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1. $P = (2, 3, 4) \Rightarrow (1/2, 3/4, 1)$

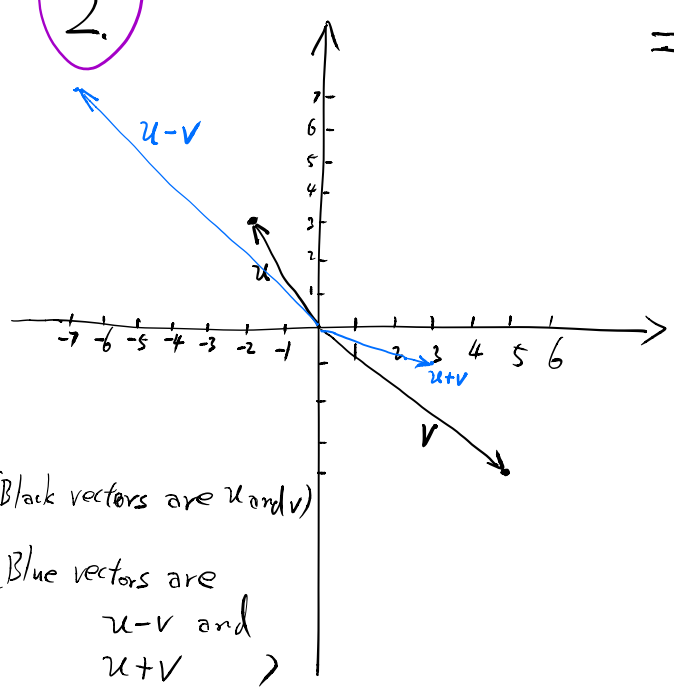
$Q = (-4, 5, 2) \Rightarrow (-2, 2.5, 1)$

$R = (2, 3, 1, 3) \Rightarrow (2/3, 1, 1/3, 1)$

$S = (-4, 5, 2, 2) \Rightarrow (-4/2, 5/2, 2/2, 1)$

(Here we assume P, Q are 2D point;
Here we assume R, S are 3D points)

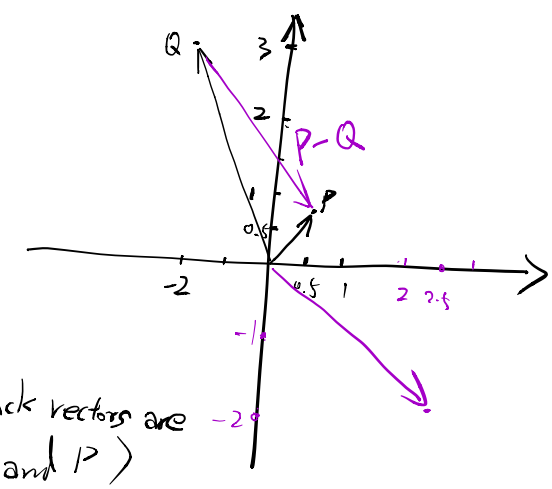
2. $u+v = (3, -1)$



(Black vectors are u and v)

(Blue vectors are
u-v and
u+v)

$\Rightarrow P - Q = (2.5, -1.75)$



(Black vectors are
Q and P)

(Purple vector is P-Q)

3. $u = (-2, 3)$; $V = (5, -4)$

$|u| = \sqrt{4+9} = \sqrt{13}$; $|V| = \sqrt{25+16} = \sqrt{41}$

$= 3.6$ $\qquad$ $= 6.4$

$u \cdot V = -10 + (-12) = -22$ (dot product)

$u \cdot V = |u| \cdot |V| \cdot \cos\theta \Rightarrow \cos\theta = \frac{-22}{\sqrt{13} \times \sqrt{41}} = \frac{-22}{23.04}$

$= -0.95$

4. $u = (-2, 3)$; $V = (5, -4)$

$|u \times V| = |u| \, |V| \sin\theta$

$u \times V = U_x \cdot V_y - U_y \cdot V_x \Rightarrow$ how dot product works

$= (-2)(-4) - (3) \cdot (5)$

$= 8 - 15 = -7$

5: $u \rightarrow 3.6$

$v \rightarrow 6.4$

$|u| = \sqrt{4+9} = \sqrt{13} = 3.6$; $|v| = \sqrt{25+16} = \sqrt{41} = 6.4$

(the way to get length)

