

Ch 14 - Problem Set 1

Calculus 3

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Section 1: Functions of Several Variables

3. Let $g(x, y) = x^2 \ln(x + y)$

(a) Evaluate $g(3, 1)$.

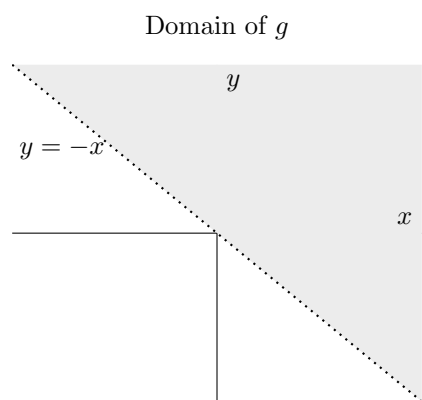
(b) Find and sketch the domain of g .

(c) Find the range of g .

Solution

a) $9 \ln 4$

b) $D : \{(x, y) \mid y > -x\}$



c) \mathbb{R}

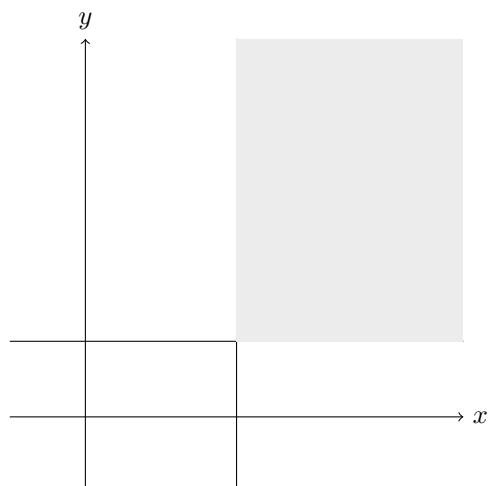
7 - 15 (odd)

Find and sketch the domain of the function.

7. $f(x, y) = \sqrt{x - 2} + \sqrt{y - 1}$

Solution

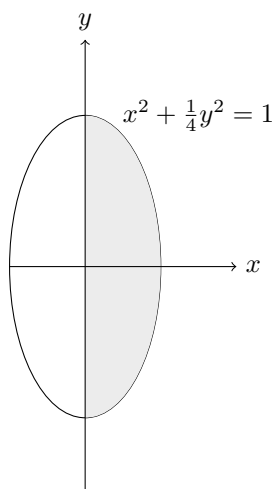
$D : \{(x, y) \mid x \geq 2, y \geq 1\}$



9. $q(x, y) = \sqrt{x} + \sqrt{4 - 4x^2 - y^2}$

Solution

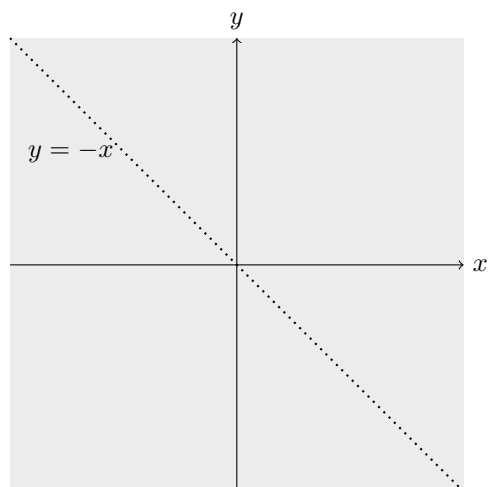
$D : \{(x, y) \mid x^2 + \frac{1}{4}y^2 \leq 1, x \geq 0\}$



11. $g(x, y) = \frac{x - y}{x + y}$

Solution

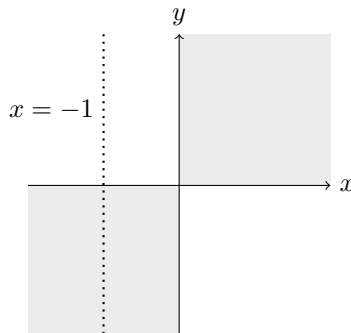
$D : \{(x, y) \mid y \neq -x\}$



13. $p(x, y) = \frac{\sqrt{xy}}{x + 1}$

Solution

$$D : \{(x, y) \mid x \neq 0, xy \geq 0\}$$



$$15. f(x, y, z) = \sqrt{4 - x^2} + \sqrt{9 - y^2} + \sqrt{1 - z^2}$$

Solution

17. A model for the surface area of a human body is given by the function

$$S = f(w, h) = 0.1091w^{0.425}h^{0.725}$$

where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.

