Florida Tech

CSE 5255

INTRODUCTION TO COMPUTER GRAPHICS CLASS NOTES

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Spring 1996

Table of Contents

- To render is reproduce or represent by artistic means
- Images are rendered to make them appear more realistic
- Images are rendered to disambiguate the scene
- Wire-frame drawings are difficult to understand — there are many techniques available to make drawings easier to interpret
  - Multiple orthographic views
  - Axonometric and oblique projections
  - Perspective projections
  - Depth cueing — decrease intensity of far objects
  - Depth clipping
  - Texture — cross hatching
  - Color
  - Hidden line elimination/ Hidden surface elimination
  - Illumination and shading
  - Shadows, transparency, and reflection
  - Stereopsis

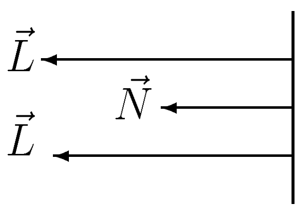
- An *illumination model (equation)* expresses the components of light reflected/transmitted from a surface
- Three light components: *ambient*, *diffuse*, and *specular*
- Light can be *reflected* or *refracted* at a surface
- Light can be diffusely or specularly reflected
- Light can be diffusely or specularly refracted
- First we will consider *local* illumination models, where we want to calculate the reflected light from surface
- Only a crude approximation to ambient light will be used to represent the *global* environment and its effect on the reflected light
- Later we will look at ray tracing and radiosity as global illumination models
- Refracted light will be considered when we study ray tracing
- Note the illumination equation must be calculated in world or view space since perspective mapping destroys the geometry of surface normal, view, and light source vectors

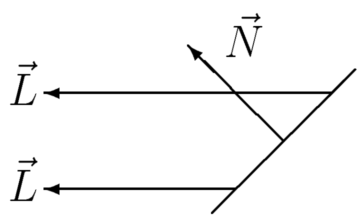
- *Ambient* light component — nondirectional light source that is the product of multiple reflections from the surrounding environment
  - Assume the intensity I_a of ambient light is constant for all objects
  - The *ambient-reflection coefficient* k_a, which ranges from 0 to 1, determines the amount of ambient light reflected by the object's surface
  - The ambient-reflection coefficient is a *material property*
  - The ambient illumination equation is

$$I = k_a I_a$$

    where I is the intensity of reflected light from a surface with ambient-reflection coefficient k_a in an environment with ambient intensity I_a

- *Diffuse* light component — light reflected from a point with equal intensity in all directions
  - Typical of dull, matte surfaces such a paper
  - Modeled by the *Lambertian reflection law*
    - Brightness depends only on the angle θ between the light source direction $\vec{L}$ and the surface normal $\vec{N}$
    - The light beam covers an area whose size is inversely proportional to the cosine of θ
    - The amount of reflected light seen by the viewer is independent of the viewer's direction and proportional to $\cos\theta$





- The diffuse illumination equation is

$$I = k_d I_p \cos\theta$$

- I_p is the point light source's intensity
- k_d is the *diffuse-reflection coefficient* which varies between 0 and 1 and depends on the surface material
- Assuming unit vectors, we can write the cosine as a dot product

$$I = k_d I_p (\vec{N} \cdot \vec{L})$$

- The illumination equation must be evaluated in world coordinates since the normalizing and perspective transformations will modify θ
- If the point light source is sufficiently distant $\vec{L}$ is constant for all points on the surface

- Two distinguish two parallel identical surfaces at different distances, the intensity from the more distance surface must be attenuated
  - The energy from a point light source obeys an inverse square law

    $$f_{att} = d_L^{-2}$$

    where d_L is the distance from the point light source to the surface
  - In practice, a more general model gives better results

    $$f_{att} = max\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

    where c_1, c_2, c_3 are user-defined constants
  - A more simple model such as

    $$f_{att} = \frac{1}{c_1 + c_2 d_L}$$

    is also often used

- Colored lights and surfaces can be modeled by sampling the intensity at discrete wavelengths

$$I_\lambda = k_a I_{a\lambda} O_{d\lambda} + f_{att} k_d I_{p\lambda} O_{d\lambda} (\vec{N} \cdot \vec{L})$$

