Reconstruction of the 3D structures of relativistic electron beam by transition radiation

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15^{th} Oct 2024

Machine Learning Seminar, UT Physics

Introduction

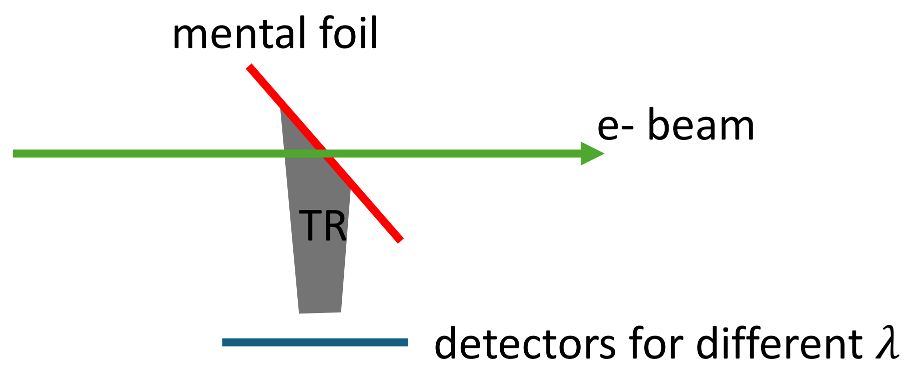
1. Relativistic electron beams ($v \approx c$) from accelerators can have:

- Transverse size ~20µm × 20µm
- Longitudinal size ~2µm (duration of ~6fs)
- Charge of ~100pC (~1e8)

2. Knowing their 3D structures, i.e. number density $\rho(x_e, y_e, z_e)$ is crucial, and we can use **transition radiation** as a diagnostic method to probe it.

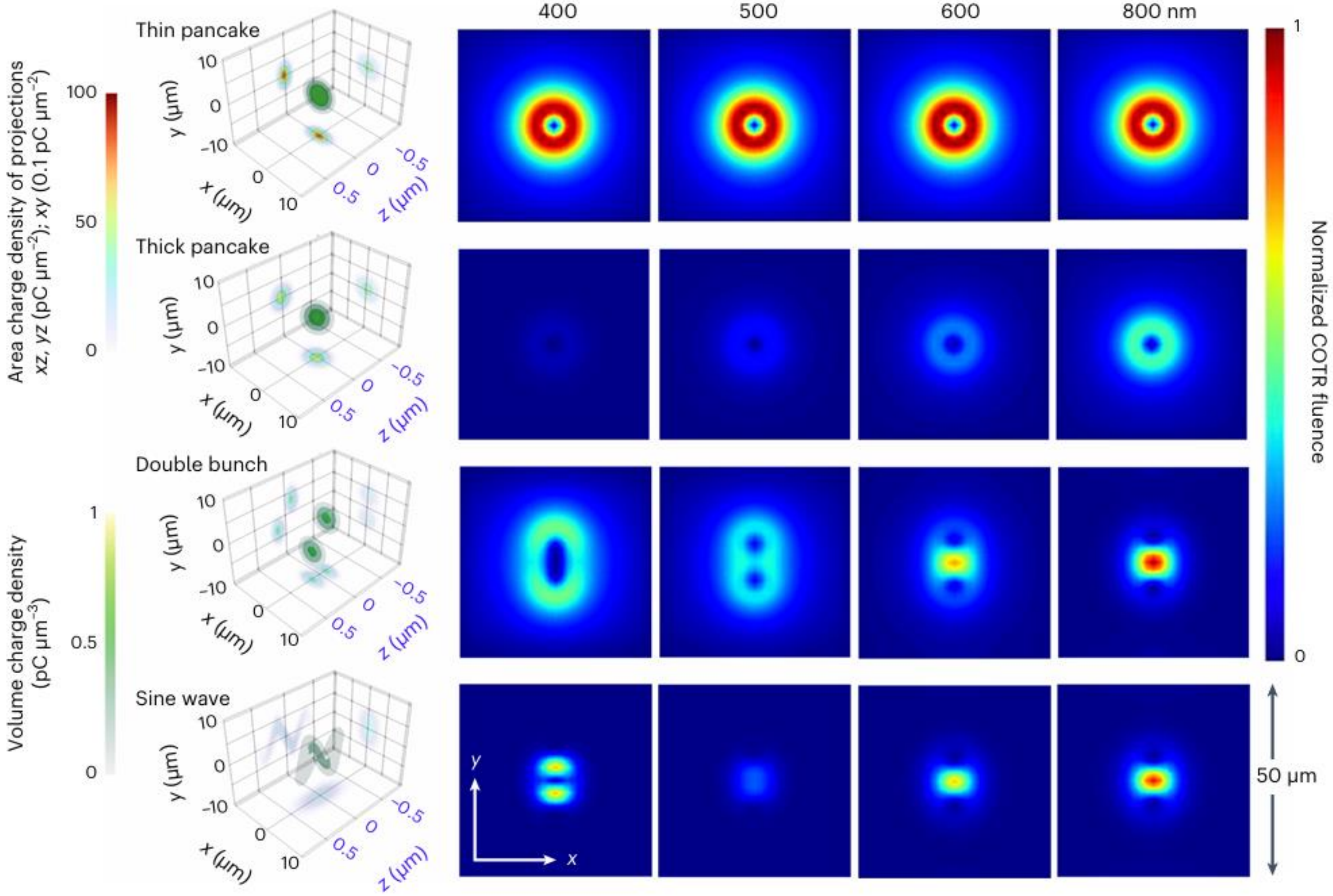
3. Transition radiation (TR) is generated when high-speed charged particles traverse thin mental foil.

