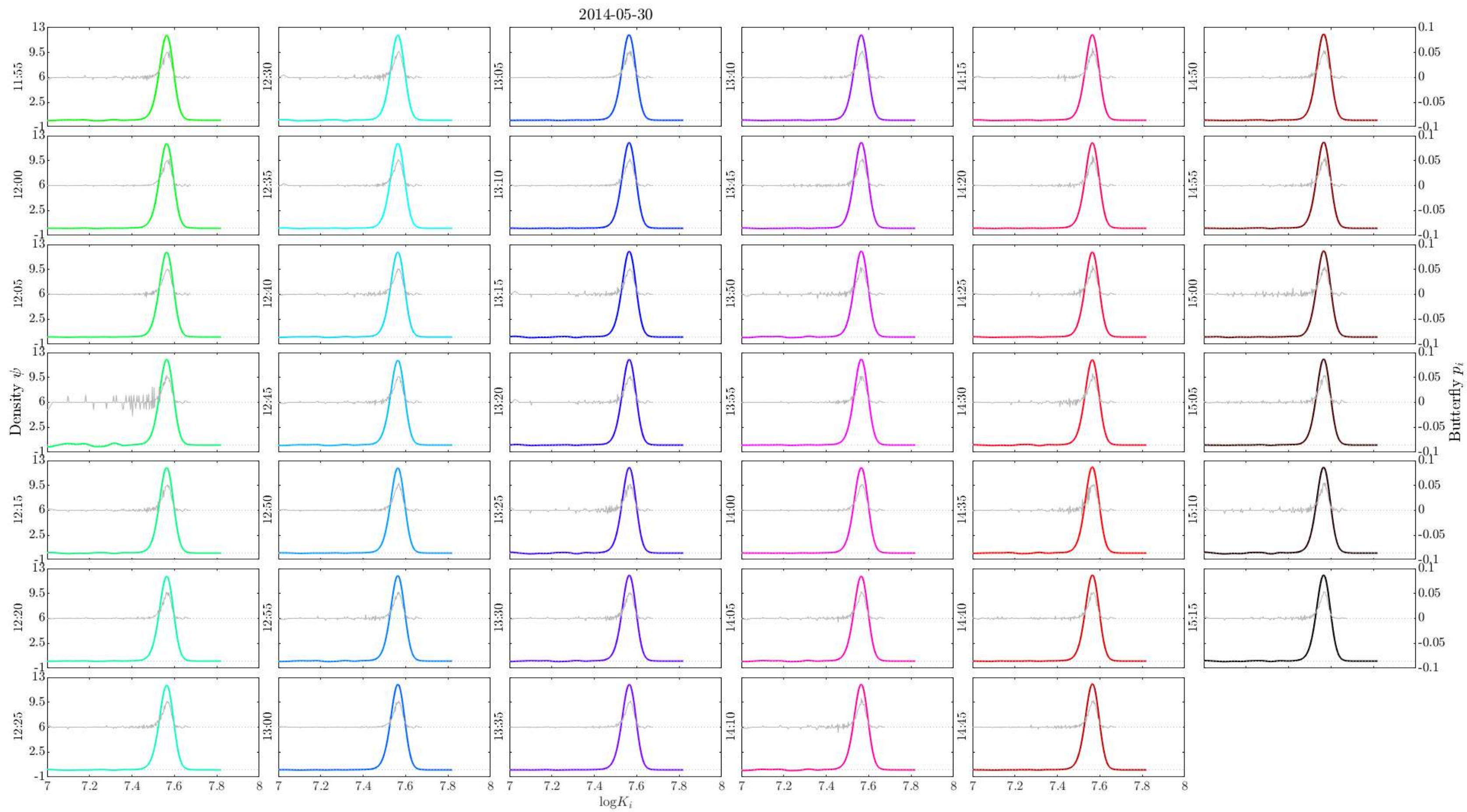

Figure 4 reports the intraday evolution of the approximate risk-neutral measure ψ as the convolution of $\widehat{p_\iota}$, the estimated cost of the sum of butterfly options, and φ, a chosen smoothing function. The estimated densities ψ for the SPX options expiring in June 2014 are shown in color for a single day, May 30[th], 2014, and for 41 5-minute intervals beginning at 11:55 am and ending 3:15 pm Central time. The moving average of $\widehat{p_\iota}$, the price of the sum of butterfly option, is shown in grey. Here $\widehat{p_\iota}$ is an estimate of a measure with point mass p_i in $log(K_i)$.

Figure A

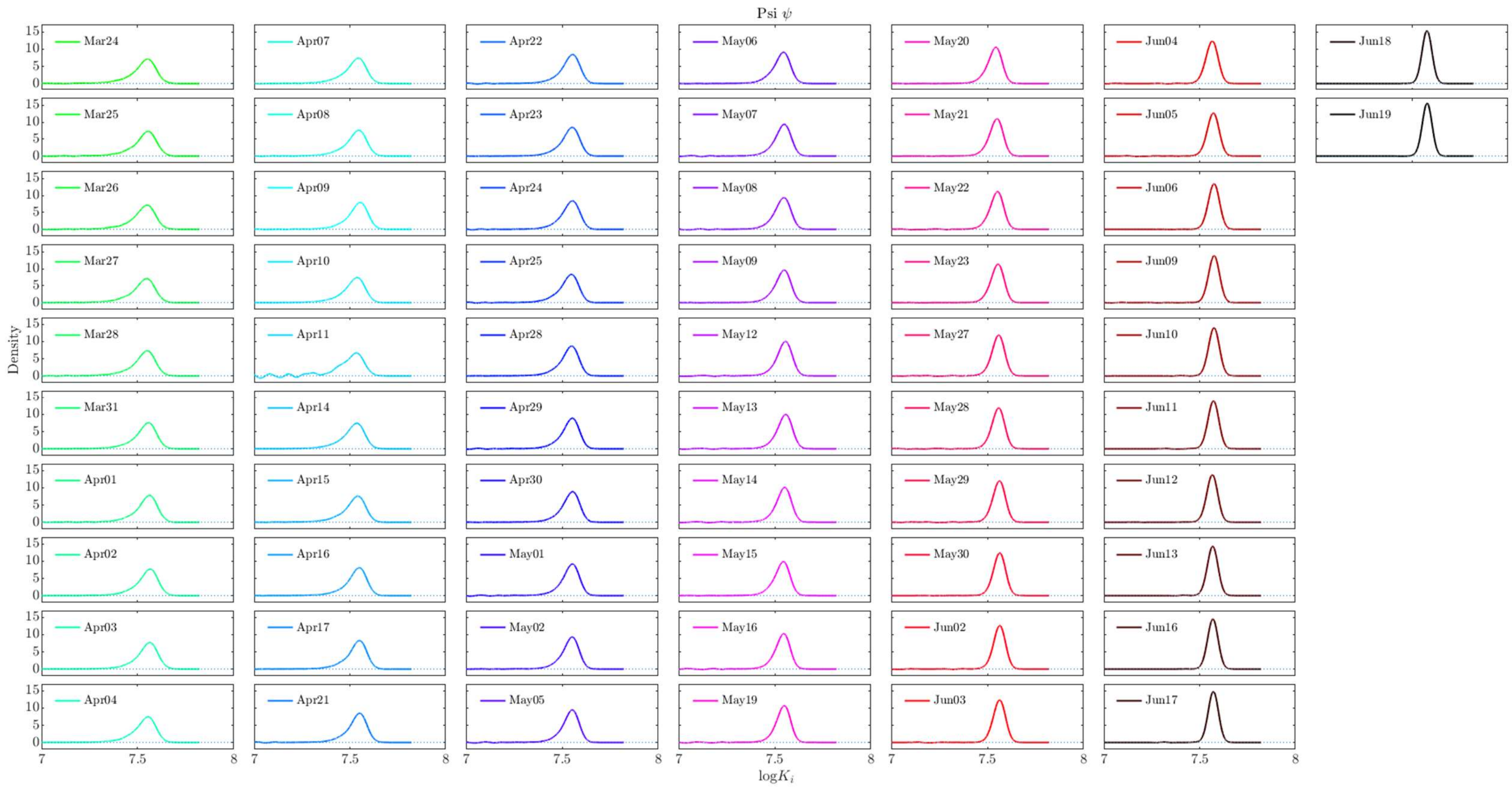

Figure A reports the daily evolution of the approximate risk-neutral measure ψ as the convolution of $\widehat{p_\iota}$, the estimated cost of the sum of butterfly options, and φ, a chosen smoothing function. The estimated densities ψ for the SPX options expiring in June 2014 are shown in color for each of $T = 62$ days in the option's life-cycle, beginning on March 24th and ending on June 19th, 2014. End-of-day estimates are reported in each panel of the figure. Each estimate is derived from prices recorded at 3:15 pm each day.

