TECHNISCHE UNIVERSITÄT DARMSTADT

PHILIPPS-UNIVERSITÄT MARBURG

Causal Identification and Estimation

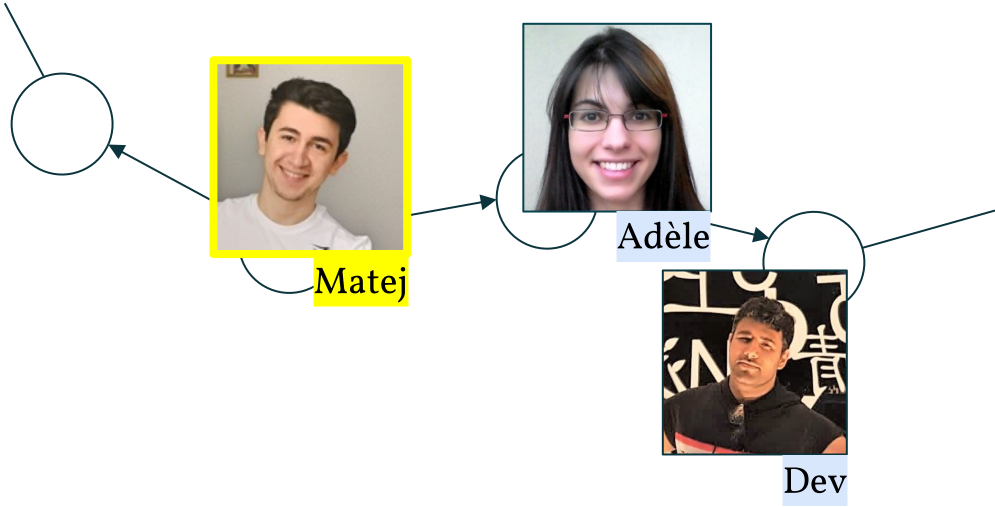
Matej Zečević

26th July 2023

ESSAI & ACAI 2023

LJUBLJANA, SLOVENIA

Machines Climbing Pearl's Ladder of Causation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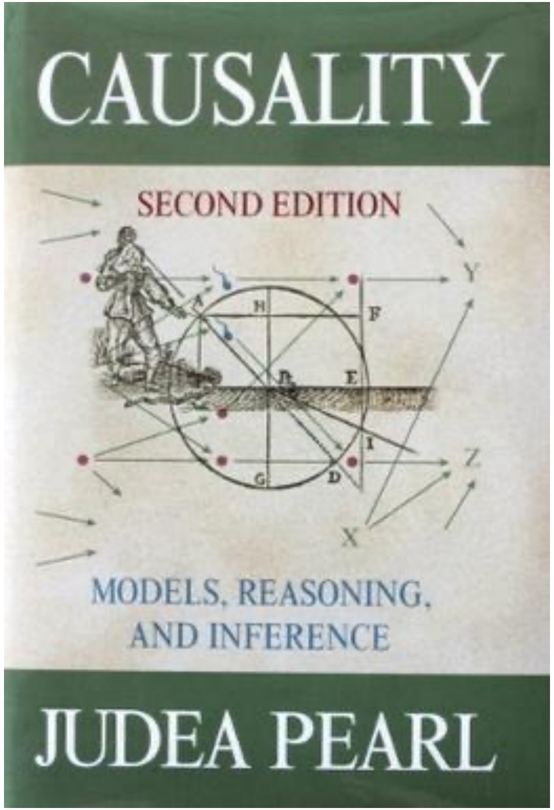
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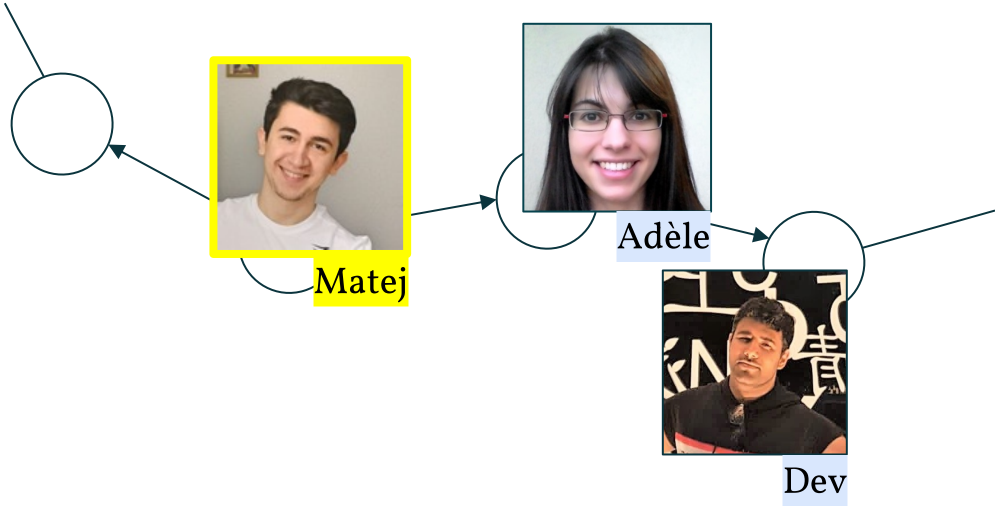
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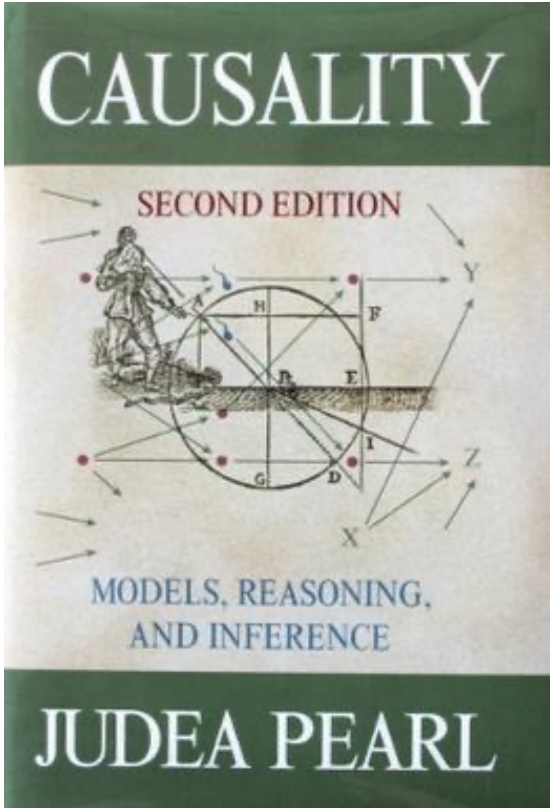
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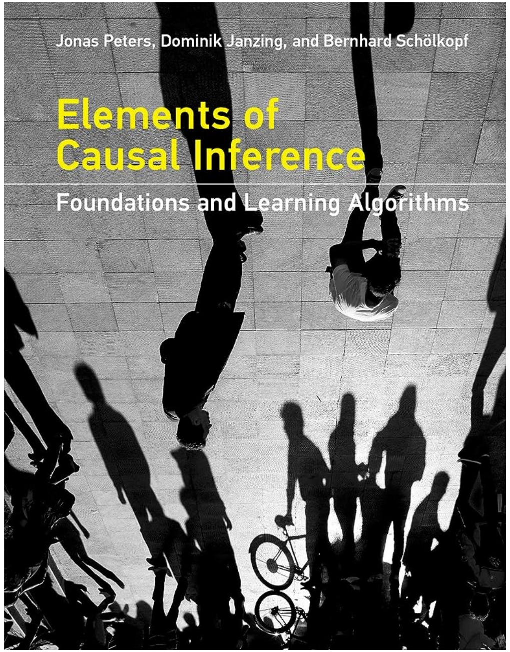
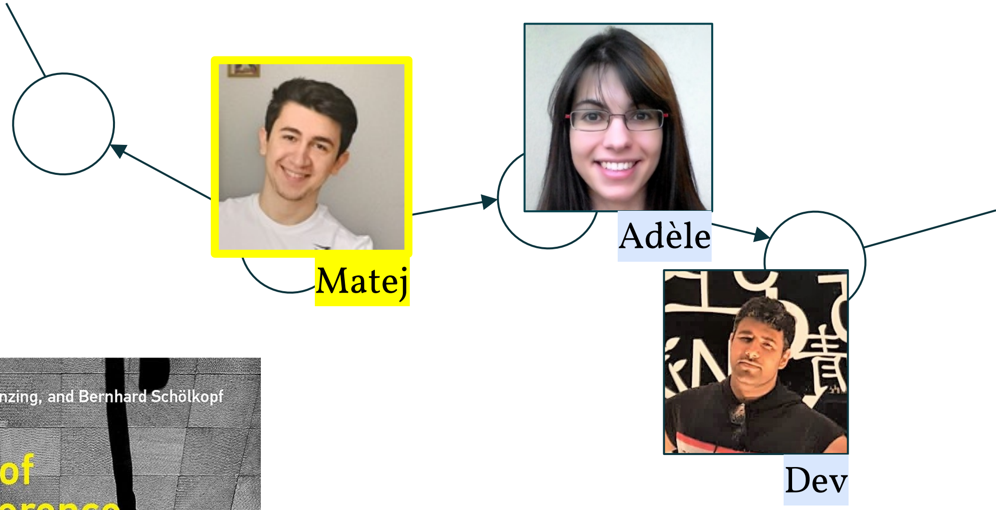

