Florida Tech

CSE 5255

INTRODUCTION TO COMPUTER GRAPHICS CLASS NOTES

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Color in Computer Graphics

- Color and gray scale required for modern computer graphics
- Complex subject – concepts from physics, physiology, psychology, art, etc.
- Interpreted color of an object depends on
  - object's surface – rough, smooth, shiny, dull
  - light source – point, linear, area, position
  - surrounding environment – outside, inside
  - human visual system – color vision, perceived range of colors

- Achromatic light runs from black through grays to white
- The *quantity* of light is the only attribute of achromatic light
- Physicists talk of *intensity* or *luminance* of the light's energy
- Psychologists talk of the *brightness* of the *perceived* intensity
- Luminance and brightness are related by not identical concepts
- Zero (0) is defined as black; One (1) as white
- Some display devices can produce gray scales (values between 0 and 1)
- Some display devices produce only *bilevel* images

- The eye is sensitive to ratios of intensity levels rather than absolute intensity
- For example, the perceived change in intensity from 0.1 to 0.11 is the same as the perceived change in intensity from 0.5 to 0.55
- To select $n+1$ intensities between 0 and 1 so that they have equal steps in brightness (perceived intensity) we have

$$I_0 = I_0,\ I_1 = rI_0,\ I_2 = rI_1 = r^2 I_0, \ldots,\ I_n = r^n I_0 = 1$$

- Therefore,

$$r = (1/I_0)^{1/(n)}$$

and

$$I_j = r^{j-1} I_0 = I_0^{(n-j)/(n)}, \quad 1 \leq j \leq n$$

- Typical values for I_0 are between 0.005 and 0.025
- The *dynamic range* $1/I_0$ of a CRT is the ratio between maximum and minimum intensities

- Display of the intensities I_j is tricky
  - Given N electrons in a beam hitting a phosphor, the intensity of light output is

    $$I = kN^\gamma$$

    where k and γ are constants, with γ typically in the range 2.2 to 2.5
  - N is proportional to the control grid voltage, which is proportional to the value V specified for the pixel, thus

    $$I = KV^\gamma, \quad V = (I/K)^{1/\gamma}$$

  - Given I, we calculate the nearest intensity I_j by

    $$I_j = r^j I_0, \quad \text{where} \quad j = \lfloor \log_r(I/I_0) + 0.5 \rfloor$$

  - The pixel value is

    $$V_j = \lfloor (I_j/K)^{1/\gamma} + 0.5 \rfloor$$

  - The value V_j is placed in the frame buffer, or j is placed in the frame buffer and V_j in entry j of the look-up table

- Multiple pixels on a device can be used to represent a single pixel from an image when the image has fewer pixels than the device
- For bilevel displays the perceived intensity range can be increased, at a cost of reduced spatial resolution, by *halftoning*
- Halftoning or clustered-dot ordered dither used variable sized black circles to produce different intensity levels
- Graphics devices can approximate variable-area circles
- For example, with a 2×2 grid, 5 different intensity levels can be achieved
- An $n \times n$ grid can produce $n^2 + 1$ intensity levels

- Selecting the set of patterns for halftoning
  - Avoid patterns that will introduce visual artifact, eg. horizontal, vertical, or diagonal line, in the image
  - Form a *growth sequence* with the patterns — any pixel intensified at level j is intensified at level k for $k > j$
  - Grow the patterns outward from the center, to create effect of increasing dot size
  - For devices that can't plot isolated dots well, eg. laser printers, all "on" pixels should be adjacent
- Dither matrices can be used to represent a sequence of patterns, eg.

$$D^{(2)} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}, \quad D^{(3)} = \begin{bmatrix} 6 & 8 & 4 \\ 1 & 0 & 3 \\ 5 & 2 & 7 \end{bmatrix}$$

