Chapter 32 Light: Reflection and Refraction

PHYS-102

Conceptual Questions

2. Archimedes is said to have burned the whole Roman fleet in the harbor of Syracuse by focusing the rays of the Sun with a huge spherical mirror. Is this reasonable?

Yes, having a huge spherical mirror, would result in many rays of the sun to be at a focus point where the object it points at will absorb these rays (and energy conversion to thermal energy)

3. What is the focal length of a plane mirror? What is the magnification of a plane mirror?

Since the mirror equation holds true for a plane mirror, the focal length is $f = \frac{r}{2}$ where $r = \infty$ since the radius of a flat surface is infinity. This would result in the focal length also to be infinity. The magnification of a plane mirror would be 1 since there is equal distance between object and image

4. An object is placed along the principal axis of a spherical mirror. The magnification of the object is -3.0. Is the image real or virtual, inverted or upright? Is the mirror concave or convex? On which side of the mirror is the image located?

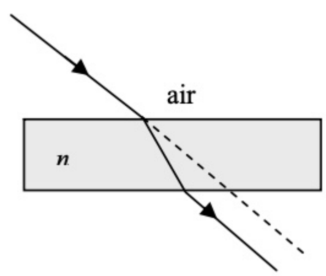
The image is real and inverted, because the magnification is negative. The mirror is concave because convex can only form virtual images. The image is on the same side of the mirror as the object; real images are formed by converging light rays and light rays cannot actually pass through a mirror

7. If a concave mirror produces a real image, is the image necessarily inverted?

Yes. When a concave mirror produces a real image, both d_0 and d_i are positive which would indicate a negative magnification by the equation for magnification $m = -\dfrac{d_i}{d_0}$. A negative magnification means the image is inverted

8. How might you determine the speed of light in a solid, rectangular, transparent object?

Rays entering the object will have an incident angle and a resulting angle of refraction. Using Snell's law, we know one of the mediums is air, and we would solve for the other medium, n_2, assuming, geometrically, we can find the angle of the refracted ray with respect to the normal. We can then find the speed of light in the object using the index of refraction and its relationship with c in a vacuum given by $n = \dfrac{c}{v} \quad \Rightarrow \quad v = \dfrac{c}{n}$

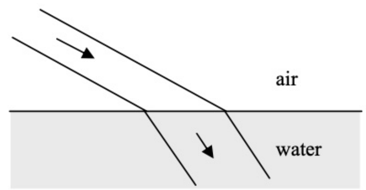


11. What is the angle of refraction when a light ray is incident perpendicular to the boundary between two transparent materials?

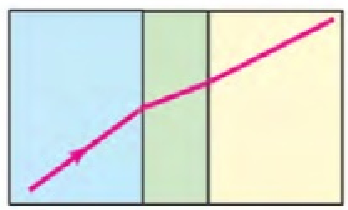
The angle of refraction and angle of incidence are both zero in this case.

14. When a wide beam of parallel light enters water at an angle, the beam broadens. Explain.

Because the broad beam hits the surface of the water at an angle, it illuminates an area of the surface that is wider than the beam width. Light form the beam bends towards the normal. The refracted beam is wider than the incident beam because one edge of the beam strikes the surface first, while the other edge travels farther in the air.



17. A ray of light is refracted through three different materials. Rank the materials according to their index of refraction, least to greatest.



Let each section be 1, 2, 3 from left to right. From 1 to 2, the beam bends towards the normal, which means that the index of refraction of 2 is greater than 1. When 2 goes to 3, the beam bends away from normal but not enough to get back to parallel with 1. So that means the index of refraction is less than 2, but greater than 1. Therefore, from least to greatest would be: 1, 3, 2.

18. Can a light ray traveling in air be totally reflected when it strikes a smooth water surface if the incident angle is chosen correctly? Explain.

No. Total Internal Reflection can only occur if traveling from a medum of higher index of refraction to lower.

20. What type of mirror is shown?

The mirror image is virtual and upright and the object is standing inside the focal point with a seemingly positive magnification so, the mirror is concave. A convex mirror woulds still result in a virtual, upright image, but it would be smaller.

Problems

32-3 Spherical Mirrors

9. A solar cooker, really a concave mirror pointed at the Sun, focuses the Sun's rays 18.8 cm in front of the mirror. What is the radius of the spherical surface from which the mirror was made?