(a) Find $f(160, 70)$ and interpret it.

(b) What is your own surface area?

Solution

23 - 31 (odd)

Sketch the graph of the function

$$23. f(x, y) = y$$

Solution

$$25. f(x, y) = 10 - 4x - 5y$$

Solution

$$27. f(x, y) = \sin x$$

Solution

$$29. f(x, y) = x^2 + 4y^2 + 1$$

Solution

$$31. f(x, y) = \sqrt{4 - 4x^2 - y^2}$$

Solution

32. Match the function with its graph (labeled I–VI). Give reasons for your choices.

(a) $f(x, y) = \frac{1}{1 + x^2 + y^2}$

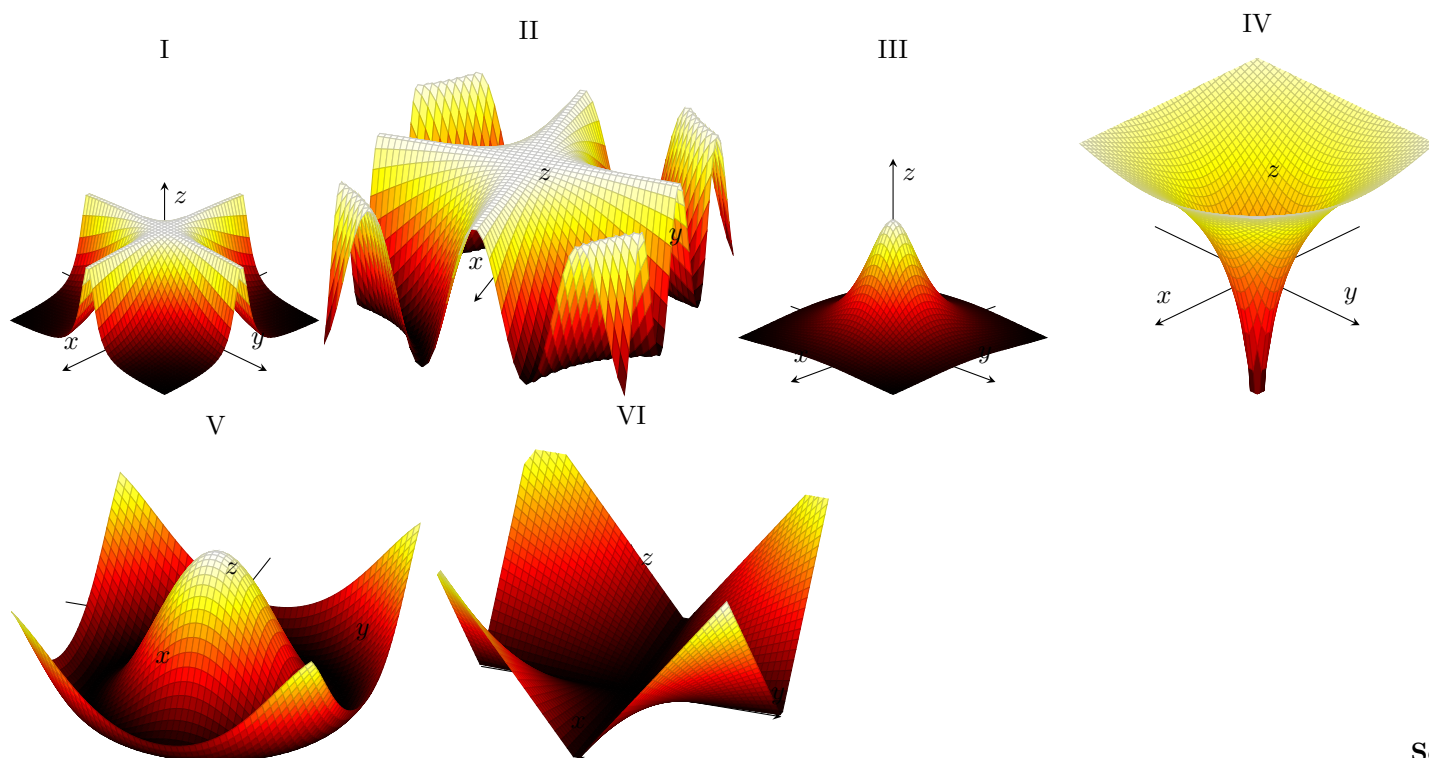
(b) $f(x, y) = \frac{1}{1 + x^2 y^2}$

(c) $f(x, y) = \ln(x^2 + y^2)$