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6. $R = (2, 3, 1, 3)$

$S = (-4, 5, 2, a)$

The affine combination

$$R + t(S - R), \quad 0 \le t \le 1$$

describe the line segment from R to S

The parametric form of a line segment is written as:

$$(1-t)R + tS, \quad 0 \le t \le 1$$

$R = (2, 3, 1, 3) \Rightarrow (2/3, 1, 1/3, 1)$

$S = (-4, 5, 2, a) \Rightarrow (-4/a, 5/a, 2/a, 1)$

$$\overrightarrow{RS} = (2/3, 1, 1/3) - (-4/a, 5/a, 2/a)$$

$$= (2/3 + 4a,\ 1 - 5/a,\ 1/3 - 2/a)$$

vector form of equation is:

$$(2/3, 1, 1/3) + t(2/3 + 4a, 1 - 5/a, 1/3 - 2/a)$$

for $t \in \mathbb{R}$

let (x, y, z) be on the line:

Then $(x, y, z) = (2/3, 1, 1/3) + t(2/3 + 4a, 1 - 5/a, 1/3 - 2/a)$

$$= \left(\tfrac{2}{3}(t+1) + 4ta,\ 1 + t - 5t/a,\ 1/3(t+1) - 2t/a\right)$$

parametric form of equation is:

$$x = \tfrac{2}{3}(t+1) + 4ta;$$

$$y = 1 + t - 5t/a;$$

$$z = 1/3(t+1) - 2t/a;$$

7 $P = (2, 3, 4, 1)$

$Q = (-4, 5, 2)$

$R = (2, 3, 1, 3)$

Since they are not colinear, the plane defined by P, Q, R is the set of points.

$$xP + yQ + zR, \quad x + y + z = 1$$

8 $Q = (-4, 5, 2, 1)$

$R = (2, 3, 1, 3)$

$S = (-4, 5, 2, 2)$

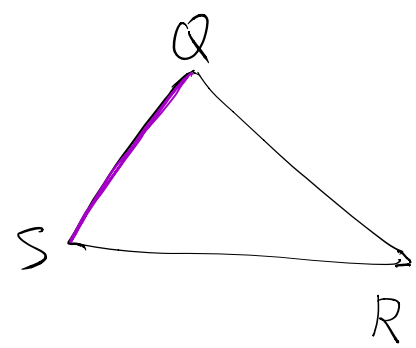
$xQ + yR + zS$, $x + y + z = 1$,

$x \in (0, 1)$

$y \in (0, 1)$

$\Rightarrow SQ$ line segment is:

$(1-\alpha)S + \alpha Q$



$\Rightarrow SR$ line segment is:

$(1-\beta)S + \beta R$

$\Rightarrow$ Now we replace S to "$(1-\alpha)S + \alpha Q$", we have

$(1-\beta)((1-\alpha)S + \alpha Q) + \beta R \Longleftarrow$ The answer!!!

9 $(x_0, y_0) \cdot (x_1, y_1) = x_0 x_1 + y_0 y_1$

10 assume $\vec{M} = (x_0, y_0)$

$\vec{N} = (x_1, y_1)$

$|\vec{M}| = \sqrt{x_0^2 + y_0^2}$, $\vec{N} = \sqrt{x_1^2 + y_1^2}$

$\vec{M} \cdot \vec{N} = |\vec{M}| |\vec{N}| \cos Q$

$$\cos Q = \frac{\vec{M} \cdot \vec{N}}{|\vec{M}| |\vec{N}|} = \frac{x_0 x_1 + y_0 y_1}{\sqrt{x_0^2 + y_0^2} \sqrt{x_1^2 + y_1^2}}$$

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$$\cos(Q) = \frac{x_0 x_1 + y_0 y_1}{\sqrt{x_0^2 + y_0^2} \sqrt{x_1^2 + y_1^2}}$$

$$Q = \cos^{-1}\left(\frac{x_0 x_1 + y_0 y_1}{\sqrt{x_0^2 + y_0^2} \sqrt{x_1^2 + y_1^2}}\right)$$

(12) let $M = (2, 3, 1)$

if $N = (x, y, z)$, then $M \cdot N = 2x + 3y + z$

if $M \cdot N = 0$, then N is orthogonal to M,

thus: $\underline{2x + 3y + z = 0}$

let $x = -3$, $y = -2$, $z = 0$, then

$$2(-3) + 3(-2) + 0 = 0$$

thus: $(-3, -2, 0)$ is one resulte

(13) Given the plane $3x - 4y + 7z - 3 = 0$, what is its surface normal?

