

# Information Arrival in Financial Markets\*

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*This article introduces a new high-frequency analysis of six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange. I quantify in real time the information contained in the probability measure implied by option prices, using concepts developed in information theory. Here information is analogous to a reduction in uncertainty surrounding the future price of the underlying security. A simple nonparametric estimator allows us to measure the amount of information gained as an option approaches maturity. I then test for jumps in the expectation of said future price. I find the intraday flow of information in a large and important market is not continuous, and often increases in discrete intervals. This fact is used to identify events in which a large amount of information is revealed to investors.*

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# 1 Introduction

How does information arrive in financial markets? In this paper, I confront the basic characterization of the process by which investors learn about the future value of an asset. The topic is of importance to much of financial economics, yet continues to be one of the least explored. Indeed, this paper is the first to quantify in real time how information drives price discovery in option markets. In doing so, the paper offers three methodological contributions to the literature on measuring the information found in option prices, and documents two empirical facts not explained by existing theoretical models. First, I find the arrival of information drives jumps in investor expectations of the future price, and second, that this process is not constant over the life of an option.

This paper joins a growing literature of high-frequency analysis of investor expectations, of which Birru and Figlewski (2012) offer another example. Both the literature and this paper estimate the distribution of future returns as implied by observed option prices. Following Cox and Ross (1976) and Cox, Ross, and Rubinstein (1979), this estimation relies on a representative investor's ability to arbitrage an option and its underlying asset. In that way, all risk except the underlying uncertainty surrounding the asset's future price may be hedged away. The resulting implied distribution is known as the "risk-neutral density". Following Harrison and Kreps (1979), if this distribution is known, then options may be priced as if all investors are "risk-neutral". In

other words, the price of an option is independent of the individual risk-preferences of an investor. Figlewski (2018) offers a recent review of the key ideas in this literature. Option prices therefore reflect investor beliefs over the probability the world will achieve some future state, and it is this information that is of vital interest to investors, researchers, and policymakers. This fact combined with the growth in derivative markets has inspired renewed interest in understanding how the beliefs of investors respond to new information.

In this paper, I provide a high-frequency analysis of the price discovery process in option markets. Using six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange, I characterize the intraday evolution of the density function implied by the price of options with the same maturity date. The analysis is done for the final 3 months of each option's life cycle, as the density is shown to become more and more concentrated over time. This paper is the first to estimate the intraday dynamics of the risk-neutral density over the life cycle, and offers the following three methodological contributions to the literature. First, I show how a simple nonparametric estimator can be used to approximate the implied density of future returns at high frequencies. Second, I show how concepts developed in information theory can be used to quantify the amount of information contained in the estimated density. Third, I show how this novel approach permits a simple testing procedure for the presence of jumps in the evolution of the risk-neutral density, coinciding with the arrival of new information. The results of

this testing procedure represent the paper's main contribution to the literature.

I find that information often arrives in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump, a result not anticipated by existing theoretical models. The testing procedure reveals both the frequency and magnitude of these jumps in investor expectations. I identify at least one jump for a majority of days, and find days without jumps contribute little to the total information gained over the life cycle. I then document two empirical facts new to the literature: First, the majority of information accrues only in the final month. I show investors learn little about the future price of an asset for much of an option's life. Second, jumps contribute a majority of information early in the life cycle. Only in the final month does information arrive often enough to contribute more to the total gained.

The paper builds on earlier work in many ways, but several features distinguish the findings from previous results. These include; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, 2. a fully non-parametric estimation technique, 3. a measure of information as a reduction in uncertainty, 4. a simple framework to test for jumps, and 5. the frequency and length of the sample of options data used. These features are discussed briefly in turn.

This paper targets the evolution of the risk-neutral density over time. In a departure from much of the earlier literature (Aït-Sahalia and Lo, 1998), among others, the density is not estimated for a fixed maturity. Instead, for

options with the same expiration date, the focus is on how the density becomes more concentrated as the maturity date approaches. The new perspective is shown to permit a simple testing procedure for the presence of jumps in the evolution of the density, and does not require the estimated density to be interpolated over time.

The paper employs an estimation procedure that is fully nonparametric. Inspired by the original procedure proposed by Breeden and Litzenberger (1978), the estimator places no restrictions on the shape of the density function or the dynamics of the underlying asset's price. In contrast to semiparametric procedures, such as those following Shimko (1993), the estimation procedure does not require the interpolation or the extrapolation of observed option prices, or the need for significant tradeoffs in measures of goodness-of-fit and smoothness.

This paper introduces concepts from information theory to quantify the uncertainty investors face about the future value of an underlying asset. The basic insight is that information can be measured as the reduction in uncertainty over time. The basic quantity of information theory is entropy. The concept is not new to economics, and both Sims (2003) and Frankel and Kamienica (2019) use entropy to model rational inattention and the information in decision problems respectively. Stutzer (2000) and Buchen and Kelly (1996) go so far as to use the concept of maximum entropy to infer the risk-neutral density from observed prices. However, this paper represents the first

application of entropy to the problem of quantifying the information gained in the density function over time.

The paper employs a framework and hypothesis test for jumps in the risk-neutral density. Here the evolution of the entropy of the density reflects the arrival of new information. The procedure is equivalent to testing for jumps in a nonstationary time series, and follows Zivot and Andrews (1992). The test derives from the literature on testing for unit roots in economic time series, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984).

The choice of procedure is threefold. First the test is straightforward and transparent. It is fast and simple to implement, and the results are easily interpreted. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items that are of immediate interest. Third, the procedure permits a statistical test for each identified jump. In this respect, the analysis differs from other high-frequency event-studies, such as Goldberg and Grisse (2013) and Andersen, Bollerslev, Diebold and Vega (2003), among others, who examine the response of interest rates and exchange rates to select economic news announcements.

Finally, the paper uses a new dataset of intraday quotes for all options written on the S&P 500 and traded on the Chicago Board of Exchange. The data was purchased with the support of the Weidenbaum Center on the Economy, Government, and Public Policy at Washington University in St. Louis. The data is novel in two respects, namely the high frequency and long

calendar span of the sample of SPX options analyzed. The intraday analysis covers six years, or nearly 1,500 trading days, beginning in January 2009 and ending in December 2014. In comparison to other high-frequency studies, Jiang and Tian (2005) and Birru and Figlewski (2012), who focus on forecasting realized variance and the change in the quantiles of the risk-neutral density during the fall of 2008, the data and analysis presented here represents a more complete picture of the intraday evolution of investor expectations.

The paper proceeds as follows. Background in section 2. Section 3 describes the high-frequency options data sample. Section 4 proposes the use of information theory to quantify the information in the estimated density. Section 5 characterizes the nonparametric estimator used to approximate the risk-neutral density. Section 6 describes the framework to test for jumps in the arrival of information. Section 7 discusses the results of the hypothesis tests. Section 8 presents select case studies around events where a large jump in information was identified. Section 9 concludes.

## 2 Background

### 2.1 Options

A derivative is a financial instrument whose value depends on the price of an underlying asset. For example, the value of the index options used in this paper “derive” from the price of an underlying stock index, the S&P 500. Introduced in 1957, the S&P 500 was the first stock index weighted by market

capitalization. Today over \$9.9 trillion dollars is benchmarked to the index, with indexed assets totaling \$3.4 trillion. The S&P 500 covers roughly 80% of total US market capitalization. Options written on the index trade worldwide, in both over-the-counter and exchange-traded markets. The largest and most liquid exchange-traded contract is the SPX, traded on the Chicago Board of Exchange (CBOE). The CBOE is the largest exchange for trading stock options. An options exchange offers standardized contracts and manages credit risk between counterparties, typically through a centralized clearing house. The CBOE began trading standardized contracts in 1973. Today, the notional value of SPX options is roughly \$5.5 trillion, with the number of open contracts exceeding 20 million. In 2019, the average daily volume exceeded 1.28 million contracts.

The SPX contracts used here are European-style call options on the underlying S&P 500 Index. The buyer of a European call option has the right to buy the underlying asset on a predetermined date ( $T$ ) and for a predetermined price ( $K$ ). The predetermined date is known as the expiration or maturity date. The price is the exercise or strike price. A single SPX contract is for 100 times the index at the given strike price. Settlement for index options is always in cash. If the buyer chooses to exercise the call option, they receive  $(X_T - K) \times 100$  from the seller where,  $X$  is the settlement value of the index. Strike prices for SPX options are defined by the exchange. Strike prices for options near the value of the underlying index are typically offered at \$5 intervals. For options far from the current value of the underlying index, strike

prices may only be available at \$10, \$25, \$50, or \$100 intervals. Trading in SPX options ends on the business day before the day on which the final settlement value of the index is calculated.

SPX options trade on a March cycle. All US stock options trade on either a January, February, or March cycle. The SPX cycle consists of options expiring every 3 months; March, June, September, and December. The standard expiration date is the third Friday of the near expiration month. For example, the “June call” is the SPX call option expiring on the third Friday of June, and trading in the June call ends on the Thursday before the third Friday. When an option expires, trading in a new option with a maturity date set in the next expiration month in the cycle begins. Regular trading in SPX options occurs every business day from 08:30 to 15:15 Central Time.

## 2.2 Risk-Neutral Valuation

Risk-neutral valuation is a general result in the pricing of derivatives. In theory, when valuing a derivative we may assume that investors are risk neutral. This assumption means that the individual risk-preferences of investors do not enter into the pricing equation. Introduced by Cox and Ross (1976) and expanded on in Cox, Ross, and Rubinstein (1979), the concept arises from the relationship between a derivative and its underlying asset. Since both are affected by the same underlying source of uncertainty, a portfolio can be constructed to arbitrage any gain or loss in the asset with an equivalent loss or

gain in the derivative. The resulting portfolio would have no risk, and its return would be the risk-free rate. The cost of constructing such a portfolio is then the price of the derivative. Harrison and Kreps (1979) show that in a market with no profitable arbitrage opportunities, the risk-neutral price is the correct price, even under risk aversion. In a complete market their result can be extended to include the risk-neutral price being unique.

The idea that information could be extracted from the observed price of an option was introduced by Black and Scholes (1973). By inverting the valuation formula, Black and Scholes were able to estimate the future volatility of an asset, as implied by the current price of its derivative. Merton (1973) extends the Black-Scholes model to continuous time. In continuous time, the evolution of the price of the underlying asset is modelled as a diffusion process. The Black-Scholes-Merton model assumes this process to be a geometric Brownian motion. This assumption implies the shape of the terminal distribution of the asset's price is lognormal. If known, this distribution may be used to price an option independent of the risk preferences of investors. This is the basic idea of risk-neutral valuation. If it is possible to arbitrage an asset and its derivative, the derivative may be priced as if it were riskless. The terminal distribution required for obtaining the riskless price is known as the "risk-neutral" density.

In practice, the ability to continuously rebalance such a portfolio is limited by transaction costs and financing requirements. The resulting arbitrage is then unlikely to be riskless. Consequently, theoretical models do not always

perform well empirically. Black and Scholes (1972) discovered this fact early on, noting that for options on the same stock the volatility implied by their model is not constant across strike prices, thus inventing the well-known Black- Scholes “volatility smile”. There now exists a large body of empirical evidence that implied volatility is not constant across strike prices and maturities. Taken together, the evidence strongly suggests the assumption of a lognormal terminal distribution implied by diffusion process that is a geometric Brownian motion does not hold in practice. A phenomenon known as the Black-Scholes-Merton anomaly. Figlewski (2018) offers a more intensive introduction to the key ideas, issues, and finding introduced here.

The Black-Scholes-Merton anomaly has led to a large literature that attempts to estimate a risk-neutral density which better fits the observed option prices, and it is to this literature that the nonparametric estimator I propose contributes. The discussion of the estimator is therefore limited to a class of nonparametric and semiparametric methods. This class of methods includes other implementations of Breeden and Litzenberger (1978), (Shimko (1993), Malz (1997), Bliss and Panigirtzoglou (2002), Weinberg (2001), and Dumas, Fleming, and Whaley (1998)), and other methods based on such concepts as maximum entropy (Buchen and Kelly (1996), and Stutzer (1996)), kernel regression (Ait-shalia and Lo (1998)), and the binomial tree method of Cox, Ross, Rubinstein (1979), (Rubinstein (1994), Rubinstein (1996), Jackwerth (1997,1999)). For an extensive review of methods for estimating the density from observed option prices see Jackwerth (1999), Jondeau and Rockinger

(2000), and Bliss and Panigirtzoglou (2002) Coutant, Jondeau, and Rockinger (2001), and Datta, Londono, and Ross (2017)).

### 3 Data

Throughout the paper I use data for a sample of options written on the S&P 500 and traded on the Chicago Board of Exchange. The data was purchased with the support of the Weidenbaum Center on the Economy, Government, and Public Policy at Washington University in St. Louis. The data is novel in two respects, principally the high frequency and long calendar span of the sample of SPX options analyzed. The data represent the most complete sample of high-frequency investor expectations to date. Here I describe the data in detail.

Daily ‘Trade And Quote’ or TAQ data were obtained from the Chicago Board of Exchange (CBOE). The raw files contain all trades and quotes for options written on the S&P 500, and traded on the CBOE. The raw data consists of daily files for each of what is typically 252 trading days per year. The sample covers six years, or approximately 1,500 trading days beginning January 2, 2009 and ending December 31, 2014. Regular trading hours occur from 08:30 to 15:15 Central Time each day. Quotes are updated throughout the day for all available strikes. Trades in SPX options are sparse relative to quotes, and are rarely observed simultaneously across strikes. To avoid introducing additional pricing-errors, quotes are used in the analysis which follows.

Each quote consists of bid and ask price, size, and the corresponding strike price, together with a timestamp, underlying index price, and flags indicating the class of option and market condition, either open or pre-open. Few quotes are observed pre-open.

The traditional SPX options chain is AM-settled on the third Friday of every month. Options trading on the March cycle consist of those expiring every 3 months; March, June, September, and December. Nontraditional SPX options are PM-settled, with varying expiration dates, including the last trading day of the month and weekly options. ‘Long-term Equity AnticiPation Securities’ or LEAPS are traditional SPX options with expiration dates up to five years in the future.

The sample consists of 24 nonoverlapping option chains, beginning with the options expiring March 21, 2009 and ending with those expiring December 20, 2014. Following convention, option chains are referred to by their expiration date. There are on average 62 trading days for each curve, with the March 2009 option having only 55 trading days observed. The sample consists of 83 5-minute intervals for each trading day, beginning at 08:25 and ending at 15:15 Central Time (CT). Intraday quotes are aggregated by strike to the corresponding 5-minute interval.

Only quotes for traditional SPX call options are used, those which are AM-settled on the third Friday of the near expiration month. Nontraditional SPX options and LEAPS are excluded from the sample. To create the 5-minute samples used in the analysis, the average of the best bid and offer were taken

for the sample of options with quotes posted in the preceding 5-minute. The reported 09:00 call price for a given strike is then the mid-price of the best bid and best offer observed from 08:55:00.000 to 08:59:59.999. In the rare instance where quotes were not observed for a particular strike, the average of the best bid and offer price were carried over from the previous interval.

There are two additional notes regarding the sample. First, trading in traditional SPX options ends on the business day before the settlement date, and before trading in a new option in the following cycle begins. No quotes for the current option chain are observed on these days, and they are excluded from the sample. Second, far out-of-the-money options are often illiquid, and I exclude call options written on strikes issued at intervals greater than \$50.

The result is a sample of 1,479 days, each day containing 83 5-minute intervals. Each interval contains the average of the best bid and best offer price for call options written on strike prices at intervals less than \$50. Table 1 reports daily summary statistics for the sample of options and the underlying index. Columns 1-3 report the average number, minimum and maximum observed strikes, call and underlying index prices observed for each day in the sample. Columns 4 and 5 report the average and standard deviation of the sample of strike, call and underlying index prices for each day. Table 1 reports summary statistics for the entire six-year sample, as well as these statistics for each year. Table A in the appendix reports the separate summary statistics for each option chain.

The number of strikes observed each day is large. Looking at Table 1, there are between 117 and 187, and on average 146, strikes observed. Strike prices range from \$150 to \$2,250 and are traded at intervals of less than \$50. The options sampled cover a large degree of moneyness, with the average ratio of the strike price to the current index price ranging from 0.3 to 1.4. Thus on a typical day, probabilities between a 70% decrease and 40% increase in the index over the following days are observed.

The average number of quotes observed each day is also large. For the options sampled, I observe on average approximately 718,000 quotes per day, ranging in price from \$0.30 to \$1,373, with an average mid-price of about \$243 and average daily standard deviation of \$219. The value of the underlying index ranges from \$674 to \$2,075, with an average price of around \$1,380, and a small average daily standard deviation of just \$0.18. Finally, the observed number of strikes and quotes increases consistently over the six years in the sample, as does the value of the underlying index. For simplicity and convenience, much of the analysis refers to options expiring in either the first or last year of the sample, 2009 and 2014. This is done to show the findings are not artifacts of any larger trends in the market. Figure 1 reports the daily level of both the underlying S&P500 Index and the CBOE Volatility Index or VIX Index for the sample period. For much of the sample, the value of the underlying index is increasing. Implied volatility as calculated by the CBOE is highest early in the sample, spikes in mid-2010 and late 2011, and declines

steadily from the beginning of 2012. In Table 1 these trends are evident in the range of strike prices available and the average value of the underlying index.

**Figure 1: Underlying Index Value and Volatility**

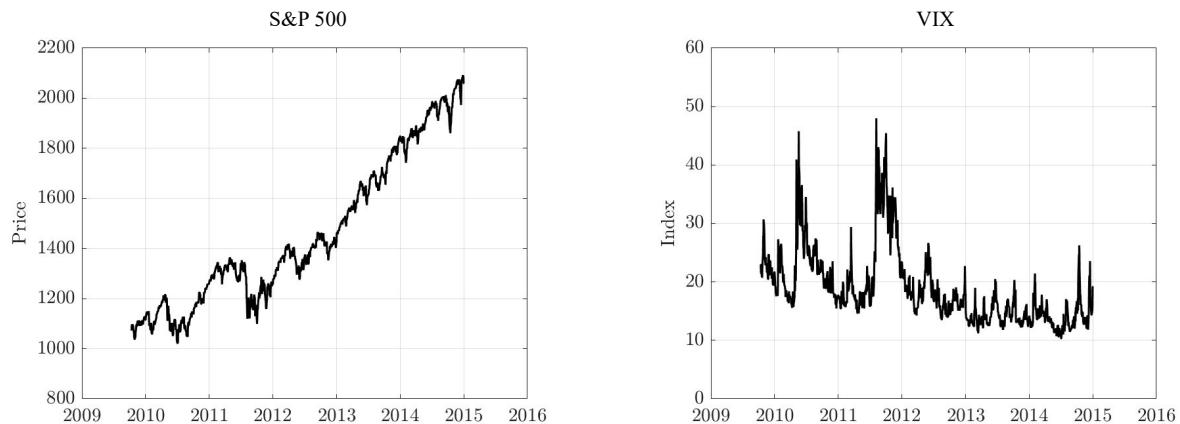


Figure 1 reports the daily level of both the underlying S&P 500 Index and the CBOE Volatility Index or VIX Index for the sample period. Source: Fred and Chicago Board Options Exchange.

**Table 1: Daily Summary Statistics**

Year		Count	Minimum	Maximum	Average	Std. Dev.
All	<i>Strikes</i>	146	150	2,250	1,185.85	265.62
	<i>Calls</i>	717,850	0.0300	1,373.14	242.59	219.01
	<i>Underlying</i>	704,148	674.1500	2,075.37	1,380.50	0.18
2009	<i>Strikes</i>	117	200	1,650	880.37	74.42
	<i>Calls</i>	713,010	0.0300	663.64	145.91	40.37
	<i>Underlying</i>	713,317	674.15	1,114.16	941.21	112.36
2010	<i>Strikes</i>	125	350	1,500	996.84	22.67
	<i>Calls</i>	461,231	0.0300	892.70	189.69	35.84
	<i>Underlying</i>	459,336	1,022.24	1,242.87	1,134.38	51.55
2011	<i>Strikes</i>	138	150	1,600	1,087.30	62.90
	<i>Calls</i>	646,719	0.0300	1,131.37	232.68	41.98
	<i>Underlying</i>	613,337	1,099.65	1,363.65	1,268.69	62.43
2012	<i>Strikes</i>	150	350	1,800	1,157.43	53.06
	<i>Calls</i>	527,438	0.0300	1,105.25	255.45	26.21
	<i>Underlying</i>	511,441	1,203.72	1,464.25	1,373.27	51.94
2013	<i>Strikes</i>	160	350	2,100	1,389.30	101.63
	<i>Calls</i>	792,863	0.0300	1,212.67	277.69	40.67
	<i>Underlying</i>	790,280	1,403.03	1,810.83	1,634.21	98.41
2014	<i>Strikes</i>	187	650	2,250	1,600.05	41.70
	<i>Calls</i>	1,170,790	0.0300	1,373.14	352.24	42.29
	<i>Underlying</i>	1,165,436	1,741.25	2,075.37	1,923.74	76.28

Table 1 reports standard summary statistics for the sample of call options used. Column 1 reports the average number of options, bids, and quotes observed each day. Columns 2 through 4 report their minimum, maximum, and average values for the sample or year listed. Column 5 reports the standard deviation of the average.

## 4 Information

### 4.1 In Option Prices

Black and Scholes (1973) introduced the idea that information could be extracted from the price of traded options. In general, option prices reflect investor beliefs over the probability the underlying asset will take a particular value. Consider two options with adjacent strikes. Intuitively, any difference in their price must reflect the likelihood that the value of the underlying asset will fall between them. This holds across all strikes. Therefore the observed difference in option prices across strikes, or state-prices, contains information about how likely investors believe different states are. Breeden and Litzenberger (1978) show how to obtain the entire risk-neutral density from these differences. Here the information contained in option prices is extracted as a probability distribution over the future value of the underlying index. This information allows for a more complete view of investor beliefs; both how they evolve over time, and how they change in response to new information or events. To extract this information, option prices are combined across strikes to mimic a security which pays \$1 if an individual state is realized, and \$0 if not. The price of such security would then be proportional to the probability of a state occurring. This idea was first introduced in time-state preference model of Arrow (1964) and Debreu (1959). The result is known as an ‘Arrow-Debreu’ security, a type of elementary claim whose value is contingent on a state being realized.

