Ch 14 - Problem Set 1

Calculus 3

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Section 1: Functions of Several Variables

3. Let $g(x, y) = x^2 \ln(x + y)$
(a) Evaluate $g(3, 1)$.
(b) Find and sketch the domain of g.
(c) Find the range of g.

Solution

7 - 15 (odd)

Find and sketch the domain of the function.

7. $f(x, y) = \sqrt{x - 2} + \sqrt{y - 1}$

Solution

9. $q(x, y) = \sqrt{x} + \sqrt{4 - 4x^2 - y^2}$

Solution

11. $g(x, y) = \dfrac{x - y}{x + y}$

Solution

13. $p(x, y) = \dfrac{\sqrt{xy}}{x + 1}$

Solution

15. $f(x, y, z) = \sqrt{4 - x^2} + \sqrt{9 - y^2} + \sqrt{1 - z^2}$

Solution

17. A model for the surface area of a human body is given by the function

$$S = f(w, h) = 0.1091 w^{0.425} h^{0.725}$$

where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.
(a) Find $f(160, 70)$ and interpret it.
(b) What is your own surface area?

Solution

23 - 31 (odd)

Sketch the graph of the function

23. $f(x, y) = y$

Solution

25. $f(x, y) = 10 - 4x - 5y$

Solution

27. $f(x, y) = \sin x$

Solution

29. $f(x, y) = x^2 + 4y^2 + 1$

Solution

31. $f(x, y) = \sqrt{4 - 4x^2 - y^2}$

Solution

32. Match the function with its graph (labeled I–VI). Give reasons for your choices.

(a) $f(x,y) = \dfrac{1}{1+x^2+y^2}$ **(b)** $f(x,y) = \dfrac{1}{1+x^2y^2}$

(c) $f(x,y) = \ln(x^2+y^2)$ **(d)** $f(x,y) = \cos\sqrt{x^2+y^2}$

(e) $f(x,y) = |xy|$ **(f)** $f(x,y) = \cos(xy)$

Solution

33. A contour map for a function f is shown. Use it to estimate the values of $f(-3,3)$ and $f(3,-2)$. What can you say about the shape of the graph?

Solution

45, 47 & 51

Draw a contour map of the function showing several level curves.

45. $f(x,y) = x^2+y^2$

Solution

47. $f(x,y) = x^2+y^2$

Solution

51. $f(x,y) = x^2+y^2$

Solution

53. Sketch both a contour map and a graph of the given function and compare them.

$$f(x,y) = x^2 + 9y^2$$

Solution

61 - 66

Match the function (a) with its graph (labeled A–F below) and (b) with its contour map (labeled I–VI). Give reasons for your choices.

61. $z = \sin(xy)$

Solution

62. $z = e^x \cos y$

Solution

63. $z = \sin(x-y)$

Solution

64. $z = \sin x - \sin y$

Solution

65. $z = (1-x^2)(1-y^2)$

Solution

66. $z = \dfrac{x-y}{1+x^2+y^2}$

Solution

67. Describe the level surfaces of the function.

$$f(x, y, z) = 2y - z + 1$$

Section 2: Limits and Continuity

5 - 11 (odd)

Find the limit

5. $\lim_{(x,y)\to(3,2)}(x^2y^3 - 4y^2)$

Solution

7. $\lim_{(x,y)\to(-3,1)} \dfrac{x^2y - xy^3}{x - y - 2}$

Solution

9. $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x - y)$

Solution

11. $\lim_{(x,y)\to(1,1)} \left(\dfrac{x^2y^3 - x^3y^2}{x^2 - y^2}\right)$

Solution

13 - 17 (odd)

Show that the limit does not exist

13. $\lim_{(x,y)\to(0,0)} \dfrac{y^2}{x^2 + y^2}$

Solution

15. $\lim_{(x,y)\to(0,0)} \dfrac{(x + y)^2}{x^2 + y^2}$

Solution

17. $\lim_{(x,y)\to(0,0)} \dfrac{y^2\sin^2 x}{x^4 + y^4}$

Solution

19 - 25 (odd)

Find the limit, if it exists, or show that the limit does not exist.

19. $\lim_{(x,y)\to(-1,-2)}(x^2y - xy^2 + 3)^3$

Solution

21. $\lim_{(x,y)\to(2,3)} \dfrac{3x - 2y}{4x^2 - y^2}$

Solution

23. $\lim_{(x,y)\to(0,0)} \dfrac{xy^2\cos y}{x^2 + y^4}$

Solution

25. $\lim_{(x,y)\to(0,0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

Solution

31 & 33

Use the Squeeze Theorem to find the limit.

31. $\lim_{(x,y)\to(0,0)} xy \sin \dfrac{1}{x^2+y^2}$

Solution

33. $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^4+y^4}$

Solution

41, 43 & 45

Determine the set of points at which the function is continuous.

41. $F(x,y) = \dfrac{xy}{1+e^{x-y}}$

Solution

43. $F(x,y) = \dfrac{1+x^2+y^2}{1-x^2-y^2}$

Solution

45. $G(x,y) = \sqrt{x} + \sqrt{1-x^2-y^2}$

Solution