- Halftone approximation can be applied to gray-scale and color devices
- Consider a gray-scale device
  - With m bits/pixel, 2^m intensities can be represented $(0, \ldots, 2^m - 1)$
  - Using an $n \times n$ pattern, $n^2 * (2^m - 1) + 1$ intensities from 0 to $n^2 * (2^m - 1)$ can be represented
- Consider a RGB color device
  - Assume m_R, m_G, and m_B bits for red, green, and blue respectively
  - Then

    $$(n^2 * (2^{m_R} - 1) + 1) * (n^2 * (2^{m_G} - 1) + 1) * (n^2 * (2^{m_B} - 1) + 1)$$

    intensities can be represented

- *Dispersed-dot ordered dithering* can be used on devices able to display individual dots
- When the device and the image have the same number of pixels, a pixel at (x, y) can be intensified if the intensity $S(x, y)$ is greater than the value in the dither matrix at the entry (i, j) corresponding to (x, y), ie. if

$$S(x, y) > D_{ij}^{(n)}, \text{ where } i = x \bmod n,\ j = y \bmod n$$

- The effect of equal image and display arrays is apparent only in areas where intensity varies

Chromatic Light

- Color perception involves three attributes: *hue*, *saturation*, and *lightness*
- Hue classifies the color as red, yellow, green, blue, or intermediate color between any contiguous pair of these colors
- Saturation (chroma) is the degree of difference from the achromatic light of the same brightness — a saturated color is a pure spectral color, an unsaturated color is pastel
- Lightness (value) is the perceived intensity of a *reflecting* object
- Brightness is the perceived intensity of a *self-luminous* object — a light source
- Colorimetry is a branch of physics which provides an objective, quantitative way of measuring color
  - Dominant wavelength corresponds to the perceptual notion of hue
  - Excitation purity corresponds to saturation
  - Luminance measures intensity or energy of light

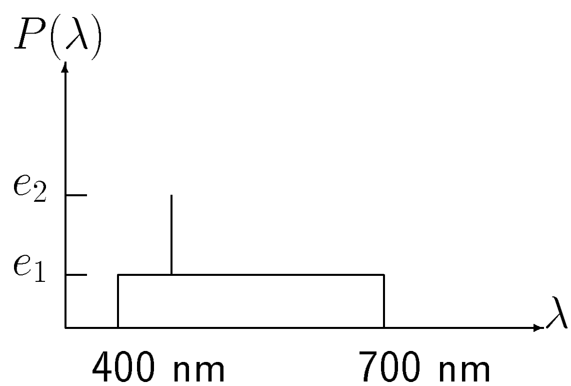
Chromatic Light

- Light is electromagnetic energy in the 400 to 700 nanometer wavelength range
- The hues range from violet to indigo to blue to green to yellow to orange to red
- Light can also be described by frequency f where f is related to wavelength λ by

$$f = \frac{c}{\lambda} \quad c = 300,000 \text{ kilometers/second}$$

- The *spectral energy distribution* $P(\lambda)$ gives the amount of energy present at each wavelength λ
- Two spectral energy distributions that “look” the same are called metamers

- The dominant wavelength λ_d is where the largest spike e_d in the spectral energy distribution occurs
- The excitation purity is the percentage difference between e_d and the uniform distribution of energy
- The luminance is proportional to the area under the spectral energy distribution curve, weighted by the *luminous-efficiency function*



- The *tristimulus theory* of color perception is based on the hypothesis that the retina has three kinds of *cones* with peak sensitivity to red, green, or blue lights
- Experiments produce *spectral-response functions* with show
  - Blue has peak response around 440 nm, with about 2% of light absorbed by the blue cones
  - Green has peak response around 545 nm, with about 20% of light absorbed by the green cones
  - Red has peak response around 580 nm, with about 19% of light absorbed by the red cones
  - The eye's response to blue light is much less strong than it is to green or red
- The luminous-efficiency function is the eye's response to light of constant luminance as the dominant wavelength is varied
- Our peak sensitivity is to yellow-green light around 550 nm