Jonas Peters, Dominik Janzing, and Bernhard Schölkopf
Elements of
Causal Inference
Foundations and Learning Algorithms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Overview

- Defining Causal Effect Identifiability

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- Identifying using: (a) Adjustment Sets,

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- Identifying using: (a) Adjustment Sets,

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  (d) optimization (Partial Identification)
- Estimation with ML, Cases: iSPN & NCM

0 Brief Recap

All Crucial (Formal) Concepts for this Lecture

Structural Causal Model

Definition

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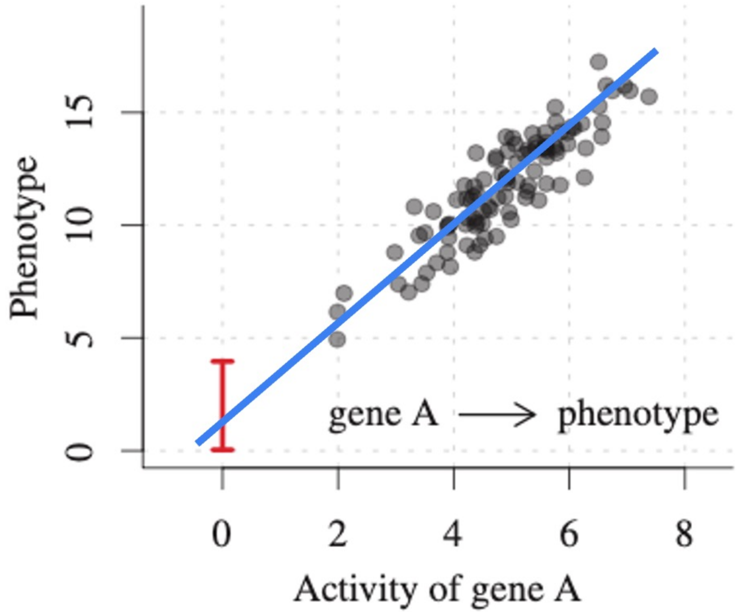
$P_{\mathbf{U}}$ is the probability distribution over U.

Assumption: $\mathcal{M}$ is recursive i.e., there are no feedback (cyclic) mechanisms

SCM as Data-Generating Processes

The Observational Distribution (Non-causal)



$$P(Y = y \mid X = x)$$

Interventions

Changing the SCM's structural equations

$$\mathcal{M} \begin{cases} \mathbf{V} = \{X, Y\} \quad \mathbf{U} = \{U_{XY}, U_X, U_Y\} \quad P_{\mathbf{U}} \\ \mathcal{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \end{cases}$$

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$$\Big\downarrow \; do(X = x)$$

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Changing the SCM's structural equations

$$\mathcal{M} \left\{ \begin{array}{l} \mathbf{V} = \{X, Y\} \quad \mathbf{U} = \{U_{XY}, U_X, U_Y\} \quad P_{\mathbf{U}} \\ \mathcal{F} = \left\{ \begin{array}{l} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{array} \right. \end{array} \right.$$

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$$P(Y = y \mid do(X = x)) \neq P(Y = y)$$

The Causal Graph

An induced property of the SCM

latent

$$\mathcal{F} = \begin{cases} X = f_X(U_X, U_{XZ}) \\ Y = f_Y(X, U_Y, U_{YZ}) \\ Z = f_Z(U_Z) \end{cases}$$

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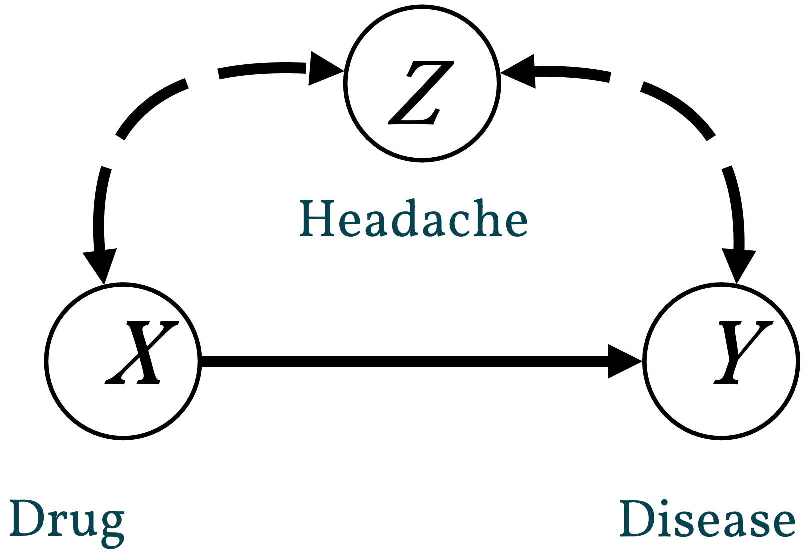
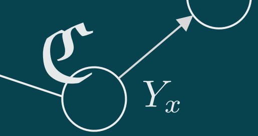
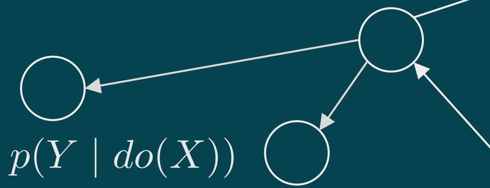
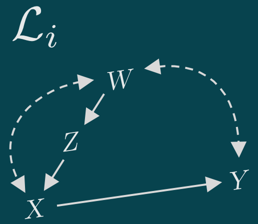






I | Causal Effect Identifiability

Definition and Intuition



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The causal effect of a (set of) treatment variable(s) X

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if the interventional distribution P(Y | do(X)) is uniquely computable.

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That is, if for every pair of SCMs M_1 and M_2

that induce G and have $P_{M_1}(V) = P_{M_2}(V) = P(V)$ with $P(V) > 0$,

it must hold: $P_{M_1}(Y \mid do(X)) = P_{M_2}(Y \mid do(X)) = P(Y \mid do(X))$.

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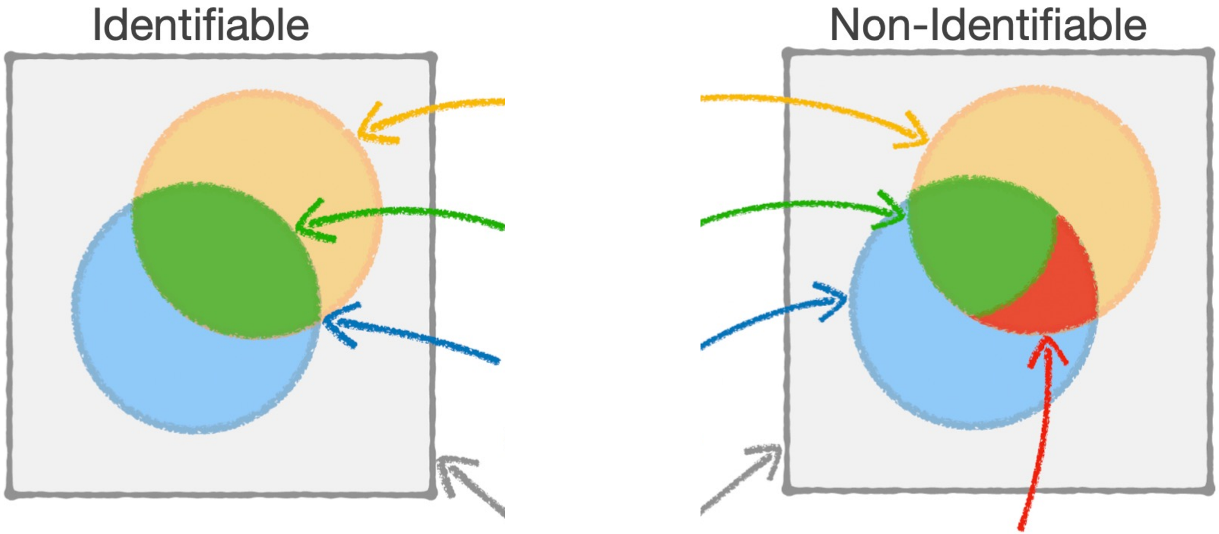
Intuition for Def.1: Set / Venn Diagram View



