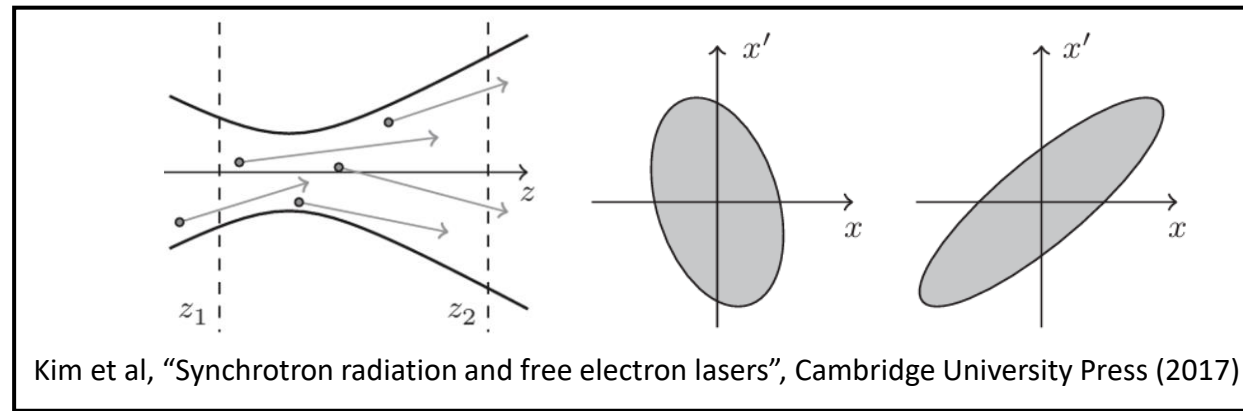


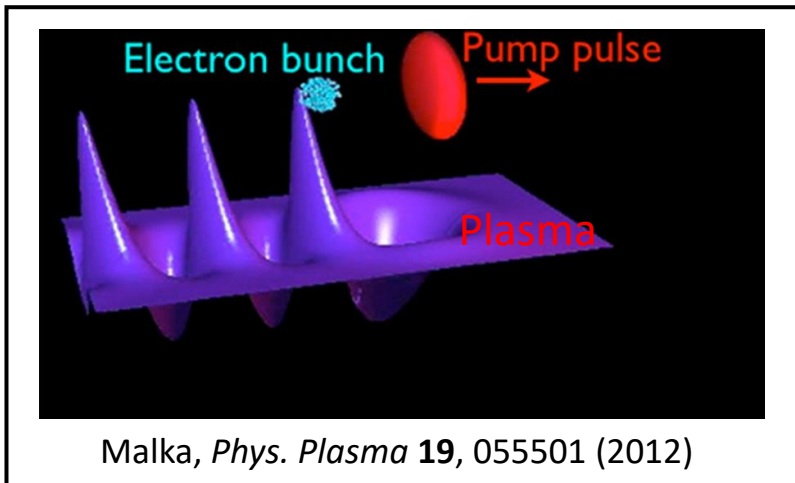


(Qualifying Exam)



Kim et al, "Synchrotron radiation and free electron lasers", Cambridge University Press (2017)

Exploring the quasi-6D structure of laser-wakefield-accelerated electron bunches with coherent optical transition radiation

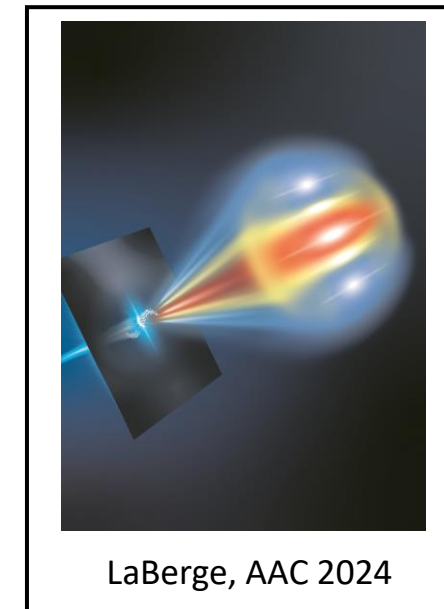


Malka, *Phys. Plasma* **19**, 055501 (2012)

Ze Ouyang

Supervisor: Michael Downer

6th Nov, 2024



LaBerge, AAC 2024

Outline

1

Introduction to LWFA and its diagnostics

2

COTR(I) and quasi-6D structure of e- bunches

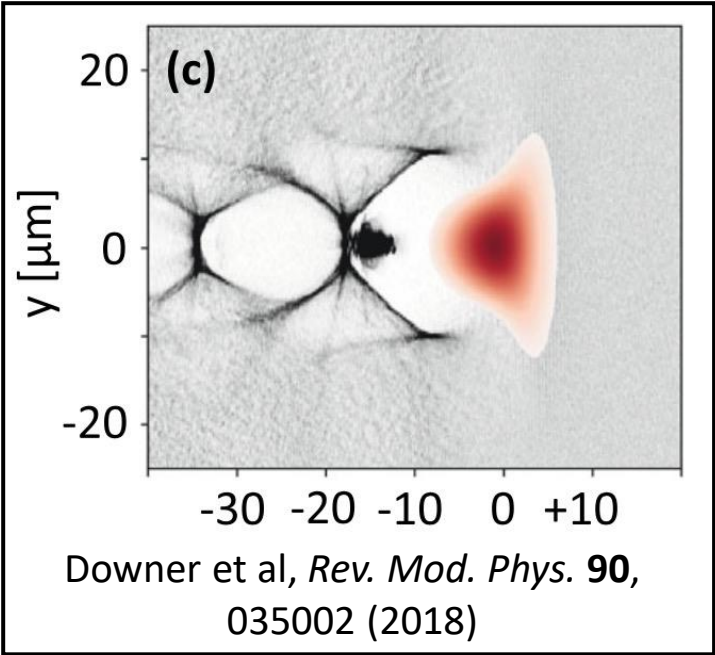
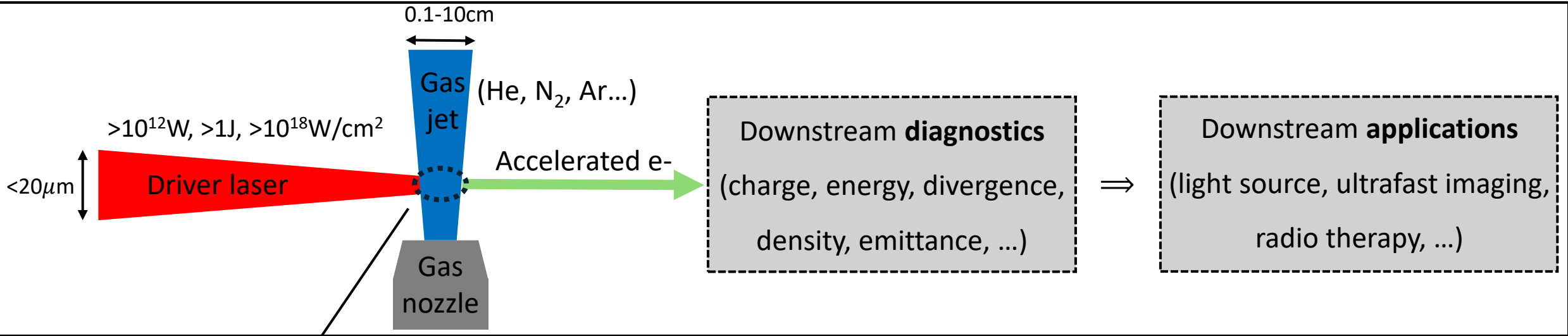
3

Future directions, experimental work & conclusion

Useful abbreviations:

- LWFA: Laser-driven WakeField Accelerator
- TR: Transition Radiation
- COTR: Coherent Optical Transition Radiation
- COTRI: Coherent Optical Transition Radiation Interferometry

Introduction: Laser-driven WakeField Accelerator



LWFA ∈ plasma-based accelerator¹

	Plasma-	Conventional (SLAC)
E	100GV/m	100MV/m
Footprint	~m	~km
Max Energy	10GeV ²	50GeV
Cost	~few \$millions	114 \$millions in 1960s

¹ Tajima et al, *Phys. Rev. Lett.* **43**, 4 (1979)

² Aniculaesei et al, *MRE* **9**, 014001 (2024)

Plasma-	Conventional
<100pC	~nC
~μm	~10μm
<10fs	~100fs
~mrad	~μrad

We need new diagnostics.

Introduction: LWFA diagnostics

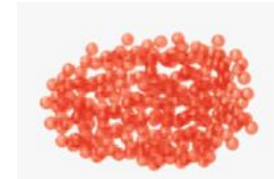
e- beams from LWFA can be:

- transversely small: $0.1 \mu\text{m} < \sigma_r < 1 \mu\text{m}$
- longitudinally short: $0.03 \mu\text{m} < \sigma_z < 3 \mu\text{m}$ ($0.1 \text{ fs} < \sigma_z/c < 10 \text{ fs}$)
- highly divergent: $1 \text{ mrad} < \sigma'_r < 10 \text{ mrad}$
 \Rightarrow transverse normalized emittance: $0.1 \text{ mm mrad} < \varepsilon_n < 1 \text{ mm mrad}$
- **microbunched**: e- grouped into subtle structure within sub- μm range
(Today's diagnostics frontier)
- bunch charge, energy spread, repetition rate, efficiency et al.

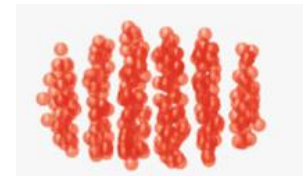
$$\text{Emittance}^1: \varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}.$$

$$\text{Normalized emittance: } \varepsilon_{x,n} = \beta_z \gamma \varepsilon_x \approx \gamma \varepsilon_x.$$

1. \propto area of e- occupied in 6D phase space
2. Conserved in ideal beam transportation



non-mb



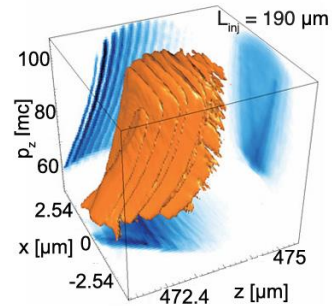
mb

Eur. Phys. J. Spec. Top. 233: 1-208 (2024)

- Microbunched e- structure (**only**) by COTR (3D)
- Transverse divergence by COTRI (2D)
- z-dependent transverse divergence by COTRI and physical constraints (quasi-1D)

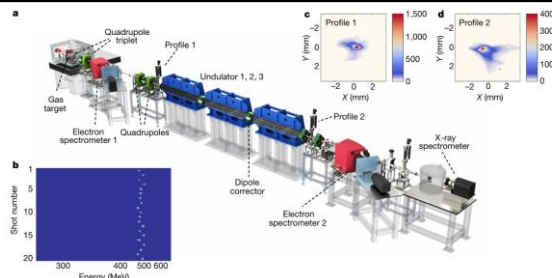
COTR \Rightarrow **quasi-6D structure**

1 Corde et al, *Rev. Mod. Phys.* **85**, 000001 (2013)



“Microbunched e- beam in LWFA”

Xu et al, *Phys. Rev. Lett* **117**, 034801 (2016)



“Microbunched e- beam in LWFA-based Free Electron Laser”

Wang et al. *Nature* **595**, 516-520 (2021)

Outline

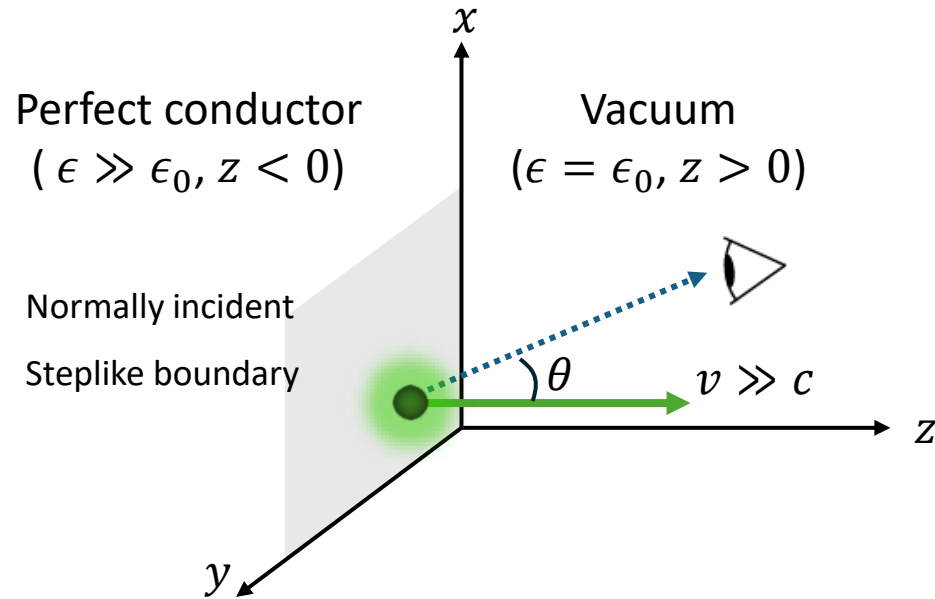
- 1 Introduction to LWFA and its diagnostics
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Useful abbreviations:

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Transition Radiation (single e-)

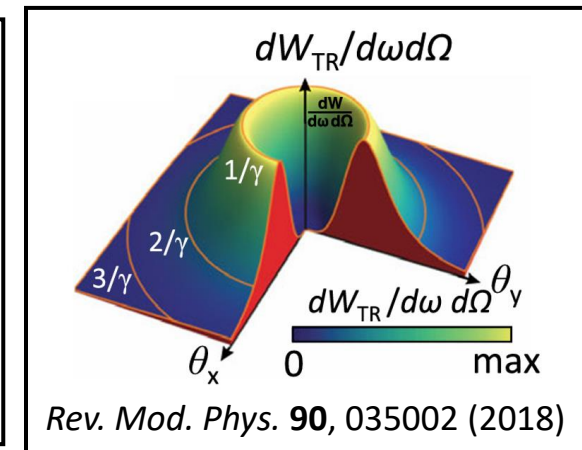
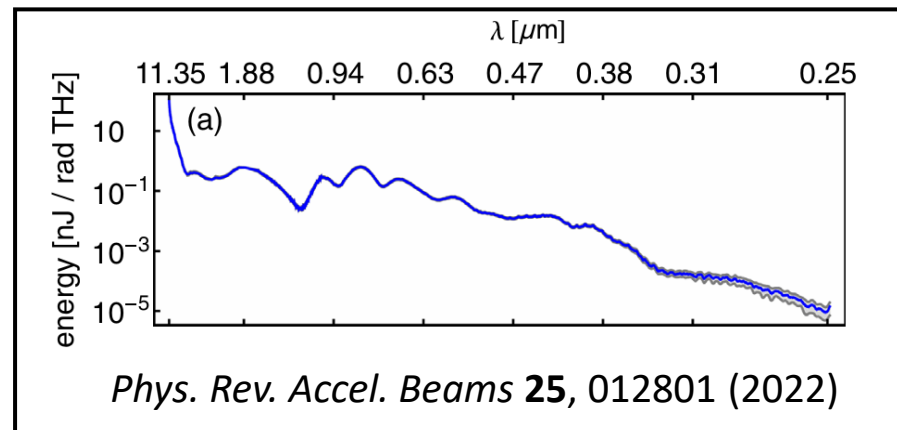
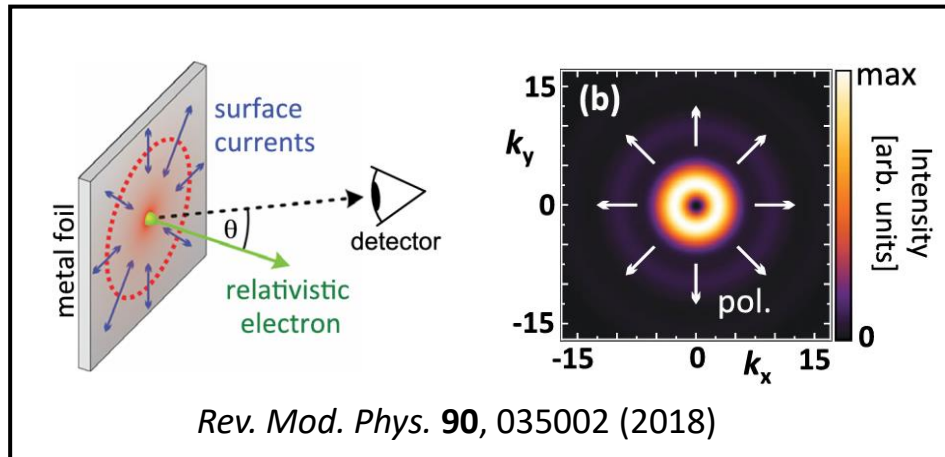
TR is emitted when charged particle passes from one medium into another with different index of refractive.