1) we find three point:

a: $(1, 0, 0, 1)$

b: $(0, 1, 1, 1)$

c: $(-3, -3, 0, 1)$

2) assume surface normal:

$$\vec{n} = (x, y, z);$$

and:

$$\vec{ab} = (-1, 1, 1);$$

$$\vec{ac} = (-4, -3, 0);$$

$$\vec{n} \cdot \vec{ab} = 0 \Rightarrow -x + y + z = 0$$

$$\vec{n} \cdot \vec{ac} = 0 \Rightarrow 4x + 3y = 0 \Rightarrow x = \frac{-3y}{4}$$

$$\Rightarrow \frac{3y}{4} + y + z = 0 \Rightarrow z = -\frac{7}{4}y$$

thus: $\vec{n} = (-3/4\, y, y, -7/4\, y)$

let $y = -4$, then

$$\vec{n} = (3, -4, 7) \Leftarrow \text{answer!!!}$$

Or:

$$\vec{n} = \vec{ab} \times \vec{ac}$$

$$= (-1, 1, 1) \times (4, 3, 0)$$

$$= \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 4 & 3 & 0 \end{vmatrix}$$

$$= (0-3, \underline{\underline{4-0}}, -3-4)$$

$$= -3, +4, -7$$

$$= (3, -4, 7) \Leftarrow \text{answer!!!}$$

Cosider a trapezoid, a square S, and a triangle T:

$$R = \{(-1, 0), (1, 0), (2, 1), (-2, 1)\}$$

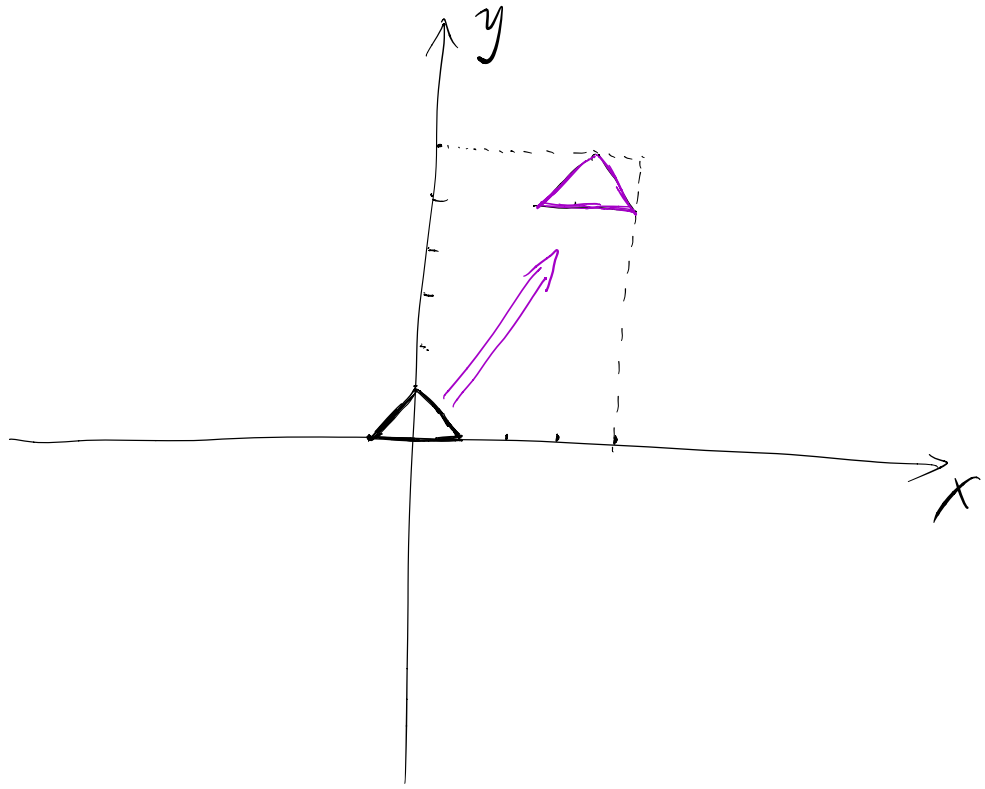
$$S = \{(0, 0), (1, 0), (1, 1), (0, 1)\}$$

$T = \{(-1, 0), (1, 0), (0, 1)\}$

(14) Translate T by 3 in x and 5 in y:

$T' = \{(-1+3, 0+5), (1+3, 0+5), (0+3, 1+5)\}$

$\{(2, 5), (4, 5), (3, 6)\}$



(15) Scale R by 2 in x and 4 in y.