Since $r = \frac{r}{2}$ and we are given f

$$r = 2f = 2(18.8\ cm)$$
$$\boxed{r = 37.6\ cm}$$

10. How far from a concave mirror (radius 24.0 cm) must an object be placed if its image is to be at infinity?

For an object to be at infinity, it must be at the focal point.

$$f = \frac{r}{2} = \frac{24.0\ cm}{2}$$
$$\boxed{f = 12.0\ cm}$$

13. You look at yourself in a shiny 9.2-cm-diameter Christmas tree ball. If your face is 25.0 cm away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?

Assuming a convex mirror (*ball*), $d_0 = 25.0$ cm, $r = \frac{1}{2}d = -4.6$ cm [convex mirror so radius is negative], $f = \frac{r}{2} = -2.3$ cm

Using the mirror equation,

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$
$$\frac{1}{25.0\ cm} + \frac{1}{d_i} = \frac{1}{-2.3\ cm}$$
$$\frac{1}{d_i} = -\frac{1}{2.3\ cm} - \frac{1}{25.0\ cm}$$
$$d_i \approx -2.1\ cm$$

If we find the magnification, we find that it is positive.

$$m = -\frac{d_i}{d_0} m = -\frac{-2.1\ cm}{25.0\ cm} \approx +0.084$$

$\boxed{\therefore \text{ we find that the image is virtual, upright and 2.1-cm behind the surface of the Christmas tree ball.}}$

16. Some rearview mirrors produce images of cars to your rear that are smaller than they would be if the mirror were flat. Are the mirrors concave or convex? What is a mirror's radius of curvature if cars 18.0 m away appear 0.33 their normal size?

Convex, and if $m = -\frac{d_i}{d_0}$,

$$\begin{aligned} 0.33 &= -\frac{d_i}{18.0\ m} \\ d_i &= -5.94\ m \end{aligned}$$

Using the mirror equation,

$$\begin{aligned} \frac{1}{d_0} + \frac{1}{d_i} &= \frac{1}{f} \\ \frac{1}{18.0\ m} - \frac{1}{5.94\ m} &= \frac{1}{f} \\ f &\approx -8.866\ m \end{aligned}$$

$$\boxed{\therefore r = 2f = 2(-8.866\ m) = -17.7\ m}$$

26. A shaving or makeup mirror is designed to magnify your face by a factor of 1.35 when your face is placed 20.0 cm in front of it. (a) What type of mirror is it? (b) Describe the type of image that it makes of your face. (c) Calculate the required radius of curvature for the mirror.

(a) To produce a larger upgright image, it needs to be a <u>concave</u>

(b) The image will be upright and virtual

(c) From the magnification,

$$\begin{aligned} m &= \frac{d_i}{d_0} \\ 1.35 &= \frac{d_i}{20.0\ cm} \\ d_i &\approx -27.0\ cm \end{aligned}$$

Using the mirror equation,

$$\begin{aligned} \frac{1}{d_0} + \frac{1}{d_i} &= \frac{1}{f} \\ \frac{1}{20.0\ cm} - \frac{1}{27.0\ cm} &= \frac{1}{f} \\ f &\approx 77.143\ cm \end{aligned}$$

$$\boxed{\therefore r = 2f = 2(77.143\ cm) \approx 154\ cm}$$

32-4 Index of Refraction

32. The speed of light in ice is 2.29×10^8 m/s. What is the index of refraction of ice?

Let $n = \frac{c}{v}$

$$n_{ice} = \frac{c}{v} = \frac{3.00 \times 10^8 \ m/s}{2.29 \times 10^8 \ m/s}$$
$$\boxed{n_{ice} \approx 1.31}$$

33. What is the speed of light in (a) ethyl alcohol, (b) lucite, (c) crown glass?

(a)

$$v = \frac{c}{n} = \frac{3.0 \times 10^8 m/s}{1.36}$$
$$\boxed{v = 2.21 \times 10^8 m/s}$$

(b)

$$v = \frac{c}{n} = \frac{3.0 \times 10^8 m/s}{1.51}$$
$$\boxed{v = 1.99 \times 10^8 m/s}$$

(c)

$$v = \frac{c}{n} = \frac{3.0 \times 10^8 m/s}{1.52}$$
$$\boxed{v = 1.97 \times 10^8 m/s}$$