Single e- TR energy¹ in far field :

$$\frac{d^2 W_1}{d\omega d\Omega} = \frac{e^2}{4\pi^3 \epsilon_0 c} \frac{\beta^2 \sin^2 \theta}{(1 - \beta^2 \cos^2 \theta)^2}$$

1. target radiating & radially polarized
2. broadband (low- and high- ω cutoff: $0.2\mu\text{m}$ - $10\mu\text{m}$)
3. narrow cone (peaked at $\theta \sim \frac{1}{\gamma}$) & weakly γ -dependent ($\gamma \gg 1$)

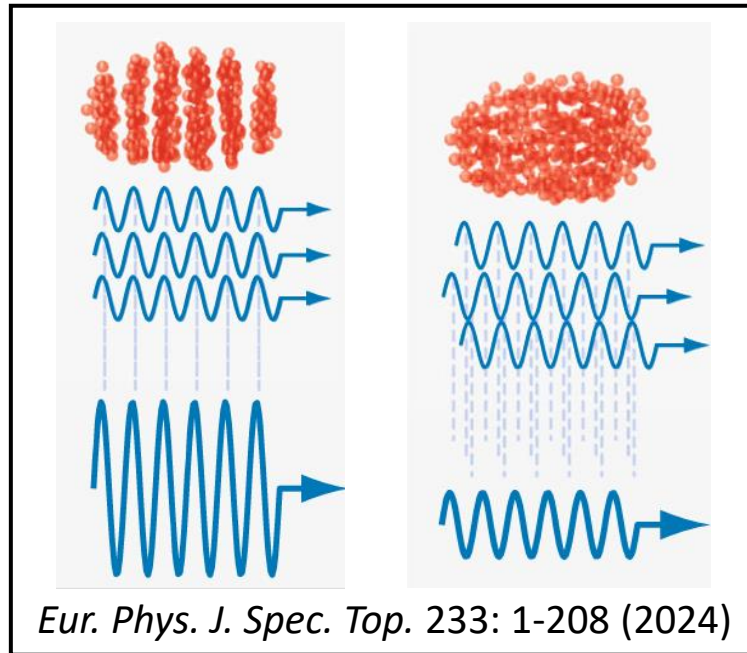


Transition Radiation (e- bunch)

In the case of multiple e-:

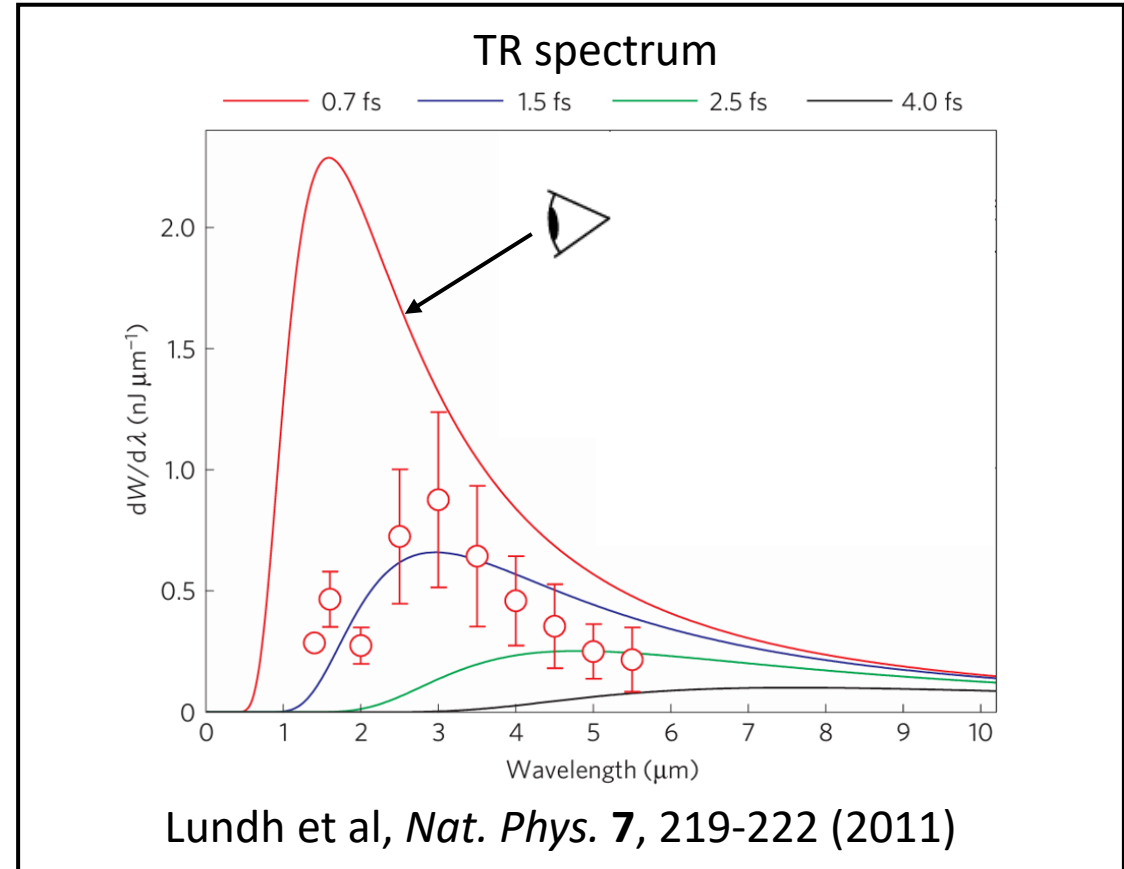
$$\frac{d^2 W_N}{d\omega d\Omega} = \underbrace{[N + N(N-1) \cdot |F(\omega, \theta)|^2]}_{\substack{\text{E field} \\ \swarrow \searrow}} \cdot \frac{d^2 W_1}{d\omega d\Omega}$$

- Out-of-phase emission $\propto N$ (incoherent)
- In-phase emission $\propto N^2$ (coherent)



where $F(\omega, \theta)$ is the **form factor** (level of coherence)

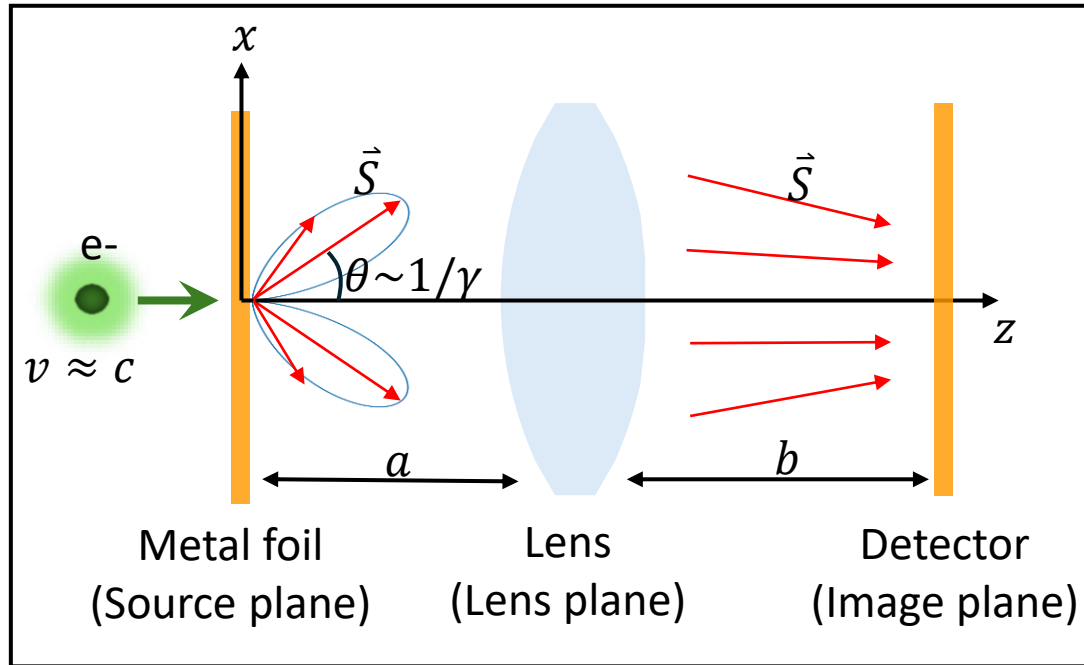
$$F(\omega, \theta) = \int \rho(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r}$$



- $\lambda > \sigma_z$: incoherent
- $\lambda < \sigma_z$: coherent
- microbunched e- beam: λ is coherent down to **optical range** (COTR) \Rightarrow structure info

Transition Radiation Imaging (single e- near field)

COTR is detected in the near field



Source plane^{1,2}:

$$E_{x,y}^s(x_s, y_s, \omega) = \frac{e\omega}{\pi v^2 \gamma} \frac{x_s, y_s}{\sqrt{x_s^2 + y_s^2}} K_1 \left(\frac{\omega}{v\gamma} \sqrt{x_s^2 + y_s^2} \right)$$

Lens plane:

$$E_{x,y}^{li}(x_s, y_s, \omega) = -\frac{ie^{ika}}{\lambda a} e^{ik\frac{x_l^2 + y_l^2}{2a}} \int dx_s dy_s E_{x,y}^s e^{-ik\frac{x_l x_s + y_l y_s}{2a}} e^{ik\frac{x_s^2 + y_s^2}{2a}}$$

$$E_{x,y}^{lo}(x_s, y_s, \omega) = E_{x,y}^{li}(x_s, y_s, \omega) e^{-ik\frac{x_l^2 + y_l^2}{2f}}$$

Image plane:

$$E_{x,y}^i(x_s, y_s, \omega) = -\frac{ie^{ikb}}{\lambda b} e^{ik\frac{x_i^2 + y_i^2}{2b}} \int dx_l dy_l E_{x,y}^{lo} e^{-ik\frac{x_l x_i + y_l y_i}{2b}} e^{ik\frac{x_l^2 + y_l^2}{2b}}$$

$$\Rightarrow \mathbf{E}(x_i, y_i) = \frac{2e}{\lambda v M} f(\theta_m, \gamma, \zeta) \mathbf{e}_r \quad \text{Field Point Spread Function (FPSF)}$$

$$\left. \begin{matrix} \lambda \\ \gamma \\ M \\ \theta_m \end{matrix} \right\} \Rightarrow S(x_i, y_i, \omega)$$

The energy flux per unit frequency interval is

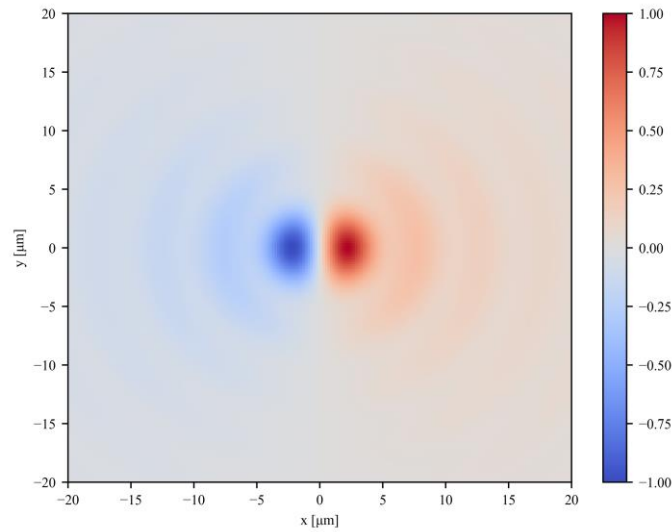
$$S(x_i, y_i, \omega) = \frac{c}{4\pi^2} (|\mathbf{E}(x_i, y_i)|^2) = \frac{d^3 W_1}{d\omega dx_i dy_i} \quad \text{Point Spread Function (PSF)}$$

¹ Castellano et al. *Phys. Rev. Accel. Beams* **1**, 062801 (1998)

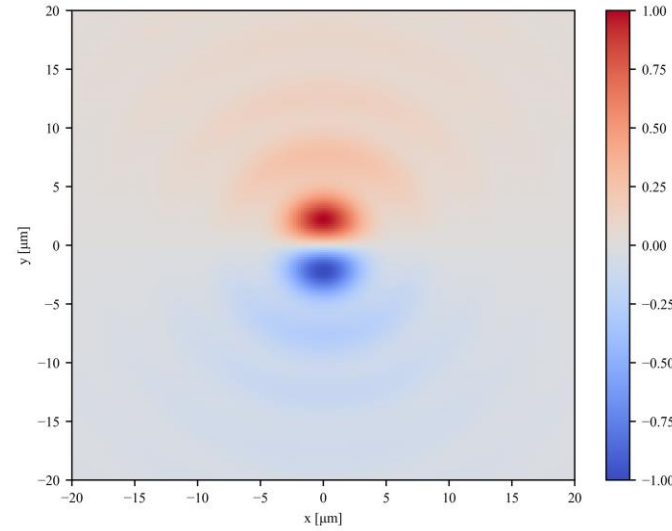
² Xiang et al. *Nucl. Instrum. Meth. A* **570**, 3 (2007)

Transition Radiation Imaging (single e- near field)

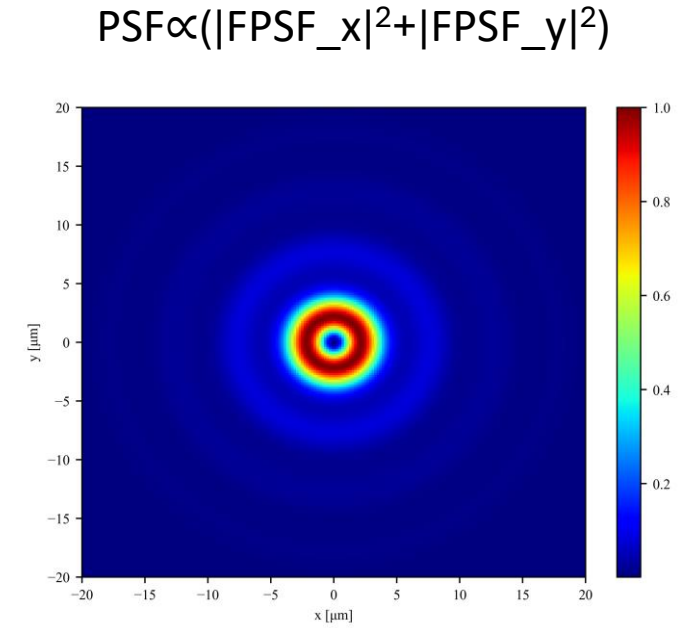
$\lambda=500\text{nm}$, $M=1$, $\gamma=391(200\text{MeV})$, and $\theta_m=0.1$