4. The detector is detecting Poynting vectors $S(x_d, y_d, \lambda)$. Theoretically, we can connect $S(x_d, y_d, \lambda)$ and $\rho(x_e, y_e, z_e)$ by a complicated function, say $S(x_d, y_d, \lambda) = f\big(\rho(x_e, y_e, z_e)\big)$.



(experimental scheme)

Motivation



LaBerge et al, *Nat. Photon.*, **18** 952-959 (2024)

Multi-wavelength TR images (2D) could help us retrieve the 3D information $\rho(x_e, y_e, z_e)$ of electron beams.

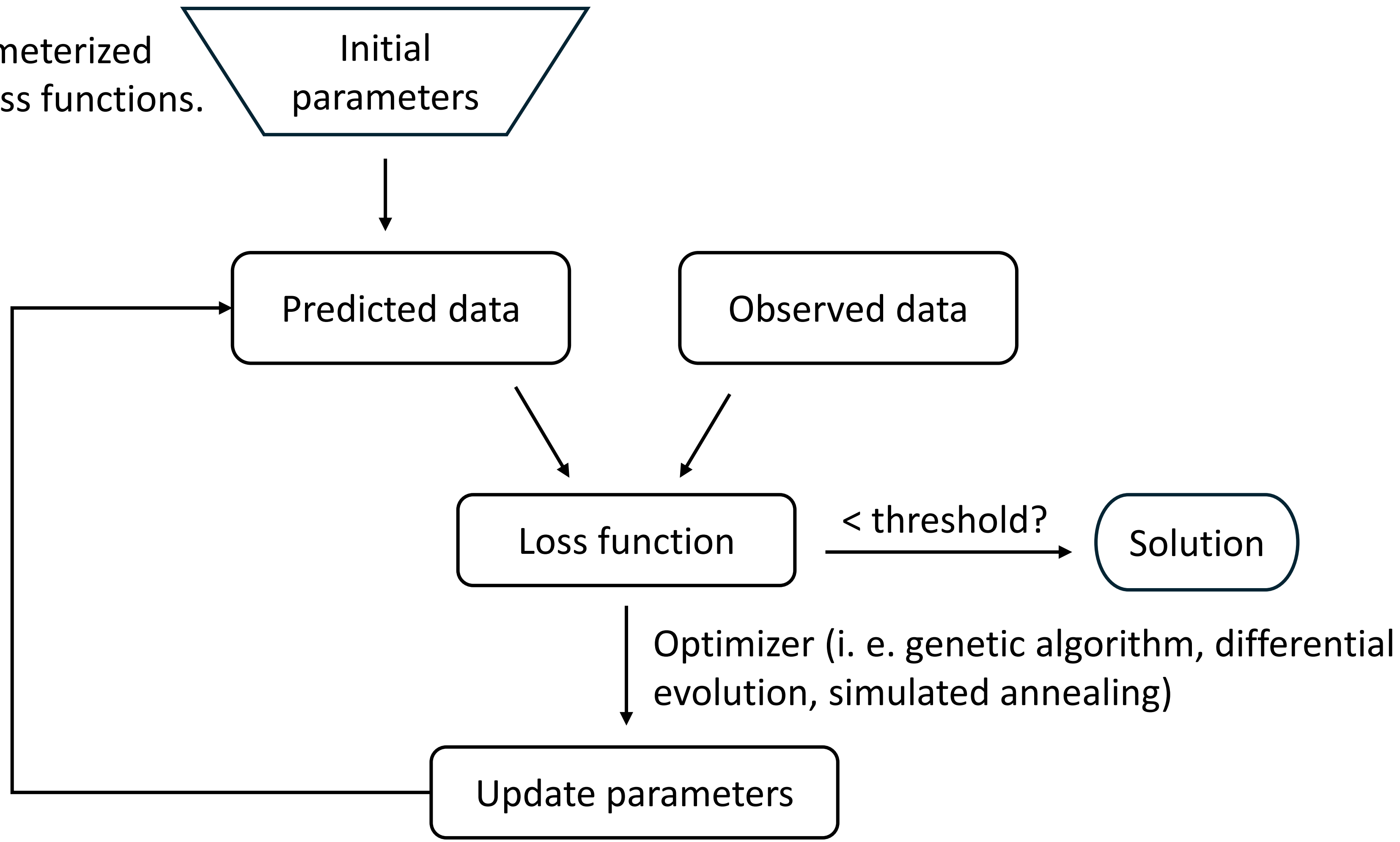
This is an inverse problem:

- Forward process: $\rho \Rightarrow S$
- Backward process: $S \Rightarrow \rho$

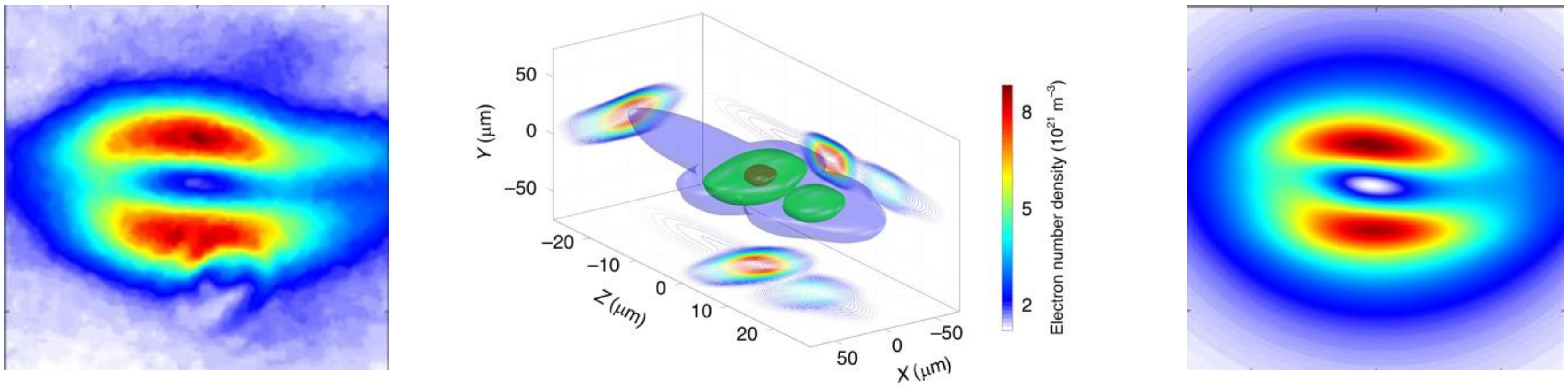
Based on measured S, how to find ρ?

