${\rm Ch~20~Second~Law~of~Thermodynamics}$

PHYS-102

Conceptual Questions

2. Can you warm a kitchen in winter by leaving the oven door open? Can you cool the kitchen on a hot summer day by leaving the refrigerator door open? Explain.

Yes, you can warm a kitchen in the winter by leaving the door open. The oven converts electrical energy to heat and leaving the door open will allow this heat to enter the kitchen. However, you cannot cool a kitchen in the summer by leaving the refrigerator door open. The refrigerator is a heat engine which (with an input of work) takes heat from the low-temperature reservoir (inside the refrigerator) and exhausts heat to the high-temperature reservoir (the room). As shown by the second law of thermodynamics, there is no "perfect refrigerator," so more heat will be exhausted into the room than removed from the inside of the refrigerator, This, leaving the refrigerator door open will actually warm the kitchen.

7. Discuss the factors that keep real engines from reaching Carnot efficiency.

The two main factors which keep real engines from Carnot efficiency are friction and heat loss to the environment

9. Describe a process in nature that is nearly reversible.

Water freezing on the surface of a body of water when the temperature is 0°C is nearly a reversible process.

11. Suppose a gas expands to twice its original volume (a) adiabatically, (b) isothermally. Which process would result in a greater change in entropy? Explain.

The isothermal process will result in a greater change in entropy. The entropy change for a reversible process is $\int \frac{dQ}{T}$. Q = 0 for an adiabatic process, so the change in entropy is also 0

13. Which do you think has the greater entropy, 1 kg of solid iron or 1 kg of liquid iron? Why?

1 kg of liquid iron will have greater entropy, since it is less ordered than solid iron and its molecules have more thermal motion. In addition, heat must be added to solid iron to melt it; the addition of heat will increase the entropy of the iron.

15. You are asked to test a machine that the inventor calls an "in-room air conditioner": a big box, standing in the middle of the room, with a cable that plugs into a power outlet. When the machine is switched on, you feel a stream of cold air coming out of it. How do you know that this machine cannot cool the room?

The machine is clearly doing work to remove heat from some of the air in the room. The waste heat is dumped back into the room, and the heat generated in the process of doing work is also dumped into the room. The net result is the addition of heat into the room by the machine

17. Suppose a lot of papers are strewn all over the floor; then you stack them neatly. Does this violate the second law of thermodynamics? Explain.

No. While you have reduced the entropy of the papers, you have increased your own entropy by doing work, for which your muscles have consumed energy. The entropy of the universe has increased as a result of your actions.

18. The first law of thermodynamics is sometimes whimsically stated as, "You can't get something for nothing," and the second law as, "You can't even break even." Explain how these statements could be equivalent to the formal statements.

The first law of thermodynamics is essentially a statement of the conservation of energy. "You can't get something for nothing" is similar to the statement that "energy can be neither created nor destroyed." If the "something" you want to get is useful work, then input energy is required. The second law concerns the direction of energy transfers. The Kelvin-Planck statement of the second law says that "no device is possible whose sole effect is to transform a given amount of heat completely into work." In other words, 100 joules of heat will result in something less than 100 joules of work, so "you can't even break even."

Problems

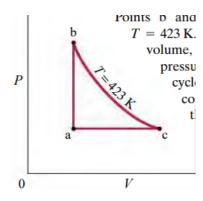
20-2 Heat Engines

1. A heat engine exhausts 7800 J of heat while performing 2600 J of useful work. What is the efficiency of this engine?

$$e = \frac{\text{Useful Work}}{Q_{in}}$$

$$\frac{2600J}{7800J} \approx 0.33 \approx 33\% \text{ efficiency}$$

6. Figure 20–17 is a PV diagram for a reversible heat engine in which 1.0 mol of argon, a nearly ideal monatomic gas, is initially at STP (point a). Points b and c are on an isotherm at $T=423\,K$. Process ab is at constant volume, process ac at constant pressure. (a) Is the path of the cycle carried out clockwise or counterclockwise? (b) What is the efficiency of this engine?



Process ab is isovolumetric, which means W=0 and $\Delta E_{int}=Q_{in}$

Recall that $\Delta E_{int} = \frac{3}{2}nRT = Q_{in}$

$$Q_{in} = \frac{3}{2}(1.0mol)(8.314 \frac{J}{mol \cdot K})(423K - 273K)$$
$$Q_{in} \approx 1870.65J$$