Interpreting the information extracted from option prices requires summarizing the characteristics of the implied probability distribution of random process. There much of early the literature focused on using the risk-neutral density to estimate the implied volatility of the underlying process, often for the purpose of forecasting realized volatility (Canina and Figlewski (1993), Weinberg (2001), and Jiang and Tian (2005), among others). Later literature has expanded the focus to examining the response of higher-order moments, principally skewness and kurtosis, to key events, see for example Birru and Figlewski (2012). The approach has known limitations, including the sensitivity of the estimated risk-neutral density to the choice of estimation methods. As a result, quantile moments are often used and are found to be a more robust (Datta, Londono, and Ross (2014), Flamouris and Giamouridis (2002) and Campa, Chang, and Refalo (2002), Birru and Figlewski (2012)).

A growing literature studies the information contained in option prices in this manner, using the moments of the risk-neutral density to examine a wide range of markets; including but not limited to international stock indexes, (Kang and Kim (2006), Shiratsuka (2001), Glatzer and Scheicher (2005), Äijö (2008), Kim and Kim (2003), and Syrdal (2002)), exchange rates, (Weinberg (2001), Bahra (1997), Campa, Chang, and Refalo (2002), and Londono and Zhou (2012)), government bonds, (Neuhaus (1995), Shiratsuka (2001), and Cheng (2010)), inflation options, (Kitsul and Wright (2012)), and oil commodities, (Datta, Londono, and Ross (2014), Melick and Thomas (1997), Flamouris and Giamouridis (2002), and Askari and Krichene (2008)).

This paper contributes to the literature analyzing the information contained in the price of index options, but departs in the methodology employed. Similar to Weinberg (2001), Jiang and Tian (2005), Kang, Kim, and Yoon (2010), and Birru and Figlewski (2012), the focus here is for options written on the S&P 500. Two features however distinguish the analysis from earlier work. This include a departure from using both a fixed horizon risk-neutral density and the moments of the estimated density function. Instead, I propose the use of information theoretic concepts to quantify the amount of information investors gain about the likely value of the underlying index at maturity over time. It is to these concepts I now turn.

## 4.2 Information Theory

The basic insight of information theory is to measure information as a reduction in uncertainty, and the basic building block is entropy. Entropy quantifies the amount of uncertainty in a random variable, here the outcome of a stochastic process (Shannon 1948). Given a random variable  $X$ , with probability density function  $p(X)$ , entropy is simply  $-E[\log(p(X))]$ . It should be noted that the concept applies whether  $X$  is a discreet or continuous random variable. That is, whether  $p(X)$  is a density with respect to a Lebesgue measure on  $\mathcal{R}^k$ , or a discreet measure on a countable set of points. By convention,  $p \log p = 0$  for any value  $X$  such that  $p = 0$ . The base of the logarithm determines only a scaling factor for the amount of information. Base 2 is often

used as intuitively the outcome of a fair coin toss contains one “*bit*” of information. Here I provided a brief introduction to the basic ideas of information theory, for a complete technical introduction see Cover and Thomas (1991).

The amount of information or entropy in an event is calculated using the probability of that event. The more deterministic or certain an event is, the less surprising or informative its realization is. The ‘surprisal’ or ‘self-information’ of a discreet event  $x$  is defined as  $h(x) = -\log_2(p(x))$ , and is 0 when  $p(X) = 1$ . Here the intuitive is clear, since no information is learned from a certain n outcome. The classic example is that of a fair coin toss, where the self-information of a single toss is  $\log_2(1/2) = 1$ . Compare this event to the outcome of a fair dice, and the intuition and idea behind entropy becomes even clearer,  $-\log_2(1/6) \approx 2.58$ . In a sentence; the more certain an event is, the less informative its outcome, and the lower its entropy.

Calculating the information in a random variable is the same as calculating the information of the probability distribution of the events of a random variable. In other words, the amount of information in a random variable is the surprisal of each event weighted by the probability of those events. In practice, estimating the entropy of a random variable is equivalent to estimating the information or surprisal for the probability distribution of events. Entropy then is the average amount of information of an event drawn from the probability distribution of a random variable.

In this paper, I propose the use of entropy to quantify the amount of information in the risk-neutral density implied by option prices. I employ two measures of information, the traditional Shannon (1948) entropy, and the generalized Rényi (1961) entropy. Shannon entropy is the expected or average amount of information for an event drawn from its distribution. Rényi entropy is equivalent to the  $L_2$  norm of the distribution. Shannon entropy can be shown to generalize to Rényi entropy. Both concepts measure how broadly distributed the outcome of a random variable is. The higher the entropy, the broader the distribution. For example, the uniform distribution is the ‘maximum entropy distribution’ on any given interval  $[a, b]$ .

Two final points of clarification, I break somewhat with convention in the analysis that follows by reporting  $-H(x)$  not  $H(x)$ , and using the natural logarithm in place of log base 2 for both Shannon and Rényi entropies. The units are then known as “*nats*”. This is done only for convenience, and does not affect the results. The negative rescaling simply allows for both entropies to be displayed on a single positive axis, and for the magnitude and direction of the gains in information, trend, and jumps to be more readily compared. For the remainder of the paper, when uncertainty declines the reported measures entropy will be shown to increase. Finally, entropy is not variance, which is the measure of variation in a random variable. No, here entropy is defined as a measure of uncertainty in the distribution of probabilities, and is not equivalent to implied volatility.

**Table 2: Entropy Summary Statistics**

Maturity	Days	Level Shannon			Standard Deviation			Level Rényi			Standard Deviation		
		3-Month	1-Month	1-Day	3-Month	2-Month	1-Month	3-Month	1-Month	1-Day	3-Month	2-Month	1-Month
2009-03-21	53	0.47	0.71	1.83	0.021	0.018	0.025	0.62	0.93	2.00	0.022	0.020	0.023
2009-06-20	62	0.27	1.03	1.98	0.017	0.014	0.022	0.44	1.24	2.15	0.016	0.014	0.019
2009-09-19	62	0.59	1.10	2.04	0.017	0.015	0.020	0.81	1.32	2.22	0.015	0.014	0.015
2009-12-19	63	0.69	1.30	2.06	0.019	0.020	0.018	0.94	1.52	2.24	0.017	0.018	0.016
2010-03-20	60	0.85	1.42	2.11	0.018	0.019	0.016	1.09	1.66	2.28	0.015	0.017	0.012
2010-06-19	62	0.99	1.16	2.07	0.026	0.023	0.031	1.24	1.42	2.24	0.025	0.022	0.030
2010-09-18	62	0.86	1.20	2.08	0.019	0.019	0.020	1.13	1.42	2.25	0.019	0.019	0.019
2010-12-18	63	0.82	1.32	2.12	0.027	0.030	0.022	1.09	1.56	2.28	0.022	0.025	0.016
2011-03-19	61	0.98	1.59	2.08	0.030	0.032	0.027	1.25	1.85	2.25	0.024	0.024	0.023
2011-06-18	62	0.92	1.48	2.11	0.026	0.029	0.019	1.20	1.72	2.27	0.022	0.025	0.017
2011-09-17	62	0.94	1.07	1.96	0.032	0.029	0.039	1.19	1.31	2.18	0.029	0.028	0.033
2011-12-17	63	0.48	1.02	2.03	0.039	0.043	0.029	0.77	1.32	2.23	0.030	0.035	0.020
2012-03-17	60	0.67	1.44	2.14	0.021	0.022	0.019	0.93	1.69	2.31	0.016	0.017	0.014
2012-06-16	62	1.07	1.23	2.09	0.030	0.030	0.030	1.34	1.51	2.26	0.022	0.021	0.023
2012-09-22	67	0.91	1.64	2.17	0.025	0.025	0.023	1.20	1.87	2.33	0.022	0.021	0.023
2012-12-22	61	1.16	1.59	2.13	0.024	0.026	0.021	1.41	1.80	2.30	0.020	0.023	0.016
2013-03-16	55	1.05	1.73	2.13	0.021	0.018	0.027	1.33	1.97	2.32	0.016	0.013	0.020
2013-06-22	67	1.25	1.67	2.08	0.017	0.016	0.020	1.49	1.89	2.27	0.014	0.013	0.016
2013-09-21	62	0.99	1.56	2.16	0.020	0.019	0.020	1.24	1.80	2.34	0.014	0.014	0.015
2013-12-21	63	1.18	1.72	2.14	0.019	0.016	0.026	1.42	1.94	2.33	0.015	0.012	0.020
2014-03-22	60	1.30	1.68	2.20	0.021	0.018	0.027	1.55	1.93	2.35	0.015	0.014	0.015
2014-06-21	62	1.25	1.73	2.19	0.018	0.018	0.020	1.51	1.96	2.36	0.014	0.014	0.013
2014-09-20	62	1.40	1.79	2.19	0.019	0.017	0.021	1.66	2.02	2.35	0.014	0.014	0.014
2014-12-20	63	1.36	1.72	2.10	0.022	0.020	0.027	1.62	1.95	2.32	0.018	0.017	0.021
<i>Average</i>	62	0.94	1.41	2.09				1.19	1.65	2.27			

Table 2 reports the level of both measures of information, Shannon and Rényi entropy, at three points in each option's life-cycle, 3-months, 2-months, and 1-day from maturity, as well as the standard deviation of each measure of information over that respective month.

**Figure 2: Level of Information**

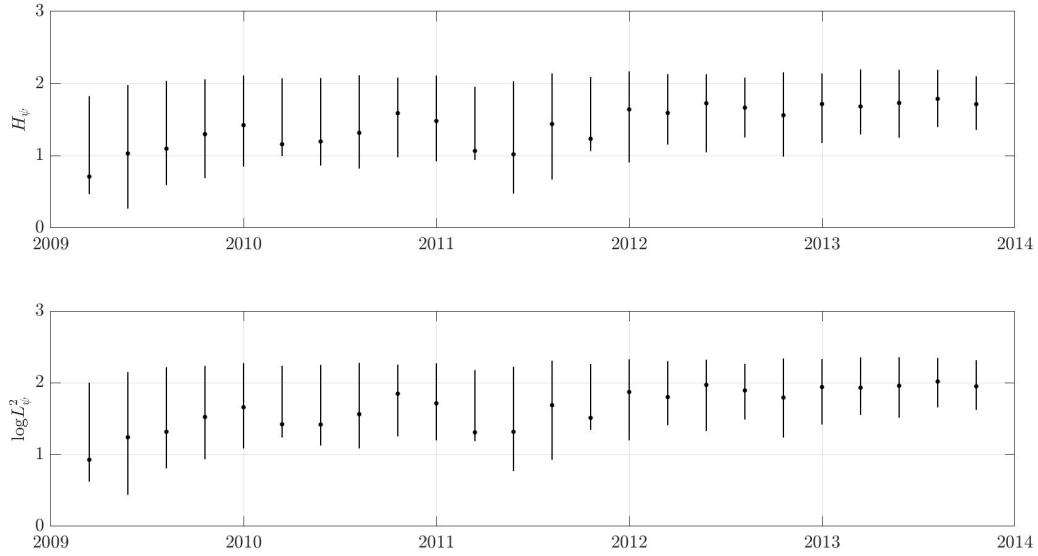


Figure 2 reports the level of Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ , entropy for each of 24 option chains over their 3-month life cycles. Each bar represents the initial 3-month (lower bound), 2-months (center dot), and final 1-day (upper bound) level of information over time.

## 5 Estimation

### 5.1 Semiparametric

Two features distinguish the following estimation from earlier work; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, and 2. a fully nonparametric estimation technique. These features are discussed briefly now.

Nonparametric estimation places no restrictions on the shape of the density function, and makes no assumptions on the dynamics of an underlying asset's

price. Applying the original Breeden and Litzenberger (1978) method requires only the assumption that markets are competitive, and that the call price function is twice differentiable. Yet in much of the literature, semiparametric methods are preferred. The reasoning for this is twofold. First, semiparametric techniques are easier to estimate and require fewer assumptions than parametric methods. Second, semiparametric methods are thought to produce estimates of the density that are smoother and more interpretable than nonparametric techniques.

Semiparametric methods require the interpolation and extrapolation of observed option prices. Since prices are available for only a finite number of discrete strikes, the call function must first be interpolated across strike-prices. Popular methods, such as those following Shimko (1993), begin by converting prices to implied volatilities and interpolating the volatility ‘smile’. The interpolated smile is then converted back into prices. This process produces a continuous and twice-differentiable call price function, but introduces additional pricing errors. Furthermore, substantial effort must be exerted to choose a curve fitting method that minimize the added pricing errors while also producing an estimate of the density function that is sufficiently smooth (Jondeau and Rockinger (2000) and Campa, Chang, and Refalo (2002)).

Semiparametric methods may also require the interpolated function to be extrapolated beyond the observed strike prices to ensure the estimated probabilities form a sufficient density, that is, sum to one. If so, another choice must be made of how to extrapolate either the interpolated call price function

or volatility smile, and additional effort must be exerted to minimize additional errors (Datta, Londono, and Ross, 2014). This is often required because prices are not observed for a sufficient range of strike prices, or the observed prices are not sufficiently liquid. Finally, options have fixed maturities, and do not mature every day. Therefore to generate a fixed-horizon density, as is common in the literature, requires interpolating the estimated functions not only across strikes, but across days. This may introduce further errors into the call price function, which requires additional tradeoffs between smoothness and goodness-of-fit.

The testing procedure employed here does not rely on a fixed-horizon estimate of the density function. In a departure from the literature, the target of the estimation is the evolution of the risk-neutral density over an option's life cycle, and not the day-to-day variation of the density for a fixed maturity. This is done for two reasons. First, targeting the dynamics removes the need for further interpolation of the call price function, and second, the approach permits a simple testing procedure for the presence of jumps in the evolution of the density. As a result, the nonparametric estimation does not require the interpolation or the extrapolation of prices across either strikes or time.

## 5.2 Nonparametric

Recall that the buyer of a call option has the right to buy the underlying asset on a predetermined date ( $T$ ) and for a predetermined price ( $K$ ). The

date is known as the expiration or maturity date, and the price as the exercise or strike price. A European call option cannot be excised before its maturity date. The payoff function of European call is then:

$$\max(0, X_T - K) \quad (5.1)$$

If the settlement price of the underlying index is less than a given strike,  $X_T \leq K$ , then the option will not be exercised and its value at time  $T$  is 0. Alternatively, if the price of the index is greater than a given strike,  $X_T > K$ , then the option is worth  $X_T - K$  at time T. At time  $t < T$ , or  $\tau$ , the price of a European call option is then:

$$C(K, X_t, t) = e^{-r\tau} E^* [ \max(X_T - K, 0) | X_t, t ] \quad (5.2)$$

$$\begin{aligned} &= e^{-r\tau} \int_{\tau} \max(X_T - K, 0) dQ(X_T) \\ &= e^{-r\tau} \int_0^{\infty} \max(X_T - K, 0) q(X_T) dX_T \end{aligned} \quad (5.2)$$

Given payoff function (5.1), the price of an option is equivalent to the expected value of its payoff under the risk-neutral measure,  $E^*$ , where  $Q(X_T)$  is the risk-neutral probability, and  $e^{-r\tau}$  is the discount factor. Here  $Q$  is the time  $T$  or terminal distribution of returns of the underlying asset, conditional

on the observed price of its derivative at time  $t < T$ . Intuitively,  $Q$  is the aggregate “belief” of the market in the distribution of the future returns of the asset at time  $t$ . Here  $q$  is the target of our nonparametric estimation.

Breeden and Litzenberger (1978) first showed that there exists a unique relationship between option prices and the risk-neutral density. Given the price function of a European call option  $C(K, X_t)$  or  $C$  for simplicity, the second derivative with respect to its strike price  $K$  is the discounted probability density function:

$$\frac{\partial C}{\partial K} = -e^{-r\tau} \int_K^\infty q(X_T) dX_T \quad (5.3)$$

$$\left. \frac{\partial^2 C}{\partial K^2} \right|_{K=X_T} = e^{-r\tau} q(X_T) \quad (5.4)$$

The result suggests the second derivative of the call price function can be used to estimate the risk-neutral density from observed option prices. However, as Breeden and Litzenberger (1978) noted, the estimation is very unstable, as small errors in observed prices are exacerbated by numerically differentiating twice. Typically these errors are small and do not represent a serious mispricing of the option or any arbitrage opportunities, and are usually a result of observing nonsynchronous prices. This problem however is not unique to lower frequency studies. Even at high-frequencies, I do not observe prices for all options simultaneously. The estimation therefore faces two challenges:

1. A finite number of observed strikes, often at non-regular intervals, and 2. nonsynchronous pricing errors.

### 5.3 Butterfly Option

The intuition behind the original procedure is straightforward. Consider any two options with adjacent strikes, the difference in their price must reflect the probability of the final price of the underlying asset falling between them. This fact holds across strikes, and price differences must reflect the probability of the different outcomes. This fact can be used to extract the targeted probability measure,  $Q$ .

Arrow (1964) and Debreu (1959) first formalized the intuition of how option prices may be used to mimic such state-contingent claims. The result is an elementary security whose return depends on the ‘state’ of the world at a particular time  $T$  in the future. Their key insight was that the price of such a claim would reflect investors’ assessment of the probability of that state occurring. Breeden and Litzenberger (1978) later showed how traded options can be combined across strikes to mimic such an Arrow-Debreu security. The goal is to construct a security which pays \$1 at time  $T$  if the underlying asset takes a value,  $X_T$ , and \$0 otherwise, using options with the same expiration date and underlying asset.

Recall that given the price of an asset  $X_t$ , a single European call option with strike price  $K$  pays  $\max(0, X_T - K)$  at maturity  $T > t$ , and at any given

time,  $t$ , we can observe  $M$  call prices for  $C_i$ , for  $i = 1, \dots, M$  options written on  $X$  across,  $K_i$ , for  $i = 1, \dots, M$  strike prices. These options can be combined to mimic an Arrow-Debreu security as follows:

First, suppose I buy  $\frac{1}{K_i - K_{i-1}}$  call options with strike price  $K_{i-1}$  and simultaneously sell the same number of call options with strike price  $K_i$ . The cost (5.5) of such a trade would be  $\frac{C_{i-1} - C_i}{K_i - K_{i-1}}$ , and at maturity I would receive a payout of (5.6). Such a trade is commonly known as a bull spread.

$$price_{bull} = \frac{C_{i-1} - C_i}{K_i - K_{i-1}} \quad (5.5)$$

$$\begin{aligned} payout_{bull} &= \frac{\max(0, X_T - K_{i-1}) - \max(0, X_T - K_i)}{K_i - K_{i-1}} \\ &= \begin{cases} 1 & \text{if } X_T \geq K_i \\ 0 & \text{if } X_T \leq K_{i-1} \\ \text{Linear} & \text{if } K_{i-1} \leq X_T \leq K_i \end{cases} \end{aligned} \quad (5.6)$$

At the same time, suppose I sell another  $\frac{1}{K_{i+1} - K_i}$  call options with strike price  $K_i$  and buy the same number of options with strike price  $K_{i+1}$ . The cost (5.7) of the trade would be  $\frac{C_i - C_{i+1}}{K_{i+1} - K_i}$ , and at maturity I would again receive a payout of (5.8). Such a trade is known as a bear spread.

$$price_{bear} = \frac{C_i - C_{i+1}}{K_{i+1} - K_i} \quad (5.7)$$

$$\begin{aligned}
payout_{bear} &= \frac{\max(0, X_T - K_i) - \max(0, X_T - K_{i+1})}{K_{i+1} - K_i} \\
&= \begin{cases} -1 & \text{if } X_T \geq K_{i+1} \\ 0 & \text{if } X_T \leq K_i \\ \text{Linear} & \text{if } K_i \leq X_T \leq K_{i+1} \end{cases} \tag{5.8}
\end{aligned}$$

The combination of these two trades is known as butterfly option or spread, and it has the following properties. First, the payout of the combined option (5.9) is \$0 for both  $X_T \leq K_{i-1}$  and for  $X_T \geq K_{i+1}$ , since in the first instance neither option is exercised, and in the second, the bull option pays \$1 and the bear option pays  $-\$1$ . Second, the spread is linearly increasing in  $K_{i-1} \leq X_T \leq K_i$ , as only the first option is exercised. Likewise the option is linearly decreasing in  $K_i \leq X_T \leq K_{i+1}$ , as both options are exercised. Finally, the option will have a payout equal to \$1 for any  $X_T = K_i$ , and the cost or price,  $p_i$ , of setting up the butterfly is (5.10).

$$payout_{butterfly} = \begin{cases} 0 & \text{if } X_T \leq K_{i-1} \\ \text{Increasing} & \text{if } K_{i-1} \leq X_T \leq K_i \\ 1 & \text{if } X_T = K_i \\ \text{Decreasing} & \text{if } K_i \leq X_T \leq K_{i+1} \\ 0 & \text{if } X_T \geq K_{i+1} \end{cases} \tag{5.9}$$

$$p_i = \frac{C_{i-1} - C_i}{K_i - K_{i-1}} - \frac{C_i - C_{i+1}}{K_{i+1} - K_i} \tag{5.10}$$

The result is an approximate Arrow-Debreu security, and the price,  $p_i$ , of constructing such an option centered at  $K_i$  can be shown to be proportional to the risk-neutral probability of that state occurring (Ross 1976). Here no interpolation is required, and it is possible to construct the spread using only the available options. In summary, an individual butterfly spread is a near binary option, whose price has point mass  $p_i$  in  $K_i$ .

