Florida Tech

CSE 5255

INTRODUCTION TO COMPUTER GRAPHICS CLASS NOTES

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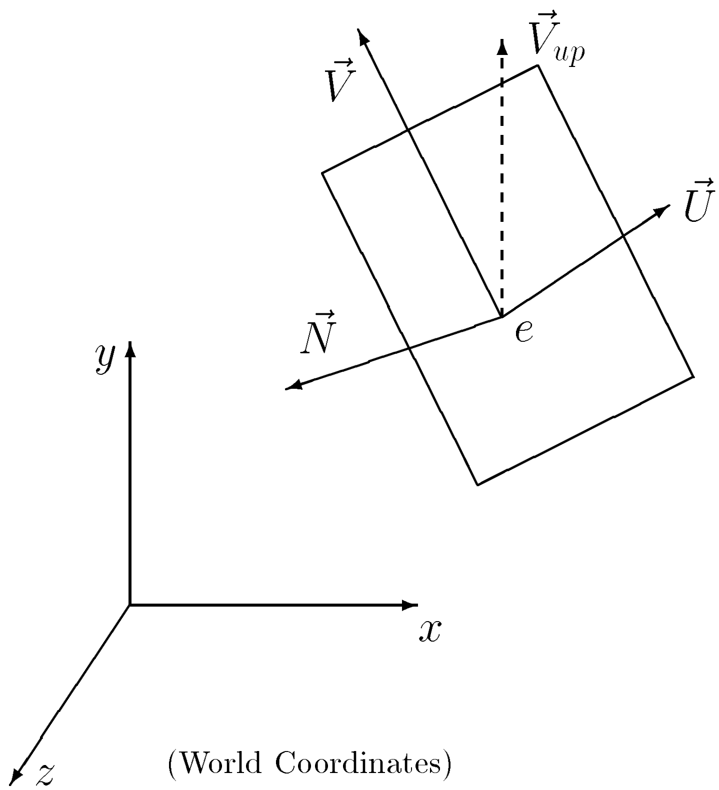
Table of Contents

- A 3D scene can be viewed from any position in 3D space
- The viewing operation is broken up into two steps
  - Define the view orientation (what we'll do in this section)
  - Projecting the view volume to normalized device coordinates (later)
- To define the view orientation we need to specify:
  - View reference point e — the origin of view coordinates
  - View plane normal $\vec{N}$ (unit length) — the z axis of view coordinates
  - View up vector $\vec{V}_{up}$ (unit length) — projects onto the y axis of view coordinates
- $\vec{N}$ and e determine a plane orthogonal to $\vec{N}$ containing e
- The perpendicular projection of $\vec{V}_{up}$ onto this plane determines $\vec{V}$ (the y axis of view coordinates)
- The x axis of view coordinates, called $\vec{U}$, is orthogonal to $\vec{V}$ and $\vec{N}$ (i.e., their cross-product)
- View coordinate system is left-handed


$\vec{V}_{up}$
$\vec{V}$
$\vec{U}$
$\vec{N}$
e
y
x
z
(World Coordinates)

- For perspective views it is convenient to think of
  - the view reference point as the eye's (camera's) position $e = (e_x,\, e_y,\, e_z)$
  - the view plane normal as a unit vector from eye to a "look-at point" $a = (a_x,\, a_y,\, a_z)$, so

    $$\vec{N} = \frac{1}{\|a - e\|}\langle a_x - e_x,\, a_y - e_y,\, a_z - e_z\rangle$$

    where
    $\|a - e\| = \sqrt{(a_x - e_x)^2 + (a_y - e_y)^2 + (a_z - e_z)^2}$
  - The view up vector as the tilt (rotation) of the head (camera)
- For parallel views it is convenient to think of the view plane normal as determining the direction of projection

- Given an object, defined in world coordinate space, we want to express its world coordinate vertices (x, y, z) in term of view coordinates (u, v, n)
- Break the transformation from world to view coordinates into a sequence of transformations:
  - Translate the view reference point e to the origin
  - Rotate about world coordinate y axis to bring the view coordinate $\vec{N}$ axis into the yz plane of world coordinates
  - Rotate about the world coordinate x axis until the z axes of both systems are aligned
  - Rotate about the world coordinate z axis to align the $\vec{V}$ axis with the y
  - Reflect relative to the xy plane, reversing sign of each z coordinate to change into a left-handed coordinate system
- The viewing transformation is

$$V = T \cdot R(y) \cdot R(x) \cdot R(z) \cdot F$$

- Let $(1, 1, 1)$ be the view reference point.
- Let $(0, 0, 0)$ be the look-at point.
- Let $(0, 1, 0)$ be the up vector.
  - Translate:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$
  - Rotate about y:
$$\begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
  - Rotate about x:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & -1/\sqrt{3} & \sqrt{2}/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
  - Rotate about z: Identity

Example of the Viewing Transformation

– Reflection:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

– Composite transform V:

$$\begin{bmatrix} \sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 2/\sqrt{6} & -1/\sqrt{3} & 0 \\ -\sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} & 1 \end{bmatrix}$$