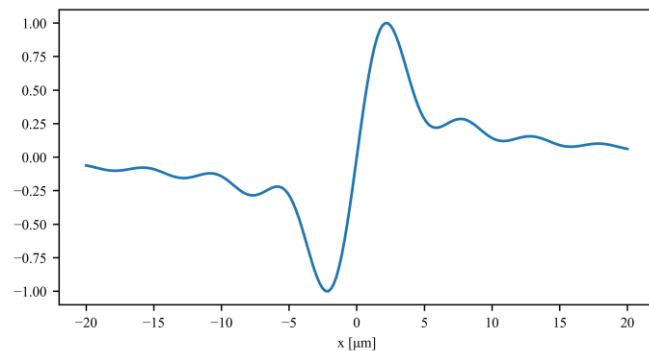
FPSF polarized in x-axis



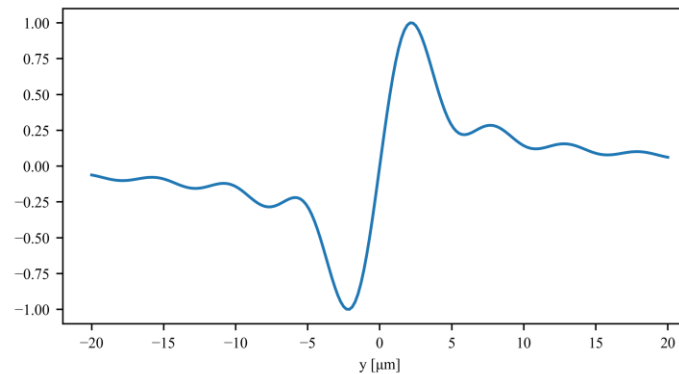
FPSF polarized in y-axis



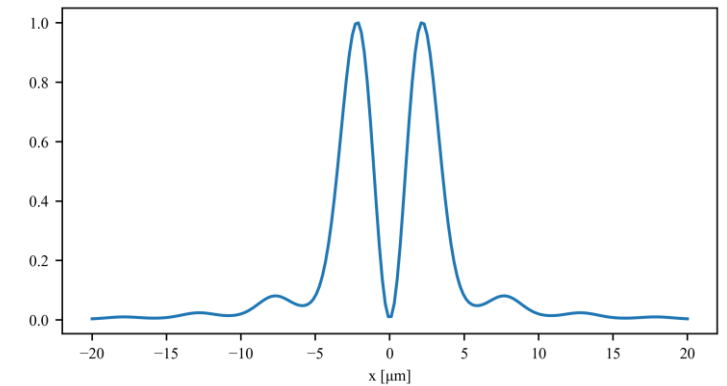
PSF



Lineout of FPSF_x at y=0

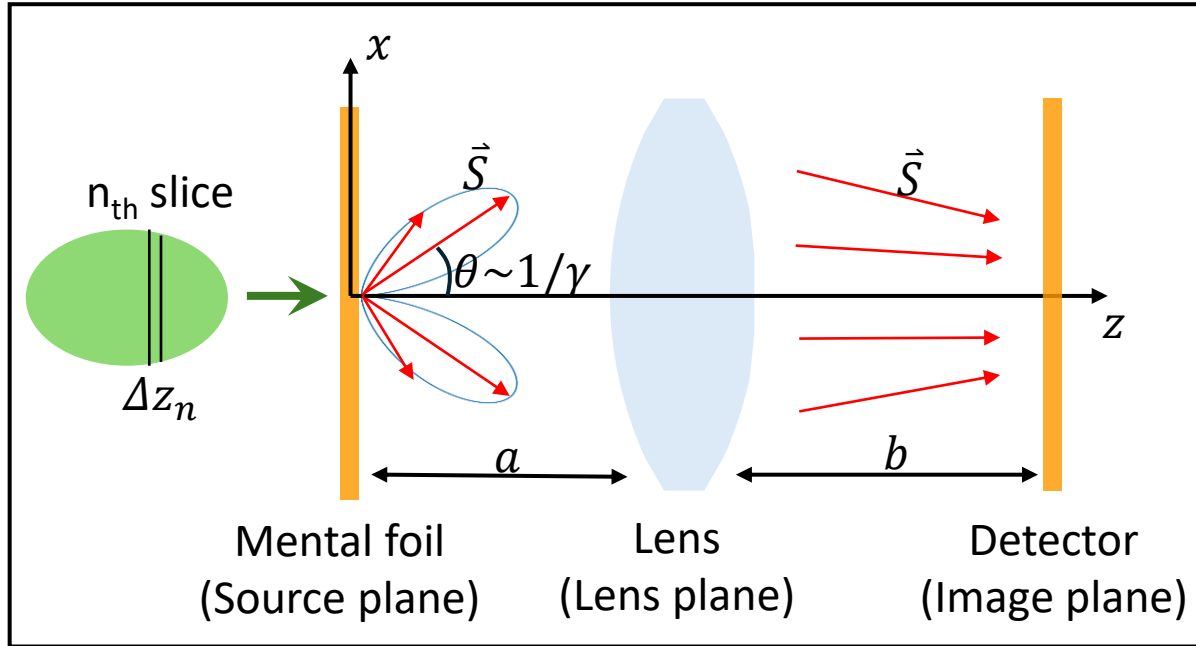


Lineout of FPSF_y at x=0



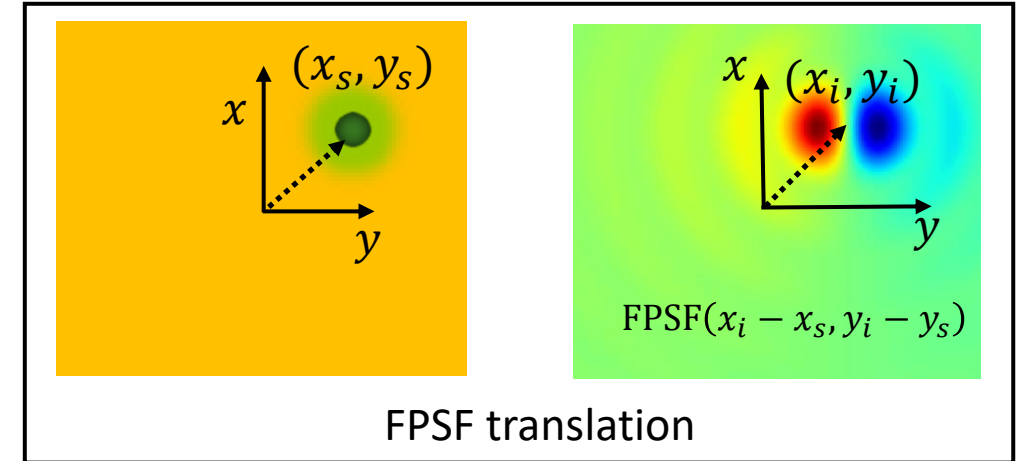
Lineout of PSF at y=0

Transition Radiation Imaging (e- bunch near field)



Electron number density $\rho(x_s, y_s, z_s)$

$$\mathbf{E}^{(n)}(x_i, y_i) = \Delta z_n \iint dx_s dy_s \rho(x_s, y_s, z_n) \text{FPSF}(x_i - x_s, y_i - y_s)$$



Each slice has a phase delay¹ e^{ikz_n}

Total \mathbf{E} field is

$$\mathbf{E}_{\text{tot}}(x_i, y_i) = \iiint dx_s dy_s dz_s \cdot \rho(x_s, y_s, z_s) \cdot e^{ikz} \cdot \text{FPSF}(x_i - x_s, y_i - y_s)$$

Total energy flux per unit frequency interval is

$$S_{\text{tot}}(x_i, y_i, \omega) = \frac{c}{4\pi^2} (|\mathbf{E}_{\text{tot}}(x_i, y_i)|^2) = \frac{d^3 W_1}{d\omega dx_i dy_i}$$

$$\left. \begin{array}{l} \lambda \\ \gamma \\ M \\ \theta_m \\ \rho \end{array} \right\} \Rightarrow S_{\text{tot}}(x_i, y_i, \omega)$$

S contains info of ρ

Revealing the $\rho(x_s, y_s, z_s)$ by COTR: an inverse problem

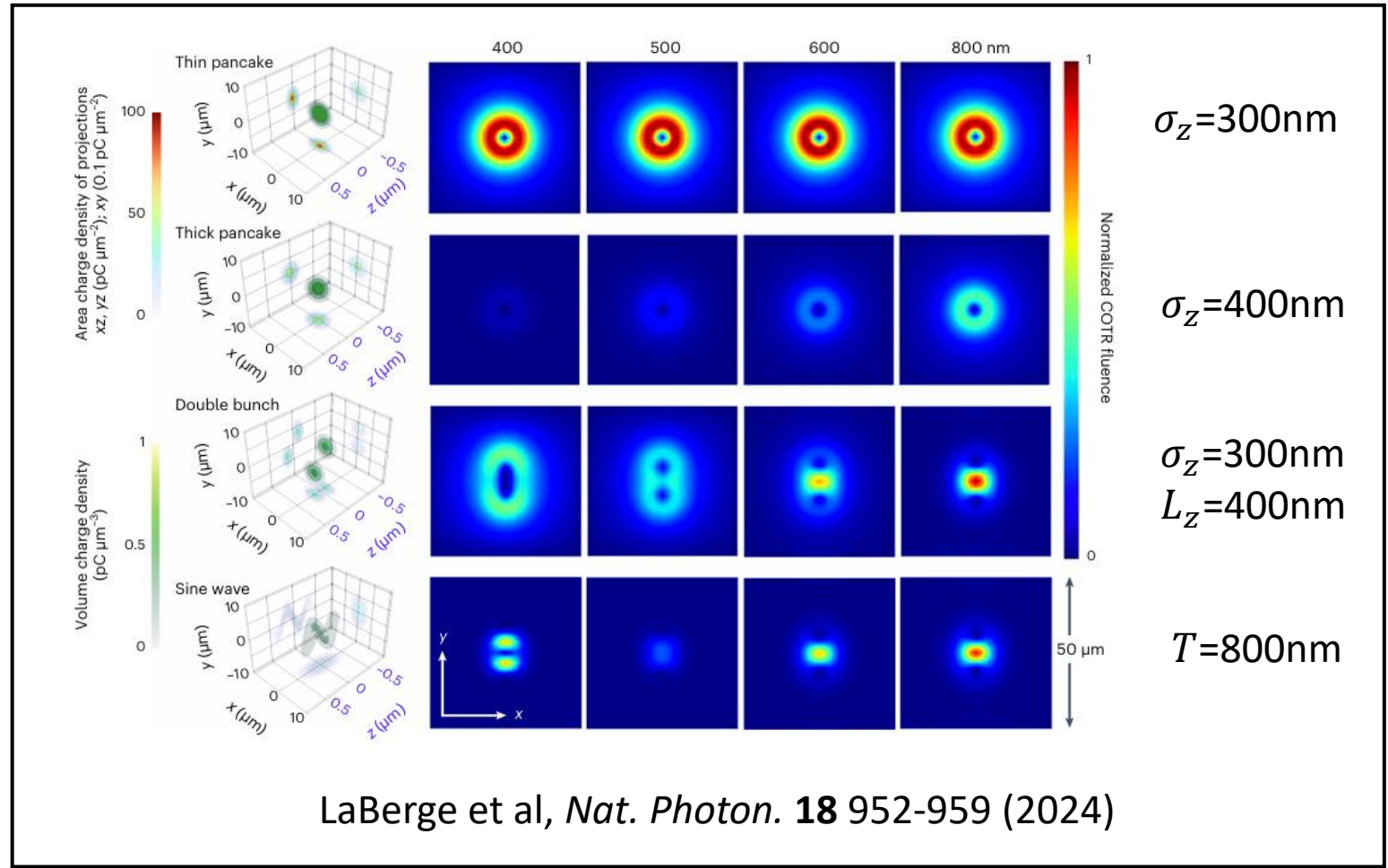
Forward process: $\rho(x_s, y_s, z_s) \Rightarrow S(x_i, y_i)$

$$\left. \begin{array}{c} \lambda \\ \gamma \\ M \\ \theta_m \\ \rho \end{array} \right\} \Rightarrow S_{\text{tot}}(x_i, y_i, \omega)$$

Backward process: $S(x_i, y_i) \Rightarrow \rho(x_s, y_s, z_s)$

$$\left. \begin{array}{c} \lambda \\ \gamma \\ M \\ \theta_m \\ S_{\text{tot}}(x_i, y_i, \omega) \end{array} \right\} \Rightarrow \rho$$

Without loss of generality, consider S as what is measured.

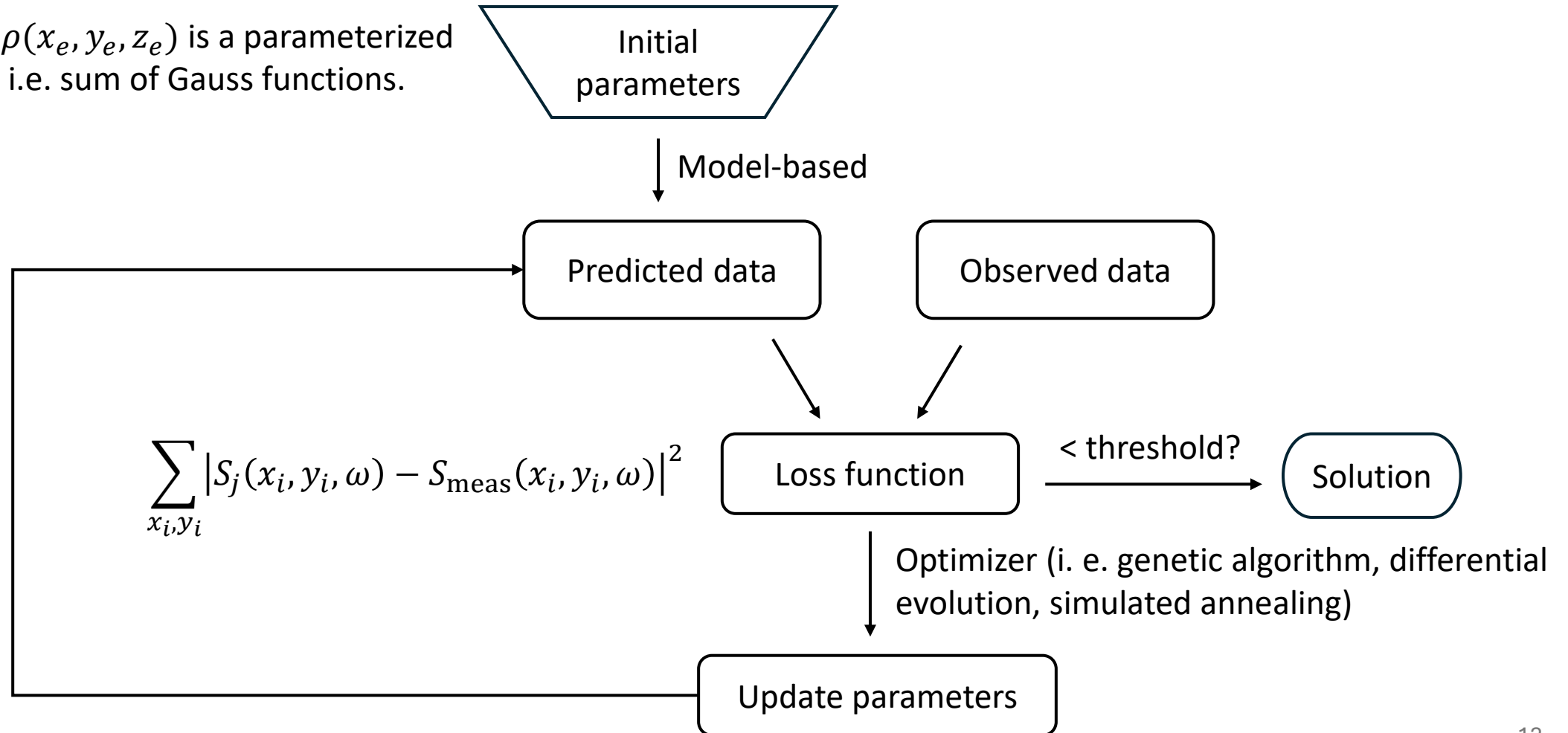


An inverse problem

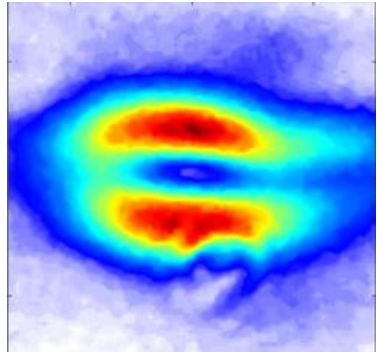
Revealing the $\rho(x_s, y_s, z_s)$ by COTR: workflow

$$\rho(x_s, y_s, z_s) = \sum_{j=1}^N N_{ej} \frac{1}{\sqrt{2\pi}\sigma_{xj}} \exp\left(-\frac{(x-\mu_{xj})^2}{2\sigma_{xj}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{yj}} \exp\left(-\frac{(y-\mu_{yj})^2}{2\sigma_{yj}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{zj}} \exp\left(-\frac{(z-\mu_{zj})^2}{2\sigma_{zj}^2}\right)$$