To approximate the entire density from these simple functions, notice that it is always possible to construct a series of butterfly options across any range of strikes  $I$  and  $J$ , that is, one option centered at each  $K_i$ , for  $i = I, \dots, J$  strike prices. The resulting sum of their payouts has the following properties; \$0 for  $X_T \leq K_{I-1}$ ,  $X_T \geq K_{J+1}$ , \$1 for  $X_T = K_I, K_{I+1}, \dots, K_J$ , and piecewise linear in between. Since at maturity  $X_T$  is a single point, the payout for the sum of butterflies must be smaller than  $\mathbb{1}_{[K_{i-1}, K_{J+1}]}$  and larger than  $\mathbb{1}_{[K_i, K_J]}$ , making it an increasing sequence of simple or step functions. Recall that any non-negative measurable function can be shown to be the pointwise limit of a sequence of such non-negative monotonically increasing simple functions.

$$\sup |K_i - K_{i-1}| \rightarrow 0 \quad (5.11)$$

In other words, it can be shown that as (5.11) tends to zero, the payoff function of the butterfly option tends to a Dirac delta function with its mass at  $X_T = K_i$ . In the limit, the constructed butterfly is equivalent to an Arrow-

Debreu security paying \$1 if  $X_T = K_i$  and \$0 for all other states,  $X_T \neq K_i$ . Assuming the target measure has a continuous cumulative distribution function, a measure constructed from functions with point mass  $p_i$  in  $K_i$  can be shown to be a very good approximation of the risk-neutral density.

For the sample of SPX options employed, the difference between available  $K_i$  is typically \$5, and here I only use options spaced at intervals of \$25 or less. If we assume condition (5.11) is fulfilled, it is then possible to work with the measures based  $p_i$ , and its modifications. Note that for convenience, I choose to work with the logarithmic scale of this measure, such that  $\log(25) < 1.4$ . After applying the log-transform, the result is a measure with point mass  $p_i$  in  $\log(K_i)$ .

## 5.4 Convolution

In the previous section, the price of a combination of butterfly options was shown to be a point mass approximation of the risk-neutral density. The goal then is to convert this point mass approximation to a continuous density function. This is complicated by the likely presence of nonsynchronous pricing errors in the data.

If the observed prices  $C_i$  contain even small pricing errors, equation (5.10) will be highly unstable. This is intuitive since the butterfly spread divides the difference of prices  $C_i$  by the difference of strikes  $K_i$ , which are already assumed to be small. In practice then, I observe  $\widehat{p}_i$ , the estimated cost of the

butterfly spread including pricing errors, and not  $p_i$ . To recover  $p_i$ , I consult the literature on inverse problems in econometrics and choose to approximate the measure as the convolution of  $\widehat{p}_i$  and a smoothing function  $\varphi$ . Convolutions also play a central role in the identification and estimation of measurement error models (Schennach 2016, 2019). Beyond econometrics, they have broad applications in statistics, physics, engineering, acoustics, image processing, and probability theory. In statistics, a convolution is a weighted moving average. In kernel density estimation, a distribution can be approximated from its sample points by a convolution with a kernel (Diggle 1995).

$$\widehat{p}_i = p_i + \varepsilon_i \quad (5.12)$$

$$\begin{aligned} \psi(x) &= \sum_i \widehat{p}_i \varphi(x - \log K_i) \\ &= \sum_i p_i \varphi(x - \log K_i) + \sum_i \varepsilon_i \varphi(x - \log K_i) \end{aligned} \quad (5.13)$$

Following Carrasco, Florens, and Renault (2014), a convolution is employed to reduce the noise in  $\widehat{p}_i$  and dampen the estimation error of  $p_i$ . Consider any non-negative function  $\varphi$  such that  $\int \varphi(x) d x = 1$ , the convolution of  $\varphi$  and  $\widehat{p}_i$  is the function  $\psi$  as defined in (5.13), where  $\psi$  has the following two components. First, a systematic component,  $\sum_i p_i \varphi(x - \log K_i)$ , here the density of a convolution of the constructed measure  $p_i$  with smoothing function  $\varphi$ . Second, an idiosyncratic component,  $\sum_i \varepsilon_i \varphi(x - \log K_i)$ , which can be shown to

converge to zero under reasonable conditions; that is,  $\varphi$  is three times differentiable, square integrable, and the pricing errors have only short-range dependence and are sufficiently tame. Stated simply, given a choice of smoothing function  $\varphi$ , if the pricing errors are not too large and not too persistent then  $\psi$  will be a good approximation of the continuous risk-neutral measure.

A few final thoughts. First, here the choice of smoothing function is the density of a  $N(0, 6.25 \times 10^{-4})$ , and  $\varphi$  is therefore three times differentiable and square integrable. Second, the target of the remaining investigation is the information contained in the convolution  $\psi$  of the approximate risk-neutral measure with this smoothing function. Third, there is no guarantee that this  $\psi$  will be positive. In practice however, the contribution of the negative portion of  $\psi < 0$  to the total variation of the measure is typically small, less than  $10^{-4}$  with few exceptions. Finally, the convolution can be shown to preserve the higher order moments, and therefore the nonlinearities, of the estimated density.

## 5.5 Initial Estimates

Figures 3 and 4 report the results of the nonparametric estimation procedure. Figure 3 reports the evolution of the intraday measures of information for each option chain covering six years, 2009-2014. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index  $\tau = (T - n)/T$  normalizes each option's life cycle.

Figure 4 reports the intraday evolution of the approximate risk-neutral measure  $\psi$  as the convolution of  $\widehat{p}_i$ , the estimated cost of the sum of butterfly options, and  $\varphi$ , a chosen smoothing function. The estimated densities  $\psi$  for the SPX options expiring in June 2014 are shown in color for a single day, May 30th, 2014, and for 83 5-minute intervals beginning at 08:25 and ending 15:15 Central Time. The moving average of  $\widehat{p}_i$ , the price of the sum of butterfly option, is shown in grey. Here  $\widehat{p}_i$  is an estimate of a measure with point mass  $p_i$  in  $\log(K_i)$ . Table 2 reports the level of both measures of information, Shannon (5.14) and Rényi (5.15) entropy, at three points in each option's life, 3-months, 2-months, and 1-day from maturity, as well as the standard deviation of each measure of information over that respective month.

$$H_\psi = \int \psi(x) \log \psi(x) \quad (5.14)$$

$$\log L_\psi^2 = \log \int (\psi(x))^2 \quad (5.15)$$

**Figure 3: Evolution of Information**

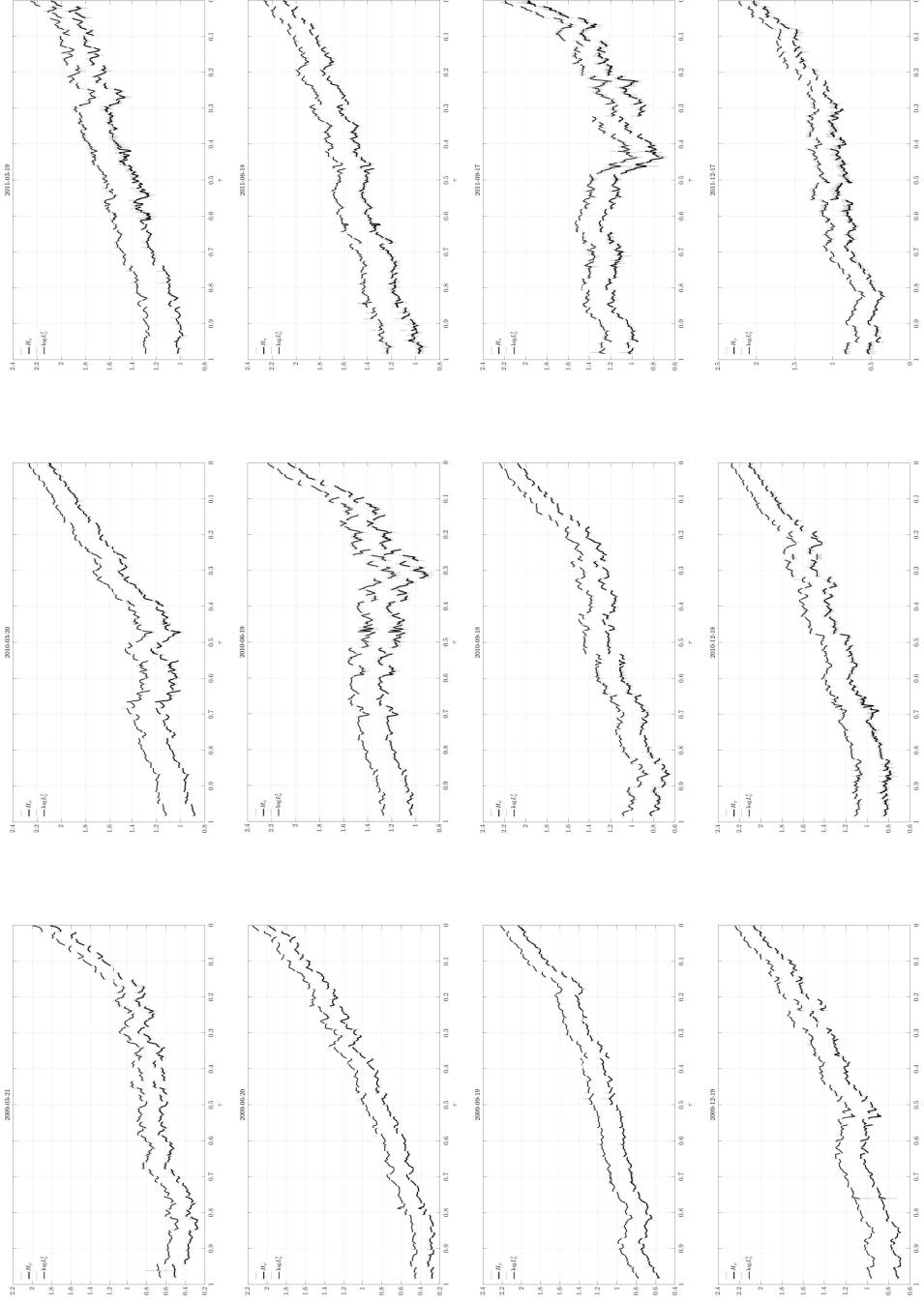


Figure 3 reports the evolution of the intraday measures of information for each option chain covering three years, 2009-2011. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index  $\tau = (T - n)/T$  normalizes each option's life cycle.

**Figure 3: Evolution of Information**

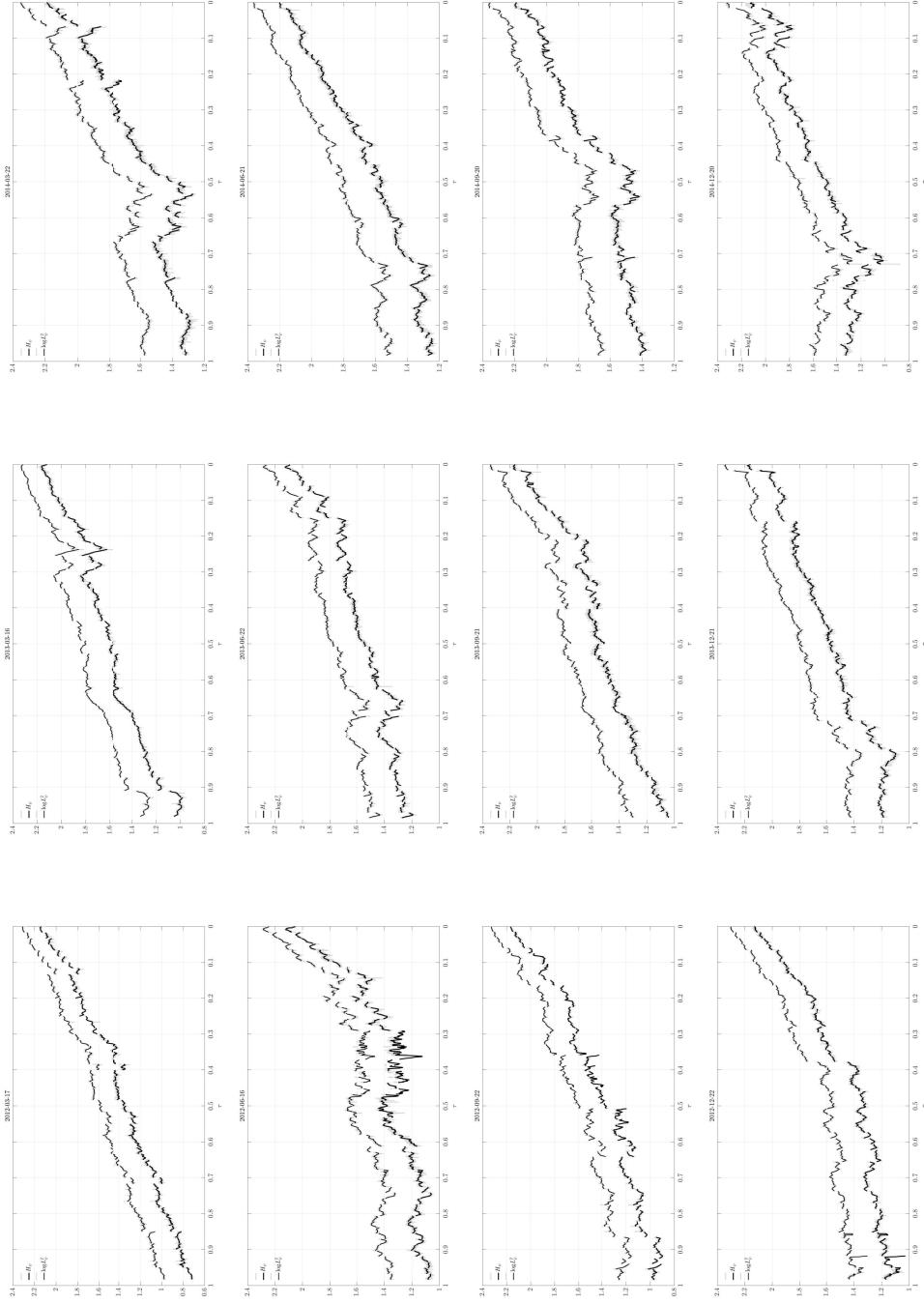


Figure 3 reports the evolution of the intraday measures of information for each option chain covering three years, 2012-2014. The 5-minute entropy measures are shaded in grey, and their 1-hour moving average in black, the index  $\tau = (T - n)/T$  normalizes each option's life cycle.

**Figure 4: Intraday Evolution**

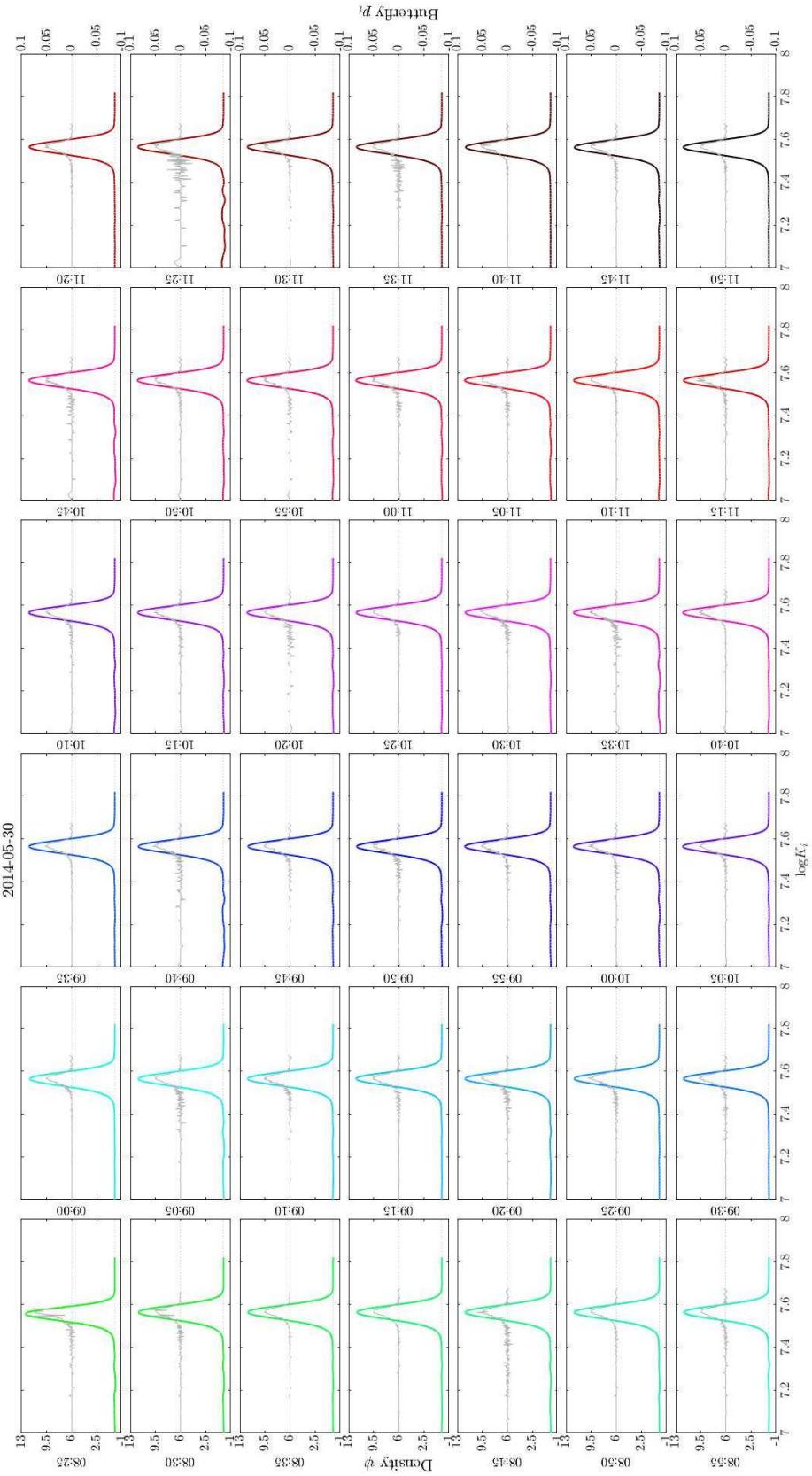


Figure 4 reports the intraday evolution of the approximate risk-neutral measure  $\psi$  as the convolution of  $\widehat{p}_i$ , the estimated cost of the sum of butterfly options, and  $\varphi$ , a chosen smoothing function. The estimated densities  $\psi$  for the SPX options expiring in June 2014 are shown in color for a single day, May 30, 2014, and for 42 5-minute intervals beginning at 08:25 and ending 11:50 Central Time. The moving average of  $\widehat{p}_i$ , the price of the sum of butterfly option, is shown in grey. Here  $\widehat{p}_i$  is an estimate of a measure with point mass  $p_i$  in  $\log(K_i)$ .

**Figure 4: Intraday Evolution**

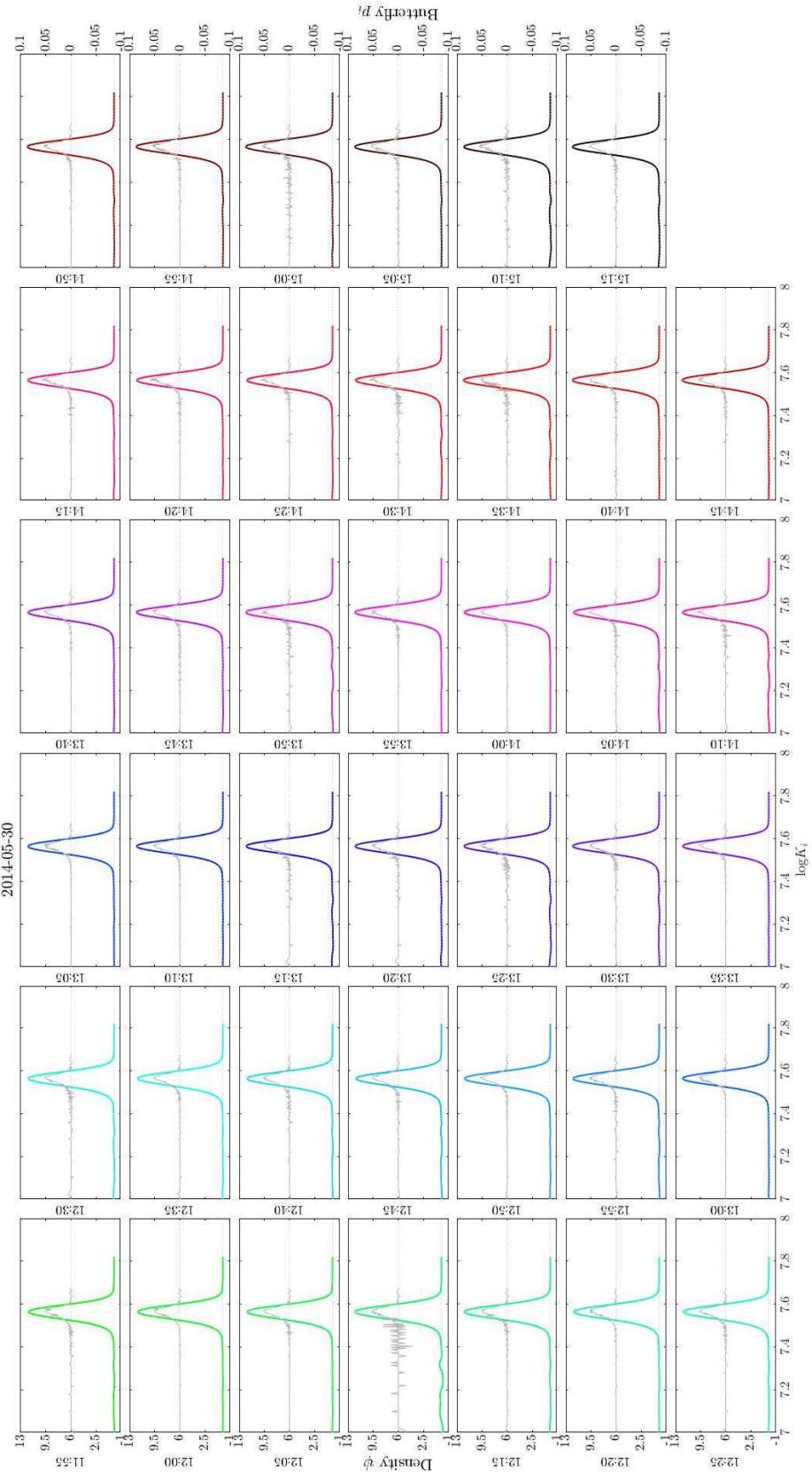


Figure 4 reports the intraday evolution of the approximate risk-neutral measure  $\psi$  as the convolution of  $\widehat{p}_i$ , the estimated cost of the sum of butterfly options, and  $\varphi$ , a chosen smoothing function. The estimated densities  $\psi$  for the SPX options expiring in June 2014 are shown in color for a single day, May 30, 2014, and for 41 5-minute intervals beginning at 11:55 and ending 15:15 Central Time. The moving average of  $\widehat{p}_i$ , the price of the sum of butterfly option, is shown in grey. Here  $\widehat{p}_i$  is an estimate of a measure with point mass  $p_i$  in  $\log(K_i)$ .

