Chapter 17 Temperature, Thermal Expansion, and Ideal Gas Law

PHYS-102

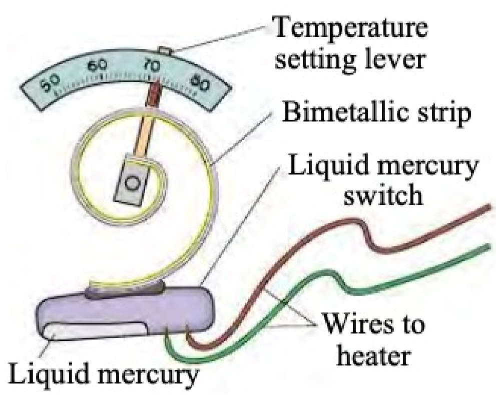
Conceptual Questions

1. Which has more atoms: 1 kg of iron or 1 kg of aluminum? See the Periodic Table or Appendix F.

1 kg of aluminum will have more atoms. Aluminum has an atomic mass less than iron. Since each Al atom is less massive than Fe atom, there will be more Al atoms than Fe atoms in 1 kg.

10. Figure 17–18 shows a diagram of a simple thermostat used to control a furnace (or other heating or cooling system). The bimetallic strip consists of two strips of different metals bonded together. The electric switch (attached to the bimetallic strip) is a glass vessel containing liquid mercury that conducts electricity when it can flow to touch both contact wires. Explain how this device controls the furnace and how it can be set at different temperatures

The bimetallic strip is made of two types of metal joined together. The metal of the outside strip has a higher coefficient of linear expansion than that of the inside strip, so it will expand and contract more dramatically. If the temperature goes above the thermostat setting, the outer strip will expand more than the inner, causing the spiral to wind more tightly and tilt the glass vessel so that the liquid mercury flows away from the contact wires and the heater turns off. If the temperature goes below the thermostat setting, the vessel tilts back as the outer strip contracts more than the inner and the spiral opens, and the heater turns on. Moving the temperature setting lever changes the initial position of the glass vessel. For instance, if the lever is set at 50, the vessel tilts with the mercury far from the contact wires. The outer strip has to shrink significantly to uncurl the spiral enough to tilt the vessel back.



13. When a cold mercury-in-glass thermometer is first placed in a hot tub of water, the mercury initially descends a bit and then rises. Explain.

When the cold thermometer is placed in hot water, the glass part of the thermometer will expand first, as heat is transferred to it first. This will cause the mercury level in the thermometer to decrease. As heat is transferred to the mercury inside the thermometer, the mercury will expand at a rate greater than the glass, and the level of mercury in the thermometer will rise

Problems

17-1 Atomic Theory

1. How does the number of atoms in a 21.5-g gold ring compare to the number in a silver ring of the same mass?

The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic masses of gold and silver from the periodic table

$$\begin{aligned}\frac{N_{Au}}{N_{Ag}} &= \frac{\frac{2.15\times 10^{-2}kg}{(196.97\frac{u}{atom})(1.66\times 10^{-27}\frac{kg}{u})}}{\frac{2.15\times 10^{-2}kg}{(107.87\frac{u}{atom})(1.66\times 10^{-27}\frac{kg}{u})}} \\ &= \frac{107.87\ atoms}{196.97\ atoms} \\ &\approx 0.548 \\ &\therefore \text{Au has about 55\% less atoms than Ag}\end{aligned}$$

17-2 Temperature and Thermometers

3. (a) "Room temperature" is often taken to be 68°F. What is this on the Celsius scale? (b) The temperature of the filament in a lightbulb is about 1900°C. What is this on the Fahrenheit scale?

a.

$$\frac{5}{9}(68^{\circ}F - 32) = 20^{\circ}C$$

b.

$$\frac{9}{5}(1900^{\circ}C + 32) = 3500^{\circ}F$$

17-4 Thermal Expansion

7. The Eiffel Tower (Fig. 17–19) is built of wrought iron approximately 300 m tall. Estimate how much its height changes between January (average temperature of 2°C) and July (average temperature of 25°C). Ignore the angles of the iron beams and treat the tower as a vertical beam

$$\Delta l = l_0 \alpha \Delta T$$
$$\Delta l = (300m)(12 \times 10^{-6} C^{-1})(25°C - 2°C)$$
$$\Delta l = 0.08m$$

8. A concrete highway is built of slabs 12 m long (15°C). How wide should the expansion cracks between the slabs be (at 15°C) to prevent buckling if the range of temperature is –30°C to ±50°C?