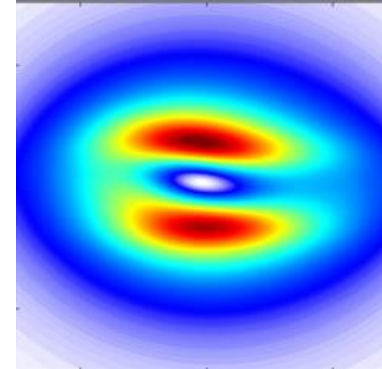
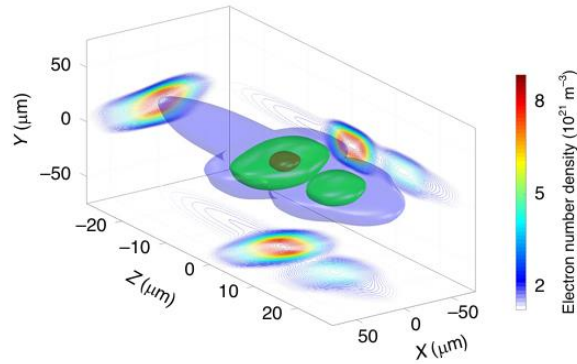
Suppose $\rho(x_e, y_e, z_e)$ is a parameterized function, i.e. sum of Gauss functions.



Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Latest results



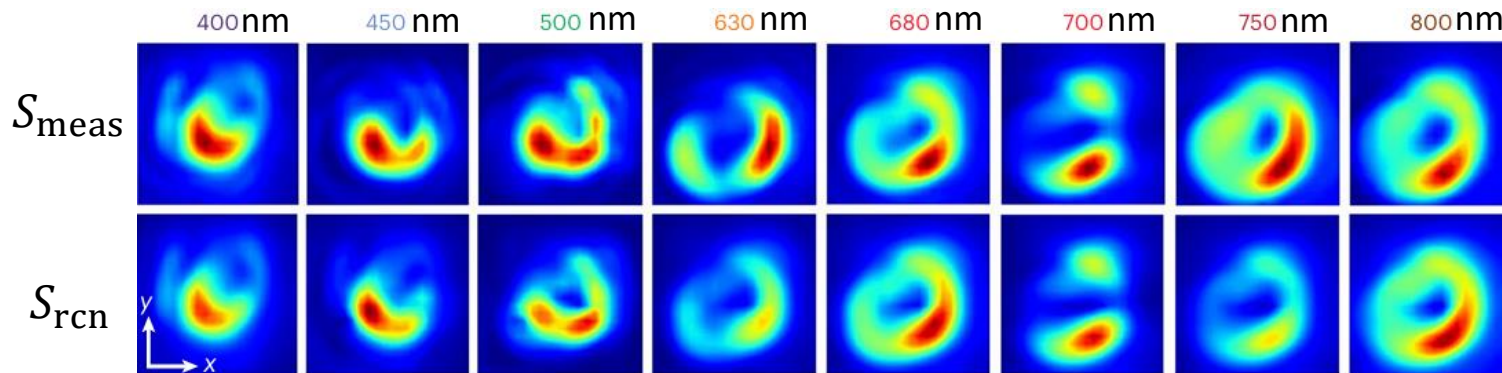
S_{meas} at 550nm



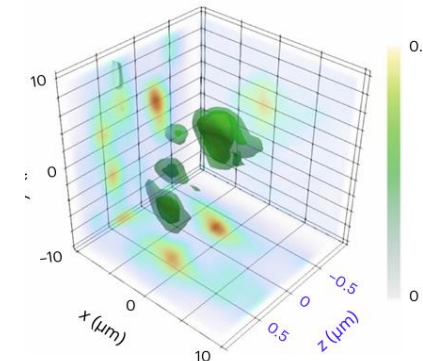
S_{rcn} at 550nm

Huang et al. *Light: sci.appl* **13**, 1 (2024)

Genetic algorithm



LaBerge et al, *Nat. Photon.* **18** 952-959 (2024)

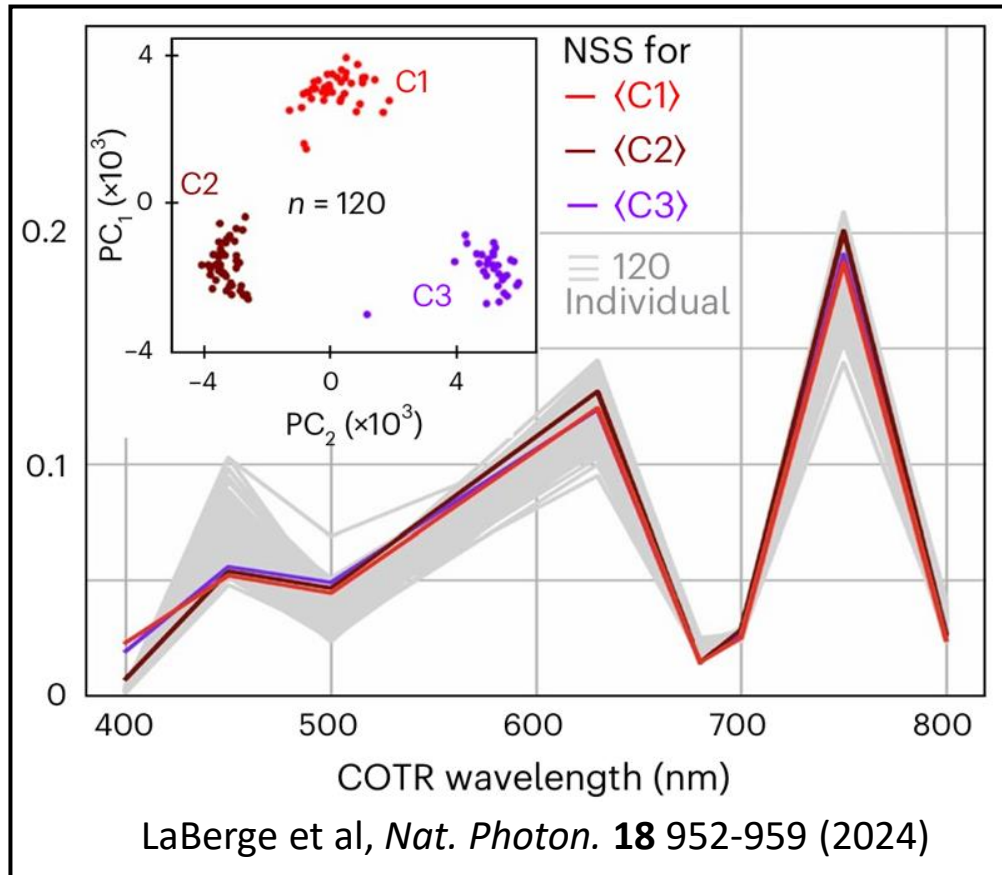


Differential evolution

Revealing the $\rho(x_s, y_s, z_s)$ by COTR: uniqueness

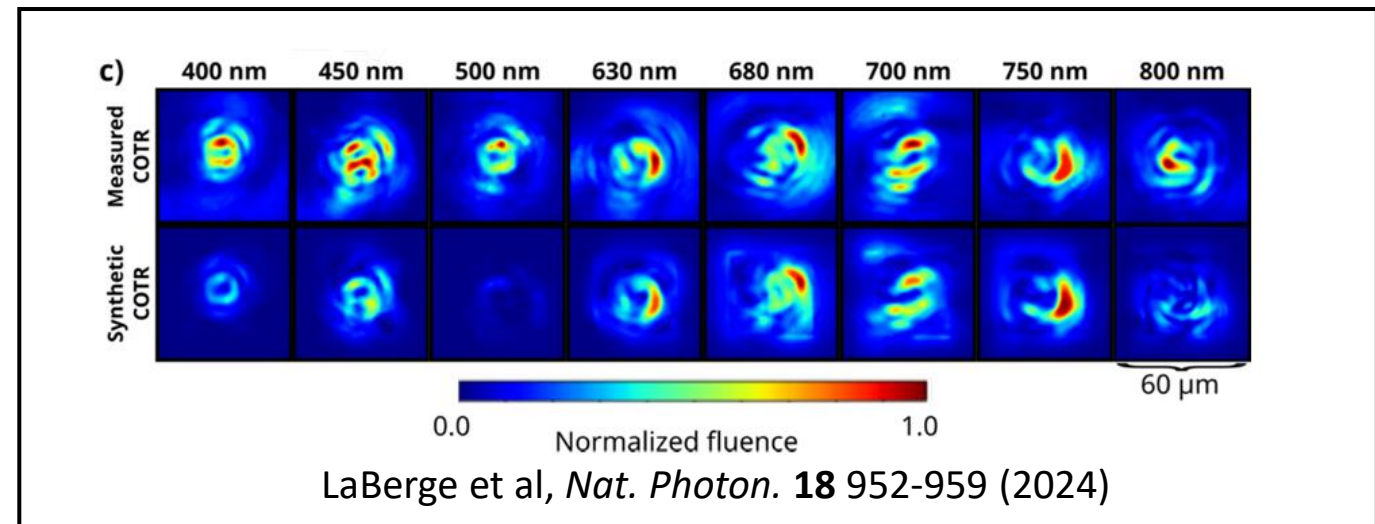
Phase info lost in the forward process \Rightarrow reconstruction is not unique

How to compress the volume of solution space \Rightarrow **Knowing longitudinal profile in advance!**



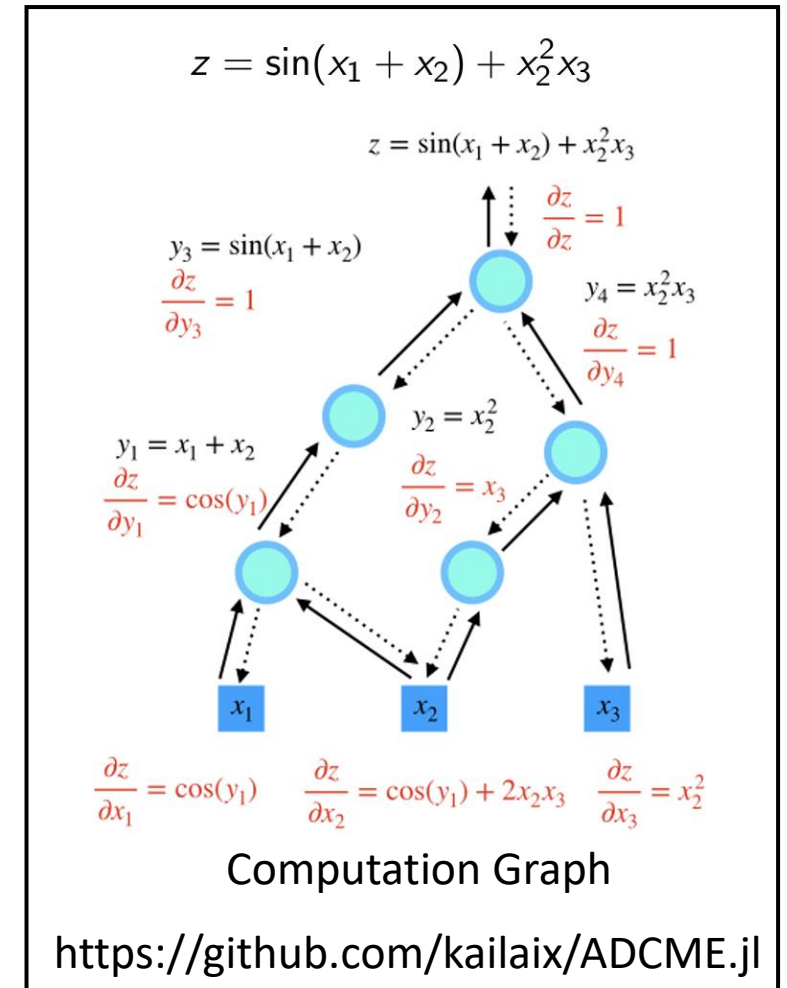
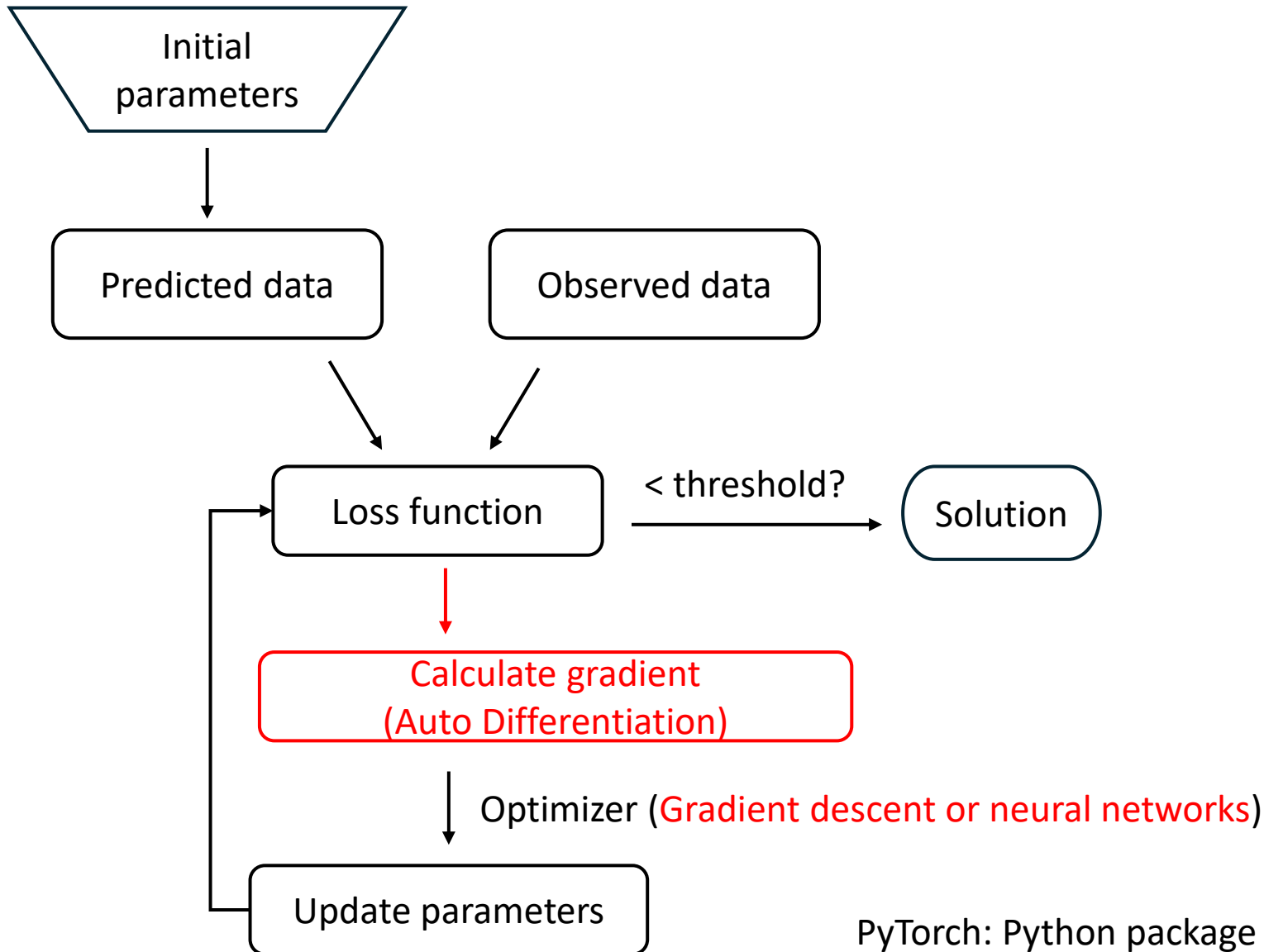
Knowledge of e- beam longitudinal is **injection-regime-dependent**:

