Chapter 33 Lenses and Optical Instruments

PHYS-102

Conceptual Questions

1. Where must the film be placed if a camera lens is to make a sharp image of an object far away?

The film must be places behind the lens at the focal length of the lens.

3. Can a diverging lens form a real image under any circumstances? Explain.

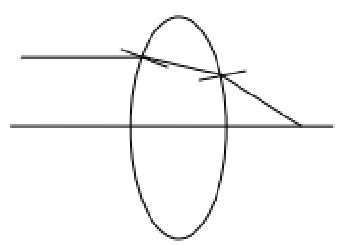
By itself, no, because by definition a diverging lens causes light rays to diverge which will not bring the rays to a focus point required for a real image. However, with the combination of a "stronger" converging lens, we can bring the rays to a focus point to form a real image.

8. Compare the mirror equation with the thin lens equation. Discuss similarities and differences, especially the sign conventions for the quantities involved.

The mirror equation and the lens equation are identical. According to the sign conventions, d ¿ 0 indicates a real object or image and d ¡ 0 indicates a virtual object or image, for both mirrors and lenses. But the positions of the objects and images are different for a mirror and a lens. For a mirror, a real object or image will be in front of the mirror and a virtual object or image will be behind the mirror. For a lens, a real image will be on the opposite side of the lens from a real object, and a virtual image will be on the same side of the lens as the real object.

14. The thicker a double convex lens is in the center as compared to its edges, the shorter its focal length for a given lens diameter. Explain.

A double convex lens causes light rays to converge because the light bends towards the normal as it enters the lens and away from the normal as it exits the lens. The result, due to the curvature of the sides of the lens, is that the light bends towards the principal axis at both surfaces. The more strongly the sides of the lens are curved, the greater the bending, and the shorter the focal length.



19. You can tell whether people are nearsighted or farsighted by looking at the width of their fa e through their glasses. If a person's face appears narrower through the glasses, is the person farsighted or nearsighted?

Nearsighted. Diverging lenses are used to correct nearsightedness and converging lenses are used to correct farsightedness. If the person's face appears narrower through the glasses, then the image of the face produced by the lenses is smaller than the face, virtual, and upright. Thus, the lenses must be diverging, and therefore the person is nearsighted.

20. The human eye is much like a camera - yet, when a camera shutter is left open and the camera is moved, the image will be blurred. But when you move your head with your eyes open, you still see clearly. Explain.

All light entering the camera lens while the shutter is open contributes to a single picture. If the camera is moved while the shutter is open, the position of the image on the film moves. The new image position overlaps the previous image position, causing a blurry final image. With the eye, new images are continuously being formed by the nervous system, so images do not "build up" on the retina and overlap with each other.

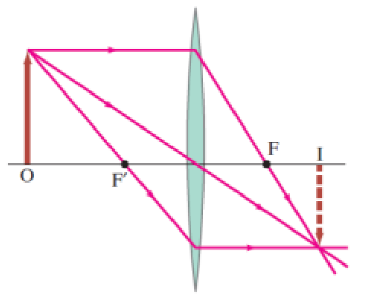
26. For both converging and diverging lenses discuss how the focal length from red light differs from that for violet light

For both converging and diverging lenses, the focal point for violet light is closer to the lens than the focal point for red light. The index of refraction for violet light is slightly greater than for red light for glass, so the violet light bends more, resulting in a smaller magnitude focal length.

Problems

33-1 and 33-2 Thin Lenses

1. A sharp image is located 373 mm behind a 215 mm focal length converging lens. Find the object distance (a) using a ray diagram, (b) by calculation.



(a)

$$\boxed{\approx 480cm}$$

(b) Converging lens so f is positive, real image so d_i is positive, negative height so image is inverted

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$$
$$d_0 = \frac{fd_i}{d_i - f} = \frac{(215\ mm)(373\ mm)}{373\ mm - 215\ mm}$$
$$\boxed{d_o \approx 508\ mm}$$

2. Sunlight is observed to focus at a point 18.5 cm behind a lens, (a) What kind of lens is it? (b) What is its power in diopters?

(a) Converging lens since it is focused at a point behind the lens

(b) Let $P = \frac{1}{f}$ (f *for and abject at infinity* $= F$)

$$P = \frac{1}{0.185\ m}$$
$$\boxed{P \approx 5.41D}$$

4. A certain lens focuses an object 1.85 m away as an image 48.3 cm on the other side of the lens. What type of lens is it, and what is its focal length? Is the image real or virtual?

Converging lens and the image is real

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$f = \frac{d_o d_i}{d_i - d_o} = \frac{(1.85\ m)(0.483\ m)}{0.483\ m - 1.85\ m}$$
$$\boxed{f \approx 0.382\ m}$$

6. A stamp collector uses a converging lens with focal length 28 cm to view a stamp 18 cm in front of the lens. (a) Where is the image located? (b) What is the magnification?

