Chapter 14 - Problem Set 2

Calculus 3

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Section 5: The Chain Rule

3-7 (odd)

Use The Chain Rule to find $\dfrac{dz}{dt}$ or $\dfrac{dw}{dt}$.

3. $z = xy^3 - x^2y, \quad x = t^2 + 1, \quad y = t^2 - 1$

5. $z = \sin x \cos y, \quad x = \sqrt{t}, \quad y = 1/t$

7. $w = xe^{y/z}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t$

11-15 (odd)

Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$

11. $z = (x - y)^5, \quad x = s^2t, \quad y = st^2$

13. $z = \ln(3x + 2y), \quad x = s \sin t, \quad y = t \cos s$

15. $z = (\sin\theta)/r, \quad r = st, \quad \theta = s^2 + t^2$

25-29 (odd)

Use the Chain Rule to find the indicated partial derivatives.

25. $z = x^4 + x^2y, \quad x = s + 2t - u, \quad y = stu^2$**;**

$\dfrac{\partial z}{\partial s}, \dfrac{\partial z}{\partial t}, \dfrac{\partial z}{\partial u}$ **when** $s = 4, t = e, u = 1$

27. $w = xy + yz + zx, \quad x = r\cos\theta, \quad y = r\cos\theta, \quad z = r\theta$**;**

$\dfrac{\partial w}{\partial r}, \dfrac{\partial w}{\partial \theta}$ **when** $r = 2, \theta = \pi/2$

29. $N = \dfrac{p+q}{p+r}, \quad p = u + vw, \quad q = v + uw, \quad r = w + uv$

$\dfrac{\partial N}{\partial u}, \dfrac{\partial N}{\partial v}, \dfrac{\partial N}{\partial w}$ **when** $u = 2, v = 3, w = 4$

Use Equation 5 to find $\dfrac{dy}{dx}$

31. $y \cos x = x^2 + y^2$

Use Equations 6 to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$

35. $x^2 + 2y^2 + 3z^2 = 1$

Section 6: Directional Derivatives and the Gradient Vector

5, 7

Find the directional derivative of f at the given point in the direction indicated by the angle θ.

5. $f(x, y) = xy^3 - x^2, \quad (1, 2), \quad \theta = \pi/3$

7. $f(x, y) = arctan(xy), \quad (2, -3), \quad \theta = 3\pi/4$

(a) Find the gradient of f

(b) Evaluate the gradient at the point P

(c) Find the rate of change of f at P in the direction of the vector $\vec{u}$

9. $f(x, y) = x/y, \quad P(2, 1), \quad \vec{u} = \frac{3}{5}\,\hat{\mathbf{i}} + \frac{4}{5}\,\hat{\mathbf{j}}$

13, 15

Find the directional derivative of the function at the given point in the direction of the vector $\vec{v}$.

13. $f(x, y) = e^x \sin y, \quad (0, \pi/3), \quad \vec{v} = \langle -6, 8 \rangle$

15. $g(s, t) = s\sqrt{t}, \quad (2, 4), \quad \vec{v} = 2\,\hat{\mathbf{i}} - \hat{\mathbf{j}}$

21, 23

Find the directional derivative of the function at the point P in the direction of the point Q.

21. $f(x, y) = x^2y^2 - y^3, \quad P(1, 2), \quad Q(-3, 5)$

23. $f(x, y) = \sqrt{xy}, \quad P(2, 8), \quad Q(5, 4)$

27, 29

Find the maximum rate of change of f at the given point and the direction in which it occurs.

27. $f(x, y) = 5xy^2, \quad (3, -2)$

29. $f(x, y) = \sin(xy), \quad (1, 0)$

37

The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(1, 2, 2)$ is $120°$

(a) Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.

(b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin

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47-51 (odd)

Find equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.

47. $2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10, \quad (3,3,5)$

49. $xy^2z^3 = 8, \quad (2,2,1)$

51. $x + y + z = e^{xyz}, \quad (0,0,1)$

Section 7: Maximum and Minimum Values

5-21 (odd)

Find the local maximum and minimum values and saddle point(s) of the function. You are encouraged to use a calculator or computer to graph the function with a domain and viewpoint that reveals all the important aspects of the function.

5. $f(x, y) = x^2 + xy + y^2 + y$

7. $f(x, y) = 2x^2 - 8xy + y^4 - 4y^3$

9. $f(x, y) = (x - y)(1 - xy)$

11. $f(x, y) = y\sqrt{x} - y^2 - 2x + 7y$

13. $f(x, y) = x^3 - 3x + 3xy^2$

15. $f(x, y) = x^4 - 2x^2 + y^3 - 3y$

17. $f(x, y) = xy - x^2y - xy^2$

19. $f(x, y) = e^x \cos y$

21. $f(x, y) = y^2 - 2y\cos x, \quad -1 \leq x \leq 7$

33-39 (odd)

Find the absolute maximum and minimum values of f on the set D.

33. $f(x, y) = x^2 + y^2 - 2x$,

D is the closed triangular region with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$

35. $f(x, y) = x^2 + y^2 + x^2y + 4$,

$D = \{(x, y) \mid |x| \leq 1, |y| \leq 1\}$

37. $f(x, y) = x^2 + 2y^2 - 2x - 4y + 1$,

$D = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$

39. $f(x, y) = 2x^3 + y^4$,

$D = \{(x, y) \mid x^2 + y^2 \leq 1\}$

43

Find the shortest distance from the point $(2, 0, -3)$ to the plan $x + y + z = 1$.

45

Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

47

Find three positive numbers whose sum is 100 and whose product is a maximum.

55

A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of cardboard used.