$R = \{(-1,0), (1,0), (2,1), (-2,1)\}$

$$R' = \{(-1\times 2, 0\times 4), \quad = \{(-2,0),$$
$$(1\times 2, 0\times 4) \qquad (2,0),$$
$$(2\times 2, 1\times 4) \qquad (4,4),$$
$$(-2\times 2, 1\times 4)\} \qquad (-4,4)\}$$

(16) Rotate S by 30° about the Z axis;

$$P' = R_z(Q)P$$

$$R = R_z(Q) = \begin{bmatrix} \cos Q & -\sin Q & 0 & 0 \\ \sin Q & \cos Q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

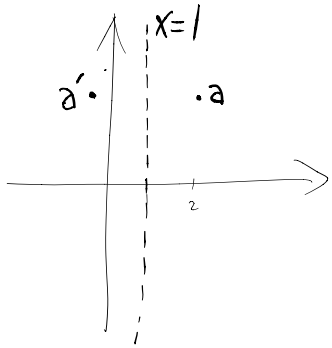
$$\cos Q = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin Q = \frac{1}{2}$$

$$P' = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} - \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} + \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Leftarrow \text{The Answer!!!!}$$

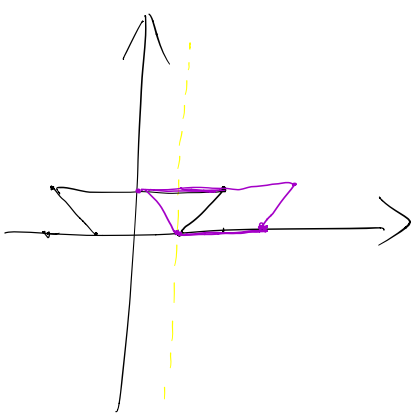
(17) Reflect R about the line $x=1$



$$a'_x = a_x - 2(a_x - 1)$$
$$= 2 - a_x$$

y keep the same:

$$R = \{(-1,0), (1,0), (2,1), (-2,1)\}$$
$$\Downarrow$$
$$\{(3,0), (1,0), (0,1), (4,1)\}$$



(18) Scale T by 2 in x and 4 in y with fixed point (1,1).

$T = \{(-1,0), (1,0), (0,1)\}$

transfer to (1, 1): $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

transfer back: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Scale element: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 4 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T' = ST = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 4 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 & -1 \\ -3 & -3 & 1 \\ 1 & 1 & 1 \end{bmatrix} \wedge$$

$$T' = \{(-3,-3), (1,-3), (-1,1)\}$$

19.

$$S = \{(0,0), (1,0), (1,1), (0,1)\}$$

1) tranfer to $(1,1)$

$$\Rightarrow \{(-1,-1), (0,-1), (0,0), (-1,0)\}$$

2) Rotation by Y axis:

$$R = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R \begin{bmatrix} -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ -1 & -1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \\ z \\ \end{array}$$

result after rotation: $\Rightarrow$

3) transfer back:

$$(1/\sqrt{2}, -1), (0, -1), (0, 0), (-1/\sqrt{2}, 0)$$

$$\Downarrow$$

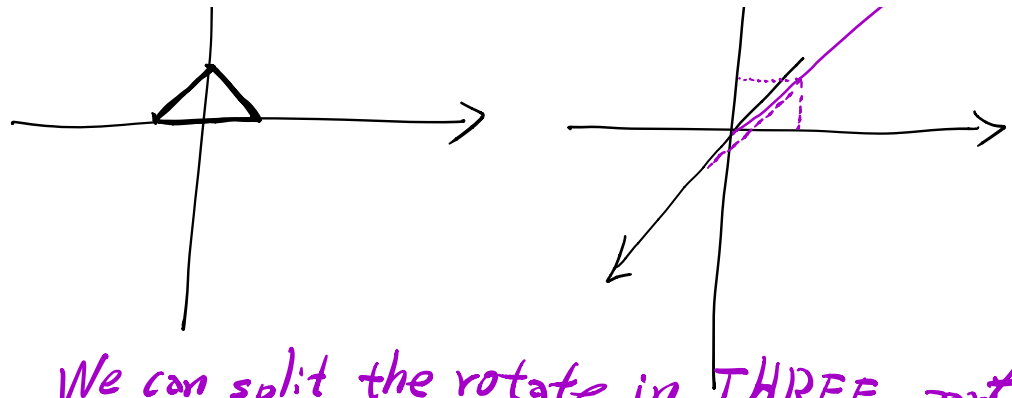
$$(1/\sqrt{2}+1, 0), (1, 0), (1, 1), (-1/\sqrt{2}+1, 1)$$