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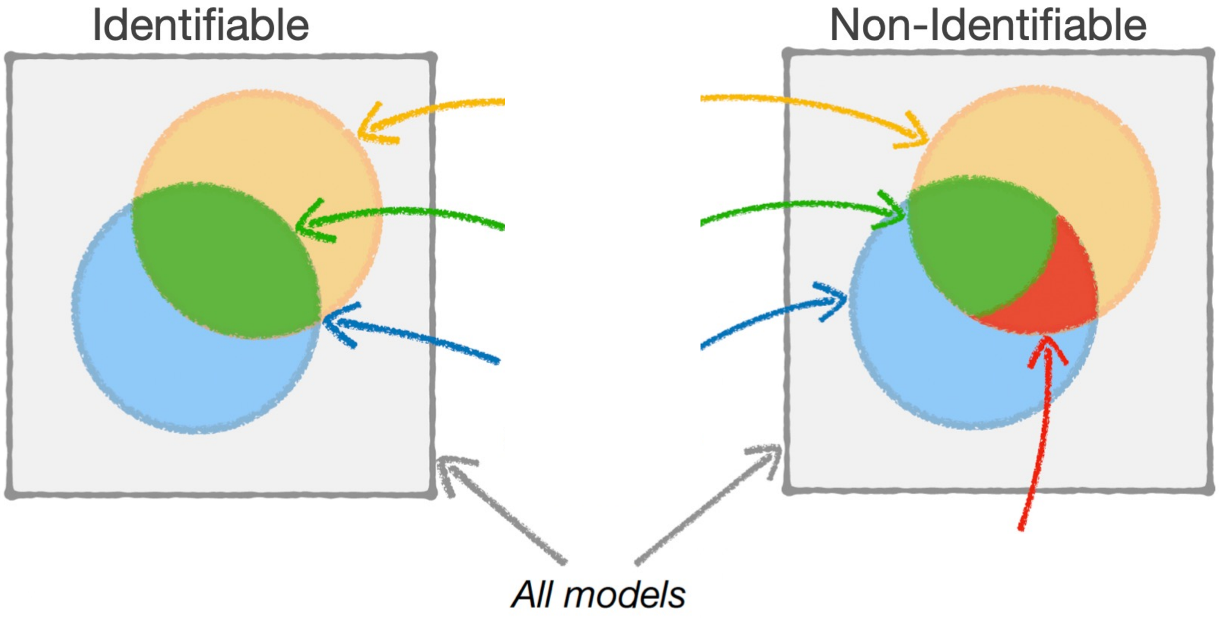

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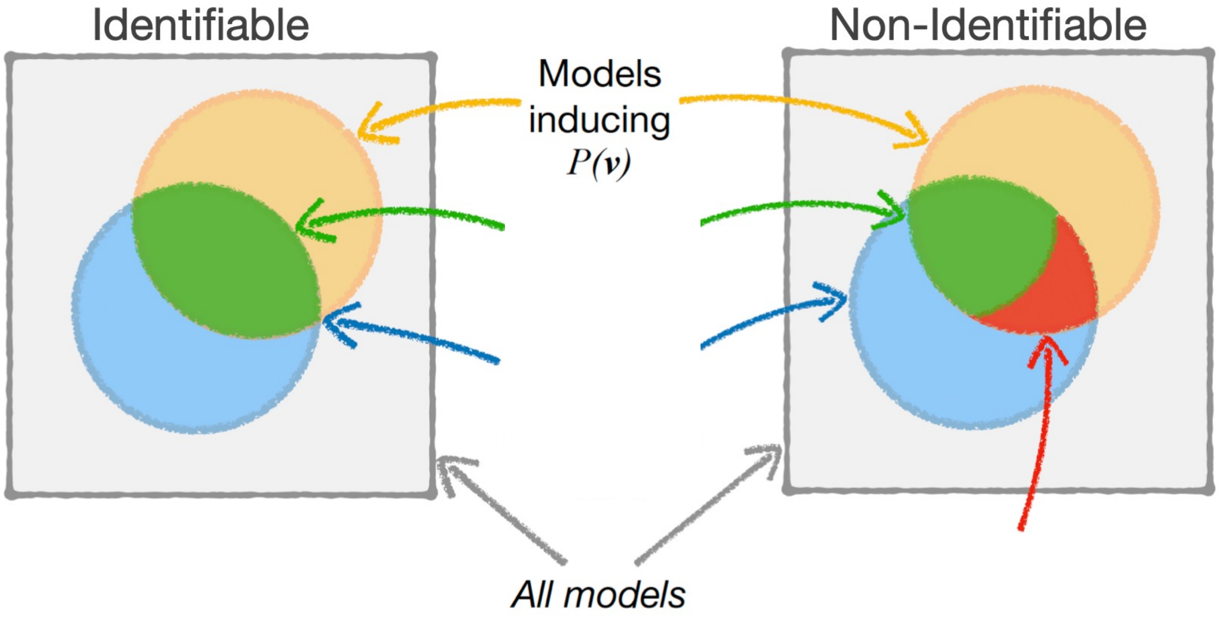

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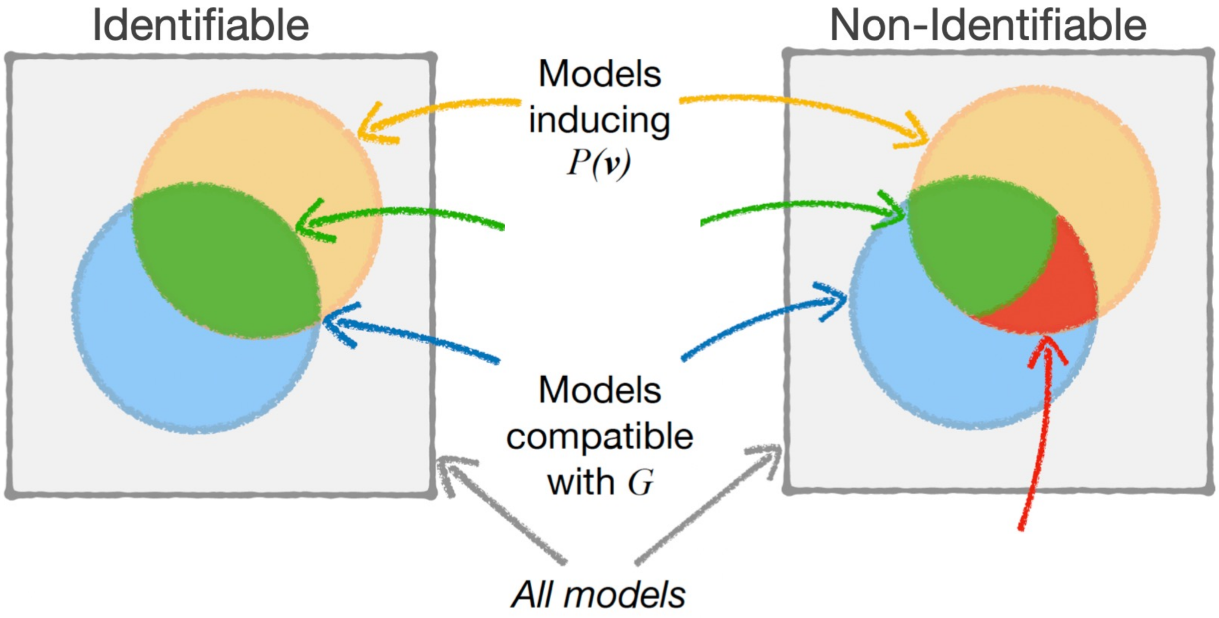

