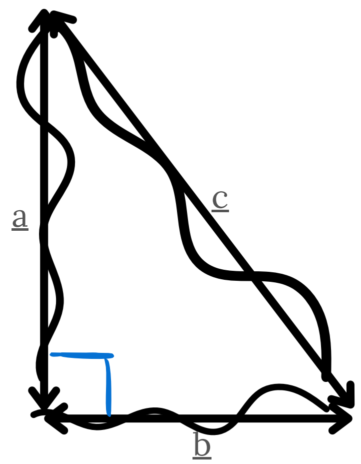
Intuitive geometry using Bithagoras Theorem

Bithagoras theorem serves as a unifying framework for various trigonometric laws by exploring infinitesimal random paths.

We first consider the simplest case of right angled triangles.



If I were to randomly, infinitesimally travel a displacement of C, what else did I do?

Any displacement corresponds to its horizontal and vertical component, **VICE VERSA.**
By randomly travelling C, I also travelled Vertical displacement A and horizontal displacement B.
Question is, did I do both randomly as well?
By randomly travelling C, you are killing two birds with one stone, or at least implicitly and **indistinguishably**. This is only possible because of the **interchangeability** of C and (A,B)

Without randomness: Travel displacement A + B = same outcome as C but different process.

With randomness: same outcome, indistinguishable process. When visualizing, you cannot visualize A then B, they are concurrent and independent. (not fixed points either)

We can derive a relationship from randomness using variance. Let X= unit length of a completed random path x.

$$Var(aX) + Var(bX) = Var(cX)$$

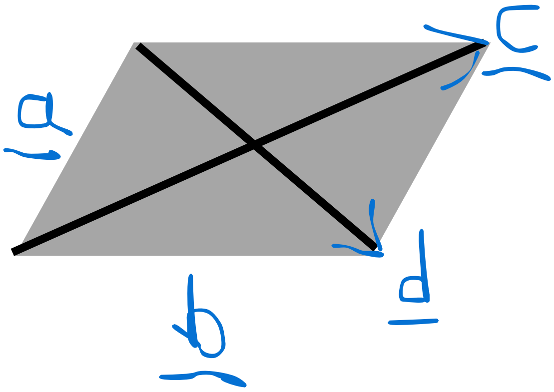
$$a^2Var(X) + b^2Var(X) = c^2Var(X)$$

$$a^2 + b^2 = c^2 .$$

Or intuitively, going in every possible direction requires AREA. By proportionality, $a^2 + b^2 = c^2$.

Random paths is incomplete for non right-angled triangles. This is because of Ambiguity.
We often imagine a vector to go 1 direction, the one we want. In a scalar equation, we can preserve the orientation of vectors relative to each other. However, we must allow the vectors to go both directions, because they are equally valid in the same equation.

A + B or A - B, it does not affect right-angled triangles because all 4 combinations result in the same displacement C. However, in other triangles, going in opposite directions can lead to 2 different displacements, it is no longer one-to-one. You need to include both in order to achieve interchangeability (respecting the equal sign)



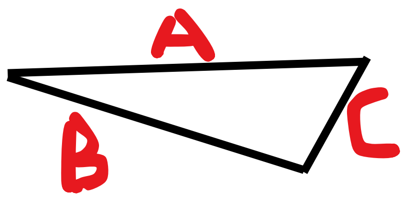
We must treat the two displacements as equally probable since each have 2 combinations. To derive a relationship, we simply compare the variances once more.

$$2\ \mathrm{Var}(aX) + 2\ \mathrm{Var}(bX) = \mathrm{Var}(cX) + \mathrm{var}(dX)$$

$$2a^2\mathrm{Var}(X) + 2b^2\mathrm{Var}(X) = c^2\mathrm{Var}(X) + d^2\mathrm{Var}(X)$$

Parallelogram Law: $2(a^2 + b^2) = c^2 + d^2$

Deriving cosine law using random displacements



Imagine random displacements for the obtuse triangle ABC
We can see that displacement A and B clearly contradict each other, which causes displacement C to be very small.

We added components of A and B that contrasted each other in overlapping paths, which caused "inflation". We can fix this by removing the contrast.

Let A be the primary path, where some component of B contrasts or supports A. Let that component be X, we know that whatever component of B that does not overlap with A is perpendicular to A, Let that be Y.

$B^2 = X^2 + Y^2$.

To fix random paths for the obtuse triangle, make them right-angled displacements.

Var(A-X) + Var(Y) = Var(C)

$(A-X)^2 + Y^2 = C^2$

If we were to reverse the direction of B, it would support A instead.

$(A+X)^2 + Y^2 = D^2$

Adding both equations gives the parallelogram law. ($B^2 = X^2 + Y^2$)

Lets expand the first equation.

$A^2 + X^2 + Y^2 - 2AX = C^2$

$A^2 + B^2 - 2AX = C^2$

We know that X is the projection of B onto A, which can be replaced as Bcos(C) Where the angle is between A and B and opposite of C.

Cosine Law: $A^2 + B^2 - 2AB\,\text{Cos}(C) = C^2$

The Bithagoras Theorem is an extension to N amount of vectors. (Generalized parallelogram law/ parseval identity)

Let $a_1, a_2, \ldots, a_n$ be vectors of a vector space.
For each element $\epsilon := (\epsilon_1, \epsilon_2, \ldots, \epsilon_n) \in \{-1, 1\}^n$ we define

$$v_\epsilon = \sum_{i=1}^{n} \epsilon_i\, a_i.$$

Then we get

$$\sum_{i=1}^{n} |a_i|^2 = \sum_{\epsilon \in \{-1,1\}^n} |v_\epsilon|^2 / 2^n$$