L. Vandenberghe ECE236B Spring 2018

Homework 2

Submit answers for problems 1–4 only.

1. Exercise T2.12 (d,g).

2. *Polar of a set.* The *polar* of a set $C \subseteq \mathbf{R}^n$ is defined as

$$C^\circ = \{y \in \mathbf{R}^n \mid y^T x \leq 1 \text{ for all } x \in C\}.$$

   (a) Show that C° is convex (even if C is not).

   (b) What is the polar of a cone?

   (c) What are the polars of the following three sets?

$$C_1 = \{x \mid \|x\|_2 \leq 1\}, \qquad C_2 = \{x \mid \|x\|_1 \leq 1\}, \qquad C_3 = \{x \mid \mathbf{1}^T x = 1,\ x \succeq 0\}.$$

   (Here $\mathbf{1}$ denotes the vector of ones.)

3. Exercise A2.10.

4. Exercise A5.8.

Problems 5–8 are additional practice problems and will not be graded.

5. Exercise T2.37.

6. *Polar of a convex set.* Suppose C is closed, convex, and $0 \in C$. Show that $(C^\circ)^\circ = C$.

   (a) By definition of C°, we have

$$y^T x \leq 1 \text{ for all } x \in C,\ y \in C^\circ.$$

   Show that this implies that $C \subseteq (C^\circ)^\circ$ (without assumptions on C).

   (b) Now suppose C is closed and convex, with $0 \in C$. Show that $(C^\circ)^\circ \subseteq C$. (Combined with the result of part (a), this proves that $(C^\circ)^\circ = C$.)
   Hint. Show that if $x \notin C$, then $x \notin (C^\circ)^\circ$. To do this you can apply the strict separating hyperplane theorem of page 49 of the textbook: If C is a closed convex set and $x \notin C$, then there exists a vector $a \neq 0$ and a scalar b such that

$$a^T x > b, \qquad a^T z \leq b \text{ for all } z \in C.$$

7. Exercise T3.1.

8. Exercise T3.18 (a).