Note that if T = $-30°C$ like sometime during the winter, there is no longer any danger of buckling, so we disregard $-30°C$ and focus on $50°C$

$$\Delta l = l_0 \alpha \Delta T$$
$$\Delta l = (12m)(12 \times 10^{-6} C^{-1})(50°C - 15°C)$$
$$\Delta l \approx 5.0 \times 10^{-3} m$$

17. It is observed that 55.50mL of water at 20°C completely fills a container to the brim. When the container and the water are heated to 60°C, 0.35 g of water is lost. (a) What is the coefficient of volume expansion of the container? (b) What is the most likely material of the container? Density of water at 60°C is 0.98324 g/mL.

Volume expansion is defined as $\Delta V = V_0 \beta \Delta T$

a.

$$
\begin{aligned}
V_{lost} &= (V_0 + \Delta V)_{H_2O} - (V_0 + \Delta V)_{container} \\
V_{lost} &= \Delta V_{H_2O} - \Delta V_{container} \\
V_{lost} &= V_0 \beta_{H_2O} \Delta T - V_0 \beta_{container} \Delta T \\
\beta_{container} &= \frac{V_0 \beta_{H_2O} \Delta T}{V_0 \Delta T} - \frac{V_{lost}}{V_0 \Delta T} \\
\beta_{container} &= \beta_{H_2 0} - \frac{V_{lost}}{V_0 \Delta T} \\
\beta_{container} &= 210 \times 10^{-6} C^{-1} - \frac{\frac{0.35g}{0.98324 \frac{g}{mL}}}{(55.50mL)(60°C - 20°C)} \\
\beta_{container} &\approx 5.0 \times 10^{-5} C^{-1}
\end{aligned}
$$

17-7 and 17-8 Ideal Gas Law

31. If 3.80 m^3 of a gas initially at STP is placed under a pressure of 3.20 atm, the temperature of the gas rises to 38.0°C. What is the volume?

$$P_1V_1 = nRT_1 \quad \Rightarrow \quad P_2V_2 = nRT_2$$
$$\frac{P_1V_1}{T_1} = nR \quad \Rightarrow \quad \frac{P_2V_2}{T_2} = nR$$

Using the transitive property,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Solving for V_2,

$$V_2 = \frac{P_1V_1T_2}{P_2T_1}$$
$$V_2 = \frac{(1atm)(1.013 \times 10^5 \frac{N}{m^2})(3.80m^3)(273K + 38)}{(3.20atm)(1.013 \times 10^5 \frac{N}{m^2})(273K)}$$
$$V_2 \approx 1.35m^3$$

34. If 14.00 mol of helium gas is at 10.0°C and a gauge pressure of 0.350 atm, calculate (a) the volume of the helium gas under these conditions, and (b) the temperature if the gas is compressed to precisely half the volume at a gauge pressure of 1.00 atm.

a. We use the Ideal Gas Law and solve for volume for find V_1

$$PV = nRT$$
$$V_1 = \frac{nRT_1}{P_1}$$

Not that we need to use absolute pressure and not gauge pressure so 1 atm + our gauge pressure

$$V_1 = \frac{(14.0mol)(8.314 \frac{J}{mol \cdot K})(283K)}{1.350atm(1.013 \times 10^5 \frac{N}{m^2})}$$
$$V_1 \approx 0.241m^3$$

b. We use the Ideal Gas Law and the transitive property to get an expression for T_2

$$nR = \frac{P_1V_1}{T_1} \quad \Rightarrow \quad nR = \frac{P_2V_2}{T_2}$$
$$T_2 = T_1\frac{P_2V_2}{P_1V_1}$$
$$T_2 = (283K)\left(\frac{(2atm)(\frac{1}{2} \cdot 0.241m^3)}{(1.350atm)(0.241m^3)}\right)$$
$$T_2 \approx 210K \approx -63°C$$

37. A storage tank at STP contains 28.5 kg of nitrogen (N_2). (a) What is the volume of the tank? (b) What is the pressure if an additional 25.0 kg of nitrogen is added without changing the temperature?

a. Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n = 28.5\ kg \left(\frac{1\ mol\ N_2}{28.01 \times 10^{-3} kg} \right) \approx 1017\ mol$$

Using Ideal Gas Law and solving for volume (note we are at STP),

$$V = \frac{nRT}{P}$$
$$V = \frac{(1017 mol)(8.314 \frac{J}{mol \cdot K})(273K)}{1.013 \times 10^5 \frac{N}{m^2}}$$
$$V \approx 22.8 m^3$$

b. T remains constant and we use the Ideal Gas Law and solve for pressure after finding the new mass in moles

$$n = 53.5\ kg \left(\frac{1\ mol\ N_2}{28.01 \times 10^{-3} kg} \right) \approx 1910\ mol$$
$$P = \frac{nRT}{V}$$
$$P = \frac{(1910 mol)(8314 \frac{J}{mol \cdot K})(273K)}{22.8 m^3}$$
$$P \approx 1.9 \times 10^5 \frac{N}{m^2} \approx 1.88\ atm$$

41. A sealed metal container contains a gas at 20.0°C and 1.00 atm. To what temperature must the gas be heated for the pressure to double to 2.00 atm? (Ignore expansion of the container.)

If there is no expansion of the container, that hints that volume is constant along with our moles and R constants.

We can use the Ideal Gas Law and the transtive property to get an expression for final temperature T_2

$$\frac{nR}{V} = \frac{P_1}{T_1} \quad \Rightarrow \quad \frac{nR}{V} = \frac{P_2}{T_2}$$
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$
$$T_2 = T_1 (\frac{P_2}{P_1})$$
$$T_2 = (293K)(\frac{2atm}{1atm})$$
$$T_2 = 586K\ or\ 313^\circ C$$

44. A helium-filled balloon escapes a child's hand at sea level and 20.0°C. When it reaches an altitude of 3600 m, where the temperature is 5.0°C and the pressure only 0.68 atm, how will its volume compare to that at sea level?

Ideal Gas Law and transitive property, then solve for the ratio of $\frac{V_2}{V_1}$

$$\begin{aligned}\frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2}\\ \frac{V_2}{V_1} &= \frac{P_1T_2}{P_2T_1}\\ \frac{V_2}{V_1} &= \frac{(1atm)(278K)}{(0.68atm)(293K)} \approx 1.4\end{aligned}$$

$\therefore$ volume will increase by a factor of 1.4 times the original volume

50. An air bubble at the bottom of a lake 37.0 m deep has a volume of 1.00 cm^3. If the temperature at the bottom is 5.5°C and at the top 18.5°C, what is the volume of the bubble just before it reaches the surface?

We use the Ideal Gas Law and transitive property to get the expression for both locations (top and bottom)

Let 1 = bot and 2 = top

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Solving for V_2

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

Note that P_1 is atomsperic pressure above + ρgy

$$\begin{aligned}V_2 &= \frac{(1atm + \rho gy)(V_1)(T_2)}{T_1P_2}\\ V_2 &= \frac{[1.013\times10^5Pa + (1000\frac{kg}{m^3})(9.8\frac{m}{s^2})(37.0m)](1.0\times10^{-6}m^3)(18°C + 273K)}{(5.5°C + 273K)(1.013\times10^5Pa)}\\ V_2 &\approx 4.79\times10^{-6}m^3 \approx 4.79cm^3\end{aligned}$$

17-9 Ideal Gas Law in Terms of Molecules; Avogadro's Number

51. Calculate the number of $\frac{molecules}{m^3}$ in an ideal gas at STP.

At STP, 1 mole of ideal gas occupies 22.4 L.

$$\frac{1mol}{22.4L}\left(\frac{6.022\times10^{23}}{1mol}\right)\left(\frac{1L}{1.0\times10^{-3}m^3}\right) \approx 2.69\times10^{25}\ molecules/m^3$$

52. How many moles of water are there in 1.000 L at STP? How many molecules?

We assume that the water is at 4°C so that its density is $1000\frac{kg}{m^3}$.

$$1.000L\left(\frac{1.0\times10^{-3}m^3}{1.000L}\right)\left(\frac{1000kg}{1m^3}\right)\left(\frac{1mol}{(15.9994+2\times1.00794)\times10^{-3}kg}\right)\approx 55.51mol$$

$$55.51mol\left(\frac{6.022\times10^{23}\ molecules}{1mol}\right)\approx 3.343\times10^{25}\ molecules$$

General Problems

69. A house has a volume of 870 m^3. (a) What is the total mass of air inside the house at 15°C? (b) If the temperature drops to –15°C, what mass of air enters or leaves the house?

To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N_2 (molecular weight 28) and 30% O_2 (molecular weight 32), then the average molecular weight is $0.70(28)+0.30(32) = 29.2$.

a. Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$\begin{aligned}
PV &= nRT \\
n &= \frac{PV}{RT} \\
n &= \frac{(1.013\times10^5\frac{N}{m^2})(870m^3)}{(8.314\frac{J}{mol\cdot})(293K)} \\
n &\approx 3.6178\times10^4\ moles \\
m &= (3.6178\times10^4\ moles)(29.2\times10^{-3}kg/mol) = 1056.4kg \approx 1100kg
\end{aligned}$$

b. Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$\begin{aligned}
n &= \frac{PV}{RT} \\
n &= \frac{(1.013\times10^5\frac{N}{m^2})(870m^3)}{(8.314\frac{J}{mol\cdot K})(263K)} \\
n &= 4.0305\times10^4\ moles \\
m &= (4.0305\times10^4\ moles)(29.2\times10^{-3}kg/mol) = 1176.9kg
\end{aligned}$$

$\therefore$ The mass entering the house is 1176.9 kg 1056.4 kg = 120.5 kg ≈ 100 kg.