- Down ramp injection: e- spectrum
- Self-truncated ionization injection: PIC simulation
- Self injection: not accessible



Other architectures to solve such an inverse question?

Revealing the $\rho(x_s, y_s, z_s)$ by COTR: ML-workflow

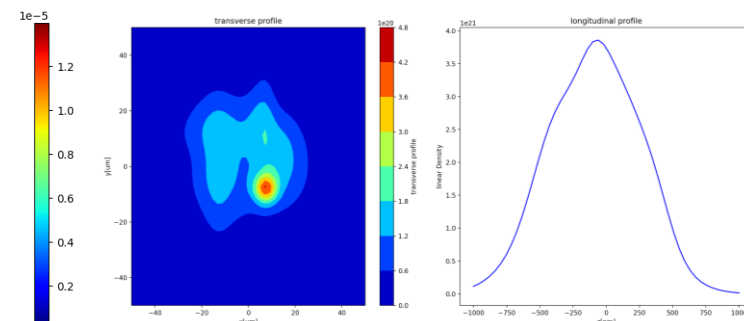
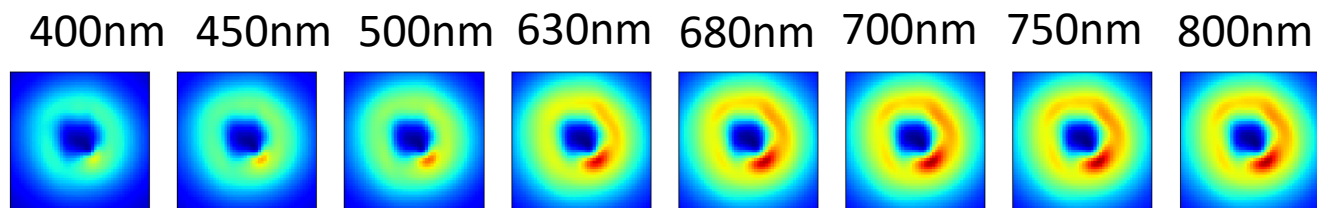


PyTorch: Python package for

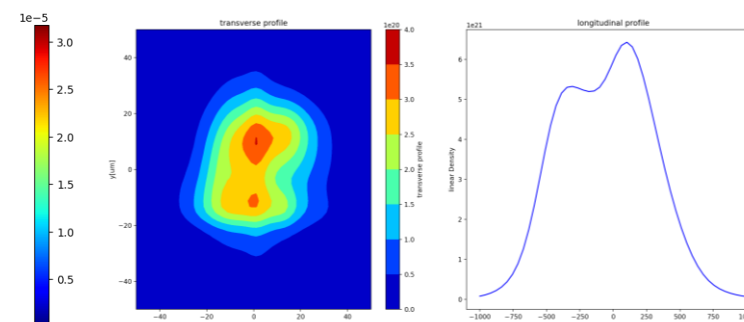
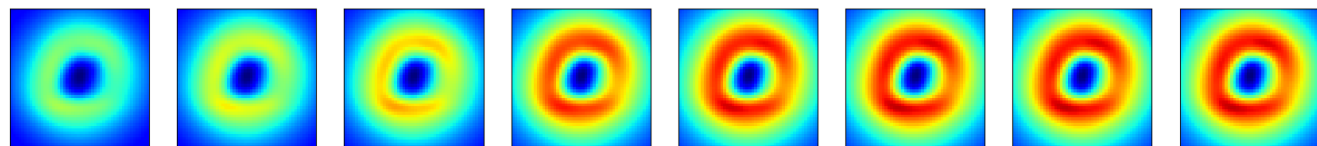
- GPU computation
- Auto Differentiation (AD)
- Machine Learning (ML)

Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Gradient descent

" S_{meas} "
26 Gauss

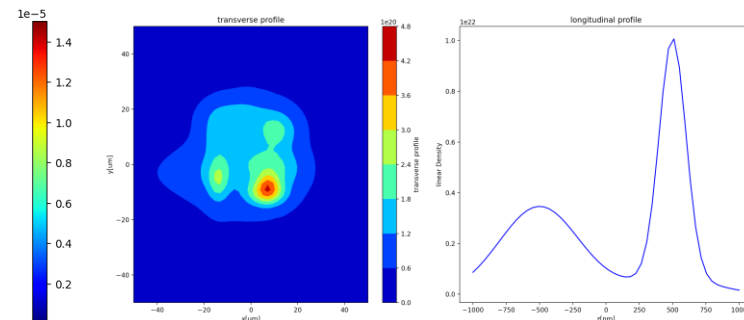
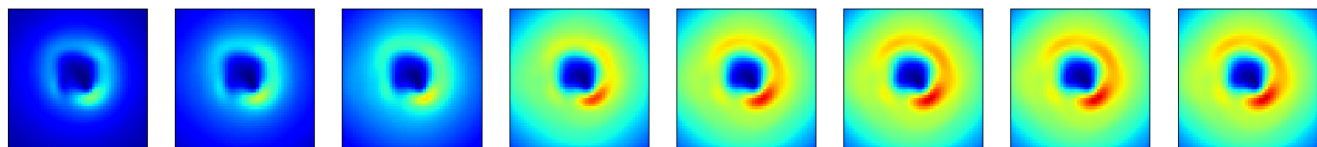


S_{seed}
50 Gauss

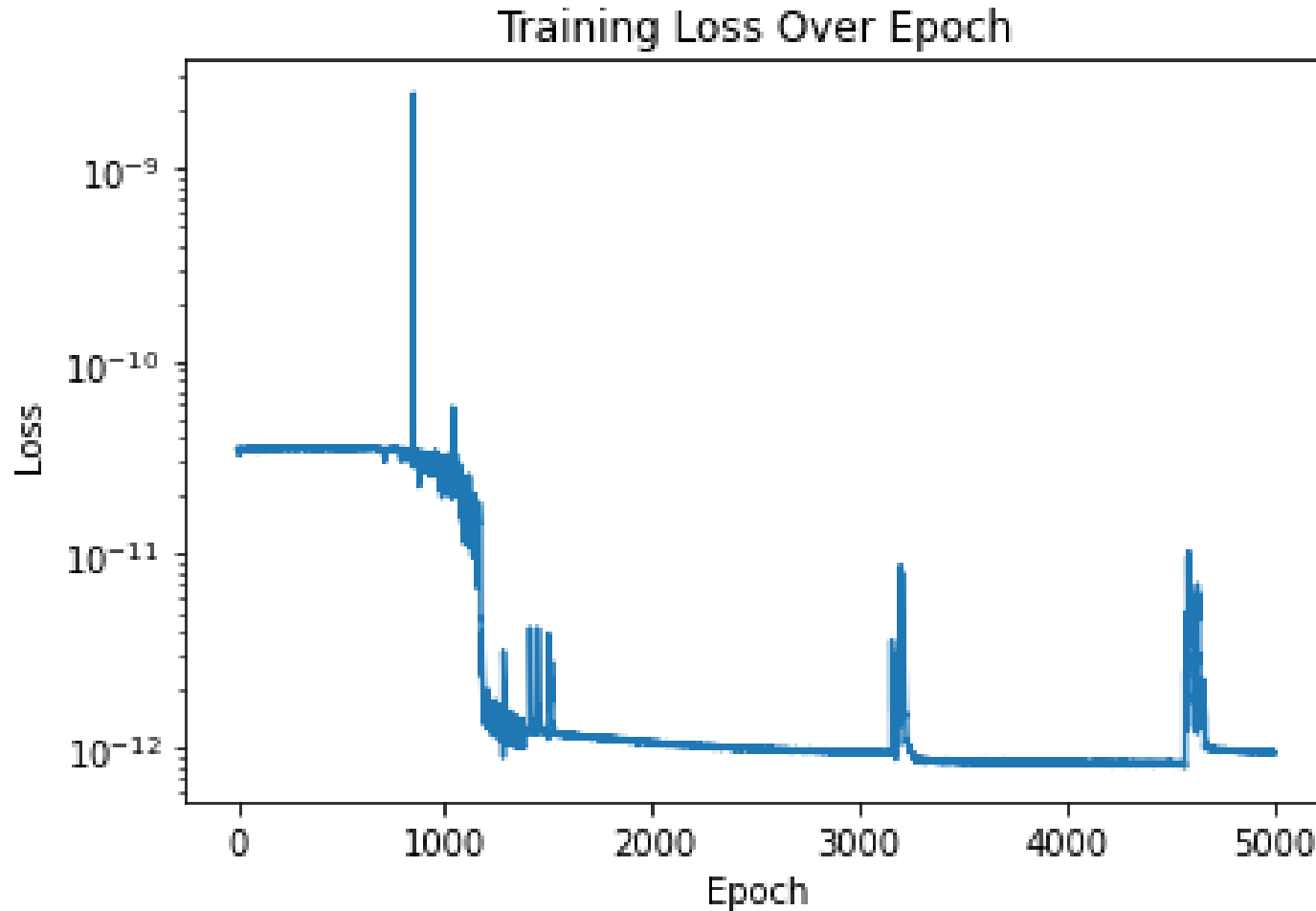


Loss function: $\sum_{x_i, y_i} |S_j(x_i, y_i, \omega) - S_{\text{meas}}(x_i, y_i, \omega)|^2$ iterate

S_{rcn}
50 Gauss

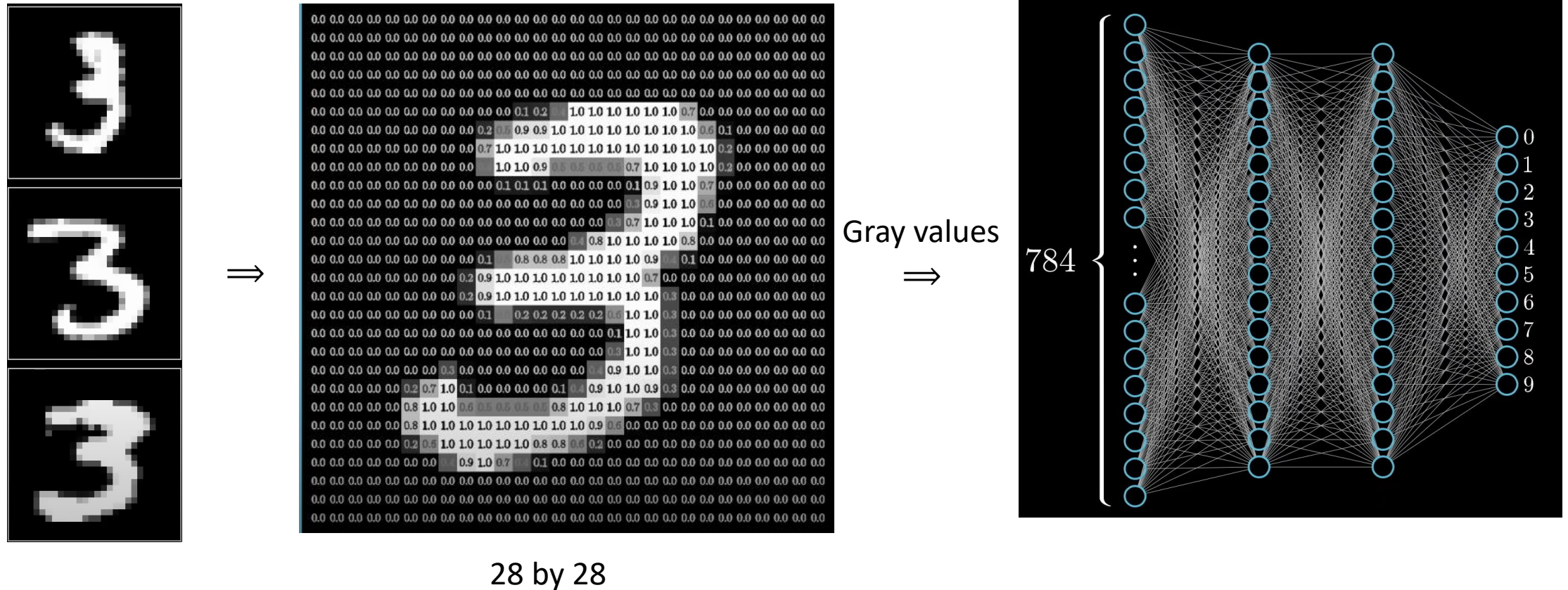


Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Training loss



- ~2 hours
- Final loss reduced to 1/50 of the initial loss

Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Neural network “vision”

