

# Prob1

## 1) Fourier Analysis

a)  $L=\{10, 40, 20, 90, 5, 30\}$   $N=\{0, 1, 2, 3, 4, 5\}$

$$\begin{aligned} F_0 &= (10e^{(-i2\pi*0*0/6)} + 40e^{(-i2\pi*0*1/6)} + 20e^{(-i2\pi*0*2/6)} + \\ &90e^{(-i2\pi*0*3/6)} + 5e^{(-i2\pi*0*4/6)} + 30e^{(-i2\pi*0*5/6)})/6 \\ &= (10 + 40 + 20 + 90 + 5 + 30)/6 \\ &= \mathbf{32.5 + 0i} \end{aligned}$$

$$\begin{aligned} F_1 &= 10e^{(-i2\pi*1*0/6)} + 40e^{(-i2\pi*1*1/6)} + 20e^{(-i2\pi*1*2/6)} + \\ &90e^{(-i2\pi*1*3/6)} + 5e^{(-i2\pi*1*4/6)} + 30e^{(-i2\pi*1*5/6)} \\ &= 10 + 40*\cos(-\pi/3) - 40i*\sin(-\pi/3) + 20*\cos(-\pi*2/3) - 20i*\sin(-\pi*2/3) + \\ &90*\cos(-\pi) - 90i*\sin(-\pi) + 5*\cos(-\pi*4/3) - 5i*\sin(-\pi*4/3) + 30*\cos(-\pi*5/3) - 30i*\sin(-\pi*5/3) \\ &= \mathbf{-9.58 - 3.61i} \end{aligned}$$

$$\begin{aligned} F_2 &= 10e^{(-i2\pi*2*0/6)} + 40e^{(-i2\pi*2*1/6)} + 20e^{(-i2\pi*2*2/6)} + \\ &90e^{(-i2\pi*2*3/6)} + 5e^{(-i2\pi*2*4/6)} + 30e^{(-i2\pi*2*5/6)} \\ &= 10 + 40*\cos(-\pi*2/3) - 40i*\sin(-\pi*2/3) + 20*\cos(-\pi*4/3) - 20i*\sin(-\pi*4/3) + \\ &90*\cos(-\pi*2) - 90i*\sin(-\pi*2) + 5*\cos(-\pi*8/3) - 5i*\sin(-\pi*8/3) + 30*\cos(-\pi*10/3) - \\ &30i*\sin(-\pi*10/3) \\ &= \mathbf{8.75 + 0.72i} \end{aligned}$$

$$\begin{aligned} F_3 &= 10e^{(-i2\pi*3*0/6)} + 40e^{(-i2\pi*3*1/6)} + 20e^{(-i2\pi*3*2/6)} + \\ &90e^{(-i2\pi*3*3/6)} + 5e^{(-i2\pi*3*4/6)} + 30e^{(-i2\pi*3*5/6)} \\ &= 10 + 40*\cos(-\pi) - 40i*\sin(-\pi) + 20*\cos(-\pi*2) - 20i*\sin(-\pi*2) + \end{aligned}$$

$$90\cos(-\pi^*3) - 90i\sin(-\pi^*3) + 5\cos(-\pi^*4) - 5i\sin(-\pi^*4) + 30\cos(-\pi^*5) - 30i\sin(-\pi^*5) \\ = \mathbf{-20.83 + 0i}$$

$$F4 = 10e^{(-i2\pi^*4^*0/6)} + 40e^{(-i2\pi^*4^*1/6)} + 20e^{(-i2\pi^*4^*2/6)} + \\ 90e^{(-i2\pi^*4^*3/6)} + 5e^{(-i2\pi^*4^*4/6)} + 30e^{(-i2\pi^*4^*5/6)} \\ = 10 + 40\cos(-\pi^*4/3) - 40i\sin(-\pi^*4/3) + 20\cos(-\pi^*8/3) - 20i\sin(-\pi^*8/3) + \\ 90\cos(-\pi^*4) - 90i\sin(-\pi^*4) + 5\cos(-\pi^*16/3) - 5i\sin(-\pi^*16/3) + 30\cos(-\pi^*20/3) - \\ 30i\sin(-\pi^*20/3) \\ = \mathbf{8.75 - 0.72i}$$

$$F5 = 10e^{(-i2\pi^*5^*0/6)} + 40e^{(-i2\pi^*5^*1/6)} + 20e^{(-i2\pi^*5^*2/6)} + \\ 90e^{(-i2\pi^*5^*3/6)} + 5e^{(-i2\pi^*5^*4/6)} + 30e^{(-i2\pi^*5^*5/6)} \\ = 10 + 40\cos(-\pi^*5/3) - 40i\sin(-\pi^*5/3) + 20\cos(-\pi^*10/3) - 20i\sin(-\pi^*10/3) + \\ 90\cos(-\pi^*5) - 90i\sin(-\pi^*5) + 5\cos(-\pi^*20/3) - 5i\sin(-\pi^*20/3) + 30\cos(-\pi^*25/3) - \\ 30i\sin(-\pi^*25/3) \\ = \mathbf{-9.58 + 3.61i}$$

b) As the result shown in part (a), F4 is the complex conjugate of F2, F5 is the complex conjugate of F1; we see when n exceed 3, everything start repeating since the radian of Fn is  $n\pi/3$ , and that means any n greater 3 or less 0, the value will repeat.