Process be is isothermal and work is done by gas, so $W = nrT \ln(\frac{V_c}{V_b})$ and Q = W

$$W = (1.0mol)(8.314 \frac{J}{mol \cdot K})(423K) \ln(\frac{?}{?})$$

Finding V_f and V_0

At a, we are at STP and using the Ideal Gas Law, we get $P_aV_a = nRT$

$$V_a = \frac{nrT}{P_a} = \frac{(1.0mol)(8.314 \frac{J}{mol \cdot K})(273K)}{1.013 \times 10^5 \frac{N}{m^2}}$$

$$V_a \approx 0.0224 \ m^3$$

At c, $P_a = P_c$ so $P = 1.013 \times 10^5 \frac{N}{m^2}$ and the process to c was isothermal so T = 423 K

$$V_c = \frac{nrT}{P} = \frac{(1.0mol)(8.314 \frac{J}{mol \cdot K})(423K)}{1.013 \times 10^5 \frac{N}{m^2}}$$

$$V_c \approx 0.0347 \ m^3$$

Going back to bc, the work done is

$$W = (1.0mol)(8.314 \frac{J}{mol \cdot K})(423K) \ln(\frac{0.0347 \ m^3}{0.0224 \ m^3})$$

$$W \approx 1539J$$

Process ca is isobaric and has work done on the gas

$$W = P\Delta V = (1.013 \times 10^5 \frac{N}{m^2})(0.0224 \ m^3 - 0.0347 \ m^3)$$

$$W \approx -1246 J$$

Finally, the efficiency would be

$$e = \frac{W}{Q_{in}} = \frac{1539J - 1246J}{1870.65J + 1539J} \approx 0.0859$$

$$e \approx 8.59\% \text{ efficiency}$$

20-3 Carnot Engine

8. What is the maximum efficiency of a heat engine whose operating temperatures are 550°C and 365°C?

Recall, that Carnot efficiency is in Kelvin not Celsius

$$\begin{split} e_{carnot} &= 1 - \frac{T_L}{T_H} = 1 - \frac{365^{\circ}C + 273K}{550^{\circ}C + 273K} = 1 - \frac{638K}{823K} \\ e_{carnot} &\approx 22.5\% \end{split}$$

13. A nuclear power plant operates at 65% of its maximum theoretical (Carnot) efficiency between temperatures of 660°C and 330°C. If the plant produces electric energy at the rate of 1.2 GW, how much exhaust heat is discharged per hour?

$$\begin{split} e_{carnot\ max} &= 1 - \frac{603K}{933K} \approx 0.354 \\ \text{Total Power} &= \frac{\text{Actual Power}}{(max\ e)(operating\ e)} = \frac{1.2GW}{(0.354)(0.65)} \\ \text{Total Power} &\approx 5.215GW \end{split}$$

Exhaust Power = Total Power - Actual Power
Exhaust Power =
$$5.215GW - 1.2GW = 4.015GW$$

$$(\frac{4.015GJ}{1s})(\frac{1.0\times10^9J}{1GJ})(\frac{3600s}{1h})\approx 1.4\times10^{13}\frac{J}{h}$$

14. A Carnot engine performs work at the rate of 520 kW with an input of 950 kcal of heat per second. If the temperature of the heat source is 560°C, at what temperature is the waste heat exhausted?

Recall that efficiency can either be $1 - \frac{T_L}{T_H}$ or $\frac{W}{Q_{in}}$, so we can relate these

$$1 - \frac{T_L}{T_H} = \frac{W}{Q_{in}}$$

$$T_L = T_H (1 - \frac{W}{Q_{in}})$$

Adding a variable of time,

$$T_L = T_H \left(1 - \frac{\frac{W}{t}}{\frac{Q_{in}}{t}}\right) = (560^{\circ}C + 273K)\left(1 - \frac{520000\frac{J}{s}}{3976700\frac{J}{s}}\right)$$
$$T_L \approx 724K \text{ or } 450^{\circ}C$$

17. A heat engine utilizes a heat source at 580°C and has a Carnot efficiency of 32%. To increase the efficiency to 38%, what must be the temperature of the heat source?

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Given that $e_{carnot} = 0.32$ we need to first solve for temperature of the waste heat

$$0.32 = 1 - \frac{T_L}{580^{\circ}C + 273K}$$
$$T_L = (0.32 - 1)(-853K)$$
$$T_L \approx 580K$$

Assuming T_L will remain the same

$$\begin{split} e_2 &= 1 - \frac{T_L}{T_H} \\ T_H e_2 &= 1 - T_L \\ T_H (e_2 - 1) &= -T_C \\ T_H &= \frac{T_C}{1 - e_2} = \frac{580K}{1 - (0.38)} \\ T_H &\approx 935K \ or \ 662^\circ C \end{split}$$

20-5 and 20-6 Entropy

33. A 7.5-kg box having an initial speed of $4.0\frac{m}{s}$ slides along a rough table and comes to rest. Estimate the total change in entropy of the universe. Assume all objects are at room temperature (293 K).

Entropy is calculated with $\Delta s = \int \frac{dQ}{T}$ and assuming the objects stay at 293K, $\Delta s = \frac{\Delta Q}{T}$

We know that the box has $KE = \frac{1}{2}mv^2 = (0.5)(7.5kg)(4.0\frac{m}{s})^2 = 60J$ of energy

If the box goes to rest then

$$\Delta E = 60J - 0J = 60J$$

$$\therefore \Delta s = \frac{\Delta Q}{T} = \frac{60J}{293K}$$

$$\Delta s \approx 0.20 \frac{J}{K}$$

34. What is the change in entropy of 1.00 m^3 of water at 0°C when it is frozen to ice at 0°C?

Frozen to ice leads me to think we have an increase in "organization" which would mean a decrease in entropy

$$\Delta s = \int \frac{dQ}{T}$$

Recall, $Q = mL_f$, L_f being the latent heat of fusion

This is a phase change so T is constant at 0°C or 273K, so $\Delta s = \frac{\Delta Q}{T}$

$$Q = (1000kg)(333000\frac{J}{kg}) = -3.33 \times 10^8 J$$

$$\therefore \Delta s = \frac{\Delta Q}{T} = \frac{-3.33 \times 10^8 J}{273K}$$

$$\Delta s \approx -1.22 \times 10^6 J$$

35. If 1.00 m^3 of water at 0°C is frozen and cooled to -10°C by being in contact with a great deal of ice at -10°C, estimate the total change in entropy of the process.

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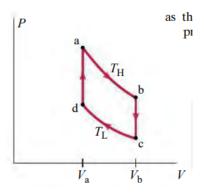
We need to find and sum the different changes of entropy at the different stages of the system

$$\begin{split} \Delta s_1 &= \frac{Q_1}{T_1} = \frac{mL_f}{T_1} = \frac{(1000kg)(333000\frac{J}{kg})}{273K} \\ \Delta s_1 &\approx -1.21978 \times 10^6 \frac{J}{K} \\ \Delta s_2 &= \frac{mc_{ice}\Delta T}{T_2} \qquad [T_2 = T_{avg} \text{ which is -5°C or 268K}] \\ \Delta s_2 &= \frac{(1000kg)(2100\frac{J}{kg \cdot K})(273K - 263K)}{268K} \\ \Delta s_2 &\approx -7.8358 \times 10^4 \frac{J}{K} \\ \Delta s_3 &= \frac{-Q_1 - Q_2}{T_3} = \frac{mL_f + mc_{ice}\Delta T_2}{T_3} \\ \Delta s_3 &= \frac{(1000kg)(333000\frac{J}{kg}) + (1000kg)(2100\frac{J}{kg \cdot K})(273K - 263K)}{-10 + 273K} \\ \Delta s_3 &\approx 1.3460 \times 10^6 \frac{J}{K} \\ \Delta s &= \Delta s_1 + \Delta s_2 + \Delta s_3 \\ &= -1.21978 \times 10^6 \frac{J}{K} - 7.8358 \times 10^4 \frac{J}{K} + 1.3460 \times 10^6 \frac{J}{K} \end{split}$$

 $\Delta s \approx 5.0 \times 10^4 \frac{J}{K}$

General Problems

77. The *Stirling cycle*, shown in Fig. 20–27, is useful to describe external combustion engines as well as solar-power systems. Find the efficiency of the cycle in terms of the parameters shown, assuming a monatomic gas as the working substance. The processes ab and cd are isothermal whereas bd and da are at constant volume. How does it compare to the Carnot efficiency?



Process ab is isothermal so Q = W

$$W_{ab} = nRT_H \ln(\frac{V_b}{V_a}) = Q_{ab}$$

Process bc is isovolumetric, so W=0 and $\Delta E_{int}=Q_{bc}$

$$Q_{bc} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(T_L - T_H)$$

Process cd is isothermal and work is done on the gas, so Q = -W

$$-W_{cd} = -1(nRT_L \ln(\frac{V_a}{V_b})) = -nRT_L \ln(\frac{V_b}{V_a}) = Q_{cd}$$

Process da is isovolumetric, so W = 0 and $\Delta E_{int} = Q_{da}$

$$Q_{da} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(T_H - T_L)$$

Now to find our Stirling efficiency,

$$\begin{split} e_{Stirling} &= \frac{\sum W}{Q_{in}} = \frac{W_{ab} - W_{cd}}{Q_{ab} + Q_{da}} \\ e_{Stirling} &= \frac{nRT_H \ln(\frac{V_b}{V_a}) - nRT_L \ln(\frac{V_b}{V_a})}{nRT_H \ln(\frac{V_b}{V_a}) + \frac{3}{2}nR(T_H - T_L)} \\ e_{Stirling} &= \frac{(T_H - T_L) \ln(\frac{V_b}{V_a})}{T_H \ln(\frac{V_b}{V_a}) + \frac{3}{2}(T_H - T_L)} \\ e_{Stirling} &= \left(\frac{T_H - T_L}{T_H}\right) \left(\frac{\ln(\frac{V_b}{V_a})}{\ln(\frac{V_b}{V_a}) + \frac{3}{2}(\frac{T_H - T_L}{T_H})}\right) \end{split}$$

We see that the Carnot efficiency is

$$e_{carnot} = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$$

This is the same term in front of our expression for the Stirling efficiency, yet the term next to it is less than 1

 $\therefore e_{Stirling} < e_{Carnot}$