Traditional way (non-ML)

Suppose $\rho(x_e, y_e, z_e)$ is a parameterized function, i.e. sum of super-Gauss functions.



Latest results using traditional way



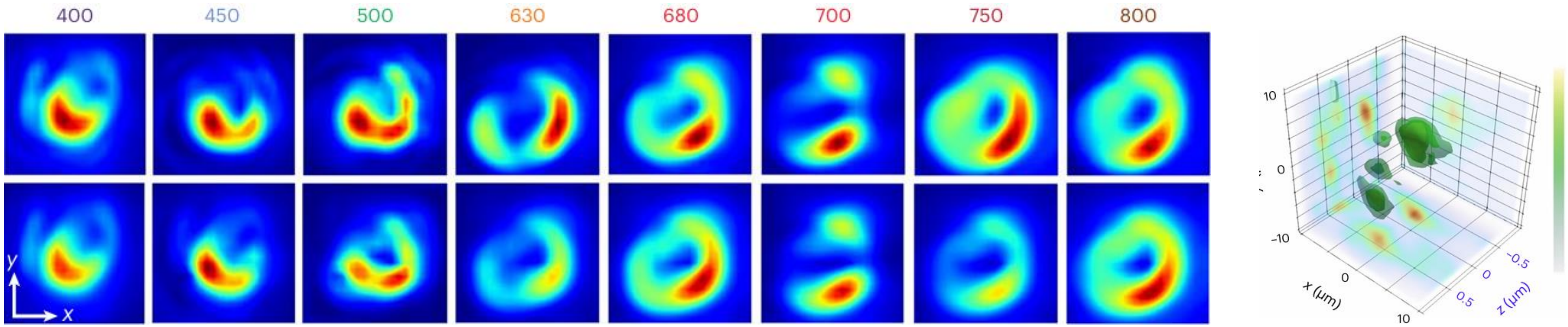
S_{meas} S_{rcn}

Huang et al. *Light: sci.appl*, **13**, 1 (2024)

Genetic algorithm

S_{meas}

S_{rcn}



LaBerge et al, *Nat. Photon.*, **18** 952-959 (2024)