- Here $O_{d\lambda}$ defines the object's *diffuse color component* at wavelength λ
- Often only three, red, green, and blue, components are sampled, which leads to color aliasing, but is acceptable for simple renderings
- That is, a red intensity I_{red}, a green intensity I_{green}, and a blue intensity, I_{blue} are used to define the color in the RGB color system
- In theory, the intensity should in integrated over the visible spectrum
- Multiple light sources can be modeled by summing the intensity reflected by the object for each source

$$I_\lambda = k_a I_{a\lambda} O_{d\lambda} + \sum_{j=1}^{n} f_{att_j} k_d I_{p\lambda j} O_{d\lambda} (\vec{N} \cdot \vec{L}_j)$$

- *Specular* reflection component — highlights caused by light reflecting primarily in one direction
  - Depends on the angle θ between $\vec{L}$ and $\vec{N}$ and the angle α between the viewer $\vec{V}$ and the reflected ray $\vec{R}$
  - Phong developed a popular approximation to the specular component

$$I = W(\theta) I_p \cos^n \alpha$$

  - $W(\theta)$ is the fraction of specularly reflected light (often set to a constant k_s)
  - n is the *specular-reflection exponent*, between 1 and several hundred (1 gives broad gentle falloff, high values give focused highlight)

- The illumination equation is

$$I_\lambda = k_a I_{a\lambda} O_{d\lambda} + f_{att} I_{p\lambda} \left[k_d O_{d\lambda} (\vec{N} \cdot \vec{L}) + k_s O_{s\lambda} (\vec{R} \cdot \vec{V})^n \right]$$

  where $O_{s\lambda}$ is the object specular color component

- A planar polygon lies in some plane with equation

$$Ax + By + Cz + D = 0$$

with normal $\vec{N} = \langle A,\ B,\ C \rangle$ (or $\langle -A,\ -B,\ -C \rangle$)

- The plane's (polygon's) normal can be computed as the cross product

$$\vec{N} = (P_2 - P_1) \times (P_3 - P_2)$$

where $P_1,\ P_2,\ P_3$ are three points in the plane (consecutive vertices for the polygon)

- Newell's technique for computing the *outward pointing surface normal* $\vec{N} = \langle N_x,\ N_y,\ N_z \rangle$

$$N_x = \sum_{k=1}^{n} (y_k - y_{k+1})(z_k + z_{k+1})$$

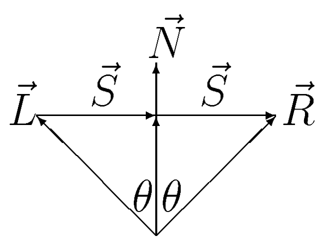
$$N_y = \sum_{k=1}^{n} (z_k - z_{k+1})(x_k + x_{k+1})$$

$$N_z = \sum_{k=1}^{n} (x_k - x_{k+1})(y_k + y_{k+1})$$

where $P_k = (x_k,\ y_k,\ z_k),\ k = 1,\ \ldots,\ n$ are the vertices of the polygon and $P_{n+1} = P_1$

Calculating the Reflection Vector

- The reflection vector $\vec{R}$ is the mirror image of the unit light vector $\vec{L}$ about the unit surface normal $\vec{N}$
- The projection of $\vec{L}$ onto $\vec{N}$ is $\vec{N}\cos\theta$
- Let $\vec{S} = \vec{N}\cos\theta - \vec{L}$
- Note that $\vec{R} = \vec{N}\cos\theta + \vec{S}$
- Thus $\vec{R} = 2\vec{N}\cos\theta - \vec{L}$
- Or $\vec{R} = 2\vec{N}(\vec{N}\cdot\vec{L}) - \vec{L}$
- And $\cos\alpha = (\vec{R}\cdot\vec{V}) = 2(\vec{N}\cdot\vec{V})(\vec{N}\cdot\vec{L}) - (\vec{L}\cdot\vec{V})$



- Blinn proposed an alternative to Phong's model that uses the *halfway vector*

$$\vec{H} = \frac{\vec{L} + \vec{V}}{\mid \vec{L} + \vec{V} \mid}$$

  which is halfway between the light source and viewer

- The halfway vector is the direction of maximum highlights

- If the surface is oriented so that the normal is aligned along $\vec{H}$, the viewer sees the brightest specular highlights

- The specular term is $(\vec{N} \cdot \vec{H})^n$

- The illumination equation is

$$I_\lambda = k_a I_{a\lambda} O_{d\lambda} + f_{att} I_{p\lambda} \left[k_d O_{d\lambda} (\vec{N} \cdot \vec{L}) + k_s O_{s\lambda} (\vec{N} \cdot \vec{H})^n \right]$$

