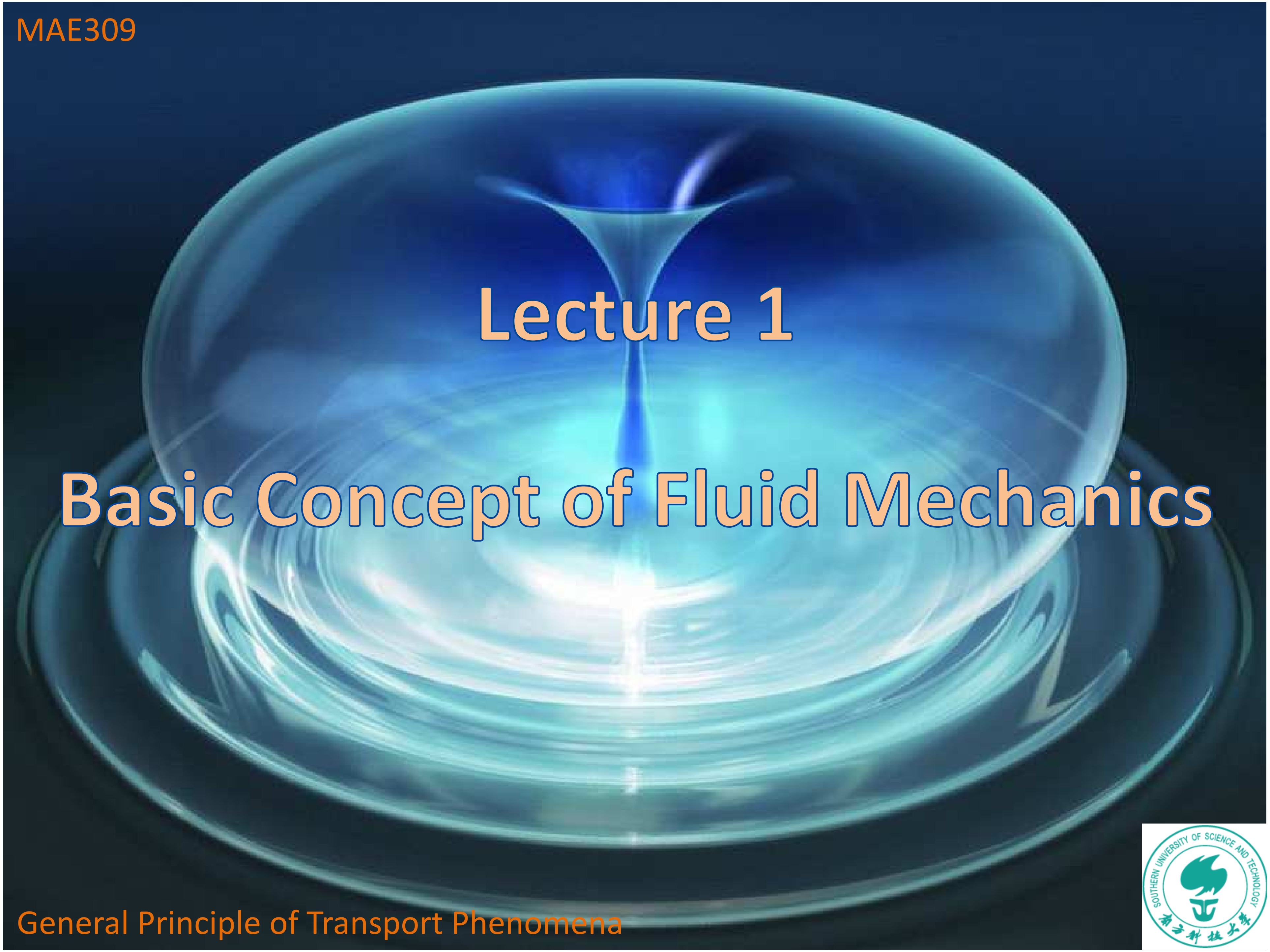
MAE309



Lecture 1

Basic Concept of Fluid Mechanics

General Principle of Transport Phenomena

Learning Objectives

- To understand
  - The concept of a fluid.
  - The wide scope of fluid mechanics.
  - The concept of a continuum and the continuum assumption.
  - The common physical properties of fluids and their units of measurement, especially viscosity.
  - Shear stress
  - Surface tension
  - Capillary effect

What is Fluid

- Fluid
  - a substance that continually deforms (flows) under an applied shear stress.
  - Including both liquid and gas.
  - Example: air, water, blood …
  - Solid can hold its shape independently of its container
  - Liquid will occupy a definite volume in the container
  - Gas fills up the whole container volume

KEY IDEAS: fluids cannot support a shear stress

What is Fluid

- Definition of Stress
  - A physical quantity that expresses the internal forces that neighboring particles of a continuous material exert on each other

剪切应力

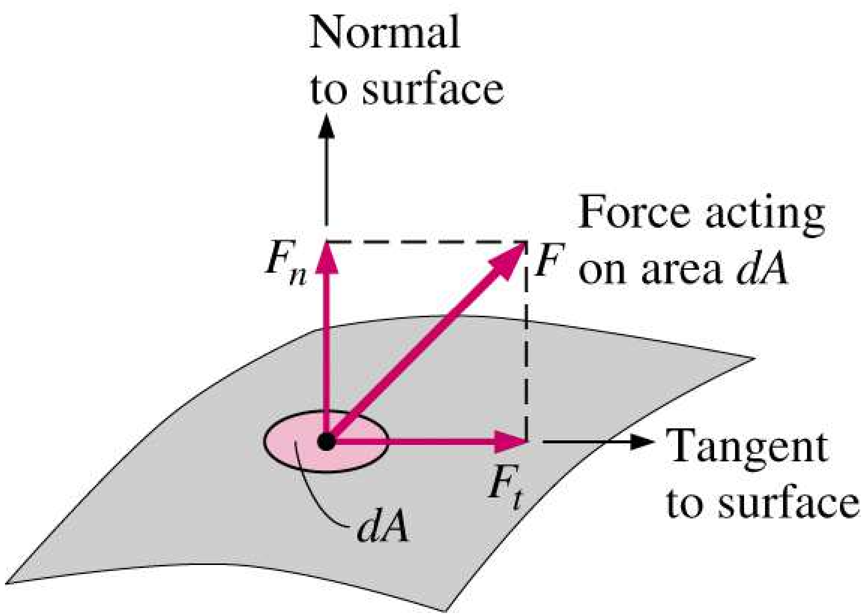
shear stress

$$\tau_t = \lim_{\delta A \to 0} \frac{\delta F_t}{\delta A}$$

法向应力

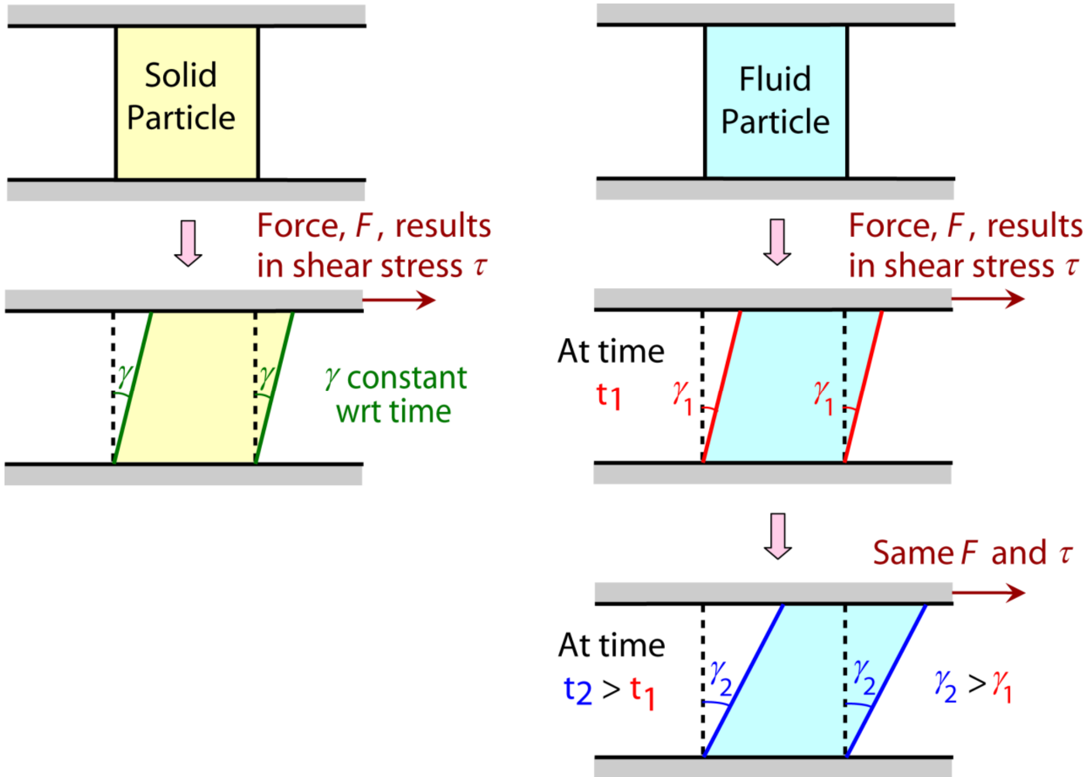
normal stress

$$\sigma_n = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A}$$



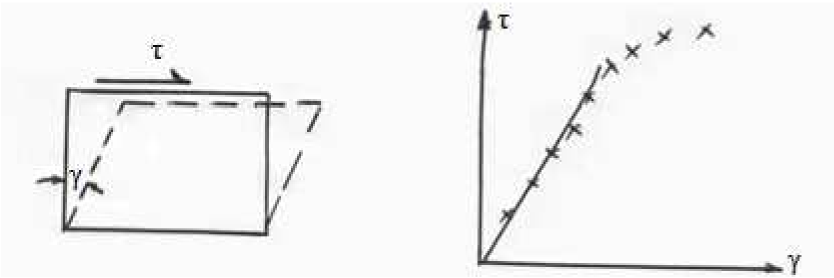
What is Fluid

- Different response to shear stress
  - A strain γ is measure of deformation representing the displacement between particles in the body relative to a reference length.



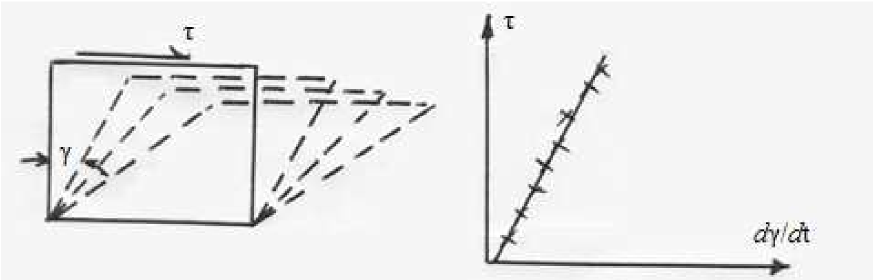
What is Fluid

- Different response to shear stress
  - Solid: the deformation (strain γ) is proportional to the shear stress τ. γ is a constant for a specified τ.



$$\gamma \propto \tau$$

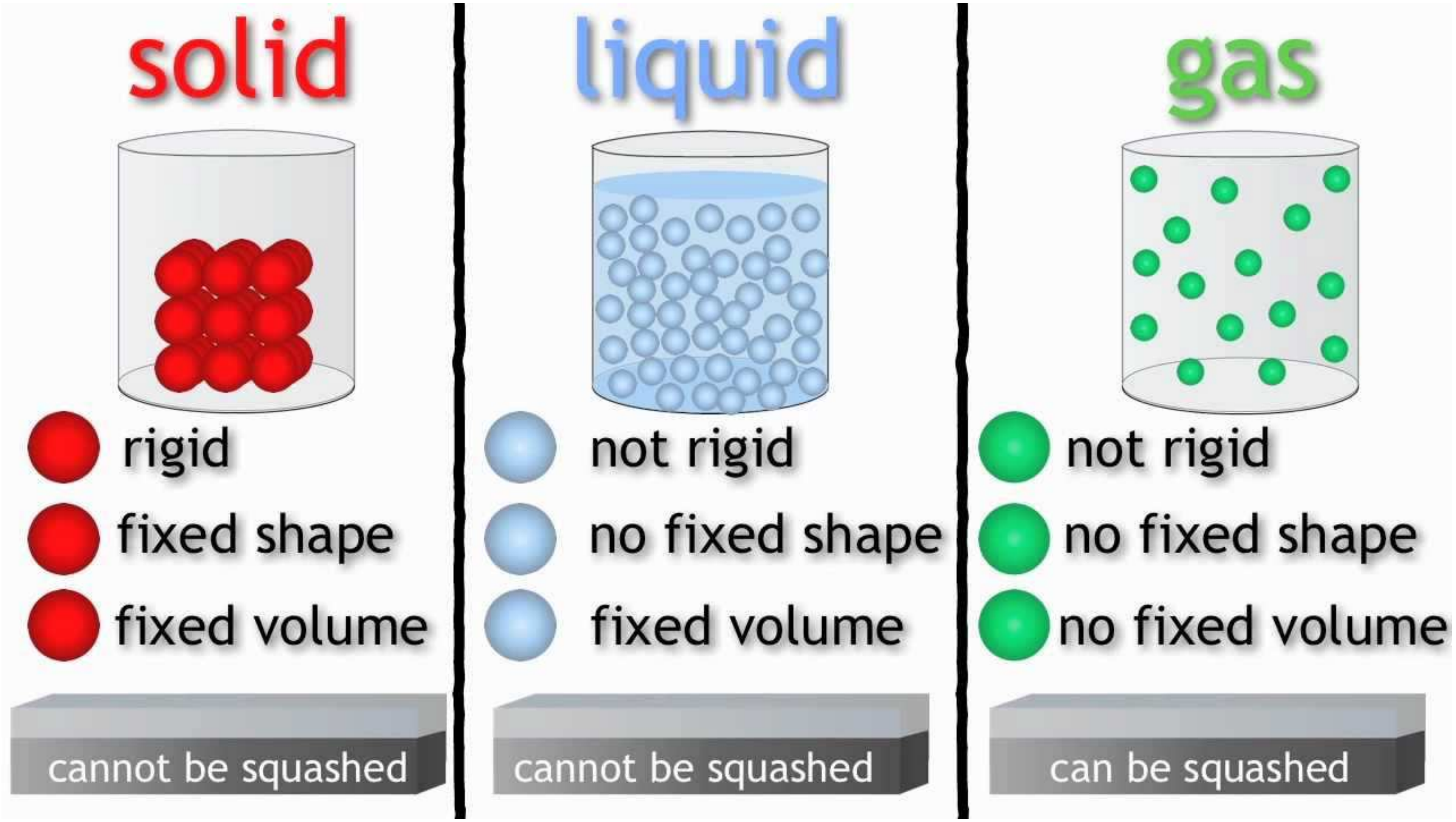
  - Fluid: the deformation rate (strain rate *d*γ/*d*t) is proportional to the shear stress τ. *d*γ/*d*t is a constant for a specified τ.



$$d\gamma/dt \propto \tau$$

What is Fluid

- Difference from molecular level



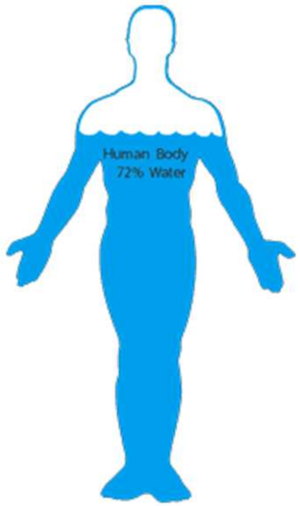
Fluid Mechanics

- Fluid Mechanics is the study of the behaviour of fluids at rest (fluid statics) or in motion (fluid dynamics)
- Fluid Mechanics can be divided into several categories
  - Hydrodynamics: study of flows of incompressible fluids (water, gases at low speeds)
  - Hydraulics: study of liquid flows in pipes and open channels
  - Gas dynamics: study of flows of gases
  - Aerodynamics: study of flow of gases (air) over bodies (aircraft, rockets, automobiles) at low and high speeds
  - Meteorology
  - Oceanography
  - Hydrology

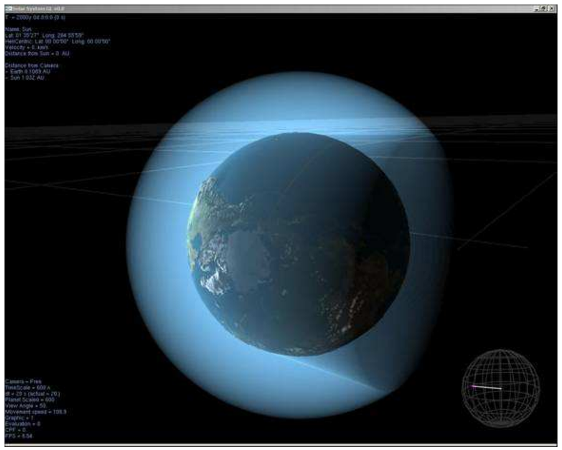
Naturally occurring flows

Fluid Mechanics

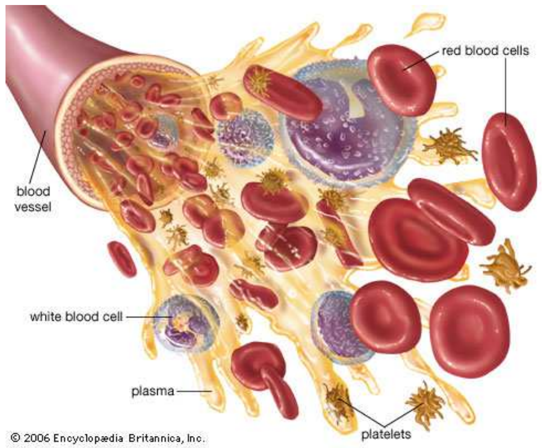
- Why study Fluid Mechanics
  - Air and water are two very important fluids
    - ✓ ~70% of human body is made up of water
    - ✓ ~70% of earth's surface is covered by water
    - ✓ ~90% of earth's atmosphere extends to an altitude of 16 km above earth's surface
  - Blood is also very important to us



Earth



Earth's Atmosphere



Blood

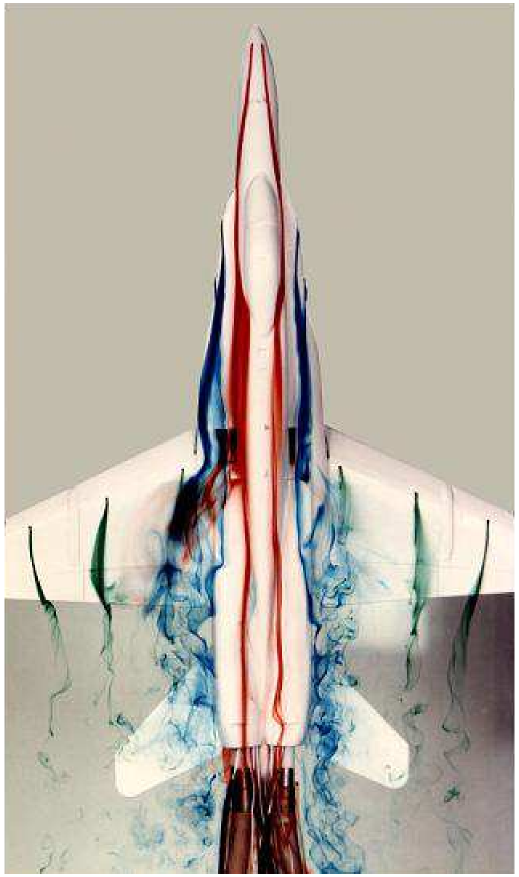
Fluid Mechanics

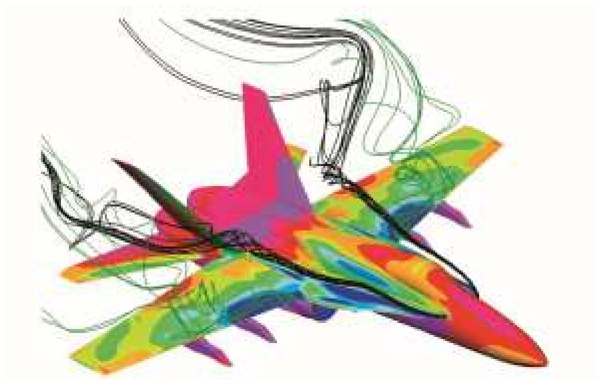
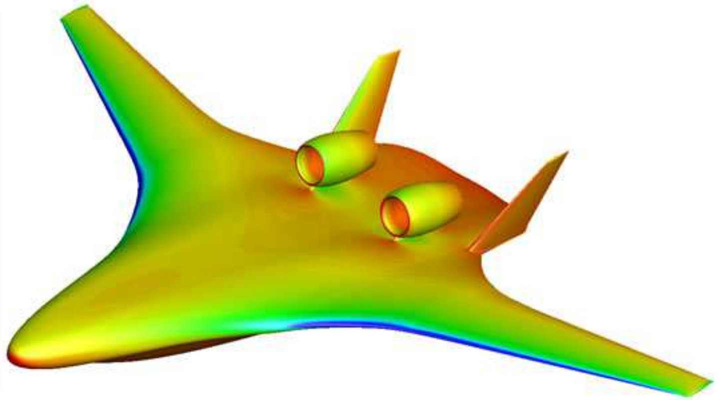
- Why study Fluid Mechanics
  - There is not a single process or event on the earth which does not somehow involve fluid mechanics

In your body	Outside your body	Environment	Movement
Blood flow	Swimming	Wind	Aircraft
Breathing	Air resistance when you walk and run	Rivers	Cars
Eating/Drinking		Ocean Currents	Hot balloon
Digestion (mixture)		Waves	Rockets