(d) $f(x, y) = \cos \sqrt{x^2 + y^2}$

(e) $f(x, y) = |xy|$

(f) $f(x, y) = \cos(xy)$



33. A contour map for a function f is shown. Use it to estimate the values of $f(-3, 3)$ and $f(3, -2)$. What can you say about the shape of the graph?

Solution

45, 47 & 51

Draw a contour map of the function showing several level curves.

45. $f(x, y) = x^2 + y^2$

Solution

47. $f(x, y) = x^2 + y^2$

Solution

51. $f(x, y) = x^2 + y^2$

Solution

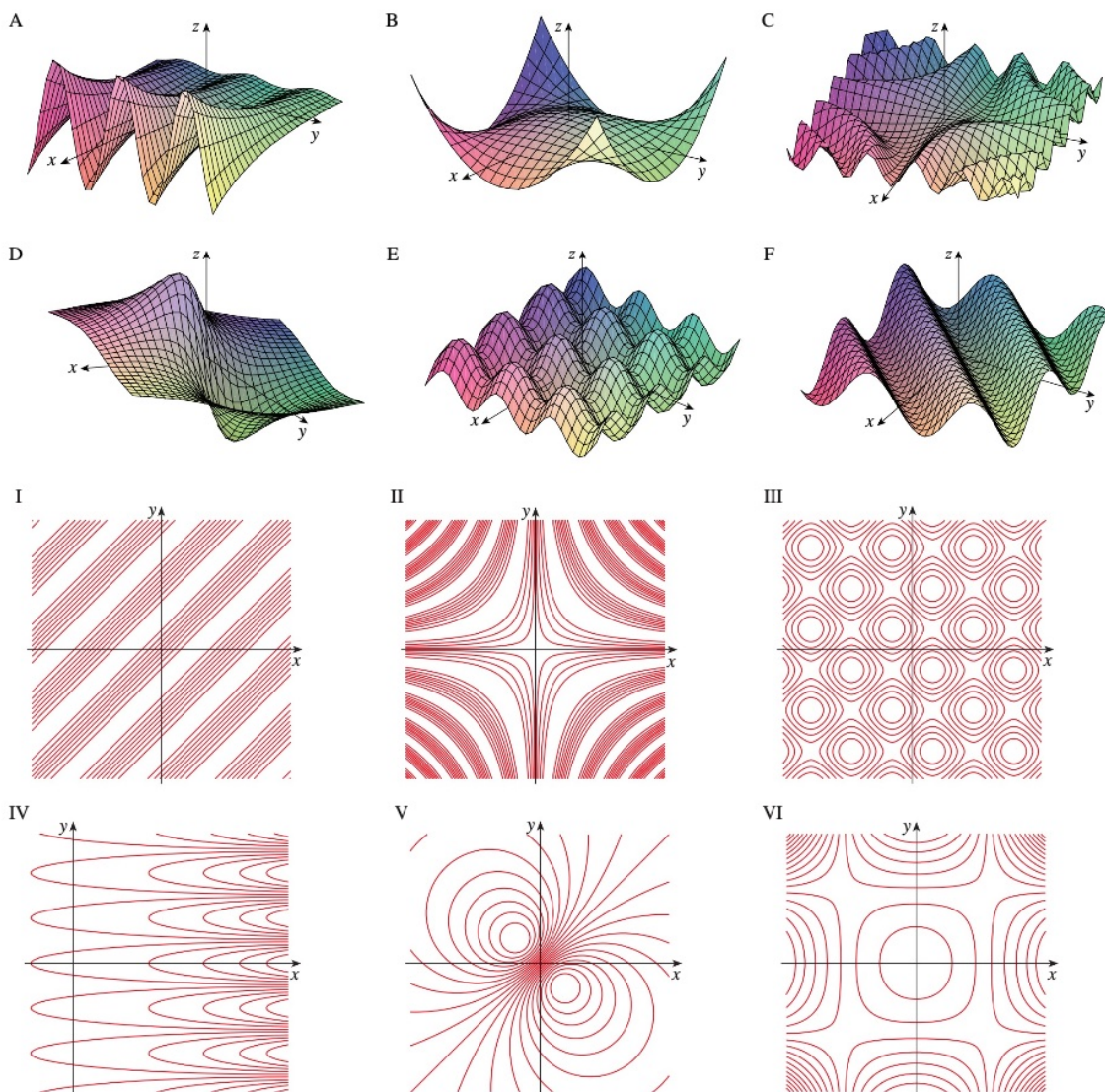
53. Sketch both a contour map and a graph of the given function and compare them.