- The illumination model (equation) developed so far is based on common-sense and works well in many situation
- Physical laws can be applied to develop better, but more expensive models
- Several definitions needed
  - *Flux* is the rate at which light energy is emitted (watts)
  - A *solid angle* is the angle at the apex of a cone and can be defined in terms of the area on a hemisphere intercepted by a cone with vertex at the hemisphere's center (just as radians are defined in terms of arc length)
  - A *steradian* is the solid angle of a cone that intercepts an area equal to the square of hemisphere's radius r (there are 2π steradians (sr) in a hemisphere)
  - *Radiant intensity* is the flux radiated into a unit solid angle in a particular direction (watts/sr)

- *Intensity* is the radiant intensity for a point light source
- *Foreshortened surface area* is computed by multiplying the surface area by $\cos\theta_r$, where θ_r is the angle of the radiated light relative to the surface normal
- *Radiance* is the radiant intensity per unit foreshortened surface area $(W/(sr \cdot m^2)$
- *Irradiance* is the flux per unit surface area (W/m^2)
- The irradiance of incident light is

$$E_i = I_i(\vec{N} \cdot \vec{L}) d\omega_i$$

  where I_i is the incident light's radiance

- The *bidirectional reflectivity* ρ is the ratio of reflected radiance (intensity) in one direction to the incident irradiance (flux density) responsible for it from another direction

$$\rho = \frac{I_r}{E_i}$$

- The reflected radiance is

$$I_r = \rho I_i(\vec{N} \cdot \vec{L}) d\omega_i$$

- Bidirectional reflectivity is composed of diffuse and specular components

$$\rho = k_d \rho_d + k_s \rho_s, \quad k_d + k_s = 1$$

- The illumination equation for the reflected intensity is from n light sources is

$$I_r = \rho_a I_a + \sum_{j=1}^{n} I_{i_j}(\vec{N} \cdot \vec{L}_j) d\omega_{i_j}(k_d \rho_d + k_s \rho_s)$$

- Developed by physicists and introduced by Blinn into computer graphics to better model the specular reflectivity
- Surface is a collection of planar *microfacets*, each a perfectly smooth reflector
- The geometry and distribution of the microfacets and the direction of the light determine the intensity and direction of specular reflection
- The Torrance-Sparrow model assumes the specular component of bidirectional reflectivity is given by

$$\rho_s = \frac{F_\lambda DG}{\pi(\vec{N} \cdot \vec{V})(\vec{N} \cdot L)}$$

  where

- D is the distribution function of the microfacet orientation
- G is the *geometrical attenuation factor* (shadowing and masking of microfacets on each other)
- F_λ is the Fresnel term which relates the incident light to reflected light for a smooth surface
- The $\vec{N} \cdot \vec{V}$ term make the equation proportional to the surface area seen by the viewer in a unit of foreshortened area
- The $\vec{N} \cdot \vec{L}$ term make the equation proportional to the surface area the light sees in a unit of foreshortened area
- The π accounts for surface roughness

- The microfacet distribution D determines the roughness of the surface
- Torrance and Sparrow assume the microfacet distribution is Gaussian

$$D = c_1 e^{-(\beta/m)^2}$$

  where c_1 is an arbitrary constant, m is the root mean square slope of the microfacets, and β is the angle between $\vec{N}$ and $\vec{H}$
- Cook and Torrance use the more theoretically correct Beckman distribution

$$D = \frac{1}{m^2 \cos^4 \beta} e^{-(\tan \beta/m)^2}$$

- When m is small, the microfacets vary slightly from the surface normal and the reflection is highly directional
- When m is large, the microfacets slopes are steep and the rough surface spreads out the light

- The geometric attenuation factor $0 \leq G \leq 1$ takes into account that some microfacets will lie in the shadow of others or light reflected from them will strike other microfacets
- If the reflected light from the microfacet is not blocked from the view and not in shadow, then $G = 1$
- Let l denote the area of a microfacet and m the area whose reflected light is blocked or is in shadow

$$G_{b|s} = 1 - \frac{m}{l}$$

- For light blocked from the viewer,

$$G_b = \frac{2(\vec{N} \cdot \vec{H})(\vec{N} \cdot \vec{V})}{(\vec{V} \cdot \vec{H})}$$