## 6 Testing

I now turn to the paper's main empirical contribution, a simple testing procedure for the presence of jumps in the arrival of information. The approach reveals both the frequency and magnitude of jumps in investor expectations, and permits the testing of several hypotheses. Building on the concept of information as a reduction in uncertainty surrounding the future price of the index, the arrival of information is reflected in the evolution of the risk-neutral density. This process can be quantified using entropy. For options with the same maturity date, the density becomes more concentrated over time. Hence both measures of entropy will increase, as investors become increasingly certain of the likely distribution of future prices. Intuitively, less weight is placed on options farther from the money, as the probability of large changes in the value of the underlying index declines near maturity.

The evolution of the entropy of the risk-neutral density reflects the arrival of information. If information arrives continuously, if investors gain information in small amounts, or if expectations of the future price evolve slowly, then entropy will increase gradually. Alternatively, if information arrives discontinuously, if investors gain information in large amounts, or if expectations of the future price change abruptly, then entropy will be shown to jump. This intuition can be formalized by specifying the dynamics of the underlying diffusion process. For example, if the underlying price were to follow a geometric

Brownian motion, then the entropy of the implied distribution would increase both continuously and at a constant rate across time.

The information theoretic framework introduces several testable hypotheses for how information drives the evolution of the risk-neutral density. First, information arrives continuously, and investors respond by constantly adjusting expectations; or information arrives discreetly, and investors update expectations in response to new information. Second, investors learn at a constant rate and information accrues consistently across the life cycle; or learning is highly variable, and information accrues at different rates across time. Third, information accrual is deterministic, and any shock to expectations will return to some trend; or information follows a random walk, and investors learn little day-to-day as shocks affect expectations indefinitely.

The results of these hypothesis tests have real-world implications. Consider a temporary negative shock to the expected future value of the index. If information follows a stochastic trend, then investors remain persistently less certain about the future value of the index. This uncertainty may persist until an option matures. Alternatively, if uncertainty is purely deterministic, investors may profit from betting information will rebound to normal levels. Likewise, if information accrues at a constant rate, or if information is found to arrive continuously, investors may try to anticipate how uncertainty and the density function will evolve following a shock. Here it is important to consider that any model which assumes the underlying price follows a continuous diffusion process also implicitly assumes that information arrives continuously,

accrues at a constant rate, and evolves following a deterministic trend. This fact can be shown for many models including the famous Black-Scholes-Merton model.

Testing the three hypotheses is equivalent to testing for jumps in a nonstationary time series. Here I follow the procedure proposed by Zivot and Andrews (1992) to test for a single jump at an unknown time. The procedure derives from the literature on testing for unit roots in economic time series, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984). Perron (1989) found the proposed augmented Dickey–Fuller procedure to be biased against rejecting the null hypothesis in the presence of jumps or structural breaks in the time series. Zivot and Andrews (1992) later endogenize Perron’s procedure to test for jumps occurring at an unknown time.

The choice of procedure is threefold. First the test is simple and transparent. It is fast and easy to implement using standard econometric tools and software, and the results are easy to interpret. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items of immediate interest in our analysis. And finally, the procedure provides a statistical test for each identified jump, a feature many event-type studies lack. I now turn to describing the procedure in detail.

Following Zivot and Andrews (1992), the null hypothesis (6.1) is a random walk with possible drift and no jump. The alternative hypothesis (6.2) is a trend-stationary process with a single jump in its trend occurring at an unknown time. The basic idea is to first estimate a sequence of alternative or

trend-stationary models, one for each interval in the series. I then select the jump which gives the most weight to a single alternative model, and test that model against the null hypothesis. Here Zivot and Andrews follow Perron's (1989) augmented Dickey–Fuller testing strategy, and use a single regression equation to test for a unit root. Operationally, their procedure identifies both the timing and magnitude of the single largest jump in each time series. Each regression also includes an optimal number of lags, typically chosen by either Schwarz's (1978) Bayesian Information Criterion or a sequence of t-tests.

Null Hypothesis—Random Walk with Drift:

$$H_0: \quad y_t = \mu + y_{t-1} + e_t \quad (6.1)$$

Alternative Hypothesis—Trend Stationary Model + Jump:

$$H_1: \quad y_t = \mu_1 + \delta t + (\mu_2 - \mu_1)DU_t + e_t \quad (6.2)$$

The number of lags included in each regression is chosen to reduce the residuals to white noise. The optimal number included is allowed to vary for each candidate model, and is selected for each regression by a sequence of t-tests, as suggested by Ng and Perron (1995) and Campbell and Perron (1991). Ng and Perron find their approach suffers fewer size distortions and retains comparable power to Schwarz's (1978) Information Criteria. In practice both Zivot and Andrews (1992) and Perron (1989) follow such a procedure; working backward from  $k = \bar{k}$  to  $k = 0$ , selecting the first  $k^*$  such that the t-stat on

$c_k$  for  $l < k$  is significant. It is their procedure I follow here, setting  $\bar{k} = \text{floor}[12\{(T + 1)/100\}^{0.25}]$  as proposed by Schwert (1989), where  $T = 83$  and in practice  $\bar{k} = 11$ .

Regression Model—Zivot and Andrews (1992):

$$y_t = \hat{\mu} + \hat{\theta}DU_t(\hat{\lambda}) + \hat{\delta}t + \hat{\alpha}y_{t-1} + \sum_{j=1}^k \hat{c}_j \Delta y_{t-j} + \hat{e}_t \quad (6.3)$$

The regression model (6.3) facilitates the testing of several hypotheses. In the model,  $\{y_t\}$  represents the amount of information measured at each interval, where  $t \in (0, 83)$  is the number of 5-minute intervals after 08:25 Central Time. The “trend” parameter  $\hat{\delta}t$  measures the gain in information as the average change in entropy over each day. The intercept  $\hat{\mu}$  captures the initial level of information for each trading day. The “jump” parameter  $\hat{\theta}$  captures the magnitude of a one-time shift in level of information occurring at a single time,  $t$ , where  $DU_t$  is indicator variable such that  $DU_t(\hat{\lambda}) = 1$  if  $t > \lambda t$ , and 0 otherwise. The Zivot and Andrews (1992) procedure is to first estimate a sequence of models, one for each possible jump, and to then select the best alternative model from that class of candidate models. The ‘best’ being the model which gives the most weight to the alternative hypothesis. Finally, the null hypothesis,  $\hat{\alpha}y_{t-1} = 1$ , is tested against the best alternative model, using the asymptotic critical values derived by Zivot and Andrews (1992). In practice, the alternative model is selected from the group of candidate models as

the single model having the smallest one-sided t-statistic against the null for the test of  $\hat{\alpha} = 1$ . Testing  $\hat{\theta}DU_t(\hat{\lambda}) = 0$  effectively tests for a jump at a given 5-minute interval; where  $\hat{\lambda} = t/T$  for  $t \in (7, T - 2)$  and solves  $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}(\lambda)$ .

In the application here, I begin with the two measures of information; Shannon and Rényi entropy. Both are measured at each of 83 5-minute intervals each day, over six years or 1,479 days in the sample, and representing 24 non-overlapping option chains. For each day, I estimate the regression model at each of 73 candidate intervals, one for 5-minute intervals between the hours of 09:00 and 15:00 Central Time. From the resulting class of 73 candidate models I select the alternative model which best fits the data, here the model with the smallest t-statistic for the test of  $\hat{\alpha} = 1$ . Finally, the random walk model is tested against the alternative trend-stationary model. The  $\hat{\theta}$  and  $\hat{\delta}t$  parameters are recorded for each day, along with the number of lags  $k^*$  selected, and the result of the main hypothesis test. For days where the null is rejected, a secondary test of  $\hat{\theta} = 0$  is performed to identify large jumps.

Much of the following analysis focuses on relative magnitude of the “trend” or  $\hat{\delta}t$  and the “jump” or  $\hat{\theta}$  parameters. It may be helpful then to review the possible outcomes of the testing procedure and the implication of each result for the analysis. First, for a given day the procedure may fail to reject the null hypothesis of a random walk. This would be the most likely outcome for days where the estimated time trend was very small, and no jump was identified. This result would indicate a day where little to no information was

gained. Second, the procedure could reject the null hypothesis but detect no jump, such that  $\hat{\delta}t > \hat{\theta}$ . This result would indicate a day where the trend or continuous gain in information dominates the arrival of any single bit of information. This result is most consistent with the model of the evolution of an asset's price as a continuous diffusion process. In practice, information is shown to arrive nearly continuously, and investors respond by making constant small adjustments in their expectations of the future price of the asset. Finally, for a given day, the procedure may reject the null hypothesis and detect a jump. This result would indicate a day with a large discrete gain in information, a jump which may dominate the more continuous gain in information, such that  $\hat{\theta} > \hat{\delta}t$ . Here information is shown to arrive discretely, and investors respond by making large revisions to their expectations.

## 7 Results

### 7.1 Full Sample

The testing procedure reveals several surprising results. Information is shown to often arrive in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump. In the data, the procedure identifies at least one jump for a majority of days, and finds days without jumps contribute little to the total gain in information. Additionally, the testing procedure reveals both the frequency and magnitude of jumps in investor expectations. The findings highlight two facts new to the literature. First, the

majority of information accrues in the final month of an option's life cycle. In fact, investors learn little about the future price of an asset for most of an option's life. Second, jumps contribute the majority of information gained during the first two months. Only in the final month does information arrive often enough for the trend to contribute more to the total gained. The size and frequency of jumps does not decline.

Information arrives in discrete intervals. The intraday analysis reveals the frequent occurrence of jumps in the risk-neutral density. Table 3 and Table 4 summarize the results of the daily Zivot and Andrews (1992) procedure for each of 24 option chains. Each table displays the results of the daily hypothesis test of  $\hat{\alpha} = 1$ . In columns 3-5, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. On average, the null hypothesis is rejected for a majority of days, and the alternative model of a trend-stationary process with a potential one-time jump in information is selected. This is true for between one-half and two-thirds of days depending on the measure of entropy and the level of significance. The result implies that investors 1. gain information about the future value of an asset over time, and 2. make large revisions to their expectations in response to new information. This is shown to occur even at high frequencies, as the procedure identifies at least one jump for a majority of days. The results also document several days where little to no information is gained. For days where the procedure fails to reject the null, no jumps are

found to have occurred. Combined with only a small gain in the trend of information, the daily process for these days would resemble a random walk.

Tables 3 and 4 also report the average daily change in total, trend, and jump estimates of information, taken over all days in each option chain, in columns 6-8. On average, the day-to-day gain in information is small, and positive; with a trend component that is on average larger than either the jump or total gain in information. On average jumps appear to coincide more with increases in uncertainty, as they are negative and nonzero. This holds for both Shannon and Rényi entropies.

The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information. Here it becomes apparent that jumps contribute to a large proportion of the total information gained, and are more often positive than their average over all days appears. On average, jumps contribute almost half of the total information gained over the cycle. Across all days, a single jump, over a single 5-minute interval, is nearly as large as the estimated trend across the entire day. This result implies that jumps occur for a majority of days, and that days without jumps likely contribute little to the total gain in information.

The testing procedure reveals the magnitude of jumps in investor expectations. Figures 5 panels 2 and 3 present histograms of the estimated daily trend and jump components for all days, and each measure of entropy. The trend component is unimodal, centered near zero, and negatively skewed. The trend in Shannon entropy is more broadly distributed and more skewed in

comparison to the trend in Rényi entropy. The estimated jump component is bimodal, centered near zero, slightly positively skewed, and not symmetric. The jumps in Shannon entropy are more broadly distributed and skewed compared to the Rényi entropy. For both measures, there are few days where the estimated jump is zero, and a large number of days the jump component is larger than the trend. To understand how the trend and jump components vary day-to-day, Figure 5 panels 1 and 4 present heat maps of the estimated daily trend and jump components for all days. For each map, the bright yellow and deep blue correspond to estimates of the daily trend and jump above the 95th percentile of each distribution, with blue-green estimates near zero.

Several patterns emerge that require further consideration. First, the size and frequency of jump estimates appears evenly distributed over the life cycle. Second, the magnitude of the trend estimate is not evenly distributed. Looking at the shading in panels 1 and 4 of both figures, a large positive trend appears more likely in the final 20 to 30 days of an option's life. This result suggests information accrues at different rates across time, that is, gains in information are not constant across the life cycle. Figure 6 panels 1 and 2 shows this in high contrast. Here, days with a strictly positive trend and jump are reported in white. Again the frequency of positive jumps appears evenly distributed over the life cycle, while the frequency of days with a strictly positive trend seems to concentrate in the final 20 days of an option's life.

**Table 3: Test Results Shannon Entropy**

Maturity	Days	Reject Null			Average Daily Change			Percent Total Change	
		10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	53	24	21	15	0.005	0.004	0.001	0.56	0.44
2009-06-20	62	31	28	15	0.001	0.003	-0.002	0.52	0.48
2009-09-19	62	40	35	23	0.012	0.012	0.000	0.51	0.49
2009-12-19	63	40	38	31	0.006	0.011	-0.004	0.51	0.49
2010-03-20	60	39	38	23	0.006	0.008	-0.003	0.53	0.47
2010-06-19	62	29	23	16	-0.002	0.000	-0.002	0.54	0.46
2010-09-18	62	29	25	19	0.005	0.006	-0.001	0.48	0.52
2010-12-18	63	47	41	31	0.004	0.008	-0.004	0.52	0.48
2011-03-19	61	34	29	19	0.006	0.008	-0.001	0.50	0.50
2011-06-18	62	37	32	23	0.002	0.007	-0.005	0.51	0.49
2011-09-17	62	26	21	13	0.007	0.005	0.002	0.51	0.49
2011-12-17	63	45	42	36	0.008	0.007	0.001	0.48	0.52
2012-03-17	60	41	39	27	0.003	0.010	-0.007	0.44	0.56
2012-06-16	62	37	35	25	0.005	0.010	-0.005	0.48	0.52
2012-09-22	67	39	34	26	0.000	0.004	-0.004	0.50	0.50
2012-12-22	61	40	36	32	-0.008	0.001	-0.009	0.46	0.54
2013-03-16	55	41	39	34	0.007	0.009	-0.002	0.51	0.49
2013-06-22	67	41	36	19	0.002	0.002	0.001	0.55	0.45
2013-09-21	62	53	49	41	0.002	0.001	0.000	0.51	0.49
2013-12-21	63	46	44	35	0.003	0.007	-0.005	0.54	0.46
2014-03-22	60	48	46	39	-0.003	-0.002	-0.001	0.54	0.46
2014-06-21	62	50	46	43	0.004	0.000	0.004	0.48	0.52
2014-09-20	62	40	39	24	0.002	0.002	0.000	0.51	0.49
2014-12-20	63	46	43	30	0.005	0.001	0.005	0.47	0.53
<i>Average</i>	62	39	36	27	0.003	0.005	-0.002	0.51	0.49

Table 3 reports the results of the Zivot and Andrews (1992) testing procedure for the Shannon measure of information. Columns 3-5, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 6-8 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

**Table 4: Test Results Rényi Entropy**

Maturity	Days	Reject Null			Average Daily Change			Percent Total Change	
		10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	53	24	20	8	0.006	0.008	-0.002	0.55	0.45
2009-06-20	62	22	21	16	0.002	0.002	0.000	0.54	0.46
2009-09-19	62	38	30	22	0.010	0.008	0.002	0.54	0.46
2009-12-19	63	39	36	23	0.005	0.007	-0.003	0.53	0.47
2010-03-20	60	30	27	16	0.006	0.005	0.001	0.54	0.46
2010-06-19	62	23	20	15	-0.003	-0.002	0.000	0.54	0.46
2010-09-18	62	28	25	14	0.006	0.005	0.001	0.47	0.53
2010-12-18	63	40	31	22	0.004	0.009	-0.005	0.52	0.48
2011-03-19	61	36	33	25	0.005	0.008	-0.002	0.52	0.48
2011-06-18	62	29	27	18	0.002	0.003	-0.002	0.50	0.50
2011-09-17	62	19	14	8	0.005	0.004	0.001	0.51	0.49
2011-12-17	63	35	32	22	0.004	0.014	-0.009	0.52	0.48
2012-03-17	60	39	35	28	0.004	0.005	-0.001	0.47	0.53
2012-06-16	62	30	26	14	0.003	0.004	-0.001	0.48	0.52
2012-09-22	67	29	28	21	0.002	0.002	0.000	0.50	0.50
2012-12-22	61	30	27	17	-0.006	0.001	-0.007	0.48	0.52
2013-03-16	55	30	26	19	0.007	0.009	-0.002	0.51	0.49
2013-06-22	67	32	25	16	0.002	0.004	-0.002	0.53	0.47
2013-09-21	62	40	38	27	0.002	0.002	0.000	0.53	0.47
2013-12-21	63	34	30	21	0.003	0.002	0.001	0.53	0.47
2014-03-22	60	37	37	23	0.000	-0.003	0.003	0.50	0.50
2014-06-21	62	39	33	26	0.005	0.005	0.000	0.53	0.48
2014-09-20	62	34	31	22	0.002	0.002	0.001	0.52	0.48
2014-12-20	63	36	30	21	0.005	0.003	0.002	0.47	0.53
<i>Average</i>	62	32	28	19	0.003	0.004	-0.001	0.51	0.49

Table 4 reports the results of the Zivot and Andrews (1992) testing procedure for the Rényi measure of information. Columns 3-5, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 6-8 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

**Figure 5: Gain in Information**

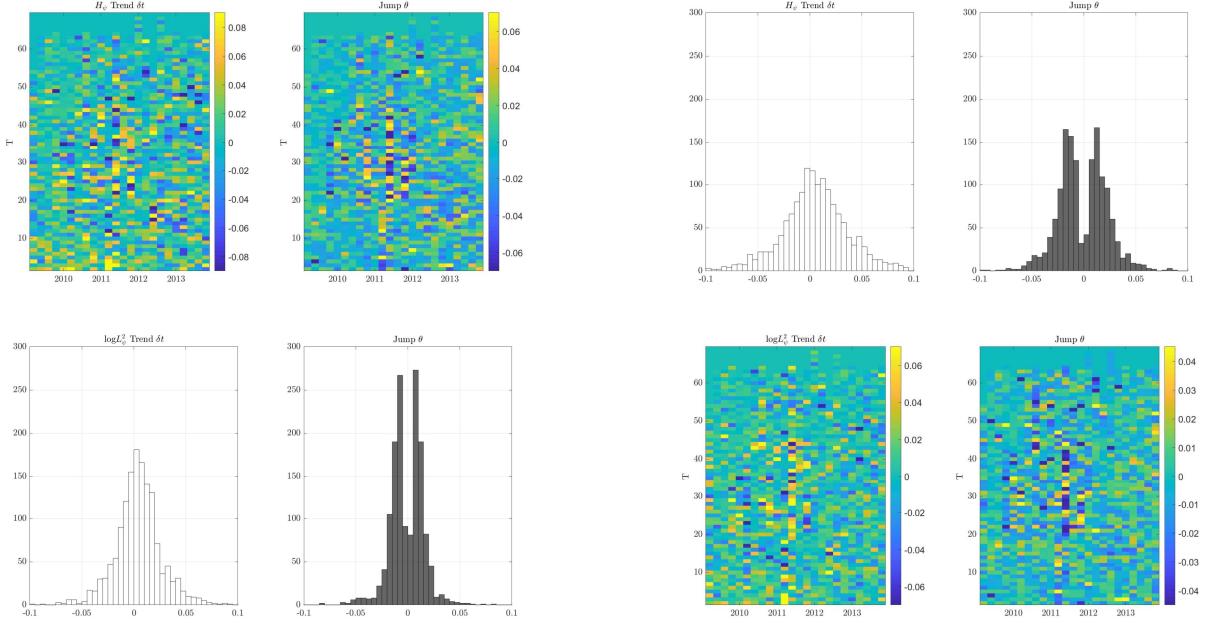


Figure 5 panels 2 and 3 present histograms of the estimated daily trend and jump components for all days, and each measure of entropy, both Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ ; panels 1 and 4 present heat maps of the estimated daily trend and jump components for all days.

The result is even more pronounced if we focus only on the last two weeks, or 10 days, of trading. Few days in the last two weeks of an option's life report a negative trend. In contrast, the magnitude and frequency of jumps does not appear to change.

When then is information gained? Figure 6 panel 3 reports the frequency of days where the trend is larger than the estimated jump for both measures of entropy. Note that there are many days where the trend exceeds the estimated jump, and Figure 6 panel 3 does not distinguish days where both are negative

or near zero from days where the trends is large and positive. However, when viewed in conjunction with Figure 6 panels 1 and 2 it becomes more apparent that information does not accrue consistently across the life cycle. Only in the final 10-20 days of an option's life cycle is the estimated trend persistently positive, and consistently larger than the estimated jumps. This explains many of the results in Tables 3 and 4. If for the majority of days the daily

**Figure 6: Relative Gain in Information**

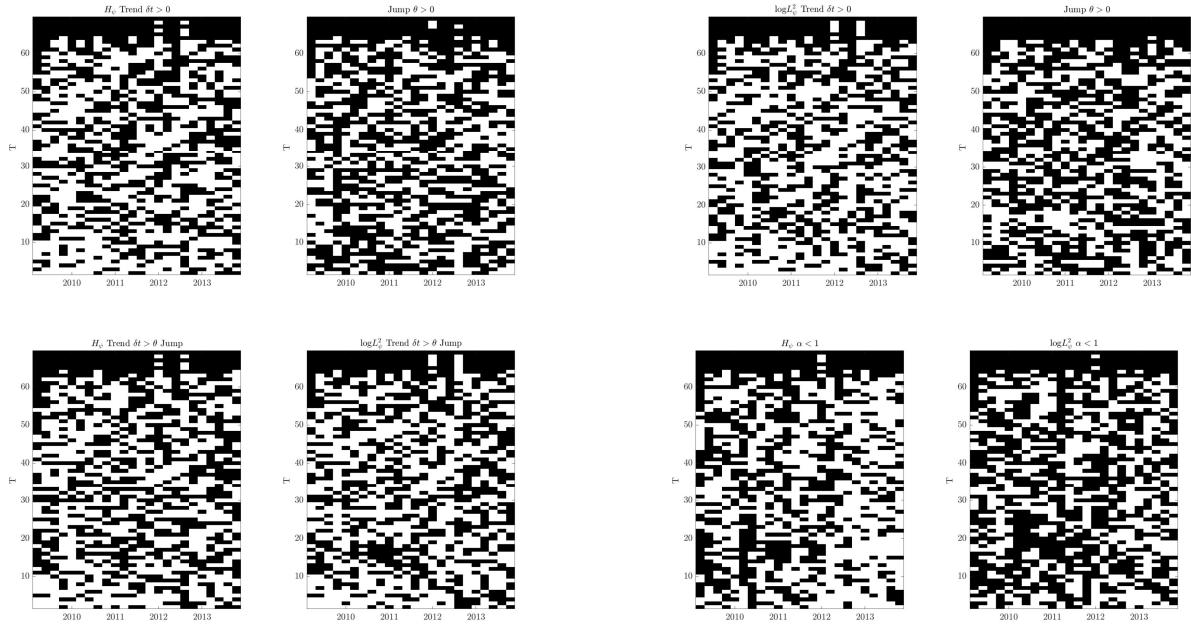


Figure 6 panels 1 and 2 shows days with a strictly positive trend and jump reported in white for both measures of information, Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$  entropy. Panel 3 reports the frequency of days where the trend is larger than the estimated jump. Panel 4 reports the days where the null hypothesis of a random walk is rejected.

gain in information is small, then jumps may contribute much to the gain in information. This effect would be particularly acute if days without jumps were found to contribute little to the total gained.