$$f(x, y) = x^2 + 9y^2$$

Solution

61 - 66

Match the function (a) with its graph (labeled A–F below) and (b) with its contour map (labeled I–VI). Give reasons for your choices.



61. $z = \sin(xy)$

Solution

62. $z = e^x \cos y$

Solution

63. $z = \sin(x - y)$

Solution

64. $z = \sin x - \sin y$

Solution

65. $z = (1 - x^2)(1 - y^2)$

Solution

66. $z = \frac{x - y}{1 + x^2 + y^2}$

Solution

67. Describe the level surfaces of the function.

$$f(x, y, z) = 2y - z + 1$$

Section 2: Limits and Continuity

5 - 11 (odd)

Find the limit

5. $\lim_{(x,y) \rightarrow (3,2)} (x^2y^3 - 4y^2)$

Solution

7. $\lim_{(x,y) \rightarrow (-3,1)} \frac{x^2y - xy^3}{x - y - 2}$

Solution

9. $\lim_{(x,y) \rightarrow (\pi, \pi/2)} y \sin(x - y)$

Solution

11. $\lim_{(x,y) \rightarrow (1,1)} \left(\frac{x^2y^3 - x^3y^2}{x^2 - y^2} \right)$

Solution

13 - 17 (odd)

Show that the limit does not exist

$$13. \lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2 + y^2}$$

Solution

$$15. \lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2 + y^2}$$

Solution

$$17. \lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$$

Solution

19 - 25 (odd)

Find the limit, if it exists, or show that the limit does not exist.

$$19. \lim_{(x,y) \rightarrow (-1,-2)} (x^2 y - xy^2 + 3)^3$$

Solution

$$21. \lim_{(x,y) \rightarrow (2,3)} \frac{3x - 2y}{4x^2 - y^2}$$

Solution

$$23. \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2 \cos y}{x^2 + y^4}$$

Solution

$$25. \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$$

Solution

31 & 33

Use the Squeeze Theorem to find the limit.

$$31. \lim_{(x,y) \rightarrow (0,0)} xy \sin \frac{1}{x^2 + y^2}$$

Solution

$$33. \lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^4 + y^4}$$

Solution

41, 43 & 45

Determine the set of points at which the function is continuous.

$$41. F(x, y) = \frac{xy}{1 + e^{x-y}}$$

Solution

$$43. F(x, y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2}$$

Solution

$$45. G(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$$

Solution