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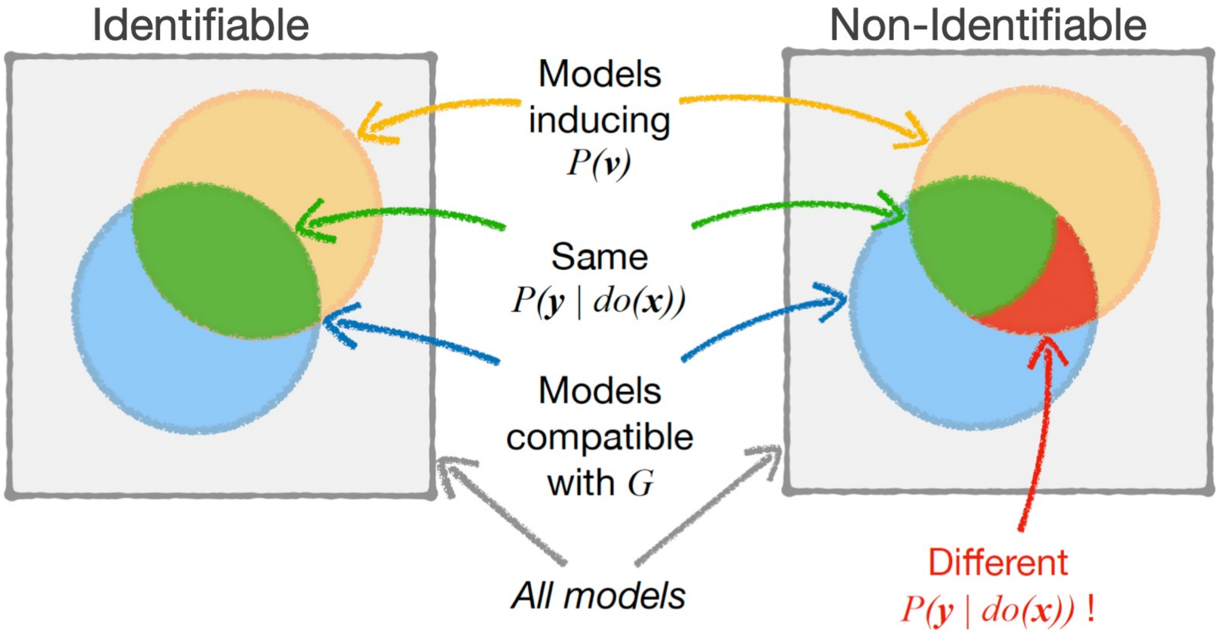


A Special Sub-class of SCM

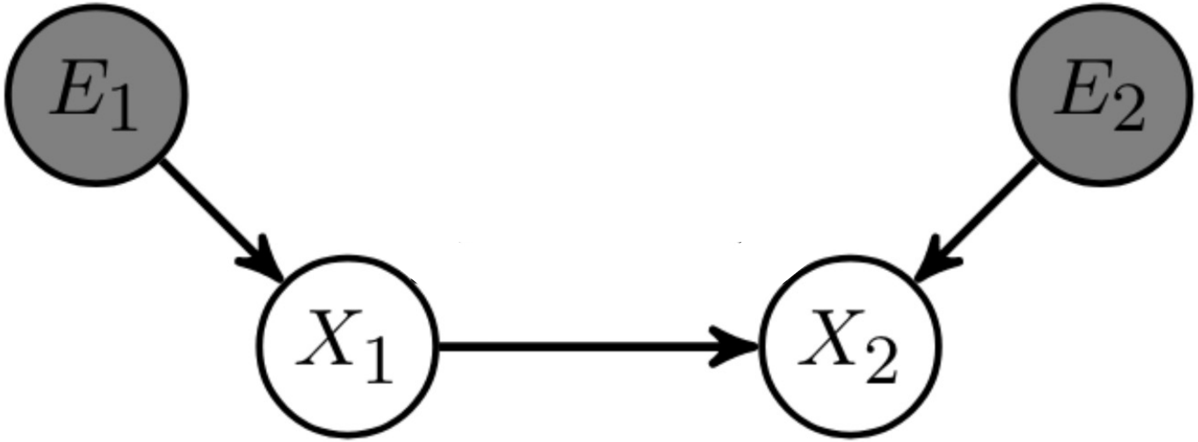
Definition 2, Markovian SCM:

Let M=(V, U, F, P(U)) be an SCM.

If P(U) is a product distribution over U,

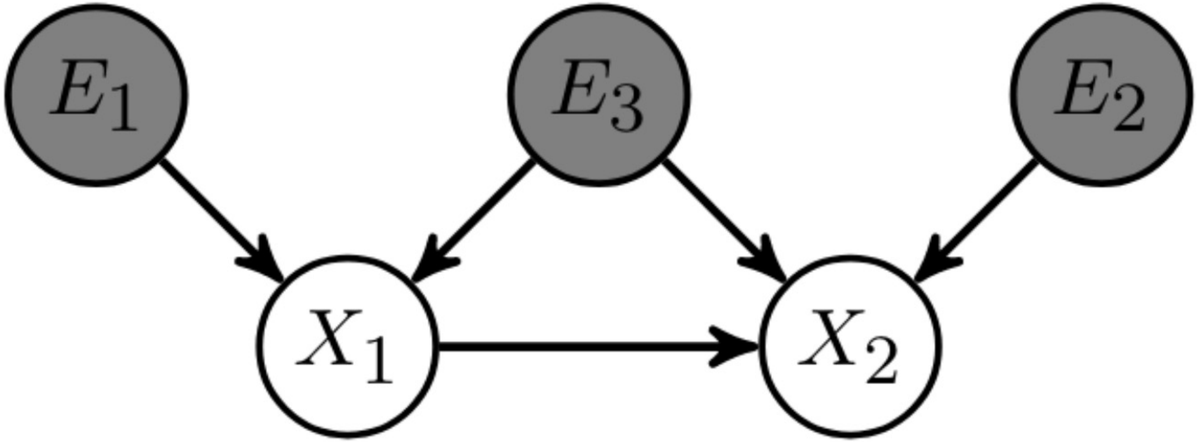
then M is called Markovian.

Intuition for Def.2: Graphical View



depicted $G_a(M)$ and Def.2 satisfied, M is Markovian

Intuition for Def.2: Graphical View



Def.2 not satisfied, M is non-/semi-Markovian

Theorem for Markovian SCM:

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Any causal effect is identifiable. $\square$

What are causal **effects**?

Definition 3, Causal Effect:

Any quantity Q derived from P(Y | do(X))

that tells us how much Y changes due to an intervention do(X).

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Example Quantity:

$$Q := E(Y \mid do(X=x')) - E(Y \mid do(X=x))$$

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$Q := E(Y \mid do(X=x')) - E(Y \mid do(X=x))$ $\qquad E(Y \mid x) := \sum_y y \cdot P(y \mid x)$

known as Average Treatment Effect (ATE)

The Identification Problem 'Big Picture'

Using Observational Data From One Population/Domain

1 **Query**

$P(y \mid do(x))$

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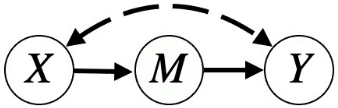

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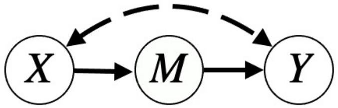

3. **Data**

   $P(x, m, y)$

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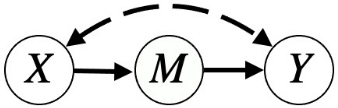

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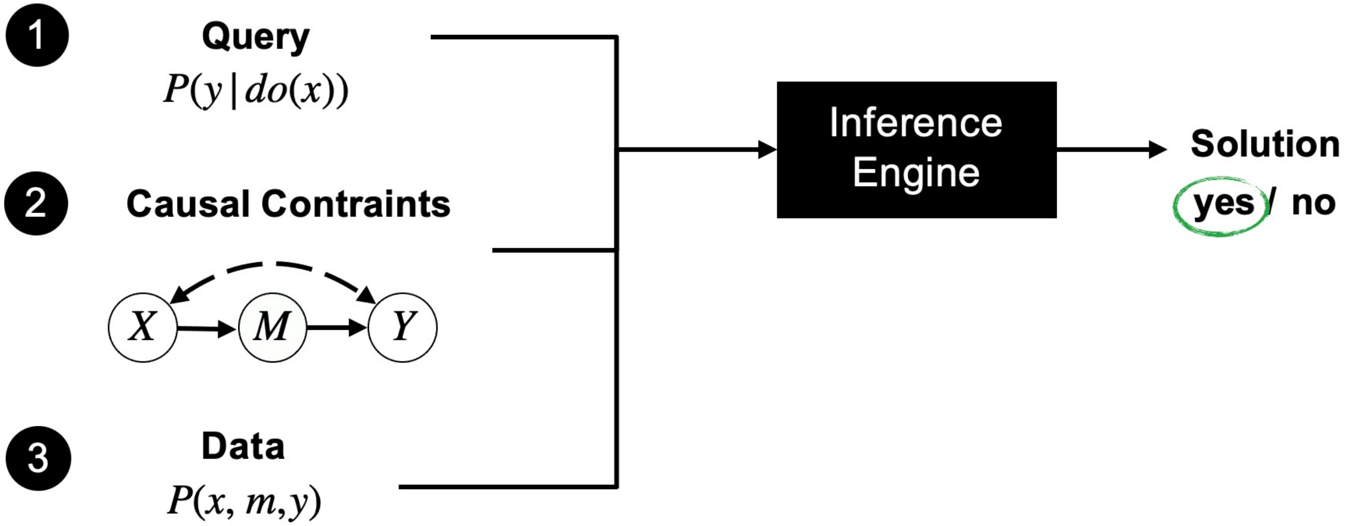

