

Ch 14 - Problem Set 1

Calculus 3

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Section 1: Functions of Several Variables

3. Let $g(x, y) = x^2 \ln(x + y)$

(a) Evaluate $g(3, 1)$.

(b) Find and sketch the domain of g .

(c) Find the range of g .

Solution

7 - 15 (odd)

Find and sketch the domain of the function.

7. $f(x, y) = \sqrt{x-2} + \sqrt{y-1}$

Solution

9. $q(x, y) = \sqrt{x} + \sqrt{4-4x^2-y^2}$

Solution

11. $g(x, y) = \frac{x-y}{x+y}$

Solution

13. $p(x, y) = \frac{\sqrt{xy}}{x+1}$

Solution

15. $f(x, y, z) = \sqrt{4-x^2} + \sqrt{9-y^2} + \sqrt{1-z^2}$

Solution

17. A model for the surface area of a human body is given by the function

$$S = f(w, h) = 0.1091w^{0.425}h^{0.725}$$

where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.

(a) Find $f(160, 70)$ and interpret it.

(b) What is your own surface area?

Solution

23 - 31 (odd)

Sketch the graph of the function

23. $f(x, y) = y$

Solution

25. $f(x, y) = 10 - 4x - 5y$

Solution

27. $f(x, y) = \sin x$

Solution

29. $f(x, y) = x^2 + 4y^2 + 1$

Solution

31. $f(x, y) = \sqrt{4-4x^2-y^2}$

Solution

32. Match the function with its graph (labeled I–VI). Give reasons for your choices.

(a) $f(x, y) = \frac{1}{1 + x^2 + y^2}$

(b) $f(x, y) = \frac{1}{1 + x^2 y^2}$

(c) $f(x, y) = \ln(x^2 + y^2)$

(d) $f(x, y) = \cos \sqrt{x^2 + y^2}$

(e) $f(x, y) = |xy|$

(f) $f(x, y) = \cos(xy)$

Solution

33. A contour map for a function f is shown. Use it to estimate the values of $f(-3, 3)$ and $f(3, -2)$. What can you say about the shape of the graph?

Solution

45, 47 & 51

Draw a contour map of the function showing several level curves.

45. $f(x, y) = x^2 + y^2$

Solution

47. $f(x, y) = x^2 + y^2$

Solution

51. $f(x, y) = x^2 + y^2$

Solution

53. Sketch both a contour map and a graph of the given function and compare them.

$$f(x, y) = x^2 + 9y^2$$

Solution

61 - 66

Match the function (a) with its graph (labeled A–F below) and (b) with its contour map (labeled I–VI). Give reasons for your choices.

61. $z = \sin(xy)$

Solution

62. $z = e^x \cos y$

Solution

63. $z = \sin(x - y)$

Solution

64. $z = \sin x - \sin y$

Solution

65. $z = (1 - x^2)(1 - y^2)$

Solution

66. $z = \frac{x - y}{1 + x^2 + y^2}$

Solution

67. Describe the level surfaces of the function.

$$f(x, y, z) = 2y - z + 1$$

Section 2: Limits and Continuity

5 - 11 (odd)

Find the limit

5. $\lim_{(x,y) \rightarrow (3,2)} (x^2y^3 - 4y^2)$

Solution

7. $\lim_{(x,y) \rightarrow (-3,1)} \frac{x^2y - xy^3}{x - y - 2}$

Solution

9. $\lim_{(x,y) \rightarrow (\pi, \pi/2)} y \sin(x - y)$

Solution

11. $\lim_{(x,y) \rightarrow (1,1)} \left(\frac{x^2y^3 - x^3y^2}{x^2 - y^2} \right)$

Solution

13 - 17 (odd)

Show that the limit does not exist

13. $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2 + y^2}$

Solution

15. $\lim_{(x,y) \rightarrow (0,0)} \frac{(x + y)^2}{x^2 + y^2}$

Solution

17. $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$

Solution

19 - 25 (odd)

Find the limit, if it exists, or show that the limit does not exist.

19. $\lim_{(x,y) \rightarrow (-1,-2)} (x^2y - xy^2 + 3)^3$

Solution

21. $\lim_{(x,y) \rightarrow (2,3)} \frac{3x - 2y}{4x^2 - y^2}$

Solution

23. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2 \cos y}{x^2 + y^4}$

Solution

25. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2} + 1} - 1$

Solution

31 & 33

Use the Squeeze Theorem to find the limit.

31. $\lim_{(x,y) \rightarrow (0,0)} xy \sin \frac{1}{x^2 + y^2}$

Solution

33. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^4 + y^4}$

Solution

41, 43 & 45

Determine the set of points at which the function is continuous.

41. $F(x, y) = \frac{xy}{1 + e^{x-y}}$

Solution

43. $F(x, y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2}$

Solution

45. $G(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$

Solution