- For light in shadow

$$G_s = \frac{2(\vec{N} \cdot \vec{H})(\vec{N} \cdot \vec{L})}{(\vec{V} \cdot \vec{H})}$$

- G is the minimum of these 3 values

$$G = \min\left\{1,\ \frac{2(\vec{N} \cdot \vec{H})(\vec{N} \cdot \vec{V})}{(\vec{V} \cdot \vec{H})},\ \frac{2(\vec{N} \cdot \vec{H})(\vec{N} \cdot \vec{L})}{(\vec{V} \cdot \vec{H})}\right\}$$

- The Fresnel equation for unpolarized light specifies the ratio of reflected light from a dielectric (nonconducting) surfaces as

$$F_\lambda = \frac{1}{2}\left(\frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} + \tan^2(\theta_i - \theta_t)\tan^2(\theta_i - \theta_t)\right)$$

where $\theta_i = \cos^{-1}(\vec{L} \cdot \vec{H})$ and θ_t is the angle of refraction

$$\sin\theta_t = (\eta_{i\lambda}/\eta_{t\lambda})\sin\theta_i$$

where $\eta_{i\lambda}$ and $\eta_{t\lambda}$ are the indices of refraction for the two media

- The Fresnel term can be written as

$$F_\lambda = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{[c(g+c)-1]^2}{[c(g-c)-1]^2}\right)$$

where $c = \vec{L} \cdot \vec{H}$, $g^2 = \eta_\lambda^2 + c^2 - 1$, and $\eta_\lambda = \eta_{i\lambda}/\eta_{t\lambda}$ is the index of refraction

- For conducting medium, the index of refraction is expressed as a complex number

- The color of the specular reflection the surface material, the incident light's wavelength and its angle of incidence
- When $\vec{H} = \vec{L} = \vec{V}$, $\theta_i = 0$, so $c = 1$, $g = \eta_\lambda$ and

$$F_{\lambda_0} = \left(\frac{\eta_\lambda - 1}{\eta_\lambda + 1}\right)^2$$

- When $\vec{H} \perp \vec{L}$, $\theta_i = \pi/2$, so $c = 0$ and

$$F_{\lambda_{\pi/2}} = 1$$

  , (the color of the material is the color of the light)
- Specular reflectance ρ_s depends on η_λ for all angles except $\pi/2$
- η_λ can be derived from F_{λ_0},

$$\eta_\lambda = \frac{1 + \sqrt{F_{\lambda_0}}}{1 - \sqrt{F_{\lambda_0}}}$$

- *Flat shading*, also called *constant shading* or *faceted shading*, is the most simple approach
  - Apply the illumination equation once for each polygon
  - Approach is valid if:
    - The light source is at infinity, so $\vec{N} \cdot \vec{L}$ is constant for a face
    - The viewer is at infinity, so $\vec{N} \cdot \vec{L}$ is constant for a face
    - The polygon is the actual surface being modeled (not an approximation to a curved surface)
- By contrast, interpolated shading computes a separate intensity for each point on the polygon
  - Assume a curved surface is approximated by a polygonal mesh
  - Interpolated can be used to smooth the facets by continuously changing the intensity across edges
  - *Mach bands* appear where intensity has a change in magnitude or slope (dark facets are perceived to be darker and light facets are perceived to be lighter

- Gouraud shading is an *intensity-interpolation* method
- The intensity is calculated at each vertex of a polygon and linearly interpolated across the polygon's edges and face
- This allows the use of polygonal models to represent smooth surfaces such as cylinders and spheres
- We approximate the normals at the polygon vertices by adding all normals for each surface that meets at the vertex
- Consider a truncated pyramid with vertices

$$\begin{aligned} V_1 &= (-1,-1,1) \\ V_2 &= (1,-1,1) \\ V_3 &= (1,1,1) \\ V_4 &= (-1,1,1) \\ V_5 &= (-2,-2,0) \\ V_6 &= (2,-2,0) \\ V_7 &= (2,2,0) \\ V_8 &= (-2,2,0) \end{aligned}$$

- And plane equations surrounding vertex V_1:

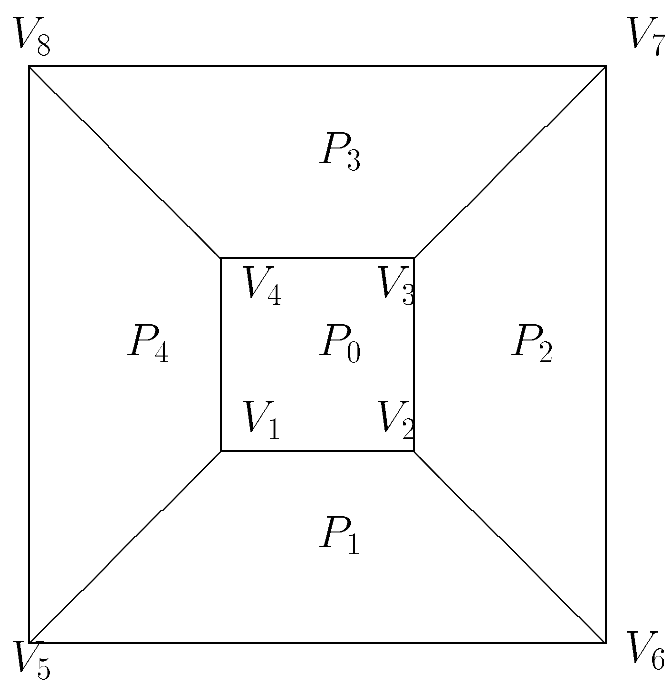
$$\begin{aligned} P_0: &\quad z - 1 &= 0 \\ P_1: &\quad -y + z - 2 &= 0 \\ P_4: &\quad -x \quad + z - 2 &= 0 \end{aligned}$$

- The normal direction at V_1 is then approximately

$$N = ([0 + 0 - 1], [0 - 1 + 0], [1 + 1 + 1]) = (-1, -1, 3)$$

or normalized

$$\hat{N} = (-1, -1, 3)/\|N\| = (-1, -1, 3)/\sqrt{11} \approx (-0.3, -0.3, 0.9)$$



- Given an approximate normal at each vertex of the polygon, we shade the image a scan line at a time.
- The intensity at Q is approximated as a linear combination of the intensities at A and B,

$$I_Q = (1-u)I_A + uI_B \quad 0 \leq u \leq 1$$

where

$$u = \|AQ\| / \|AB\|$$

- Similarly at R, we approximate the intensity by

$$I_R = (1-w)I_B + wI_C \quad 0 \leq w \leq 1$$

where

$$w = \|BR\| / \|BC\|$$

- To compute the intensity at P between Q and R we interpolate the intensities at Q and R,

$$I_P = (1-t)I_Q + tI_R \quad 0 \leq t \leq 1$$

where

$$t = \|QP\| / \|QR\|$$

- The intensity calculation along a scan line can be done incrementally.
- For two adjacent pixels P_1 and P_2 we have

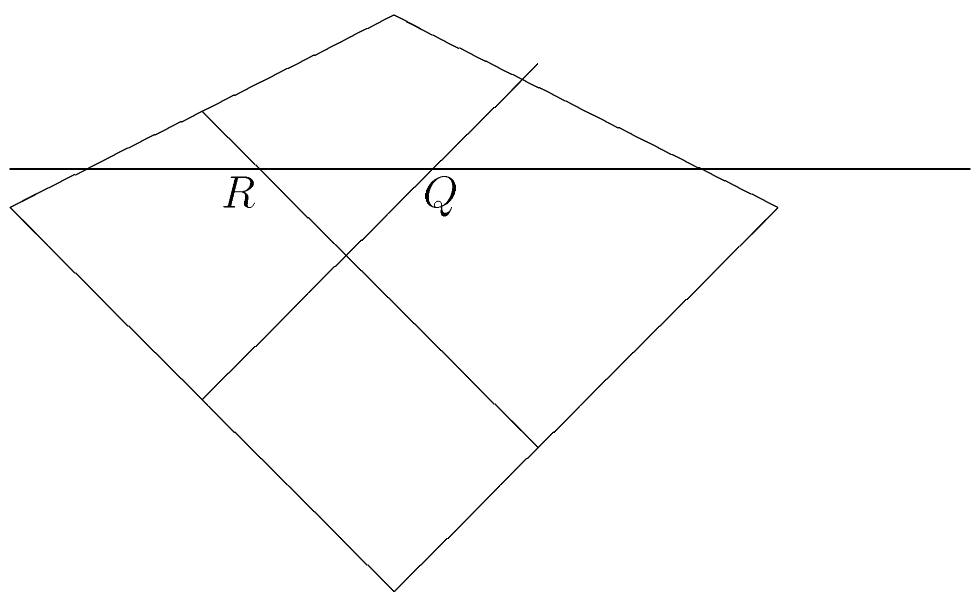
$$I_{P_2} = (1 - t_2)I_Q + t_2 I_R$$

and

$$I_{P_1} = (1 - t_1)I_Q + t_1 I_R$$

Subtracting yields

$$I_{P_2} = I_{P_1} + (I_R - I_Q)(t_2 - t_1) = I_{P_1} + \triangle I \triangle t$$