Figure 6 panel 4 reports the days where the null hypothesis of a random walk is rejected, in a test of  $\hat{\alpha} = 1$ . Again it can be seen that jumps are quite common, and that the alternative model is selected more often in the final month of an option's life. Indeed, days where little to no information is learned (days where the test fails to reject the null) appear concentrated in the first two month of the sample. Given that the trend is less pronounced during this period, days without jumps likely contribute little to the total gain in information during much of an option's early life. This is not to say that information commonly follows a random walk, or to suggest investors learn nothing day-to-day. No, information accrual is decidedly deterministic. The trend is small, but I find little evidence that shocks affect expectations or the level of information indefinitely.

Taken together, the initial results suggest learning is highly variable, and information about the final price of the asset accrues at different rates across time. The findings contradict the assumption that investors are less certain about the future price of an asset mostly as a function of time, something a continuous diffusion process implicitly assumes. Instead, the arrival of information in discreet intervals results in jumps in the level of uncertainty. This fact, combined with a small trend for much of an options life cycle, and varying levels of initial uncertainty, explains the results of Tables 3 and 4. To

verify this intuition, I turn to the question of quantifying these differences across the life cycle.

## 7.2 Split Sample

The following findings highlight two facts new to the literature. First, the majority of information accrues in the final month of an option's life cycle. Table 2 and Figure 2 show that on average nearly 60 percent of gain in entropy accrues in the final month. Given that this represents only one-third of an option's life, the gain in information in the last month is both large and unexpected. The finding suggests that investors learn little about the future price of an asset for much of an option's life. This result runs contrary to the assumption that investors become more certain about the likely final price of an asset at close to a constant rate. The finding also raises new questions regarding what drives the differences in the rate of the accrual of information in the final month of an option's life.

Tables 5 and 6 document the source of the difference in the accrual rate of information. Each table reports the results of the intraday Zivot and Andrews (1992) procedure for each day of the 24 option chains. Here the sample is split into thirds, with the results of the unit root test and gains in information reported separately for the first 2 months and final 1 month of each option chain. On average there are 40 trading days in the first two months, and 21 trading days in the final month; a ratio 2:1, representing on average 66 and

34 percent of each life cycle. Column 3 reports the fraction of days in each subsample for each option. Each table displays the results of the daily hypothesis test of  $\hat{\alpha} = 1$ . In columns 4-6, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. On average, the null hypothesis is rejected for a similar fraction of days in both subsamples, with the alternative model being selected for between one-half and two-third of days.

Tables 5 and 6 report the average daily change in total, trend, and jump estimates of information for each period. Columns 7-9 report that the average day-to-day gain in information remains small and positive, but is noticeably larger during the final month of the sample. This holds for both Shannon and Rényi entropies. Indeed, the average trend estimate triples for the Shannon entropy and doubles for the Rényi entropy in the last month. In contrast, the average jump estimate does not change. Figure 7 demonstrates this result visually. Here I plot the average absolute trend and jump estimates for the first 2 months in white, and the final month in black. The results match those as reported in columns 8 and 9 respectively. In almost all cases the average trend for the last month of the life cycle is large and positive, particularly early on in the sample when the initial level of uncertainty is highest. By comparison, the average jump estimate does not appear systematically larger in final month. This result holds for both measures of information.

**Table 5: Test Results Shannon Entropy Split Sample**

Maturity	Days	Fraction	Reject Null			Gain in Information		Percent Contribution		
			10	5	1	Total	Trend	Jump	Trend	Jumps
2009-03-21	33	0.62	18	17	12	0.003	0.003	0.001	0.54	0.46
	20	0.38	6	4	3	0.009	0.006	0.003	0.58	0.42
2009-06-20	40	0.65	21	19	8	0.004	0.004	0.000	0.51	0.49
	22	0.35	10	9	7	-0.004	0.002	-0.006	0.53	0.47
2009-09-19	40	0.65	24	22	14	0.006	0.008	-0.002	0.50	0.50
	22	0.35	16	13	9	0.021	0.018	0.003	0.53	0.47
2009-12-19	42	0.67	29	28	24	0.008	0.009	-0.001	0.55	0.45
	21	0.33	11	10	7	0.003	0.014	-0.010	0.44	0.56
2010-03-20	40	0.67	24	23	16	0.002	0.002	0.000	0.52	0.48
	20	0.33	15	15	7	0.013	0.021	-0.008	0.55	0.45
2010-06-19	40	0.65	21	16	12	-0.007	-0.001	-0.006	0.49	0.51
	22	0.35	8	7	4	0.007	0.003	0.004	0.63	0.37
2010-09-18	40	0.65	23	21	16	0.002	0.002	0.000	0.44	0.56
	22	0.35	6	4	3	0.011	0.013	-0.003	0.54	0.46
2010-12-18	42	0.67	31	27	20	0.000	0.004	-0.004	0.50	0.50
	21	0.33	16	14	11	0.013	0.016	-0.003	0.54	0.46
2011-03-19	42	0.69	24	20	11	0.008	0.004	0.003	0.49	0.51
	19	0.31	10	9	8	0.003	0.015	-0.012	0.52	0.48
2011-06-18	40	0.65	26	23	18	0.001	0.009	-0.008	0.48	0.52
	22	0.35	11	9	5	0.005	0.003	0.002	0.57	0.43
2011-09-17	40	0.65	14	11	7	-0.006	-0.007	0.001	0.52	0.48
	22	0.35	12	10	6	0.029	0.026	0.003	0.50	0.50
2011-12-17	42	0.67	30	28	24	0.002	0.004	-0.002	0.49	0.51
	21	0.33	15	14	12	0.021	0.015	0.006	0.46	0.54
2012-03-17	40	0.67	29	28	20	0.002	0.013	-0.011	0.48	0.52
	20	0.33	12	11	7	0.006	0.005	0.000	0.36	0.64
2012-06-16	40	0.65	27	26	21	0.001	0.006	-0.004	0.49	0.51
	22	0.35	10	9	4	0.012	0.018	-0.006	0.47	0.53
2012-09-22	45	0.67	25	21	16	-0.003	0.003	-0.006	0.48	0.52
	22	0.33	14	13	10	0.007	0.007	0.000	0.54	0.46
2012-12-22	40	0.66	20	17	14	-0.017	-0.003	-0.013	0.43	0.57
	21	0.34	20	19	18	0.008	0.008	0.000	0.50	0.50
2013-03-16	36	0.65	24	23	19	0.008	0.009	-0.001	0.49	0.51
	19	0.35	17	16	15	0.005	0.007	-0.002	0.56	0.44
2013-06-22	45	0.67	27	25	13	0.004	0.006	-0.002	0.54	0.46
	22	0.33	14	11	6	-0.001	-0.006	0.006	0.58	0.42
2013-09-21	40	0.65	33	31	28	-0.003	-0.004	0.002	0.52	0.48
	22	0.35	20	18	13	0.009	0.011	-0.002	0.51	0.49
2013-12-21	42	0.67	29	29	22	0.005	0.008	-0.004	0.55	0.45
	21	0.33	17	15	13	-0.002	0.006	-0.007	0.50	0.50
2014-03-22	40	0.67	30	28	23	-0.003	-0.001	-0.002	0.53	0.47
	20	0.33	18	18	16	-0.003	-0.004	0.002	0.55	0.45
2014-06-21	40	0.65	32	29	28	0.000	-0.007	0.007	0.51	0.49
	22	0.35	18	17	15	0.012	0.013	-0.001	0.43	0.57
2014-09-20	40	0.65	26	26	16	0.000	-0.002	0.002	0.53	0.47
	22	0.35	14	13	8	0.007	0.010	-0.003	0.47	0.53
2014-12-20	42	0.67	30	28	19	0.007	0.001	0.006	0.45	0.55
	21	0.33	16	15	11	0.002	0.000	0.002	0.52	0.48
<i>Averages</i>										
2-Month	40	0.66	26	24	18	0.001	0.003	-0.002	0.50	0.50
1-Month	21	0.34	14	12	9	0.008	0.009	-0.001	0.52	0.48

Table 5 reports the results of the Zivot and Andrews (1992) testing procedure for the Shannon measure of information. Column 3 reports the fraction of days in each subsample. Columns 4-6, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 7-9 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

**Table 6: Test Results Rényi Entropy Split Sample**

Maturity	Days	Fraction	Reject Null			Gain in Information			Percent Contribution		
			10	5	1	Total	Trend	Jump	Trend	Jumps	
2009-03-21	33	0.62	16	14	8	0.006	0.009	-0.003	0.54	0.46	
	20	0.38	8	6	1	0.007	0.007	-0.001	0.58	0.42	
2009-06-20	40	0.65	15	14	11	0.002	0.003	-0.001	0.52	0.48	
	22	0.35	7	7	5	0.002	0.000	0.002	0.59	0.41	
2009-09-19	40	0.65	24	17	14	0.008	0.007	0.001	0.54	0.46	
	22	0.35	14	13	8	0.014	0.012	0.002	0.53	0.47	
2009-12-19	42	0.67	28	26	17	0.005	0.007	-0.002	0.56	0.44	
	21	0.33	11	10	7	0.003	0.008	-0.004	0.48	0.52	
2010-03-20	40	0.67	20	18	13	0.004	0.002	0.002	0.50	0.50	
	20	0.33	10	9	3	0.010	0.012	-0.002	0.62	0.38	
2010-06-19	40	0.65	17	15	12	-0.004	-0.001	-0.004	0.51	0.49	
	22	0.35	6	5	4	0.000	-0.005	0.005	0.61	0.39	
2010-09-18	40	0.65	21	18	10	0.006	0.002	0.003	0.41	0.59	
	22	0.35	7	7	5	0.007	0.010	-0.003	0.57	0.43	
2010-12-18	42	0.67	27	19	14	0.003	0.011	-0.008	0.50	0.50	
	21	0.33	13	12	9	0.007	0.004	0.003	0.56	0.44	
2011-03-19	42	0.69	26	25	20	0.007	0.009	-0.003	0.51	0.49	
	19	0.31	10	8	7	0.002	0.003	-0.002	0.54	0.46	
2011-06-18	40	0.65	23	21	14	0.000	0.004	-0.003	0.49	0.51	
	22	0.35	6	6	4	0.004	0.003	0.001	0.52	0.48	
2011-09-17	40	0.65	11	7	3	-0.001	-0.004	0.003	0.49	0.51	
	22	0.35	8	7	5	0.017	0.019	-0.002	0.54	0.46	
2011-12-17	42	0.67	26	25	19	0.004	0.012	-0.008	0.54	0.46	
	21	0.33	9	7	4	0.006	0.018	-0.012	0.49	0.51	
2012-03-17	40	0.67	27	25	22	0.003	0.007	-0.004	0.48	0.52	
	20	0.33	12	10	7	0.006	0.001	0.005	0.45	0.55	
2012-06-16	40	0.65	22	22	14	0.003	0.004	-0.001	0.44	0.56	
	22	0.35	8	4	2	0.003	0.004	-0.001	0.55	0.45	
2012-09-22	45	0.67	16	15	12	0.001	0.003	-0.002	0.48	0.52	
	22	0.33	13	13	10	0.004	0.001	0.002	0.52	0.48	
2012-12-22	40	0.66	17	16	12	-0.016	-0.003	-0.013	0.45	0.55	
	21	0.34	13	11	7	0.012	0.009	0.003	0.54	0.46	
2013-03-16	36	0.65	21	18	14	0.006	0.009	-0.002	0.51	0.49	
	19	0.35	9	8	7	0.007	0.009	-0.001	0.52	0.48	
2013-06-22	45	0.67	21	14	8	0.002	0.005	-0.003	0.50	0.50	
	22	0.33	11	11	8	0.001	0.001	0.000	0.59	0.41	
2013-09-21	40	0.65	26	25	20	0.001	-0.001	0.002	0.51	0.49	
	22	0.35	14	13	8	0.005	0.008	-0.003	0.58	0.42	
2013-12-21	42	0.67	21	19	14	0.004	0.003	0.001	0.54	0.46	
	21	0.33	13	11	8	0.001	-0.001	0.002	0.53	0.47	
2014-03-22	40	0.67	24	24	16	0.000	-0.003	0.004	0.50	0.50	
	20	0.33	13	13	9	-0.001	-0.002	0.001	0.49	0.51	
2014-06-21	40	0.65	28	22	16	0.003	0.002	0.001	0.55	0.45	
	22	0.35	11	11	10	0.010	0.010	0.000	0.48	0.52	
2014-09-20	40	0.65	22	21	16	0.001	0.000	0.001	0.51	0.49	
	22	0.35	12	10	7	0.006	0.005	0.001	0.53	0.47	
2014-12-20	42	0.67	23	19	13	0.006	0.002	0.004	0.45	0.55	
	21	0.33	13	11	8	0.003	0.004	-0.002	0.50	0.50	
<i>Averages</i>											
2-Month	40	0.66	22	19	14	0.002	0.004	-0.001	0.50	0.50	
1-Month	21	0.34	10	9	6	0.006	0.006	0.000	0.54	0.46	

Table 6 reports the results of the Zivot and Andrews (1992) testing procedure for the Rényi measure of information. Column 3 reports the fraction of days in each subsample. Columns 4-6, report the total number of days where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. Columns 7-9 report the average daily change in total, trend, and jump estimates of information. The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information.

**Figure 7: Contribution to Gain**

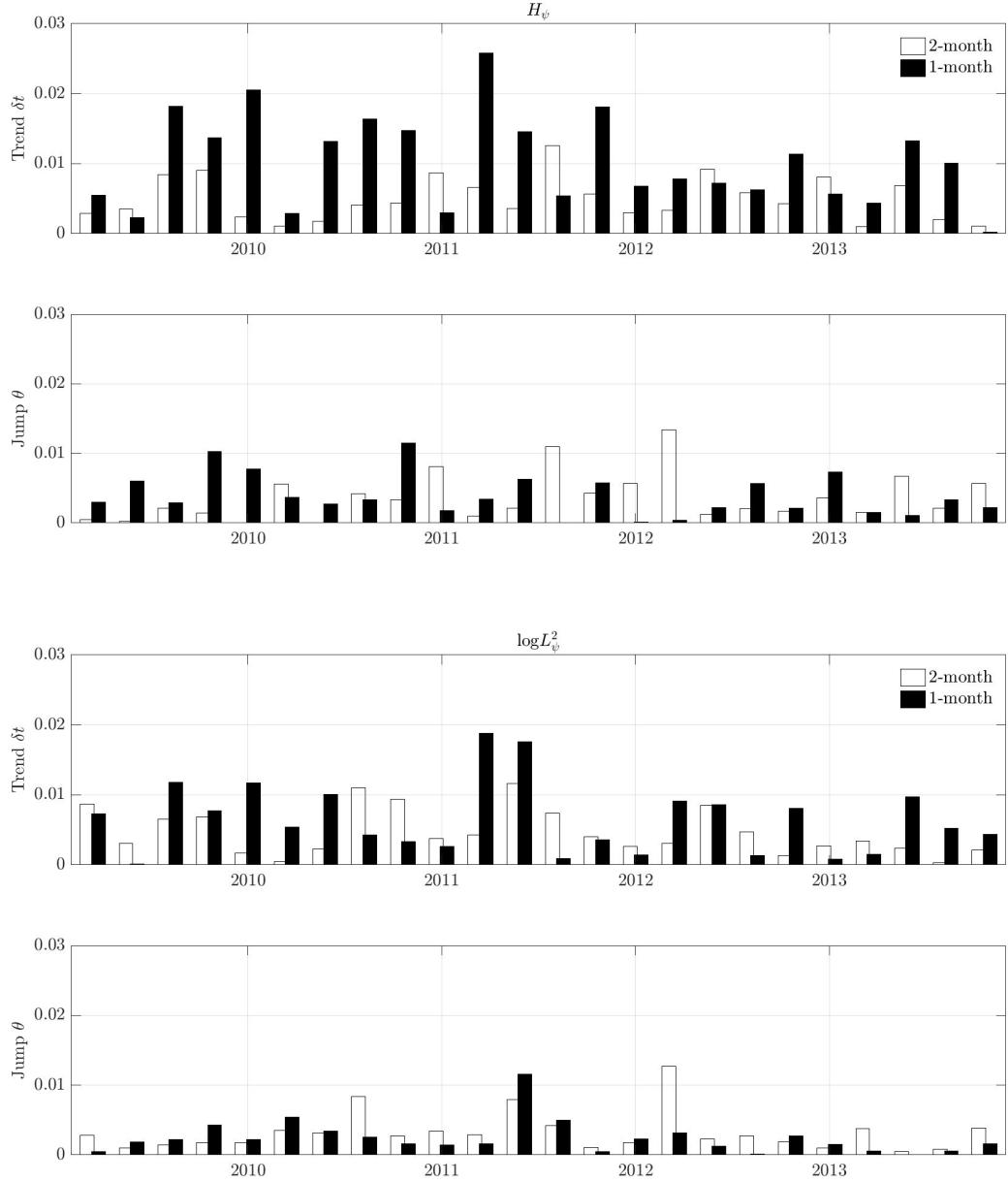


Figure 7 reports the average absolute trend and jump estimates for the first 2 months in white, and the final month in black for Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ , entropy. Figure 7 plots columns 8 and 9 of Table 5 and 6 respectively.

Only in the final month does the trend contribute more to the total gained. The final two columns of Tables 5 and 6 report the average daily contribution of jump and trend estimates as a percentage of the total gain in information, across the first two and final months of the sample. On average, the trend contributes to a larger percentage gain in total information in the final month. As a result, jumps are shown to contribute a smaller percentage gain. That is not to say the importance of jumps diminishes. On average, a single jump continues to contribute nearly an equivalent amount of information as the estimated trend across the entire day. This effect is simply more pronounced earlier in the sample when the trend estimates are on average smaller.

The testing procedure reveals the change in magnitude of the trend and jump estimates in the final month of an option's life. Figure 8 present histograms of the estimated daily trend and jump components for each subsample and measure of entropy. In panel 1, the trend and jumps in Shannon entropy are shown for all 3-months in white, and the final month in grey. Panel 2 reports the trend and jumps in Shannon entropy for the first 2-months in white and the final month in grey. Looking across both panels, the shape of the distribution of jumps does not change in the final month of the sample. The shaded subsample remains bimodal, centered near zero, and slightly asymmetric. By comparison, the shape of the distribution of estimated trends is noticeably more skewed in the final month. In all cases the distribution of

**Figure 8: Gain in Information Split Sample**

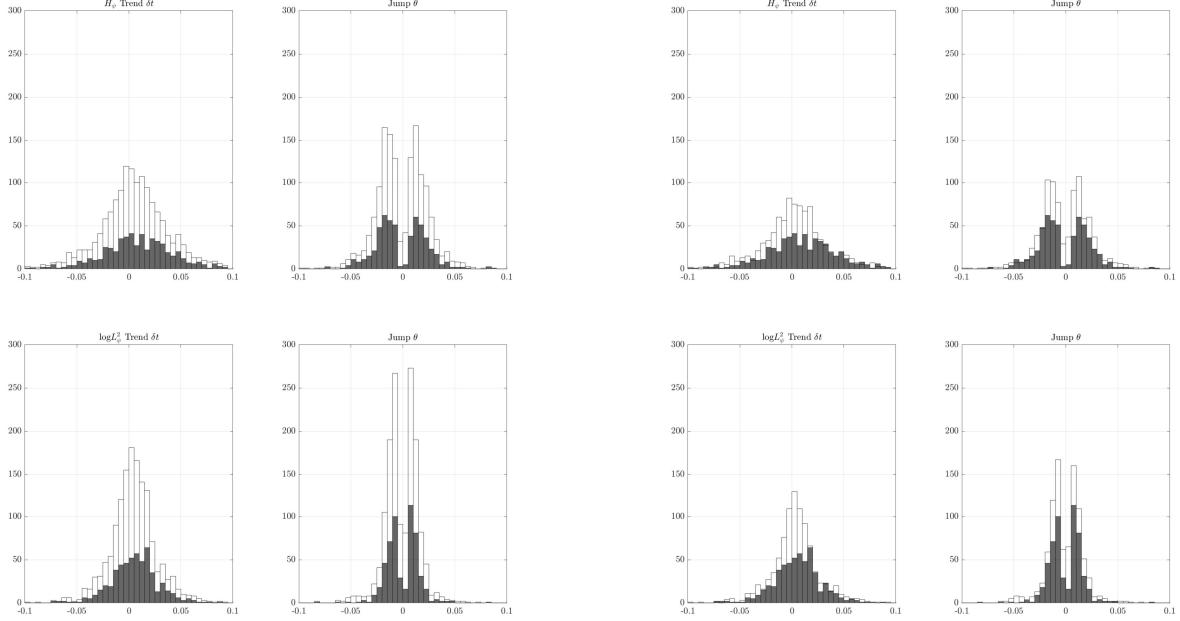


Figure 8 reports the estimated daily trend and jump estimates for each subsample and measure of entropy. In panel 1 and 3, the trend and jumps in Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ , entropy are shown for all 3-months in white, and the final month in grey, respectively. Panels 2 and 4 reports the trend and jumps in Shannon entropy for the first 2-months in white and the final 1-month in grey.

trend estimates remain unimodal, with the final month being negatively skewed and centered in the positive quadrant. This effect is most obvious in panel 2, where the occurrence or frequency of days with a large positive trend is near equal between the first two and final month of the sample, a counter-intuitive result given the relative lengths of the subsamples. The finding supports the conclusion that a large positive trend occurs predominately in the final 20 days of an option's life. Similar patterns emerge in the Rényi entropy.