Applications of Fluid Mechanics

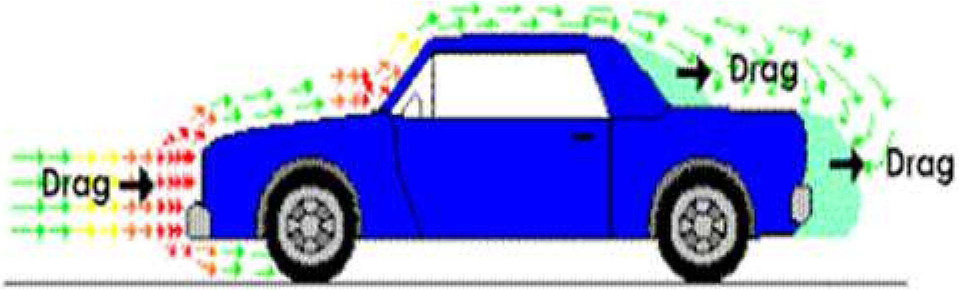
- Aircrafts

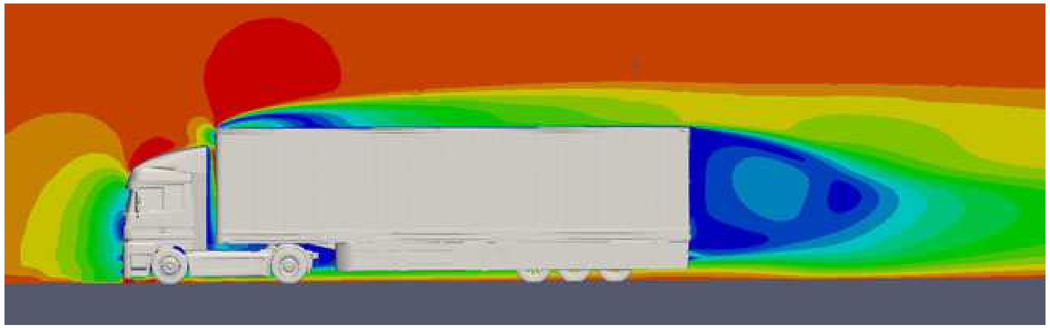


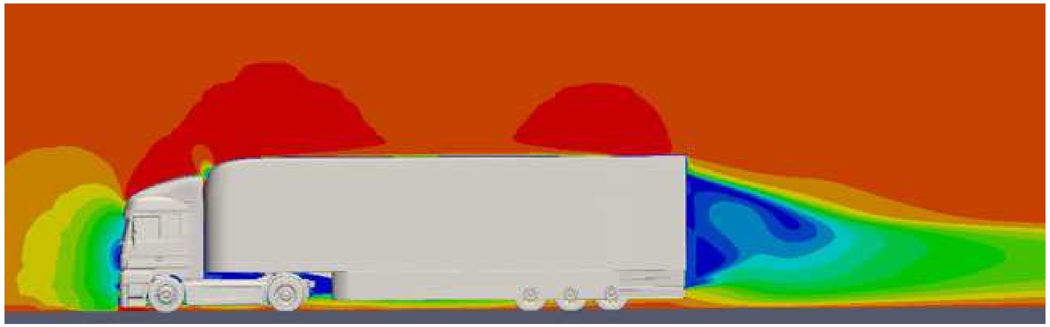




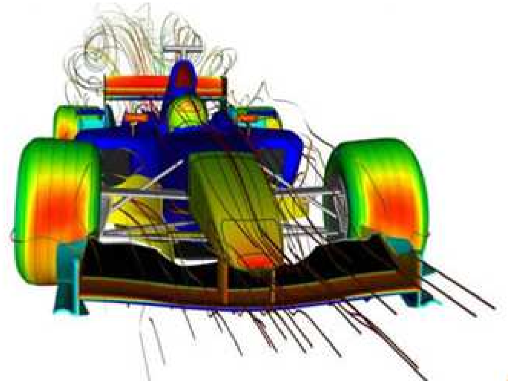
Applications of Fluid Mechanics

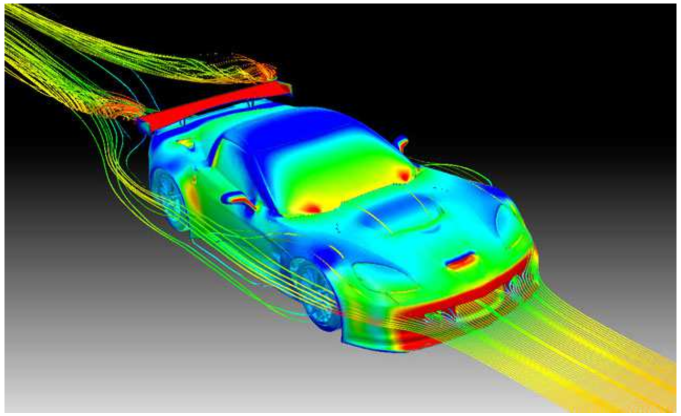
- Vehicles





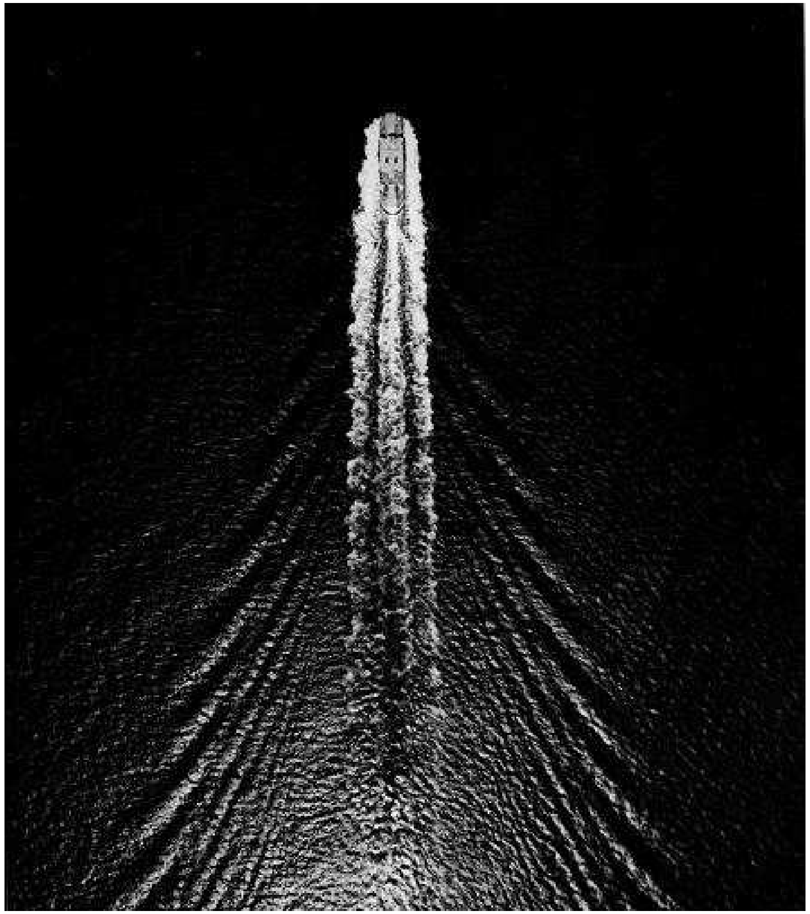


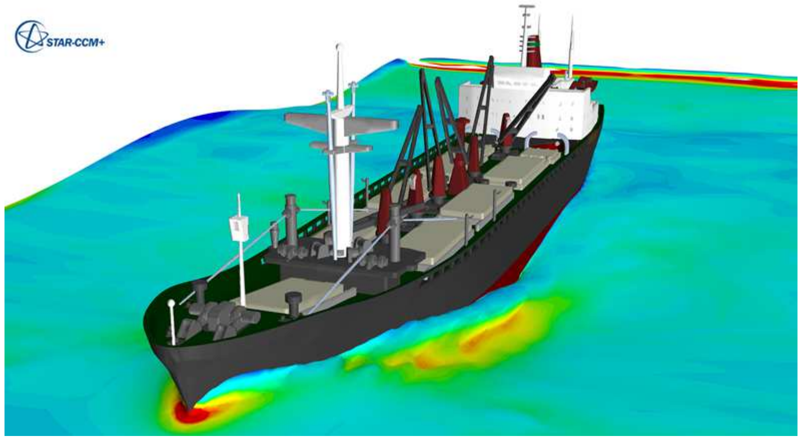


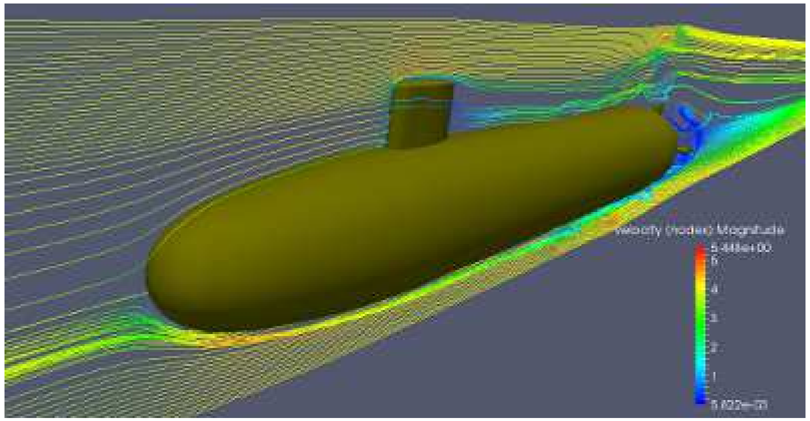


Applications of Fluid Mechanics

- Ships

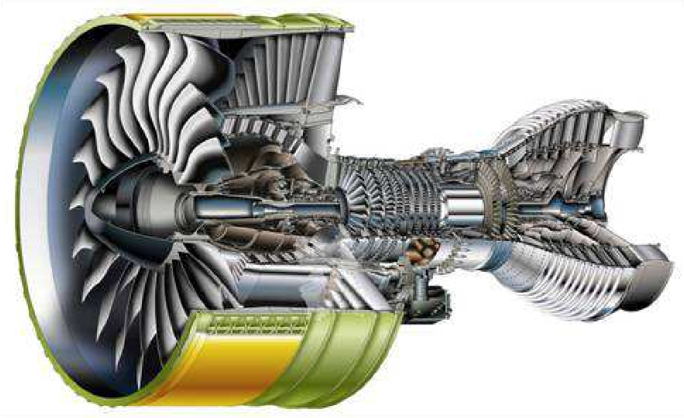




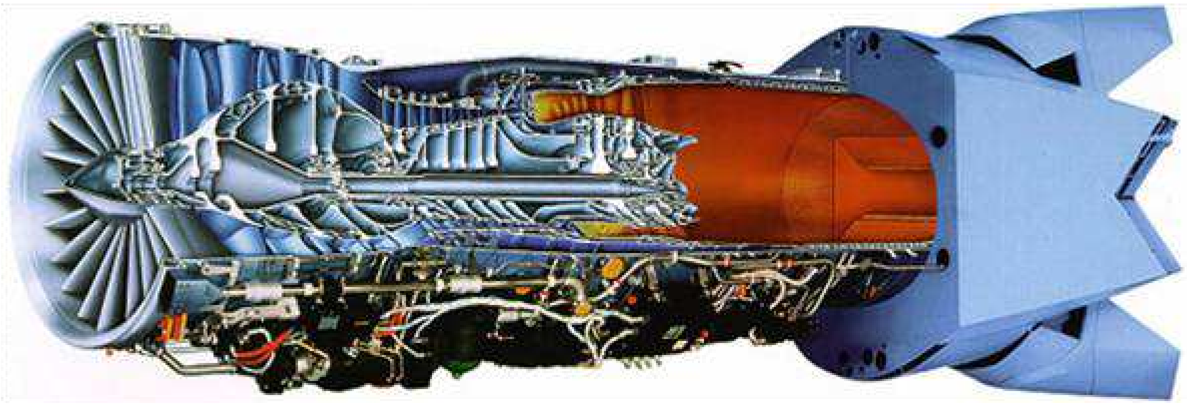


Applications of Fluid Mechanics

- Aerospace Propulsion



Jet engine for commercial aircraft

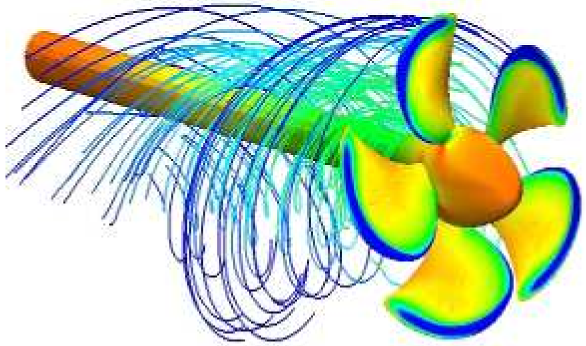


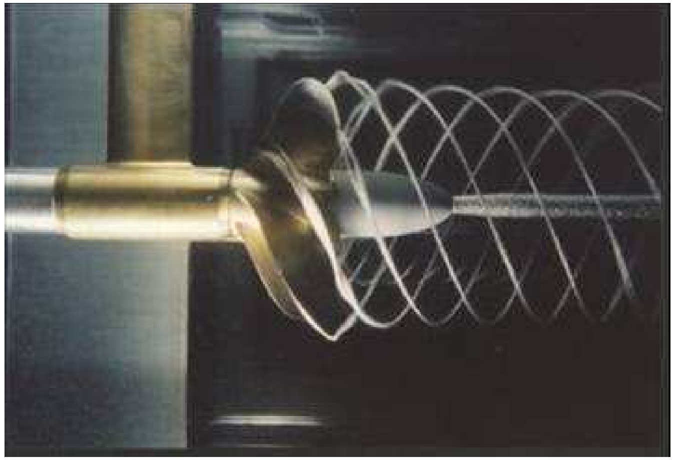
Jet engine for fighter aircraft

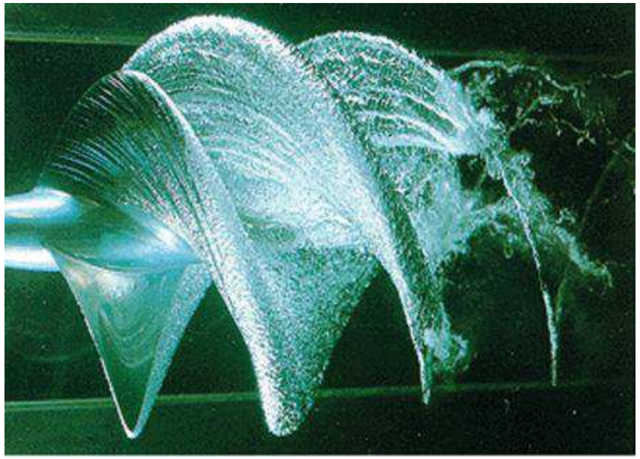
Rocket propulsion

Applications of Fluid Mechanics

- Marine Propulsion



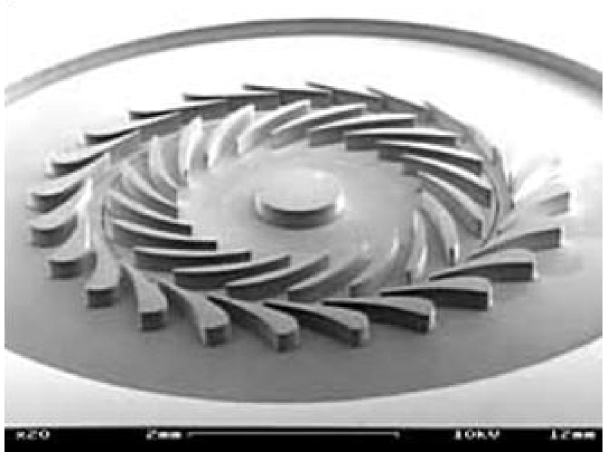




Cavitation in marine propellers

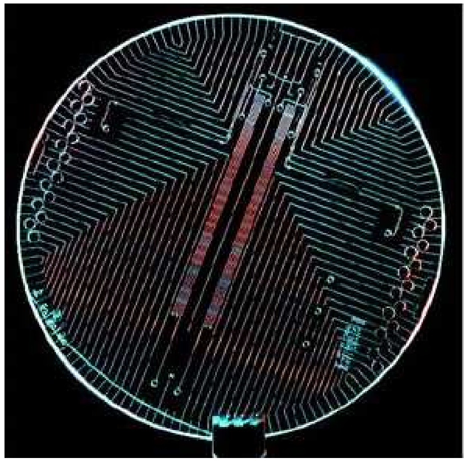
Applications of Fluid Mechanics

- Microfluidics

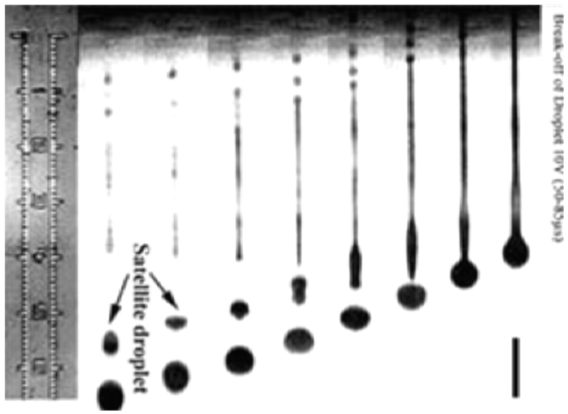


Microengine

Microrocket



Integrated microfluidic bioprocessor



Inkjet printer

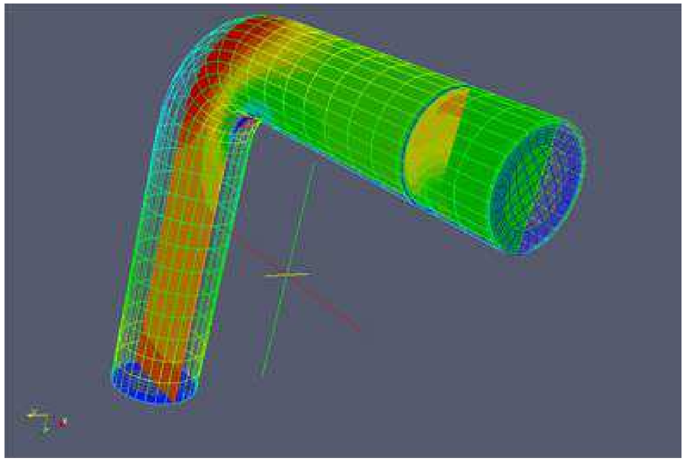
Applications of Fluid Mechanics

- Pipelines

Pipe network

Oil refinery

Water pipeline

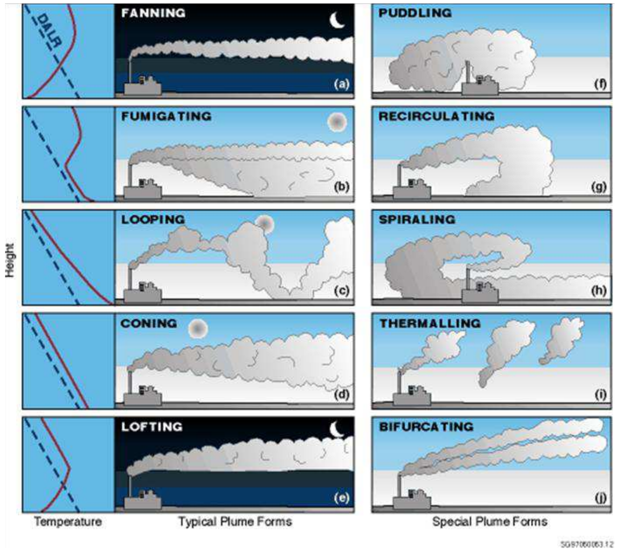


Computer simulation of pipe flow

Applications of Fluid Mechanics

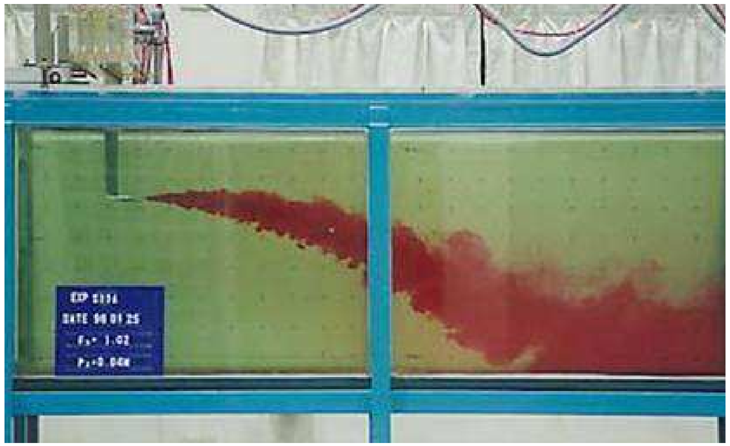
- Environmental Fluid Mechanics

Atmospheric pollution



Plume dispersion

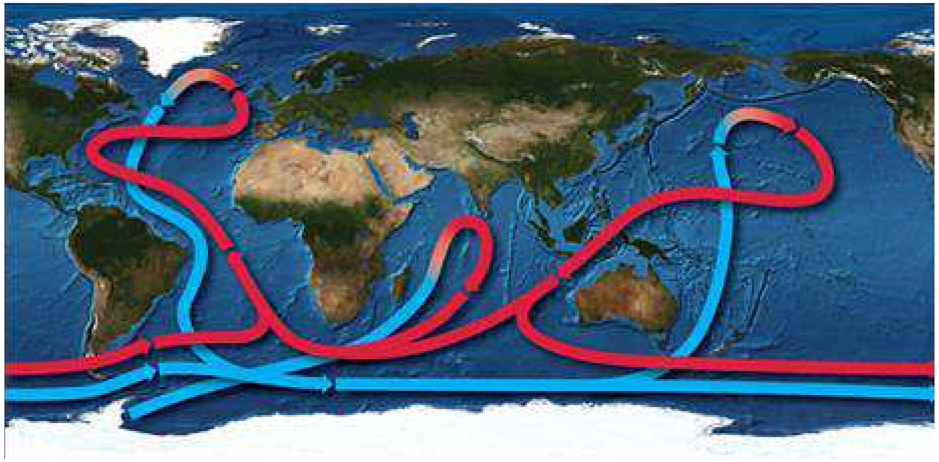
River pollution and sedimentation



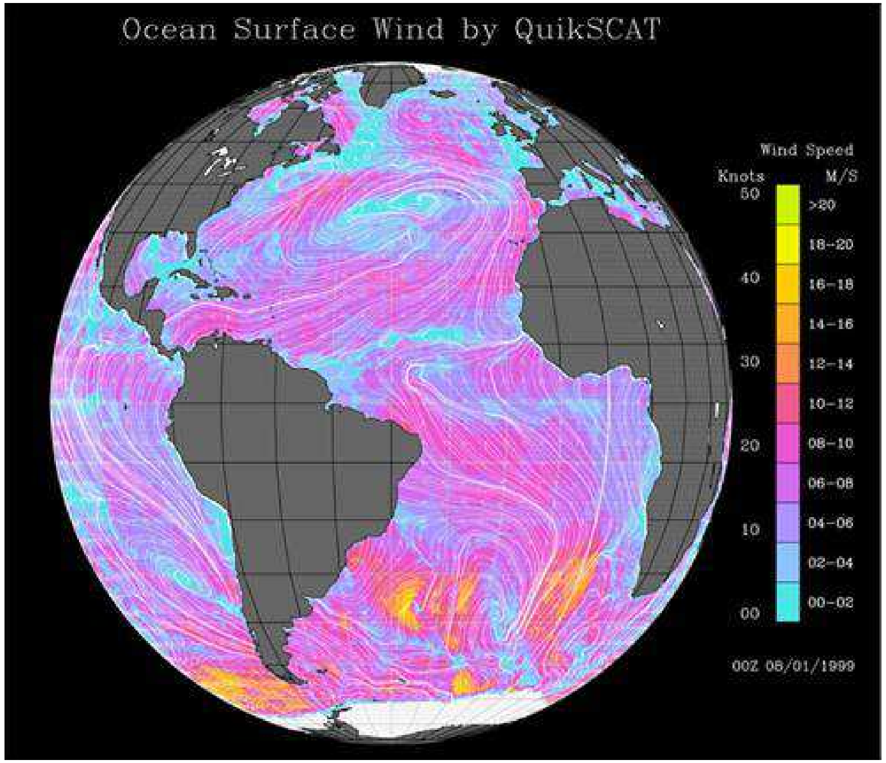
Pollutant sedimentation and dispersion

Applications of Fluid Mechanics

- Geophysical Fluid Dynamics: Ocean



Circulation system of the ocean

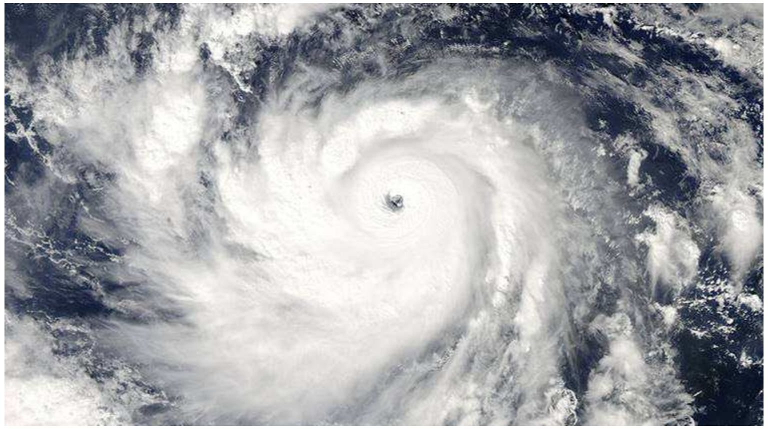


Ocean surface wind

Ocean waves

Applications of Fluid Mechanics

- Geophysical Fluid Dynamics: Atmosphere / Weather



Hurricane

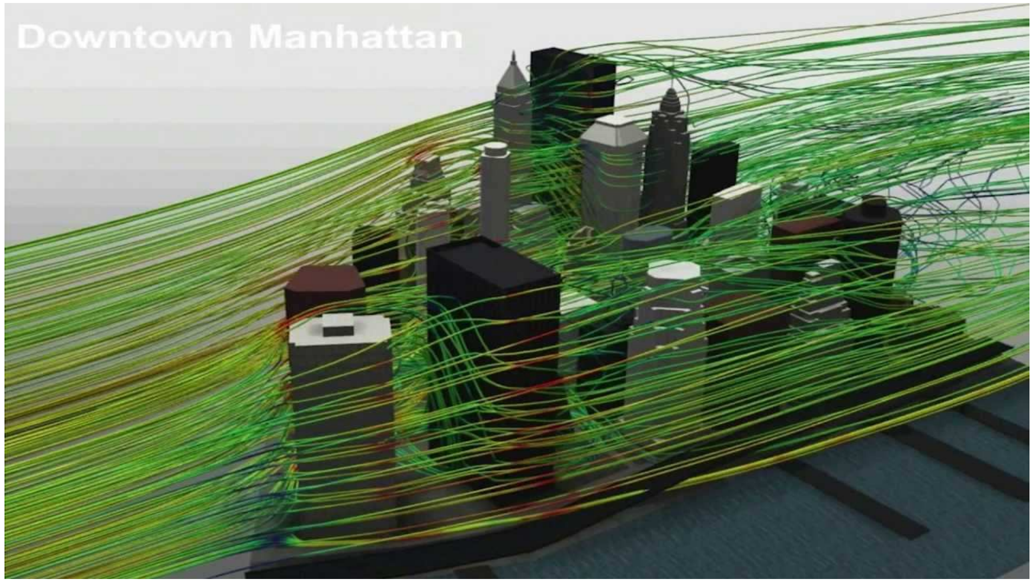
Cloud Karman vortex street

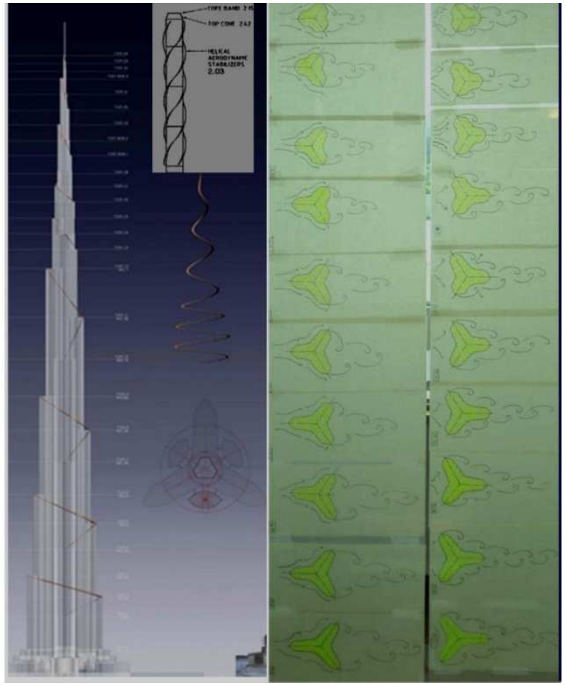
Waterspout

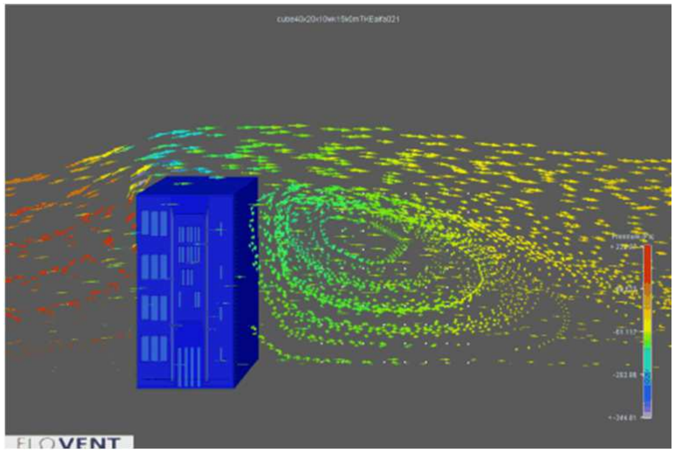
Tornado

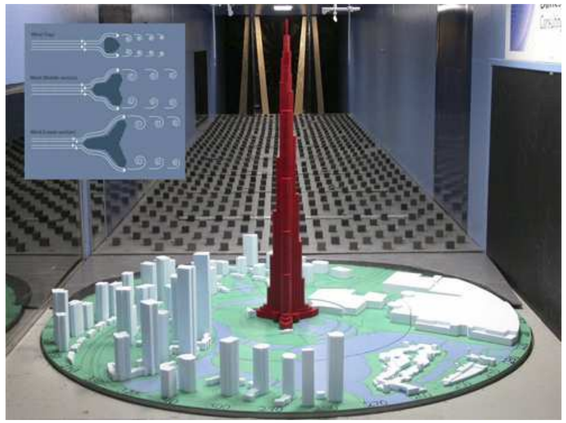
Applications of Fluid Mechanics

- Builds



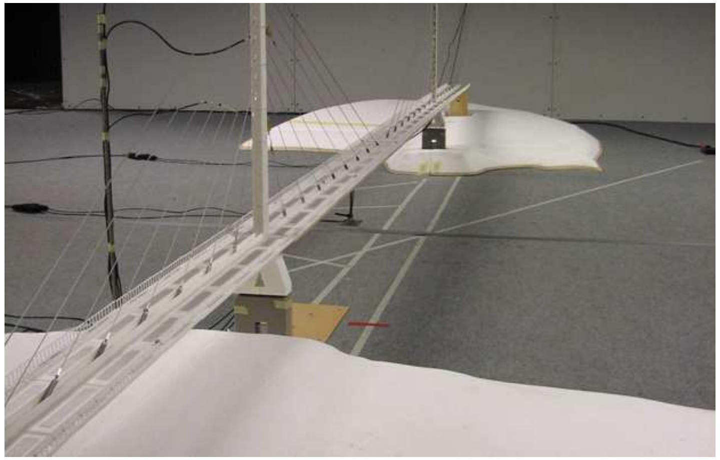


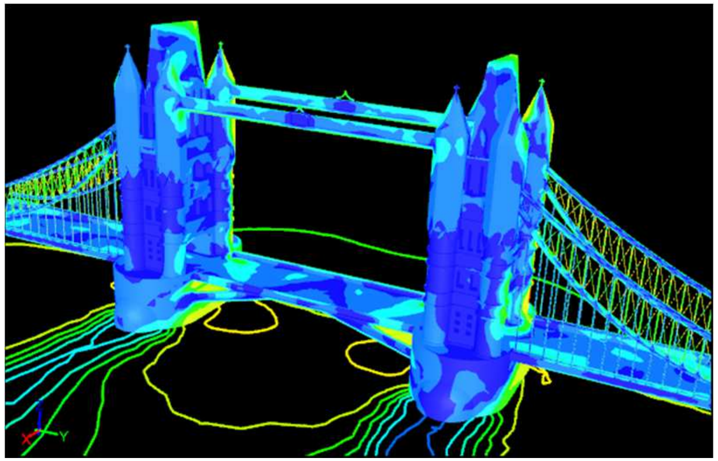




Applications of Fluid Mechanics

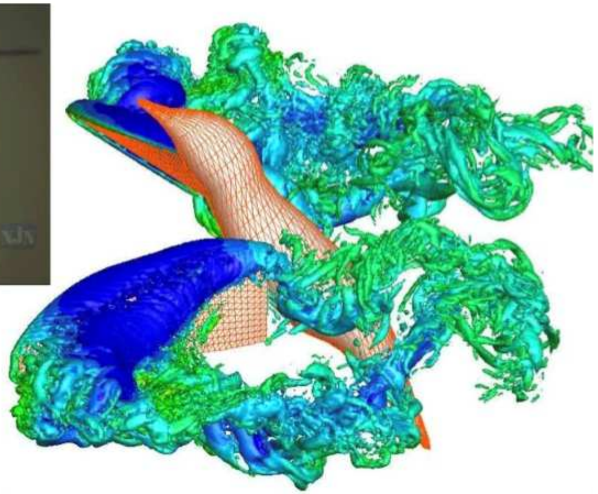
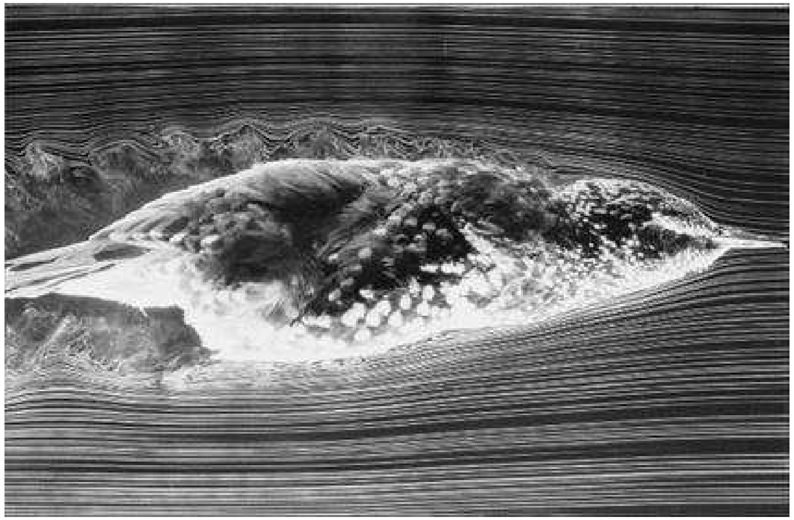