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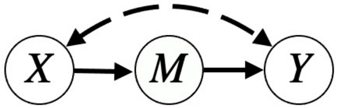

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Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_{m} P(m \mid x) \sum_{x'} P(y \mid m, x') P(x')$$

Interventional Distribution ← Available Distributions

👍

Solving the Ident. Problem

Underlying Model | Corresponding Ident. Approach

- For Markovian Models use Truncated Factorization

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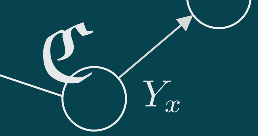
Underlying Model — Corresponding Ident. Approach

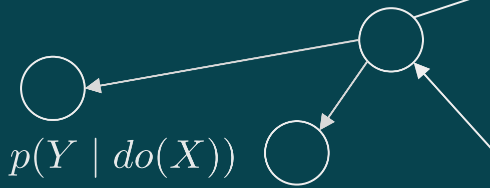
- For Markovian Models use Truncated Factorization
- For certain special graphs use Adjustment Sets

Solving the Ident. Problem

Underlying Model — Corresponding Ident. Approach

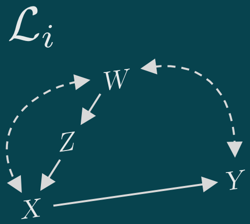
- For Markovian Models use Truncated Factorization
- For certain special graphs use Adjustment Sets
- For General SCM use Pearl’s do-Calculus





2 Identifying using

Adjustment Sets, Truncated Factorization, Pearl's do-Calculus and Optimization



Simpson's Paradox

$$P(R \mid A) < P(R \mid B)$$

	Overall
Treatment *a*: Open surgery	78% (273/350)
Treatment *b*: Percutaneous nephrolithotomy	**83%** (289/350)

Simpson's Paradox

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	Overall	Patients with small stones	Patients with large stones
Treatment *a*: Open surgery	78% (273/350)	**93%** (81/87)	**73%** (192/263)
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however $P(R \mid do(A)) > P(R \mid do(B))$

When Correlation ≠ Causation

Definition 4, Confounding:

If $P(Y \mid do(X)) \neq P(Y \mid X)$, then the causal mechanism from X to Y is called confounded.

When Correlation ≠ Causation*

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* confounding is not 'evil' per se! Synonymous to "common cause."

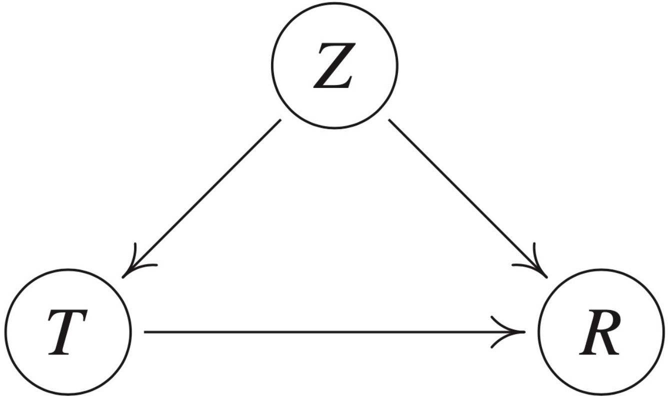
The Kidney Stone Example

For any patient we are given (binary) records on

Z, the kidney stone size

T, the treatment and

R, the recovery.



Conditioning ≠ Intervening

Example Calculation:

$P(R=1 \mid do(T=A)) = \ldots$

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Example Calculation:

$$P(R=1 \mid do(T=A)) = 0.93 \cdot (357/700) + 0.73 \cdot (343/700) = 0.832$$

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Conditioning ≠ Intervening

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$P(R=1 \mid T=A) = 0.78$

analogously for T=B

Conditioning $\neq$ Intervening

Example Calculation:

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$P(R=1 \mid T=A) = 0.78$

analogously for T=B

$P(R=1 \mid do(T=A))$ - $P(R=1 \mid do(T=B)) = 0.832 - 0.782$

$P(R=1 \mid T=A)$ - $P(R=1 \mid T=B) = 0.78 - 0.83$

Density View

$$P(R \mid do(T)) =$$

$$\Sigma_T\, P(R \mid T, Z) \cdot P(Z)$$

Density View

$$P(R \mid do(T)) =$$

$$\sum_T P(R \mid T, Z) \cdot P(Z) \neq \sum_T P(R \mid T, Z) \cdot P(Z \mid T)$$

$$= P(R \mid T)$$

Special Sets that yield Identification

Theorem 1, Adjustment Set:

For any pair (X,Y) with $Y \notin pa(X)$ the set Z will satisfy the equality

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(i) $Z := pa(X)$,

(ii) $Z := W$ where W contains no descendants of X and blocks all backdoor paths,.

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$P(Y \mid do(X)) = \sum_Z P(Y \mid X, Z) \cdot P(Z)$, if either:

(i) $Z := pa(X)$,

(ii) $Z := W$ where W contains no descendants of X and blocks all backdoor paths,

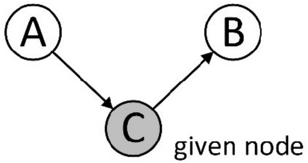
or (iii) $Z := W$ where W contains no descendant of any node

on a directed path from X to Y and blocks all non-directed paths from X to Y.

d-Separation & Conditional Independence

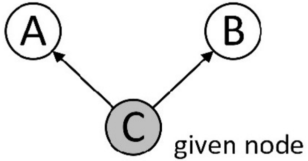
Graphical Tools

G



$$(A \perp B \mid C)_G$$

G



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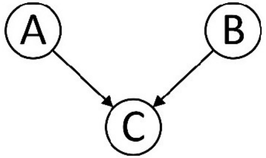

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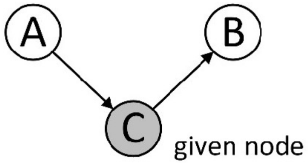

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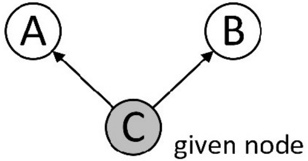

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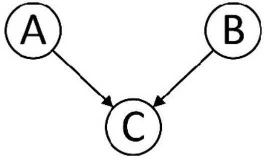


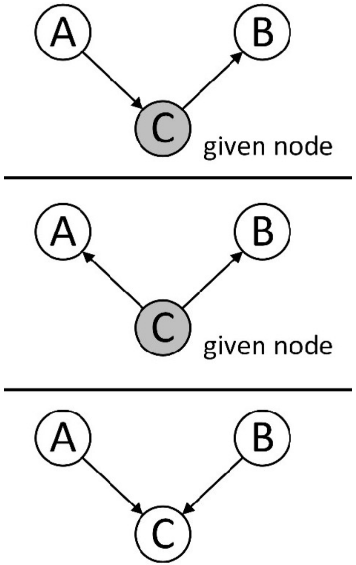
$$(A \perp B \mid \emptyset)_G$$

global Markov property

$$(\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})_G \implies (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})_P$$

d-Separation & Conditional Independence

Graphical Tools



G $\quad (A \perp B \mid C)_G$

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faithfulness

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TECHNISCHE UNIVERSITÄT DARMSTADT

d-Separation & Conditional Independence

Graphical Tools

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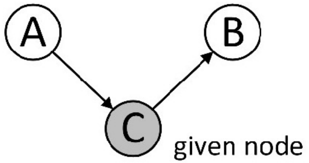

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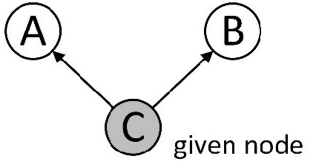

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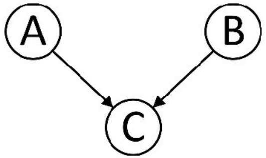

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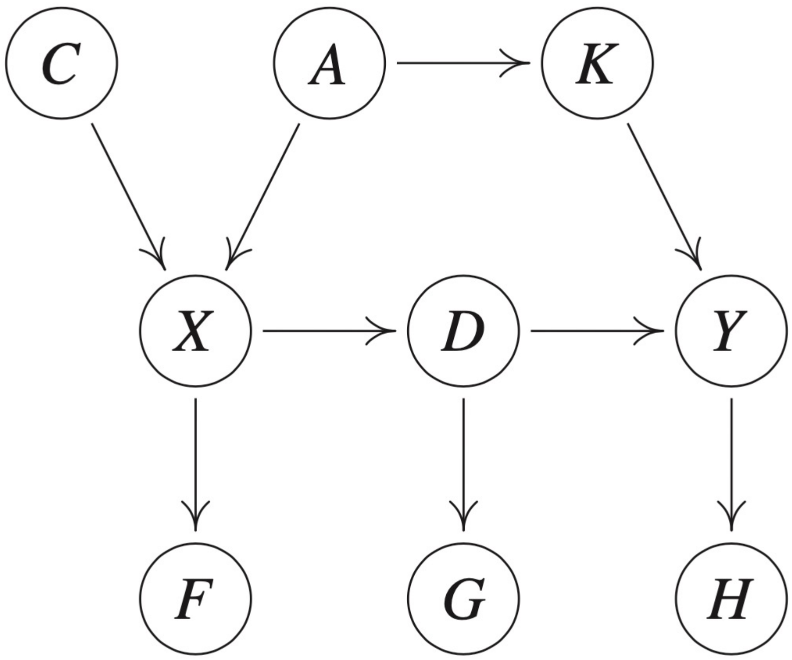
local factorization

$$P(\mathbf{v}) = \sum_{\mathbf{u}} P(\mathbf{u}) \prod_i P(v_i \mid \mathbf{pa}_i, u_i)$$

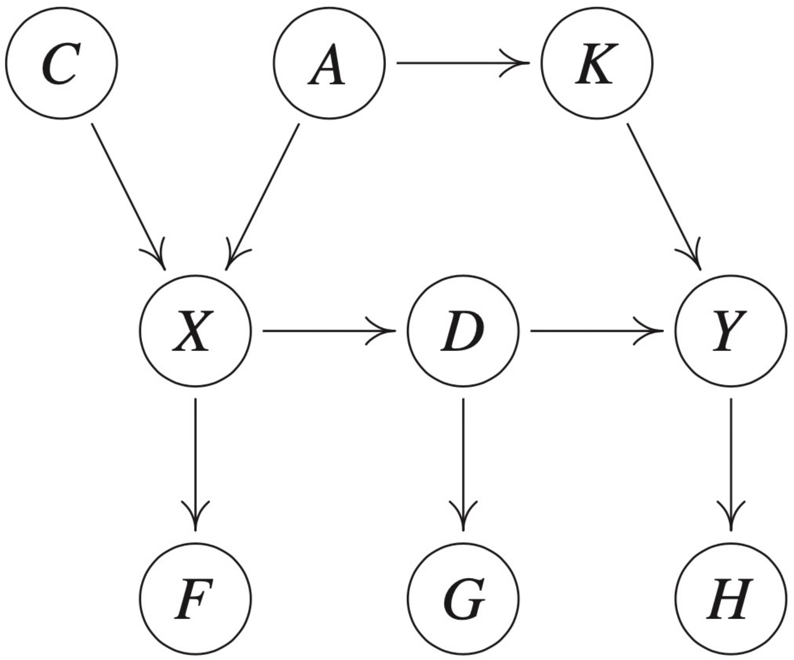
Special Sets that yield Identification

Theorem 1 gives us backdoor adjustment set Z = {K}

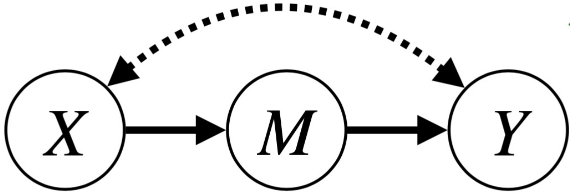


Special Sets that yield Identification

Theorem 1 gives us backdoor adjustment set Z = {K}, alternate non-backdoor valid adjustment set is Z = {F, C, K}

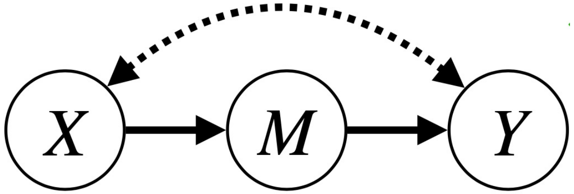


Plenty of 'Special' Graphs with corresponding Criteria

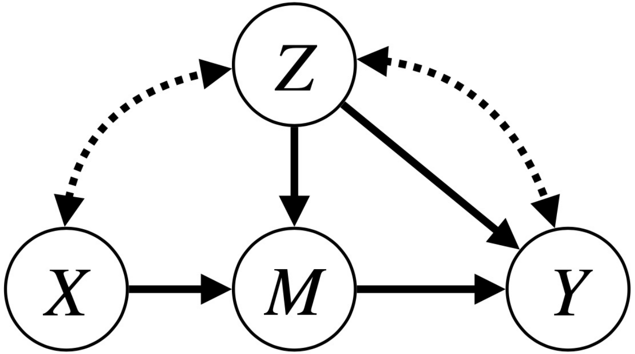


Frontdoor

Plenty of 'Special' Graphs with corresponding Criteria

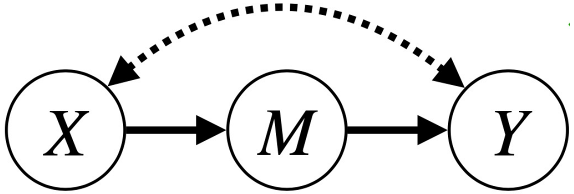


Frontdoor

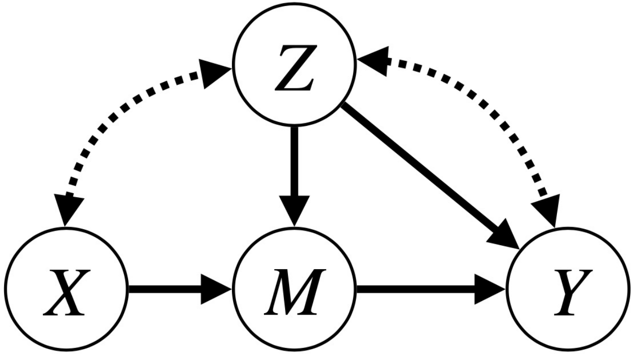


Conditional Frontdoor

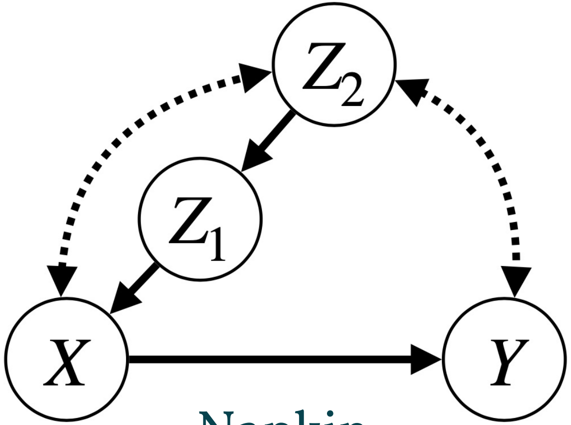
Plenty of 'Special' Graphs with corresponding Criteria



Frontdoor



Conditional Frontdoor



Napkin

Switching the approach to identification next

Adjustment Sets,

Truncated Factorization and Pearl's do-Calculus

‘Hard’ interventions

Definition 5, Hard Intervention:

Given $x=g(z)$ for some $Z \subset V$ in $do(X=x)$,

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Given $x=g(z)$ for some $Z \subset V$ in do($X=x$),

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Ex.: do($X=4$) fixes the value of X to 4

Identifying Markovian SCM

Theorem 2, Truncated Factorization*:

* also known G-computation or manipulation theorem

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Let V denote all variables,

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Identifying Markovian SCM

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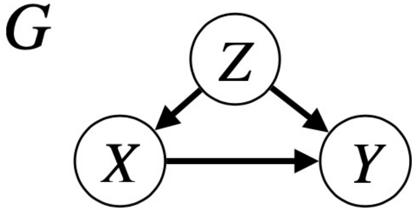
Ex.: $v := (x=0)$ is inconsistent with $do(X=1)$

Intuition: Graphical Decomposition



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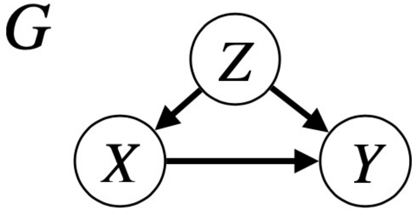

$$P(x, y, z) = P(z)P(x \mid z)P(y \mid x, z)$$

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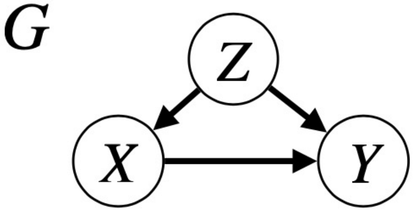
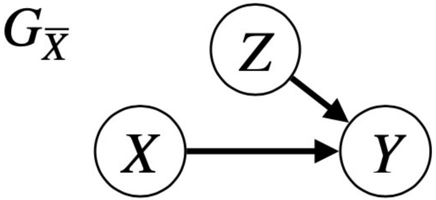


$do(X = x)$



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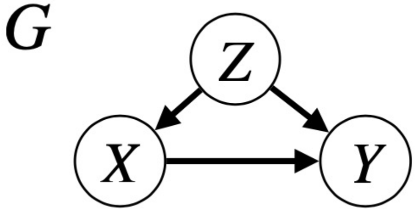

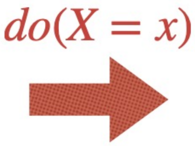

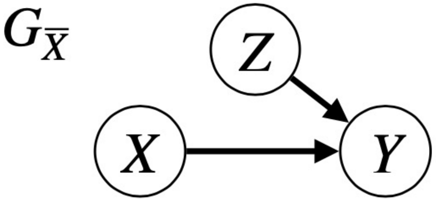

$$P(x, y, z) = P(z)P(x \mid z)P(y \mid x, z)$$

$$P(y, z \mid do(x)) = P(z)P(y \mid x, z)$$

Theorem 2 gives $P(y \mid do(x)) = \sum_z P(z) \cdot P(y \mid x, z)$

Switching the approach to identification again

Adjustment Sets,
Truncated Factorization and Pearl’s do-Calculus

Teaching the Causal Calculus on Twitter

> **Andrew Heiss, geriatric millennial** @andrewheiss · Apr 20, 2020
> Naive question: are backdoor and front door adjustment in DAG world just specific applications of the 3 rules of do-calculus, or are they a separate thing?
> Show this thread

Teaching the Causal Calculus on Twitter *

Judea Pearl @yudapearl

Naive questions are everyone's questions. Ans. The backdoor and frontdoor are logical consequences of do-calculus. So why do we decorate them with names? Because they are easily recognizable in the DAG, so we store them explicitly in our arsenal of tools, so skip re-deriving them

> **Andrew Heiss, geriatric millennial** @andrewheiss · Apr 20, 2020
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* Off-topic:
Elon probably somewhere
“put that bird back in its cage”

Identifying **Anything** Identifiable

Theorem 3, do-Calculus: For disjoint variable sets X, Y, Z, W we have

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Rule 1: Invariance to Observation

$$P(Y \mid do(W), Z, X) = P(Y \mid do(W), Z) \qquad \text{if } (Y \perp X \mid Z, W) \text{ in } G_{*W}$$

G_{*W} Graph with **incoming** arrows deleted, G_{W*} vice versa for **outgoing**

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Example Derivation: Confounded Frontdoor Graph

$P(Y \mid do(X))$

$= \ldots$

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$= \Sigma_M P(Y \mid do(M)) \cdot P(M \mid X)$ Rule 3

$= \ldots = \Sigma_{X'} \Sigma_M P(Y \mid M, X') \cdot P(X' \mid M) \cdot P(M \mid X)$ (…)

Intuition: Graphical Example for Rule 1

Rule 1: Invariance to Observation

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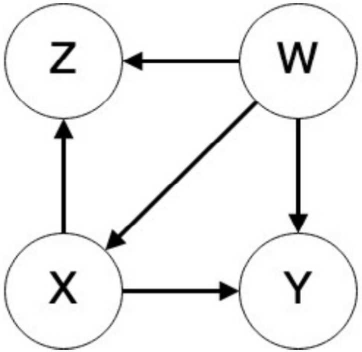
$G :=$



sorry for the confusion (red unequal black)
but the sets from the rules are instantiated
as follows in above example X:={Z}, Y:={Y}, Z:={W}, W:={X}

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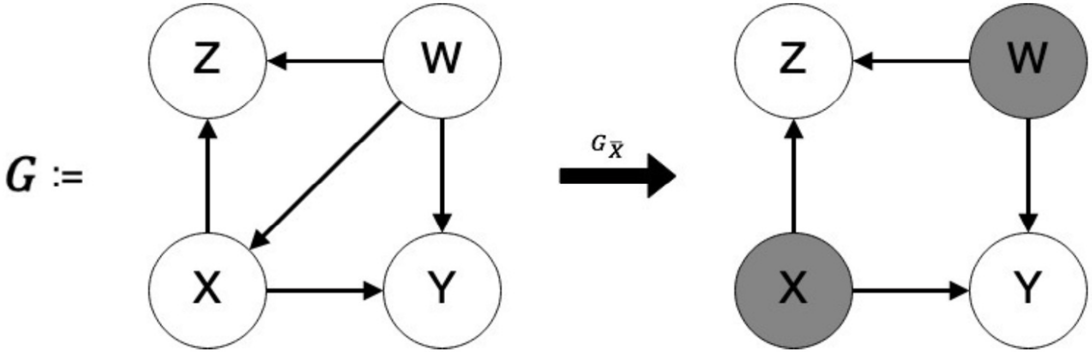

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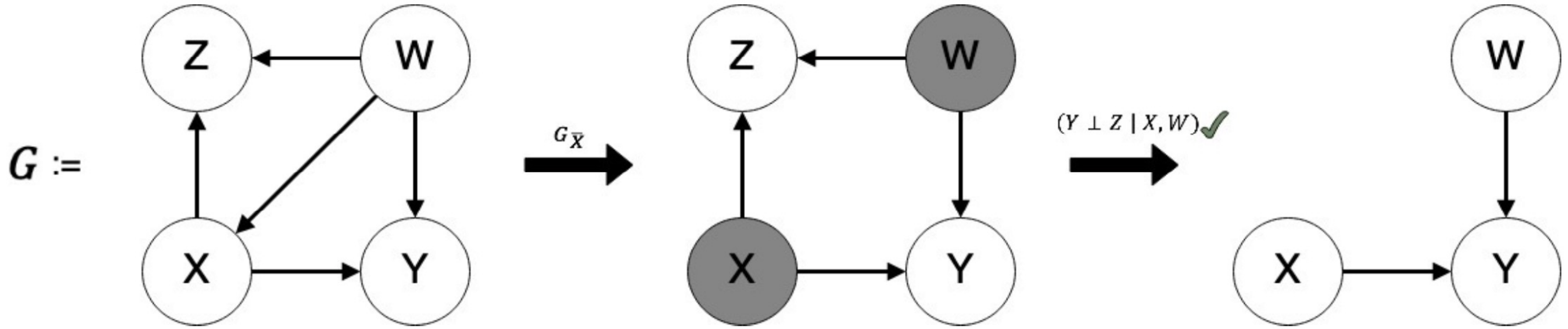

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but the sets from the rules are instantiated
as follows in above example $\quad X := \{Z\}$, $Y := \{Y\}$, $Z := \{W\}$, $W := \{X\}$

Switching the approach to partial identification

Adjustment Sets,
Truncated Factorization and Pearl's do-Calculus

Motivation behind Partial Identification

- Regular identification (a-c) requires conditions such as **sufficiency**

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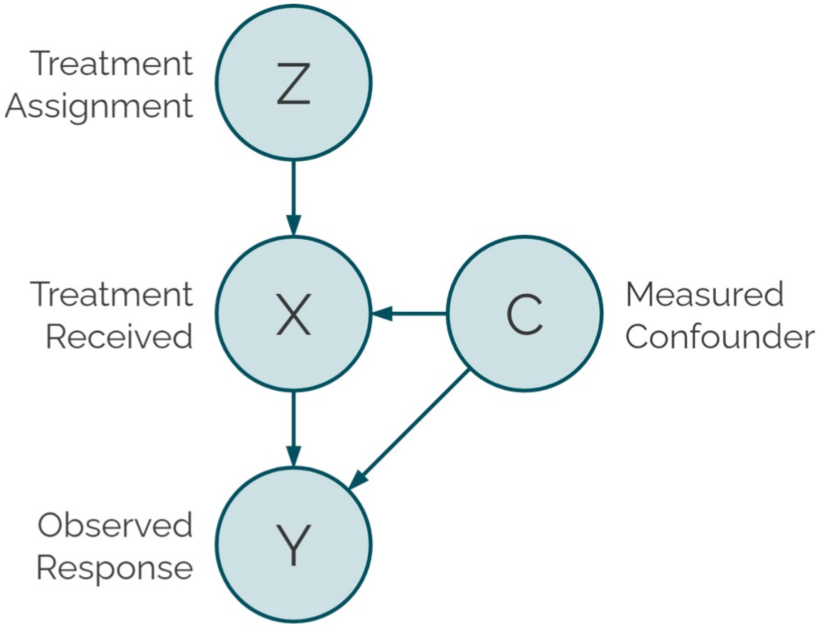
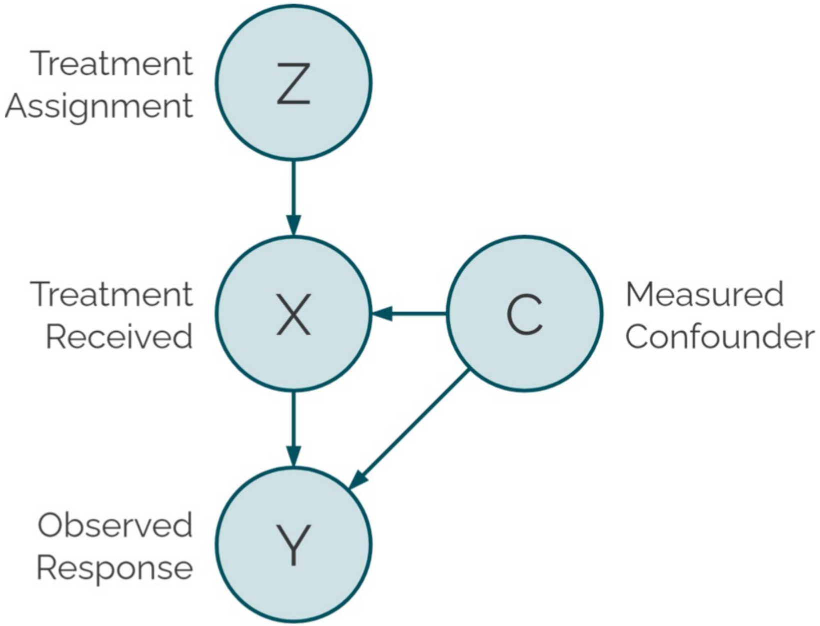
  Qualitative vs. Quantitative insight