In panels 2 and 4, the contribution of the final month to the right tail of the histogram is pronounced. Both results suggest that the gain in information is not constant across the life cycle. Indeed, Figure 8 clearly shows that information accrues at different rates across the life cycle. For good measure, Table 7 reports the percentiles of the empirical cumulative distribution functions for panels 2 and 4, and Figure C in the appendix plots these functions. For both measures of entropy the estimated trend or continuous gain in information in the final month is on average larger, more frequent, and persistently more positive. Likewise, the estimated daily jumps or discrete gains in information are frequent, nonzero, and larger than the trend for many days. The result supports the conclusion that jumps contribute much to the total gain in information and, given the small trend, contribute the majority of information which accrues during the first two months.

**Table 7: Gain in Information**

Entropy			Percentile								
			1	5	10	25	50	75	90	95	99
Shannon	Trend	3m	-0.090	-0.052	-0.034	-0.013	0.005	0.024	0.047	0.061	0.090
		Jumps	-0.071	-0.039	-0.028	-0.017	-0.005	0.015	0.027	0.035	0.061
Rényi	Trend	3m	-0.063	-0.033	-0.023	-0.008	0.004	0.016	0.032	0.043	0.070
		Jumps	-0.050	-0.024	-0.018	-0.011	-0.003	0.010	0.016	0.021	0.043
Shannon	Trend	2m	-0.092	-0.055	-0.035	-0.014	0.003	0.020	0.042	0.055	0.087
		1m	-0.081	-0.049	-0.034	-0.013	0.009	0.031	0.050	0.068	0.132
	Jumps	2m	-0.079	-0.038	-0.027	-0.016	-0.003	0.014	0.026	0.036	0.056
		1m	-0.055	-0.040	-0.030	-0.018	-0.007	0.016	0.027	0.035	0.084
Rényi	Trend	2m	-0.062	-0.034	-0.022	-0.007	0.003	0.014	0.030	0.042	0.069
		1m	-0.064	-0.031	-0.023	-0.008	0.006	0.018	0.033	0.043	0.071
	Jumps	2m	-0.051	-0.027	-0.018	-0.010	-0.003	0.010	0.016	0.022	0.044
		1m	-0.039	-0.021	-0.017	-0.011	-0.003	0.010	0.016	0.020	0.041

Table 7 reports the percentiles of the empirical cumulative distribution functions for the daily trend and jump estimates for each subsample, full and split, and both measures of information, Shannon and Rényi entropy. Figure C in the appendix plots the cdfs.

## 8 Case Studies

The results in the previous section demonstrate that information often arrives in discreet intervals, and in response investors often make large revisions to their expectations. In this section, large jumps in entropy are used to identify events in which significant new information is revealed to investors. In a first application of these methods, the case study demonstrates the ability to identify, quantify, and test for the effect of new information on the beliefs of market participants at high frequencies.

This section features three case studies, each highlighting a different category of discreet events: speeches by individual policymakers, economic news announcements, and flash crashes in the price of the underlying index. The first case study features President Obama's speech in January 2010 on tightening regulations on US banks; Chairman Bernanke's July 2010 testimony to Congress describing the Federal Open Market Committee's (FOMC) economic outlook; and statements released by the FOMC in March 2009, September 2012, and December 2013, each affecting their use of unconventional monetary policies. The second case study covers the introduction of the stimulus package passed by Congress in January 2009; the August 2011 deal to raise the US debt ceiling; and the deadlock in Congressional budget super committee in November 2011. The final case study highlights the May 2010 flash crash and the April 2013 Twitter crash, following reports of an explosion at the White House.

The events in each study are distinguished by how the arrival of new information affects uncertainty, whether market participants anticipated the information, and whether the information's effect on uncertainty was temporary or persistent. To identify these events, the largest jumps were selected from each of the 24 option chains. Concurrent jumps in both measures of information were then compared to a simple Factiva news search to find events which correspond to the arrival of significant new information. Here, the frequency and calendar span of the sample present two advantages. The high frequency allows us to identify the source of the new information, and the long calendar span allows us to compare the effect of that information to other events in the sample. The testing procedure introduced in Section 6 presents a third advantage, allowing us to test for jumps at an unknown time in the series. When used in conjunction with the frequency and span of the sample, the framework provides a novel way to identify, quantify, and test for the effect of new information on the beliefs of market participants.

The events analyzed here were not pre-selected or assumed to have affected the distribution of investor expectations. In a departure from much of the event-study literature, a hypothesis test was used to identify days where the potential arrival of new information resulted in a large and sudden change in the level of uncertainty. The approach therefore differs from other high-frequency event-studies, such as Goldberg and Grisse (2013) and Andersen, Bollerslev, Diebold and Vega (2003), among others, who examine the response of interest rates and exchange rates to select economic news announcements.

**Table 8: Large Information Events**

Desc	Date	Time	ttm	Shannon Entropy						Rényi Entropy					
				Trend	Jump	t-stat	Time	Pctile	t-stat	Trend	Jump	t-stat	Time	Pctile	t-stat
<i>Case I—Policymakers</i>															
President Obama	2010-01-21	10:35	41	0.004	-0.054	-6.26	09:25	98	-5.88	0.001	-0.033	-5.65	09:25	98	-5.51
Chairman Bernanke	2010-07-21	13:00	42	0.028	-0.035	-6.87	13:00	94	-6.25	0.013	-0.022	-6.30	13:00	95	-5.86
FOMC QE I	2009-03-18	13:15	03	0.000	-0.031	-3.49	13:15	93	-5.50	0.005	-0.011	-1.92	13:00	78	-2.94
QE III	2012-09-13	11:30	07	-0.034	0.084	7.02	11:40	99	-6.29	-0.014	0.037	8.28	11:35	98	-6.85
Taper	2013-12-18	13:00	03	-0.036	0.102	4.45	13:50	99	-5.59	-0.044	0.082	7.65	13:10	99	-6.52
<i>Case II—Legislatures</i>															
Stimulus package	2009-01-15	12:15-13:15	45	0.008	0.024	3.23	13:20	87	-5.82	0.002	0.022	4.12	12:40	95	-6.03
Debt ceiling	2011-08-01	08:30-09:00	34	0.020	-0.028	-4.29	09:00	91	-4.72	0.017	-0.023	-4.33	09:00	95	-4.77
Super committee	2011-11-17	11:00-11:30	21	0.046	-0.072	-3.20	11:25	99	-5.49	0.024	-0.083	-5.50	11:30	99	-6.13
<i>Case III—Flash Crashes</i>															
Flash crash	2010-05-06	13:42	31	-0.094	-0.075	-3.73	13:35	99	-5.24	-0.101	-0.083	-3.15	13:35	99	-4.22
Twitter crash	2013-04-23	12:07	43	0.034	-0.027	-3.82	11:50	90	-6.72	0.059	-0.048	-6.35	12:05	99	-6.66

Table 8 reports a summary of the results for each case study. Columns 2-4 report the date, time, and time-to-maturity, in days, for the events corresponding to large jumps in information. Columns 5 and 11 report estimates of the trend in Shannon and Rényi entropy for those days. Columns 6-9 and 12-15 report estimates, timing, and percentiles of the identified jumps in information. Percentiles are in absolute value. Columns 10 and 16 report the t-statistic for the test of the null of a random walk against the alternative, a trend stationary process with a single jump, for each day.

Table 8 reports the results of the testing procedure for each event in the three case studies. Figures 9-12 report the intraday evolution of information, the timing of the identified jump, and the intraday price of the underlying index and implied volatility index for each day. Figures D-G in the appendix report the t-statistics for each candidate model, the jump corresponding to the model with the minimum t-statistic, as well as the daily volume and price of the underlying index, and daily implied volatility for each option chain. The timing of all events is reported in Central Time (CT), -5 or -6 hours Coordinated Universal Time (UTC). Percentiles are in absolute value.

## 8.1 Policymakers

The first of three categories of discreet events demonstrates how new information arrives in the statements of key policymakers. The category includes speeches by both President Obama and Federal Reserve Chairman Ben Bernanke, and the release of three statements by the Federal Open Market Committee (FOMC). The events are distinguished by their effect on uncertainty, timing, and persistence. The case study presents evidence that significant new information is revealed in the statements of policymakers, and that uncertainty may trend or jump in anticipation of large news events.

When Federal Reserve Chairman Ben Bernanke began his semiannual report before the Senate Banking Committee, shortly after 13:00 CT on July 21, 2010, the S&P 500 was little changed for the day. Following his statement

that the economic outlook remained ‘unusually uncertain’ the index quickly dropped 1 percent and would end the day down 1.3 percent. Here the effect of new information on expectations is clear. The testing procedure identifies a sharp increase in uncertainty coinciding with Chairman’s opening remarks. Figure 9 shows both measures of information decline suddenly at 13:00 and continue falling until 13:30. The event presents evidence that significant new information was revealed in the statement, as investors rapidly revised their expectations in response to the Chairman’s subdued forecast for the economic recovery. The effect was large and persistent. Neither Shannon nor Rényi entropy recovered for the session. Table 8 reports estimates of theta as -0.035 and -0.022 nats respectively, just below the 95th percentile for the sample, and nearly half of the observed decline in information for the day. Referring to Tables 3 and 4, this single jump is 10 times larger than the average gain in information over the entire day.

The President’s speech began shortly before 10:40 CT on January 21, 2010. The day before, the S&P 500 had closed down 1.1 percent. At 07:30 that morning, initial jobless claims arrived at 482 thousand, higher than the 440 thousand expected. At 09:00 the Philadelphia Fed’s Business Outlook index was released, declining to 15.2 and below expectations of 18. Against this economic backdrop, President Obama announced his intention that morning to strengthen financial regulation by introducing a “Volcker Rule” to limit the trading activities of large US banks. His speech lasted less than 10 minutes. The S&P closed down 1.9 percent later that day, led down by financial stocks.

**Figure 9: Policymakers**

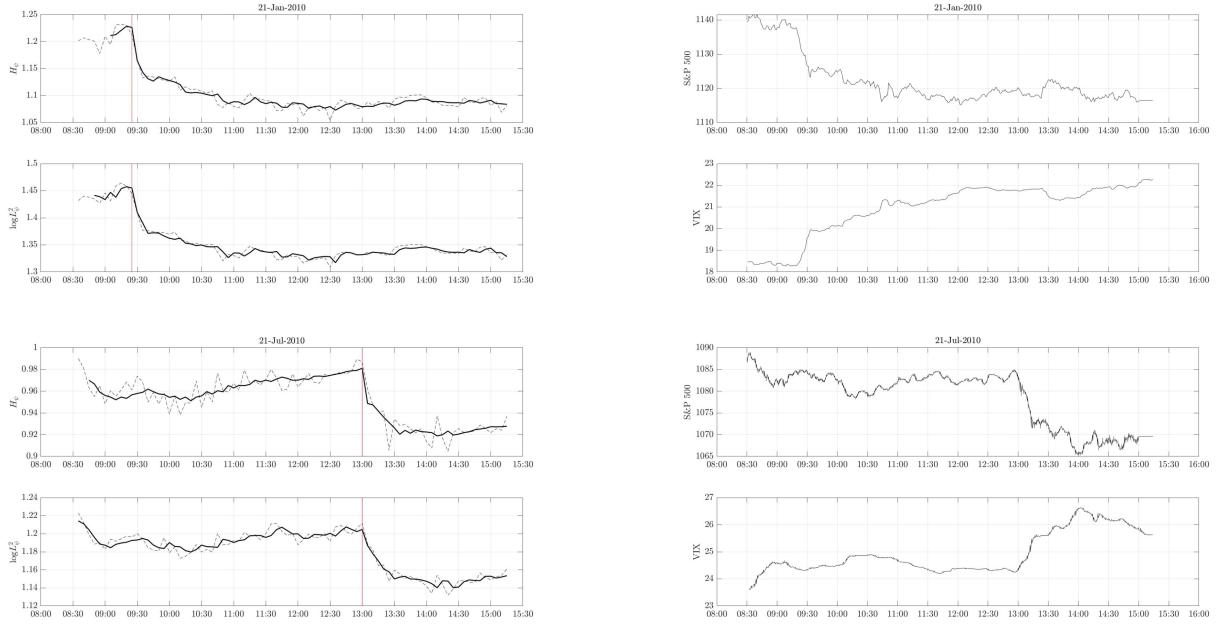


Figure 9 reports the results for the jumps identified on January 21, 2010 at 09:25 CT and July 21, 2010 at 13:00 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , and the intraday level of both the underlying S&P 500 index and CBOE Volatility or VIX Index. Panels 1 and 3 report the fitted model for each entropy in black and the detected jump  $\hat{\theta}$  in red.

In comparison, the testing procedure identifies a sharp increase in uncertainty just before 09:30. The timing of the jump does not coincide with the President's speech or an economic data release. Here the frequency of the analysis offers a more nuanced picture of how investor expectations evolved over the day. Looking at Figure 9, a large negative jump is detected in both measures of information at 09:25. The figure confirms this corresponds with

a sharp decline in the spot price of the underlying index, as expectations appear to have jumped in anticipation of the President's proposal. By 10:15 the S&P 500 was down 1.1 percent and the decline in information had already levelled off. The effect on uncertainty was large and persistent. Looking at Table 8, estimates of theta are -0.055 and -0.033 nats, representing a 98th percentile decline. Information did not recover following the President's speech, ending the day down over 0.10 nats.

The event presents evidence that uncertainty may jump in anticipation of events. Here the testing procedure identifies the exact timing of the shift in investor expectations. The jump coincides with the arrival of information prior to the President's scheduled remarks later that morning. The evidence suggests a lower frequency analysis risks misattributing this jump to a single data release, speech, or ambiguous combination of events.

Looking at both events, each occurred approximately 40 trading days or two calendar months from the expiration date of two separate option chains. The level of entropy is noticeably lower in July 2010, reflecting the growing macroeconomic uncertainty. The estimated trend is also dramatically larger, reflecting how market beliefs were evolving prior to the Chairman's testimony that day. The event demonstrates how information may also trend in anticipation of events. In this case, the jump reversed this trend, reflecting the uncertainty the Chairman's statement added to expectations.

Several statements by the Federal Open Market Committee (FOMC) are identified by the testing procedure. These include statements released in

March 2009, September 2012, and December 2013 affecting the Federal Reserve's use of quantitative easing to stimulate the US economy. On March 18, 2009, the FOMC announced new efforts to purchase long-term US government debt and expanded their purchases of mortgage-backed securities. On September 13, 2012, they expanded these purchases further, and would not announce any reduction in their efforts until December 18, 2013. Official statements from the FOMC were released at 13:15, 11:30, and at 13:00 CT on those days. Looking at Figure 10, the timing of each jump coincides with the release. From Table 8, each announcement arrived less than two weeks before the current option chain expired. In all three instances, the spot price of the underlying index increased, and finished the day between 1.6 and 2.1 percent higher. For the two later dates, the testing framework estimates a sharp increase in information with theta ranging from 0.084 and 0.037 nats to 0.102 and 0.082 nats respectively. Both jumps exceed the 98th percentile, as the S&P 500 reached multi-year and record highs in 2012 and 2013. The effect is large and persistent. The negative trend in information reflects the elevated level of near-term uncertainty as investors awaited the announcement.

In comparison, the flat trend and low level of information for the earlier date reflects the extreme level of uncertainty during the Global Financial Crisis. Here the FOMC announcement drove the spot price 3 percent higher. Estimates of theta however are negative, -0.031 and -0.011 nats, documenting a large increase in uncertainty. The effect was temporary, as the level of

**Figure 10: Policymakers**

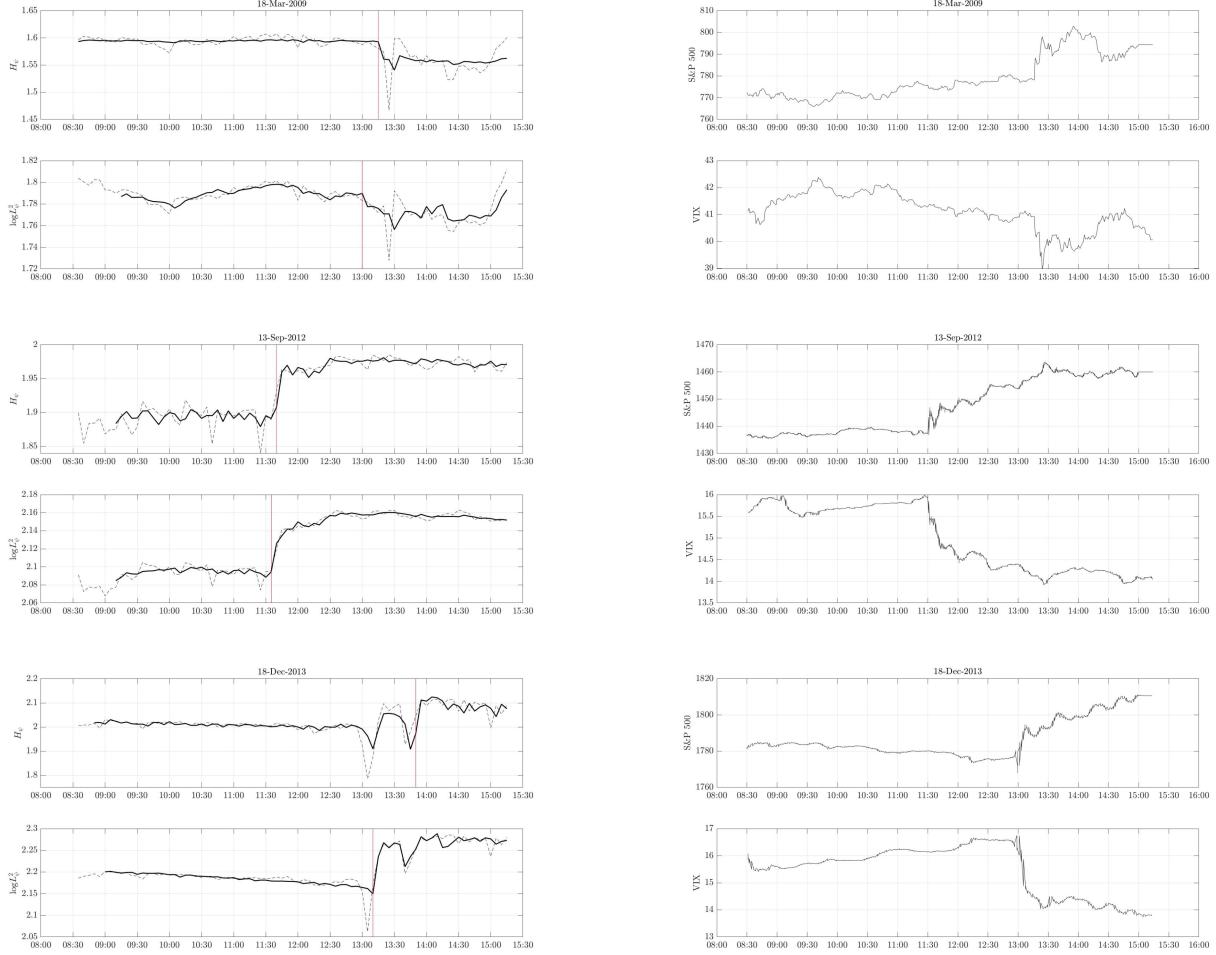


Figure 10 reports the results for the jumps identified on March 18, 2009, September 13, 2012, and December 18, 2013 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , and the intraday level of both the underlying S&P 500 index and CBOE Volatility or VIX Index. Panels 1, 3, and 5 report the fitted model for each entropy in black and the detected jump  $\hat{\theta}$  in red.

information recovered later that day. Yet the event suggests that investors consider both short and long-term implications of new information. For example, whether the expanded stimulus efforts also revealed a more pessimistic view of the recovery.

In summary, the first case study presents evidence that significant new information is revealed in the statements of policymakers. Events are distinguished by their effect on uncertainty, timing, and persistence. The case study demonstrates that uncertainty may trend or jump in anticipation of large news events.

## 8.2 Legislatures

The second case study demonstrates how information arrives in the actions of lawmakers. The category includes news related to the progress of legislation in the United States Congress and focuses on the 2009 stimulus package and 2011 debt ceiling increase. Events are distinguished by a lack of a reliable timestamp and their effect on uncertainty concerning the state of fiscal policy. The case study presents evidence that information is revealed in the action of Congress and demonstrates the testing procedure's ability to identify the timing of significant news events.

Several legislative actions are identified by the testing procedure. These events include the introduction of an economic stimulus package in mid-January 2009, the deal to raise the government's borrowing limit in early August

2011, and the deadlock in the congressional budget ‘super committee’ in mid-November 2011.