VI. Testing

I now turn to the paper's main empirical contribution, a simple testing procedure for the presence of jumps in the arrival of information. The approach reveals both the frequency and magnitude of jumps in investor expectations, and permits the testing of several hypotheses. Building on the concept of information as a reduction in uncertainty surrounding the future price of the index, the arrival of information is reflected in the evolution of the risk-neutral density. This process can be quantified using entropy. For options with the same maturity date, the density becomes more concentrated over time. Hence both measures of entropy will increase, as investors become increasingly certain of the likely distribution of future prices. Intuitively, less weight is place on options farther from the money, as the probability of large changes in the value of the underlying index declines near maturity.

The evolution of the entropy of the risk-neutral density reflects the arrival of information. If information arrives continuously, if investors gain information in small amounts, or if expectations of the future price evolve slowly, then entropy will increase gradually. Alternatively, if information arrives discontinuously, if investors gain information in large amounts, or if expectations of the future price change abruptly, then entropy will be shown to jump. This intuition can be formalized by specifying the dynamics of the underlying diffusion process. For example, if the underlying price were to follow a geometric

Brownian motion, then the entropy of the implied distribution would increase both continuously and at a constant rate across time.

The information theoretic framework introduces several testable hypotheses for how information drives the evolution of the risk-neutral density. First, information arrives continuously, and investors respond by constantly adjusting expectations; or information arrives discreetly, and investors update expectations in response to new information. Second, investors learn at a constant rate and information accrues consistently across the lifecycle; or learning is highly variable, and information accrues at different rates across time. Third, information accrual is deterministic, and any shock to expectations will return to some trend; or information follows a random walk, and investors learn little day-to-day as shocks affect expectations indefinitely.

The results of these hypothesis tests have real-world implications. Consider a temporary negative shock to the expected future value of the index. If information follows a stochastic trend, then investors remain persistently less certain about the future value of the index. This uncertainty may persist until an option matures. Alternatively, if uncertainty is purely deterministic, investors may profit from betting information will rebound to normal levels. Likewise, if information accrues at a constant rate, or if information is found to arrive continuously, investors may try to anticipate how uncertainty and the density function will evolve following such a shock. Here it is important to consider that any model which assumes the underlying price follows a continuous diffusion process also implicitly assumes that information arrives continuously, accrues at a constant rate, and evolves following a deterministic trend. This fact can be shown for many models including the Black-Scholes-Merton model.

Testing the three hypotheses is equivalent to testing for jumps in a nonstationary time series. Here I follow the procedure proposed by Zivot and Andrews (1992) to test for a single jump at an unknown time. The procedure derives from the literature on testing for unit roots in economic time series, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984). Perron (1989) found the proposed augmented Dickey–Fuller procedure to be biased against rejecting the null hypothesis in the presence of jumps or structural breaks in the time series. Zivot and Andrews (1992) later endogenize Perron's procedure to test for jumps occurring at an unknown time.

The choice of procedure is threefold. First the test is simple and transparent. It is fast and easy to implement using standard econometric tools and software, and the results are easy to interpret. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items of immediate interest in our analysis. And finally, the procedure provides a statistical test for each identified jump, a feature many event-type studies lack. I now turn to describing the procedure in detail.

Following Zivot and Andrews (1992), the null hypothesis (5.1) is a random walk with possible drift and no jump. The alternative hypothesis (5.2) is a trend-stationary process with a single jump in its trend occurring at an unknown time. The basic idea is to first estimate a sequence of alternative or trend-stationary models, one for each interval in the series. I then select the jump which gives the most weight to a single alternative model, and test that model against the null hypothesis. Here Zivot and Andrews follow Perron's (1989) augmented Dickey–Fuller testing strategy, and use a single regression equation to test for a unit root. Operationally, their procedure identifies both the timing and magnitude of the single largest jump in each time series. Each regression also includes an optimal number of lags, typically chosen by either Schwarz's (1978) Bayesian Information Criterion or a sequence of t-tests.

Null Hypothesis—Random Walk with Drift:

$$H_0: \qquad y_t = \mu + y_{t-1} + e_t \tag{5.1}$$

Alternative Hypothesis—Trend Stationary Model + Jump:

$$H_1: \qquad y_t = \mu_1 + \delta t + (\mu_2 - \mu_1)DU_t + e_t \tag{5.2}$$

The number of lags included in each regression is chosen to reduce the residuals to white noise. The optimal number included is allowed to vary for each candidate model, and is selected for each regression by a sequence of t-tests, as suggested by Ng and Perron (1995) and Campbell and Perron (1991). Ng and Perron find their approach suffers fewer size distortions and retains comparable power to Schwarz's (1978) Information Criteria. In practice both Zivot and Andrews (1992) and Perron (1989) follow such a procedure; working backward from k = $\bar{k}$ to k = 0, selecting the first k^* such that the t-stat on

c_k for $1 < k$ is significant. It is their procedure I follow here, setting $\bar{k} = \text{floor}\left[12\{(T+1)/100\}^{0.25}\right]$ as proposed by Schwert (1989), where $T = 83$ and in practice $\bar{k} = 11$.

Regression Model—Zivot and Andrews (1992):

$$y_t = \hat{\mu} + \hat{\theta} DU_t(\widehat{\lambda}) + \hat{\delta} t + \hat{\alpha} y_{t-1} + \sum_{j=1}^{k} \widehat{c_j} \Delta y_{t-j} + \widehat{e_t} \tag{5.3}$$

The regression model (5.3) facilitates the testing of several hypotheses. In the model, $\{y_t\}$ represents the amount of information measured at each interval, where $t \in (0,83)$ is the number of 5-minute intervals after 8:25 am Central time. The "trend" parameter $\hat{\delta} t$ measures the gain in information as the average change in entropy over each day. The intercept $\hat{\mu}$ captures the initial level of information for each trading day. The "jump" parameter $\hat{\theta}$ captures the magnitude of a one-time shift in level of entropy occurring at a single time, t, where DU_t is indicator variable such that $DU_t(\hat{\lambda}) = 1$ if $t > \lambda t$, and 0 otherwise. The Zivot and Andrews (1992) procedure is to first estimate a sequence of models, one for each possible jump, and to then select the best alternative model from that class of candidate models. The 'best' being the model which gives the most weight to the alternative hypothesis. Finally, the null hypothesis, $\hat{\alpha} y_{t-1} = 1$, is tested against the best alternative model, using the asymptotic critical values derived by Zivot and Andrews (1992). In practice, the alternative model is selected from the group of candidate models as the single model having the smallest one-sided t-statistic against the null for the test of $\hat{\alpha} = 1$. Testing $\hat{\theta}\mathrm{DU_t}(\hat{\lambda}) = 0$ effectively tests for a jump at a given 5-minute interval; where $\hat{\lambda} = t/T$ for $t \in (7, \mathrm{T} - 2)$ and solves $\inf\limits_{\lambda \in \Lambda} t_{\hat{\alpha}}(\lambda)$.

In the application here, I begin with the two measures of information; Shannon and Rényi entropy. Both are measured at each of 83 5-minute intervals each day, over six years or 1,479 days in the sample, and representing 24 non-overlapping option chains. For each day, I estimate the regression model at each of 73 candidate intervals, one for 5-minute intervals between the hours of 9:00 and 3:00 pm Central time. From the resulting class of 73 candidate models I select the alternative model which best fits the data, here the model with the smallest t-statistic for the test of $\hat{\alpha} = 1$. Finally, the random walk model is tested against the alternative trend-stationary model. The $\hat{\theta}$ and $\hat{\delta}\mathrm{t}$ parameters are recorded for each day, along with the number of lags k^{*} selected, and the result of the main hypothesis test. For days where the null is rejected, a secondary test of $\hat{\theta} = 0$ is performed to identify large jumps.

Much of the following analysis focuses on relative magnitude of the "trend" or $\hat{\delta}\mathrm{t}$ and the "jump" or $\hat{\theta}$ parameters. It may be helpful then to review the possible outcomes of the testing procedure and the implication of each result for the analysis. First, for a given day the procedure may fail to reject the null hypothesis of a random walk. This would be the most likely outcome for days where the estimated time trend was very small, and no jump was identified. This result would indicate a day where little to no information was gained. Second, the procedure could reject the null hypothesis but detect no jump, such that $\hat{\delta}t > \hat{\theta}$. This result would indicate a day where the trend or continuous gain in information dominates the arrival of any single bit of information. This result is most consistent with the model of the evolution of an asset's price as a continuous diffusion process. In practice, information is shown to arrive nearly continuously, and investors respond by making constant small adjustments in their expectations of the future price of the asset. Finally, for a given day, the procedure may reject the null hypothesis and detect a jump. This result would indicate a day with a large discreet gain in information, a jump which may dominate the more continuous gain in information, such that $\hat{\theta} > \hat{\delta}t$. Here information is shown to arrive discreetly, and investors respond by making fewer but larger revisions to their expectations.