- Imagine viewing a test color with the left eye and have 3 knob to control the red, green, and blue components of a second color, seen with the right eye (suppose the knobs turn in 1 nm increments)
- As the test color's dominant wavelength is varied, three color matching functions, r_λ, g_λ, and b_λ, showing the amount of red, green, and blue needed to match the test color are generated
- For some test colors, the values of r_λ, g_λ, or b_λ may be negative — indicating the light needs to be moved to the test color side
- Not all colors can be represented by positive RGB mixes
- The human eye can distinguish hundreds of thousands of colors
- Near the ends of the spectrum colors of noticeably different hues may be about 10 nm apart
- Most distinguishable hues are with 4 nm
- About 128 fully saturated hues can be distinguished
- The eye is less sensitive to saturated light

- In 1931, the *Commission Internationale de l'Éclairage* (CIE) defined three primaries called $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ to be used in color matching
- Color matching functions x_λ, y_λ, and z_λ are all positive for matching any visible color
- The $\mathbf{Y}$ primary's color matching function y_λ exactly matches the luminous-efficiency function
- These color matching functions are tabulated at 1 nm intervals using color samples that subtend a 2° field of view on the retina
- The amount of $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ needed to match a color with spectral energy function $P(\lambda)$ are:

  $$X = k \int P(\lambda) x_\lambda d\lambda, \quad Y = k \int P(\lambda) y_\lambda d\lambda, \quad Z = k \int P(\lambda) z_\lambda d\lambda$$

- For self-luminous objects, k is 680 lumens/watt
- For reflecting objects

  $$k = \frac{100}{\int P_w(\lambda) y_\lambda d\lambda}$$

  where $P_w(\lambda)$ is the spectral energy function of "standard" white

The CIE Color Model

- The visible colors are contained in a cone-shaped region of $\mathbf{XYZ}$ space that extends from the origin into the positive octant

- Let color $\mathbf{C}$ be matched by

$$C = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}$$

- Define *chromaticity values* by

$$x = \frac{X}{X+Y+Z},\ y = \frac{Y}{X+Y+Z},\ z = \frac{Z}{X+Y+Z}$$

- Given (x, y, Y) we can recover (X, Y, Z)

$$X = \frac{x}{y}Y, \quad Y = Y, \quad Z = \frac{1-x-y}{y}Y$$

- Note $X + Y + Z$ can be thought of as the total amount of light energy

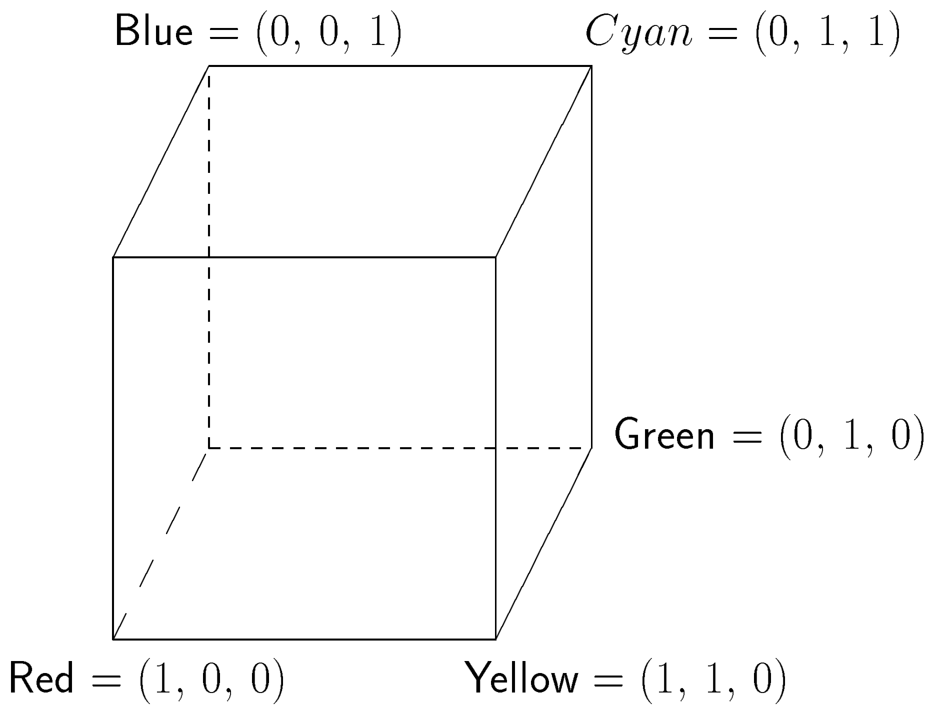
- The chromaticities depend only on dominant wavelength and excitation purity (they are independent of luminous energy)