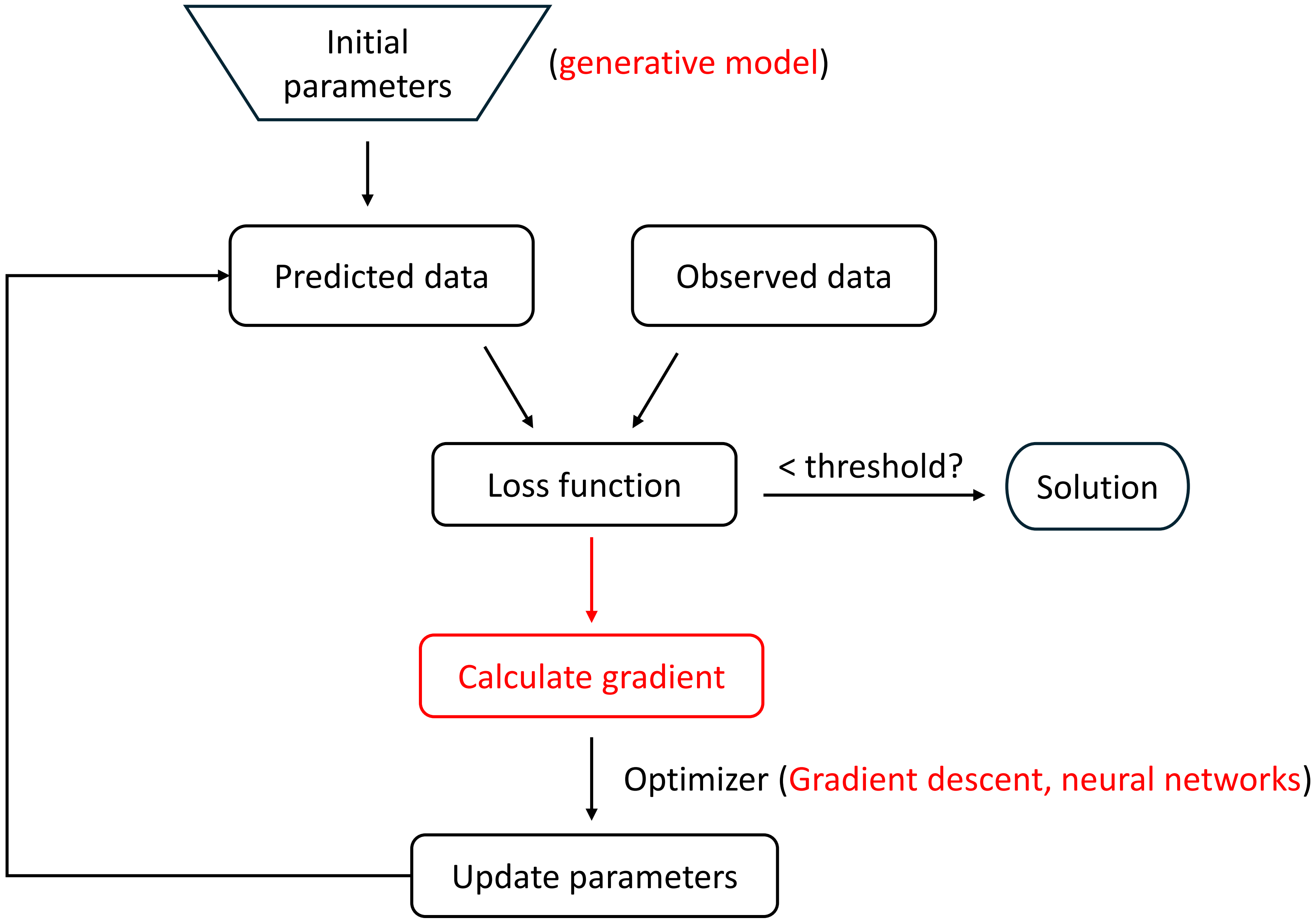
Differential

evolution

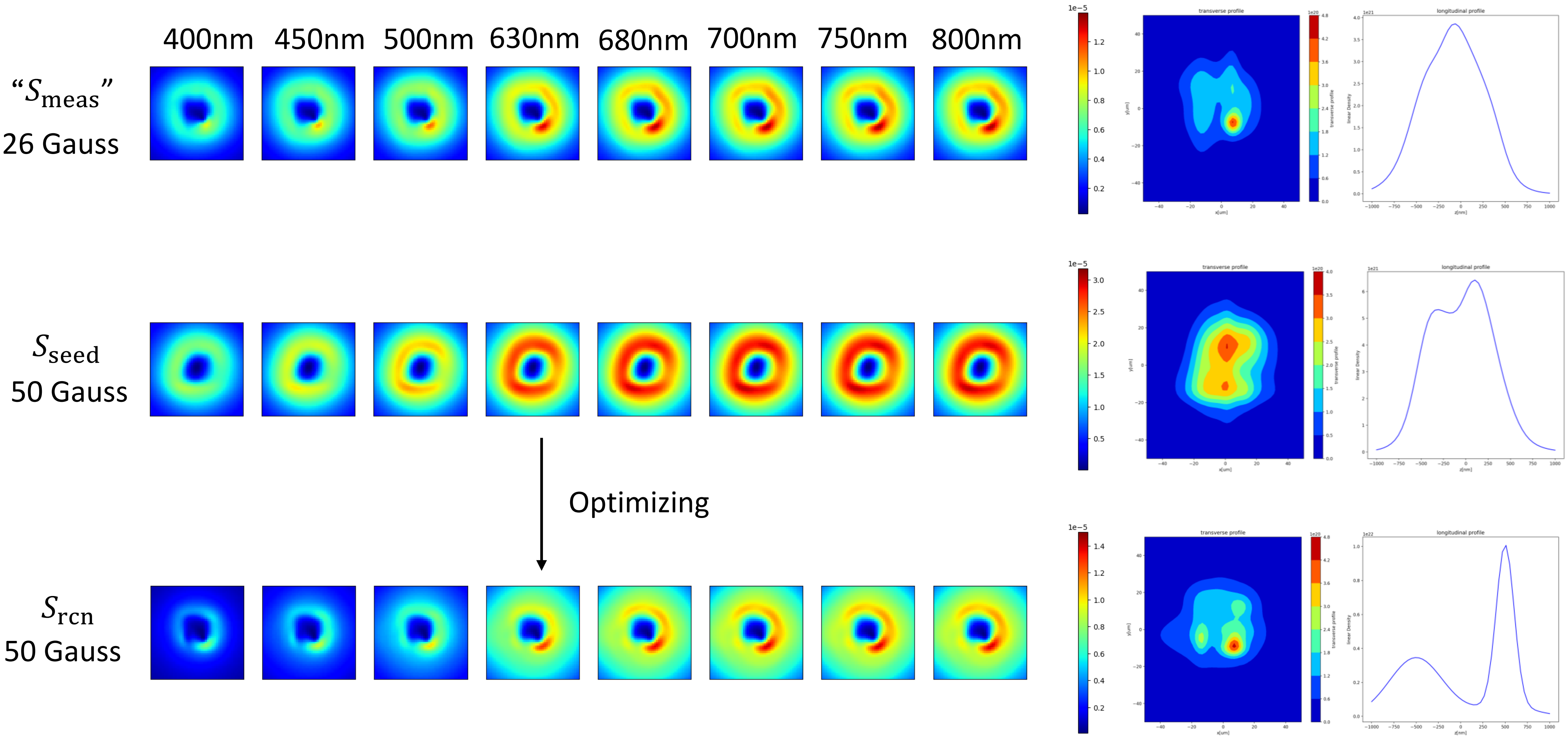
ML ways

1. **Automatic differential** forward process for gradient descent optimization. (Adam)
2. Using **generative model** to reduce parameter space.
3. **Neural-network** based reconstruction.