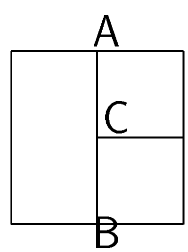
Gouraud Shading

- Gouraud shading gives a improvement over constant shading
- Silhouette edges belie the smooth appearance
- Mach bands can still appear
- Normals along faces can point in radically different direction, but the average may all point in the same direction giving a flat appearance to a non flat object.
- The orientation of the polygon affects its shading
- Specular effects are not represented well

- Phong shading is normal-vector interpolation shading
- The normals at vertices are computed as in Gouraud shading
- The normals are interpolated across polygon edges and between edges on a scan line
- The interpolation can be done incrementally (as the intensities were for Gouraud shading
- At each pixel, the normal is made to be unit vector and inversed mapped into world coordinates
- The illumination equation is evaluated to determine the intensity at the pixel
- Phong shading is better than Gouraud when specular components appear in the illumination model
  - Gouraud shading may miss highlight altogether if they don't appear at vertices
  - Gouraud shading spread highlights at a vertex across the surface
  - Mach bands are reduced in most cases
- Phong shading is expensive

Problems with Interpolated Shading

- Polygonal silhouette edges still appear, for example, in the approximation of a sphere
- Perspective distortion occurs because interpolation is not performed in world coordinates
- The interpolated values depend on the object's orientation
- Shading discontinuities occur when two adjacent polygons fail to share a vertex that lies along their common edge



- Vertex normals may not represent the surface's geometry, for example, averaged normals may all point in the same direction

- Not all surfaces are opaque — some transmit light, e.g. glass, water
- Light ray is "bent" as it passes from one medium to another
- Snell's law states that the refracted (transmitted) ray lies in the same plane as the incident ray and governed by the relationship

$$\eta_i \sin \theta_i = \eta_t \sin \theta_t$$

  where η_i and η_t are the indices of refraction of the two media, and θ_i is the angle between the incident ray and the surface normal and θ_t is the angle between the transmitted ray and the surface normal
- Light can be transmitted specularly (transparent) or diffusely (translucent)

- *Interpolated transparency* computes the intesity at a pixel where transparent surface 1 covers surface 2 by

$$I_\lambda = (1 - k_{t1}) I_{\lambda 1} + k_{t1} I_{\lambda 2}$$

where $I_{\lambda 1}$ and $I_{\lambda 2}$ are the intensities of the two surfaces and k_{t1} is the transmission coefficient of surface 1 ($1 - k_{t1}$ is the surface's *opacity*)

- *Filtered transparency* is modeled by

$$I_\lambda = I_{\lambda 1} + k_{t1} O_{t\lambda} I_{\lambda 2}$$

where $O_{t\lambda}$ is polygon 1's *transparency color*

- Some visible surface algorithms can implement these transparency equations easily, other visible surface algorithm make it difficult to implement transparency

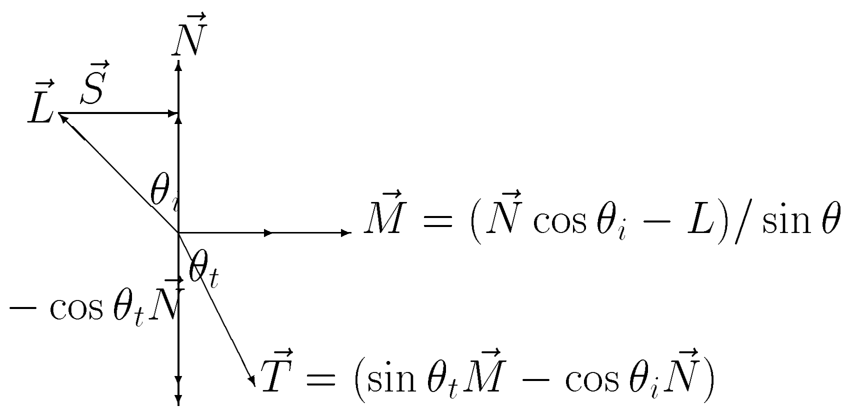
- The (unit) refraction vector is calculated as

$$\vec{T} = \sin\theta_t \vec{M} - \cos\theta_t \vec{N}$$

where $\vec{M}$ is a unit vector perpendicular to $\vec{N}$ in the plane of the incident light ray $\vec{L}$ and $\vec{N}$