20 Rotate T by 60° around the vector (1, 1, 1)

b. show the actual mathematical operations.

$$T = \{(-1,0), (1,0), (0,1)\}$$

Vector(1, 1, 1)



We can split the rotate in THREE parts:

1) part One: $(1, 0, 0) \Rightarrow X$ Axis;

2) part two: $(0, 1, 0) \Rightarrow Y$ axis;

3) part three: $(0, 0, 1) \Rightarrow Z$ axis;

→ assume vector V will rotate around vector $\vec{n} = (1, 1, 1)$

→ V can be break into $V_{\parallel}$ and $V_{\perp}$

→ $V_{\parallel}$ is the mapping on n, $V_{\perp}$ is the orthorganal vector to n.

→ $V_{\parallel}$ will not rotate, only $\underline{\underline{V_{\perp}}}$ will rotate

$$V_{\parallel} = (V^T n) n, \quad V_{\perp} = V - (V^T n) n$$

now we let $w = n \times V_{\perp}$ ⇐ w is orthorganal to n and $V_{\perp}$

$$V_{\perp}' = V_{\perp} \cos\theta + w \sin\theta = [V - (V^T n) n] \cos\theta + (n \times V) \sin\theta$$

$$V' = V_{\perp}' + V_{\parallel}$$ (Because $V_{\parallel}$ never change)

$$= [V - (V^T n) n] \cos\theta + (n \times V) \sin\theta + (V^T n) n$$

if we let $v=\begin{bmatrix}1\\0\\0\end{bmatrix}$, or $\begin{bmatrix}0\\1\\0\end{bmatrix}$, or $\begin{bmatrix}0\\0\\1\end{bmatrix}$, we get different

v' value, and we have that transformation Matrix is:

$$R=\begin{bmatrix} n_x^2(1-\cos\theta)+\cos\theta & n_xn_y(1-\cos\theta)-n_z\sin\theta & n_xn_z(1-\cos\theta)+n_y\sin\theta \\ n_xn_y(1-\cos\theta)+n_z\sin\theta & n_y^2(1-\cos\theta)+\cos\theta & n_yn_z(1-\cos\theta)-n_x\sin\theta \\ n_xn_z(1-\cos\theta)-n_y\sin\theta & n_yn_z(1-\cos\theta)+n_x\sin\theta & n_z^2(1-\cos\theta)+\cos\theta \end{bmatrix}$$

here: $n=(1,1,1)$,

$\theta=60$,

$\cos 60^\circ=\frac{1}{2}$, $\sin 60^\circ=\sqrt{3}/2$, $1-\cos\theta=\frac{1}{2}$

thus: $$R=\begin{bmatrix} 1 & \frac{1}{2}-\sqrt{3}/2 & \frac{1}{2}+\frac{\sqrt{3}}{2} \\ \frac{1}{2}+\sqrt{3}/2 & 1 & \frac{1}{2}-\sqrt{3}/2 \\ \frac{1}{2}-\sqrt{3}/2 & \frac{1}{2}+\sqrt{3}/2 & 1 \end{bmatrix}$$

$$T=\begin{array}{c} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}\begin{array}{c} x \\ y \\ \\ \end{array}$$

$$T'=RT=\begin{bmatrix} 1 & \frac{1}{2}-\sqrt{3}/2 & \frac{1}{2}+\frac{\sqrt{3}}{2} \\ \frac{1}{2}+\sqrt{3}/2 & 1 & \frac{1}{2}-\sqrt{3}/2 \\ \frac{1}{2}-\sqrt{3}/2 & \frac{1}{2}+\sqrt{3}/2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & \frac{1}{2} - \sqrt{3}/2 \\ -(\frac{1}{2}+\sqrt{3}/2) & (\frac{1}{2}+\sqrt{3}/2) & 1 \\ -\frac{1}{2}+\sqrt{3}/2 & \frac{1}{2}-\frac{\sqrt{3}}{2} & \frac{1}{2}+\frac{\sqrt{3}}{2} \end{bmatrix}$$

after removing z element, it becomes:

$$\begin{bmatrix} -1 & 1 & \frac{1}{2} - \sqrt{3}/2 \\ -\frac{1}{2}-\frac{\sqrt{3}}{2} & \frac{1}{2}+\frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

$$\left\{\left(-1, -\frac{1}{2}-\frac{\sqrt{3}}{2}\right), \left(1, \frac{1}{2}+\frac{\sqrt{3}}{2}\right), \left(\left(\frac{1}{2}-\sqrt{3}/2\right), 1\right)\right\}$$

final answer:

21) find inverse of the matrix:

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{matrix}$$

$$A^{-1} = \frac{A^*}{|A|} \text{ — } \det A$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \frac{A^*}{|A|}$$

$$A^* = \begin{pmatrix} b_2c_3 - c_2b_3 & c_1b_3 - b_1c_3 & b_1c_2 - c_1b_2 \\ c_2a_3 - a_2c_3 & a_1c_3 - c_1a_3 & a_2c_1 - a_1c_2 \\ a_2b_3 - b_2a_3 & b_1a_3 - a_1b_3 & a_1b_2 - a_2b_1 \end{pmatrix}$$

$$|A| = a_1(b_2c_3 - c_2b_3) - a_2(b_1c_3 - c_1b_3) + a_3(b_1c_2 - c_1b_2)$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

$$|A| = \det A$$

$$= 1\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1 - 2 = -1$$

$$A^* \Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$$

$$A = \frac{A^*}{|A|} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix} \Leftarrow \text{Answer!!!!}$$

22

$$A = \begin{vmatrix} 5 & 0 & 1 \\ 0 & 0.5 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$$|A| = 5\begin{vmatrix} 0.5 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} + 1\begin{vmatrix} 0 & 0.5 \\ 0 & 0 \end{vmatrix}$$

$$= |2.5|$$

$$A^* \Rightarrow \begin{vmatrix} 0.5 & 0 & 0 \\ 0 & 5 & 0 \\ -0.5 & -5 & 2.5 \end{vmatrix} \Rightarrow \begin{vmatrix} 0.5 & 0 & -0.5 \\ 0 & 5 & -5 \\ 0 & 0 & 2.5 \end{vmatrix}$$

$$A = \frac{A^*}{|A|} = \begin{vmatrix} 1/5 & 0 & -1/5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{vmatrix} \Leftarrow \text{Answer!}$$

23 $A = \begin{vmatrix} 0.5 & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & 0.5 & 0 \end{vmatrix}$

$$\begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$$|A| = 0.5 \begin{vmatrix} 0.5 & 0 \\ 0 & 1 \end{vmatrix} - \left(-\frac{\sqrt{3}}{2}\right) \begin{vmatrix} \frac{\sqrt{3}}{2} & 0 \\ 0 & 1 \end{vmatrix}$$

$$+ 0 \begin{vmatrix} \frac{\sqrt{3}}{2} & 0.5 \\ 0 & 0 \end{vmatrix}$$

$$= 0.25 + \frac{\sqrt{3}}{2}\,\frac{\sqrt{3}}{2} = 0.25 + \frac{3}{4} = 1$$

$$A^* \Rightarrow \begin{vmatrix} 0.5 & -\frac{\sqrt{3}}{2} & 0 \\ +\frac{\sqrt{3}}{2} & 0.5 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0.5 & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 0.5 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Leftarrow \text{Answer!!!}$$

(24) for example, we have a vector: (x, y)

→ we translate to $(\underline{2, 2}) \Rightarrow (x+2, y+2)$

→ we scale $(\underline{4, 4}) \Rightarrow (4x+8, 4y+8)$

However, if

→ we scale first $\Rightarrow (4x, 4y)$

→ then we translate $\Rightarrow (4x+2, 4y+2)$

$(4x+2, 4y+2) \; != (4x+8, 4y+8)$

normally, we translate first, and we scale.

if T means translate, S means scale:

$$T \cdot S \; != S \cdot T$$

they are not commute!

25 fix point: (3, 2, 5)

Scale: (5, 7, 9)

translate to scale translat back

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 & 0 & -3 \\ 0 & 7 & 0 & -2 \\ 0 & 0 & 9 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0 & 0 & 12 \\ 0 & 7 & 0 & 12 \\ 0 & 0 & 9 & 40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer!