Gravitation Collapse: simulating interactions between many bodies

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Introduction

Gravitational simulations can be used to model systems of many bodies, that only interact using the force of gravity, in situations for which no analytical solutions can be found. These numerical methods have given physicists an alternative way to investigate astronomical systems and events beyond observation which was historically the only probe we had into the universe. Therefore, simulations have contributed to many areas of research including galaxy formation, dark matter models and gravitational collapse. Here we have outlined how a leapfrog method of approximation can be used to model a gravitational system and have applied this to model a simple solar system consisting of Jupiter, the Earth and the Sun.

The Leapfrog Method

In our simulation, we have used the leapfrog finite difference approximation method [1]. In this aptly named scheme, we increment the time variable, *t*, in steps of *dt* and calculate the positions and velocities half a timestep out of phase so one can be used to promote the other. For a timestep *n* and each particle *i,* we use the positions of all the other particles to calculate the force on *i* using Newton's law of gravity. We then vector sum these forces to get a net force which we use to calculate the velocity at the half timestep $n + \frac{1}{2}$,

$$v_i^{n+\frac{1}{2}} = v_i^{n-\frac{1}{2}} + F_i \frac{dt}{m_i}, \qquad (1)$$

where m_i, is the mass of the particle. This new velocity is then used to update the positions of the particles to the timestamp *n+1,*

$$x_i^{n+1} = x_i^{n+1} + v_i^{n+\frac{1}{2}} dt. \qquad (2)$$

This allows a system to be evolved over a chosen number of timesteps.

In this method it is very important to specify the initial velocity of a particle at a time of $-\frac{1}{2}dt$ as seen from equation 1. Small changes in initial conditions can be amplified into large errors as the system evolves. Therefore, to refine our initial conditions, we have used the binary system mass ratio relationships between 2 bodies in orbit around a common point:

$$\frac{m_1}{m_2} = \frac{a_1}{a_2} = \frac{v_1}{v_2}, \qquad (3)$$

where *m, a* and *v* are the mass, semi-major axis and velocity respectively. A very good estimation for the initial conditions of the Sun comes from assuming that the sun is on a separate binary orbit with both Jupiter and the Earth and summing the initial vectors we would expect for the sun in these orbits to find an accurate initial position and velocity. Fig. 1 demonstrates how vital these modifications are to the accuracy of our simulation.

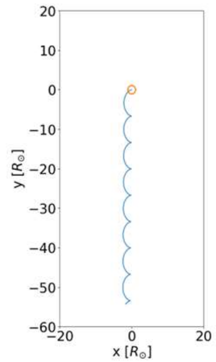

Fig. 1 Comparing how the orbit of the sun evolves over a 100-year simulation when it is taken to start stationary, plotted in orange, against when its initial conditions are specified using the binary mass ratio relationships, plotted in blue. The small circular orbit given by these relationships is as we would expect.

Convergence Of The Model

In an ideal model of the solar system: the total energy and total momentum of the system should stay constant, and the separation between the Sun and the planets should be periodic (as we expect the orbits to have a slight eccentricity) with a constant small amplitude of variation. Fig. 2 shows us the effect of choosing different time steps on our model. We see that the separation between the Sun and the Earth fluctuates regardless of the chosen timestep. However, we can see that as *dt* decreases, the separation converges to a pattern, so there is only a slight difference in orbit evolution when using a timestep of 10 days against any smaller timestep. In scientific computing, it is important to balance accuracy against computational complexity. We therefore assume a timestep of 10 days gives us almost as good of a simulation as any smaller timestep and proceed to use it for our solar system model.

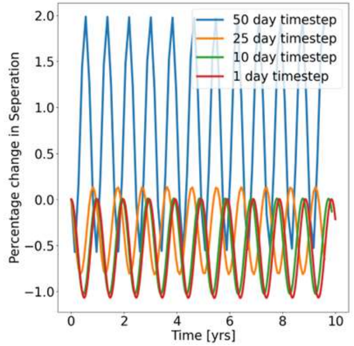

Fig. 2 Plot of the fractional change in separation between the Earth and the Sun over 10 years for different timesteps.

Scan this QR code to watch an animation of the solar system model!

Diagnosing With Conserved Quantities

Continuing the hunt for an ideal model, we turn our attention to the mentioned conserved quantities. As gravity is a conservative force our isolated system should conserve both energy and momentum over time [1]. Recording how these quantities change as the system evolves provides a further check to the validity of the simulation in approximating the real system; this is done in Fig. 3 and Fig. 4. We see that total energy varies by less than 1% and total momentum never rises above $2 \cdot 10^{-13}$% of the sum of the initial momentum magnitude for the 3 individual bodies. This further confirms that our simulation is a good model of the real 3-body Sun, Jupiter and Earth System.

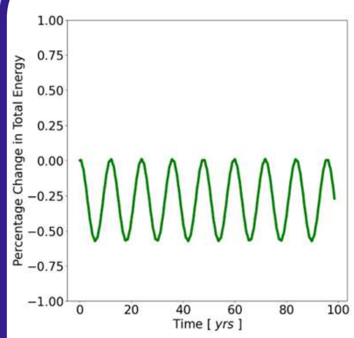

Fig. 3 Fractional change in total energy of the 3-body system over 100 years using a timestep of 10 days.

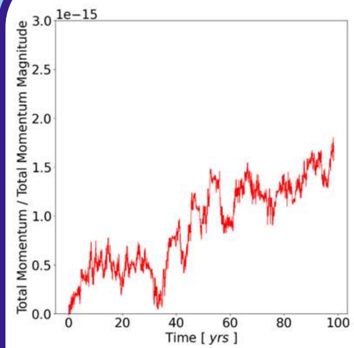

Fig. 4 The total momentum of the 3-body system divided by the summed linear momentum of the 3 bodies, over 100 years using a timestep of 10 days.

Extension

Having established in this poster that our simulation is successful at modelling gravitational systems, the next step is to use it to investigate another, more complex system. The Andromeda and Milky Way galaxies are on course for a collision in roughly 4.5 Billion years[2], We would like to use this N-body simulation to model what the dynamics of this event will be and how the final combined system would behave. These galaxies are thought to contain dark matter halos at their centres, so to correctly approximate this event our simulation needs to be adapted to allow for the existence of masses with spatial extent as well as point masses, such as a Navarro-Frenk-White profile which is a mass distribution for a dark matter halo [3].

[1] Binney, James, and Scott Tremaine. *Galactic dynamics*. Vol. 20. Princeton University Press, 2011.
[2] Cox, T. J., and Abraham Loeb. "The collision between the Milky Way and Andromeda." *Monthly Notices of the Royal Astronomical Society* 386.1 (2008): 461-474.
[3] Navarro, Julio F., Carlos S. Frenk, and Simon DM White. "A universal density profile from hierarchical clustering." *The Astrophysical Journal* 490.2 (1997): 493.