

1A.

Let $b=1.470\text{mm}$ L =the length of glass plate

$$2\sigma L = \rho_w g h L b$$

$$\rightarrow 2\sigma = \rho_w g h b$$

$$\rightarrow h = 2\sigma / (\rho_w g b)$$

$$\sigma = 0.0735, \rho_w = 1 \times 10^3 \text{ kg} \cdot \text{m}^{-3}, g = 10 \text{ m} \cdot \text{s}^{-2}, b = 1.470 \text{ mm}$$

$$h = \frac{2 \times 0.0735}{1 \times 10^3 \times 10 \times 1.470 \times 10^{-3}} \text{ m} = 0.1 \text{ m} = 10 \text{ mm}$$

1B.

$$P_{atm}(\pi R^2) + 2 \times (2\pi R)\sigma = P_{int}(\pi R^2)$$

$$\rightarrow \Delta P = P_{int} - P_{atm} = \frac{4\sigma}{R}$$

$$D=5\text{mm} \rightarrow R=2.5\text{mm} \quad \sigma = 0.025 \text{ (N/m)}$$

$$\Delta P = \frac{4 \times 0.025}{2.5 \times 10^{-3}} \text{ Pa} = 40 \text{ Pa}$$

5A.

Part (a): velocity of water leaving (3)

$$z_1 - z_3 = 4 \text{ m}$$

$$p_1 = p_3 = 0 \text{ (atmospheric pressure, 0 gage pressure)}$$

$$v_1 \approx 0 \text{ (large tank)}$$

Applying Bernoulli equation between (1) and (3)

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + g z_1 = \frac{p_3}{\rho} + \frac{v_3^2}{2} + g z_3$$

$$\frac{v_3^2}{2} = g(z_1 - z_3)$$

$$v_3 = \sqrt{2g(z_1 - z_3)} = \sqrt{2 \times 10 \times 4} = 4\sqrt{5} = 4 \times 2.24 = 8.96 \text{ m/s}$$

Part (b): water pressure in tube at (4)

Applying continuity equation between (4) and (3):

$$A_4 v_4 = A_3 v_3$$

$$\text{Since } A_4 = A_3, v_4 = v_3 = 8.96 \text{ m/s}$$

Applying Bernoulli equation between (4) and (3)

$$\frac{p_4}{\rho} + \frac{v_4^2}{2} + g z_4 = \frac{p_3}{\rho} + \frac{v_3^2}{2} + g z_3$$

$$\frac{p_4}{\rho} = g(z_3 - z_4) \quad (z_3 - z_4) = 4 \text{ m}$$

$$P_4 = \rho g(z_3 - z_4) = 1000 \times 10 \times (-4) \text{ Pa} = -40 \text{ kPa}$$

5B

Part (a): velocity of water leaving (3)

$$z_1 - z_3 = 4 \text{ m}$$

$$p_1 = p_3 = 0 \text{ (atmospheric pressure, 0 gage pressure)}$$

$$v_1 \approx 0 \text{ (large tank)}$$

Applying Bernoulli equation between (1) and (3)

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_3}{\rho} + \frac{v_3^2}{2} + gz_3$$

$$\frac{v_3^2}{2} = g(z_1 - z_3)$$

$$v_3 = \sqrt{2g(z_1 - z_3)} = \sqrt{2 \times 10 \times 4} = 4\sqrt{5} = 4 \times 2.24 = 8.96 \text{ m/s}$$

Part (b): water pressure in tube at (2)

Applying continuity equation between (2) and (3):

$$A_2 v_2 = A_3 v_3$$

$$\text{Since } A_2 = A_3, v_2 = v_3 = 8.96 \text{ m/s}$$

Applying Bernoulli equation between (2) and (3)

$$\frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2 = \frac{p_3}{\rho} + \frac{v_3^2}{2} + gz_3$$

$$\frac{p_2}{\rho} = g(z_3 - z_2) \quad (z_2 - z_3) = 6 \text{ m}$$

$$P_2 = \rho g(z_3 - z_2) = 1000 \times 10 \times (-6) \text{ Pa} = -60 \text{ kPa}$$

