Solutions L2 + L6

Solution 2. A

2.121 The uniform beam in the figure is of size L by h by b, with b,h $\ll$ L. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

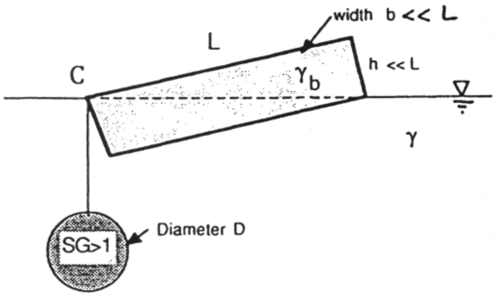


Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{sphere} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that} \quad D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG - 1)}}\right]^{1/3} \quad \textit{Ans. (b)}$$

Solution 2. B

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

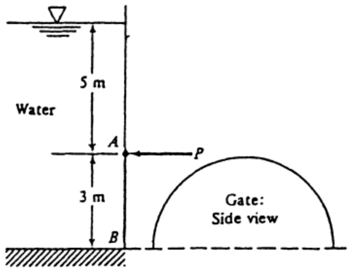
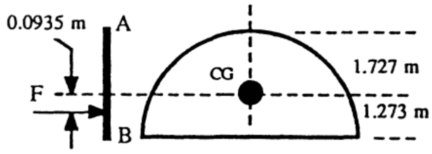


Fig. P2.65



Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is

$$F = \gamma h_{CG} A = (9790)(5.0 + 1.727)\frac{\pi}{2}(3)^2$$
$$= 931000 \text{ N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(0.10976)(3)^4 \sin 90°}{(5 + 1.727)(\pi/2)(3)^2} = -0.0935 \text{ m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \quad \text{or:} \quad P = \mathbf{366000\ N} \quad \textit{Ans.}$$

Solution 6. A
Write the given relation and count variables:

$$Q = f(R, \mu, \frac{\mathrm{d}p}{\mathrm{d}x}) \quad \text{four variables} \, (n = 4)$$

Make a list of the dimensions of these variables using the $\{MLT\}$ system: There are three primary

Q	R	μ	$\mathrm{d}p/\mathrm{d}x$
L^3T^{-1}	L	$ML^{-1}T^{-1}$	$ML^{-2}T^{-2}$

dimensions (M, L, T), hence $j = 3$. By trial and error we determine that $R, \mu,$ and $\mathrm{d}p/\mathrm{d}x$ cannot be combined into a pi group. Then $j = 3$, and $n - j = 4 - 3 = 1$. There is only *one* pi group, which we find by combining Q in a power product with the other three:

$$\Pi_1 = R^a \mu^b (\frac{\mathrm{d}p}{\mathrm{d}x})^c Q^1 = (L)^a (ML^{-1}T^{-1})^b (ML^{-2}T^{-2})^c (L^3T^{-1}) = M^0 L^0 T^0$$

Equate exponents:

$$\begin{cases} b + c = 0 \\ a - b - 2c + 3 = 0 \\ -b - 2c - 1 = 0 \end{cases}$$

Solving simultaneously, we obtain $a = -4$, $b = 1$, and $c = -1$. Then

$$\Pi_1 = R^{-4} \mu^1 (\frac{\mathrm{d}p}{\mathrm{d}x})^{-1} Q$$

or

$$\Pi_1 = \frac{Q\mu}{R^4(\mathrm{d}p/\mathrm{d}x)} = \text{const}$$

Solution 6. B
The functional relationship is $\delta = f(x, U, \mu, \rho)$, with $n = 5$ variables and $j = 3$ primary dimensions (M, L, T). Thus we expect $n - j = 5 - 3 = 2$ Pi groups:

$$\Pi_1 = \rho^a x^b \mu^c \delta = M^0 L^0 T^0 \quad \text{if } a = 0, b = -1, c = 0 : \Pi_1 = \frac{\delta}{x}$$

$$\Pi_2 = \rho^a x^b \mu^c U = M^0 L^0 T^0 \quad \text{if } a = 1, b = 1, c = -1 : \Pi_2 = \frac{\rho U x}{\mu}$$

Thus $\delta/x = f(\rho U x/\mu) = f(Re_x)$.