HACETTEPE UNIVERSITY

GEOMATICS ENGINEERING DEPARTMENT

GMT312
GLOBAL NAVIGATION SATELLITE SYSTEMS

Assignment 3

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1. INTRODUCTION

Global Navigation Satellite Systems (GNSS) constitute a core application in modern geodesy, navigation, and positioning applications through the delivery of accurate position, velocity, and time (PVT) determination anywhere on the planet. Proper positioning of a GNSS receiver essentially relies on accurate knowledge of the spatial geometry between the receiver and satellite at the exact time of signal emission—referred to as the emission time. However, observation data obtained at the receiver stored in the RINEX format corresponds to the time of reception of the signal. This emission reception time difference necessitates precise modeling of propagation delay of the signal and satellite clock bias corrections to retrieve the position of the satellite at the correct epoch.

The trajectory of GNSS satellites is highly dynamic and typically has speeds of more than 3.9 km/s in MEO, and hence even a tiny timing error can lead to huge position errors on the order of several kilometers. Additionally, the Earth rotates as the signal propagates from satellite to receiver and needs to be accounted for in order to accurately convert the satellite coordinates from emission to reception reference frames. For this, precise ephemerides (SP3 files) and satellite clock corrections and high-order interpolation techniques such as Lagrange interpolation are used.

This exercise involves the computation of the end Earth-Centered Earth-Fixed (ECEF) coordinates of two GPS satellites (G06 and G11) for the MERS GNSS station on April 1, 2025. The objective is to compute the time of signal emission from code pseudorange measurements and satellite clock corrections, interpolate the satellites' positions at that instant, and add a rotation correction to account for Earth's angular motion. All the calculations are performed programmatically with Python, integrating numerical methods with satellite geodesy theory. The end coordinates are the actual spatial geometry between receiver and satellites at the moment the signals were sent and become a foundation for subsequent position estimation procedures.

2. METHODOLOGY

The approximate receiver coordinates were taken from the RINEX file header as: [4239146.6414, 2886967.1245, 3778874.4800] meters in ECEF.
The reception time is calculated by multiplying the sum of the digits of the student ID by 960 seconds. If the result is divisible by 900, 720 seconds are added. In this assignment, the final reception epoch is:

t_rec = 27840 seconds = 07:44:00 UTC

Pseudorange measurements are taken from the observation file:

PRN G06: 22719869.219 meters.
PRN G11: 21509336.237 meters

The satellite clock corrections and positions were extracted from the SP3 file. Ten epochs centered around the reception time were used (five before and five after).
To compute the **signal emission time**, the following function is used:

```
# Emission time calculation
"""
    Computes signal emission time using Lagrange interpolation.
    Inputs:
        trec - reception time (seconds)
        pc - pseudorange (meters)
        clk - 10x2 array: [time, clock correction]
    Output:
        tems - emission time (seconds)
 """
def emist(trec, pc, clk):

    t_vals = clk[:, 0]
    clk_vals = clk[:, 1]
    dt_sat = lagrange_interpolation(trec, t_vals, clk_vals)
    tems = trec - (pc / c) - dt_sat
    return tems
```

To interpolate satellite positions at emission time, the following structure is applied for each of X, Y, Z coordinates:

```
    tems = emist(trec, pc, clk_mat)
# inner function to interpolate satellite coordinates at emission time

    def interpolate_xyz(t_interp, sp3_data):
        pos = []
        for i in range(1, 4):  # X, Y, Z
            pos.append(lagrange_interpolation(t_interp, sp3_data[:, 0], sp3_data[:, i]))
        return np.array(pos)
```

To compensate for Earth’s rotation during the signal travel time, the angle θ is computed as: $\theta = \omega E \times \Delta t$, where Δt is the geometric distance between the satellite and the receiver divided by the speed of light.
The final satellite position is then obtained using the function:

```python
def sat_pos(trec, pc, sp3, r_apr):
    """
   computes the final ECEF coordinates of a satellite at the reception time,
   correcting for Earth's rotation during signal travel.

   parametres:
   trec: reception time (s)
   pc: pseudorange measurement (m)
   sp3: SP3 satellite data [time, X, Y, Z, clock] (10x5)
   r_apr: approximate receiver position [x, y, z] in meters

   Returns:
   r_sat_final: final satellite coordinates (ECEF, m)
   """
# extarct satellite clock data

    clk_mat = sp3[:, [0, 4]]
# compute signal emission time

    tems = emist(trec, pc, clk_mat)
# inner function to interpolate satellite coordinates at emission time

    def interpolate_xyz(t_interp, sp3_data):
        pos = []
        for i in range(1, 4):  # X, Y, Z
            pos.append(lagrange_interpolation(t_interp, sp3_data[:, 0], sp3_data[:, i]))
        return np.array(pos)
# approximate satellite coordinates at emission time

    r_sat_apr = interpolate_xyz(tems, sp3)
# compute signal travel time from satellite to receiver

    delta_t = np.linalg.norm(r_sat_apr - r_apr) / c
# compute earths rotation angle durinf that time

    theta = wE * delta_t
# rotation matrix for correcting earth rotation around z- axis

    R3 = np.array([
        [np.cos(theta), np.sin(theta), 0],
        [-np.sin(theta), np.cos(theta), 0],
        [0, 0, 1]
    ])
# apply rotation to get final satellite coordinates at reception time

    r_sat_final = R3 @ r_sat_apr
    return r_sat_final
```

This function was executed separately for PRN G06 and PRN G11. All computations were performed using NumPy.

3. RESULTS AND DISCUSSIONS

The final Earth-Centered Earth-Fixed coordinates of the satellites at the moment of signal emission are:

Final Position (in meters)	X	Y	Z
G06	6,511,118.29	23,724,917.84	-10,702,070.27
G11	11,636,060.19	14,505,918.22	–17,232,014.23

```
Final ECEF Coordinates (G06): [  6511118.28854048  23724917.84071328
-10702070.27429111]
Final ECEF Coordinates (G11): [ 11636060.19217986  14505918.22350372
-17232014.22628454]
```

These coordinates represent the satellites' locations in the ECEF frame after correcting for signal travel time and Earth's rotation. The satellites travel at high speeds (~3.9 km/s), and even during short propagation delays (~70 ms), the position can change by hundreds of meters. Earth's rotation introduces further deviation in the horizontal plane, which must be corrected to ensure coordinate precision at centimeter level.

The use of Lagrange interpolation on SP3 ephemeris and clock data allowed smooth and accurate estimation of both position and timing. Ten epochs were used for interpolation to ensure reliability. The rotation matrix R3(θ) effectively converted emission-time coordinates to reception-time ECEF frame, as required by GNSS modeling.

The results confirm the necessity of using emission-time coordinates instead of reception-time data, especially in post-processing or high-accuracy applications.

REFERENCES

SP3c File Format Specification by IGS
RINEX 2.11 Format Description
NumPy Python Library Documentation