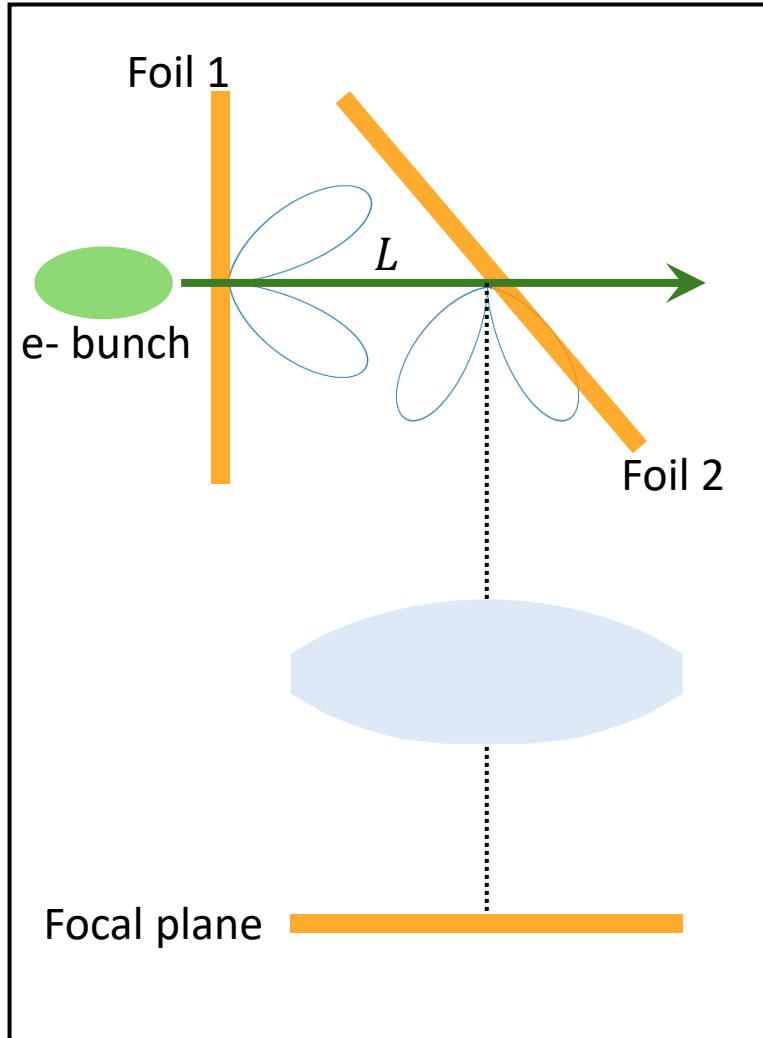


Training repository: paired ρ and S for NN(neural network) to learn

Test repository: paired ρ and S . Given the S , to see if the NN could deduce ρ close to the right one

COTRI Imaging

COTRI is detected in the far field

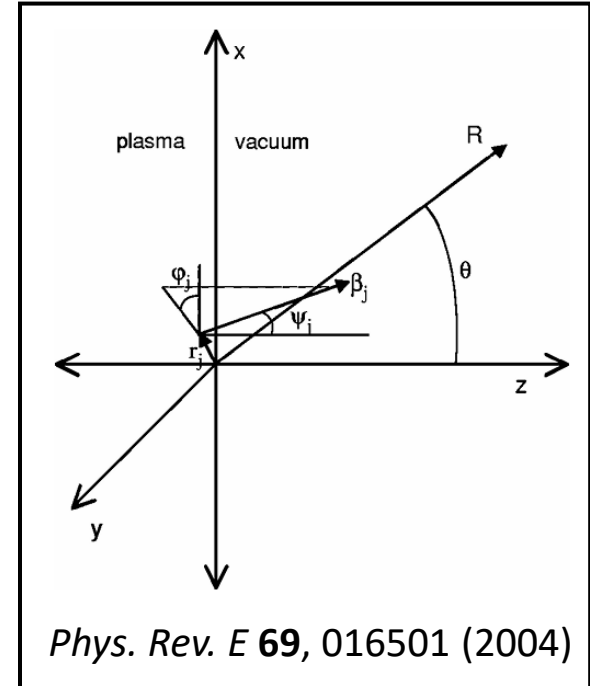
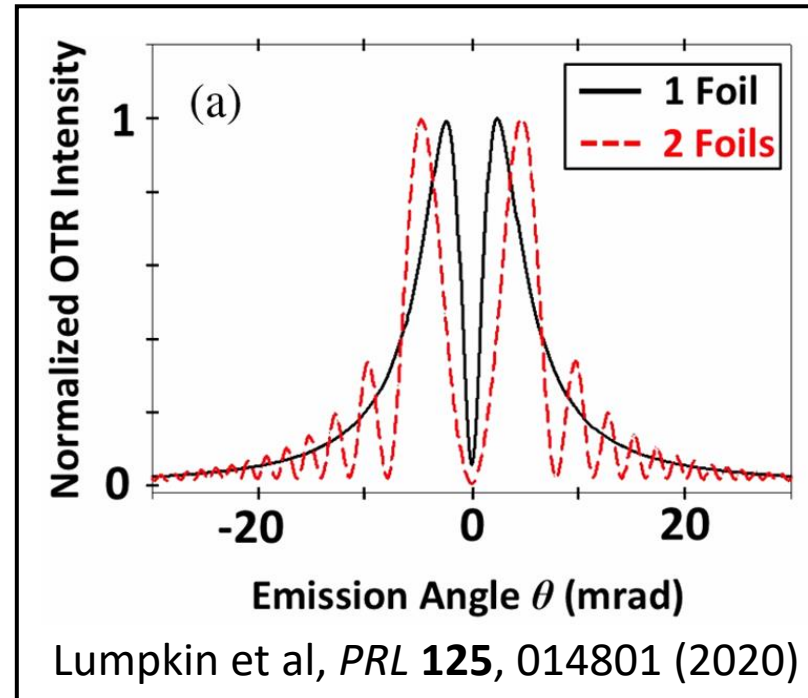


Divergence \Leftrightarrow Angle of incidence $\psi \Rightarrow$ Far-field Interferometry

$$\text{Field point spread function}^1: E = \frac{e}{\pi\sqrt{c}} \frac{\psi - \theta}{\gamma^{-2} + |\psi - \theta|^2}$$

$$\text{Total E field: } E_{\text{tot}} = E * \underline{h(\mathbf{r}, \mathbf{p})} e^{ikr}$$

6D phase space distribution



Fringes contain info of ...

Revealing divergence by COTRI

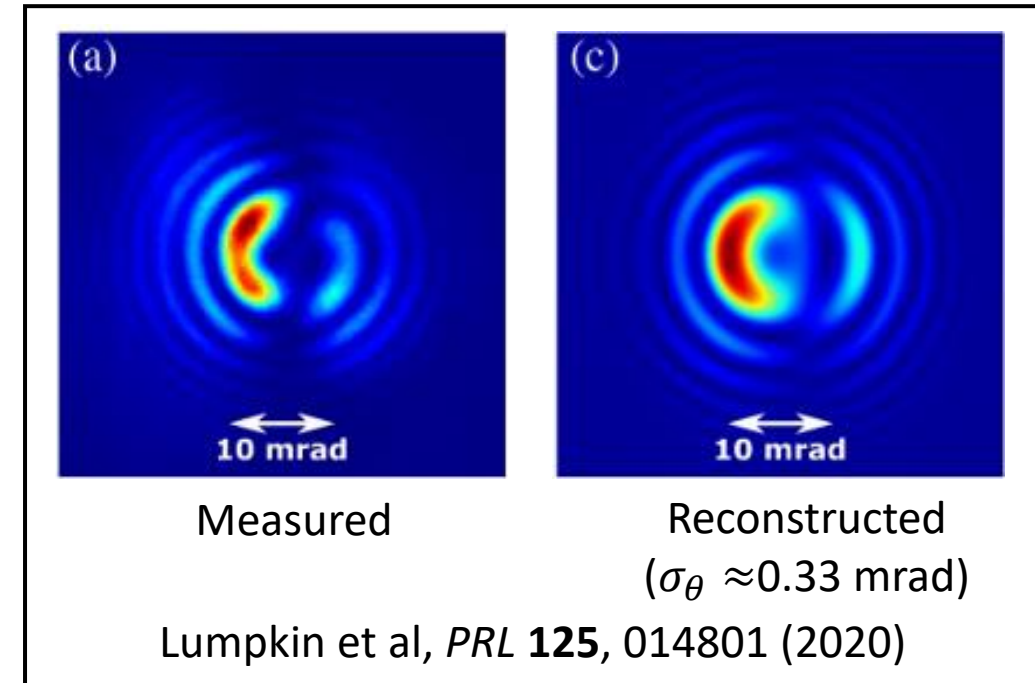
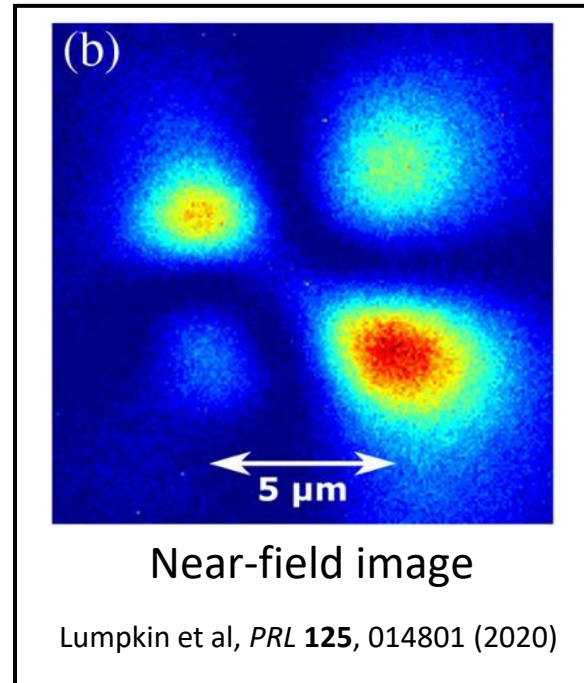
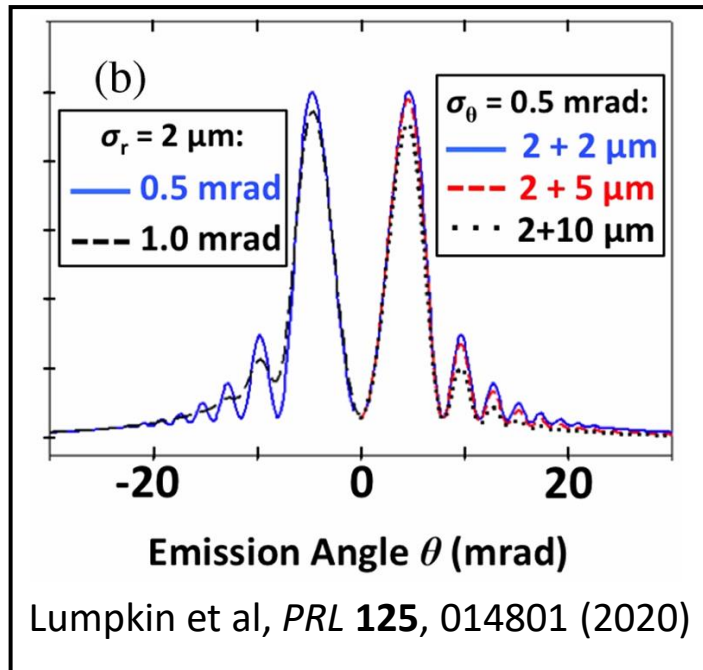
Fringes are sensitive to:

- Optical detection bandwidth $\Delta\lambda$
- Energy bandwidth $\Delta\gamma$
- Transverse size σ_r
- Divergence σ_θ

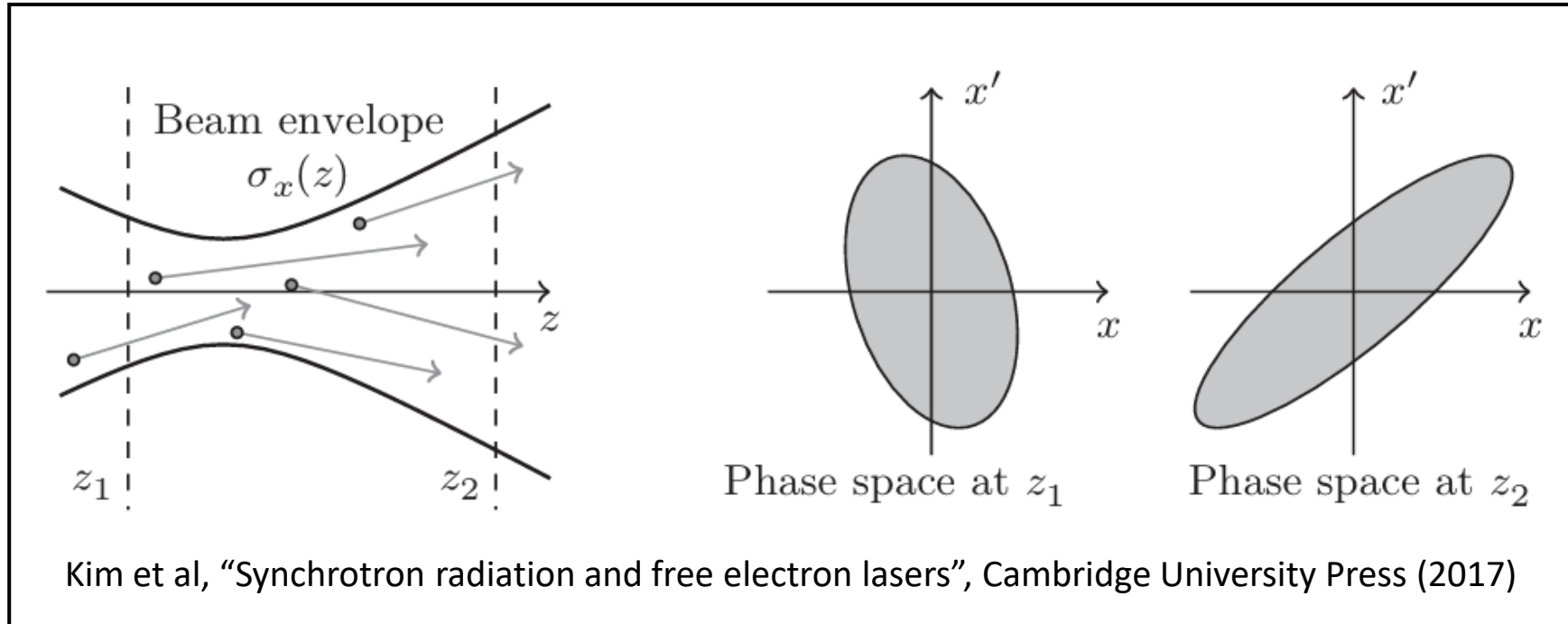
By choosing $\Delta\lambda$, $\Delta\gamma$, and L

σ_r and σ_θ can be dominant

Transverse divergence could be revealed!