The CIE Color Model

- The chromaticities are on the plane $\mathbf{X} + \mathbf{Y} + \mathbf{Z} = 1$
- The orthographic projection of this plane onto the $\mathbf{XY}$ plane is called the *CIE chromaticity diagram*
- The CIE chromaticity diagram is a plot of x and y for all visible colors
- 100% spectrally pure colors are on the boundary of the CIE diagram
- A standard white light, (approximately sunlight) called *illuminant C* is near where $x = y = z = 1/3$

- *Complementary colors* when mixed produce white — they lie on opposite sides of a line through C
- The dominant wavelength of a *nonspectral* color F is found by intersecting the line from the color through C with the "horseshoe" part of the CIE diagram
- The excitation purity is the ratio of length CF to CG, where G is the closest point on the boundary along the line through C and F
- A *color gamut* is the set of all colors that can be formed by mixing the colors — it is the convex hull of the colors in the CIE diagram
- Color gamuts of different color devices can be compared using the chromaticity diagram
- Color gamuts for monitors are triangular, film and print gamuts may be more complex shapes
- The CIE LUV uniform color space was developed in 1976 to solve the problem that changes $\triangle C$ in color are not perceived to be equal

- The red, green, and blue (RGB) color model is used in color CRT monitors
- The model is represented by a unit cube
- Red, green, and blue are at corners $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$
- Black is at $(0, 0, 0)$ with grays along the diagonal to white at $(1, 1, 1)$
- The RGB primaries are *additive*
  - Cyan $(0, 1, 1)$ is formed by adding green and blue
  - Magenta $(1, 0, 1)$ is formed by adding red and blue
  - Yellow $(1, 1, 0)$ is formed by adding red and green
  - Red and cyan are complementary; green and magenta are complementary; blue and yellow are complementary


Blue $= (0,\ 0,\ 1)$
$Cyan = (0,\ 1,\ 1)$
Green $= (0,\ 1,\ 0)$
Red $= (1,\ 0,\ 0)$
Yellow $= (1,\ 1,\ 0)$

- The color gamut of the RGB model is defined by the chromaticities of a CRT's phosphors
- Let (x_r, y_r), (x_g, y_g), and (x_b, y_b) be the chromaticities for the vertices of a monitor's RGB color gamut
- Let Y_r, Y_g, Y_b be the luminances of maximum-brightness red, green and blue
- Let $C_r = Y_r/y_r = X_r + Y_r + Z_r$,
  $C_g = Y_g/y_g = X_g + Y_g + Z_g$,
  $C_b = Y_b/x_b = X_b + Y_b + Z_b$
- The transformation from RGB to CIE is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_r C_r & x_g C_g & x_b C_b \\ y_r C_r & y_g C_g & y_b C_b \\ z_r C_r & z_g C_g & z_b C_b \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

  where $z_r = 1 - x_r - y_r$ and similarly for z_g, z_b
- If M_1 maps monitor 1's RGB to CIE and M_2 maps monitor 2's RGB to CIE, then $M_2^{-1} M_1$ maps monitor 1's RGB to monitor 2's RGB
- Of course, some values may not be displayable on both monitors

- Cyan, magenta, and yellow are used with hardcopy devices that deposit colored pigment onto paper
- The CMY model describes this model with *subtractive* primaries
- Cyan subtract (filters) red from reflected white light allowing only green and blue to be be seen
- Magenta absorbs green; yellow absorbs blue
- The CMY model uses a unit cube with a relabeling of corners; colors are specified by what is subtracted from white light
- The (ideal) transformations between CMY and RGB are given by