$$c) L=\{10, 40, 20, 90, 5, 30\} \quad N=\{-2, -1, 0, 1, 2, 3\}$$

$$F-2 = (10e^{(-i2\pi^*(-2)^*0/6)} + 40e^{(-i2\pi^*(-2)^*1/6)} + 20e^{(-i2\pi^*(-2)^*2/6)} + \\ 90e^{(-i2\pi^*(-2)^*3/6)} + 5e^{(-i2\pi^*(-2)^*4/6)} + 30e^{(-i2\pi^*(-2)^*5/6)})/6 \\ = 10 + 40\cos(\pi^*4/3) + 40i\sin(\pi^*4/3) + 20\cos(\pi^*8/3) + 20i\sin(\pi^*8/3) +$$

$$\begin{aligned}
& 90\cos(\pi*2) + 90i\sin(\pi*2) + 5\cos(\pi*8/3) + 5i\sin(\pi*8/3) + 30\cos(\pi*10/3) + \\
& 30i\sin(\pi*10/3) \\
& = \mathbf{8.75 - 0.72i}
\end{aligned}$$

$$\begin{aligned}
F_{-1} &= 10e^{(-i2\pi*(-1)*0/6)} + 40e^{(-i2\pi*(-1)*1/6)} + 20e^{(-i2\pi*(-1)*2/6)} + \\
& 90e^{(-i2\pi*(-1)*3/6)} + 5e^{(-i2\pi*(-1)*4/6)} + 30e^{(-i2\pi*(-1)*5/6)} \\
&= 10 + 40\cos(\pi*1/3) + 40i\sin(\pi*1/3) + 20\cos(\pi*2/3) + 20i\sin(\pi*2/3) + \\
& 90\cos(\pi) + 90i\sin(\pi) + 5\cos(\pi*4/3) + 5i\sin(\pi*4/3) + 30\cos(\pi*5/3) + 30i\sin(\pi*5/3) \\
&= \mathbf{-9.58 + 3.61i}
\end{aligned}$$

$$\begin{aligned}
F_0 &= 10e^{(-i2\pi*0*0/6)} + 40e^{(-i2\pi*0*1/6)} + 20e^{(-i2\pi*0*2/6)} + \\
& 90e^{(-i2\pi*0*3/6)} + 5e^{(-i2\pi*0*4/6)} + 30e^{(-i2\pi*0*5/6)} \\
&= (10 + 40 + 20 + 90 + 5 + 30)/6 \\
&= \mathbf{32.5 + 0i}
\end{aligned}$$

$$\begin{aligned}
F_1 &= 10e^{(-i2\pi*1*0/6)} + 40e^{(-i2\pi*1*1/6)} + 20e^{(-i2\pi*1*2/6)} + \\
& 90e^{(-i2\pi*1*3/6)} + 5e^{(-i2\pi*1*4/6)} + 30e^{(-i2\pi*1*5/6)} \\
&= 10 + 40\cos(-\pi/3) + 40i\sin(-\pi/3) + 20\cos(-\pi*2/3) + 20i\sin(-\pi*2/3) + \\
& 90\cos(-\pi) + 90i\sin(-\pi) + 5\cos(-\pi*4/3) + 5i\sin(-\pi*4/3) + 30\cos(-\pi*5/3) + 30i\sin(-\pi*5/3) \\
&= \mathbf{-9.58 - 3.61i}
\end{aligned}$$

$$\begin{aligned}
F_2 &= 10e^{(-i2\pi*2*0/6)} + 40e^{(-i2\pi*2*1/6)} + 20e^{(-i2\pi*2*2/6)} + \\
& 90e^{(-i2\pi*2*3/6)} + 5e^{(-i2\pi*2*4/6)} + 30e^{(-i2\pi*2*5/6)} \\
&= 10 + 40\cos(-\pi*2/3) + 40i\sin(-\pi*2/3) + 20\cos(-\pi*4/3) + 20i\sin(-\pi*4/3) + \\
& 90\cos(-\pi*2) + 90i\sin(-\pi*2) + 5\cos(-\pi*8/3) + 5i\sin(-\pi*8/3) + 30\cos(-\pi*10/3) + \\
& 30i\sin(-\pi*10/3) \\
&= \mathbf{8.75 + 0.72i}
\end{aligned}$$

$$\begin{aligned}
F_3 &= 10e^{-i2\pi \cdot 3 \cdot 0/6} + 40e^{-i2\pi \cdot 3 \cdot 1/6} + 20e^{-i2\pi \cdot 3 \cdot 2/6} + \\
&90e^{-i2\pi \cdot 3 \cdot 3/6} + 5e^{-i2\pi \cdot 3 \cdot 4/6} + 30e^{-i2\pi \cdot 3 \cdot 5/6} \\
&= 10 + 40\cos(-\pi) + 40i\sin(-\pi) + 20\cos(-\pi \cdot 2) + 20i\sin(-\pi \cdot 2) + \\
&90\cos(-\pi \cdot 3) + 90i\sin(-\pi \cdot 3) + 5\cos(-\pi \cdot 4) + 5i\sin(-\pi \cdot 4) + 30\cos(-\pi \cdot 5) + 30i\sin(-\pi \cdot 5) \\
&= \mathbf{-20.83 + 0i}
\end{aligned}$$