Solutions L2 + L6

Solution 2. A

2.121 The uniform beam in the figure is of size L by h by b , with $b, h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at $L/2$ from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at $L/3$ from the left corner. Sum moments about the left corner, point C:

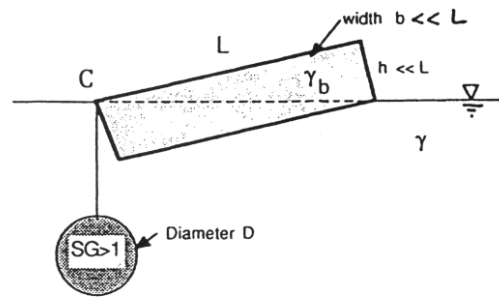


Fig. P2.121

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or} \quad \gamma_b = \gamma/3 \quad \text{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lhb/2 - \gamma_b Lhb - T, \quad \text{or} \quad T = \gamma Lhb/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{\text{sphere}} = (SG - 1)\gamma \frac{\pi}{6} D^3, \quad \text{so that} \quad D = \left[\frac{Lhb}{\pi(SG - 1)} \right]^{1/3} \quad \text{Ans. (b)}$$

Solution 2. B

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is

$$F = \gamma h_{CG} A = (9790)(5.0 + 1.727) \frac{\pi}{2} (3)^2$$

$$= 931000 \text{ N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A} = -\frac{(0.10976)(3)^4 \sin 90^\circ}{(5 + 1.727)(\pi/2)(3)^2} = -0.0935 \text{ m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \text{ or: } P = \mathbf{366000 \text{ N}} \quad \text{Ans.}$$

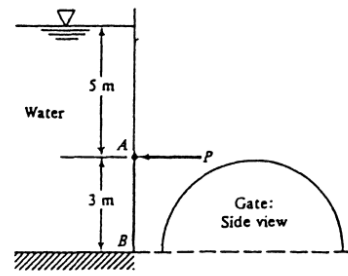
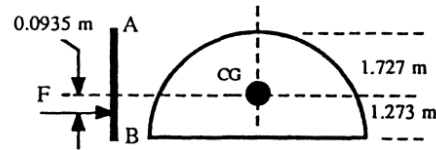


Fig. P2.65



Solution 6. A

Write the given relation and count variables:

$$Q = f(R, \mu, \frac{dp}{dx}) \quad \text{four variables } (n = 4)$$

Make a list of the dimensions of these variables using the $\{MLT\}$ system: There are three primary

Q	R	μ	dp/dx
L^3T^{-1}	L	$ML^{-1}T^{-1}$	$ML^{-2}T^{-2}$

dimensions (M, L, T) , hence $j = 3$. By trial and error we determine that R, μ , and dp/dx cannot be combined into a pi group. Then $j = 3$, and $n - j = 4 - 3 = 1$. There is only *one* pi group, which we find by combining Q in a power product with the other three:

$$\Pi_1 = R^a \mu^b (\frac{dp}{dx})^c Q^1 = (L)^a (ML^{-1}T^{-1})^b (ML^{-2}T^{-2})^c (L^3T^{-1}) = M^0 L^0 T^0$$

Equate exponents:

$$\begin{cases} b + c = 0 \\ a - b - 2c + 3 = 0 \\ -b - 2c - 1 = 0 \end{cases}$$

Solving simultaneously, we obtain $a = -4$, $b = 1$, and $c = -1$. Then

$$\Pi_1 = R^{-4} \mu^1 (\frac{dp}{dx})^{-1} Q$$

or

$$\Pi_1 = \frac{Q\mu}{R^4(dp/dx)} = \text{const}$$

Solution 6. B

The functional relationship is $\delta = f(x, U, \mu, \rho)$, with $n = 5$ variables and $j = 3$ primary dimensions (M, L, T) . Thus we expect $n - j = 5 - 3 = 2$ Pi groups:

$$\Pi_1 = \rho^a x^b \mu^c \delta = M^0 L^0 T^0 \quad \text{if } a = 0, b = -1, c = 0 : \Pi_1 = \frac{\delta}{x}$$

$$\Pi_2 = \rho^a x^b \mu^c U = M^0 L^0 T^0 \quad \text{if } a = 1, b = 1, c = -1 : \Pi_2 = \frac{\rho U x}{\mu}$$

Thus $\delta/x = f(\rho U x / \mu) = f(Re_x)$.

