mid-term exam

2016.11.17

1. let $t = 0$

$$\frac{dx}{dt} = x \tag{1}$$

$$\frac{dy}{dt} = -y \tag{2}$$

$\Rightarrow$

$$x = c_x e^t \tag{3}$$

$$y = c_y e^{-t} \tag{4}$$

let $t = 0$ we have

$$c_x = -1 \tag{5}$$

$$c_y = -1 \tag{6}$$

so the streamline is

$$xy = 1 \tag{7}$$

$$\frac{dx}{dt} = x + t \tag{8}$$

$$\frac{dy}{dt} = -y + t \tag{9}$$

$\Rightarrow$

$$x = c_x e^t - t - 1 \tag{10}$$

$$y = c_y e^{-t} + t - 1 \tag{11}$$

use the condition of $x = -1, y = -1$ when $t = 0 \Rightarrow$

$$x + y + 2 = 0 \tag{12}$$

2.

$$u = (a + 1)e^t - 1 \tag{13}$$

$$v = (b + 1)e^t - 1 \tag{14}$$

$\Rightarrow$

$$x = c_x e^t - t - 1 \tag{15}$$

$$y = c_y e^{-t} - t - 1 \tag{16}$$

use the condition $x = a, y = b$ when $t = 0$

$$c_x = a + 1 \tag{17}$$

$$c_y = b + 1 \tag{18}$$

so

$$u = (a + 1)e^t - 1 \tag{19}$$

$$v = (b + 1)e^t - 1 \tag{20}$$

3.

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A u)}{\partial x} = 0 \tag{21}$$

4.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{22}$$

$\Rightarrow$

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \rho = 0 \tag{23}$$

using the incompressible condition

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0 \tag{24}$$

$\Rightarrow$

$$\nabla \cdot \mathbf{u} = 0 \tag{25}$$