- Let $e = (e_x, e_y, e_z)$ be the eye position (view reference point)
- Let $a = (a_x, a_y, a_z)$ be the "look-at" point.
- Unit length view plane normal is

$$\vec{N} = \frac{a - e}{\|a - e\|}$$

- Let M be a 3×3 rotation matrix that maps the view plane normal $\vec{N}$ to the z axis.

$$\vec{N}M = (0, 0, 1)$$

Thus, the third column of M is $\vec{N}$

$$\vec{N} = (0, 0, 1)M^T = (m_{1,3}, m_{2,3}, m_{3,3}) = m_3$$

- Let $\vec{V}_{up}$ be the unit length view up vector. We want M to map $\vec{V}_{up}$ into the yz plane. Thus,

$$\vec{V}_{up}M = (0,\ A,\ B)$$

for some A, B such that $A^2 + B^2 = 1$.

$$\vec{V}_{up} = (0,\ A,\ B)M^T = Am_2 + Bm_3$$

where m_2 and $m_3 = \vec{N}$ are the second and third columns of M.

- Inner product of $\vec{V}_{up}$ and m_3 gives

$$\vec{V}_{up} \cdot m_3 = A(m_2 \cdot m_3) + B(m_3 \cdot m_3) = B$$

- Both $\vec{V}_{up}$ and $m_3 = \vec{N}$ are known, so B is known and

$$A = \sqrt{1 - B^2}$$

. Thus, solving for m_2

$$m_2 = \frac{\vec{V}_{up} - B\vec{N}}{\sqrt{1 - B^2}}$$

$$m_2 = \frac{\vec{V}_{up} - (\vec{V}_{up} \cdot \vec{N})\vec{N}}{\sqrt{1 - (\vec{V}_u p \cdot \vec{N})^2}}$$

- Finally, we can compute the first column of M via cross products

$$\begin{aligned} m_1 &= m_3 \times m_2 \\ &= \frac{m_3 \times \vec{V}_{up}}{\sqrt{1 - (\vec{V}_{up} \cdot \vec{N})^2}} \end{aligned}$$

(Note: (1) the order of the cross product produces a left-handed system, (2) a vector crossed with itself is zero.)

Constructing the Viewing Transform

- First translate view reference point to the origin

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -e_x & -e_y & -e_z & 0 \end{bmatrix}$$

- Second construct the rotation matrix M
- The viewing transform is given by

$$V = T \begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{21} & m_{22} & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Let $(1,\ 1,\ 1)$ be the view reference point.
- Let $(0,\ 0,\ 0)$ be the look-at point.
- Let $\vec{V}_{up} = \langle 0,\ 1,\ 0 \rangle$ be the up vector.
- Translate:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

- compute third column of M
  - step 1: view plane normal

$$\begin{aligned} \vec{N} &= \frac{a - e}{\|a - e\|} \\ &= (-1/\sqrt{3},\ -1\sqrt{3},\ -1\sqrt{3}) \\ &= m_3 \end{aligned}$$

- compute second column of M
  - step 1:

$$(\vec{V}_{up} \cdot m_3) = -1/\sqrt{3} = B$$

  - step 2:

$$\begin{aligned} m_2 &= \frac{\vec{V}_{up} - B\vec{N}}{\sqrt{1 - B^2}} \\ m_2 &= (-1/\sqrt{6},\ 2/\sqrt{6},\ -1/\sqrt{6}) \end{aligned}$$

Example – Constructing Viewing Transform

- compute first column of M

$$m_1 = m_3 \times m_2 = (\sqrt{2}/2,\ 0,\ -\sqrt{2}/2)$$

- The matrix M is

$$\begin{bmatrix} \sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 2/\sqrt{6} & -1/\sqrt{3} & 0 \\ -\sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- compute $V = TM$:

$$\begin{bmatrix} \sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 2/\sqrt{6} & -1/\sqrt{3} & 0 \\ -\sqrt{2}/2 & -1/\sqrt{6} & -1/\sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} & 1 \end{bmatrix}$$

Problems

1. Find the viewing transformation matrix given a view reference point of $(1,\ 2,\ 1)$, a look-at point $(0,\ 1,\ 0)$, and a view-up vector $\vec{V}_{up} = \langle 1,\ 0,\ 0 \rangle$.
2. Find the viewing transformation given eye position $e = (1,\ 1,\ 1)$, look-at point $e = (1,\ 0,\ 0)$ and up-vector $\vec{V}_{up} = \langle 0,\ 1,\ 0 \rangle$.
3. Find the viewing transformation given eye position $e = (1,\ 1,\ 1)$, look-at point $e = (1,\ 0,\ 0)$ and up-vector $\vec{V}_{up} = \langle 1,\ 0,\ 0 \rangle$.
4. Find the viewing transformation given eye position $e = (1,\ 0,\ 0)$, look-at point $e = (0,\ 1,\ 0)$ and up-vector $\vec{V}_{up} = \langle 0,\ 0,\ 1 \rangle$.
5. If r_1 and r_2 are two different rows of a rotation matrix (a) what is their inner product, (b) what can be said about their cross product?