Gravitation Collapse: simulating interactions between many bodies

Durham University

Introduction

Gravitational simulations can be used to model a system of many bodies, who only interact using the force of gravity,
in situations for which no analytical solutions can be found. These numerical methods have given physicists an alternative way to investigate astronomical systems and events beyond observation which was historically the only probe we head into the universe. Therefore, simulations have contributed to many areas of research including galaxy formation, dark matter models and gravitational collapse. Here we have outlined how a leapfrog method of simulation can be used to model the orbits of a solar system consisting of Jupiter, the Earth and the sun.

Leapfrog Method

In our simulation we have used the leapfrog finite difference approximation method [1]. In this scheme we increment the time variable, *t*, in steps of *dt* and calculate the positions and velocities half a timestep out of phase where one is used to promote the other. For a timestep *n* and each particle *i,* we use the positions of the other particles to calculate the force, on the particle using Newton's law of gravity. We use this to calculate the velocity at half the timestep,

$$v_i^{n+\frac{1}{2}} = v_i^{n-\frac{1}{2}} + F_i \frac{dt}{m_i} (1)$$

Where m_i, is the mass of the particle. This new velocity is then used to update the particles positions to the timestamp *n+1,*

$$x_i^{n+1} = x_i^{n+1} + v_i^{n+\frac{1}{2}} dt. \ (2)$$

This allows a system to be evolved over a chosen number of timesteps.

In this method it is very important to specify the initial velocity of a particle at a time of $-\frac{1}{2}dt$ as seen from equation 1. small changes in initial conditions can be amplified into large errors as the system evolves. To refine our initial conditions, we have used the binary system mass ratio relationships between 2 bodies in orbit around a common point:

$$\frac{m_1}{m_2} = \frac{a_1}{a_2} = \frac{v_1}{v_2}, \qquad (3)$$

where *m, a* and *v* are the mass, semi-major axis and velocity respectively. We assume that the sun is on a binary orbit with both Jupiter and the Earth and sum the vectors we would expect for the sun in these orbits to find an accurate initial position and velocity. Fig. 1 demonstrates how vital these modifications are to the accuracy of our simulation.

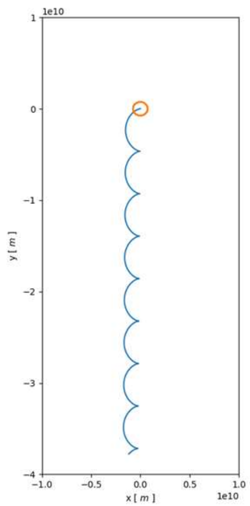

Fig. 1 Comparing how the orbit of the sun evolves over a 100-year simulation when it is taken to start stationary, plotted in blue, against when its initial conditions are specified using the binary mass ratio relationships, plotted in orange. The small circular orbit given by these relationships is as we would expect.

Convergence

Fig. 2 shows us the effect of choosing different time steps on our model. We see that the separation between the sun and the Earth fluctuates regardless of the chosen timestep, this is caused by the slight eccentricity to the Earth's orbit. However, we can see that as *dt* decreases, the separation converges to a specific time dependent pattern; furthermore, there is little difference between the timestep of 10 days and any smaller timestep, so we can assume a timestep of 10 days is good enough for our solar system simulation and we proceed to use it in further investigations.

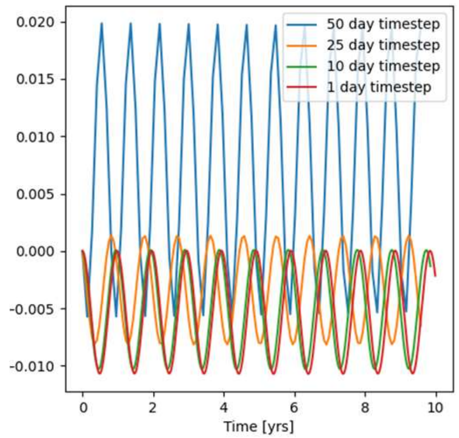

Fig. 2 Plot of the fractional change in separation between the Earth and the Sun over 10 years for different timesteps.

Conserved Quantities

As gravity is a conservative force our isolated system should conserve both energy and momentum over time. Recording how these quantities change as the system evolves provides a further check as to the validity of the simulation in approximating the real system; this is shown in Fig. 3 and Fig. 4. We see that Total energy varies by less than 1% and total momentum never rises above $2 \cdot 10^{-13}\%$ of the momentum of the 3 individual bodies. This confirms that our simulation is a good model of the real 3-body Sun, Jupiter and Earth System.

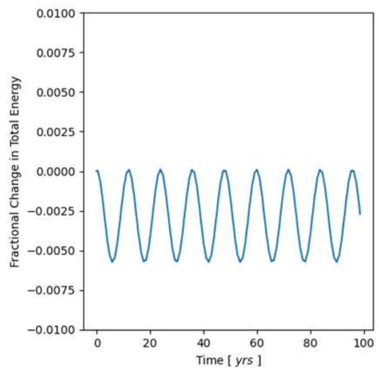

Fig. 3 Fractional change in total energy of the 3-body system over 100 years

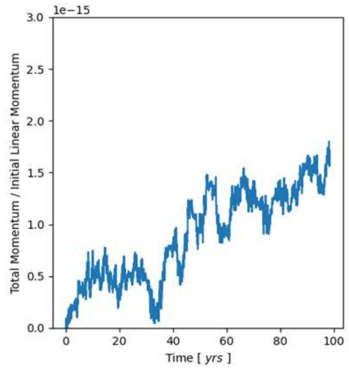

Fig. 4 The total momentum of the 3-body system divided by the summed linear momentum of the 3 bodies, over 100 years.

Extension

Having established in this poster that our simulation is successful at modelling gravitational systems, the next step is to apply it to another, more complex system to investigate further properties. The Andromeda and Milky Way galaxies are on course for a collision in roughly 4.5 Billion years[2], We would like to use this N-body simulation to model what the dynamics of this event will be and how the final combined system would behave.

References

[1] Binney, James, and Scott Tremaine. *Galactic dynamics*. Vol. 20. Princeton university press, 2011
[2] Cox, T. J., and Abraham Loeb. "The collision between the Milky Way and Andromeda." *Monthly Notices of the Royal Astronomical Society* 386.1 (2008): 461-474