Quasi-6D structures explored by COTR(I)



So far, we have obtained the 5D structures:

- 3D density profile (by COTR)
- 2D transverse divergence (by COTR(I))

With reasonable physical assumptions,
some phase spaces can be ruled out¹

eg: microbunched portion have
lower divergence

Obtain an **upper limit** on transverse
emittance on each slice (quasi-1D)

Outline

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Introduction to LWFA and its diagnostics

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COTR(I) and quasi-6D structure of e- bunches

3

Future directions, experimental work & conclusion

- Measurement of form factor
- Extension to Smith-Purcell Radiation
- Monitoring the microbunched e- in Free Electron Lasers
- Combination with Diffraction Radiation

Measurement of form factor

$$\frac{d^2 W_N}{d\omega d\Omega} = [N + N(N-1) \cdot |F(\omega, \theta)|^2] \cdot \frac{d^2 W_1}{d\omega d\Omega}$$

$$F(\omega, \theta) = \int \rho(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r} \text{ (Form factor)}$$

With longitudinal and transverse profile separatable:

$$F(\omega, \theta) = F_{\perp}(\omega, \theta) F_z(\omega, \theta) = \int \rho_{\perp}(\mathbf{r}_{\perp}) e^{i\mathbf{k}_{\perp}\mathbf{r}_{\perp}} d\mathbf{r}_{\perp} \int \rho_z(z) e^{ik_z z} dz$$

Suppose the e- bunch takes a bi-Gaussian shape:

$$\rho(\mathbf{r}) = \rho_{\perp}(\mathbf{r}_{\perp}) \rho_z(z) = \frac{1}{\sqrt{2\pi}^3 \sigma_{\perp}^2 \sigma_z} e^{-\frac{r_{\perp}^2}{2\sigma_{\perp}^2}} e^{-\frac{z^2}{2\sigma_z^2}}$$

$$\text{We have } |F_{\perp}(\omega, \theta)| = e^{-2\pi^2 \frac{\sigma_{\perp}^2}{\lambda^2} \sin^2 \theta} \quad (\text{close to unity if } \sigma_{\perp} \ll \gamma\lambda)^1$$

$$|F_z(\omega, \theta)| = e^{-2\pi^2 \frac{\sigma_z^2}{\lambda^2} \cos^2 \theta}$$

$$|F(\omega, \theta)| \approx |F_z(\omega, \theta)|$$

With inverse Fourier transform:

$$\rho_z(z) = \frac{1}{2\pi} \int F(\omega, \theta) e^{\frac{i\omega z}{c}} d\omega$$

- **With the knowledge of form factor, we can reconstruct the longitudinal profile of the e- beam.**
- The only general method to go down to sub-fs resolution

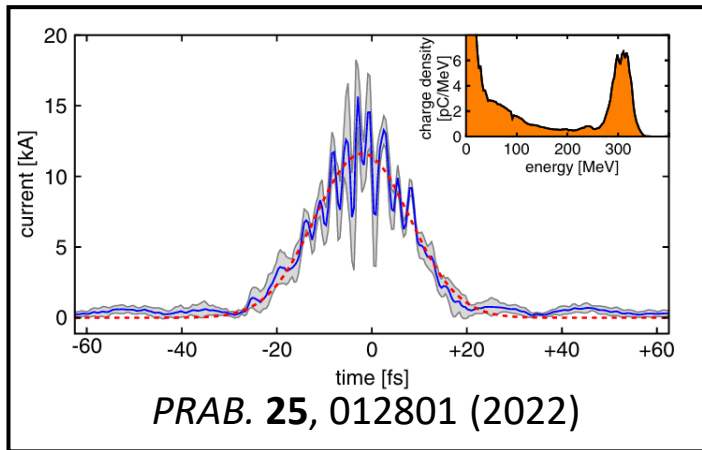
Measurement of form factor: complex value

$F(\omega, \theta)$ is a complex value: $\rho_z(z) = \frac{1}{2\pi} \int F(\omega, \theta) e^{\frac{i\omega z}{c}} d\omega$

Measurement of the absolute value^{1,2}

$$|F(\omega, \theta)| = \frac{\frac{dW_N}{d\omega} \cdot \frac{dW_1}{d\omega} - N \frac{dW_1}{d\omega}}{N(N-1)}$$

1. Interpolation & extrapolation
2. Phase retrieval algorithm
3. Physical constraints



Measurement of the phase:

The phase is closely related to the phase of E field

$$\mathbf{E}_{\text{tot}}(\omega, \theta) = \int \text{FPSF}(\omega, \theta) \rho_z(z) e^{\frac{i\omega z}{c} \cos \theta} dz$$

$|\mathbf{E}_{\text{tot}}(\omega)|$ is captured by the camera, phase $\varphi(\omega)$?



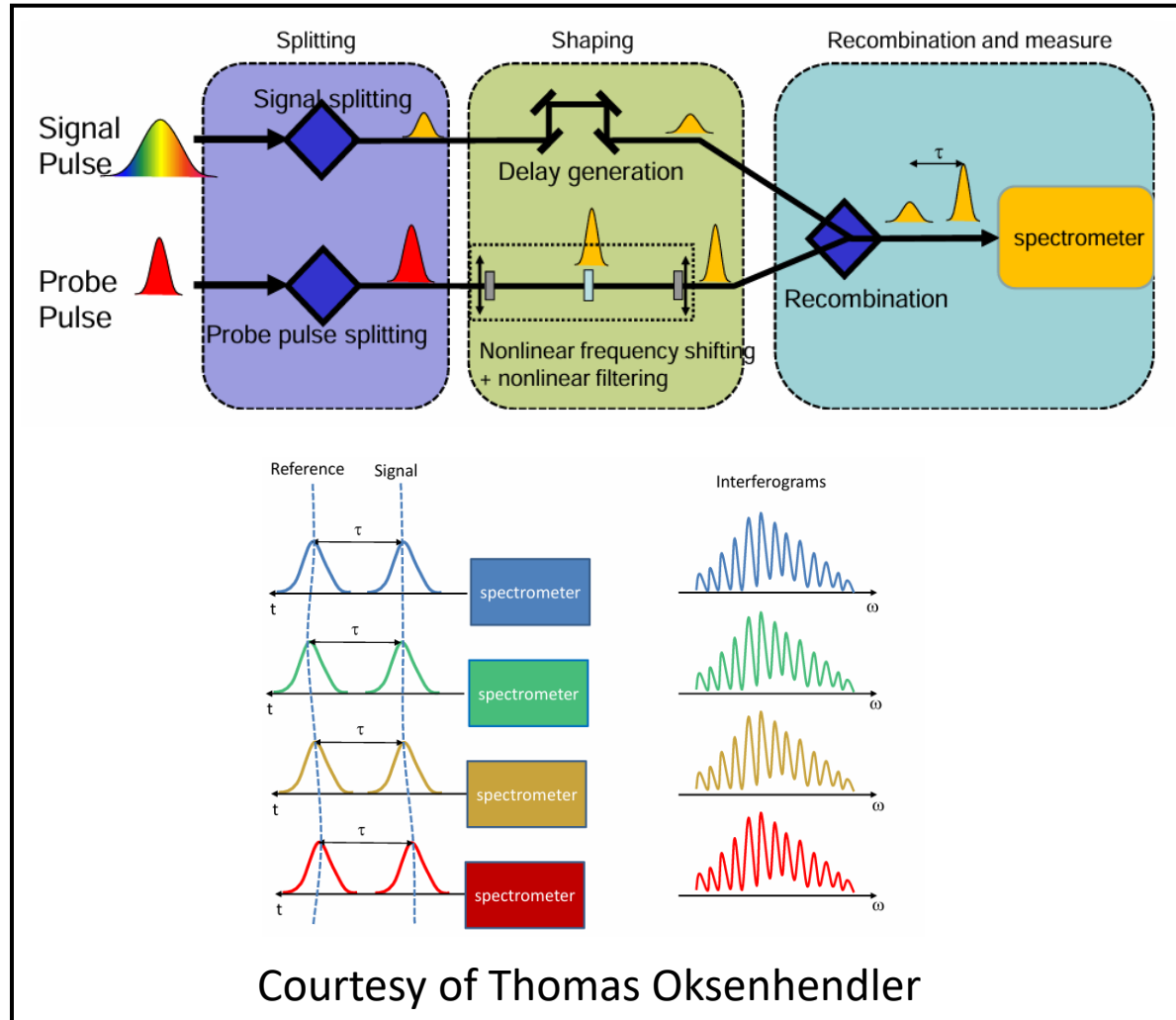
To build **phase-sensitive** detectors!

1 Lai et al, *Phys. Rev. E* **50**, 5 (1994)

2 Lai et al, *Phys. Rev. E* **50**, 6 (1994)

Measurement of form factor: spectral interferometry

Self-referenced spectral interferometry¹



\tilde{E}_{ref} is well characterized in amplitude and phase²

How to detect \tilde{E}_{sig} ? From Interferometry

$$\tilde{S}(\omega) = |\tilde{E}_{\text{ref}} + \tilde{E}_{\text{sig}}|^2 = \tilde{S}_0(\omega) + \tilde{f}(\omega)e^{i\omega\tau} + \tilde{f}^*(\omega)e^{-i\omega\tau}$$

$$\tilde{S}_0(\omega) = |\tilde{E}_{\text{ref}}|^2 + |\tilde{E}_{\text{sig}}|^2 \text{ (DC term)}$$

$$\tilde{f}(\omega) = \tilde{E}_{\text{ref}}\tilde{E}_{\text{sig}}^* \text{ (AC term)}$$

$$|\tilde{E}_{\text{ref}}(\omega)| = \frac{1}{2} \left(\sqrt{\tilde{S}_0(\omega) + 2|\tilde{f}(\omega)|} + \sqrt{\tilde{S}_0(\omega) - 2|\tilde{f}(\omega)|} \right)$$

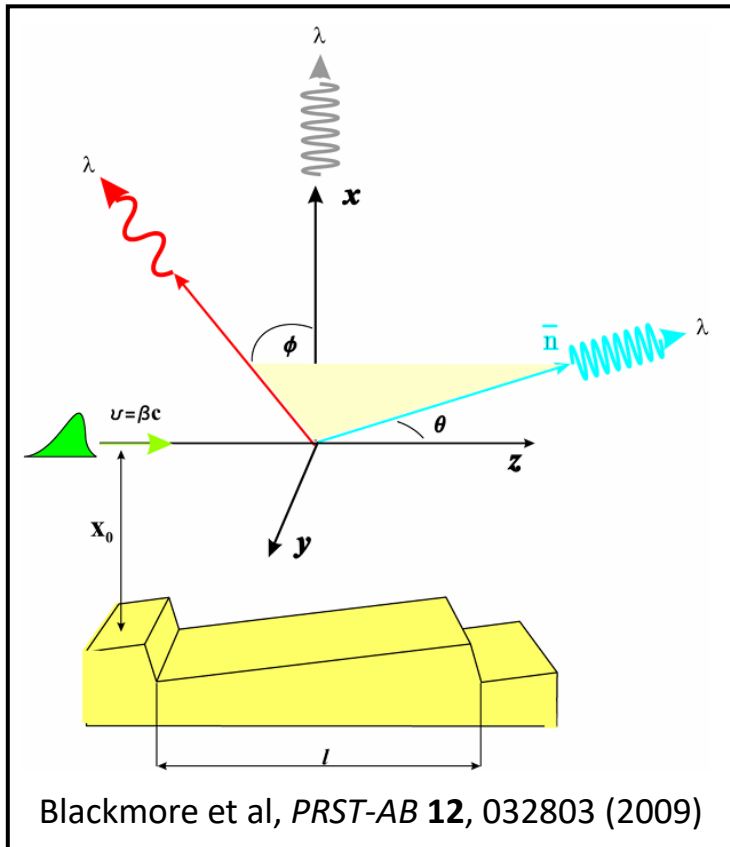
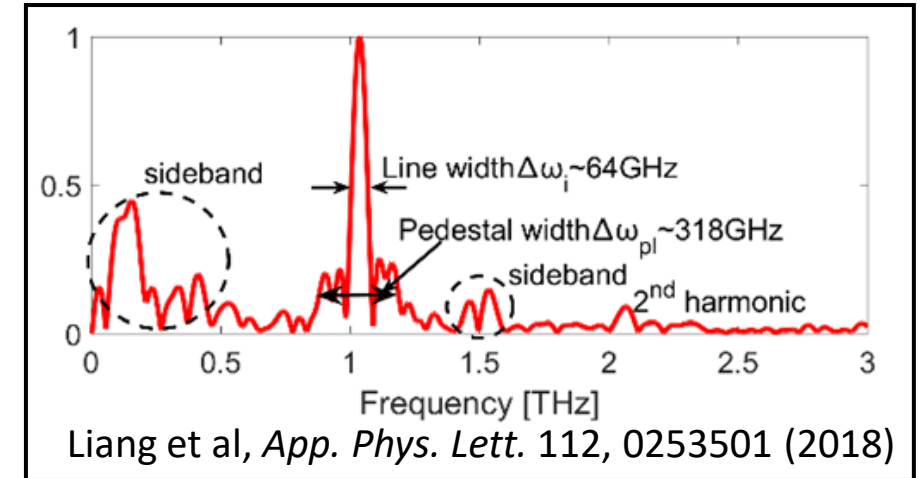
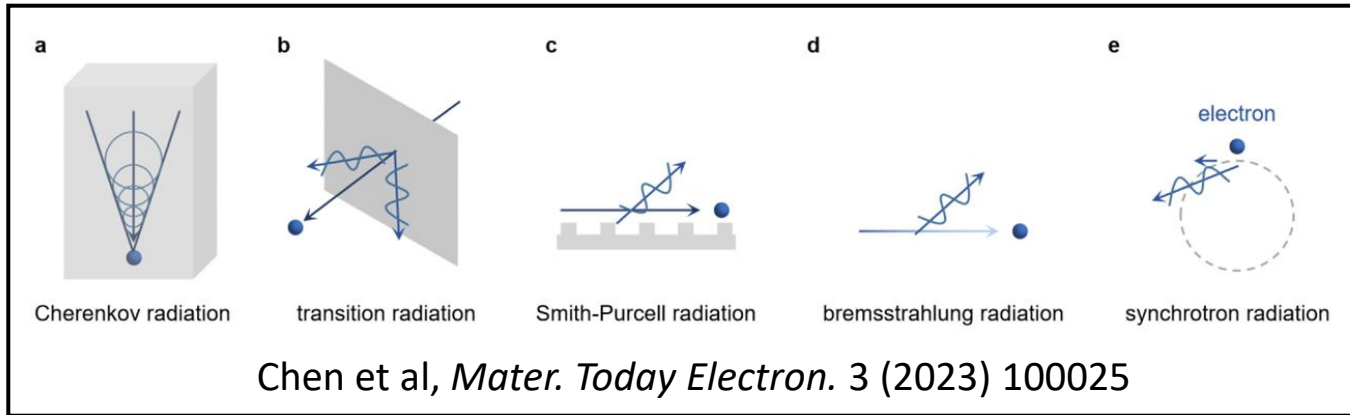
$$|\tilde{E}_{\text{sig}}(\omega)| = \frac{1}{2} \left(\sqrt{\tilde{S}_0(\omega) + 2|\tilde{f}(\omega)|} - \sqrt{\tilde{S}_0(\omega) - 2|\tilde{f}(\omega)|} \right)$$

$$\varphi_{\text{sig}}(\omega) = \varphi_{\text{ref}}(\omega) - \arg(\tilde{f}(\omega))$$

¹ Oksenhendler et al, *Appl. Phys. B* **99**, 7-12 (2001)

² Pariente et al, *Nat. Photon.* **10**, 547-553 (2016)

Extension to Smith-Purcell radiation



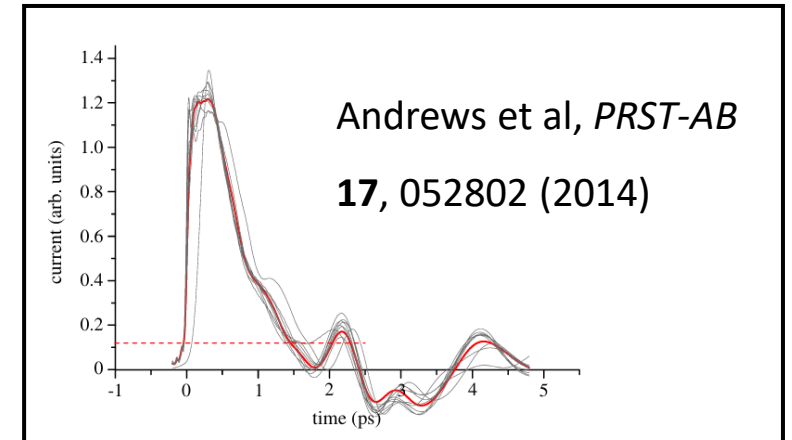
SPR angle-wavelength condition

$$\lambda = \frac{l}{n} \left(\frac{1}{\beta} - \cos\theta \right)$$

$$\frac{dW_1}{d\Omega} = 2\pi e^2 \frac{Z}{l} \frac{n^2 \beta^3}{(1 - \beta \cos\theta)^3} e^{-\frac{2x_0}{\lambda_e} R^2}$$

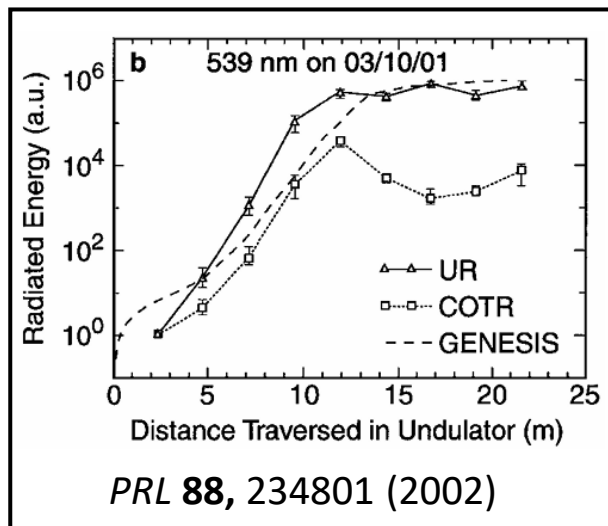
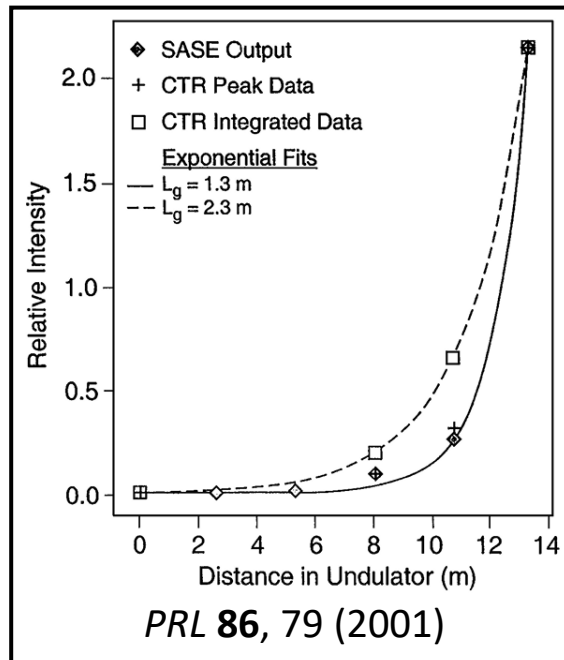
Coherent emission $\frac{dW_N}{d\Omega} \cong \frac{dW_1}{d\Omega} N^2 S_{\text{coh}}$

where $S_{\text{coh}} = \left| \int T e^{-i\omega t} dt \right|^2$



1. Another source of THz radiation
 2. Possesses microbunching info
 3. Help to reveal the temporal profile (cross-calibration with COTR)
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Monitoring the microbunching in Free Electron Lasers



Seed laser or noise radiation interacting with electrons¹

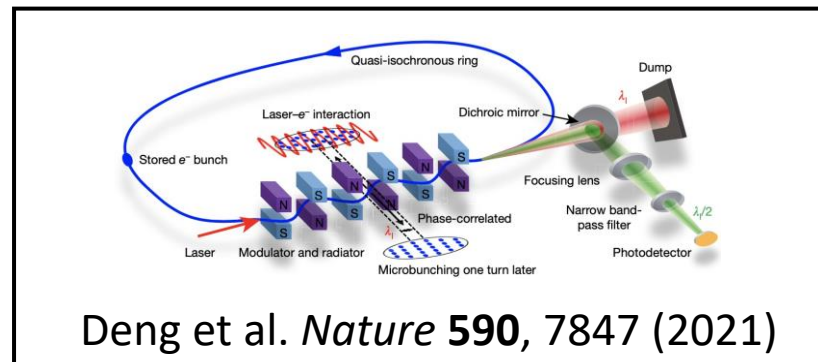
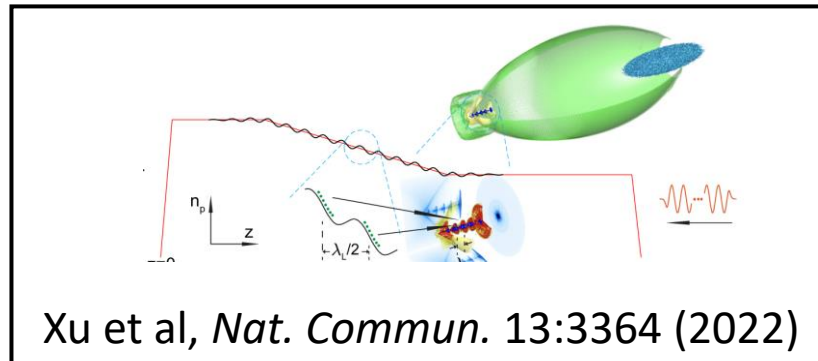


Radiation amplified linearly & e- microbunching growth

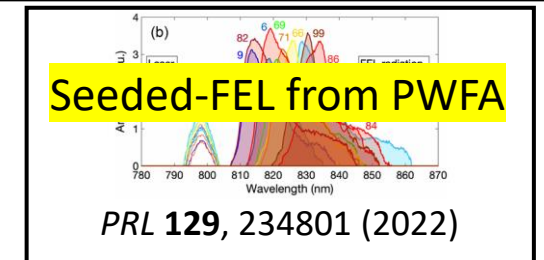
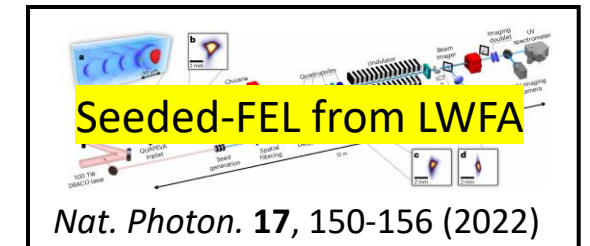
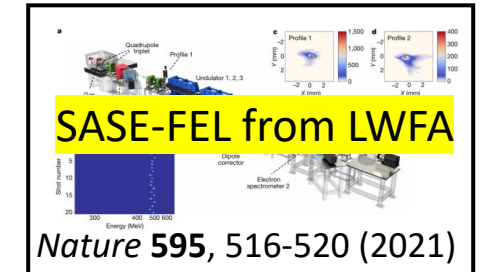


Exponential gain regime & microbunched e-

Monitoring the pre-microbunching



Monitoring the microbunching in FEL

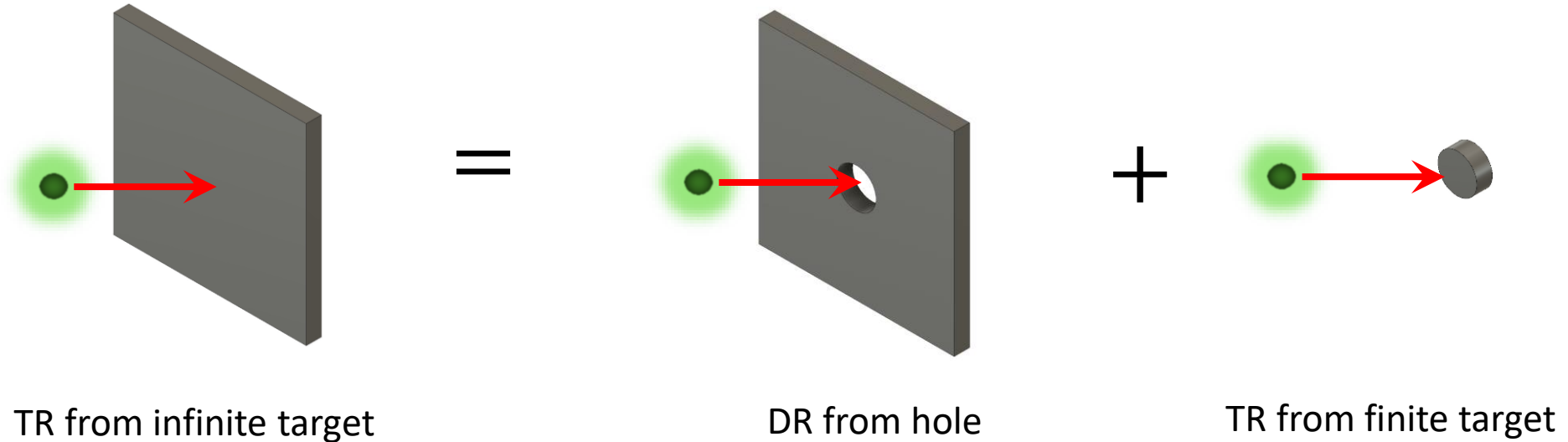


Invasive?

Combined with diffraction radiation (DR)¹

Single-shot & **Non-invasive** diagnostics

Babinet's principle²:



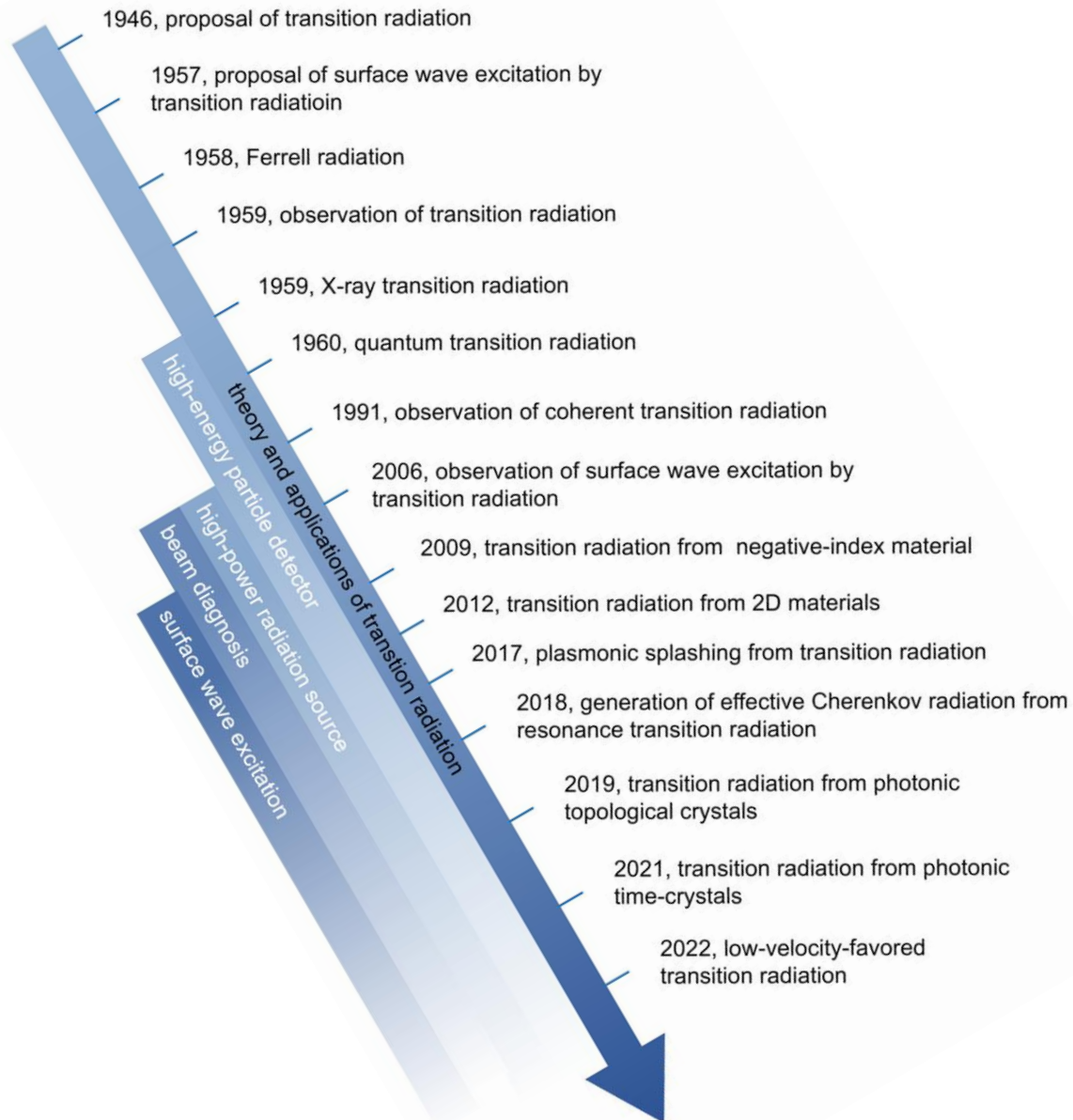
TR from a finite screen can be analytically calculated

$$E_{x,y}^{li}(x_s, y_s, \omega) = -\frac{ie^{ika}}{\lambda a} e^{ik\frac{x_l^2+y_l^2}{2a}} \int dx_s dy_s E_{x,y}^s e^{-ik\frac{x_l x_s + y_l y_s}{2a}} e^{ik\frac{x_s^2+y_s^2}{2a}}$$

¹ Potylitsyn et al. *Diffraction Radiation from Relativistic Particles*, Springer (2010)

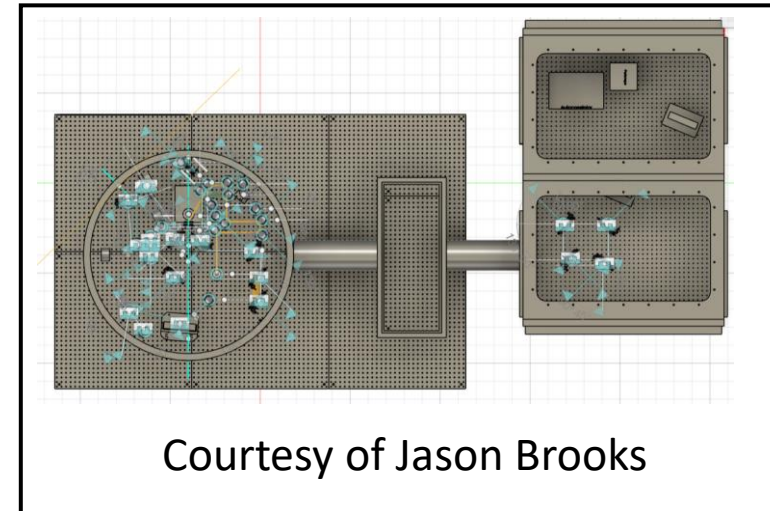
² Fiorito et al, *Proceedings of BIW08*, 316-322 (2008)

Upcoming Experimental Work & Conclusion



Chen et al, *Mater. Today Electron.* 3 (2023) 100025

Future experimental COTR(I) work is scheduled in UT³ lab.



Conclusion

1. Introduction on LWFA, and COTR-related diagnostics \Rightarrow quasi-6D structure
2. Several possible directions in the future

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Courtesy of Google image & Ross