(a)

$$\begin{aligned}\frac{1}{d_i} &= \frac{1}{f} - \frac{1}{d_o} = \frac{fd_o}{d_o - f}\\ d_i &= \frac{(28\ cm)(18\ cm)}{18\ cm - 28\ cm}\\ &\boxed{d_i \approx -50\ cm}\end{aligned}$$

∴ The image is virtual and is behind the lens

(b)

$$\begin{aligned}m &= -\frac{d_i}{d_o} = -\frac{50\ cm}{18\ cm}\\ &\boxed{m = +2.8\times}\end{aligned}$$

∴ The image is upright and is roughly three times larger

10. (a) How far from a 50.0 mm focal length lens must an object be placed if its image is to be magnified 2.50X and be real? (b) What if the image is to be virtual and magnified 2.50X?

(a) For the image to be real, the image distance has to be positive, so the magnification has to be negative

$$-2.50 = -\frac{d_i}{d_o} \quad \Rightarrow \quad d_i = 2.50d_o$$

Using the thin lens equation,

$$\begin{aligned}\frac{1}{50\ mm} &= \frac{1}{d_o} + \frac{1}{2.50d_o} \quad \Rightarrow \quad \frac{1}{50\ mm} = \frac{1}{d_o}\left(1 + \frac{1}{2.50}\right)\\ d_o &= 50\ mm\left(1 + \frac{1}{2.50}\right)\\ &\boxed{d_o \approx 70\ mm}\end{aligned}$$

(b) Same process but instead of a negative magnification, we need a positive one, meaning d_i is negative

$$\begin{aligned}d_i &= -2.50d_o\\ \frac{1}{50\ mm} &= \frac{1}{d_o} - \frac{1}{2.50d_o} \quad \Rightarrow \quad \frac{1}{50\ mm} = \frac{1}{d_o}\left(1 - \frac{1}{2.50}\right)\\ d_o &= 50\ mm\left(1 - \frac{1}{2.50}\right)\\ &\boxed{d_o \approx 30\ mm}\end{aligned}$$

12. (a) A 2.80 cm-high insect is 1.30 m from a 135 mm focal length lens. Where is the image, how high is it, and what type is it? (b) What if f = -135 mm?

(a) Using the lens equation to find the image distance,

$$d_i = \frac{fd_o}{d_o - f} = \frac{(0.135\ m)(1.30\ m)}{1.30\ m - 0.135\ m}$$
$$\boxed{d_i \approx 0.151\ m}$$
$$m = -\frac{d_i}{d_o} \quad \Rightarrow \quad m = \frac{h_i}{h_o} \quad \Rightarrow \quad \therefore -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$
$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(0.151\ m)(2.80\ cm)}{1.30\ m}$$
$$\boxed{h_i \approx -0.324\ m}$$

$\therefore$ The image is 0.151 m behind the lens, is real, and is inverted

(b) A negative focal length hints that it is now a diverging lens

Using the lens equation (for a diverging lens) to find the image distance

$$d_i = \frac{fd_o}{f - d_o} = \frac{(0.135\ m)(1.30\ m)}{0.135\ m - 1.30\ m} \approx -0.151\ m$$
$$m = -\frac{d_i}{d_o} \quad \Rightarrow \quad m = \frac{h_i}{h_o} \quad \Rightarrow \quad \therefore -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$
$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(-0.151\ m)(2.80\ cm)}{1.30\ m}$$
$$\boxed{h_i \approx +0.324\ m}$$

13. A bright object and a viewing screen are separated by a distance of 86.0 cm. At what location(s) between the object and the screen should a lens of focal length 16.0 cm be placed in order to produce a sharp image on the screen? [Hint, first draw a diagram.]

The sum of the object and image distances must be the distance between object and screen, which we label as d_T. We solve this relationship for the image distance, and then use that to find object distance

$$d_T = d_i + d_o \quad \Rightarrow \quad d_i = d_T - d_o; \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_T - d_o}$$
$$\frac{1}{d_o} + \frac{1}{d_T - d_o} - \frac{1}{f} = 0$$
$$\frac{f(d_T - d_o) + fd_o - d_o(d_T - d_o)}{fd_o(d_T - d_o)} = \frac{fd_T - fd_o + fd_o - d_T d_o + d_o^2}{fd_o(d_T - d_o)} = 0$$
$$\frac{d_o^2 - d_T d_o + fd_T}{fd_o(d_T - d_o)} = 0$$

so,

$$d_o^2 - d_T d_o + fd_T = 0$$

Using the quadratic formula,

$$d_o = \frac{d_T \pm \sqrt{d_T^2 - 4fd_T}}{2}$$
$$d_o = \frac{(86.0\ cm) \pm \sqrt{(86.0\ cm)^2 - 4(16.0\ cm)(86.0\ cm)}}{2}$$
$$\boxed{d_o = 64.7\ cm,\ 21.3\ cm}$$

Note that $d_T^2 - 4fd_T > 0 \quad \Rightarrow \quad d_T > 4f$ so, $D = \{(d_T, f) \mid d_T > 4f\}$

14. How far apart are an object and an image formed by an 85 cm focal length converging lens if the image is 2.95X larger than the object and is real?

