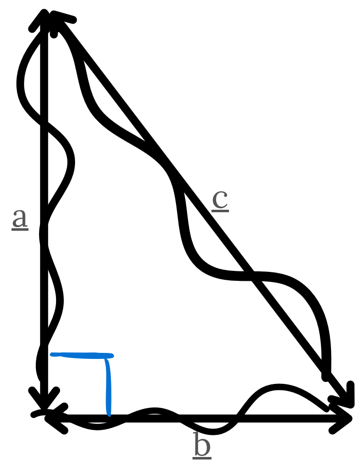
# Reframing of Pythagoras theorem

Bithagoras theorem states that the sum of squared norms of a set of vectors is equal to the average squared length of all their signed combinations.

In order to intuitively derive the relationship, we first consider the simplest case of right angled triangles.




If I were to randomly travel a displacement of C, what else did I do?

Any displacement corresponds to its horizontal and vertical component, **VICE VERSA.**
By randomly travelling C, I also travelled Vertical displacement A and horizontal displacement B.
Question is, did I do both randomly as well?
By randomly travelling C, you are killing two birds with one stone, or at least implicitly and **indistinguishably**. This is only possible because of the **interchangeability** of C and (A,B)

We can derive a relationship from randomness using variance.

Let X= unit length of a random path.

$$Var(aX) + Var(bX) = Var(cX)$$

$$a^2Var(X) + b^2Var(X) = c^2Var(X)$$

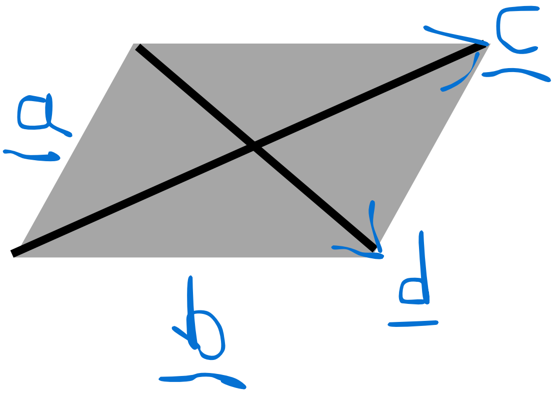
$$a^2 + b^2 = c^2 .$$

Intuitively, going in every possible direction requires AREA.

By proportionality, $a^2 + b^2 = c^2$ .

This relationship is wrong for non right-angled triangles.
This is because of Ambiguity.
We often imagine a vector to go 1 direction, the one we want.
In an equation, we can preserve the orientation of vectors relative to each other. However, we must allow the vectors to go both directions, because we cannot control it.

A to B or B to A, it does not affect right-angled triangles because all 4 combinations result in the same displacement C.
However, in other triangles, going in opposite directions can lead to 2 different displacements, it is no longer injective.




We must treat the two displacements as equally probable since each have 2 combinations. To derive a relationship, we simply compare the variances once more.

$$2 Var(aX) + 2 Var(bX) = Var(cX) + var(dX)$$

$$2a^2Var(X) + 2b^2Var(X) = c^2Var(X) + d^2Var(X)$$

**Parallelogram Law:** $\mathbf{2(a^2 + b^2) = c^2 + d^2}$

The Bithagoras Theorem is simply  an extension for N amount of vectors.

Let $a_1, a_2, \ldots, a_n$ be vectors of a vector space.
For each element $\epsilon := (\epsilon_1, \epsilon_2, \ldots, \epsilon_n) \in \{-1, 1\}^n$ we define

$$v_\epsilon = \sum_{i=1}^{n} \epsilon_i \, a_i.$$

Then we get

$$\sum_{i=1}^{n} |a_i|^2 = \sum_{\epsilon \in \{-1,1\}^n} |v_\epsilon|^2 / 2^n$$