Modern Physics Final Study Guide

PHYS-122

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Contents

1 Special Relativity

1.1 Key Concepts

- Postulates: Laws of physics are the same in all inertial frames; the speed of light is constant in a vacuum for all observers.
- Relativity of Simultaneity: two events that appear to happen at the same time for one observer may not appear simultaneous to another observer moving relative to the first.
- Relativistic velocity addition: These problems generally have 3 objects: A stationary object and 2 moving objects. Notation is usually (or u and v reversed):
  - S = stationary frame
  - S' = 2nd moving object reference frame
  - v = velocity of 2nd object with respect to stationary frame
  - u = velocity of 1st object with respect to stationary frame
  - u' = velocity of 1st object with respect to 2nd object
- Relativistic Conservation Laws:
  - In an isolated system of particles, the relativistic total energy (kinetic energy plus rest energy) remains constant
  - In an isolated system of particles, the total linear momentum remains constant

1.2 Essential Equations

- Time dilation: $\Delta t = \gamma \Delta t_0$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$.
- Length contraction: $L = L_0 \gamma^{-1} = L_0 \sqrt{1 - v^2/c^2}$.
- Relativistic addition of velocities: $u' = \frac{u+v}{1+uv/c^2}$.
- Lorentz transformation (time): $t' = \gamma(t - \frac{vx}{c^2})$
- Lorentz transformation (space): $x' = \gamma(x - vt)$
- Doppler Shift:
  - Redshift (receeding): $f_{obs} = \sqrt{\frac{1 - v/c}{1 + v/c}} f_{source}$ and $\lambda_{obs} = \sqrt{\frac{1 + v/c}{1 - v/c}} \lambda_{source}$
  - Blueshift (approaching): $f_{obs} = \sqrt{\frac{1 + v/c}{1 - v/c}} f_{source}$ and $\lambda_{obs} = \sqrt{\frac{1 - v/c}{1 + v/c}} \lambda_{source}$
- Kinetic energy: $KE = (\gamma - 1) m_0 c^2$
- Total energy: $E = \gamma m_0 c^2$
- Energy-momentum relation: $E^2 = (pc)^2 + (m_0 c^2)^2$.

2 Early Quantum Theory and Quantum Mechanics

2.1 Key Concepts

- Blackbody radiation: Explained using Planck's quantum hypothesis $E = nhf$.
- Photoelectric effect: Energy of emitted electrons depends on frequency, not intensity.
- Wave-particle duality: Matter exhibits both particle and wave properties.
- Schrödinger equation: Governs quantum mechanical behavior of particles.

2.2 Essential Equations

- Planck's law: $E = hf$
- Photoelectric equation: $KE_{\text{max}} = hf - \phi$
- De Broglie wavelength: $\lambda = \frac{h}{p}$
- Schrödinger equation (time-independent):

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

- 3D Schrödinger equation (spherical coordinates):

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi = E\psi$$

where

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

- General solution to the wave function:

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$$

where:

  - $R(r)$: Radial part of the wave function, satisfying a differential equation dependent on the potential $V(r)$.
  - $Y(\theta,\phi)$: Angular part, given by spherical harmonics:

$$Y_l^m(\theta,\phi) = (-1)^m\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

  where P_l^m are associated Legendre polynomials, and l, m are quantum numbers.

3 Statistical Mechanics

3.1 Key Concepts

- Microstates and macrostates: Connection between microscopic and macroscopic descriptions.
- Boltzmann distribution: $P_i = \frac{e^{-E_i/k_BT}}{Z}$.
- Partition function: Summation of states to understand thermodynamic properties, $Z = \sum_i e^{-E_i/k_BT}$.
- Occupation numbers: Average number of particles in a state, $n_i = \frac{1}{e^{(E_i-\mu)/k_BT} \pm 1}$ (for fermions and bosons).
- Degeneracy: The number of microstates corresponding to a single energy level.
- Distinguishable vs. indistinguishable particles: Boltzmann distribution for distinguishable particles; Fermi-Dirac and Bose-Einstein distributions for indistinguishable particles.

3.2 Essential Equations

$$\text{Boltzmann distribution: } P_i = \frac{e^{-E_i/k_BT}}{Z}$$

$$\text{Partition function: } Z = \sum_i g_i e^{-E_i/k_BT}$$

$$\text{Fermi-Dirac distribution: } n_i = \frac{1}{e^{(E_i-\mu)/k_BT} + 1}$$

$$\text{Bose-Einstein distribution: } n_i = \frac{1}{e^{(E_i-\mu)/k_BT} - 1}$$

$$\text{Average energy: } \langle E \rangle = \sum_i P_i E_i = -\frac{\partial \ln Z}{\partial \beta}$$

$$\text{Probability of a state: } P(E) = \frac{g_i e^{-E/k_BT}}{Z}$$

3.3 Units and Unit Conversions

- Energy: 1 eV $= 1.602 \times 10^{-19}$ J.
- Temperature: Convert between Kelvin and energy using $k_B = 1.38 \times 10^{-23}$ J/K.
- Wavelength and frequency: $E = hf$, $\lambda = \frac{c}{f}$, with $hc = 1240$ eV · nm.

4 Quick Reference

4.1 Mathematical Tools

- Trigonometric Identities:

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$
$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$
$$\sin(2x) = 2 \sin x \cos x$$

- Series:
  - Taylor Series Expansion: $\displaystyle\sum_{n=0}^{\infty} C_n (x - a)^n$ where $C_n = \dfrac{f^{(n)}(a)}{n!}$
  - Binomial Series Expansion: $(1 \pm x)^r = 1 \pm rx \pm \ldots$

- Integration Tips:
  - For even functions: $\int_{-a}^{a} f(x)dx = 2 \int_{0}^{a} f(x)dx$
  - For odd functions: $\int_{-a}^{a} f(x)dx = 0$
  - Gaussian integrals often appear in wave packets

- Complex Numbers Review
  - Complex conjugate: $z^* = a - bi$ for $z = a + bi$
  - Modulus: $|z| = \sqrt{a^2 + b^2}$
  - Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$
  - Argument: $\arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

- Second Order Differential Equations
  - If ODE has this form: $\frac{d^2 f}{dx^2} = k^2 f$ where k is a constant

    The solution is:
    $$f(x) = Ae^{kx} + Be^{-kx}$$

4.2 Important Constants

- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot K^4$
- Planck's constant: $h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s} = 4.136 \times 10^{-15}$ eV $\cdot$ s
- Reduced Planck's constant: $\hbar = 1.055 \times 10^{-34}\,\mathrm{J \cdot s}$
- Electron mass: $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
- Speed of light: $c = 3.0 \times 10^8\,\mathrm{m/s}$
- Bohr radius: $a_0 = 0.0529 \times 10^{-9}\,\mathrm{m}$
- Compton wavelength of the electron: $\frac{h}{m_e c} = 0.002426$ nm

4.3 Unit Conversions

- 1 eV $= 1.602 \times 10^{-19}$ J
- $hc = 1240$ eV $\cdot$ nm
- $\hbar c = \frac{1240}{2\pi}$ eV $\cdot$ nm