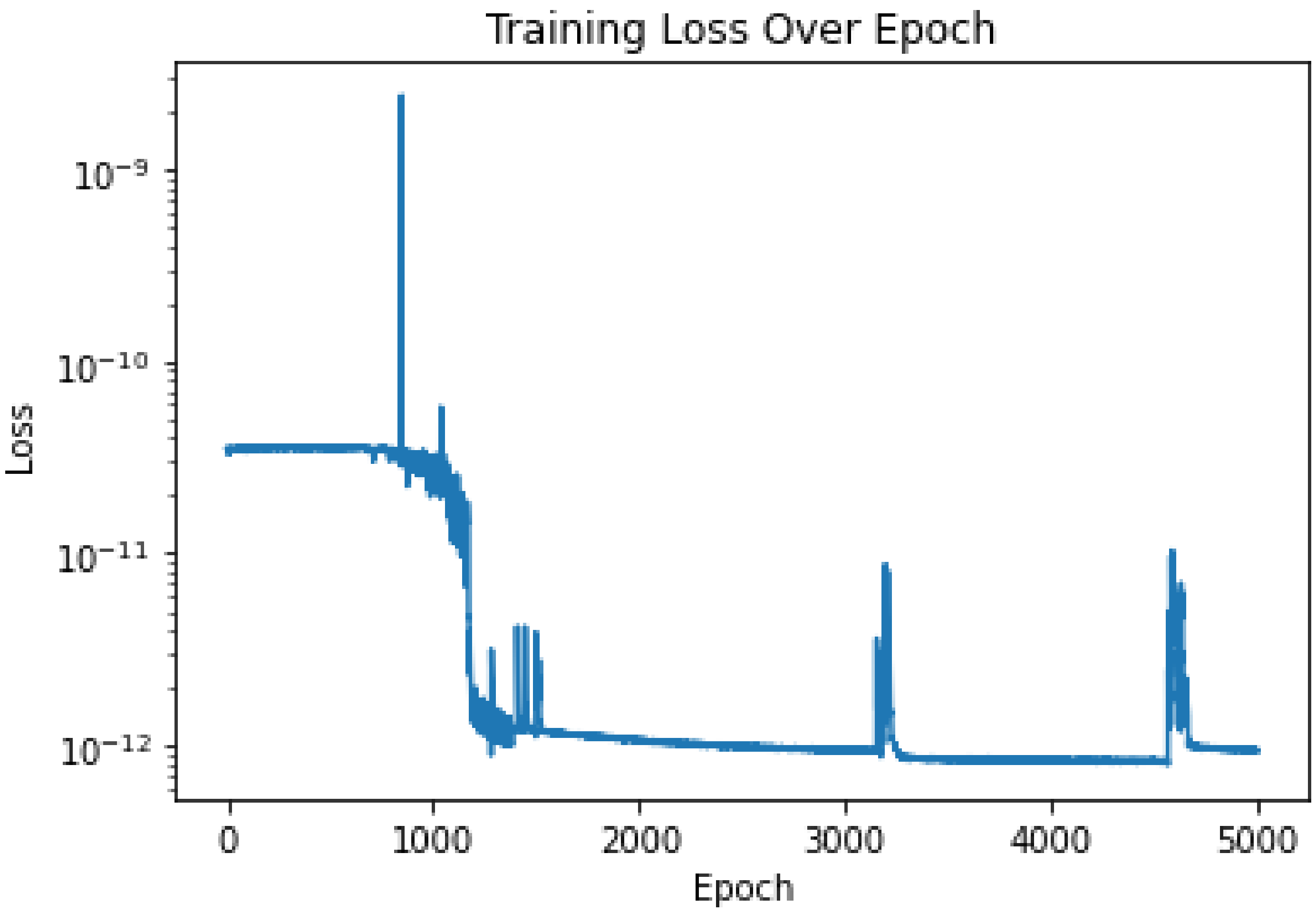
ML ways


Initial parameters
(generative model)
Predicted data
Observed data
Loss function
< threshold?
Solution
Calculate gradient
Optimizer (Gradient descent, neural networks)
Update parameters

ML ways (Differentiable forward process)



ML ways



~2 hours;
Loss reduced by 50 times lower;

Transition Radiation by electron beams

$$S(x_d, y_d, \lambda) = \frac{c}{4\pi^2}\left(\left|\iiint \mathrm{d}x_e \mathrm{d}y_e dz_e \cdot \rho(x_e, y_e, z_e) \cdot \exp(ikz_e) \cdot \mathrm{FPSF}_x(x_d - x_e, y_d - y_e)\right|^2 + \left|\iiint \mathrm{d}x_e \mathrm{d}y_e dz_e \cdot \rho(x_e, y_e, z_e) \cdot \exp(ikz_e) \cdot \mathrm{FPSF}_y(x_d - x_e, y_d - y_e)\right|^2\right)$$

where

$$\mathrm{FPSF}_x(x_d, y_d, \lambda) = \frac{2qk}{Mv} f(\theta_m, \gamma, \zeta)\cos(\varphi)\boldsymbol{e_x}$$
$$\mathrm{FPSF}_y(x_d, y_d, \lambda) = \frac{2qk}{Mv} f(\theta_m, \gamma, \zeta)\sin(\varphi)\boldsymbol{e_y}$$

Traditional way (non-ML)

Suppose $\rho(x_e, y_e, z_e)$ is a parameterized function, i.e. sum of super-Gauss functions. Our goal is then **to find those parameters**, such that the resultant S is close to measured S_{meas}.

Randomly set initial $\rho_i(x_e, y_e, z_e)$, then calculate initial $S_i(x_d, y_d, \lambda)$. Define a cost function i.e. $|S_i(x_d, y_d, \lambda) - S_{meas}(x_d, y_d, \lambda)|^2$, use global optimization method like genetic algorithm, differential evolution or simulated annealing to minimize the cost, by adjusting parameters.

(We are fitting it)

Find the **parameters**, the cost is converged, and $\rho(x_e, y_e, z_e)$ is determined after many iterations.