- Note that $\vec{M} = \vec{S}/\sin\theta_i = (\vec{N}\cos\theta_i - L)/\sin\theta_i$, so

$$\vec{T} = \frac{\sin\theta_t}{\sin\theta_i}(\vec{N}\cos\theta_i - \vec{L}) - \cos\theta_t \vec{N}$$



- The index of refraction is

$$\eta_\lambda = \eta_{i\lambda}/\eta_{t\lambda} = \sin\theta_t/\sin\theta_i$$

- Thus

$$\vec{T} = (\eta_\lambda \cos\theta_i - \cos\theta_t)\vec{N} - \eta_\lambda \vec{L}$$

- Note that

$$\cos\theta_i = \vec{N}\cdot\vec{L}$$

and

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\eta_\lambda^2\sin^2\theta_i} = \sqrt{1-\eta_\lambda^2(1-(\vec{N}\cdot\vec{L})^2)}$$

- Thus

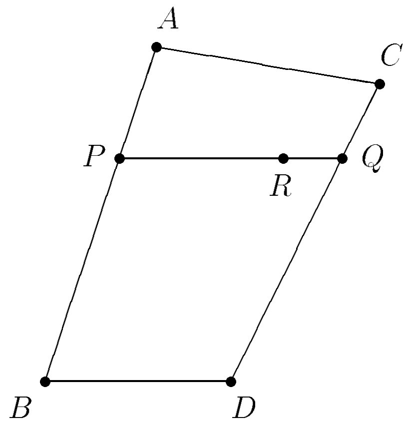
$$\vec{T} = \left(\eta_\lambda(\vec{N}\cdot\vec{L}) - \sqrt{1-\eta_\lambda^2(1-(\vec{N}\cdot\vec{L})^2)}\right)\vec{N} - \eta_\lambda\vec{L}$$

Problems

1. Given a polygon with vertices $(1, 1, -1)$, $(8, 0, -9)$, $(-1, 1, 0)$ and $(2, 2, 3)$ determine a unit normal vector to the polygon and an equation for the plane in which the polygon lies.
2. Given the plane equation $2x - y + 3 = 0$ in xyz space, what is a normal vector to the plane?
3. Given a surface normal $\vec{N} = \langle 1, 2, 1 \rangle$ at a point P being illuminated from a light source in direction $\vec{L} = \langle 3, 1, 4 \rangle$, what is the direction of the specular reflected ray?
4. A surface with normal vector $\vec{N} = \langle 3/5, 0, 4/5 \rangle$ is illuminated by a light source in the direction $\vec{L} = \langle 0, -4/5, 3/5 \rangle$. Determine the reflection vector $\vec{R}$.
5. Define ambient light.
6. How is ambient reflected light modeled in the standard illumination equation?
7. Define diffuse light.
8. How is diffuse reflected light modeled in the standard illumination equation?
9. Define specular light.
10. How is specular reflected light modeled in the standard illumination equation?
11. What is the Phong specular exponent?
12. What is the purpose of the Phong specular exponent?
13. What is the main concept behind Gouraud shading?
14. What incremental technique can be used in Gouraud shading?
15. Suppose that you know that point R lies $4/5$ of the distance from P to Q along a scan line. In addition, P is $1/3$ of the distance from A to B and Q is $1/4$ of the distance from C to D, where A, B, C, D are vertices of a polygon.

    a) Given the intensity of light $I_A = 0.3$, $I_B = 0.48$, $I_C = 0.6$, $I_D = 0.4$ at each of the polygon vertices, find the interpolated intensity I_R at R.

    b) Given a step size of $\triangle t = 0.1$ between points on the scan line what would be the intensity at the point two step to the right of R?



16. Why is Phong shading more expensive than Gouraud shading?