VII. Results

A. Full Sample

The testing procedure reveals several surprising results. Information is shown to often arrive in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump. In the data, the procedure identifies at least one jump for a majority of days, and finds days without jumps contribute little to the total gain in information. Additionally, the testing procedure reveals both the frequency and magnitude of jumps in investor expectations. The findings highlight two facts new to the literature. First, the majority of information accrues in the final month of an option's life-cycle. In fact, investors learn little about the future price of an asset for most of an option's life. Second, jumps contribute the majority of information gained during the first two months. Only in the final month does information arrive often enough for the trend to contribute more to the total gained. The size and frequency of jumps does not decline.

Information arrives in discrete intervals. The intraday analysis reveals the frequent occurrence of jumps in the risk-neutral density. Table 3 and Table 4 summarize the results of the daily Zivot and Andrews (1992) procedure for each of 24 option chains. Each table displays the results of the daily hypothesis test of $\hat{\alpha} = 1$. In columns 3-5, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. On average, the null hypothesis is rejected for a majority of days, and the alternative model of a trend-stationary process with a potential one-time jump in information is selected. This is true for between one-half and two-thirds of days depending on the measure of entropy and the level of significance. The result implies that investors 1. gain information about the future value of an asset over time, and 2. make large revisions to their expectations in response to new information. This is shown to occur even at high frequencies, as the procedure identifies at least one jump for a majority of days. The results also document several days where little to no information is gained. For days where the procedure fails to reject the null, no jumps are found to have occurred. Combined with only a small gain in the trend of information, the daily process for these days would resemble a random walk.

Tables 3 and 4 also report the average daily change in total, trend, and jump estimates of information, taken over all days in each option chain, in columns 6-8. On average, the day-to-day gain in information is small, and positive; with a trend component that is on average larger than either the jump or total gain in information. On average jumps appear to coincide more with increases in uncertainty, as they are negative and nonzero. This holds for both Shannon and Rényi entropies.

The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information. Here it becomes apparent that jumps contribute to a large proportion of the total information gained, and are more often positive than their average over all days appears. On average, jumps contribute to almost half of the total information gained over the cycle. Across all days, a single jump, over a single 5-minute interval, is nearly as large as the estimated trend across the entire day. This result implies that jumps occur for a majority of days, and that days without jumps likely contribute little to the total gain in information.

The testing procedure reveals the magnitude of jumps in investor expectations. Figures 5 panels 2 and 3 present histograms of the estimated daily trend and jump components for all days, and each measure of entropy. The trend component is unimodal, centered near zero, and negatively skewed. The trend in Shannon entropy is more broadly distributed and more skewed in comparison to the trend in Rényi entropy. The estimated jump component is bimodal, centered near zero, slightly positively skewed, and not symmetric. The jumps in Shannon entropy are more broadly distributed and skewed compared to the Rényi entropy. For both measures, there are few days where the estimated jump is zero, and a large number of days the jump component is larger than the trend. To understand how the trend and jump components vary day-to-day, Figure 5 panels 1 and 4 present heat maps of the estimated daily trend and jump components for all days. For each map, the bright yellow and deep blue correspond to estimates of the daily trend and jump in the 95th percentile of each distribution, with blue-green estimates near zero.

Several patterns emerge that require further consideration. First, the size and frequency of jump estimates appears evenly distributed over the life-cycle. Second, the magnitude of the trend estimate is not evenly distributed. Looking at the shading in panels 1 and 4 of both figures, a large positive trend appears more likely in the final 20 to 30 days of an option's life. This result suggests information accrues at different rates across time, that is, gains in information are not constant across the life-cycle. Figure 6 panels 1 and 2 shows this in high contrast. Here, days with a strictly positive trend and jump are reported in white. Again the frequency of positive jumps appears evenly distributed over the life-cycle, while the frequency of days with a strictly positive trend seems to concentrate in the final 20 days of an option's life.

[Insert Tables 3 and 4 Here]

Table 3: Test Results Shannon Entropy—Full Sample

		Reject Null			Average Daily Change			Percent Total Change	
Maturity	Days	10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	53	24	21	15	0.005	0.004	0.001	0.56	0.44
2009-06-20	62	31	28	15	0.001	0.003	-0.002	0.52	0.48
2009-09-19	62	40	35	23	0.012	0.012	0.000	0.51	0.49
2009-12-19	63	40	38	31	0.006	0.011	-0.004	0.51	0.49
2010-03-20	60	39	38	23	0.006	0.008	-0.003	0.53	0.47
2010-06-19	62	29	23	16	-0.002	0.000	-0.002	0.54	0.46
2010-09-18	62	29	25	19	0.005	0.006	-0.001	0.48	0.52
2010-12-18	63	47	41	31	0.004	0.008	-0.004	0.52	0.48
2011-03-19	61	34	29	19	0.006	0.008	-0.001	0.50	0.50
2011-06-18	62	37	32	23	0.002	0.007	-0.005	0.51	0.49
2011-09-17	62	26	21	13	0.007	0.005	0.002	0.51	0.49
2011-12-17	63	45	42	36	0.008	0.007	0.001	0.48	0.52
2012-03-17	60	41	39	27	0.003	0.010	-0.007	0.44	0.56
2012-06-16	62	37	35	25	0.005	0.010	-0.005	0.48	0.52
2012-09-22	67	39	34	26	0.000	0.004	-0.004	0.50	0.50
2012-12-22	61	40	36	32	-0.008	0.001	-0.009	0.46	0.54
2013-03-16	55	41	39	34	0.007	0.009	-0.002	0.51	0.49
2013-06-22	67	41	36	19	0.002	0.002	0.001	0.55	0.45
2013-09-21	62	53	49	41	0.002	0.001	0.000	0.51	0.49
2013-12-21	63	46	44	35	0.003	0.007	-0.005	0.54	0.46
2014-03-22	60	48	46	39	-0.003	-0.002	-0.001	0.54	0.46
2014-06-21	62	50	46	43	0.004	0.000	0.004	0.48	0.52
2014-09-20	62	40	39	24	0.002	0.002	0.000	0.51	0.49
2014-12-20	63	46	43	30	0.005	0.001	0.005	0.47	0.53
Average	62	39	36	27	0.003	0.005	-0.002	0.51	0.49

Table 3 reports the results of the Zivot and Andrews (1992) testing procedure for the Shannon measure of information. Columns 3-5, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 6-8 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

Table 4: Test Results Rényi Entropy—Full Sample

		Reject Null			Average Daily Change			Percent Total Change	
Maturity	Days	10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	53	24	20	8	0.006	0.008	-0.002	0.55	0.45
2009-06-20	62	22	21	16	0.002	0.002	0.000	0.54	0.46
2009-09-19	62	38	30	22	0.010	0.008	0.002	0.54	0.46
2009-12-19	63	39	36	23	0.005	0.007	-0.003	0.53	0.47
2010-03-20	60	30	27	16	0.006	0.005	0.001	0.54	0.46
2010-06-19	62	23	20	15	-0.003	-0.002	0.000	0.54	0.46
2010-09-18	62	28	25	14	0.006	0.005	0.001	0.47	0.53
2010-12-18	63	40	31	22	0.004	0.009	-0.005	0.52	0.48
2011-03-19	61	36	33	25	0.005	0.008	-0.002	0.52	0.48
2011-06-18	62	29	27	18	0.002	0.003	-0.002	0.50	0.50
2011-09-17	62	19	14	8	0.005	0.004	0.001	0.51	0.49
2011-12-17	63	35	32	22	0.004	0.014	-0.009	0.52	0.48
2012-03-17	60	39	35	28	0.004	0.005	-0.001	0.47	0.53
2012-06-16	62	30	26	14	0.003	0.004	-0.001	0.48	0.52
2012-09-22	67	29	28	21	0.002	0.002	0.000	0.50	0.50
2012-12-22	61	30	27	17	-0.006	0.001	-0.007	0.48	0.52
2013-03-16	55	30	26	19	0.007	0.009	-0.002	0.51	0.49
2013-06-22	67	32	25	16	0.002	0.004	-0.002	0.53	0.47
2013-09-21	62	40	38	27	0.002	0.002	0.000	0.53	0.47
2013-12-21	63	34	30	21	0.003	0.002	0.001	0.53	0.47
2014-03-22	60	37	37	23	0.000	-0.003	0.003	0.50	0.50
2014-06-21	62	39	33	26	0.005	0.005	0.000	0.53	0.48
2014-09-20	62	34	31	22	0.002	0.002	0.001	0.52	0.48
2014-12-20	63	36	30	21	0.005	0.003	0.002	0.47	0.53
Average	62	32	28	19	0.003	0.004	-0.001	0.51	0.49

Table 4 reports the results of the Zivot and Andrews (1992) testing procedure for the Rényi measure of information. Columns 3-5, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 6-8 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

Figure 5: Gain in Information

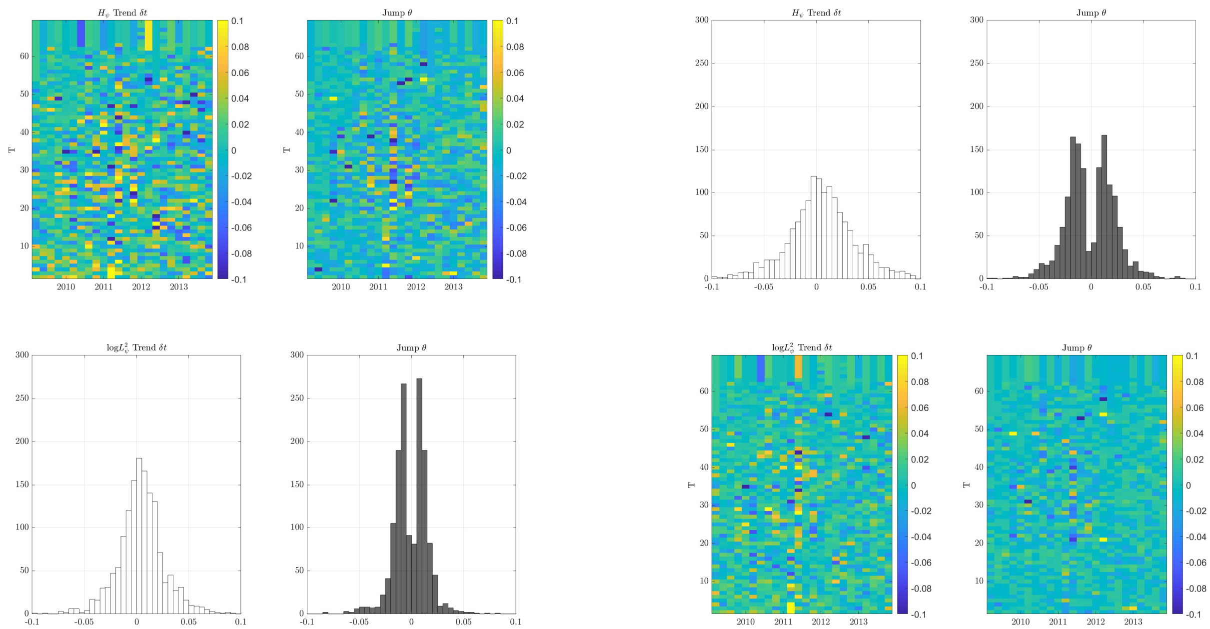

Figure 5 panels 2 and 3 present histograms of the estimated daily trend and jump components for all days, and each measure of entropy, both Shannon, H_ψ, and Rényi, $logL_\psi^2$; panels 1 and 4 present heat maps of the estimated daily trend and jump components for all days.

The result is even more pronounced if we focus only on the last two weeks, or 10 days, of trading. Few days in the last two weeks of an option's life report a negative trend. In contrast, the magnitude and frequency of jumps does not appear to change.

When then is information gained? Figure 6 panel 3 reports the frequency of days where the trend is larger than the estimated jump for both measures of entropy. Note that there are many days where the trend exceeds the estimated jump, and Figure 6 panel 3 does not distinguish days where both are negative or near zero from days where the trends is large and positive. However, when viewed in conjunction with Figure 6 panels 1 and 2 it becomes more apparent that information does not accrue consistently across the lifecycle. Only in the final 10-20 days of an option's life cycle is the estimated trend persistently positive, and consistently larger than the estimated jumps. This explains many of the results in Tables 3 and 4. If for the majority of days the daily

Figure 6: Relative Gain in Information

Figure 6 panels 1 and 2 shows days with a strictly positive trend and jump reported in white for both measures of information, Shannon, H_ψ, and Rényi, $logL_\psi^2$ entropy. Panel 3 reports the frequency of days where the trend is larger than the estimated jump. Panel 4 reports the days where the null hypothesis of a random walk is rejected.

gain in information is small, then jumps may contribute much to the gain in information. This effect would be particularly acute if days without jumps were found to contribute little to the total gained.

Figure 6 panel 4 reports the days where the null hypothesis of a random walk is rejected, in a test of $\hat{\alpha} = 1$. Again it can be seen that jumps are quite common, and that the alternative model is selected more often in the final month of an option's life. Indeed, days where little to no information is learned (days where the test fails to reject the null) are concentrated in the first two month of the sample. Given that the trend is less pronounced during this period, days without jumps likely contribute little to the total gain in information during much of an option's early life. This is not to say that information commonly follows a random walk, or to suggest investors learn nothing day-to-day. No, information accrual is decidedly deterministic. The trend is small, but I find little evidence that shocks affect expectations or the level of information indefinitely.

Taken together, the initial results suggest learning is highly variable, and information about the final price of the asset accrues at different rates across time. The testing procedure makes this clear. The findings contradict the assumption that investors are less certain about the future price of an asset mostly as a function of time, something a continuous diffusion process implicitly assumes. Instead, the arrival of information in discreet intervals results in jumps in the level of uncertainty. This fact, combined with a small trend for much of an options life cycle, and varying levels of initial uncertainty, explains the results of Tables 3 and 4. To verify this intuition, I turn to the question of quantifying these differences across the life-cycle.

B. Split Sample

The following findings highlight two facts new to the literature. First, the majority of information accrues in the final month of an option's life-cycle. Table 2 and Figure 2 show that on average nearly 60 percent of gain in entropy accrues in the final month. Given that this represents only one-third of an option's life, the observed gain in information in the last month is both large and unexpected. The finding suggests that investors learn little about the future price of an asset for much of an option's life. This result runs contrary to the common assumption that investors become more certain about the likely final price of an asset at something close to a constant rate. The finding also raises new questions regarding what drives the differences in the rate of the accrual of information in the final month of an option's life.

Tables 5 and 6 document the source of the difference in the accrual rate of information. Each table reports the results of the daily Zivot and Andrews (1992) procedure for each of the 24 option chains. Here the sample is split into thirds, with the results of the unit root test and gains in information reported separately for the first 2 months and final 1 month of each option chain. On average there are 40 trading days in the first two months, and 21 trading days in the final month; a ratio 2:1, representing on average 66 and 34 percent of each life-cycle. Column 3 reports the fraction of days in each subsample for each option. Each table displays the results of the daily hypothesis test of $\hat{\alpha} = 1$. In columns 4-6, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. On average, the null hypothesis is rejected for a similar fraction of days in both subsamples, with the alternative model selected for between one-half and two-third of days depending on the measure of entropy and the level of significance.

Tables 5 and 6 report the average daily change in total, trend, and jump estimates of information for each period. Columns 7-9 report that the average day-to-day gain in information remains small and positive, but is noticeably larger during the final month of the sample. This holds for both Shannon and Rényi entropies. Indeed, the average trend estimate triples for the Shannon entropy and doubles for the Rényi entropy in the last month. In contrast, the average jump estimate does not change. Figure 7 demonstrate this result visually. Here I plot the average absolute trend and jump estimates for the first 2 months in white, and the final month in black. The results match those as reported in columns 8 and 9 respectively. In almost all cases the average trend for the last month of the life-cycle is large and positive, particularly early on in the sample when the initial level of uncertainty is highest. By comparison, the average jump estimate does not appear systematically larger in final month. This result holds for both measures of information.

[Insert Tables 5 and 6 Here]

Table 5: Test Results Shannon Entropy—Split Sample

Maturity	Days	Fraction	Reject Null 10	5	1	Gain in Information Total	Trend	Jump	Percent Contribution Trend	Jumps
2009-03-21	33	0.62	18	17	12	0.003	0.003	0.001	0.54	0.46
	20	0.38	6	4	3	0.009	0.006	0.003	0.58	0.42
2009-06-20	40	0.65	21	19	8	0.004	0.004	0.000	0.51	0.49
	22	0.35	10	9	7	-0.004	0.002	-0.006	0.53	0.47
2009-09-19	40	0.65	24	22	14	0.006	0.008	-0.002	0.50	0.50
	22	0.35	16	13	9	0.021	0.018	0.003	0.53	0.47
2009-12-19	42	0.67	29	28	24	0.008	0.009	-0.001	0.55	0.45
	21	0.33	11	10	7	0.003	0.014	-0.010	0.44	0.56
2010-03-20	40	0.67	24	23	16	0.002	0.002	0.000	0.52	0.48
	20	0.33	15	15	7	0.013	0.021	-0.008	0.55	0.45
2010-06-19	40	0.65	21	16	12	-0.007	-0.001	-0.006	0.49	0.51
	22	0.35	8	7	4	0.007	0.003	0.004	0.63	0.37
2010-09-18	40	0.65	23	21	16	0.002	0.002	0.000	0.44	0.56
	22	0.35	6	4	3	0.011	0.013	-0.003	0.54	0.46
2010-12-18	42	0.67	31	27	20	0.000	0.004	-0.004	0.50	0.50
	21	0.33	16	14	11	0.013	0.016	-0.003	0.54	0.46
2011-03-19	42	0.69	24	20	11	0.008	0.004	0.003	0.49	0.51
	19	0.31	10	9	8	0.003	0.015	-0.012	0.52	0.48
2011-06-18	40	0.65	26	23	18	0.001	0.009	-0.008	0.48	0.52
	22	0.35	11	9	5	0.005	0.003	0.002	0.57	0.43
2011-09-17	40	0.65	14	11	7	-0.006	-0.007	0.001	0.52	0.48
	22	0.35	12	10	6	0.029	0.026	0.003	0.50	0.50
2011-12-17	42	0.67	30	28	24	0.002	0.004	-0.002	0.49	0.51
	21	0.33	15	14	12	0.021	0.015	0.006	0.46	0.54
2012-03-17	40	0.67	29	28	20	0.002	0.013	-0.011	0.48	0.52
	20	0.33	12	11	7	0.006	0.005	0.000	0.36	0.64
2012-06-16	40	0.65	27	26	21	0.001	0.006	-0.004	0.49	0.51
	22	0.35	10	9	4	0.012	0.018	-0.006	0.47	0.53
2012-09-22	45	0.67	25	21	16	-0.003	0.003	-0.006	0.48	0.52
	22	0.33	14	13	10	0.007	0.007	0.000	0.54	0.46
2012-12-22	40	0.66	20	17	14	-0.017	-0.003	-0.013	0.43	0.57
	21	0.34	20	19	18	0.008	0.008	0.000	0.50	0.50
2013-03-16	36	0.65	24	23	19	0.008	0.009	-0.001	0.49	0.51
	19	0.35	17	16	15	0.005	0.007	-0.002	0.56	0.44
2013-06-22	45	0.67	27	25	13	0.004	0.006	-0.002	0.54	0.46
	22	0.33	14	11	6	-0.001	-0.006	0.006	0.58	0.42
2013-09-21	40	0.65	33	31	28	-0.003	-0.004	0.002	0.52	0.48
	22	0.35	20	18	13	0.009	0.011	-0.002	0.51	0.49
2013-12-21	42	0.67	29	29	22	0.005	0.008	-0.004	0.55	0.45
	21	0.33	17	15	13	-0.002	0.006	-0.007	0.50	0.50
2014-03-22	40	0.67	30	28	23	-0.003	-0.001	-0.002	0.53	0.47
	20	0.33	18	18	16	-0.003	-0.004	0.002	0.55	0.45
2014-06-21	40	0.65	32	29	28	0.000	-0.007	0.007	0.51	0.49
	22	0.35	18	17	15	0.012	0.013	-0.001	0.43	0.57
2014-09-20	40	0.65	26	26	16	0.000	-0.002	0.002	0.53	0.47
	22	0.35	14	13	8	0.007	0.010	-0.003	0.47	0.53
2014-12-20	42	0.67	30	28	19	0.007	0.001	0.006	0.45	0.55
	21	0.33	16	15	11	0.002	0.000	0.002	0.52	0.48
Averages										
2-Month	40	0.66	26	24	18	0.001	0.003	-0.002	0.50	0.50
1-Month	21	0.34	14	12	9	0.008	0.009	-0.001	0.52	0.48

Table 5 reports the results of the Zivot and Andrews (1992) testing procedure for the Shannon measure of information. Column 3 reports the fraction of days in each subsample. Columns 4-6, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 7-9 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

Table 6: Test Results Rényi Entropy—Split Sample

			Reject Null			Gain in Information			Percent Contribution	
Maturity	Days	Fraction	10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	33	0.62	16	14	8	0.006	0.009	-0.003	0.54	0.46
	20	0.38	8	6	1	0.007	0.007	-0.001	0.58	0.42
2009-06-20	40	0.65	15	14	11	0.002	0.003	-0.001	0.52	0.48
	22	0.35	7	7	5	0.002	0.000	0.002	0.59	0.41
2009-09-19	40	0.65	24	17	14	0.008	0.007	0.001	0.54	0.46
	22	0.35	14	13	8	0.014	0.012	0.002	0.53	0.47
2009-12-19	42	0.67	28	26	17	0.005	0.007	-0.002	0.56	0.44
	21	0.33	11	10	7	0.003	0.008	-0.004	0.48	0.52
2010-03-20	40	0.67	20	18	13	0.004	0.002	0.002	0.50	0.50
	20	0.33	10	9	3	0.010	0.012	-0.002	0.62	0.38
2010-06-19	40	0.65	17	15	12	-0.004	-0.001	-0.004	0.51	0.49
	22	0.35	6	5	4	0.000	-0.005	0.005	0.61	0.39
2010-09-18	40	0.65	21	18	10	0.006	0.002	0.003	0.41	0.59
	22	0.35	7	7	5	0.007	0.010	-0.003	0.57	0.43
2010-12-18	42	0.67	27	19	14	0.003	0.011	-0.008	0.50	0.50
	21	0.33	13	12	9	0.007	0.004	0.003	0.56	0.44
2011-03-19	42	0.69	26	25	20	0.007	0.009	-0.003	0.51	0.49
	19	0.31	10	8	7	0.002	0.003	-0.002	0.54	0.46
2011-06-18	40	0.65	23	21	14	0.000	0.004	-0.003	0.49	0.51
	22	0.35	6	6	4	0.004	0.003	0.001	0.52	0.48
2011-09-17	40	0.65	11	7	3	-0.001	-0.004	0.003	0.49	0.51
	22	0.35	8	7	5	0.017	0.019	-0.002	0.54	0.46
2011-12-17	42	0.67	26	25	19	0.004	0.012	-0.008	0.54	0.46
	21	0.33	9	7	4	0.006	0.018	-0.012	0.49	0.51
2012-03-17	40	0.67	27	25	22	0.003	0.007	-0.004	0.48	0.52
	20	0.33	12	10	7	0.006	0.001	0.005	0.45	0.55
2012-06-16	40	0.65	22	22	14	0.003	0.004	-0.001	0.44	0.56
	22	0.35	8	4	2	0.003	0.004	-0.001	0.55	0.45
2012-09-22	45	0.67	16	15	12	0.001	0.003	-0.002	0.48	0.52
	22	0.33	13	13	10	0.004	0.001	0.002	0.52	0.48
2012-12-22	40	0.66	17	16	12	-0.016	-0.003	-0.013	0.45	0.55
	21	0.34	13	11	7	0.012	0.009	0.003	0.54	0.46
2013-03-16	36	0.65	21	18	14	0.006	0.009	-0.002	0.51	0.49
	19	0.35	9	8	7	0.007	0.009	-0.001	0.52	0.48
2013-06-22	45	0.67	21	14	8	0.002	0.005	-0.003	0.50	0.50
	22	0.33	11	11	8	0.001	0.001	0.000	0.59	0.41
2013-09-21	40	0.65	26	25	20	0.001	-0.001	0.002	0.51	0.49
	22	0.35	14	13	8	0.005	0.008	-0.003	0.58	0.42
2013-12-21	42	0.67	21	19	14	0.004	0.003	0.001	0.54	0.46
	21	0.33	13	11	8	0.001	-0.001	0.002	0.53	0.47
2014-03-22	40	0.67	24	24	16	0.000	-0.003	0.004	0.50	0.50
	20	0.33	13	13	9	-0.001	-0.002	0.001	0.49	0.51
2014-06-21	40	0.65	28	22	16	0.003	0.002	0.001	0.55	0.45
	22	0.35	11	11	10	0.010	0.010	0.000	0.48	0.52
2014-09-20	40	0.65	22	21	16	0.001	0.000	0.001	0.51	0.49
	22	0.35	12	10	7	0.006	0.005	0.001	0.53	0.47
2014-12-20	42	0.67	23	19	13	0.006	0.002	0.004	0.45	0.55
	21	0.33	13	11	8	0.003	0.004	-0.002	0.50	0.50
Averages										
2-Month	40	0.66	22	19	14	0.002	0.004	-0.001	0.50	0.50
1-Month	21	0.34	10	9	6	0.006	0.006	0.000	0.54	0.46

Table 6 reports the results of the Zivot and Andrews (1992) testing procedure for the Rényi measure of information. Column 3 reports the fraction of days in each subsample. Columns 4-6, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 7-9 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

Figure 7: Contribution to Gain

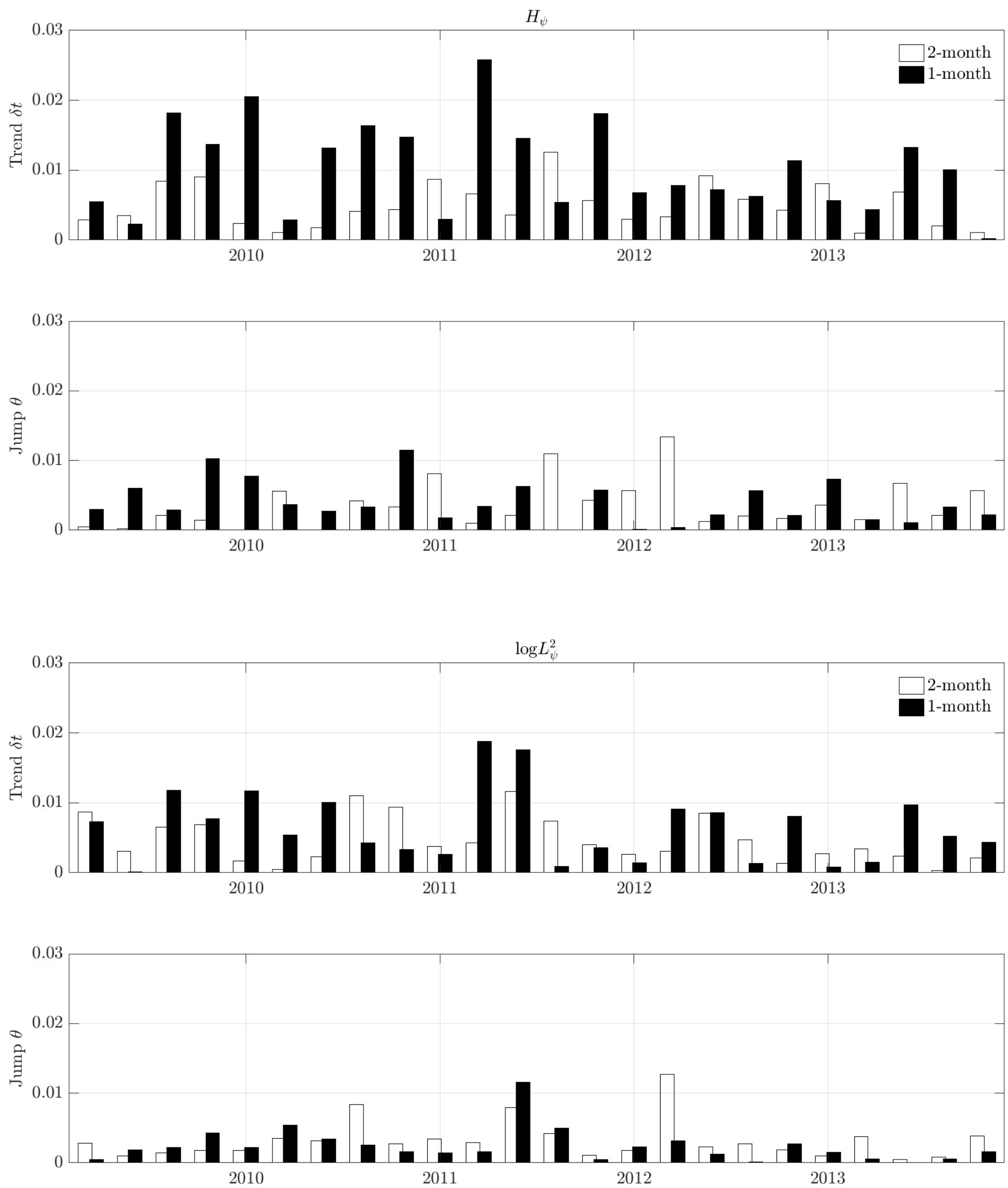

Figure 7 reports the average absolute trend and jump estimates for the first 2 months in white, and the final month in black for Shannon, H_ψ, and Rényi, $logL_\psi^2$, entropy. Figure 7 plots columns 8 and 9 of Table 5 and 6 respectively.

Only in the final month does information arrive often enough for the trend to contribute more to the total gained. The final two columns of Tables 5 and 6 report the average daily contribution of jump and trend estimates as a percentage of the total gain in information, across the first two and final months of the sample. On average, the trend contributes to a larger percentage gain in total information in the final month. As a result, jumps are shown to contribute a smaller percentage gain. That is not to say the importance of jumps diminishes. On average, a single jump continues to contribute nearly an equivalent amount of information as the estimated trend across the entire day. This effect is simply more pronounced earlier in the sample when the trend estimates are on average smaller.

The testing procedure reveals the change in magnitude of the trend and jump estimates in the final month of an option's life. Figure 8 present histograms of the estimated daily trend and jump components for each subsample and measure of entropy. In panel 1, the trend and jumps in Shannon entropy are shown for all 3-months in white, and the final month in grey. Panel 2 reports the trend and jumps in Shannon entropy for the first 2-months in white and the final month in grey. Looking across both panels, the shape of the distribution of jumps does not change in the final month of the sample. The shaded subsample remains bimodal, centered near zero, and slightly asymmetric. By comparison, the shape of the distribution of estimated trends is noticeably more skewed in the final month. In all cases the distribution of

Figure 8: Contribution to Gain

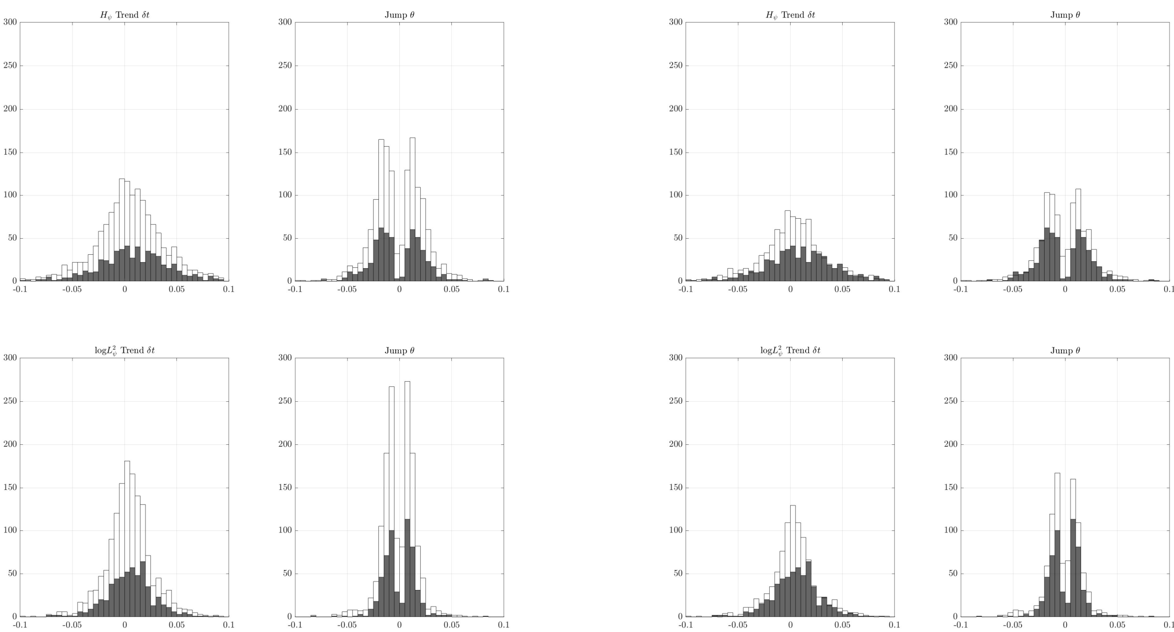

Figure 8 reports the estimated daily trend and jump estimates for each subsample and measure of entropy. In panel 1 and 3, the trend and jumps in Shannon, H_ψ, and Rényi, $logL^2_\psi$, entropy are shown for all 3-months in white, and the final month in grey, respectively. Panels 2 and 4 reports the trend and jumps in Shannon entropy for the first 2-months in white and the final 1-month in grey.

trend estimates remain unimodal, with the final month being negatively skewed and centered in the positive quadrant. This effect is most obvious in panel 2, where the occurrence or frequency of days with a large positive trend is near equal between the first two and final month of the sample, a counter-intuitive result given the relative lengths of the subsamples. The finding supports the conclusion that a large positive trend occurs predominately in the final 20 days of an option's life. Similar patterns emerge in the Rényi entropy.

In panel 3 and 4, the contribution of the final month to the right tail of the histogram is pronounced. Both results suggest that the gain in information is not constant across the life-cycle. Indeed, Figure 8 clearly shows that information accrues at different rates across the life-cycle. For both measures of entropy the estimated trend or continuous gain in information in the final month is on average larger, more frequent, and persistently more positive. Likewise, the estimated daily jumps or discreet gains in information are frequent, nonzero, and larger than the trend for many days. The result supports the conclusion that jumps contribute much to the total gain in information and, given the small trend, contribute the majority of information which accrues during the first two months.

VIII. Case-Studies

[Under Revision]

A. *S&P 500 Sets Record High*

[Under Revision]

Table 7: Identified Large Jumps—June 2014

	Jump $\hat{\theta}$		Time	
Date	Shannon H_ψ	Rényi $logL^2_\psi$	Shannon H_ψ	Rényi $logL^2_\psi$
30-May-14	0.061	0.020	12:55	12:50
28-Apr-14	0.041	0.021	13:50	13:50
20-May-14	0.035	0.016	12:05	11:55
8-May-14	0.023	0.019	10:30	12:40
29-May-14	0.039	0.020	10:40	10:40
7-Apr-14	0.029	0.019	13:45	13:45
23-May-14	0.031	0.014	9:00	9:00
3-Apr-14	0.029	0.018	14:40	14:30

Table 7 reports the jumps identified by the Zivot and Andrews (1992) testing procedure for the option expiring June 2014 option. Columns 2-5 report the magnitude and timing of each jump and for each measure of information.

[Under Revision]

Figure 9

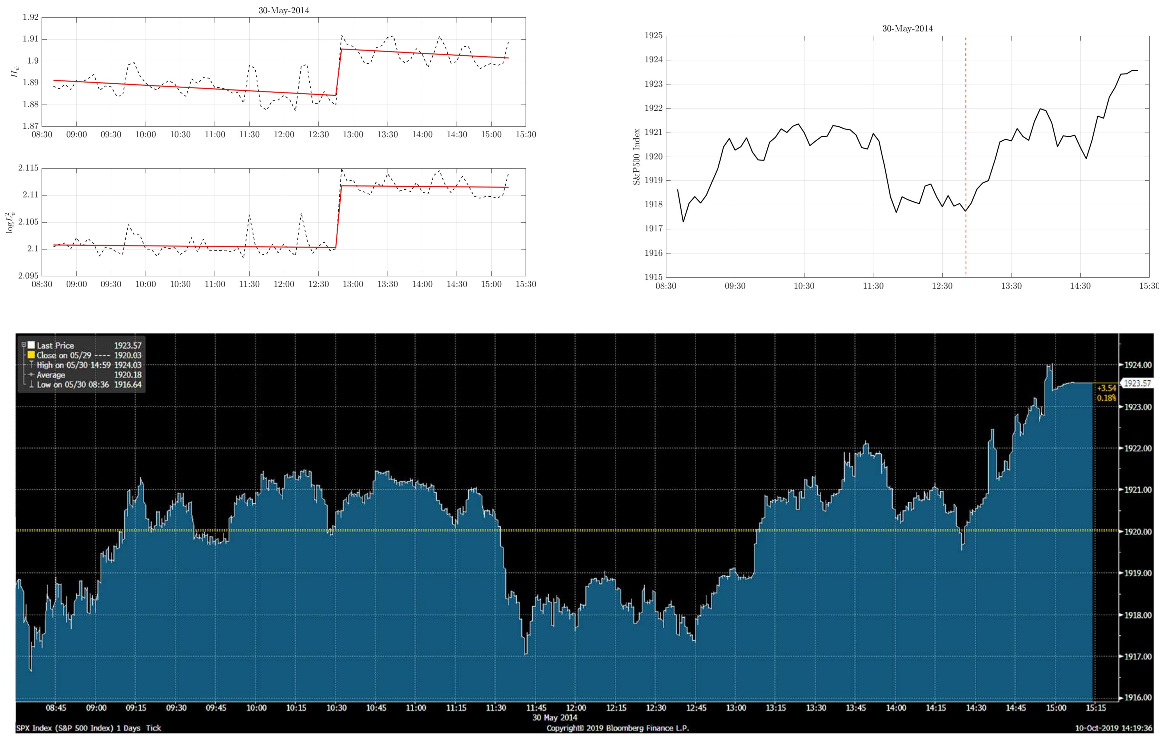

Figure 9 reports the case-study results for the jump identified on 30-May-2014 at 12:45 pm for both measures of information, Shannon entropy, H_ψ, and Rényi entropy, $logL_\psi^2$. Panel 1 shows the intraday evolution of each model of entropy in black, and the jump $\hat{\theta}$ and trend $\hat{\delta}t$ decomposition in red. The arrival of new information is shown both in the jump in Panel 1, and in the red line in Panel 2 for the 5-minute price level of the S&P 500 Index. Panel 3 shows the same intraday price information for the S&P 500 at the level of individual ticks.

[Under Revision]

IX. Conclusions

[Under Revision]

In this paper, I confront the basic characterization of the process by which investors learn about the future value of an asset. The paper is the first to quantify in real time how information drives price discovery in option markets. In doing so, I offer three methodological contributions to the literature on measuring the information found in option prices, and document two empirical facts not explained by existing theoretical models.

I provide a high-frequency analysis of the price discovery process in option markets. Using six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange, I characterize the intraday evolution of the density function implied by the price of options with the same maturity date. The analysis is done for the final 3 months of each option's life-cycle, as the density is shown to become more and more concentrated over time. This paper is the first to estimate the intraday dynamics of the risk-neutral density over the life-cycle, and offers the following three methodological contributions to the literature. First, I show how a simple nonparametric estimator can be used to approximate the implied density of future returns at high-frequencies. Second, I show how concepts developed in information theory can be used to quantify the amount of information contained in the estimated density. Third, I show how this novel approach permits a simple testing procedure for the presence of jumps in the evolution of the risk-neutral density, coinciding with the arrival of new information. The results of this testing procedure represent the paper's main contribution to the literature.

I investigate 'how information arrives in financial markets', and I find that information often arrives in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump, a result not anticipated by existing theoretical models. The testing procedure reveals both the frequency and magnitude of these jumps in investor expectations. I identify at least one jump for a majority of days, and find days without jumps contribute little to the total information gained over the life-cycle. I then document two empirical facts new to the literature: First, the majority of information accrues only in the final month. I show investors learn little about the future price of an asset for much of an option's life. Second, jumps contribute a majority of information early in the life-cycle. Only in the final month does information arrive often enough to contribute more to the total gained.

The paper builds on earlier work in many ways, but several features distinguish the findings from previous results. These include; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, 2. a fully non-parametric estimation technique, 3. a measure of information as a reduction in uncertainty, 4. a simple framework to test for jumps, and 5. the frequency and length of the sample of options data used.

REFERENCES

Äijö, Janne. 2008. "Impact of US and UK Macroeconomic News Announcements on the Return Distribution Implied by FTSE-100 Index Options." *International Review of Financial Analysis* 17 (2): 242–58.

Aït-sahalia, Yacine, and Andrew W Lo. 1998. "Nonparametric Estimation of State-Price." *The Journal of Finance* 53 (2): 499–547.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Clara Vega. 2003. "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange." *American Economic Review* 93 (1): 38–62.

Arrow, K. J. 1964. "The Role of Securities in the Optimal Allocation of Risk-Bearing." *The Review of Economic Studies* 31 (2): 91.

Askari, Hossein, and Noureddine Krichene. 2008. "Oil Price Dynamics (2002-2006)." *Energy Economics* 30 (5): 2134–53.

Bahra, Bhupinder. 1997. "Implied Risk-Neutral Probability Density Functions from Option Prices: Theory and Application." *Bank of England Working Papers*.

Benatia, David, Marine Carrasco, and Jean Pierre Florens. 2017. "Functional Linear Regression with Functional Response." *Journal of Econometrics* 201 (2): 269–91.

Birru, Justin, and Stephen Figlewski. 2012. "Anatomy of a Meltdown: The Risk Neutral Density for the S&P 500 in the Fall of 2008." *Journal of Financial Markets* 15 (2): 151–80.

Black, Fischer. 1976. "The Pricing of Commodity Contracts." *Journal of Financial Economics* 3 (1–2): 167–79.

Black, Fischer, and Myron Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (3): 637–57.

Bliss, Robert R., and Nikolaos Panigirtzoglou. 2002. “Testing the Stability of Implied Probability Density Functions.” *Journal of Banking and Finance* 26 (2–3): 381–422.

Breeden, Douglas T., and Robert H. Litzenberger. 1978. “Prices of State-Contingent Claims Implicit in Option Prices.” *The Journal of Business* 51 (4): 621.

Buchen, Peter W., and Michael Kelly. 1996. “The Maximum Entropy Distribution of an Asset Inferred from Option Prices.” *The Journal of Financial and Quantitative Analysis* 31 (1):

Campa, José M., P. H.Kevin Chang, and James F. Refalo. 2002. “An Options-Based Analysis of Emerging Market Exchange Rate Expectations: Brazil’s Real Plan, 1994-1999.” *Journal of Development Economics* 69 (1): 227–53.

Campbell, John Y., and Pierre Perron. 1991. “Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots.” *NBER Macroeconomics Annual* 6 (January): 141–201.

Canina, Linda, and Stephen Figlewski. 1993. “The Informational Content of Implied Volatility.” *Review of Financial Studies* 6 (3): 659–81.

Cheng, Kevin C. 2010. “A New Framework to Estimate the Risk-Neutral Probability Density Functions Embedded in Options Prices.” *IMF Working Papers* 10 (181):

Coutant, Sophie, Eric Jondeau, and Michael Rockinger. 2001. “Reading PIBOR Futures Options Smiles: The 1997 Snap Election.” *Journal of Banking and Finance* 25 (11): 1957–87.

Cover, Thomas M., and Joy A. Thomas. 2005. *Elements of Information Theory*. *Elements of Information Theory*.

Cox, John C., and Stephen A. Ross. 1976. “The Valuation of Options for Alternative Stochastic Processes.” *Journal of Financial Economics*.

Cox, John C, Stephen A Ross, Mark Rubinstein, Stephen Ross, Franco Modigliani, and Edward Lear Publishing. 1979. "Option Pricing: A Simplified Approach †."

Datta, Deepa Dhume, Juan M. Londono, and Landon J. Ross. 2017. "Generating Options-Implied Probability Densities to Understand Oil Market Events." *Energy Economics* 64 (1122): 440–57.

Debreu, G. 1959. *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*.

Dickey, David A., and Wayne A. Fuller. 1979. "Distribution of the Estimators for Autoregressive Time Series With a Unit Root." *Journal of the American Statistical Association* 74 (366): 427.

Diggle, Peter. 1985. "A Kernel Method for Smoothing Point Process Data." *Applied Statistics* 34 (2): 138.

Dumas, Bernard, Jeff Fleming, and Robert E. Whaley. 1998. "Implied Volatility Functions: Empirical Tests." *The Journal of Finance* 53 (6): 2059–2106.

Figlewski, Stephen. 2018. "Risk Neutral Densities : A Review Risk Neutral Densities : A Review."

Flamouris, Dimitris, and Daniel Giamouridis. 2002. "Estimating Implied PDFs from American Options on Futures: A New Semiparametric Approach." *Journal of Futures Markets* 22 (1): 1–30.

Frankel, Alexander, and Emir Kamenica. 2019. "Quantifying Information and Uncertainty." *American Economic Review* 109 (9): 3650–80.

Glatzer, Ernst, and Martin Scheicher. 2005. "What Moves the Tail? The Determinants of the Option-Implied Probability Density Function of the DAX Index." *Journal of Futures Markets* 25 (6): 515–36.

Goldberg, Linda S., and Christian Grisse. 2013. "Time Variation in Asset Price Responses to Macro Announcements." *SSRN Electronic Journal*, no. 626.

Harrison, J Michael, and David M Kreps. 1979. “Martingales and Arbitrage in Multiperiod Securities Markets.” *Journal of Economic Theory* 20 (3): 381–408.

Jackwerth, Jens Carsten. 1999. “Option-Implied Risk-Neutral Distributions and Implied Binomial Trees: A Literature Review.” *Journal of Derivatives*. Institutional Investor, Inc.

Jiang, George J., and Yisong S. Tian. 2005. “The Model-Free Implied Volatility and Its Information Content.” *Review of Financial Studies* 18 (4): 1305–42.

Jondeau, Eric, and Michael Rockinger. 2000. “Reading the Smile: The Message Conveyed by Methods Which Infer Risk Neutral Densities.” *Journal of International Money and Finance* 19 (6): 885–915.

Kang, Byung Jin, and Tong Suk Kim. 2006. “Option-Implied Risk Preferences: An Extension to Wider Classes of Utility Functions.” *Journal of Financial Markets* 9 (2): 180–98.

Kang, Byung Jin, Tong Suk Kim, and Sun-Joong Yoon. 2009. “Information Content of Volatility Spreads.” *Journal of Futures Markets*.

Kim, In Joon, and Sol Kim. 2003. “On the Usefulness of Implied Risk-Neutral Distributions – Evidence from the Korean KOSPI 200 Index Options Market.” *The Journal of Risk* 6 (1): 93–110.

Kitsul, Yuriy, and Jonathan H. Wright. 2013. “The Economics of Options-Implied Inflation Probability Density Functions.” *Journal of Financial Economics* 110 (3): 696–711.

Londono, Juan-Miguel, and Hao Zhou. 2012. “Variance Risk Premiums and the Forward Premium Puzzle.” *SSRN Electronic Journal*.

Malz, Allan M. 1997. “Option-Implied Probability Distributions and Currency Excess Returns.” *Staff Reports*.

Melick, William R., and Charles P. Thomas. 1997. "Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil during the Gulf Crisis." *The Journal of Financial and Quantitative Analysis* 32 (1): 91.

Merton, Robert C. 1973. "Theory of Rational Option Pricing." *Bell J Econ Manage Sci* 4 (1): 141–83.

Neuhaus, Holger. 1995. "The Information Content of Derivatives for Monetary Policy Implied Volatilities and Probabilities."

Ng, Serena, and Pierre Perron. 1995. "Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag." *Journal of the American Statistical Association* 90 (429): 268–81.

Perron, Pierre. 1989. "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57 (6): 1361.

Rényi, Alfréd. 1961. "On Measures of Entropy and Information." *Fourth Berkeley Symposium on Mathematical Statistics and Probability* 1 (c): 547–561.

Rubinstein, Mark. 1994. "Implied Binomial Trees." *The Journal of Finance* 49 (3): 771–818.

Rubinstein, Mark, and Jens Carsten Jackwerth. n.d. "Recovering Probability Distributions from Option Prices."

Said, Said E., and David A. Dickey. 1984a. "Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order." *Biometrika* 71 (3): 599–607.

Schennach, Susanne M. 2016. "Recent Advances in the Measurement Error Literature." *Annual Review of Economics* 8 (1): 341–77.

———. 2019. "Convolution without Independence." *Journal of Econometrics* 211 (1): 308–18.

Schwarz, Gideon. 1978. “Estimating the Dimension of a Model.” *The Annals of Statistics* 6 (2): 461–64.

Shannon, C. E. 1948. “A Mathematical Theory of Communication.” *Bell System Technical Journal* 27 (3): 379–423.

Shimko, David. 1993. “Bounds of Probability.” *Risk Publications* 6 (4): 33–37.

Shiratsuka, Shigenori. 2001. “Information Content of Implied Probability Distributions: Empirical Studies of Japanese Stock Price Index Options.”

Sims, Christopher A. 2003. “Implications of Rational Inattention.” *Journal of Monetary Economics* 50 (3): 665–90.

Stutzer, Michael J. 2000. “Simple Entropic Derivation of a Generalized Black-Scholes Option Pricing Model.” *Entropy* 2 (2): 70–77.

Syrdal, Stig Arild. 2002. “Working Paper Securities Markets and International Finance Department A Study of Implied Risk-Neutral Density Functions in the Norwegian Option Market.”

Weinberg, Steven Aric. 2001. “Interpreting the Volatility Smile: An Examination of the Information Content of Option Prices.” *SSRN Electronic Journal*.

Zivot, Eric, and Donald W.K. Andrews. 1992. “Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis.” *Journal of Business and Economic Statistics* 10 (3): 251–70.