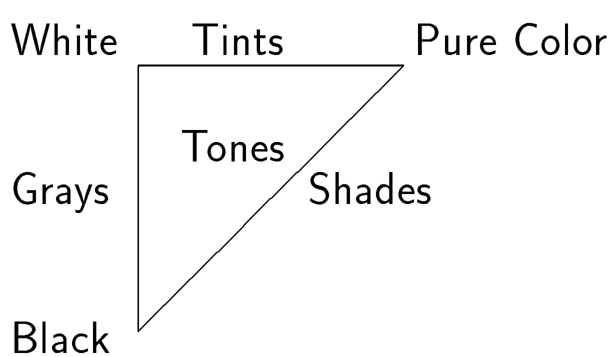
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

The CMY Color Model

- Four color printing uses the CMYK color model where K represents black
- The (ideal) conversion from CMY to CMYK is

$$
\begin{aligned}
&K = \min(C, M, Y)\\
&C = C - K\\
&M = M - K\\
&Y = Y - K
\end{aligned}
$$

- A system closer to the artist's model of color mixing
- Hue, Saturation, and Value (brightness)
- The model is defined on a *hexcone*, or six-sided pyramid
- The top of the cone, given by $V = 1$, contains the brightest colors
- The apex of the cone corresponds to black
- The hue is measured by angles around the cone's vertical axis (Red $= 0°$, Green $= 120°$, Blue $= 240°$)
- The saturation ranges from 0 on the V axis to 1 on the cone's faces
- The top of the hexcone is the projection of the RGB cube looking along the principal diagonal



- Conversion from RGB to HSV — determines the value as the largest RGB component, the saturation as the relative range of RGB values, and the hue as the relative angular displacement from the largest RGB component

$$
\begin{array}{l}
\textit{RGB-to-HSV}(\textbf{float}\, r,\ g,\ b,\ h,\ s,\ v) \\
\{ \\
\quad max = \text{Maximum}(r,\ g,\ b); \\
\quad min = \text{Minimum}(r,\ g,\ b); \\
\quad v = max; \\
\quad \textbf{if } max \neq 0 \textbf{ then } s = (max\text{-}min)/max; \\
\quad \textbf{else } s = 0; \\
\quad \textbf{if } s == 0 \textbf{ then } h = \mathit{UNDEFINED}; \\
\quad \textbf{else } \{ \\
\qquad delta = max \text{ - } min; \\
\qquad \textbf{if } r = max \textbf{ then } h = (g - b)/delta; \\
\qquad \textbf{else if } g = max \textbf{ then } h = 2 + (b - r)/delta; \\
\qquad \textbf{else } h = 4 + (r - g)/delta; \\
\qquad h = h * 60; \\
\qquad \textbf{if } h < 0 \textbf{ then } h = h + 360; \\
\quad \} \\
\}
\end{array}
$$

The HLS Color Model

- The hue, lightness, and saturation color model uses a double hexcone
- White is at $L = 1$ and black at $L = 0$
- Hues are measured in angles from $0°$ to $360°$
- Saturation varies from 0 on the L axis to 1 on the face of the cone
- Grays have $S = 0$
- Maximally saturated color have $S = 1$, $L = 0.5$

```
RGB-to-HLS(float r, g, b, h, l, s)
{
   max = Maximum(r, g, b);
   min = Minimum(r, g, b);
   l = (max + min)/2;
   if max = min then s = 0; h = UNDEFINED;
   else {
      if l < 0.5 then s = (max-min)/(max+min);
      else s = (max-min)/(2-max-min);
      delta = max - min;
      if r = max then h = (g - b)/delta;
      else if g = max then h = 2 + (b - r)/delta;
      else h = 4 + (r - g)/delta;
      h = h * 60;
      if h < 0 then h = h + 360;
   }
}
```