By midday on January 15, 2009, the S&P 500 was down 2.6 percent, weighed down by the news of Bank of America’s request for additional capital from the Troubled Asset Relief Program (TARP). President Obama was not due to take office for another week, and it remained uncertain whether the new government would be able to prevent the recession from deepening. On this day, the testing procedure detected a large gain in information around 12:40 CT. Estimates of theta are 0.024 and 0.022 nats, near the 90th percentile. The effect was large and persistent. Figure 11 shows the jump corresponds with a steady increase in the spot price, reversing earlier declines. The underlying index finished the session up 0.1 percent. The event presents evidence that information is revealed in the action of Congress. Looking at reports for the day, the timing of the jump coincides with news that House Democrats were ready to introduce a stimulus package to Congress which included tax cuts and billions in new spending and aid to states. At the time, the incoming President had already announced his intention to sign the new legislation by mid-February

A second jump in uncertainty was detected on August 1, 2011, shortly after 09:00 CT. Estimates of theta are -0.028 and -0.023 nats, near the 90th and 95th percentiles. At first glance, reports for the day are dominated by the news that President Obama and Congressional leaders had reached a deal late Sunday night to increase the government’s borrowing limit. Stocks had opened

**Figure 11: Legislatures**

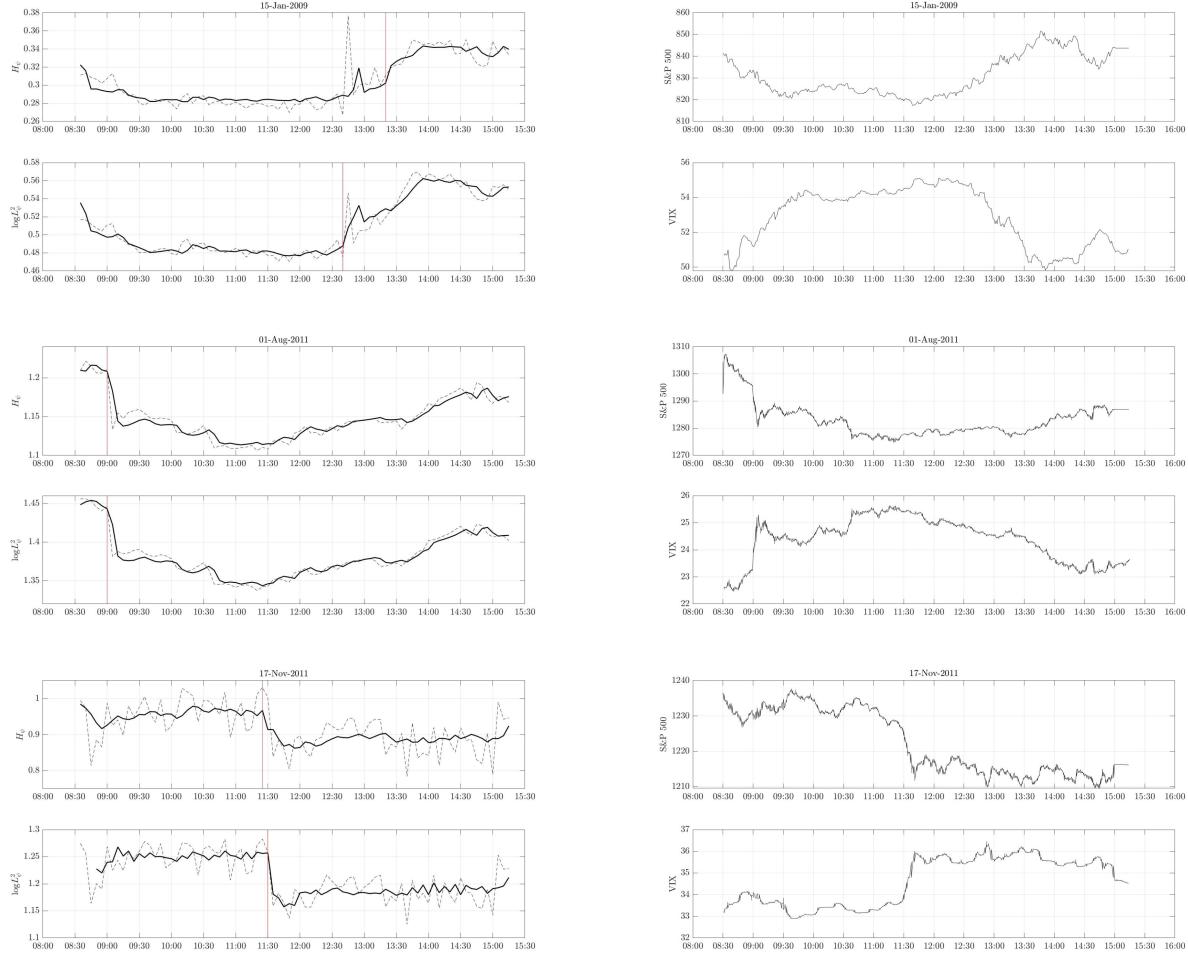


Figure 11 reports the results for the jumps identified on January 15, 2009, August 1 2011, and November 17, 2011 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , and the intraday level of both the underlying S&P 500 index and CBOE Volatility Index. Panels 1, 3, and 5 report the fitted model for each entropy in black and the detected jump  $\hat{\theta}$  in red.

sharply higher on this news. However, looking at Figure 11, the detected jump does not coincide with the open and by 10:00 the S&P 500 was down 1.3 percent. This event demonstrates the procedure's ability to identify the source, effect, and persistence of new information. Looking at all news for the day, weak manufacturing data was released at 09:00, with the ISM manufacturing index coming in at 50.9, well below expectations of 54.5. The release coincides with the jump identified at 09:00. The evidence suggests the information was significant and unexpected, as uncertainty increased in response. The effect was also persistent, as information did not recover for the day, despite a large and positive trend in both measures.

A third jump in uncertainty was detected shortly before 11:30 CT on November 11, 2011, on a morning the index had been mostly flat. The day before, stocks had fallen on news that US banks were increasingly vulnerable to contagion from eurozone debt. A week earlier, uncertainty had jumped as the yield on the 10-year Italian bond closed above 7.1 percent. This can be seen in the initial level of entropy for the day. When news reports began to arrive that the Congressional budget 'super committee' was deadlocked days before their deadline information crashed. The effect on uncertainty was large and persistent. Estimates of theta are in the 99th percentile, over -0.072 and -0.083 nats. In 10 minutes, the spot price had fallen almost 2 percent. Figure 11 shows that neither information nor the underlying index recovered that day.

The second case study presents evidence that information about the state of fiscal policy is revealed in the action of legislators. Measuring the effect of that information however requires being able to detect its arrival to investors, a fact that is not trivial given the lack of reliable timestamp. The second category of discreet events demonstrates the testing procedure's ability to identify the source and timing of such information.

### 8.3 Flash Crashes

The final case study demonstrates the effect of a flash crash in the underlying index on information. The category includes two sudden, large, and extremely volatile declines in the spot price of the S&P 500. Each event is distinguished by its effect on uncertainty, timing, and persistence. The events occurred in May 2010 and April 2013. The study presents evidence that the rapid arrival of unanticipated information led to a dramatic spike in uncertainty, and that information did not recover until the cause of volatility was known.

On the morning of May 6, 2010, the S&P 500 opened flat as markets awaited the European Central Bank's (ECB) decision on interest rates. News that day was dominated by concerns over Europe's sovereign debt, with spreads on Portuguese and Spanish bonds approaching all-time highs. Just the day before, Greece had passed the austerity measures necessary to access the euro-zone bailout. Premarket, US economic data was mixed; with first-quarter

**Figure 12: Flash Crashes**

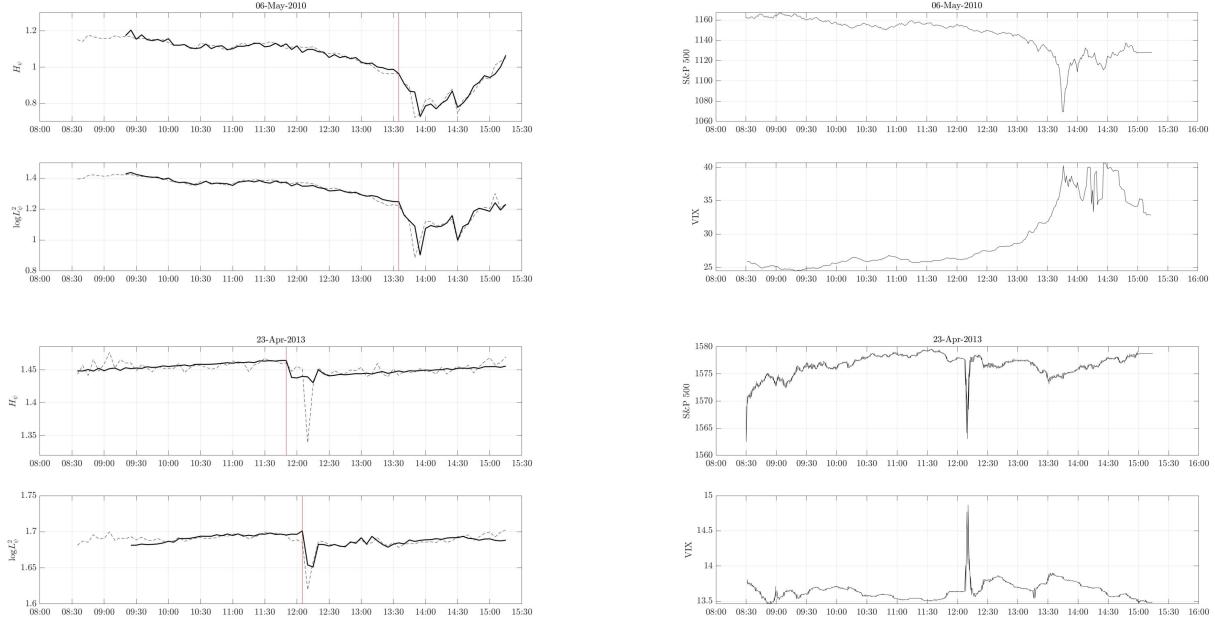


Figure 12 reports the results for the jumps identified on May 6, 2010 and April 23, 2013 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , and the intraday level of both the underlying S&P 500 index and CBOE Volatility or VIX Index. Panels 1 and 3 report the fitted model for each entropy in black and the detected jump  $\hat{\theta}$  in red.

productivity beating expectations, while unemployment and retail sales fell short of forecasts. Selling began in earnest shortly after 08:00 CT. At 08:30 the ECB announced its decision to leave interest rates unchanged. By 13:00 the underlying index was down 1.6 percent on reports that markets had turned their attention to the risk of contagion within Europe's periphery. By 13:40 the index was down 4.1 percent. Then information collapsed.

In 5 minutes, the S&P 500 fell another 4.5 percent, bottoming out at 13:46, down almost 8.6 percent. From 13:40 to 13:45, Shannon entropy fell 0.14 nats and Rényi entropy dropped 0.19 nats, two of the largest declines in the sample. As markets scrambled to identify the cause of the panic, the selling reversed. Five minutes later the index had recovered. Information however had not, and volatility continued throughout the session.

The event presents evidence that uncertainty remained elevated while the cause of volatility was unknown. The testing procedure places the beginning of the crash at 13:35 CT, and estimates of theta are -0.075 and -0.083 nats, both in the 99th percentile. The effect on uncertainty was dramatic and persistent. The estimated trend is both large and negative. By 14:00 information had fallen 0.40 nats and would not recover until after 14:30. Looking at Figure 12, neither measure of information returned to pre-crash levels that day. The S&P 500 closed down 3.2 percent.

At the time, the source of volatility could not be found. Initial reports blamed the crash on a ‘flight to safety’ amid fears that Europe’s debt problems would end the economic recovery. As the day closed however, attention turned to reports that a trading error may have caused the massive sell-off. By the end of the week, regulators had turned their focus to the role of high-frequency traders in exacerbating price declines. The official report on the events leading up to the May 6th flash crash would not be published until September 2010.

By comparison, the cause of the flash crash in April 2013 was immediately clear. “Breaking: Two Explosions in the White House and Barak Obama is

injured” tweeted the Associated Press (@AP) just after 12:00 CT on April 23, 2013. As the news hit markets, the S&P 500 fell 1 percent in 3 minutes. The testing procedure identifies the crash at 12:05 and estimates the loss to be near -0.027 and -0.048 nats, in the 90th and 99th percentiles. From 12:00 to 12:05, Shannon entropy fell 0.10 nats and Rényi entropy sank 0.06 nats. Minutes later the index rebounded as information arrived that the AP’s twitter account had been hacked. Reports of an explosion at the White House were false. The effect on uncertainty then was large but temporary. By 12:20 information had returned to pre-crash levels. Following the event, the estimated trend is large and positive. Looking at Figure 12, both measures of information recovered quickly, with the underlying index finishing the day up 1 percent.

The final case study presents evidence that the rapid arrival of unanticipated information led to a dramatic spike in uncertainty. Each flash crash was defined by a sudden, large, and volatile declines in the spot price of the underlying index. The May 2010 and April 2013 events are distinguished by their effect on uncertainty and persistence. Both are large, and the events present evidence that information did not recover until the cause of volatility was known.

In summary, the events in each case study are defined by how the arrival of new information affects uncertainty, whether that information was anticipated, and if its effect was persistent. In a first application, each study demonstrates the method’s ability to identify, quantify, and test for the effect of new

information on the beliefs of market participants. The results demonstrate the need for a more nuanced understanding of how investor expectations respond to information. In practice, investors, researchers, and policymakers should consider carefully how they quantify uncertainty and identify events of interest to their analysis. Going forward, similar methods could be used to identify jumps in the underlying asset and to determine whether market participants view such jumps as temporary or persistent over time. Current work is underway to investigate the ability of these measures of information to forecast realized variance, model the term structure of volatility, and quantify the effect of different sources of information on the beliefs of market participants.

## 9 Conclusions

In this paper, I confront the basic characterization of the process by which investors learn about the future value of an asset. The paper is the first to quantify in real time how information drives price discovery in option markets. In doing so, I offer three methodological contributions to the literature on measuring the information found in option prices, and document two empirical facts not explained by existing theoretical models.

I provide a high-frequency analysis of the price discovery process in option markets. Using six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange, I characterize the intraday evolution of the density function implied by the price of options with the same

maturity date. The analysis is done for the final 3 months of each option's life cycle, as the density is shown to become more and more concentrated over time. This paper is the first to estimate the intraday dynamics of the risk-neutral density over the life cycle, and offers the following three methodological contributions to the literature. First, I show how a simple nonparametric estimator can be used to approximate the implied density of future returns at high frequencies. Second, I show how concepts developed in information theory can be used to quantify the amount of information contained in the estimated density. Third, I show how this novel approach permits a simple testing procedure for the presence of jumps in the evolution of the risk-neutral density, coinciding with the arrival of new information. The results of this testing procedure represent the paper's main contribution to the literature.

I investigate how information arrives in financial markets, and I find that information often arrives in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump, a result not anticipated by existing theoretical models. The testing procedure reveals both the frequency and magnitude of these jumps in investor expectations. I identify at least one jump for a majority of days, and find days without jumps contribute little to the total information gained over the life cycle. I then document two empirical facts new to the literature: First, the majority of information accrues only in the final month. I show investors learn little about the future price of an asset for much of an option's life. Second, jumps contribute a majority of

information early in the life cycle. Only in the final month does information arrive often enough to contribute more to the total gained.

The paper builds on earlier work in many ways, but several features distinguish the findings from previous results. These include; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, 2. a fully non-parametric estimation technique, 3. a measure of information as a reduction in uncertainty, 4. a simple framework to test for jumps, and 5. the frequency and length of the sample of options data used.

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# Appendix

**Table A: Daily Summary Statistics**

Maturity		Count	Minimum	Maximum	Average	Std. Dev.	Option Ex Date		Count	Minimum	Maximum	Average	Std. Dev.
2009-03-21	<i>Strikes</i>	110	200	1,600.00	857.65	82.57	2012-03-17	<i>Strikes</i>	154	400	1,550.00	1,080.10	14.59
	<i>Calls</i>	730,566	0.03	630.69	89.90	14.44		<i>Calls</i>	311,975	0.03	1,001.60	270.89	33.50
	<i>Underlying</i>	724,788	674	934.39	807.95	66.88		<i>Underlying</i>	293,866	1,204	1,402.61	1,323.45	44.76
2009-06-20	<i>Strikes</i>	118	300	1,600.00	802.55	25.08	2012-06-16	<i>Strikes</i>	157	450	1,750.00	1,156.40	27.44
	<i>Calls</i>	948,751	0.03	645.55	155.58	26.68		<i>Calls</i>	613,567	0.03	962.40	251.98	14.47
	<i>Underlying</i>	950,198	786	947.71	880.67	44.19		<i>Underlying</i>	583,803	1,278	1,419.18	1,360.77	40.63
2009-09-19	<i>Strikes</i>	114	450	1,650.00	899.55	41.57	2012-09-22	<i>Strikes</i>	131	400	1,600.00	1,169.96	6.13
	<i>Calls</i>	666,463	0.03	617.52	149.95	31.37		<i>Calls</i>	423,603	0.03	1,064.22	245.27	26.85
	<i>Underlying</i>	669,623	879	1,068.76	976.62	53.17		<i>Underlying</i>	412,665	1,314	1,464.25	1,387.58	39.40
2009-12-19	<i>Strikes</i>	125	450	1,550.00	958.43	16.40	2012-12-22	<i>Strikes</i>	157	350	1,800.00	1,222.06	15.78
	<i>Calls</i>	508,810	0.03	663.64	180.09	21.49		<i>Calls</i>	768,118	0.03	1,105.25	254.77	19.29
	<i>Underlying</i>	510,326	1,025	1,114.16	1,080.27	24.07		<i>Underlying</i>	758,723	1,353	1,462.10	1,420.25	26.64
2010-03-20	<i>Strikes</i>	134	400	1,500.00	974.39	12.82	2013-03-16	<i>Strikes</i>	156	350	1,800.00	1,217.09	9.21
	<i>Calls</i>	326,984	0.03	765.84	199.05	21.50		<i>Calls</i>	640,450	0.03	1,212.67	303.08	28.06
	<i>Underlying</i>	328,884	1,057	1,166.21	1,116.48	27.72		<i>Underlying</i>	630,545	1,403	1,563.25	1,498.68	35.88
2010-06-19	<i>Strikes</i>	124	550	1,500.00	1,024.83	15.83	2013-06-22	<i>Strikes</i>	149	900	1,850.00	1,397.21	14.06
	<i>Calls</i>	592,298	0.03	662.63	175.34	25.00		<i>Calls</i>	911,785	0.03	766.75	236.32	30.46
	<i>Underlying</i>	579,694	1,051	1,217.16	1,146.51	50.43		<i>Underlying</i>	908,343	1,542	1,669.34	1,603.54	39.20
2010-09-18	<i>Strikes</i>	116	600	1,400.00	987.32	14.19	2013-09-21	<i>Strikes</i>	171	650	2,000.00	1,415.36	13.62
	<i>Calls</i>	603,780	0.03	525.39	156.42	26.13		<i>Calls</i>	775,906	0.03	1,074.66	295.04	32.17
	<i>Underlying</i>	605,497	1,022	1,128.25	1,086.60	27.73		<i>Underlying</i>	774,767	1,573	1,725.28	1,067.12	33.00
2010-12-18	<i>Strikes</i>	127	350	1,500.00	999.74	7.97	2013-12-21	<i>Strikes</i>	163	850	2,100.00	1,505.10	6.25
	<i>Calls</i>	315,365	0.03	892.70	228.38	21.24		<i>Calls</i>	813,782	0.03	959.51	283.22	33.18
	<i>Underlying</i>	316,956	1,124	1,242.87	1,187.25	31.12		<i>Underlying</i>	814,691	1,656	1,810.83	1,753.01	44.79
2011-03-19	<i>Strikes</i>	132	350	1,600.00	1,077.37	6.22	2014-03-22	<i>Strikes</i>	187	650	2,150.00	1,536.28	5.75
	<i>Calls</i>	398,175	0.03	990.37	256.34	19.85		<i>Calls</i>	1,004,186	0.03	1,226.51	331.89	32.44
	<i>Underlying</i>	385,724	1,247	1,343.01	1,294.92	25.64		<i>Underlying</i>	994,344	1,741	1,878.04	1,832.26	32.15
2011-06-18	<i>Strikes</i>	123	550	1,550.00	1,154.30	7.48	2014-06-21	<i>Strikes</i>	179	900	2,150.00	1,595.54	9.14
	<i>Calls</i>	545,121	0.03	810.19	212.51	14.07		<i>Calls</i>	1,036,519	0.03	1,058.53	320.98	33.35
	<i>Underlying</i>	524,984	1,266	1,363.65	1,322.02	24.30		<i>Underlying</i>	1,052,149	1,816	1,959.47	1,889.78	34.05
2011-09-17	<i>Strikes</i>	132	550	1,550.00	1,124.09	28.17	2014-09-20	<i>Strikes</i>	187	700	2,200.00	1,617.38	2.62
	<i>Calls</i>	861,006	0.03	799.34	188.95	29.96		<i>Calls</i>	666,667	0.03	1,312.05	378.46	21.67
	<i>Underlying</i>	817,286	1,119	1,353.83	1,245.15	72.85		<i>Underlying</i>	645,371	1,908	2,011.64	1,972.74	24.53
2011-12-17	<i>Strikes</i>	164	150	1,550.00	993.27	6.88	2014-12-20	<i>Strikes</i>	196	700	2,250.00	1,648.19	14.01
	<i>Calls</i>	778,565	0.03	1,131.37	273.30	32.09		<i>Calls</i>	1,959,596	0.03	1,373.14	377.08	42.93
	<i>Underlying</i>	732,673	1,100	1,285.92	1,213.10	43.48		<i>Underlying</i>	1,942,217	1,862	2,075.37	1,996.52	58.27

Table A reports standard summary statistics for the sample of call options used. Column 1 reports the average number of options, bids, and quotes observed each day. Columns 2 through 4 report their minimum, maximum, and average values for the sample or option listed. Column 5 reports the standard deviation of the average.