7. for planar couette flow, $\frac{\partial}{\partial x} = 0, \frac{\partial}{\partial z} = 0,$

So for mass conservation, $\frac{\partial v}{\partial y} = 0.$

For momentum conservation, $\frac{d^2 u}{dy^2} = 0.$

So, we can deduce that $u_1 = c_1 y + c_2$ in fluid 1, and $u_2 = c_3 y + c_4$ in fluid 2, and the boundary conditions are:

$$y = 0, u = 0; y = d, u_1 = u_2; y = h, u = U;$$

And in h, we have $\tau_1 = \tau_2, \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y}$

We have

$$\begin{cases} c_4 = 0 \\ c_3 d = c_1 d + c_2 \\ U = c_1 h + c_2 \\ \mu_1 c_1 = \mu_2 c_2 \end{cases}$$

$$\text{So } c_1 = \frac{U}{h-d+\frac{\mu_1 d}{\mu_2}}, c_2 = \frac{(\frac{\mu_1}{\mu_2}-1)dU}{h-d+\frac{\mu_1 d}{\mu_2}}, c_3 = \frac{\mu_1}{\mu_2} \frac{U}{h-d+\frac{\mu_1 d}{\mu_2}}$$

$$\text{So in fluid 1, } u_1 = \frac{Uy+(\frac{\mu_1}{\mu_2}-1)dU}{h-d+\frac{\mu_1 d}{\mu_2}}, u_2 = \frac{\mu_1}{\mu_2} \frac{U}{h-d+\frac{\mu_1 d}{\mu_2}} y$$

8.

$$\mu = \rho \nu = 900 \times 0.0002 = 0.18 \text{ kg}/(\text{m} \cdot \text{s})$$

$$z = 10 \times \sin 40^\circ = 6.43 \text{ m}$$

To calculate the head loss, we have

$$z_2 + \frac{p_2}{\rho g} + h_f = z_1 + \frac{p_1}{\rho g}$$

$$\text{So } h_f = 4.68 \text{ m}$$

V=2.63, according to the equation given in the hint,

$$\text{So } \text{Re} = \frac{Vd}{\nu} = 790$$

7.

$$\begin{cases} \frac{Du_x}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u_x}{dx^2} + \frac{\partial^2 u_x}{dy^2} + \frac{\partial^2 u_x}{dz^2} \right) \\ \frac{Du_y}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 u_y}{dx^2} + \frac{\partial^2 u_y}{dy^2} + \frac{\partial^2 u_y}{dz^2} \right) \\ \frac{Du_z}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u_z}{dx^2} + \frac{\partial^2 u_z}{dy^2} + \frac{\partial^2 u_z}{dz^2} \right) \end{cases}$$

Apply the condition, $\frac{\partial u_y}{\partial} = 0, \frac{\partial u_z}{\partial} = 0$, so

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 u_x}{dy^2}$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$8. v = \frac{Q}{A} = \frac{\frac{11}{3600}}{\frac{\pi}{4} \times 0.03^2} = 4.32 \text{ m/s}$$

$$Re = \frac{\rho v d}{\mu} = 129681$$

For head loss , we have

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

Take the surface of tank and the outlet of the pipe as reference ,

$$p_1 = p_2 = p_{atm}, z_1 = 4, v_1 = 0$$

$$h_f = z_1 - \frac{v_2^2}{2g} = 3.07$$

And $h_f = f \frac{L}{d} \frac{v^2}{2g}$, so $f = 0.0197$, from moody chart ,we can get $\frac{\epsilon}{d} = 0.0004$

$$\text{So } \epsilon = 0.012 \text{ mm}$$