- The Color Naming System is based on natural language color categories
  - Lightness is chosen from *very dark*, *dark*, *medium*, *light* , or *very light*
  - Saturation can be *grayish*, *moderate*, *strong*, or *vivid*
  - Thirty-one hues are formed from seven generic hues: *red*, *orange*, *brown*, *yellow*, *green*, *blue*, and *purple*
  - Halfway hues are denoted by hyphenation, eg, green-blue
  - Quarterway hues with an *ish* suffix, eg yellowish-green
- The Munsell color-order system is a set of published standard colors organized in a space of hue, value, and chroma (saturation)
- Each color in the Munsell system is named and ordered to have an equally perceived "distance" in color space from its neighbors

- Color interpolation needed for (1) Gouraud shading, (2) antialiasing, (3) blending images (fade-in, fade-out)
- Results of interpolation depend on color model
- If map from one color model to another is linear, then linear interpolation in both models will be the same (RGB, CMY, CIE, YIQ).
- Map from RGB to HSV or HLS is non-linear
- Example:
  - Consider $(1-t)(1,\,0,\,0)+t(0,\,1,\,0)$ which linearly interpolates from red to green in RGB space
  - At $t=0.5$ the color $(0.5,\,0.5,\,0)$ maps to $(60°,1,\,0.5)$ in HSV
  - Consider $(1-t)(0°,\,1,\,1)+t(120°,\,1,\,1)$ which linearly interpolates from red to green in HSV space
  - At $t=0.5$ the color $(60°,\,1,\,1)$
- For Gouraud shading any model can be use because the two interpolants are so close together.
- For antialiasing and fade-in, fade-out then RGB is appropriate.

- Spatial integration of RGB triad on color monitors and 4-color (CMY+Black) printing
- Dithering – use 2×2 (or larger grids of pixels)
  - Extend color range, but lose resolution
  - If 3-bits/pixel (1 for red, green and blue), then there are 5 different reds, greens and blues or 125 color combinations.
- Xerography, ink-jets and thermal color copiers actually mix subtractive color pigments.

- Given a color image with n bits/pixel to be displayed on a device with $m < n$ bits/pixel
  - Which 2^m colors should be displayed?
  - What map from the 2^n colors in the image to the 2^m colors should be used?
  - Solution 1: Use a fixed set of display colors and a fixed mapping (eg. $m = 8$, use 3 bit for red and green, 2 bits for blue)
  - Solution 2: *popularity algorithm* — create histogram of the image's color and use the 2^m most frequent colors.
  - Solution 3: *median-cut algorithm* — recursively fit a box around the colors used in the image, splitting the box along its longer dimension at the median until 2^m boxes have been created. Use the centroid of the box as the display color for all image colors in the box.
- Accurate color reproduction is difficult.

- Select colors by some method, eg. traversing a smooth curve or restricting to a plane in color space
- Avoid the simultaneous display of highly saturated, spectrally extreme colors
- Pure blue should be avoided for text, thin lines, and small shapes
- Avoid adjacent colors that differ only in the amount of blue
- Older operators need higher brightness levels to distinguish colors
- Colors change in appearance as the ambient light level changes
- The magnitude of a detectable change in color varies across the spectrum
- It is difficult to focus upon edges created by color alone

- Avoid red and green in the periphery of large-scale displays
- Opponent colors go well together
- For color-deficient observers, avoid single-color distinctions
- Random selection of different hues is usually garish
- If a display contains only a few colors, the complement of one color should be used as the background
- A neutral (gray) should be used for the background for an image with many colors
- If two adjoining colors are not harmonious, place thin black border between them

Problems

1. Define achromatic light and chromatic light.
2. Explain how can grey shades be achieved on a bi-level (black and white) display?
3. Carefully describe the CIE and RGB color models.
4. How would you specify a medium gray using the RGB color model?
5. How would you specify a dark blue using the RGB color model?
6. How would you specify a medium gray using the HSV color model?
7. How would you specify a dark blue using the HSV color model?
8. What are hue and dominant wavelength and how do they differ?
9. What is the color gamut of colors C_1, C_2 and C_3?
10. Distinguish additive and subtractive color models?