**Figure A: Daily Evolution**

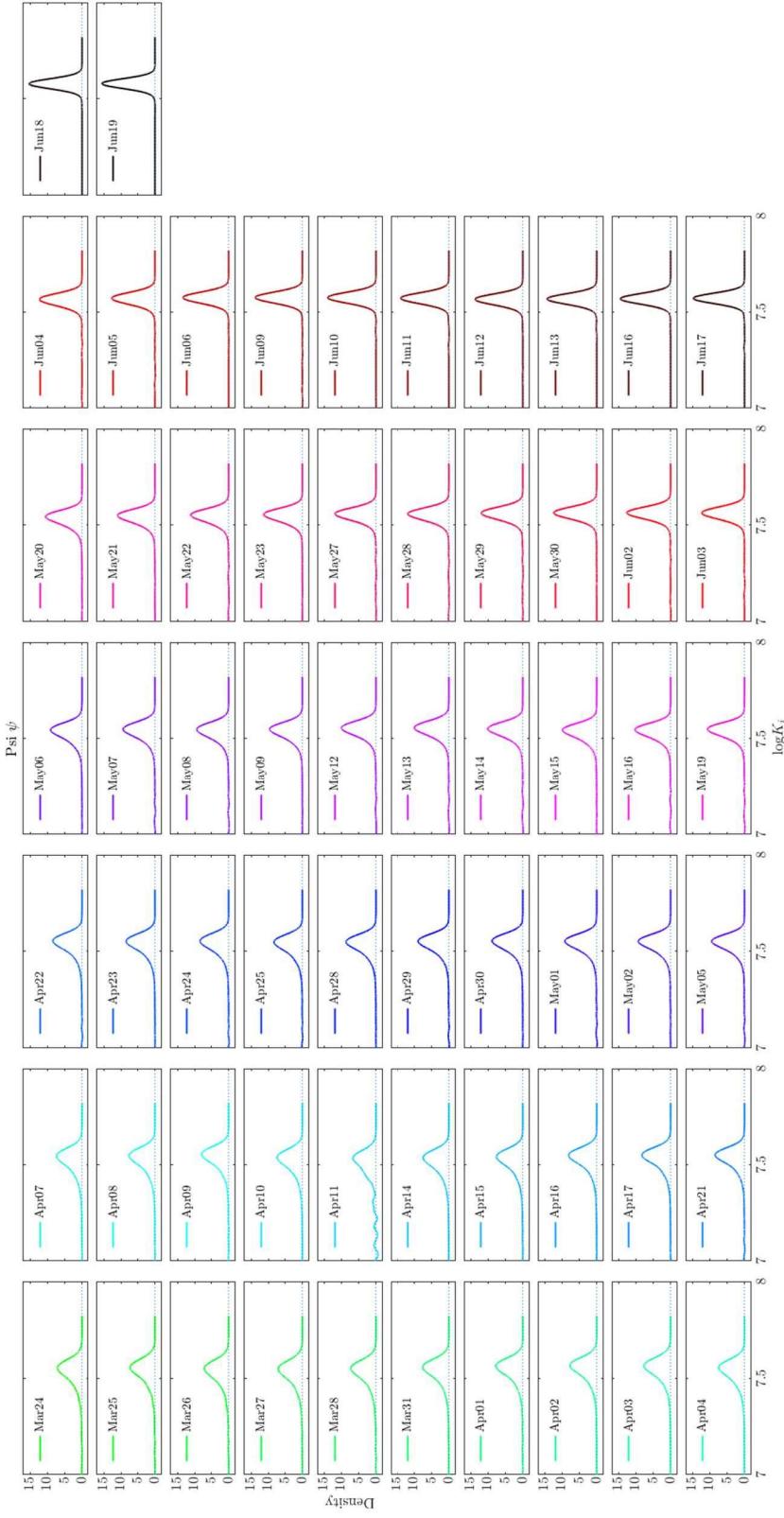


Figure A reports the daily evolution of the approximate risk-neutral measure  $\psi$  as the convolution of  $\widehat{p}_i$ , the estimated cost of the sum of butterfly options, and  $\varphi$ , a chosen smoothing function. The estimated densities  $\psi$  for the SPX options expiring in June 2014 are shown in color for each of  $T = 62$  days in the option's life cycle, beginning on March 24 and ending on June 19, 2014. End-of-day estimates are reported in each panel of the figure. Each estimate is derived from prices recorded at 15:15 each day.

**Figure B: Timing of Information**

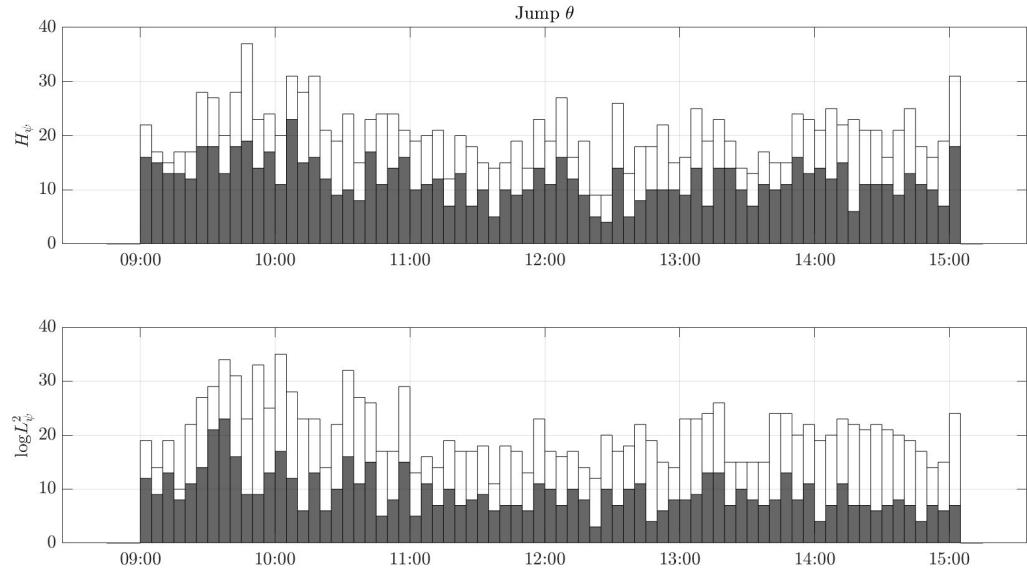


Figure B presents histograms of the timing of the estimated daily jump in information for all days, and both measures of information, Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ , entropy for all days in white and for days where the null hypothesis of a random walk is rejected in grey.

**Figure C: Gain in Information Split Sample**

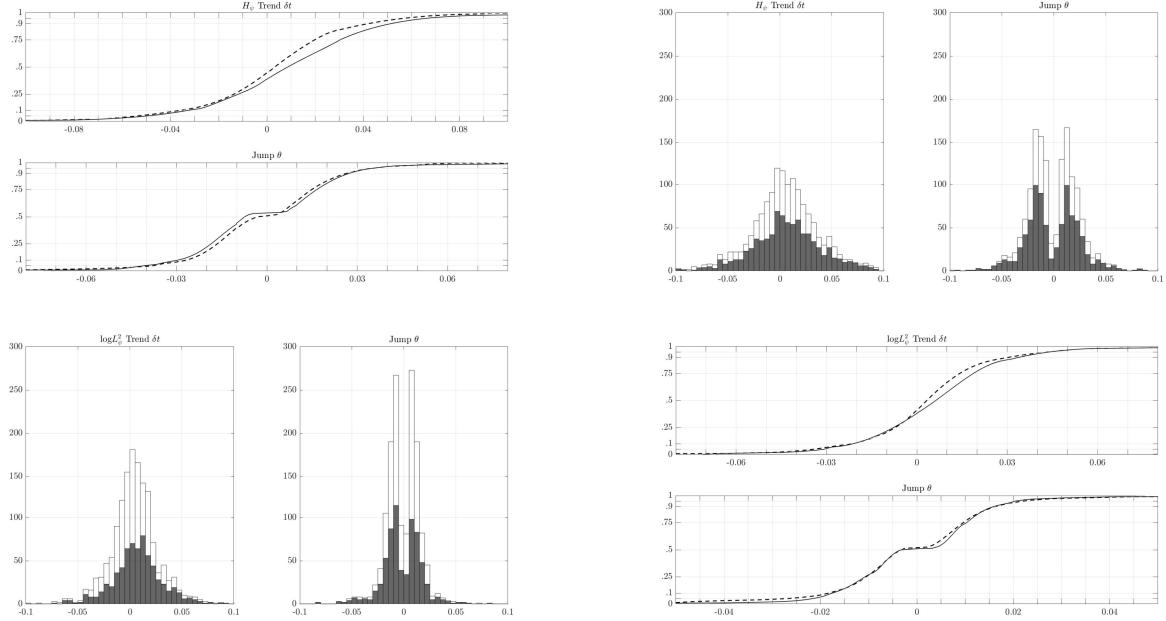


Figure C reports the estimated daily trend and jump estimates for separate subsamples and measures of entropy. In panels 1 and 4, the empirical cumulative distribution functions for the trend and jumps in Shannon,  $H_\psi$ , and Rényi,  $\log L_\psi^2$ , entropy are shown for the first 2-months, dashed, and the final month in black. Panels 2 and 3 report the histogram of trend and jumps in Shannon and Rényi entropy for all days in white and for days where the null hypothesis of a random walk is rejected in grey.

**Figure D: Policymakers**

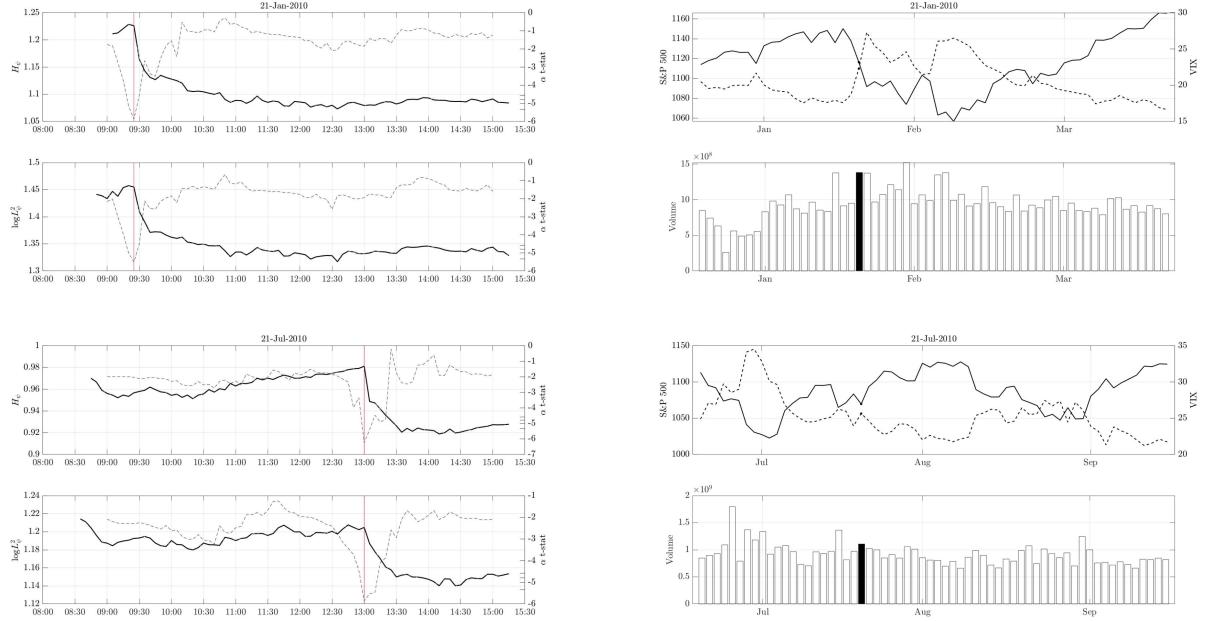


Figure D reports the results for the jumps identified on January 21, 2010 at 09:25 CT and July 21, 2010 at 13:00 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , the daily level of both the underlying S&P 500 index and CBOE Volatility or VIX Index, and volume. Panels 1 and 3 report the t-statistics for each candidate model, the minimum t-statistic in red, and fitted alternative model for each entropy in black.

## Figure E: Policymakers

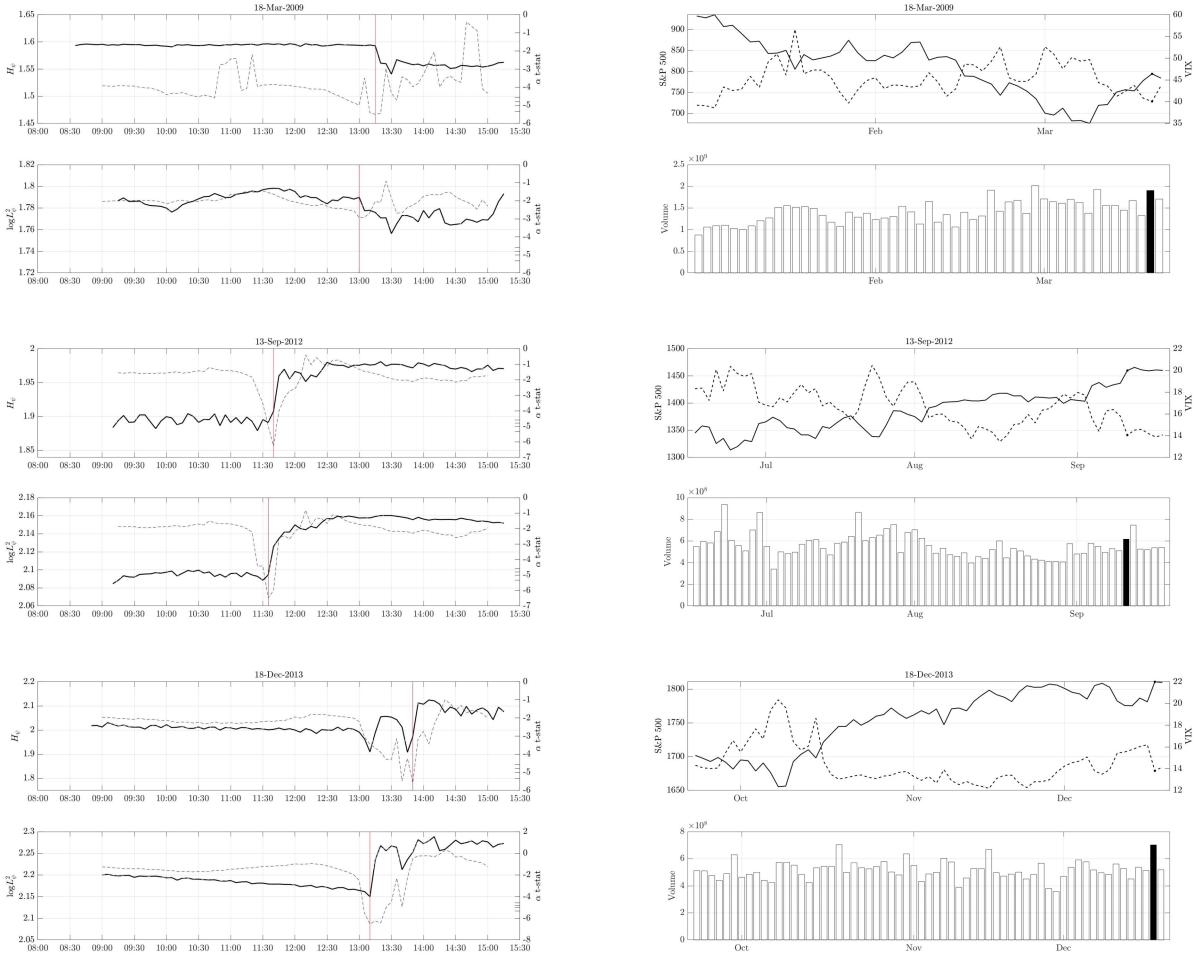


Figure E reports the results for the jumps identified on March 18, 2009, September 13, 2012, and December 18, 2013 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , the daily level of both the underlying S&P 500 index and CBOE Volatility Index, and volume. Panels 1, 3, and 5 report the t-statistics for each candidate model, the minimum t-statistic in red, and fitted alternative model for each entropy in black.

**Figure F: Legislatures**

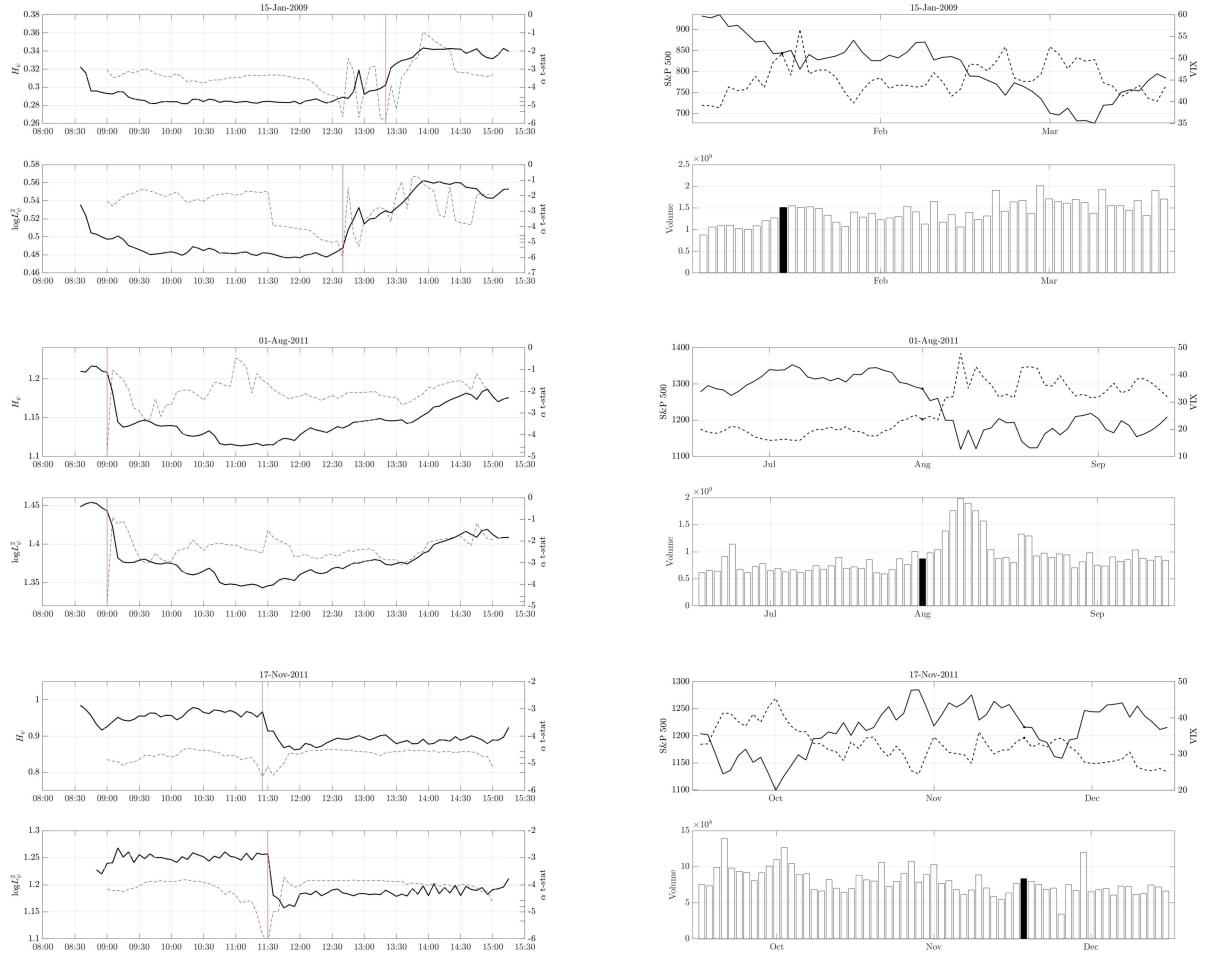


Figure F reports the results for the jumps identified on January 15, 2009, August 1 2011, and November 17, 2011 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , the daily level of both the underlying S&P 500 index and CBOE Volatility Index, and volume. Panels 1, 3, and 5 report the t-statistics for each candidate model, the minimum t-statistic in red, and fitted alternative model for each entropy in black.

**Figure G: Flash Crashes**

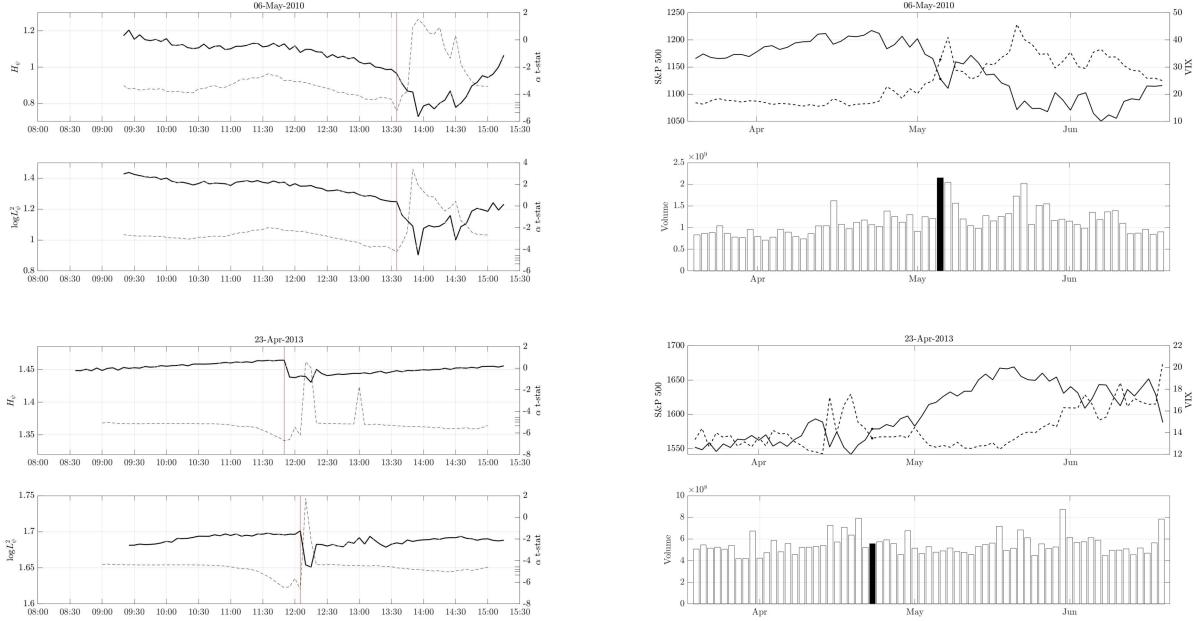


Figure G reports the results for the jumps identified on May 6, 2010 and April 23, 2013 for both measures of information, Shannon entropy,  $H_\psi$ , and Rényi entropy,  $\log L_\psi^2$ , the daily level of both the underlying S&P 500 index and CBOE Volatility Index, and volume. Panels 1 and 3 report the t-statistics for each candidate model, the minimum t-statistic in red, and fitted alternative model for each entropy in black.