34. Our nearest star (other than the Sun) is 4.2 light years away. That is, it takes 4.2 years for the light to reach Earth. How far away is it in meters?

distance = rate x time,

$$d = c\Delta t$$
$$d = \frac{3.00 \times 10^8 \ m}{1s} \times \frac{4.2 \ yr}{1} \times \frac{3.16 \times 10^7 s}{1 \ yr}$$
$$\boxed{d \approx 4.0 \times 10^{16} \ m}$$

35. How long does it take light to reach us from the Sun, 1.50×10 km away?

$$d = c\Delta t$$
$$\Delta t = \frac{d}{c} = \frac{(1.50 \times 10^8 \ km)(1000 \frac{m}{km})}{3.0 \times 10^8 \ m/s}$$
$$\boxed{\Delta t \approx 5.00 \times 10^2 s \approx 8.33 \ min}$$

36. The speed of light in a certain substance is 88% of its value in water What is the index of refraction of that substance?

Index of refraction of water is $n_1 = 1.33$ and our mystery substance $n_2 =?$

$$v_? = 0.88(v_{h_2o}) \qquad \left[v = \frac{c}{n}\right]$$

$$\frac{c}{n_2} = 0.88\left(\frac{c}{n_1}\right)$$

$$\frac{1}{n_2} = \frac{0.88}{n_1}$$

$$\frac{1}{n_2} = \frac{0.88}{1.33}$$

$$\boxed{n_2 \approx 1.51 \text{ most likely lucite}}$$

32-5 Snell's Law

38. A diver shines a flashlight upward from beneath the water at a 38.5° angle to the vertical. At what angle does the light leave the water?

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, angle that light leaves water is θ_2

water(n_1) $\Rightarrow$ air(n_2)

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1.33}{1.00}\sin 38.5^\circ\right)$$
$$\boxed{\theta_2 \approx 55.9^\circ}$$

39. A flashlight beam strikes the surface of a pane of glass (n = 1.56) at a 63° angle to the normal. What is the angle of refraction?

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, angle of refraction is θ_2

air(n_1) $\Rightarrow$ glass(n_2)

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1.00}{1.56}\sin 63^\circ\right)$$
$$\boxed{\theta_2 \approx 34.8^\circ \approx 35^\circ}$$

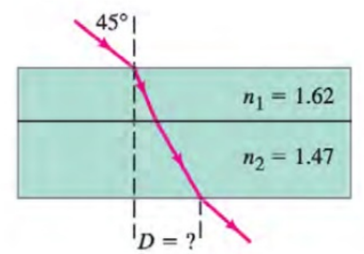
41. A light beam coming from an underwater spotlight exits the water at an angle of 56.0°. At what angle of incidence did it hit the air-water interface from below the surface?

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, incident angle is θ_1

air(n_1) $\Rightarrow$ water(n_2)

$$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)$$
$$\theta_1 = \sin^{-1}\left(\frac{1.00}{1.33}\sin 56.0^\circ\right)$$
$$\boxed{\theta_1 \approx 38.6^\circ}$$

43. A light beam strikes a 2.0-cm-thick piece of plastic with a refractive index of 1.62 at a 45° angle. The plastic is on top of a 3.0-cm-thick piece of glass for which n = 1.47. What is the distance D?



Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, multiple iterations needed

$\text{air}(n_1) \Rightarrow n_2 = 1.62$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1}{1.62}\sin 45°\right)$$
$$\theta_2 \approx 25.87987°$$

$n_1 = 1.62 \Rightarrow n_2 = 1.47$

$$\theta_2 = \sin^{-1}\left(\frac{1.62}{1.47}\sin 25.87987°\right)$$
$$\theta \approx 28.75237°$$

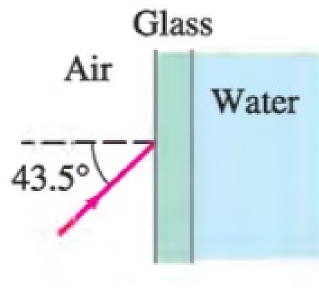
Using trigonometric identities,

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta_1 = \frac{D_1}{h_1} \qquad \tan\theta_2 = \frac{D_2}{h_2}$$
$$D_1 = h_1\tan\theta_1 \qquad D_2 = h_2\tan\theta_2$$

If $D = D_1 + D_2$,

$$D = h_1\tan\theta_1 + h_2\tan\theta_2$$
$$D = (2.0\ cm)\tan 25.87987° + (3.0\ cm)\tan 28.75237°$$
$$\boxed{D \approx 2.6\ cm}$$

44. An aquarium filled with water has flat glass sides whose index of refraction is 1.56. A beam of light from outside the aquarium strikes the glass at a 43.5° angle to the perpendicular. What is the angle of this light ray when it enters (a) the glass, and then (b) the water? (c) What would be the refracted angle if the ray entered the water directly?