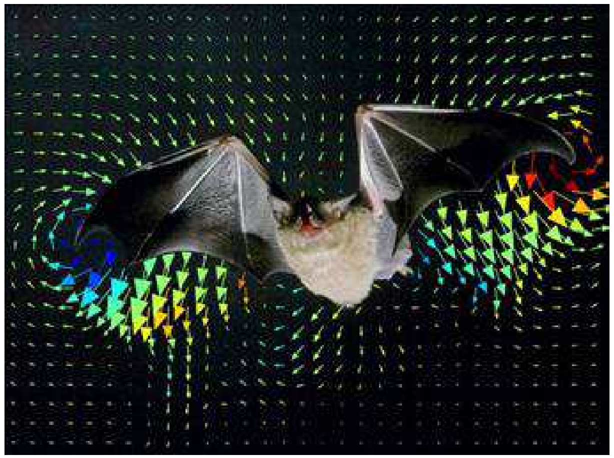
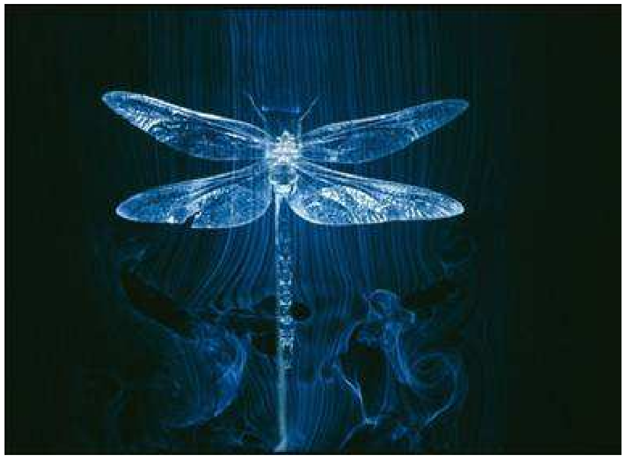
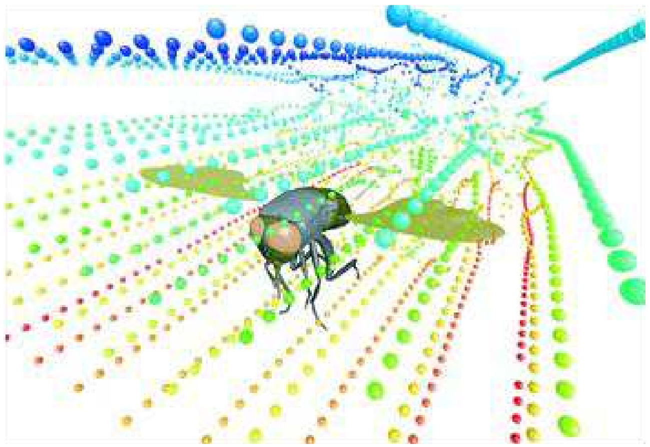
- Bridges





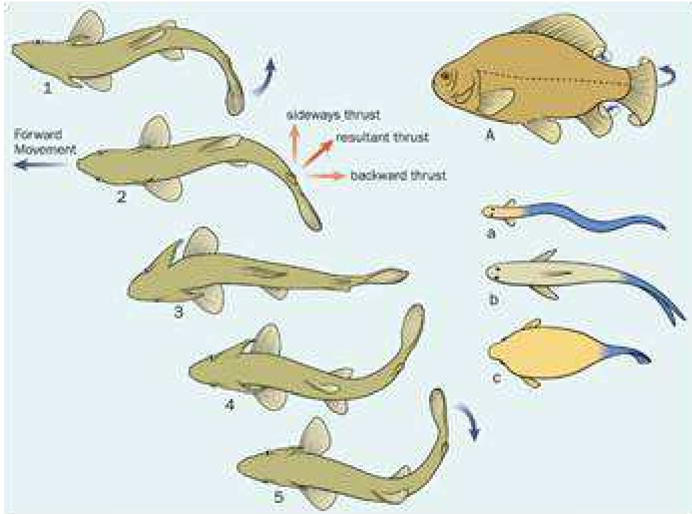
Applications of Fluid Mechanics

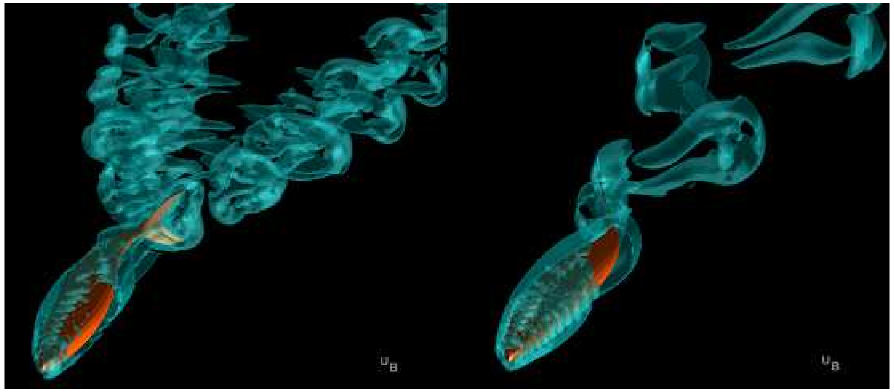
- Biofluids: flying

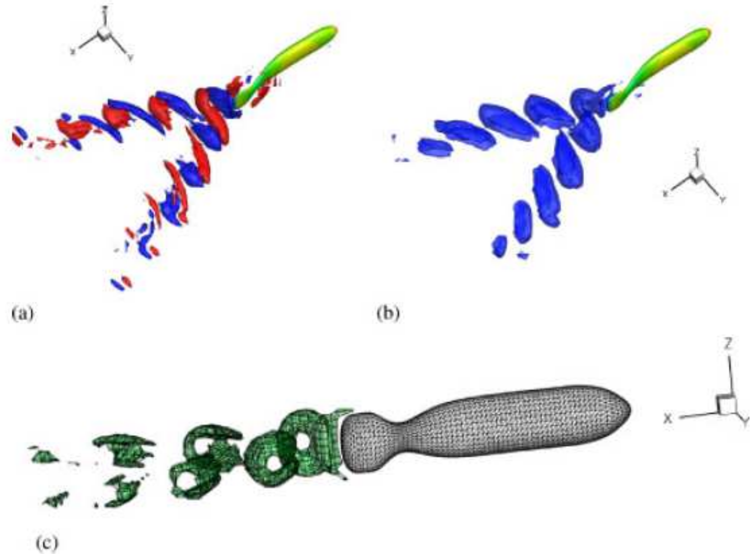







Applications of Fluid Mechanics

- Biofluids: swimming

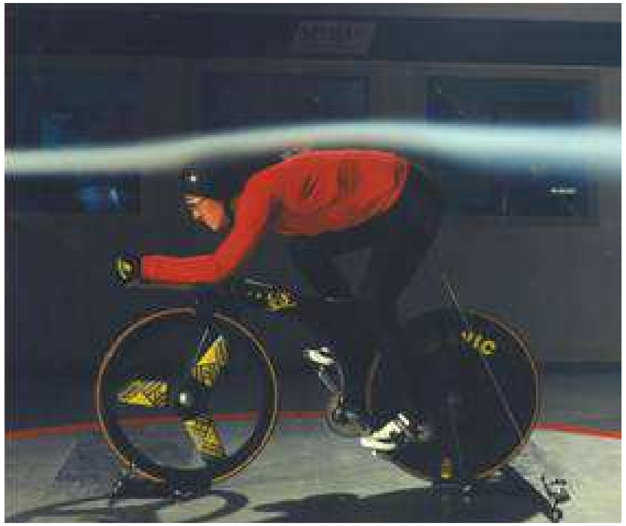


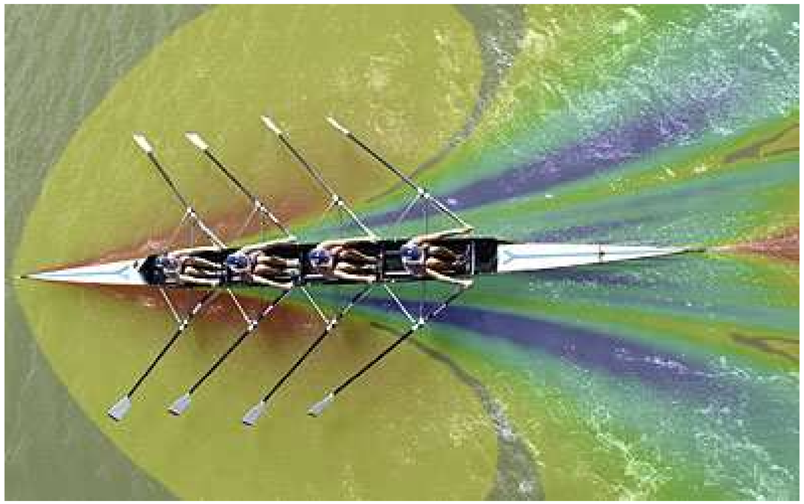


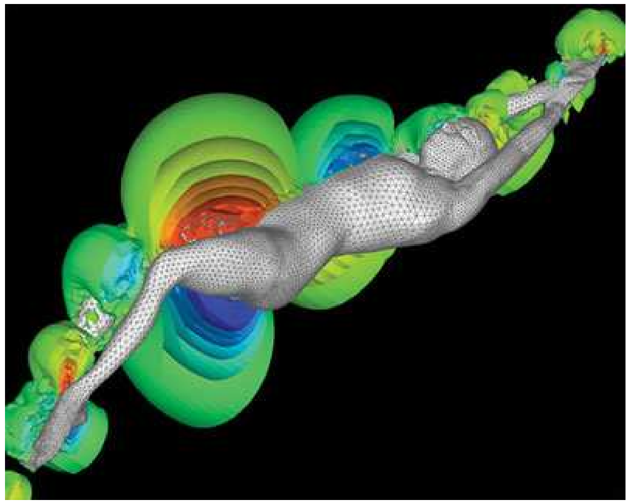


Applications of Fluid Mechanics

- Sports

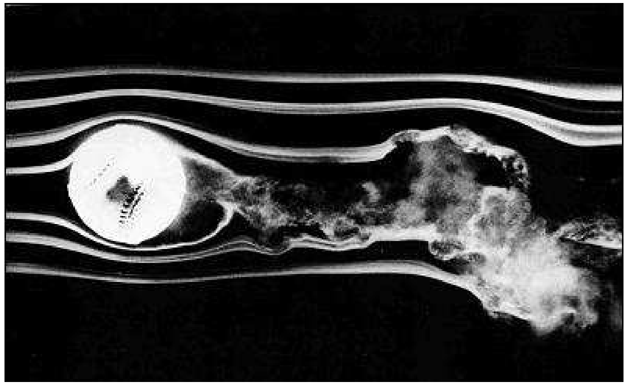




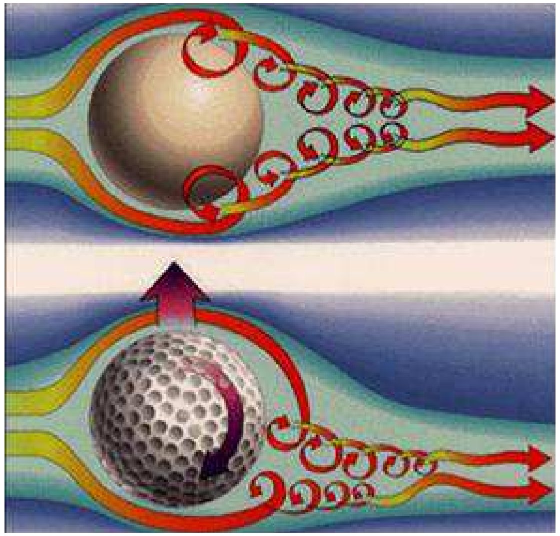


Applications of Fluid Mechanics

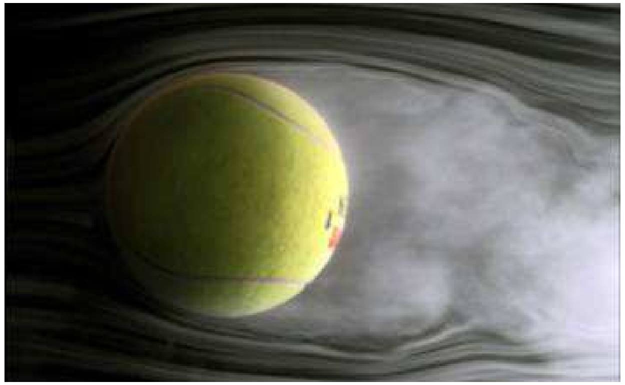
- Sports: balls



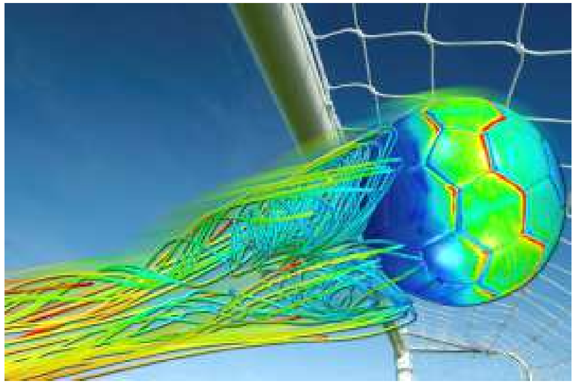
cricket ball



golf ball



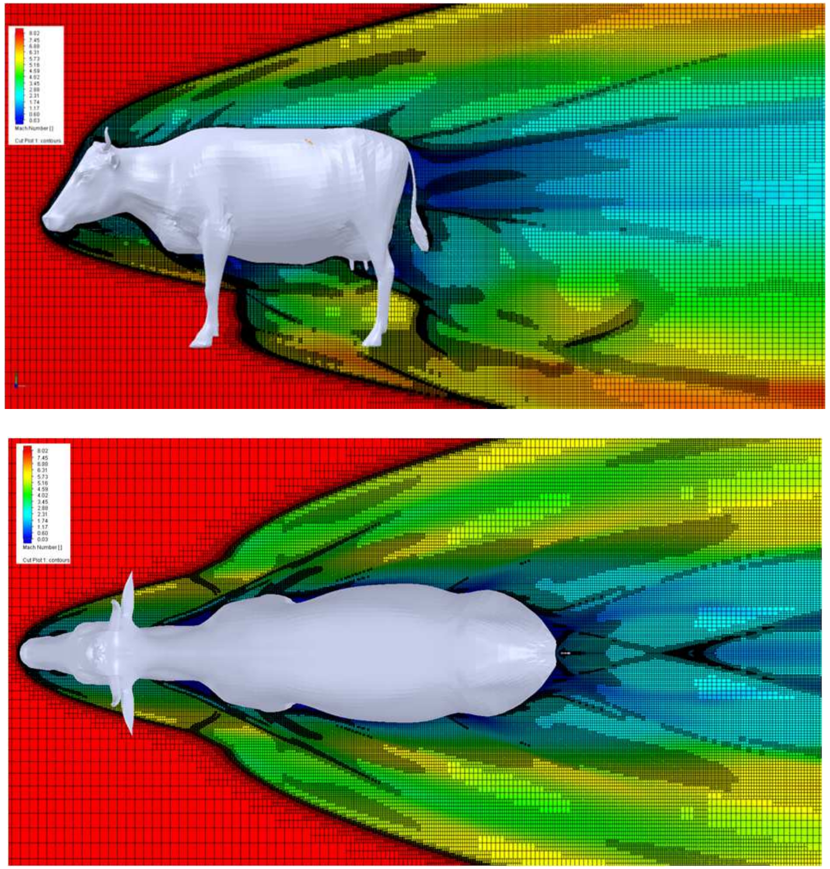
tennis ball



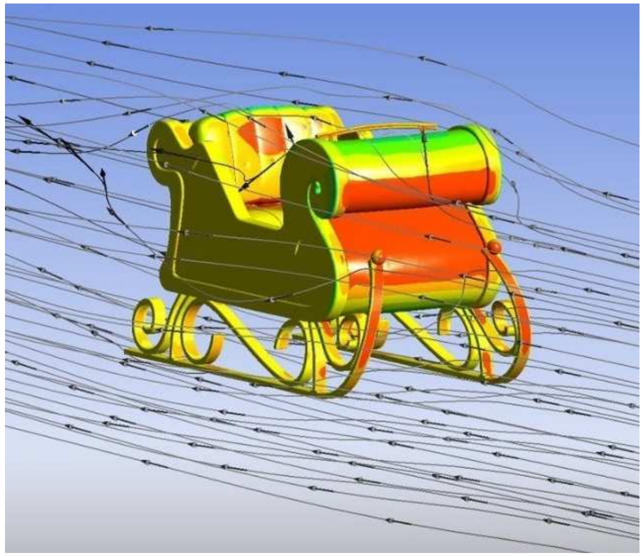
foot ball

Applications of Fluid Mechanics

- Funny applications

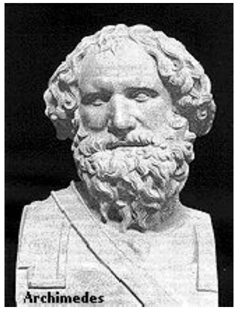


Can a cow fly?



Sled of Santa Claus

History of Fluid Mechanics



Archimedes (287-212 BC)

Newton (1642-1727)

Pitot (1695-1771)

d'Alembert (1717-1783)

Bernoulli (1667-1748)

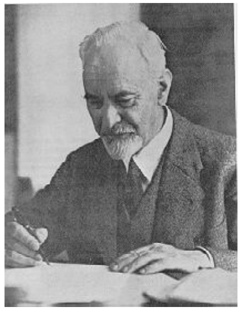
Euler (1707-1783)

Navier (1785-1836)

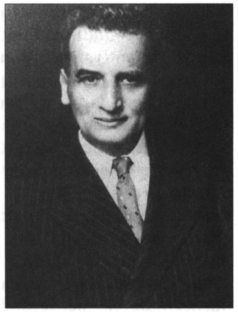
Stokes (1819-1903)

Reynolds (1842-1912)

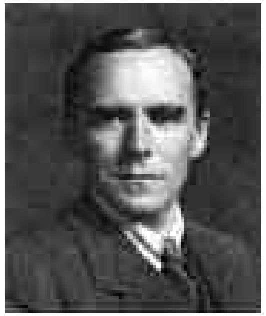
Rayleigh (1842—1919)



Prandtl (1875-1953)



Karman (1881-1963)



Taylor (1886-1975)

Batchelor (1920-2000)

History of Fluid Mechanics

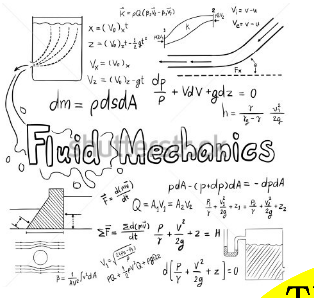
周培源
（1902–1993）

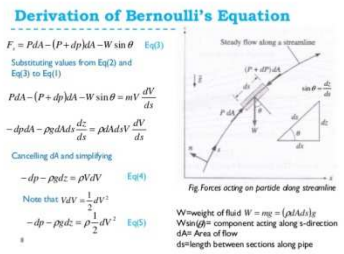
郭永怀
（1909–1968）

钱学森
（1911–2009）

钱伟长
（1912–2010）

Methodology to Study Fluid Mechanics

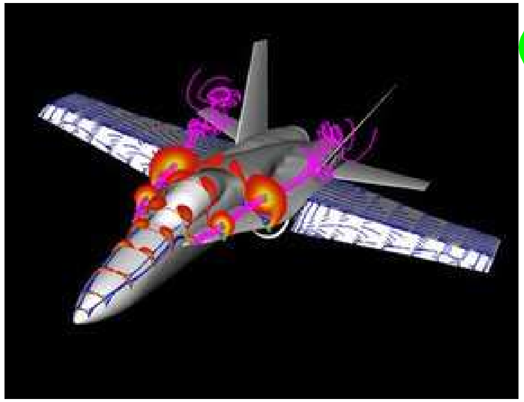


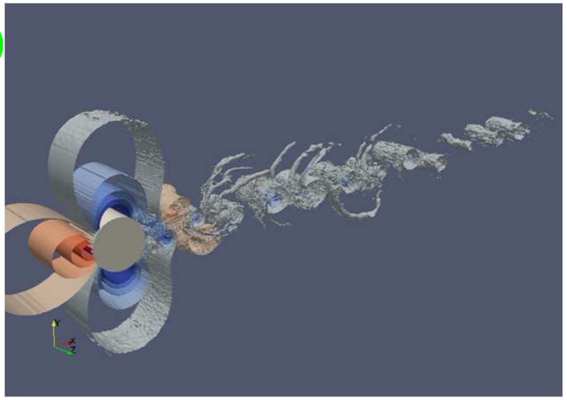


Experiments in Fluid Mechanics

Theoretical Fluid Mechanics

Computational Fluid Dynamics(CFD)





Methodology to Study Fluid Mechanics

- Theoretical Fluid Mechanics
  - Build up the mathematical model based on the physical problem
  - Solve the model using mathematical tools
  - Obtain analytical or approximated solution according

Example:

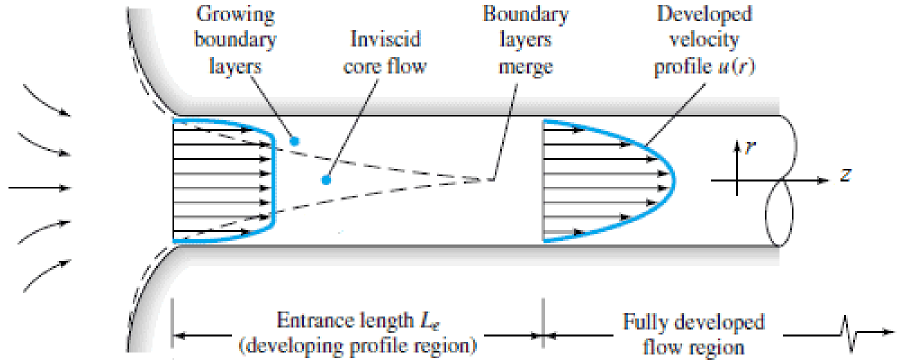
- Laminar pipe flow

Assumption:

- Fully developed laminar flow

Method:

1) Simplify the momentum equation;
2) Apply calculus to solve the simplified equation;
3) Apply boundary condition to determine the integral constant



Velocity Profile of Fluid in a Pipe (laminar Flow)

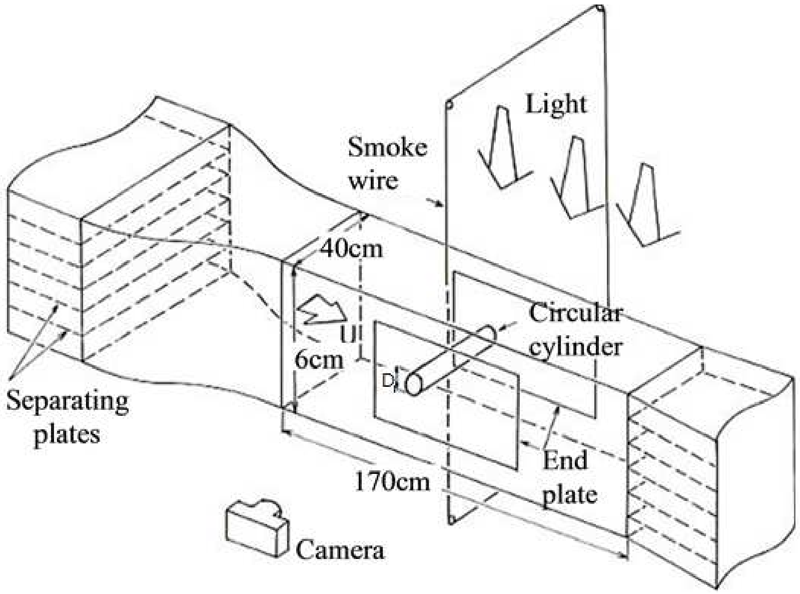
$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_\theta\frac{1}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)=\frac{dp}{dz}$$

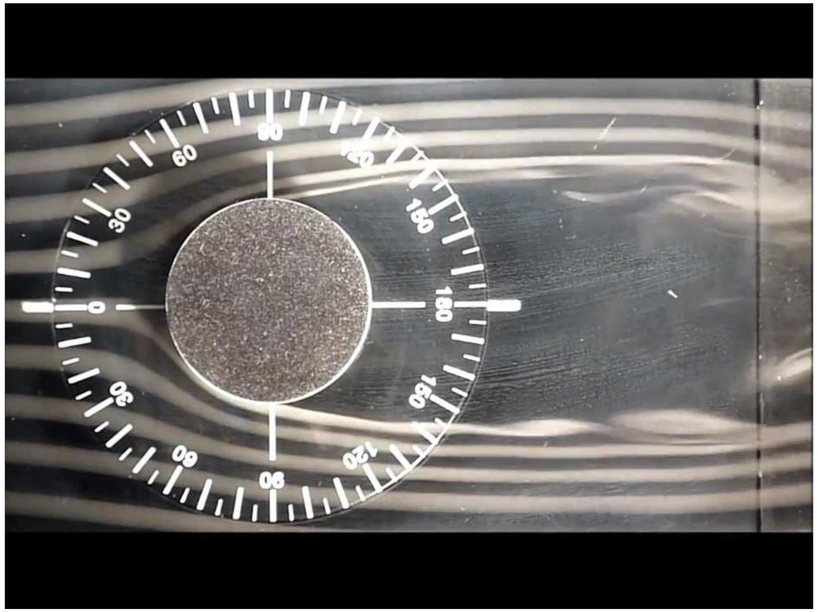
$$v_z=\left(-\frac{dp}{dz}\right)\frac{1}{4\mu}\left(\frac{D^2}{4}-r^2\right)$$

Methodology to Study Fluid Mechanics

- Experimental Fluid Mechanics
  - Experimental setup based on the physical problem
  - Choose proper working fluid such as air, water, oil …
  - Choose proper measurement equipment
  - Data acquisition and analysis



Experimental setup

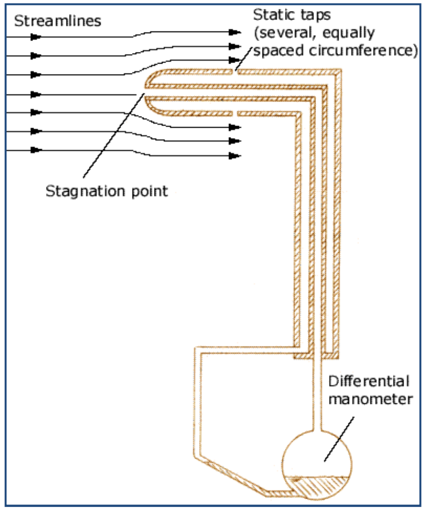


Methodology to Study Fluid Mechanics

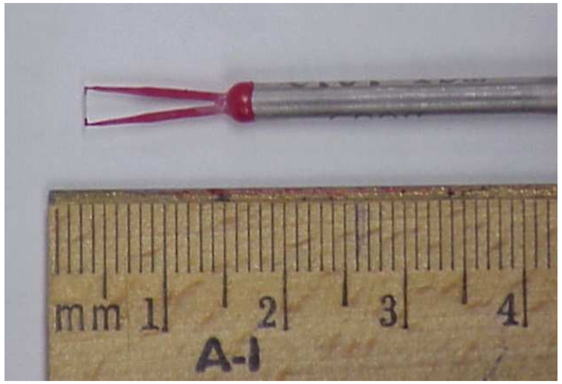
- Experimental facility and equipment

Water channel

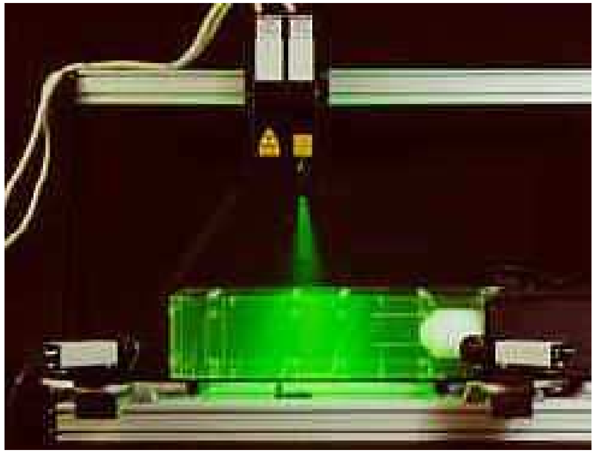
Wind tunnel



Pitot tube



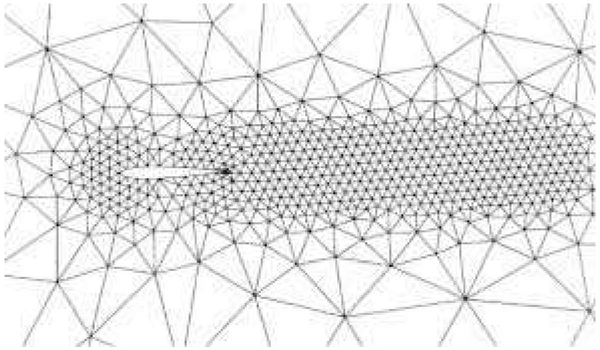
Hot wire



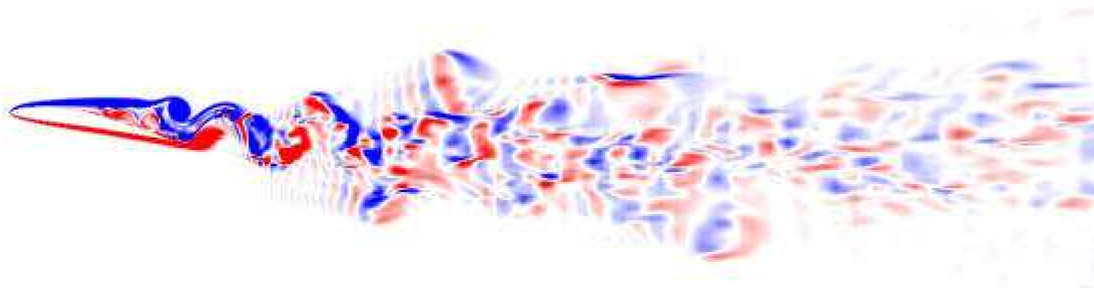
Particle image velocimetry

Methodology to Study Fluid Mechanics

- Computational Fluid Dynamics (CFD)
  - Define computation domain
  - Generate mesh
  - Discretize governing equations (FVM, FEM etc.)
  - Solve algebra equation system (time & memory consuming)
  - Post-processing: visualization



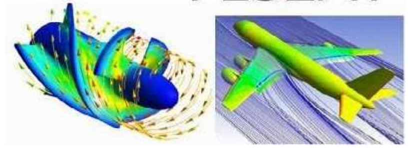
Sample of mesh



Vorticity contours

Methodology to Study Fluid Mechanics

- CFD software
  - FLUENT: http://www.ansys.com/Products/Fluids/ANSYS-Fluent
  - CFDRC: http://www.cfdrc.com
  - STAR-CD: http://www.cd-adapco.com
- Grid Generation software
  - Pointwise: http://www.pointwise.com
  - GridPro: http://www.gridpro.com
- Visualization software
  - Tecplot: http://www.tecplot.com/
  - Fieldview: http://www.ilight.com



Fluid as a Continuum

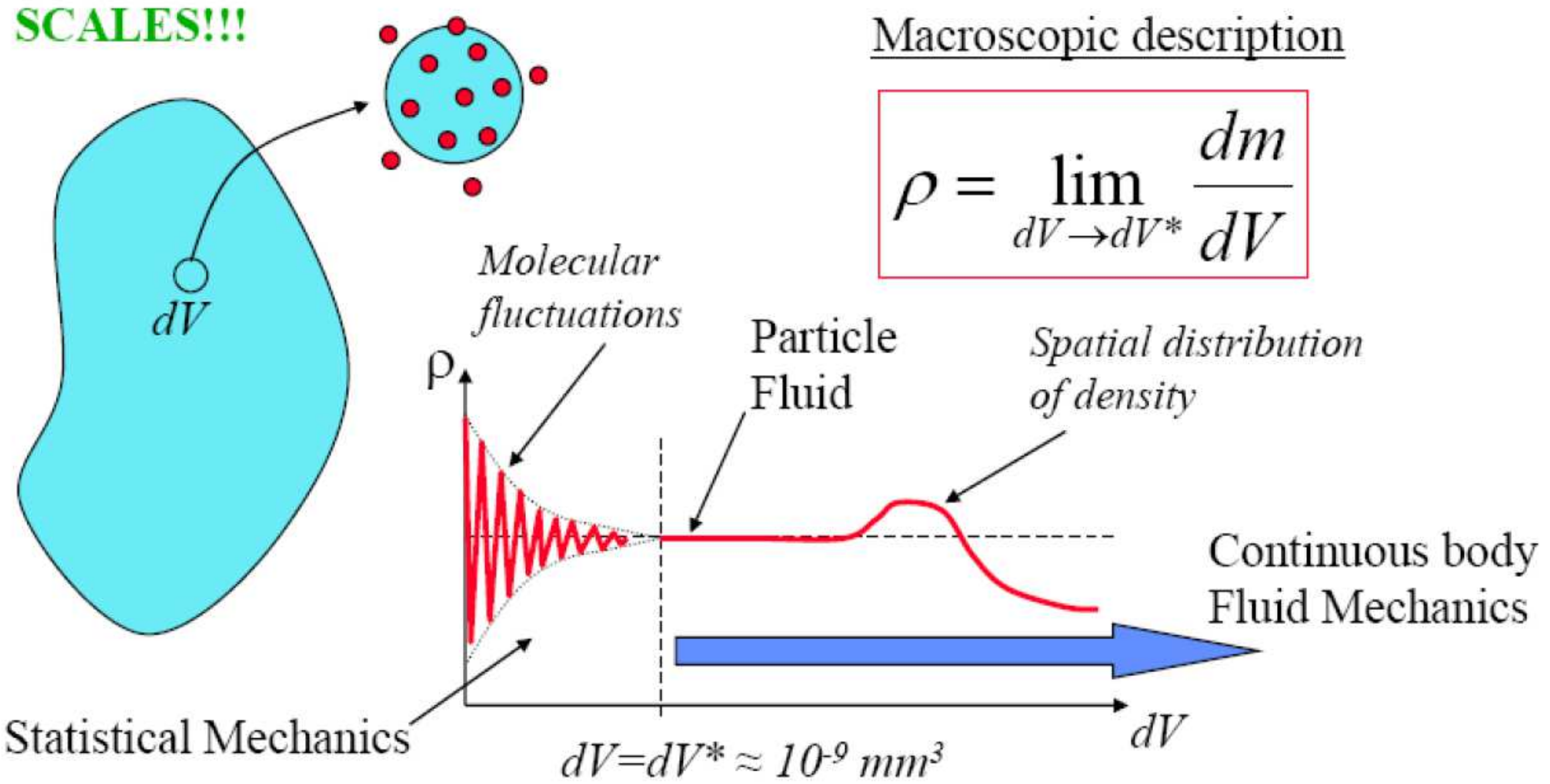
- Continuum Theory
  - In almost all fluid mechanics applications, it is convenient to neglect the molecular nature of the fluid; instead we consider the fluid to be a continuous, homogeneous medium (continuum assumption), capable of infinitely sub-division.
  - Fluid is a continuum, which means that the substance of the object completely fills the space it occupies.

Fluid as a Continuum

- Continuum Theory
  - At each point of the region of the fluid it is possible to construct one volume small enough compared to the region of the fluid and still big enough compared to the molecular mean free path.
  - A fluid volume dV can be shrunk down to infinitesimally small in size, and yet the fluid in this volume still have a definite property, down to a mathematical point (fluid particle).

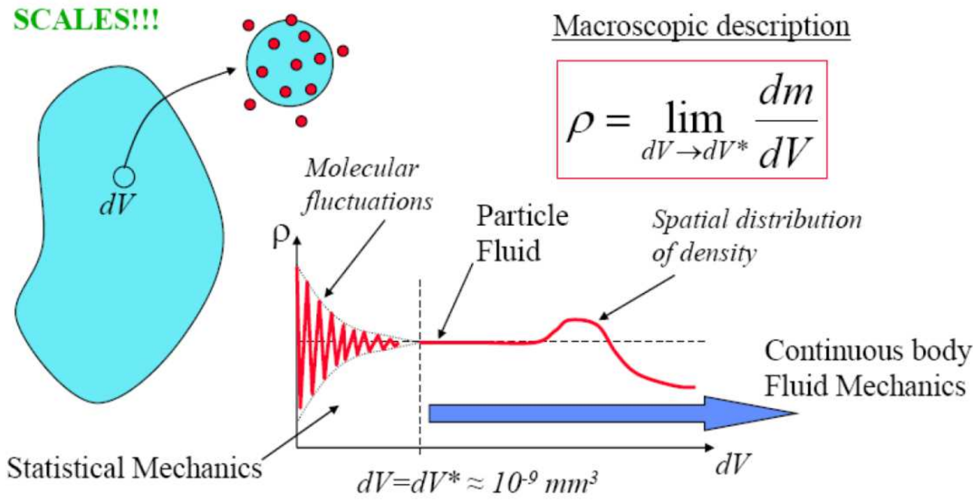
Fluid as a Continuum

- Fluid Particle



Fluid as a Continuum

- Fluid Particle
  - $dV < dV^*$ too few molecules to yield statistically meaningful value for the density ρ
  - dV must be sufficiently large to yield statistically meaningful and reproducible result for ρ and yet small enough to be regarded as a “point”
  - $dV^* \approx 10^{-9}$ mm^3 for all liquids and for all gases at atmospheric pressure (mean free path of typical gases)
  - Density at “point” C thus defined as



Fluid as a Continuum

- Fluid Particle
  - Physical volumes much larger than 10^{-9} mm^3 in most engineering problems → density is essentially a point function → fluid properties assumed to vary continuously throughout fluid → continuum assumption
  - Continuum assumption is valid as long as characteristic length of system is much larger than mean free path of molecules.
  - With continuum assumption, the variations in fluid properties are smooth so that differential calculus can be used.
  - A fluid particle is a collection of a sufficiently large number of fluid molecules such that the continuum assumption is valid, but it is also small enough to be regarded as a "point".

Properties of Fluid

Quantities物理量	SI Unit单位	Dimensions量纲
长度（Length）	米，meter，[m]	L
质量（Mass)	千克，Kilogram，[kg]	M
时间（Time)	秒，Second，[s]	T
温度（Temperature)	开尔文，Kelvin，[K]	θ（温度差）
密度（Density）	$[kg/m^3]$	ML^{-3}
速度（Velocity）	[m/s]	LT^{-1}
力（Force）	牛顿，Newton，$[N]=[kg\cdot m/s^2]$	MLT^{-2}
加速度（Acceleration)	$[m/s^2]$	MT^{-2}
压强或切应力（Pressure、stress）	帕斯卡，Pascal，$[N/m^2]=[Pa]$	$ML^{-1}T^{-2}$
能量（Energy）	焦耳，Joule, $[J]=[N\cdot m]$	ML^2T^{-2}
动力粘性（Dynamic viscosity）	$[N\cdot s/m^2]=[kg/(m\cdot s)]$	$ML^{-1}T^{-1}$
表面张力（Surface tension）	$[N/s]=[kg/s^2]$	MT^{-2}
压缩率（Compressible rate)	$[Pa^{-1}]$	$M^{-1}LT^2$
膨胀系数（expansion coefficient)	$[K^{-1}]$	θ^{-1}

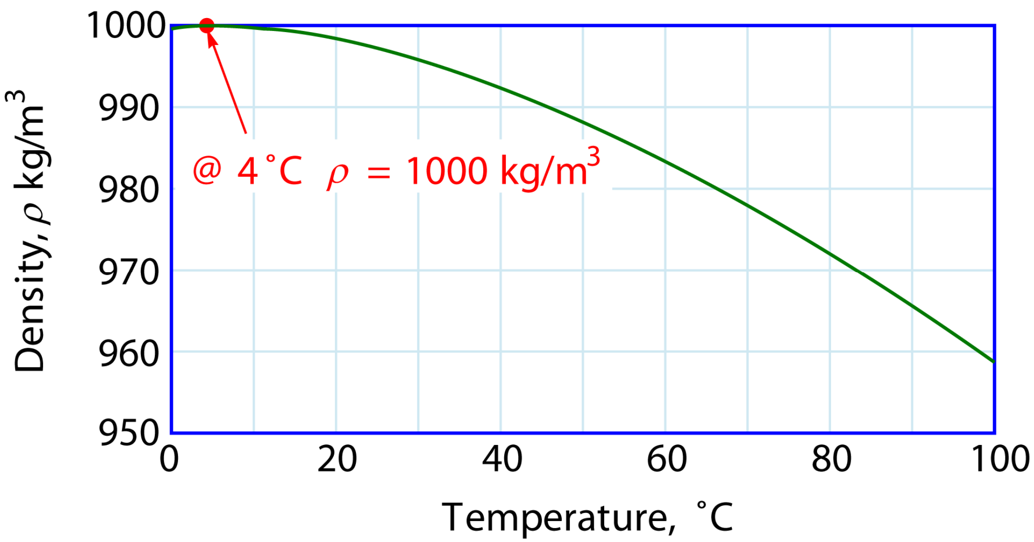
Properties of Fluid

- Density ρ
  - Density is defined as mass per unit volume

$$\rho = dM/dV$$

  - For uniform fluid, $\rho = M/V$
  - For liquid, variations in pressure and temperature generally have only small effect on ρ

Water



Properties of Fluid

- Density ρ
  - Density of some liquids at 1 atm and 20 °C

Liquid	Density ρ, kg/m^3
Ammonia	608
Benzene	881
Carbon tetrachloride	1,590
Ethanol	789
Ethylene glycol	1,117
Freon 12	1,327
Gasoline	680
Glycerin	1,260
Kerosene	804

Liquid	Density ρ, kg/m^3
Mercury	13,550
Methanol	791
SAE 10W oil	870
SAE 10W30 oil	876
SAE 30W oil	891
SAE 50W oil	902
Water	998
Seawater (30%)	1,025

Properties of Fluid

- Density ρ
  - For gas, density is strongly influenced by both pressure and temperature. Ideal-gas law or equation of state (constitutive equation): $P = \rho RT$
  - Density of some gases at 1 atm and 20 °C

Gas	Density ρ, kg/m^3
H_2	0.084
He	0.166
H_2O	0.749
Ar	1.662
Dry air	1.203
CO_2	1.825
CO	1.162

Gas	Density ρ, kg/m^3
N_2	1.162
O_2	1.336
NO	1.234
N_2O	1.825
Cl_2	2.947
CH_4	0.667

Properties of Fluid

- Specific weight
  - defined as $\gamma = \rho g$, where g is the acceleration due to gravity. γ has units Kg $m^{-2}s^{-1}$ which is (of course) Nm^{-3}.

- Specific gravity
  - defined as $SG = \rho/\rho_{ref}$, where ρ_{ref} is the density of water at 20^0C. Sometimes the reference density used is that of water at 4^0C.
  - SG is not used much because of the risk of using the wrong reference density.

Properties of Fluid

- Compressibility of Fluid
  - Compressibility is a measure of the relative volume change of a fluid as a response to a pressure change.
  - Isothermal compressibility (constant temperature *T*)

$$\beta_T = \lim_{\Delta p \to 0}\left(-\frac{\delta V / V}{\delta p}\right)_T = -\frac{1}{V}\left(\frac{dV}{dp}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T \qquad (1/\mathrm{Pa})$$

  - Isentropic compressibility (constant entropy *S*)

$$\beta_S = \lim_{\Delta p \to 0}\left(-\frac{\delta V / V}{\delta p}\right)_S = -\frac{1}{V}\left(\frac{dV}{dp}\right)_S = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_S \qquad (1/\mathrm{Pa})$$

  - Bulk modulus (the inverse of the compressibility)

$$K = \frac{1}{\beta_T} \qquad (\mathrm{Pa})$$

Properties of Fluid

- Coefficient of Thermal Expansion
  - Thermal expansion is the tendency of fluid to change in volume in response to a change in temperature, through heat transfer.
  - The degree of expansion divided by the change in temperature is called the material's coefficient of thermal expansion and generally varies with temperature

$$\alpha_V = \lim_{\Delta p \to 0} \left(-\frac{\delta V / V}{\delta T} \right)_P = -\frac{1}{V} \left(\frac{dV}{dT} \right)_P = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P \quad (1/K)$$

Properties of Fluid

- Compressible and Incompressible Fluids
  - Incompressible flow refers to a flow in which the material density is constant within a fluid parcel.
  - An equivalent statement that implies incompressibility is that the divergence of the flow velocity is zero.
  - Compressible flow (gas dynamics) is the branch of fluid mechanics that deals with flows having significant changes in fluid density.
  - the Mach number (the ratio of the speed of the flow to the speed of sound) is often applied to distinguish between compressible and incompressible flow

  - Incompressible flow: liquid flow and low speed gas flow
  - Compressible flow: high-speed aircraft, jet engines, rocket motors, high-speed entry into a planetary atmosphere, gas pipelines etc.

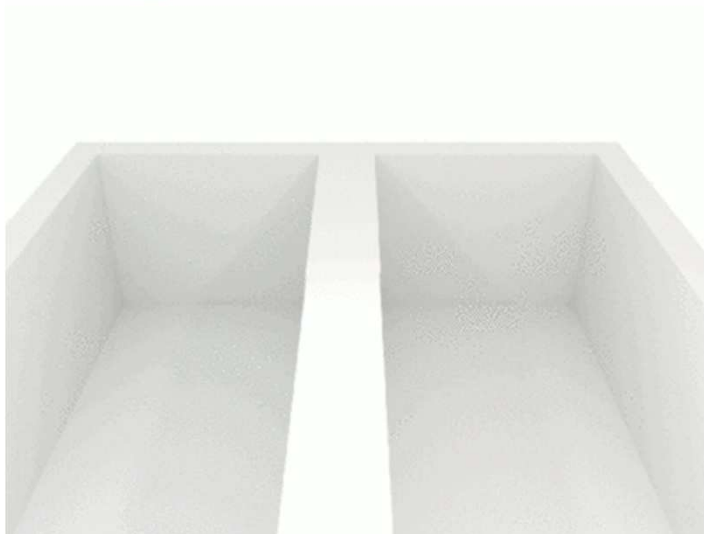
Properties of Fluid

- Viscosity μ
  - a measure of the amount of friction developed inside a fluid when it is in motion.
  - a property arising from collisions between neighboring particles in a fluid that are moving at different velocities.
  - typically the larger the intermolecular forces, the higher the viscosity

Substance	Formula	Viscosity (kg/m-s)
Hexane	$CH_3CH_2CH_2CH_2CH_2CH_3$	$3.26 * 10^{-4}$
Heptane	$CH_3CH_2CH_2CH_2CH_2CH_2CH_3$	$4.09 * 10^{-4}$
Octane	$CH_3CH_2CH_2CH_2CH_2CH_2CH_2CH_3$	$5.42 * 10^{-4}$
Nonane	$CH_3CH_2CH_2CH_2CH_2CH_2CH_2CH_2CH_3$	$7.11 * 10^{-4}$
Decane	$CH_3CH_2CH_2CH_2CH_2CH_2CH_2CH_2CH_2CH_3$	$1.42 * 10^{-3}$

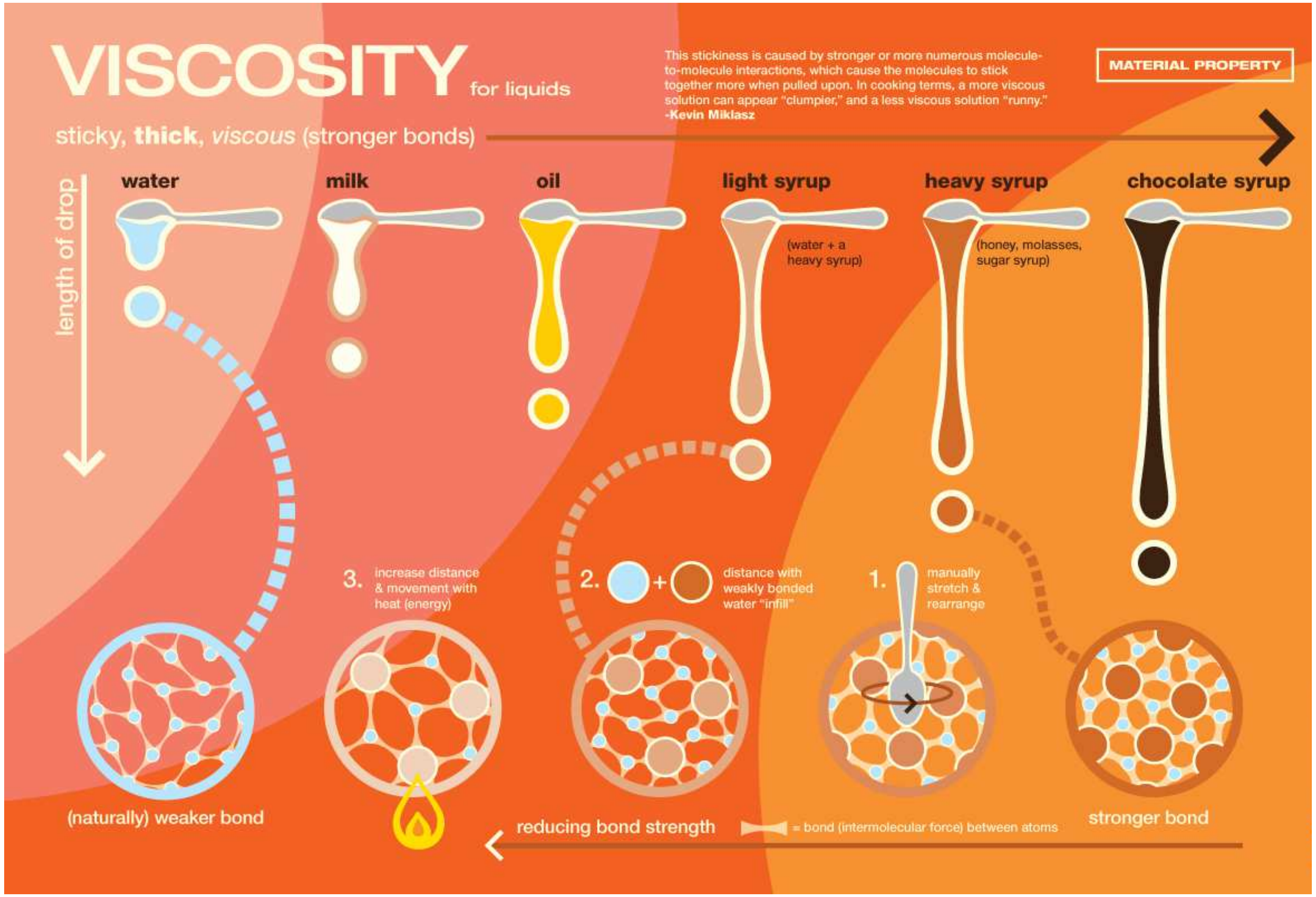
Properties of Fluid

- Viscosity μ



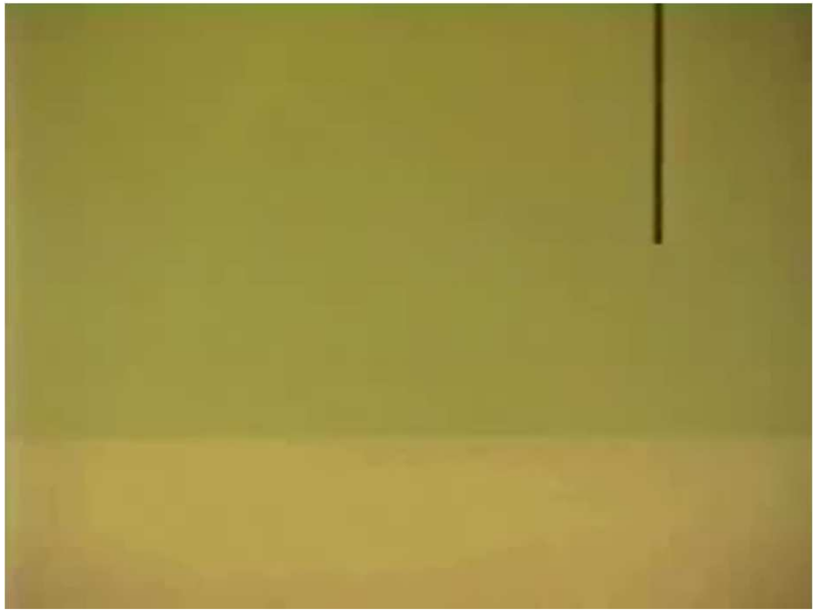
Properties of Fluid

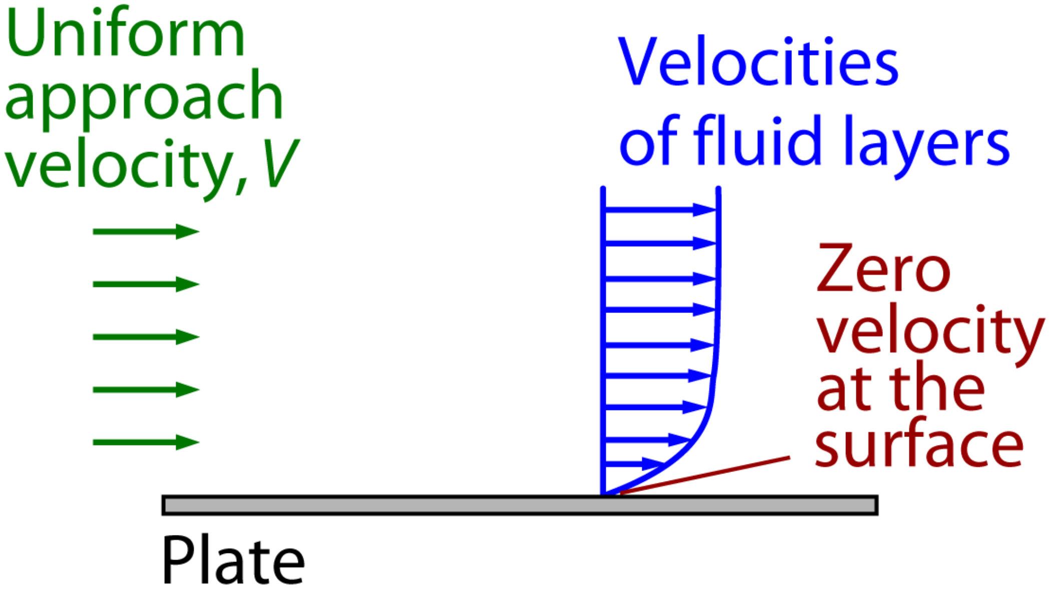
- Viscosity μ



Properties of Fluid

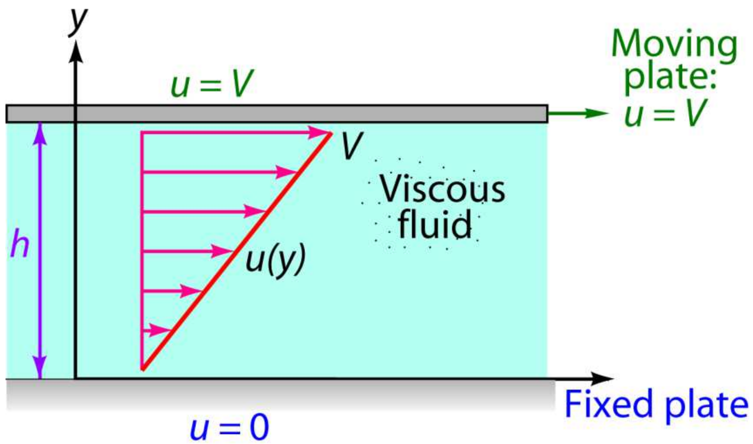
- Viscosity μ
  - No-slip condition.
    - ✓ For a real, viscous fluid, the velocity of the fluid at the wall is zero relative to the wall.
    - ✓ Fluid in direct contact with solid boundary has same velocity as boundary.





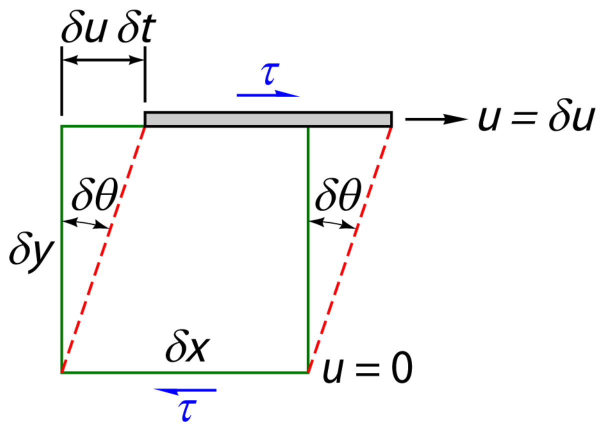
Properties of Fluid

- Viscosity μ
  - Simple shear flow between two parallel plates
    - ✓ No-slip conditions on bottom and top walls:
    - ✓ $u = 0$ at $y = 0$ (bottom stationary wall)
    - ✓ $u = V$ at $y = h$ (top moving wall)
    - ✓ Velocity u varies in y-direction, $u = u(y)$
    - ✓ Velocity distribution also known as velocity profile
    - ✓ Velocity gradient or strain rate du/dy



Properties of Fluid

- Viscosity μ
  - To relate strain rate $\dot{\gamma}$ of fluid particle to velocity gradient du/dy
    - ✓ Consider two layers of fluid, acting on by a shear stress τ, a distance δy apart, with the upper layer moving at velocity δu relative to the lower layer:



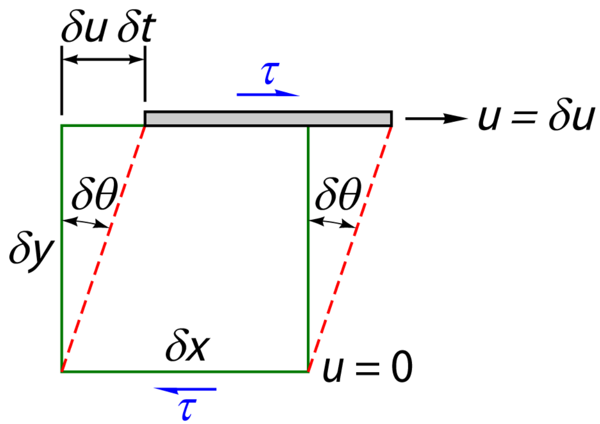
    - ✓ In time δt, fluid element deforms by infinitesimal angle $\delta\theta$

Properties of Fluid

- Viscosity μ
  - To relate strain rate $\dot{\gamma}$ of fluid particle to velocity gradient du/dy
    - ✓ From geometry, for small angle $\delta\theta$ in the limit of small δy:

$$\delta\theta \approx \tan\delta\theta = \frac{\delta u \delta t}{\delta y}$$

$$\frac{\delta\theta}{\delta t} = \frac{\delta u}{\delta y}$$



- ✓ Rate of deformation or shear strain rate:

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\theta}{\delta t}$$

$$\dot{\gamma} = \frac{d\theta}{dt} = \frac{du}{dy}$$

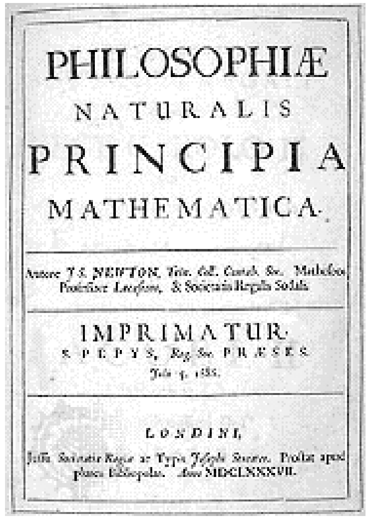
Properties of Fluid

- Viscosity μ
  - Newton's law of viscosity
    - ✓ In a fluid, stress is proportional to strain rate.
    - ✓ The constant μ is called the dynamic viscosity or viscosity of the fluid.

$$\tau = \mu \frac{du}{dy}$$

    - ✓ SI units for μ: N·s/m^2, kg/(m·s) or Pa·s.
    - ✓ Examples of Newtonian fluids: water, oil, gasoline, glycerin, air。
    - ✓ Constant viscosity is a consequence of Newtonian assumption, but it is not a sufficient condition。

Sir Isaac Newton
1642-1727



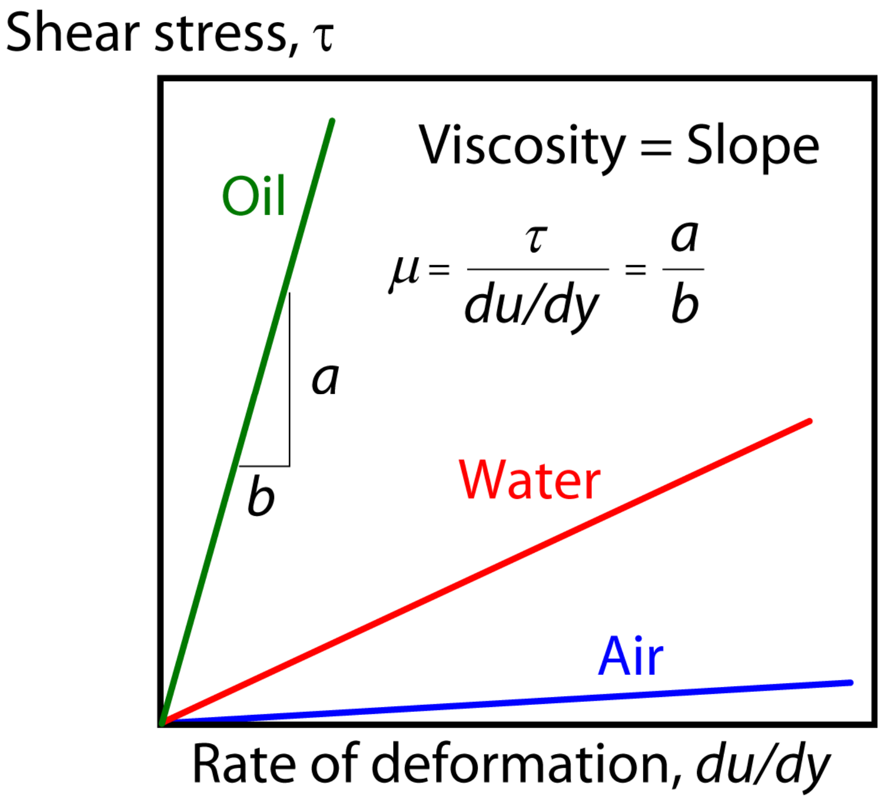
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

IMPRIMATUR

LONDINI,

Properties of Fluid

- Viscosity μ
  - Viscosity of different fluids



$$\mu_{oil} > \mu_{water} > \mu_{air}$$

Properties of Fluid

- Viscosity μ
  - Viscosity of different fluids at 1 atm and 20 °C

Fluid	Dynamic viscosity μ, kg/(m • s)
Glycerin:	
-20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015

Fluid	Dynamic viscosity μ, kg/(m • s)
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.0040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

Note 1 kg/(m·s) = 1kg·s·m/(m^{2}s^2) = 1 N·s/m^2= 1 Pa·s

Properties of Fluid

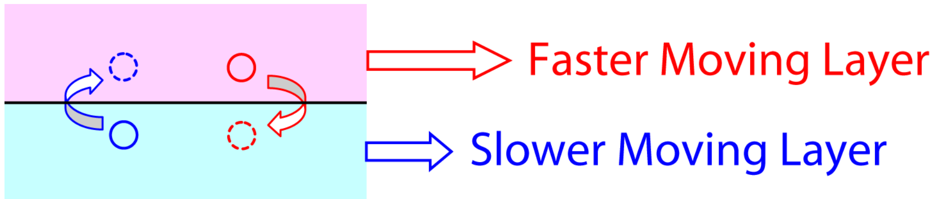
- Viscosity μ
  - Liquids
    - ✓ Resistance to relative motion between adjacent layers of liquid caused by cohesive forces (intermolecular forces of attraction) between liquid molecules.
    - ✓ As temperature ↑ cohesive forces ↓ liquid viscosity ↓
    - ✓ Liquid molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. Therefore, the energized liquid molecules move more freely
    - ✓ Can be approximated using Andrade's equation:

$$\eta = ae^{b/T}$$

where T is the absolute temperature and a and b are experimentally determined constants (eg. a = 0.0016 kg/(m·s), b = 1903 K for water)

Properties of Fluid

- Viscosity μ
  - Gases
    - Resistance to relative motion between adjacent layers of gas arises due to exchange of momentum of gas molecules (negligible intermolecular forces of attraction)
    - Random motion of gas molecules:



- ➢ Red molecule from faster layer joins slower layer $\Rightarrow$ loses momentum to slower layer
- ➢ Blue molecule from slower layer joins faster layer $\Rightarrow$ gains momentum from faster layer
- ➢ Exchange or transfer of momentum between layers with different bulk velocities

Properties of Fluid

- Viscosity μ
  - Gases
    - Gas viscosity ↑ as temperature ↑

      ⇨ random motion of gas molecules enhanced at high temperatures
      ⇨ more molecular collisions
      ⇨ more exchange or transfer of momentum
      ⇨ greater resistance to flow

    - Maybe correlated using Sutherland's Law:

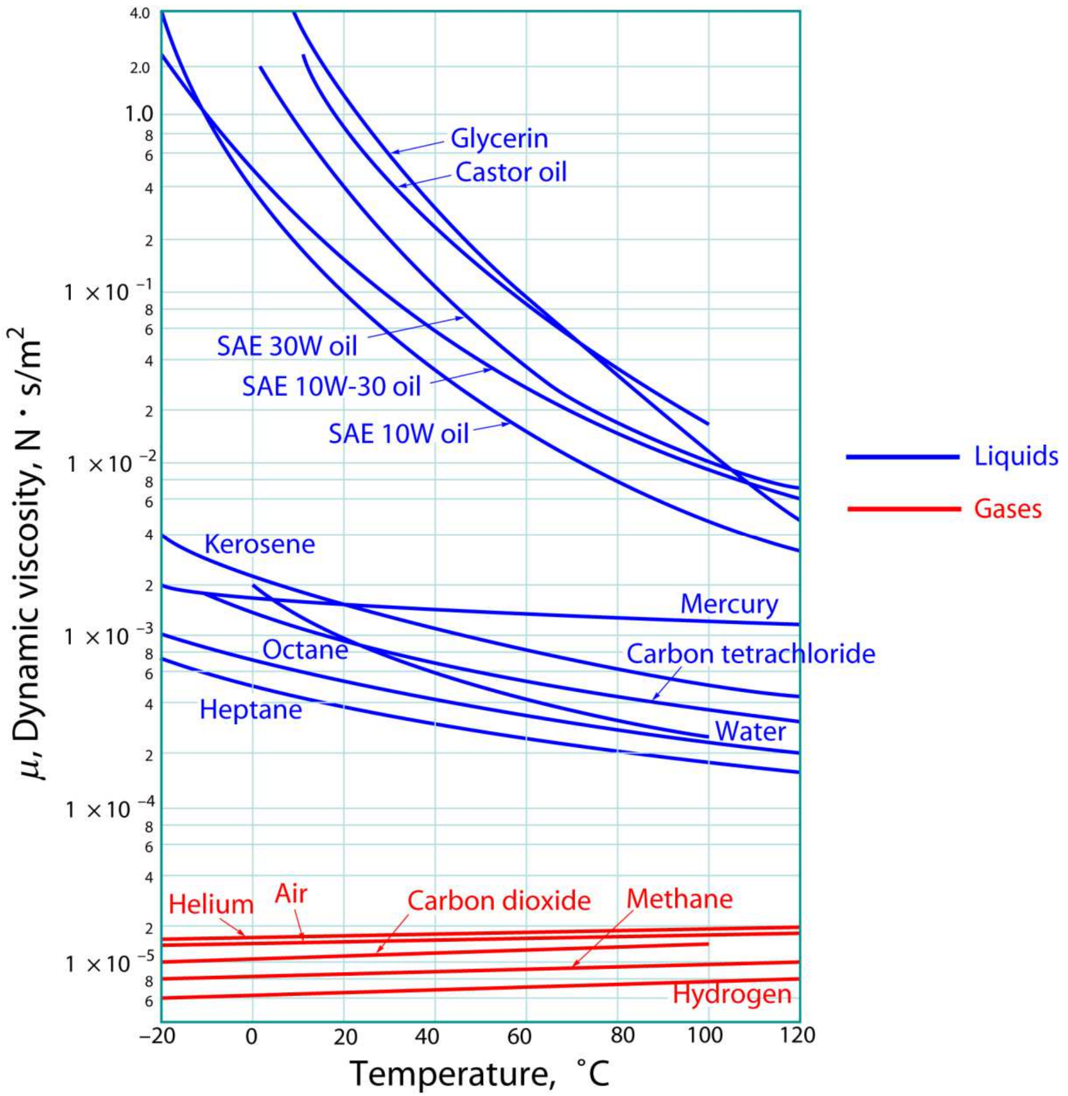
$$\mu = \frac{AT^{1/2}}{1 + B/T}$$

where T is the absolute temperature and A and B are experimentally determined constants (eg. $A = 1.458 \times 10^{-6}$ kg/(m·s·K$^{1/2}$) and $B = 110.4$ K for air)

Properties of Fluid

- Viscosity μ
  - Temperature effect

Temperature ↑
liquid viscosity ↓
gas viscosity ↑



Properties of Fluid

- Dynamic Viscosity μ and Kinematic Viscosity v
  - μ is viscosity or dynamic viscosity
  - Kinematic viscosity v is the ratio of the dynamic viscosity to density ρ

$$v = \frac{\mu}{\rho}$$

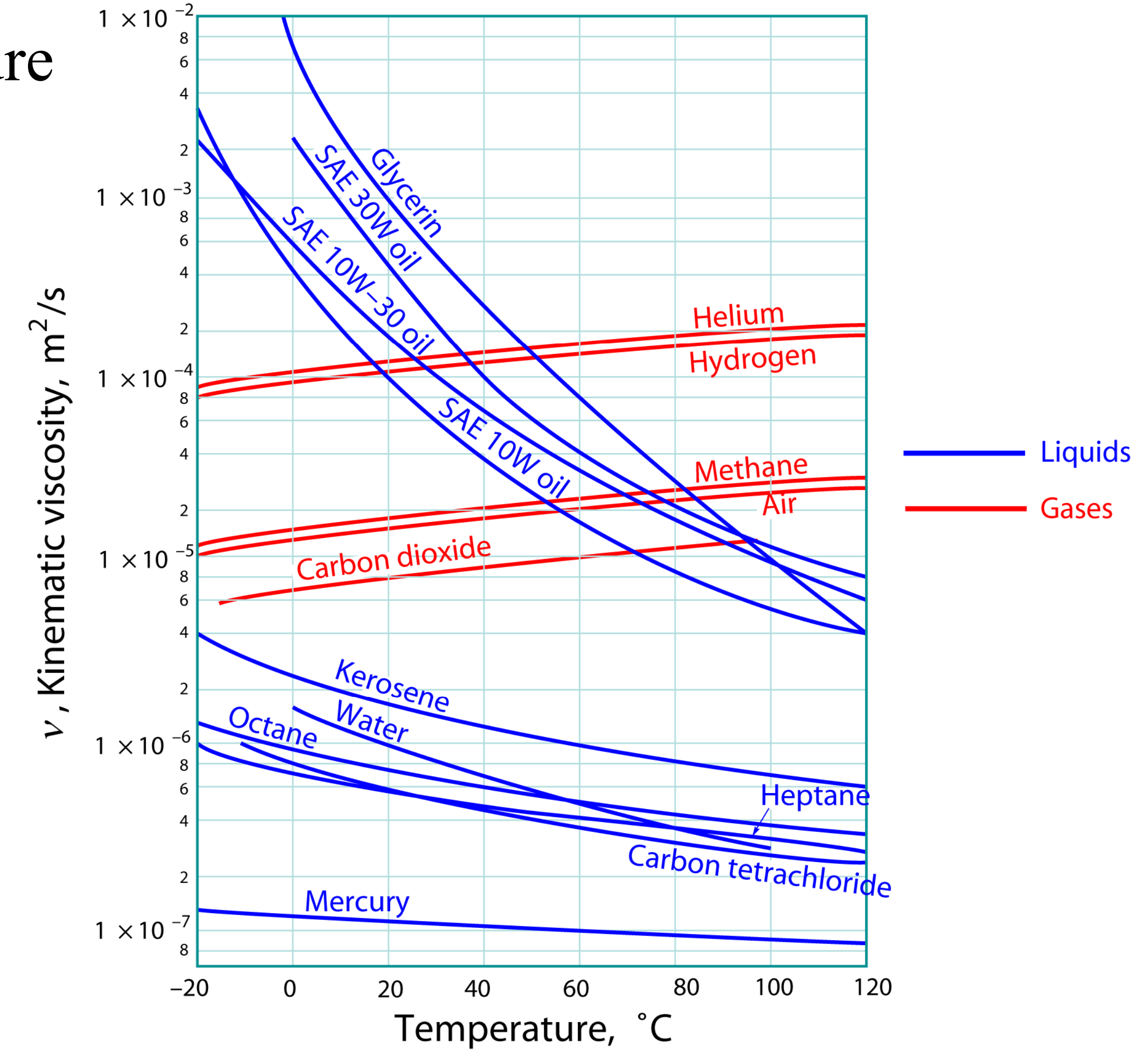
  - SI Units for v : kg/(s·m) (m^3/kg) = m^2/s

μ : Greek symbol "mu"
v : Greek symbol "nu"

Properties of Fluid

- Kinematic Viscosity ν
  - Effect of Temperature

Temperature ↑
    liquid kinematic viscosity ↓
    gas kinematic viscosity ↑



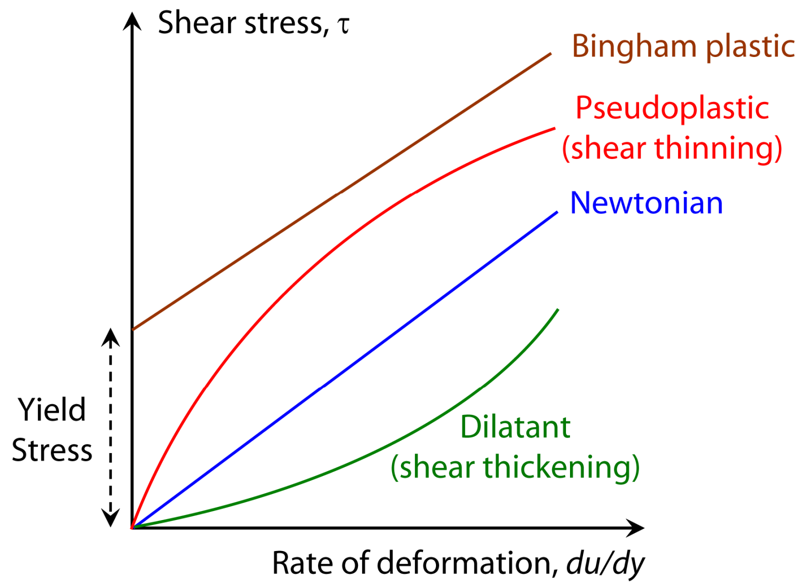
Note

$$\mu_{\text{water}} > \mu_{\text{air}}$$

$$\nu_{\text{water}} < \nu_{\text{air}}$$

Properties of Fluid

- Viscosity
  - Non-Newtonian fluids
    - ✓ Stress is proportional to strain rate and viscosity μ is constant.
  - Non-Newtonian fluids
    - ✓ Fluids not obeying Newtonian law.
    - ✓ Non-Newtonian is much more than viscosity behaviour!



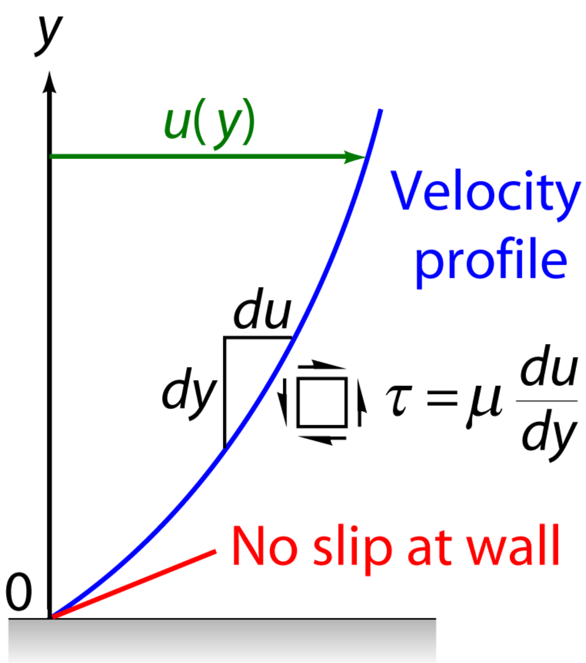
Properties of Fluid

- Non-Newtonian Fluids
  - Apparent viscosity μ_{app}
    - ✓ Local slope of the shear stress τ vs. shear rate du/dy graph
  - Newtonian fluids $\mu_{app} = \mu =$ constant
  - Dilatant or shear thickening
    - ✓ $\mu_{app} \uparrow$ as shear rate $\uparrow$
    - ✓ Example: corn starch solutions.
  - Pseudoplastic or shear shining
    - ✓ $\mu_{app} \downarrow$ as shear rate $\uparrow$
    - ✓ Example: paints, polymer solution.
  - Bingham plastic
    - ✓ shear stress must reach a certain minimum yield stress before flow commences
    - ✓ Example: toothpaste, mayonnaise, ketchup.

Properties of Fluid

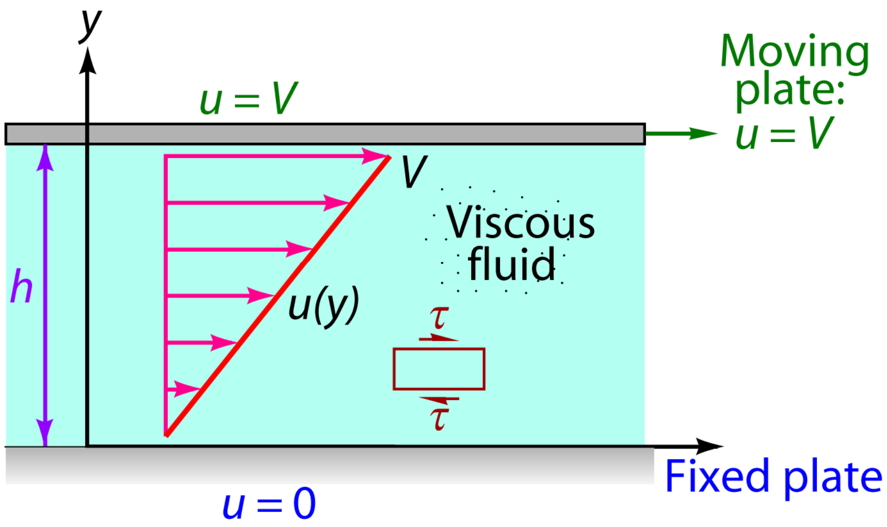
- Shear Stress for Newtonian Fluids
  - If the local velocity gradient *du/dy* is known, the shear stress can be determined using

$$\tau = \mu \frac{du}{dy}$$



Properties of Fluid

- Shear Flow Between Parallel Plates
  - Fluid acceleration is zero (not necessary to assume this)
  - Pressure uniform everywhere
  - Force balance on small fluid element indicates that shear stress τ is constant throughout fluid:



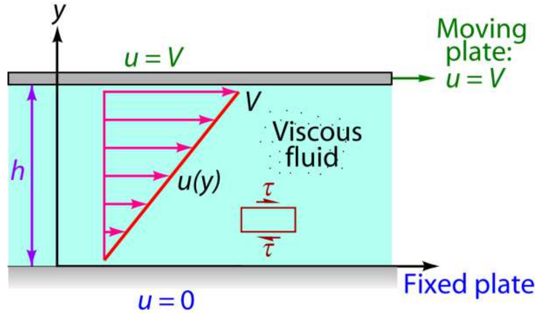
$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const} = B$$

$$u = A + By$$

Properties of Fluid

- Shear Flow Between Parallel Plates
  - No-slip conditions on the top of bottom walls
    - ✓ $u = 0$ at $y = 0$
    - ✓ $u = V$ at $y = h$

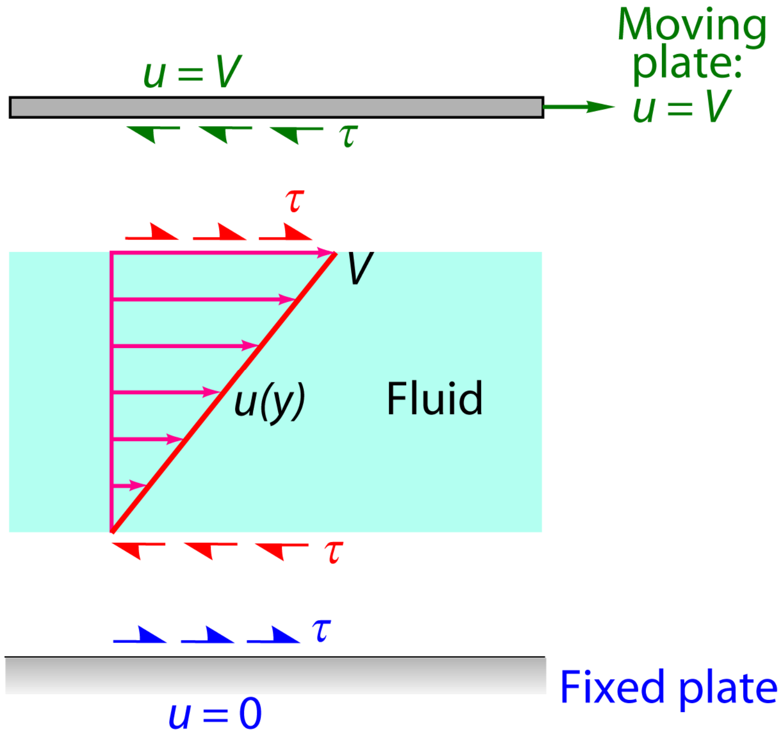
    $\Rightarrow$ $A = 0$ and $B = Vh$



  - Linear velocity profile: $u = V\dfrac{y}{h}$
  - Shear stress: $\tau = \mu\dfrac{du}{dy} = V\dfrac{y}{h}$
  - Top plate experiences shear force to the left (i.e. a resistance force), since it is doing work trying to drag the fluid along with it to the right
  - Fluid at the top experiences equal and opposite shear force to the right
  - Bottom plate experiences shear force to the right, since fluid is trying to pull bottom plate with it to the right

Properties of Fluid

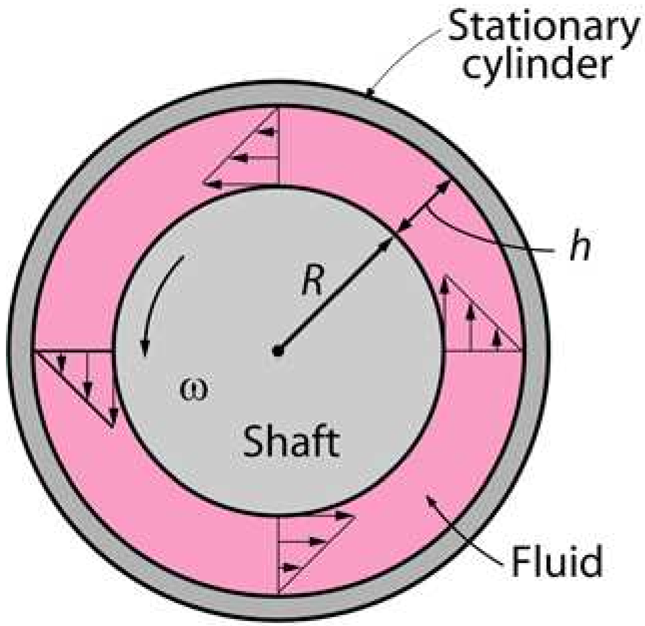
- Shear Flow Between Parallel Plates
  - Fluid at bottom experiences equal and opposite shear force to the left
  - Note: flow is also called a simple shear, or Couette flow



Maurice Couette
1858-1943

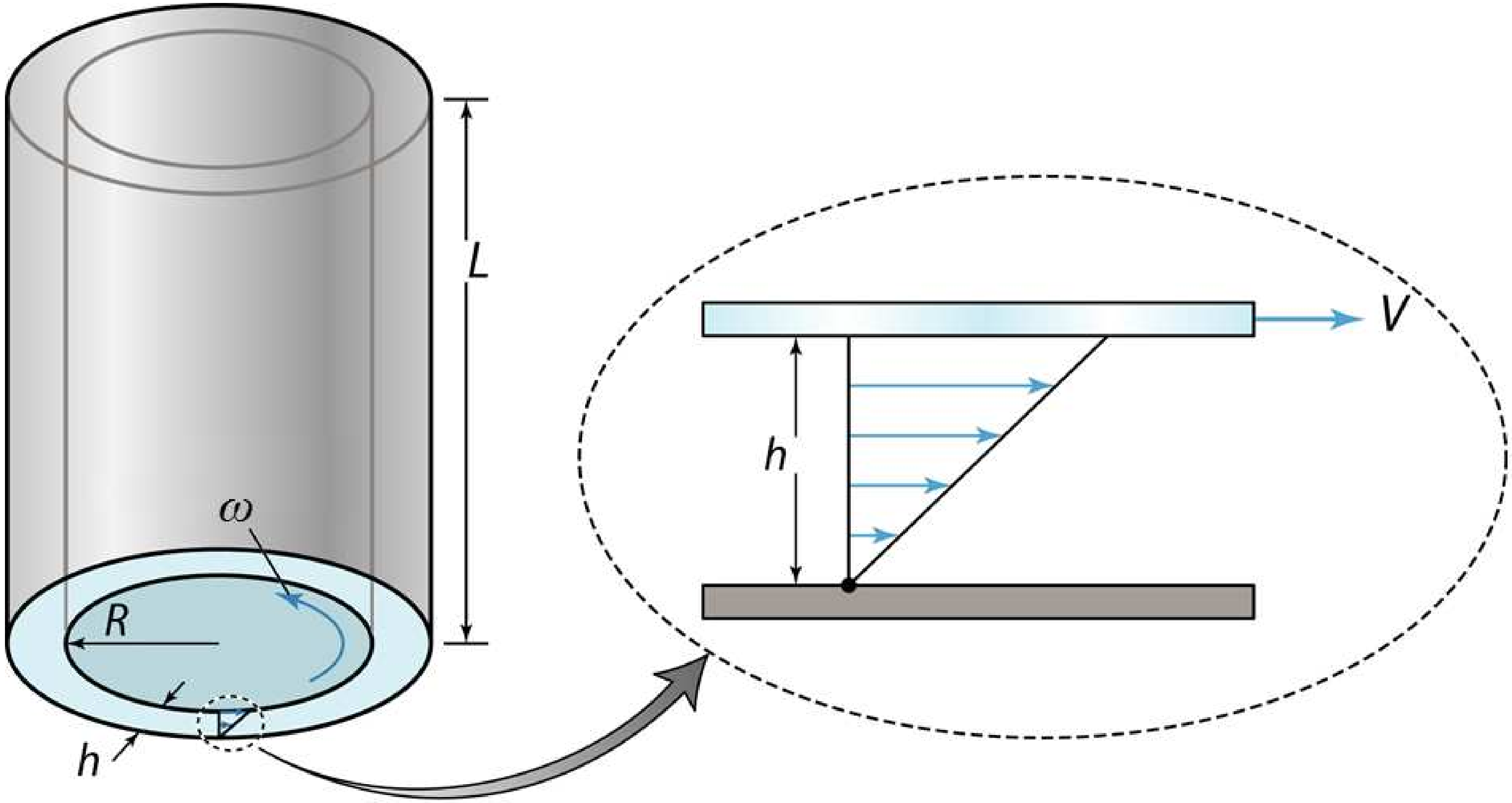
Properties of Fluid

- Couette Flow between two Concentric Cylinders
  - Two concentric cylinders with length L separated by small gap h
  - Fluid is filled in the gap
  - Rotating inner cylinder of radius R; rotating speed is ω
  - Fixed outer cylinder of radius $R+h$



Properties of Fluid

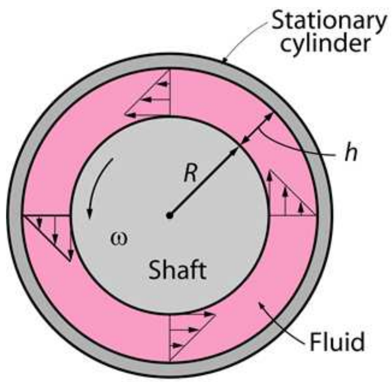
- Couette Flow between two Concentric Cylinders
  - If $h/R << 1$, then cylinders can be modelled as flat plates and velocity profile across gap can be assumed linear



Properties of Fluid

- Couette Flow between two Concentric Cylinders
  - $r = R$, $u = \omega R$; $r = R+h$, $u = 0$; thus, velocity profile is

$$u = \omega R \frac{R - r}{h} + \omega R$$



  - Wall shear stress of inner cylinder

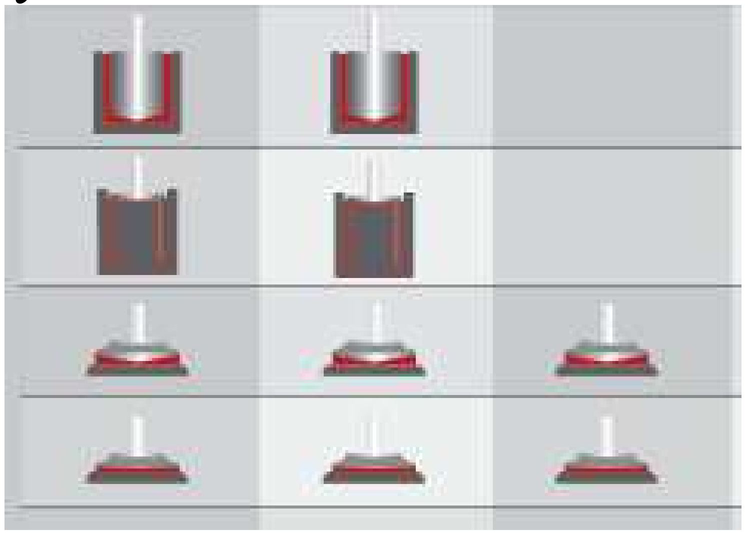
$$\tau = \mu \frac{\omega R}{h}$$

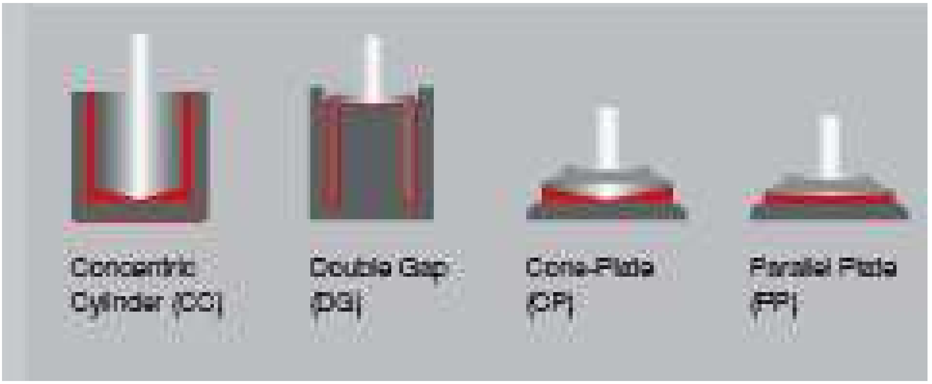
  - Wetted surface area of inner cylinder is $A = 2\pi RL$
  - Shear force F and torque T are:

$$F = \tau A = 2\pi\mu \frac{\omega R^2 L}{h} \qquad T = FR = 2\pi\mu \frac{\omega R^3 L}{h}$$

Properties of Fluid

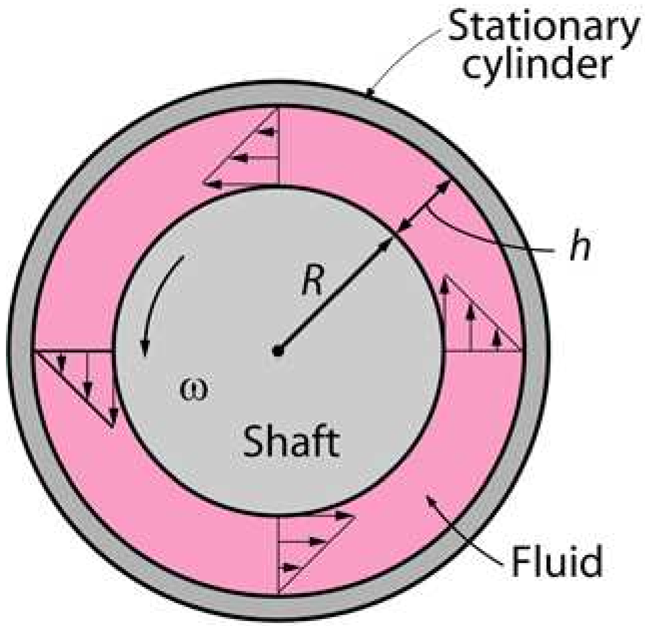
- Applications: Viscometers
  - Instruments to measure viscosity and more





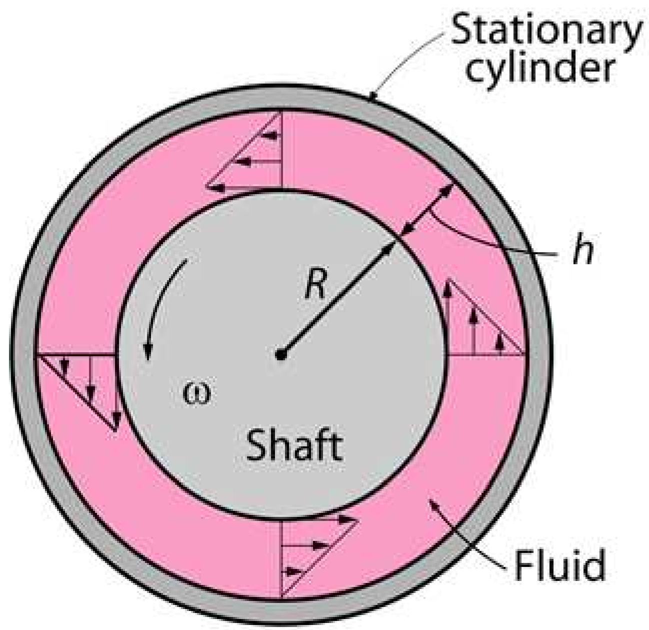
Properties of Fluid

- Application
  - Geometry parameters: L = 30 cm; R = 8 cm; h =0.1 cm;
  - Inner cylinder rotates at constant speed ω = 300 rpm
  - Torque measured T = 2.0 N·m
  - Determine the dynamic viscosity of fluid



Properties of Fluid

- Application
  - Solution



$$T = FR = 2\pi\mu \frac{\omega R^3 L}{h}$$

$$\mu = \frac{Th}{2\pi\omega R^3 L}$$

$$= \frac{2.0 \times 0.1 \times 10^{-2}}{2\pi \times \left(\frac{300 \times 2\pi}{60}\right) \times \left(8 \times 10^{-2}\right)^3 \times 30 \times 10^{-2}}$$

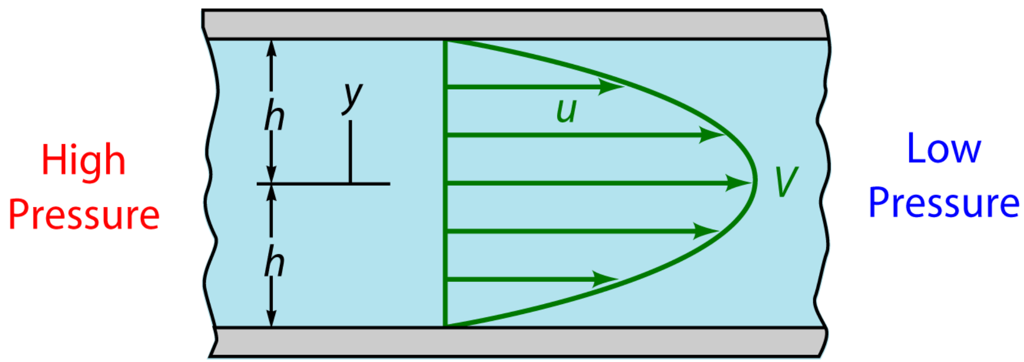
$$= 6.6 \times 10^{-2}\ \text{Pa} \cdot \text{s}$$

Properties of Fluid

- Question
  - Newtonian fluid flows between two wide, parallel plates
  - Flow driven by pressure difference
  - Parabolic velocity profile given by

$$u = V\left[1-\left(y/h\right)^2\right]$$

  where V is the maximum velocity (along channel centerline $y = 0$)
  - Determine (a) shear stress acting on bottom wall, and (b) shear stress acting on a plane parallel to the walls and passing through the centerline (mid-plane, $y = 0$)



Properties of Fluid

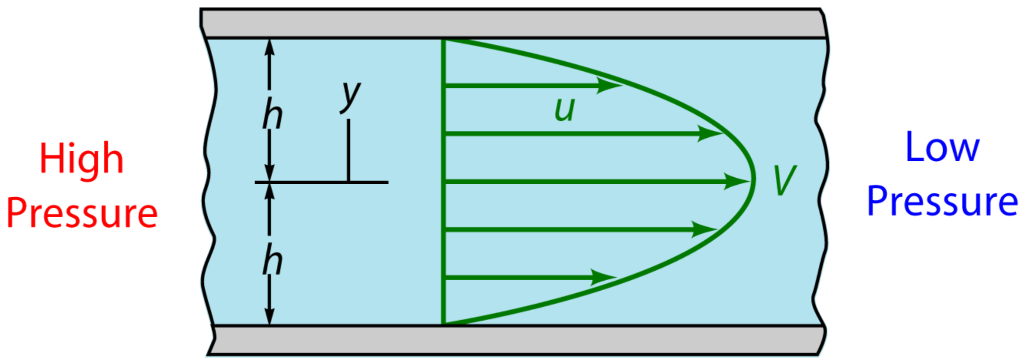
- Question
  - Solution:
    - ✓ Velocity profile

$$u = V\left[1 - \left(y/h\right)^2\right]$$

    - ✓ Shear stress

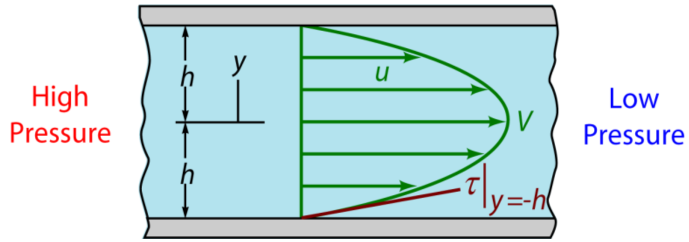
$$\tau = \mu \frac{du}{dy} = \mu\left(-2V\frac{y}{h^2}\right) = -\frac{2\mu V y}{h^2}$$

<u>Recall</u>:
$u = 0$ @ $y = \pm h$ ⇨ no-slip condition satisfied



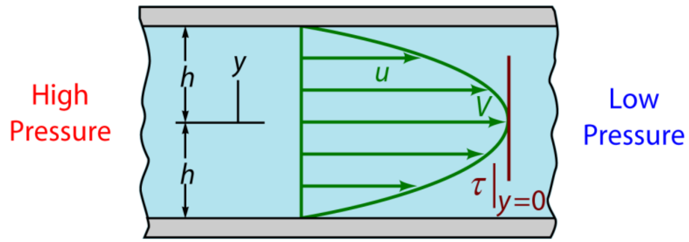
Properties of Fluid

- Question
  - Solution:
    - ✓ Along bottom wall, $y = -h$, shear stress



$$\tau = \frac{2\mu V}{h}$$

    - ✓ Along midplane, $y = 0$, shear stress:



$$\tau = 0$$

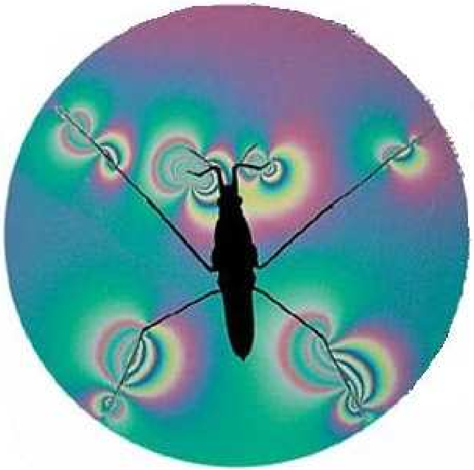
    - ✓ This flow is also called a plane Poiseuille flow

Surface Tension

- Introduction to Surface Tension
  - Examples:
    - ✓ Some insects can walk on water
    - ✓ Steel needle (heavier than water) is able to float

Steel paper clip floating on water

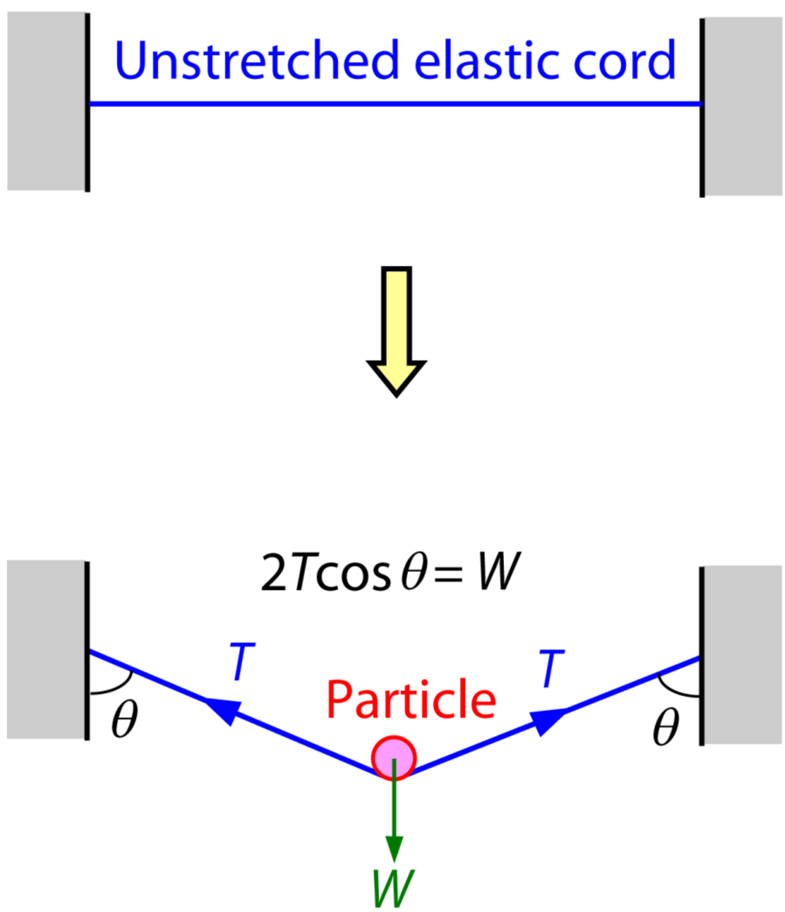
Trampoline

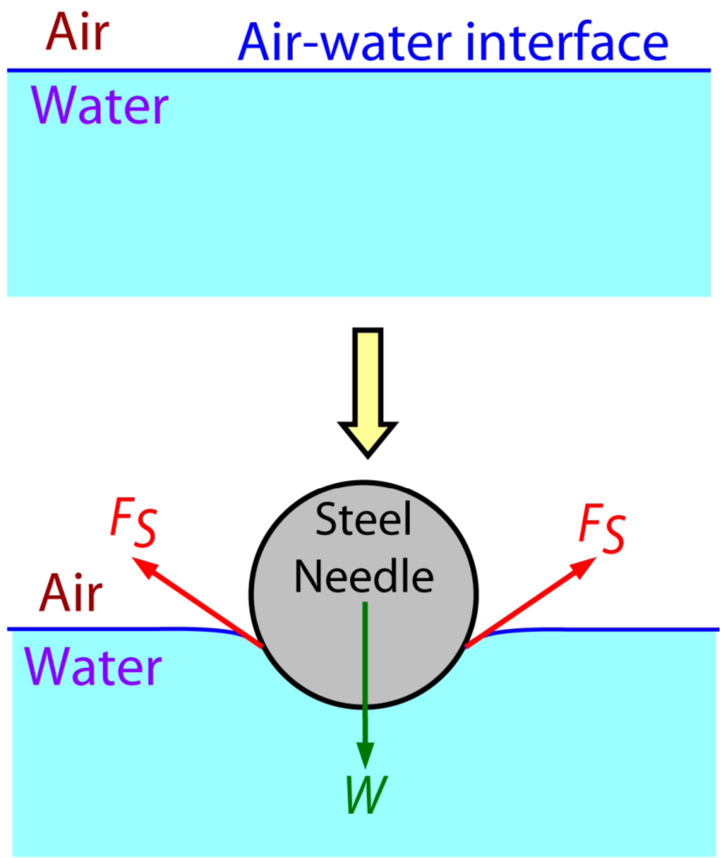


Water strider walking on water

Surface Tension

- Analogy



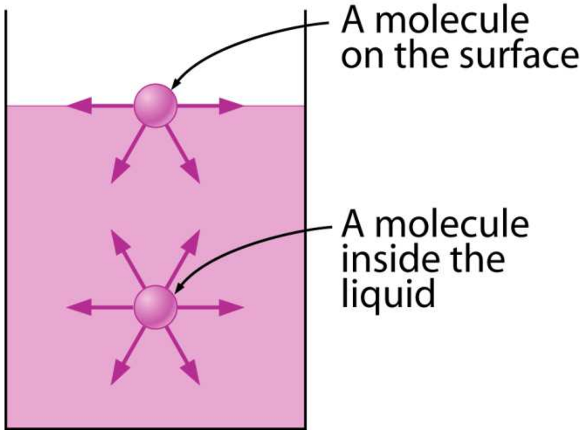


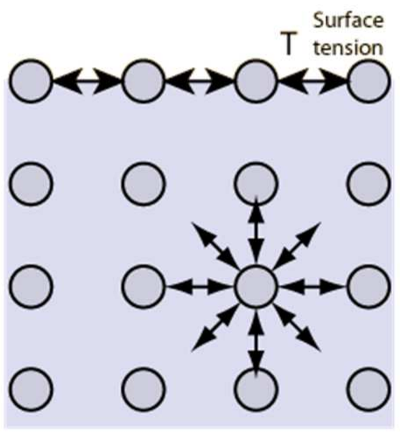
Surface Tension

- Analogy
  - Due to surface tension, indentations or dimples are formed on water surface by the object
  - Liquid surface behaves like a membrane in tension
  - With surface tension, the liquid surface behaves like a trampoline
  - When a person stands on a trampoline, it stretches downward a bit and, in so doing, exerts an upward force on the person, due to its elasticity. The upward force balances the person's weight
  - Liquid surface behaves in a similar way
  - A water strider can walk on the water surface just as a person can walk on a trampoline
  - When a liquid is in contact with a gas or another immiscible liquid, an **interface** develops which acts like a stretched elastic membrane, creating a surface tension

Surface Tension

- Origin of Surface Tension
  - Molecules repel one another when they are close by, attract each other when far apart, and at equilibrium at some distance apart. This attraction forces fall off rapidly, over a distance of roughly 10μm, the "attraction zone"
  - In the interior of a liquid, they slightly repel each other slightly, to compensate for the pressure from the surrounding environment. Molecules on the surface tend to be a bit further than the equilibrium distance, resulting as a slight attraction, or surface tension. This is because for a liquid molecule whose attraction zone lies totally within a fluid, the molecule is attracted in all directions statistically equally, resulting in no force on it. On or near a surface, this attraction zone is partially outside, where the gas molecules are smaller in number, resulting in a force that pulls the molecule toward the liquid side, attempting to minimize the surface





Surface Tension

- Origin of Surface Tension
  - This attractive force is balanced by repulsive forces from molecules below the surface
  - Net effect of this attraction on all the surface molecules is to make the surface of the liquid contract, thus minimizing the surface area of the liquid
  - Small liquid droplets tend to be spherical, which has the minimum surface area for a given volume
  - Interface thus behaves like a stretched skin or membrane $\Rightarrow$ each portion of liquid surface exerts tension on adjacent portions of the surface or on objects in contact with liquid surface $\Rightarrow$ interfacial or surface tension

Surface Tension

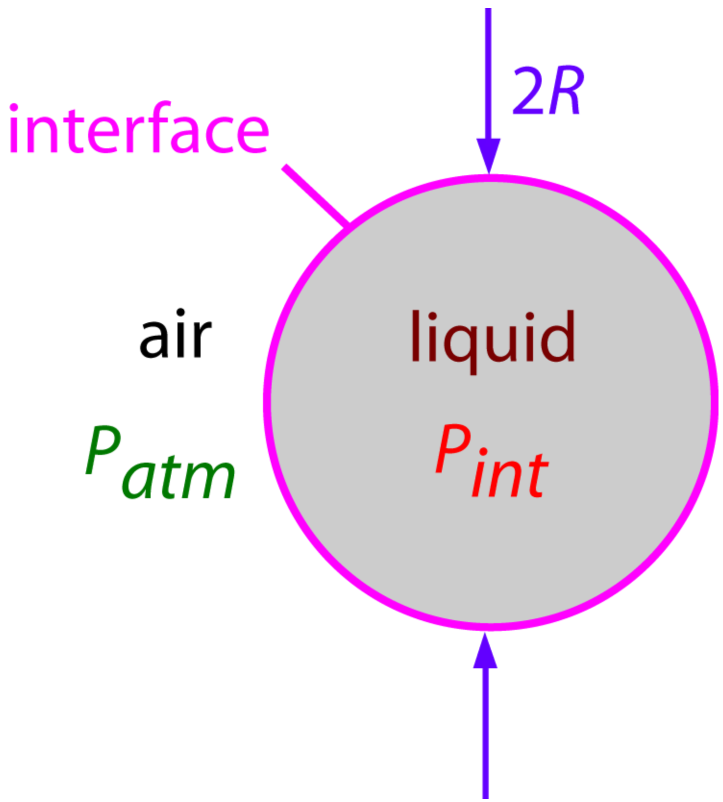
- Definition of Surface Tension
  - Coefficient of surface tension (or simply surface tension) σ is the magnitude of the interfacial tension force F divided by the length L along the interface over which the force acts:

$$\sigma = \frac{F}{L}$$

  - SI units for σ : N/m
  - An equivalent measure of surface tension is surface energy, expressed as J/m^2

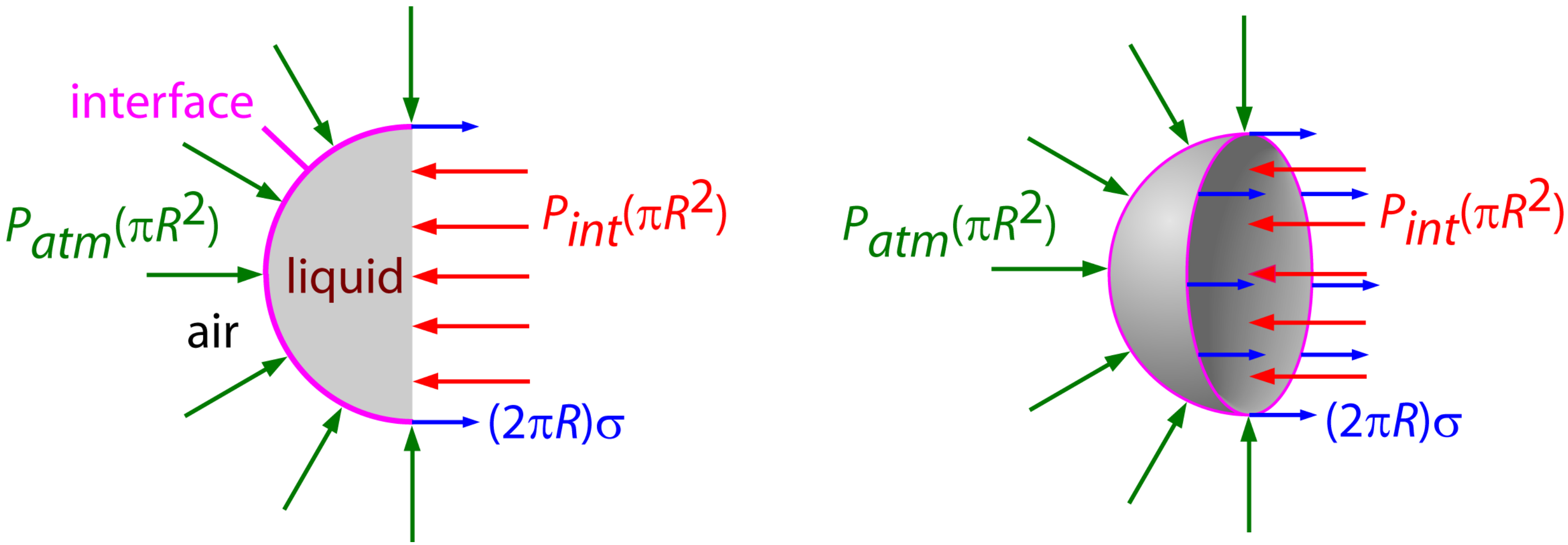
Surface Tension

- Liquid Droplets
  - Consider a spherical liquid droplet of radius R in air:



Surface Tension

- Liquid Droplets
  - Sectioning the droplet through its center:



  - External pressure P_{atm} acts on droplet surface $\Rightarrow$ horizontal force acting on droplet surface due to external pressure = $P_{atm}(\pi R^2) \rightarrow$
  - Pressure within droplet is P_{int} $\Rightarrow$ horizontal force due to internal pressure = $P_{int}(\pi R^2) \leftarrow$

Surface Tension

- Liquid Droplets
  - On the sectioned surface, surface tension acts along the interface $\Rightarrow$ horizontal force due to surface tension = $(2\pi R)\,\sigma \rightarrow$
  - Equilibrium of forces in the horizontal direction:

$$\underset{\rightarrow}{P_{atm}\left(\pi R^2\right)} + \underset{\rightarrow}{\left(2\pi R\right)\sigma} = \underset{\leftarrow}{P_{\text{int}}\left(\pi R^2\right)}$$

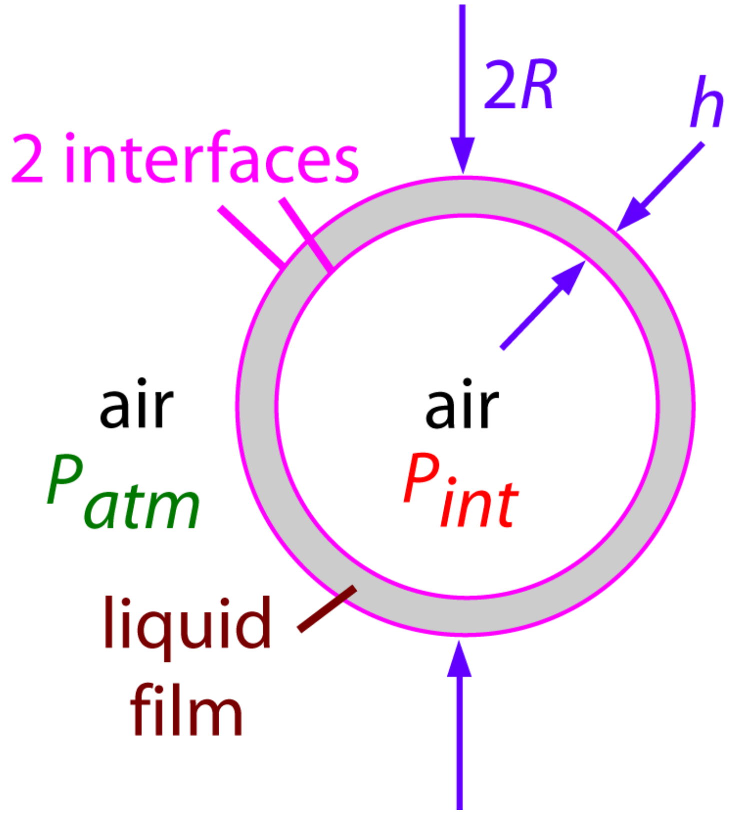
  - Pressure difference:

$$\Delta P = P_{\text{int}} - P_{atm} = \frac{2\sigma}{R}$$

  - Pressure inside droplet is greater than pressure outside droplet

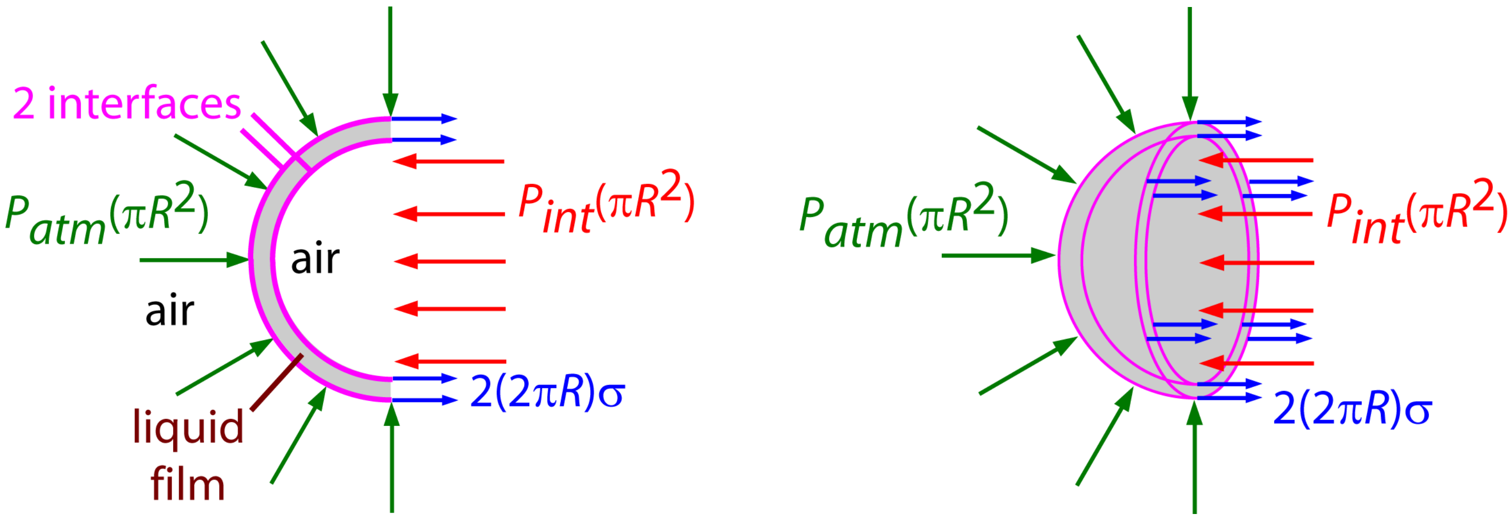
Surface Tension

- Bubbles
  - Consider a soap bubble of radius R in air
  - Bubble consists of a liquid film of thickness h with 2 surfaces or interfaces (inner and outer)
  - Assume $h << R$



Surface Tension

- Bubbles
  - Sectioning the bubble through its center:



  - External pressure P_{atm} acts on bubble surface $\Rightarrow$ horizontal force acting on bubble surface due to external pressure = $P_{atm}(\pi R^2) \rightarrow$
  - Pressure within bubble is P_{int} $\Rightarrow$ horizontal force due to internal pressure = $P_{int}(\pi R^2) \leftarrow$

Surface Tension

- Bubbles
  - On the sectioned surface, surface tension acts along the 2 interfaces $\Rightarrow$ horizontal force due to surface tension = $2(2\pi R)\,\sigma \rightarrow$
  - Equilibrium of forces in the horizontal direction:

$$\underset{\rightarrow}{P_{atm}\left(\pi R^2\right)} + \underset{\rightarrow}{2\left(2\pi R\right)\sigma} = \underset{\leftarrow}{P_{\text{int}}\left(\pi R^2\right)}$$

  - Pressure difference:

$$\Delta P = P_{\text{int}} - P_{atm} = \frac{4\sigma}{R}$$

  - Pressure inside bubble is greater than pressure outside droplet

Surface Tension

- Surface tension of fluids in air at 1 atm and 20 °C

Fluid	Surface tension σ, N/m	Fluid	Surface tension σ, N/m
Water:		Ethyl alcohol	0.023
0°C	0.076	Acetone	0.024
20°C	0.073	Blood, 37°C	0.058
100°C	0.059	Gasoline	0.022
300°C	0.014	Ammonia	0.021
Glycerin	0.063	Soap solution	0.025
SAE 30 oil	0.035	Kerosene	0.028
Mercury	0.485	Helium (4 K)	0.0001

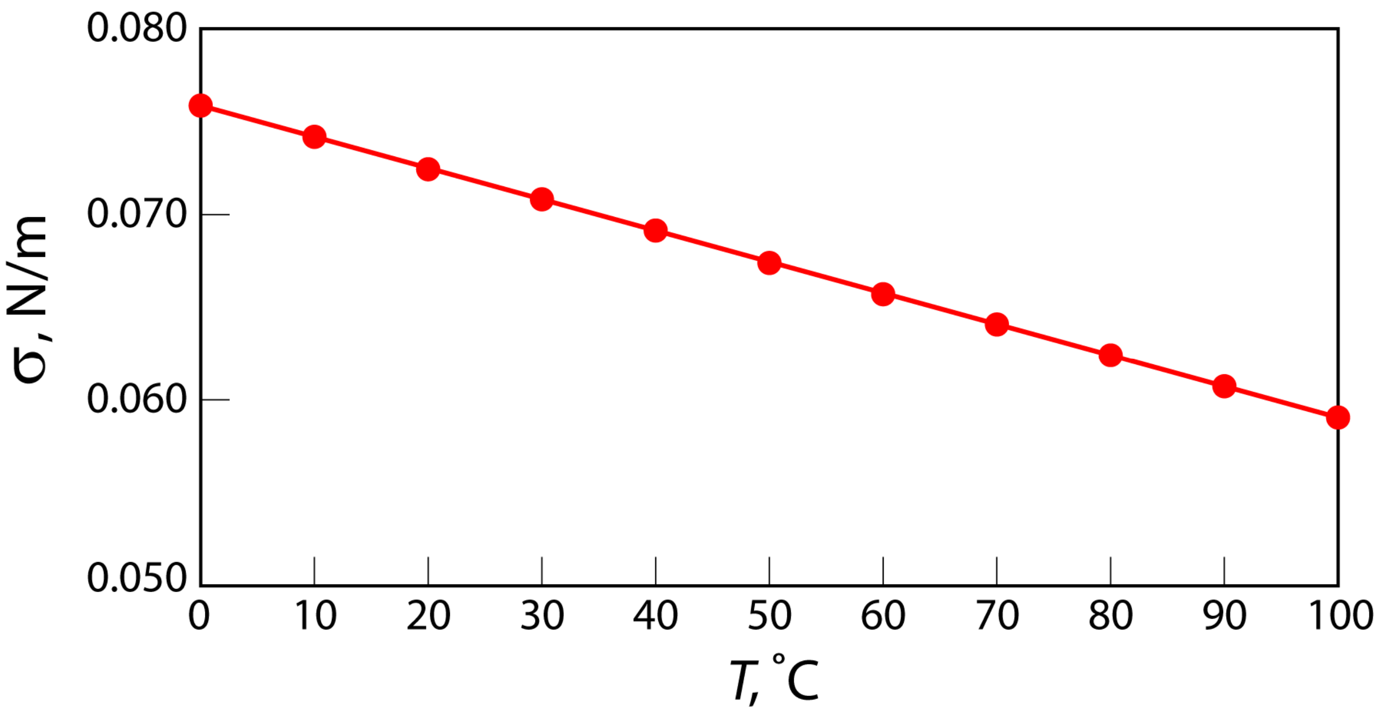
http://en.wikipedia.org/wiki/Surface_tension_values
http://www.surface-tension.de/

Surface Tension

- Surface Tension
  - Water has a higher surface tension than most other liquids (with the exception of liquid metals)
  - The very high surface tension of mercury explains why mercury droplets form spherical balls that can be rolled like a solid ball on a surface without wetting the surface
  - Surface tension varies with temperature for a given substance, in general, surface tension of a liquid decreases with temperature - it becomes easier to pull molecules up into the surface
  - Surface tension becomes zero at the critical point (and thus there is no distinct liquid–vapor interface at temperatures above the critical point)

Surface Tension

- Surface Tension



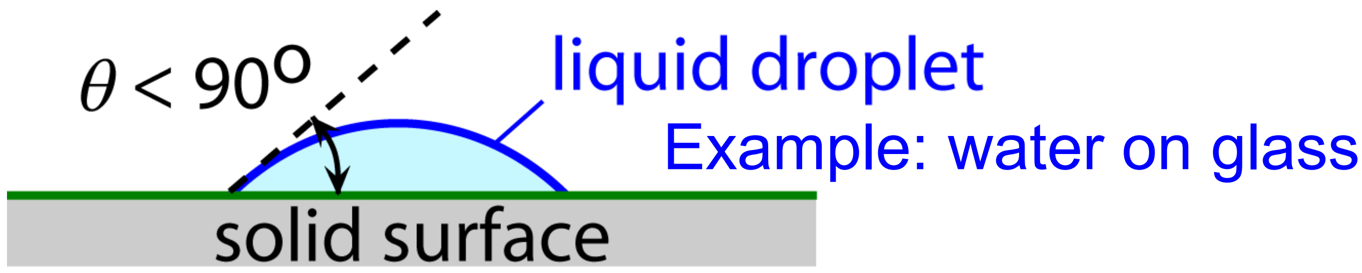
Surface tension of clean air-water interface

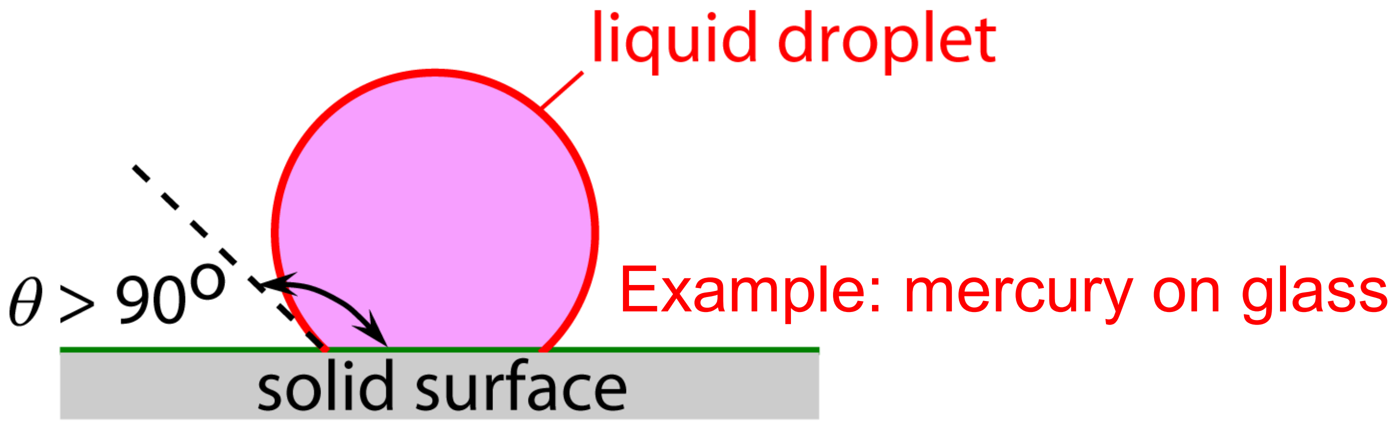
Surface Tension

- Surface Tension
  - Effect of pressure on surface tension is usually negligible
  - Surface tension of a substance can be changed considerably by impurities $\Rightarrow$ certain chemicals, called surfactants, can be added to a liquid to decrease its surface tension $\Rightarrow$ e.g., soaps and detergents lower the surface tension of water and enable it to penetrate through the small openings between fibers for more effective washing

Contact Angle

- Introduction
  - A second important surface effect is the contact angle θ which appears when a liquid/vapor (gas) interface intersects with a solid surface:



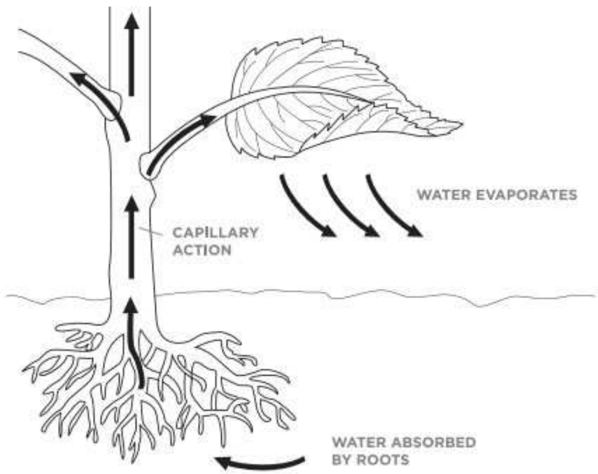


Contact Angle

- Definition
  - Contact angle θ is defined as the angle measured from the solid surface through the liquid (or the denser medium) to the surface
  - $\theta < 90^\circ$: liquid wets the solid surface or liquid is wetting
  - $\theta > 90^\circ$: liquid does not wet the solid surface or liquid is nonwetting
  - Examples
    - ✓ Water wets soap but does not wet wax
    - ✓ Water on a clean glass surface $\Rightarrow \theta \approx 0^\circ$
    - ✓ Clean mercury-air-glass interface $\Rightarrow \theta \approx 130^\circ$
  - Like σ , θ is sensitive to the actual conditions of the solid-liquid interface

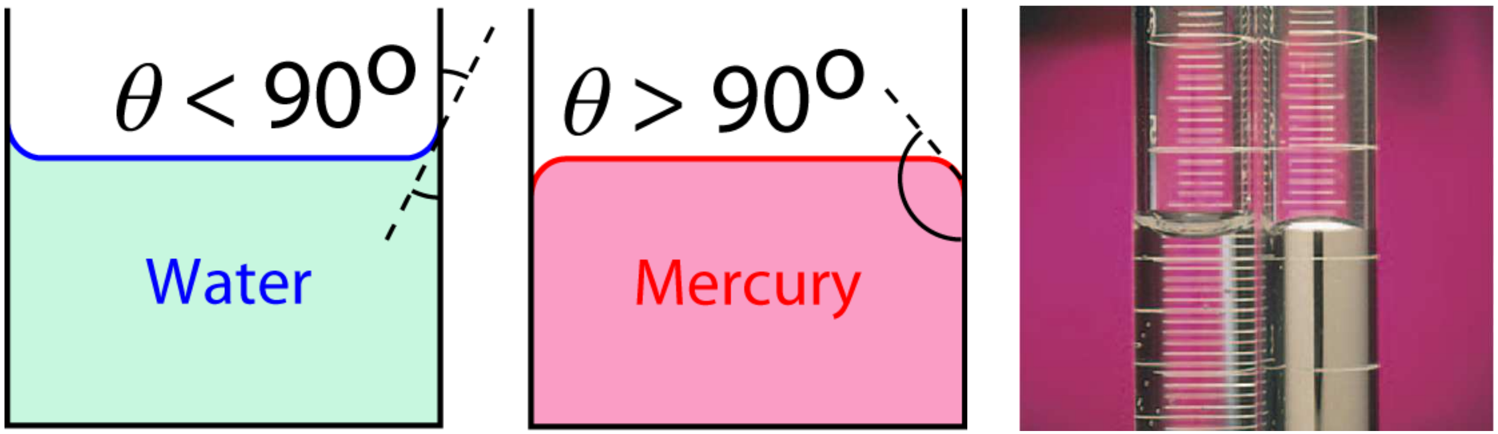
Capillary Effect

- Capillary Effect
  - Rise or fall of a liquid in a small-diameter tube inserted into the liquid
  - Examples
    - ✓ Rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp
    - ✓ Rise of water to the top of tall trees



Capillary Effect

- Meniscus
  - Curved free surface of a liquid in a capillary tube



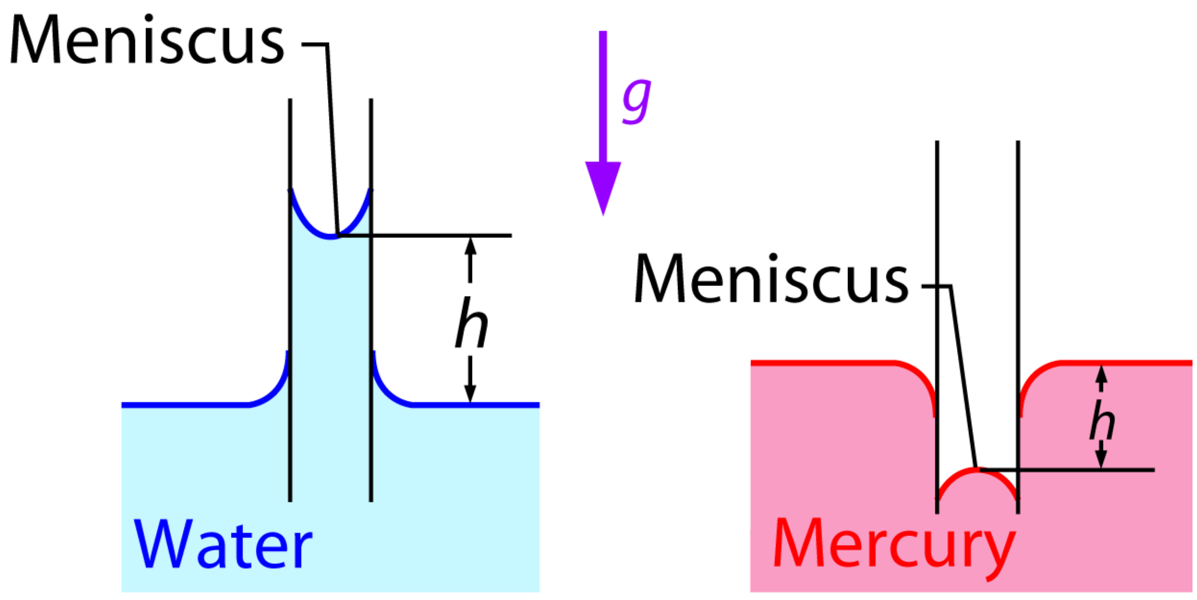
(a) Wetting fluid

(b) Nonwetting fluid

H_2O Hg

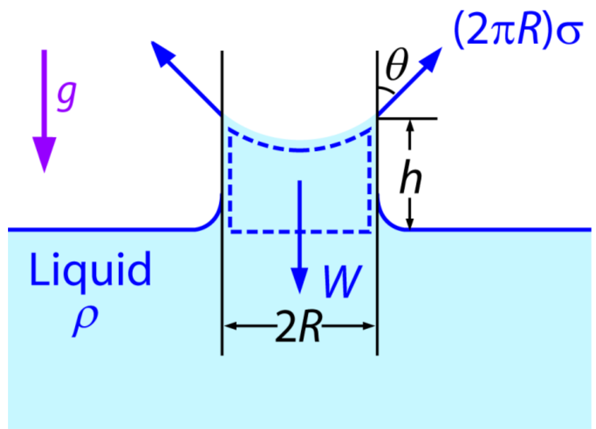
Capillary Effect

- Meniscus
  - Insert a capillary tube in a liquid
    - ✓ $\theta < 90^\circ$ (wetting) $\Rightarrow$ capillary rise $\Rightarrow h > 0$
    - ✓ $\theta > 90^\circ$ (nonwetting) $\Rightarrow$ capillary depression $\Rightarrow h < 0$

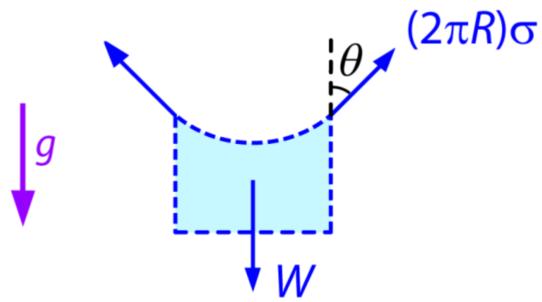


Capillary Effect

- Capillary rise *h* of wetting liquid in a tube
  - Force balance on cylindrical liquid column of height h in tube:



  - Free-body diagram



Capillary Effect

- Capillary rise *h* of wetting liquid in a tube
  - Weight of liquid column:

$$W = mg = \rho V g = \rho\left(\pi R^2 h\right) g$$

  - Vertical (upward) component of surface tension force:

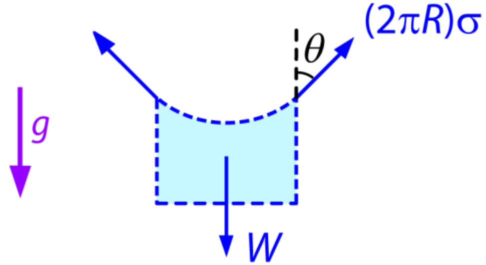
$$F_S = \left(2\pi R\right)\sigma\cos\theta$$

  - Equating vertical forces acting on liquid column:

$$W = F_S$$

$$\rho\left(\pi R^2 h\right) g = \left(2\pi R\right)\sigma\cos\theta$$

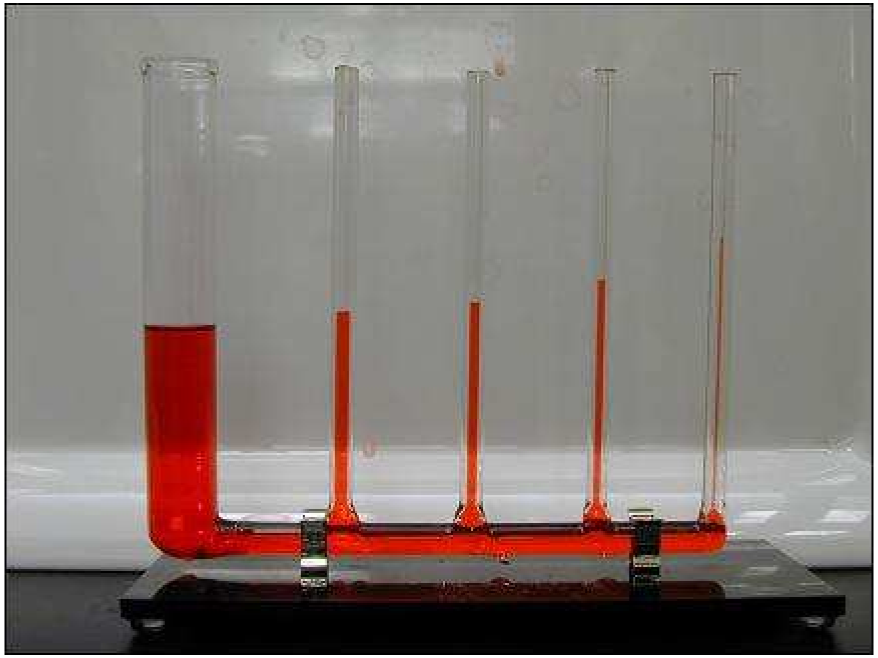
$$h = \frac{2\sigma}{\rho g R}\cos\theta$$



Capillary Effect

- Capillary rise h of wetting liquid in a tube
  - $R\uparrow h\downarrow$

$$h=\frac{2\sigma}{\rho g R}\cos\theta$$



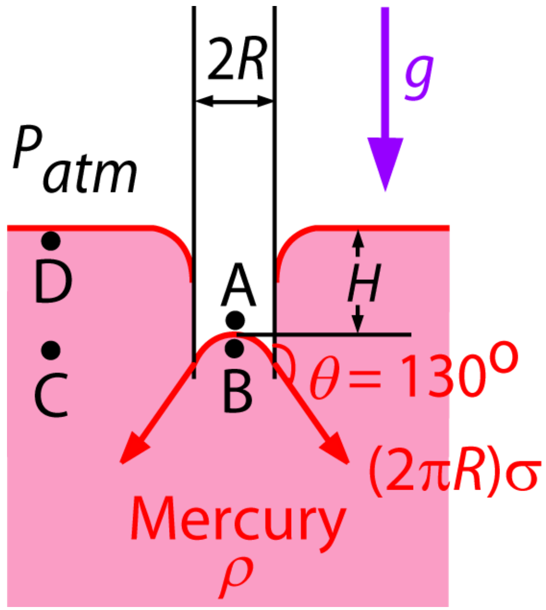
Capillary Effect

- Example 1
  - Determine absolute value of capillary depression *H* for mercury in a 1.0 mm diameter glass tube
  - Solution:
    - ✓ A is a point just above the meniscus

$$P_A = P_{atm}$$

    - ✓ B is a point just below the meniscus
    - ✓ C and B are at the same horizontal level
    - ✓ D is a point just below the flat mercury surface

$$P_D = P_{atm}$$



Capillary Effect

- Example 1
  - Solution (cont'd):

✓ Hydrostatic relation

$$P_B = P_C = P_D + \rho g H = P_{atm} + \rho g H$$

✓ Pressure difference across interface or meniscus

$$P_B - P_A = \left(P_{atm} + \rho g H\right) - P_{atm} = \rho g H$$

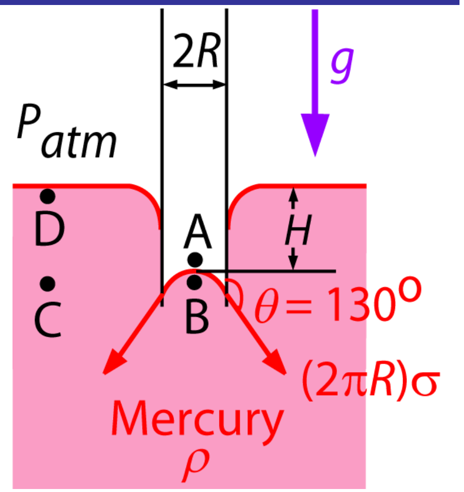
✓ Vertical upward force due to pressure difference across meniscus:

$$F_P = \left(P_B - P_A\right)\pi R^2 = \left(\rho g H\right)\pi R^2$$

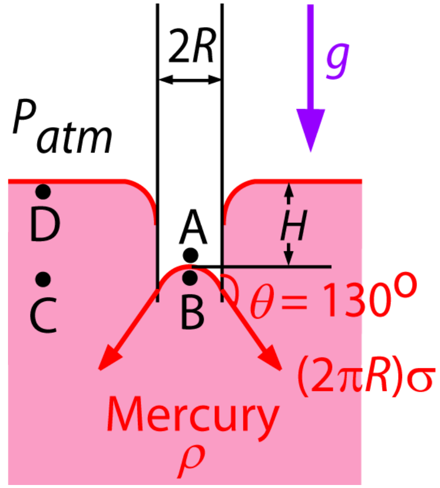
Note: (πR^2 is the projected area)

✓ Vertical (downward) component of surface tension force:

$$F_S = \left(2\pi R\right)\sigma\cos\left(180° - \theta\right) = -\left(2\pi R\right)\sigma\cos\theta$$



Capillary Effect



- Example 1
  - Solution (cont'd):
    - ✓ Equating vertical forces acting on meniscus:

$$F_P = F_S$$

$$(\rho g H)\pi R^2 = -(2\pi R)\sigma\cos\theta$$

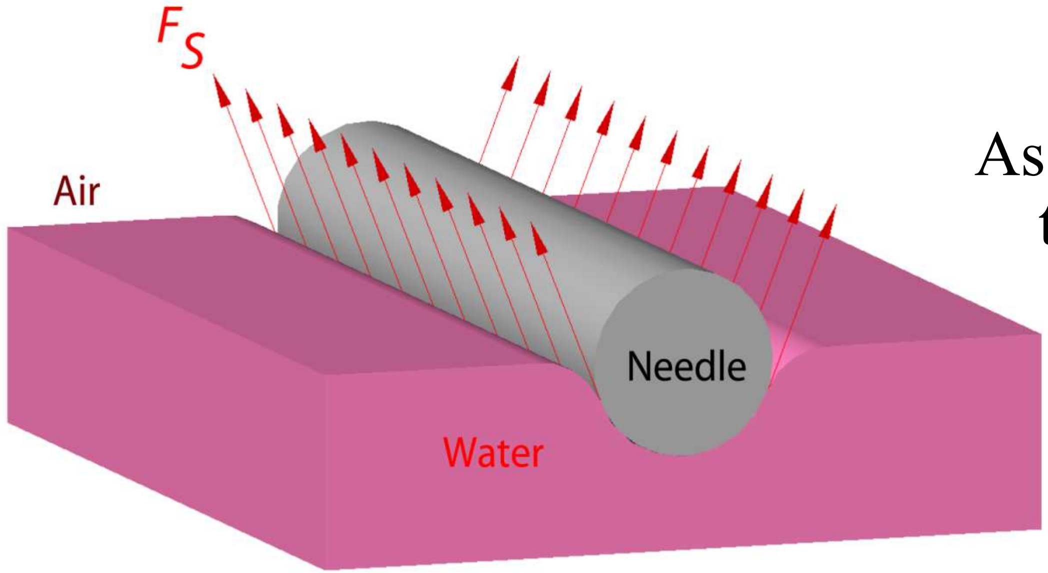
$$H = -\frac{2\sigma}{\rho g R}\cos\theta$$

$$H = -\frac{2\times 0.485}{13550\times 9.81\times 0.0005}\times\cos(130^\circ)$$

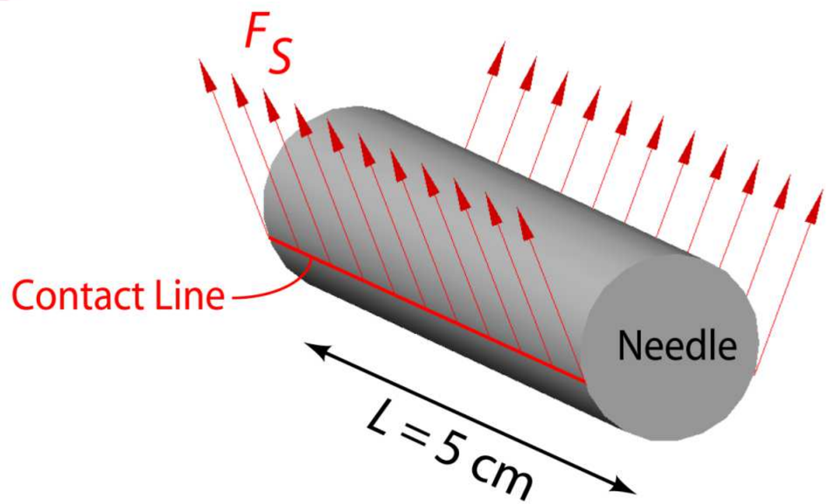
$$H = 9.38 \text{ mm}$$

Capillary Effect

- Example 2: Steel needle floats on water

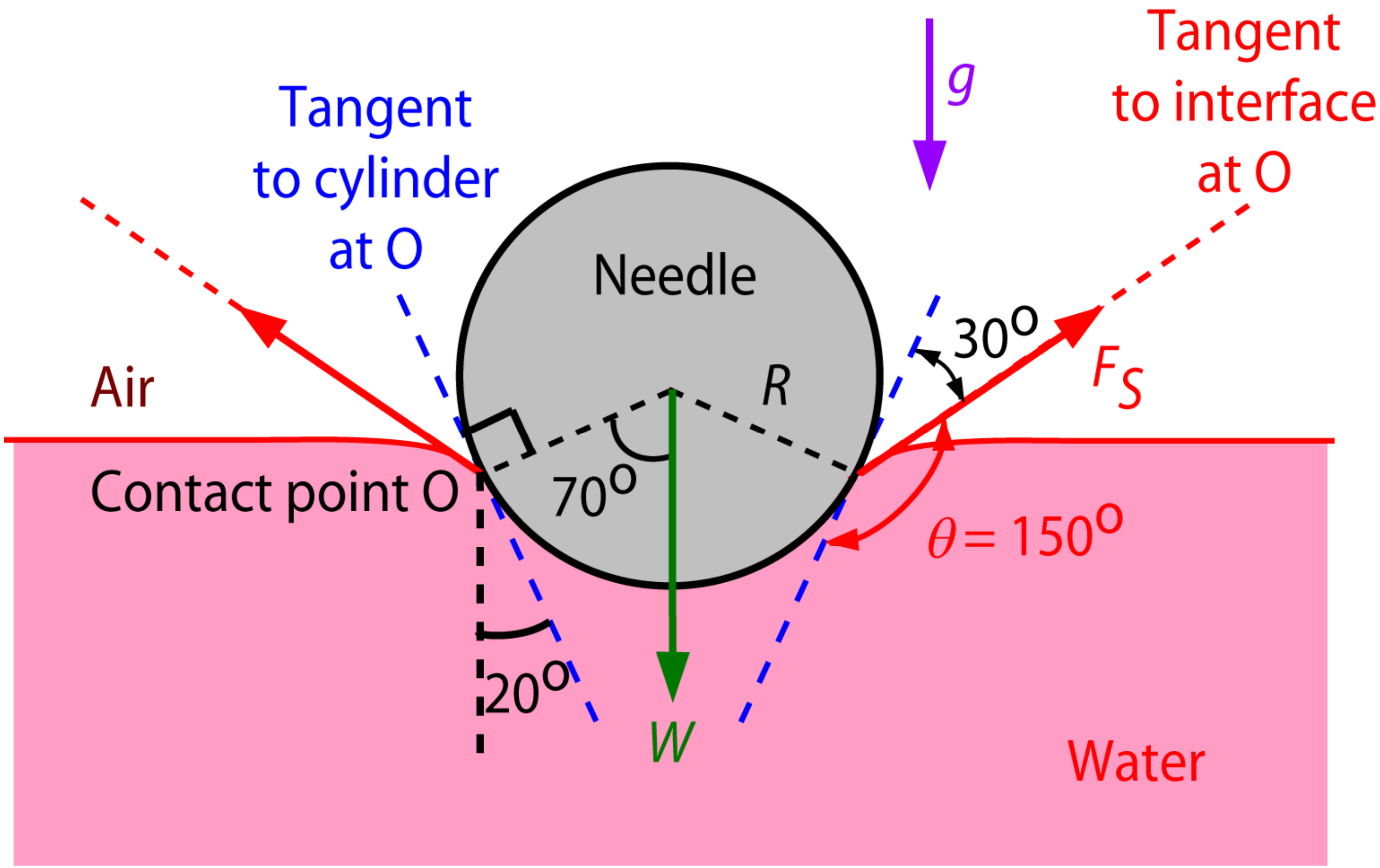


Assumption:
the needle is a circular cylinder



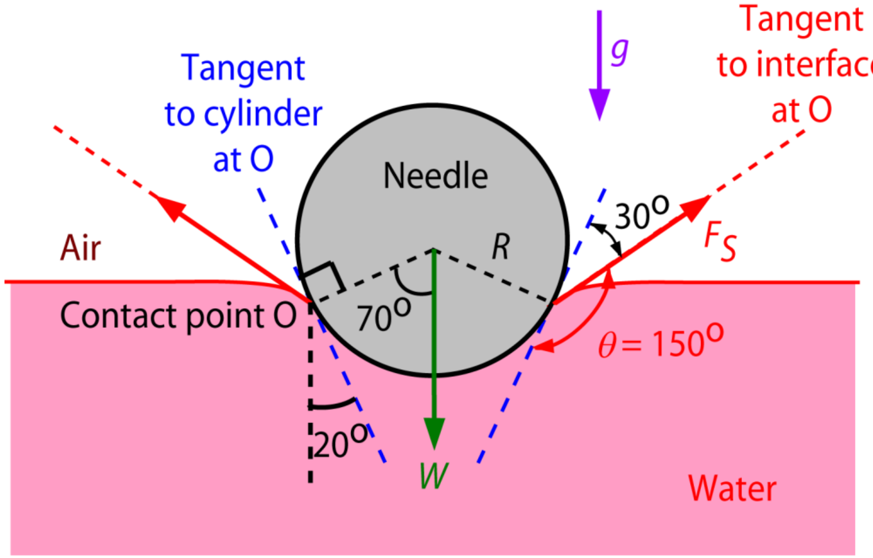
Capillary Effect

- Example 2: Steel needle floats on water



Capillary Effect

- Example 2: Steel needle floats on water
  - Problem statement:
    - ✓ Length and radius of needle are L = 5 cm and R = 1 mm
    - ✓ O: contact point of water-air interface with cylinder
    - ✓ Contact angle $\theta = 150°$
    - ✓ Assume $\sigma = 0.073$ N/m
    - ✓ Determine weight of needle?

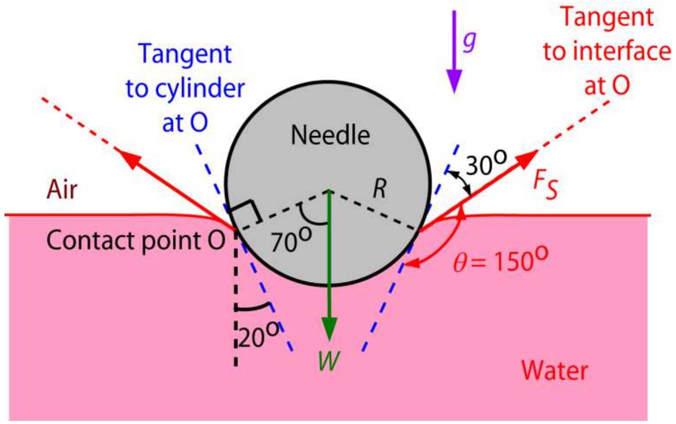


Capillary Effect

- Example 2: Steel needle floats on water
  - Solution:
    - ✓ Surface tension force F_S makes an angle of (150-20) ° = 130 ° with downward vertical direction
    - ✓ Vertical component of surface tension force (upward):

$$F_S = \sigma L \cos\left(180° - 130°\right) \times 2$$

(factor of 2 arises because the air-water interface is in contact with 2 sides of cylinder parallel to length of cylinder) – **Neglect the ends of the cylinder**



- ✓ Equating vertical forces acting on meniscus:

$$W = F_S = 2\sigma L \cos\left(50°\right) = 2 \times 0.073 \times 0.05 \times \cos(50°)$$

$$W = 4.69 \times 10^{-3} \text{ N}$$

Summary

- Fluid
- Fluid Mechanics
- Continuum Assumption
- Fluid Particle/Parcel
- Methodology to Study Fluid Mechanics
- Properties: density, compressibility, thermal expansion
- Viscosity, Kinematic viscosity, Newton's law of viscosity, Non-newton fluid
- Shear Stress, Application: viscometer
- Surface Tension, droplets, bubbles
- Contact angle, capillary effect


Thank You for Your Attention!
Any Questions?