Thin lens equation to find image distance using the relationship for image distance using magnification (for a real image $d_o = +$, $d_i = +$, so $m = -$)

$$m = -\frac{d_i}{d_o} \quad \Rightarrow \quad d_i = -md_o = -(-2.95)d_o = 2.95d_o$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{2.95d_o}$$
$$\frac{1}{85\ cm} = \frac{2.95+1}{2.95d_o} \quad \Rightarrow \quad d_o = \frac{3.95}{2.95}(85\ cm) \approx 113.8\ cm$$

so,

$$d_i = 2.95d_o = 2.95(113.8\ cm) \approx 335.8\ cm$$
$$\boxed{d_T = d_i + d_o = 113.8\ cm + 335.8\ cm \approx 450\ cm}$$

33-3 Lens Combinations

20. A diverging lens with f = 33.5 cm is placed 14.0 cm behind a converging lens with f = 20.0 cm. Where will an object at infinity be focused?

The image distance of the first lens will be the object distance for the second lens. Object at $\infty \rightarrow d_o = \infty$ for first lens.

$$\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} \quad \Rightarrow \quad \frac{1}{d_{i1}} = \frac{1}{20.0\ cm} - \frac{1}{\infty}$$
$$d_{i1} = 20.0\ cm$$

Since this image is behind the second lens, the object distance for the second lens is negative, and so, $d_{o2} = 14.0\ cm - 20.0\ cm = -6.0\ cm$

$$\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}} \quad \Rightarrow \quad d_{i2} = \frac{f_2 d_{o2}}{d_{o2} - f_2}$$
$$d_{i2} = \frac{(-33.5\ cm)(-6.0\ cm)}{-6.0\ cm - (-33.5\ cm)} \approx 7.31\ cm$$

$\boxed{\therefore \text{ The image is 7.31 cm behind the second lens and is real}}$

21. Two 25.0 cm focal length converging lenses are placed 16.5 cm apart. An object is placed 35.0 cm in front of one lens. Where w ill the final image formed by the second lens be located? What is the total magnification?

Let $d_{o1} = 35.0\ cm$, $f = 25.0\ cm$ for lens 1

$$d_i = \frac{f_1 d_{o1}}{d_{o1} - f_1} = \frac{(25.0\ cm)(35.0\ cm)}{35.0\ cm - 25.0\ cm} \approx 87.5\ cm$$
$$m_1 = -\frac{d_{i1}}{d_{o1}} = -\frac{87.5\ cm}{35.0\ cm} = -2.5\times$$

Let $d_{o2} = 16.5\ cm - 87.5\ cm = -71.0\ cm$, $f = 25.0\ cm$ for lens 2

$$d_i = \frac{f_1 d_{o1}}{d_{o1} - f_1} = \frac{(25.0\ cm)(-71.0\ cm)}{-71.0\ cm - 25.0\ cm} \approx 18.5\ cm$$
$$m_2 = -\frac{d_{i1}}{d_{o1}} = -\frac{18.5\ cm}{-71.0\ cm} = +0.260\times$$

Total magnification,

$$m_T = m_1 m_2 = (-2.5\times)(0.260\times) = -0.651\times$$

$\boxed{\therefore \text{ The final image is 18.5 cm behind the second lens, real, inverted, and about 35\% smaller}}$

26. A diverging lens is placed next to a converging lens of focal length f_C, as in Fig. 33 15. If f_T represents the focal length of the combination, show that the focal length of the diverging lens, f_D, is given by

$$\frac{1}{f_D} = \frac{1}{f_T} - \frac{1}{f_C}$$

Assuming that the initial object is at $d_{o1} = \infty$

$$\frac{1}{f_C} = \frac{1}{\infty} + \frac{1}{d_{i1}} \quad \Rightarrow \quad f_C = d_{i1}$$

The image distance of the first lens becomes the object distance of lens 2 (note that the image is behind the second lens, so $d_{o2} = -f_C$)