(a) air(n_1) $\Rightarrow$ glass(n_2)

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1.00}{1.56}\sin 43.5^\circ\right)$$
$$\boxed{\theta_2 = 26.2^\circ}$$

(b) glass(n_1) $\Rightarrow$ water(n_2)

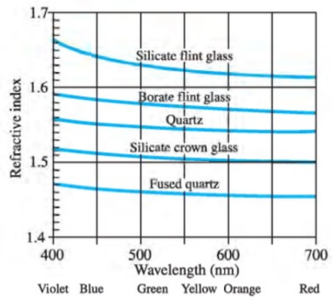
$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1.56}{1.33}\sin 26.2^\circ\right)$$
$$\boxed{\theta_2 = 31.2^\circ}$$

(c) air(n_1) $\Rightarrow$ water(n_2)

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1.00}{1.33}\sin 43.5^\circ\right)$$
$$\boxed{\theta_2 = 31.2^\circ}$$

32-6 Visible Spectrum; Dispersion

52. By what percent is the speed of blue light (450 nm) less than the speed of red light (680 nm), in silicate flint glass



$$\frac{v_{red} - v_{blue}}{v_{red}} = \frac{\dfrac{c}{n_{red}} - \dfrac{c}{n_{blue}}}{\dfrac{c}{n_{red}}} = \frac{\dfrac{1}{1.613} - \dfrac{1}{1.643}}{\dfrac{1}{1.613}}$$
$$\boxed{\approx 0.01826 \approx 1.8\%}$$

32-7 Total Internal Reflection

57. What is the critical angle for the interface between water and diamond? To be internally reflected, the light must start in which material?

θ_c is given by $\sin\theta_c = \dfrac{n_2}{n_1}\sin 90^\circ = \dfrac{n_2}{n_1}$

water(n_1) $\Rightarrow$ diamond(n_2)

$$\theta_c = \sin -1\left(\frac{n_2}{n_1}\right)$$
$$\theta_c = \sin -1\left(\frac{2.42}{1.33}\right)$$

Error at this angle which means there is total internal reflection

diamond(n_1) $\Rightarrow$ water(n_2)

$$\theta_c = \sin -1\left(\frac{n_2}{n_1}\right)$$
$$\theta_c = \sin -1\left(\frac{1.33}{2.42}\right)$$
$$\theta_c = 33.3^\circ$$

$$\boxed{\therefore \text{To be internally reflected, the light must start with diamond}}$$

58. The critical angle for a certain liquid-air surface is 49.6°. What is the index of refraction of the liquid?

liquid$(n_1) \Rightarrow$ air(n_2)

$$\sin 49.6^\circ = \frac{n_2}{n_1}$$
$$n_1 = \frac{1.00}{\sin 49.6^\circ}$$

$$\boxed{n_1 \approx 1.31}$$

61. A beam of light is emitted 8.0 cm beneath the surface of a liquid and strikes the surface 7.6 cm from the point directly above the source. If total internal reflection occurs, what can you say about the index of refraction of the liquid?

$$\tan \theta_1 = \frac{7.6\ cm}{8.0\ cm}$$
$$\text{Incident Angle} = \theta_1 = \tan^{-1}\left(\frac{7.6\ cm}{8.0\ cm}\right)$$
$$\theta_1 \approx 43.5^\circ$$

Relationship for the max incident angle for refraction from liquid into air

$$n_1 \sin \theta_{max} = n_2 \sin 90^\circ$$
$$n_1 \sin \theta_{max} = n_2$$

So,

$$\sin \theta_1 \geq \sin \theta_{max} = \frac{1.00}{n_{liquid}}$$
$$\sin \theta_1 \geq \frac{1.00}{n_{liquid}}$$
$$0.688355 \geq \frac{1.00}{n_{liquid}}$$

$$\boxed{\therefore n_{liquid} \geq 1.5}$$