

# MAE 263C Homework #1

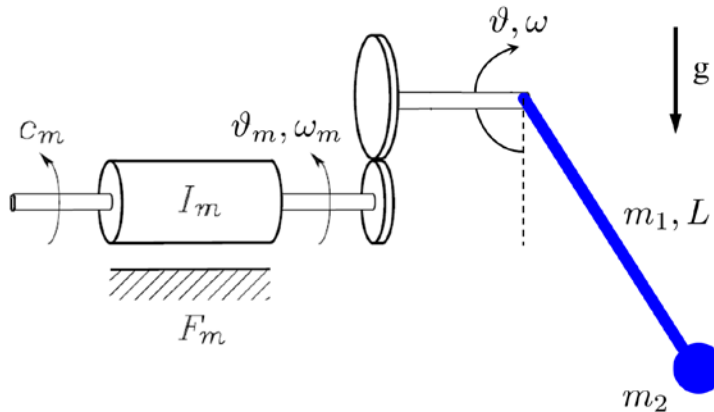
(Due online by 5pm on Friday, 4/13)

## 1. Motor-load dynamics with gear reduction

Consider a motor with a rotor mass moment of inertia  $I_m = 0.03 \text{ Kg-m}^2$  and a maximum torque  $c_m = 12 \text{ N-m}$  that is connected to a uniformly distributed link of mass  $m_l = 3 \text{ kg}$  and length  $L = 0.5 \text{ m}$  with a concentrated mass  $m_2 = 2 \text{ kg}$  at its end. Consider a configuration in which the link lies in the horizontal plane with an angular position  $\nu = 270^\circ$  as measured from the vertical equilibrium position (see  $\nu$  in figure below). The inertia of the pair of reduction gears and viscous friction in the system is negligible.

- Determine the total inertia  $I_{tot}$  felt by the motor and the maximum angular acceleration  $\dot{\omega}_m$  that the motor can develop if the gear ratio  $k_r$  is *i)* 5 and *ii)* 50. Note the mass moment of inertia for the load should include terms for both the rotation of the link and the point mass.
- Compare the total inertia values for the two gear ratio scenarios. Compare the maximum angular acceleration values for the two gear ratio scenarios. Explain why your answers make physical sense.

Hint: Draw a free body diagram to determine the reaction torque on the load axis  $c_l$ .



## 2. Motor-load dynamics with gear reduction

Repeat Problem 1, but now consider that the gear attached to the motor output shaft has an inertia of  $0.002 \text{ Kg-m}^2$  and the larger gear that transfers torque to the link has an inertia of  $0.004 \text{ Kg-m}^2$ . Viscous friction in the system remains negligible.

- Determine the total inertia  $I_{tot}$  felt by the motor and the maximum angular acceleration  $\dot{\omega}_m$  that the motor can develop if the gear ratio  $k_r$  is *i)* 5 and *ii)* 50.
- Compare the total inertia values for Problems 1 and 2. Compare the maximum angular acceleration values for Problems 1 and 2. Comment on the effects of including the inertia of the gear reduction system in your model. Explain why your answers make physical sense.

Hint: The contribution to  $I_{tot}$  of the larger gear that transfers torque to the link will be affected by the gear ratio.

### 3. Stiffness of transmission systems

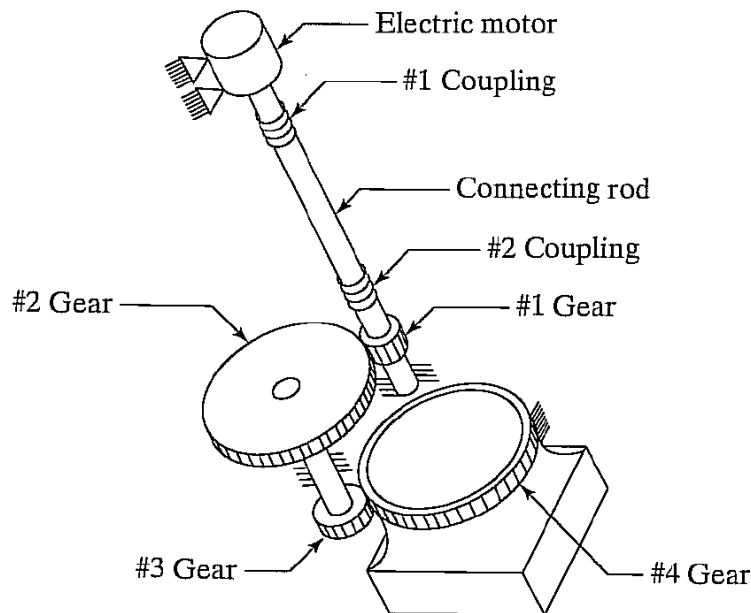
Prove that, if two transmission systems are connected serially, then the equivalent stiffness

$k_{series}$  of the overall system is given by  $\frac{1}{k_{series}} = \frac{1}{k_1} + \frac{1}{k_2}$ .

Hint: Consider the serial connection of two linear springs having stiffness coefficients  $k_1$  and  $k_2$ .

### 4. Stiffness of transmission systems and joints

Consider the simplified schematic of the drive system of a joint shown below. The torsional stiffness of the couplings is 100 N-m/rad each, that of the connecting rod is 400 N-m/rad, and each of the reduction pairs has an output stiffness of 2000 N-m/rad with its input gears fixed. Each gear reduction pair has a gear ratio  $k_r = 6$ . Assuming that the structure and bearing are perfectly rigid, what is the stiffness of the joint (i.e., when the motor's shaft is locked)? In other words, what is the stiffness at Gear #4? Hint: You can leverage the principles from Problem 3.



### 5. Modeling a DC servomotor

Consider the given block diagram model for a DC servomotor. Use MATLAB Simulink to perform a computer simulation of the current  $I_a$  and angular velocity  $\Omega_m$  response to a unit step voltage input  $V_c$ . for  $t = [0, 0.15]$  sec using a sampling time of 1 ms.

- Plot current (A) vs. time (sec).
- Plot angular velocity (rad/s) vs. time (sec).
- Submit plots, custom MATLAB scripts, and a screenshot of your Simulink block diagram.

Be sure to use good code-writing and plotting practices.

- MATLAB scripts should have your name and be well commented.
- Plots should have titles and axis labels, including units.
- Sparse plots should plot individual data points in addition to trend lines.

Constants:

|  |                                  |
|--|----------------------------------|
| Current regulator transfer function      | $C_i = 1$                        |
| Power amplifier voltage gain             | $G_v = 1$                        |
| Power amplifier time constant            | $T_v = 0.1$ ms                   |
| Armature inductance                      | $L_a = 2$ mH                     |
| Armature resistance                      | $R_a = 0.2$ Ohm                  |
| Transducer gain of current feedback loop | $k_i = 0$                        |
| Motor torque constant                    | $k_t = 0.2$ N-m/A                |
| Motor voltage constant                   | $k_v = 0.2$ V-s/rad              |
| Motor moment of inertia                  | $I_m = 0.0014$ kg-m <sup>2</sup> |
| Motor coefficient of viscous friction    | $F_m = 0.01$ N-m-s/rad           |
| Load reaction torque                     | $C_l = 0$                        |