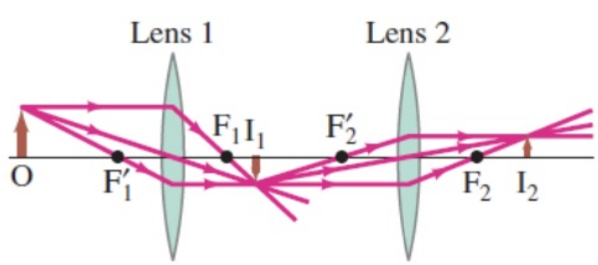
$$\frac{1}{f_D} = -\frac{1}{f_C} + \frac{1}{d_{i2}}$$

Since the object was at ∞ the second image distance must be at the combined focal length, $\therefore$ $d_{i2} = f_T$

$$\boxed{\frac{1}{f_D} = \frac{1}{f_T} - \frac{1}{f_C}}$$

27. A lighted candle is placed 36 cm in front of a converging lens of focal length $f_1 = 13$ cm, which in turn is 56 cm in front of another converging lens of focal length $f_2 = 16$ cm. (a) Draw a ray diagram and estimate the location and the relative size of the final image. (b) Calculate the position and relative size of the final image.

(a) We see that the image is real and upright. We estimate that it is 30 cm beyond the second lens, and that the final image height is half the original object height.



(b) First lens,

$$\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} \quad \Rightarrow \quad d_{i1} = \frac{f_1 d_{o1}}{d_{o1} - f_1} = \frac{(13\ cm)(36\ cm)}{36\ cm - 13\ cm} \approx 23.4\ cm$$

Image for lens 1 becomes the object for lens 2 (note the distance between the lenses, and that d_{i1} describes the distance of the image from lens 1, not the distance from lens 2), so, $d_{o2} = 56\ cm - 20.35\ cm = 35.65\ cm$

$$d_{i2} = \frac{f_2 d_{o2}}{d_{o2} - f_2} = \frac{(16\ cm)(35.65\ cm)}{35.65\ cm - 16\ cm} \approx 29\ cm$$

$$m_T = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \left(-\frac{20.35\ cm}{36\ cm}\right)\left(-\frac{29.028\ cm}{35.65\ cm}\right) \approx 0.46\times$$

$\boxed{\therefore \text{ The final image is about 29 cm behind the second lens, real, upright, and about 54\% smaller}}$

33-4 Lensmaker's Equation

28. A double concave lens has surface radii of 33.4 cm and 28.8 cm. What is the focal length if n = 1.58

Using the lensmaker's equation, (note that a double concave lens means a diverging lens, so f is negative meaning radii are negative as well $f = \frac{R}{2}$)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{R_2 + R_1}{R_1 R_2}\right) \quad \Rightarrow \quad f = \frac{R_1 R_2}{(n-1)(R_2 + R_1)}$$

$$f = \frac{(-33.4\ cm)(-28.8\ cm)}{(1.58-1)(-33.4\ cm + (-28.8\ cm))}$$

$$\boxed{f \approx -26.7\ cm}$$

29. Both surfaces of a double convex lens have radii of 31.4 cm. If the focal length is 28.9 cm, what is the index of refraction of the lens material?

33. A prescription for a corrective lens calls for +3.50 diopters. The lensmaker grinds the lens from a "blank" with n = 1.56 and convex front surface of radius of curvature of 30.0 cm. What should be the radius of curvature of the other surface?

33-6 Eye and Corrective Lenses

41. A person struggles to read by holding a book at arm's length, a distance of 55 cm away. What power of reading glasses should be prescribed for her, assuming they will be placed 2.0 cm from the eye, and she wants to read at the "normal" near point of 25 cm?

42. Reading glasses of what power are needed for a person whose near point is 105 cm, so that he can read a computer screen at 55 cm? Assume a lens-eye distance of 1.8 cm.

45. A person's right eye can see objects clearly only if they are between 25 cm and 78 cm away, (a) What power of contact lens is required so that objects far away are sharp? (b) What w ill be the near point with the lens in place?

46. A person has a far point of 14 cm. What power glasses would correct this vision if the glasses were placed 2.0 cm from the eye? What power contact lenses, placed on the eye, would the person need?

48. What is the focal length of the eye lens system when viewing an object (a) at infinity, and (b) 38 cm from the eye? Assume that the lens retina distance is 2.0 cm.

33-8 Telescopes

60. What is the magnification of an astronomical telescope whose objective lens has a focal length of 78 cm, and whose eyepiece has a focal length of 2.8 cm? What is the overall length of the telescope when adjusted for a relaxed eye?

63. An astronomical telescope has an objective with focal length 75 cm and a +35 D eyepiece. What is the total magnification?

General Problems

87. A small object is 25.0 cm from a diverging lens. A converging lens with a focal length of 12.0 cm is 30.0 cm to the right of the diverging lens. The two lens system forms a real inverted image 17.0 cm to the right of the converging lens. What is the focal length of the diverging lens?