Treatment Assignment
Z
Treatment Received
X
C
Measured Confounder
Observed Response
Y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Z
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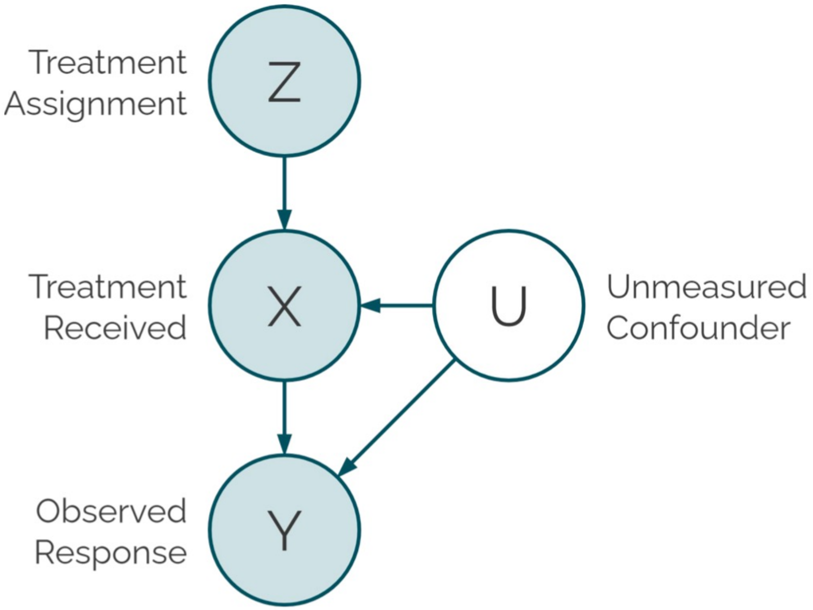
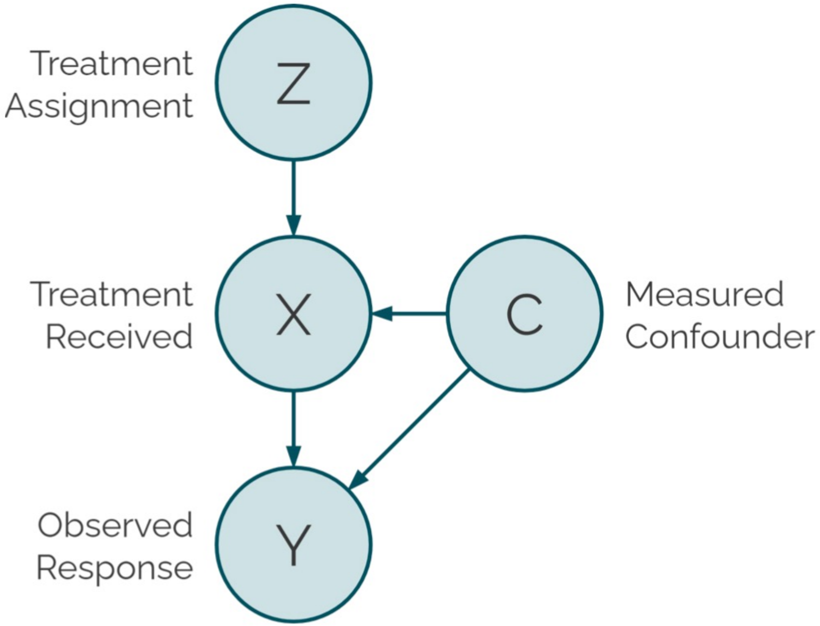
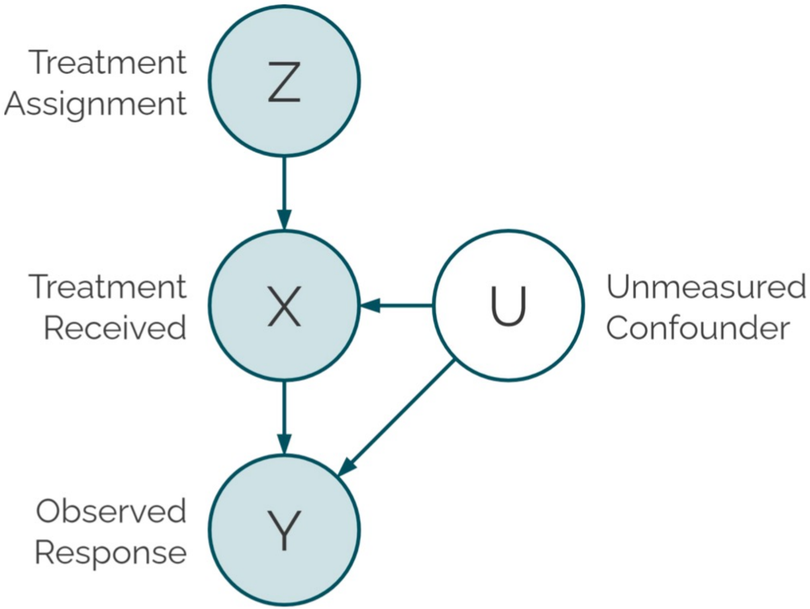

Treatment Assignment
Z
Treatment Received
X
U
Unmeasured Confounder
Observed Response
Y

the ‘standard’ real-world situation





Causal Relations as **Enumerable** Function Sets

Definition 6, Response Function Variable:

$\mathrm{Dom}(R_V) := \{1, \ldots, N_v\}$ where $N_v := |V|^{|pa(V)|}$, then R_V is called a RFV.

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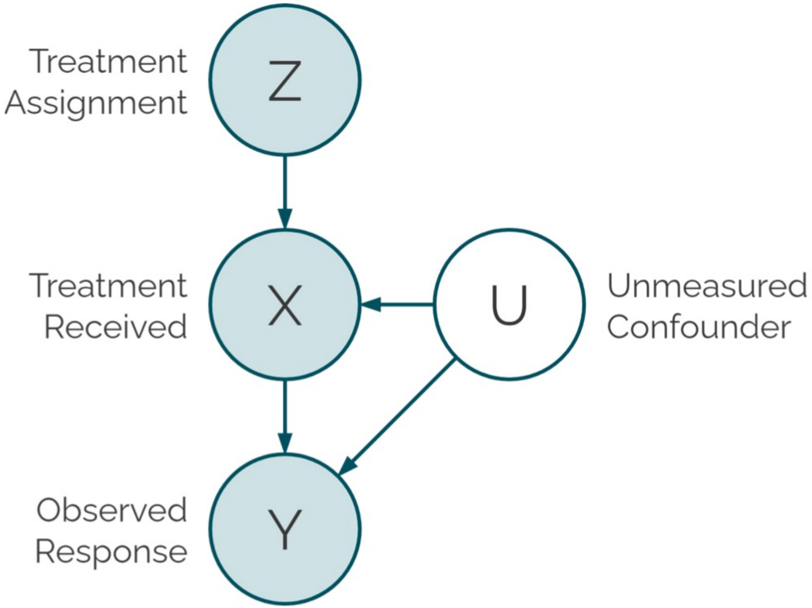
Theorem 4, RFV Distribution:

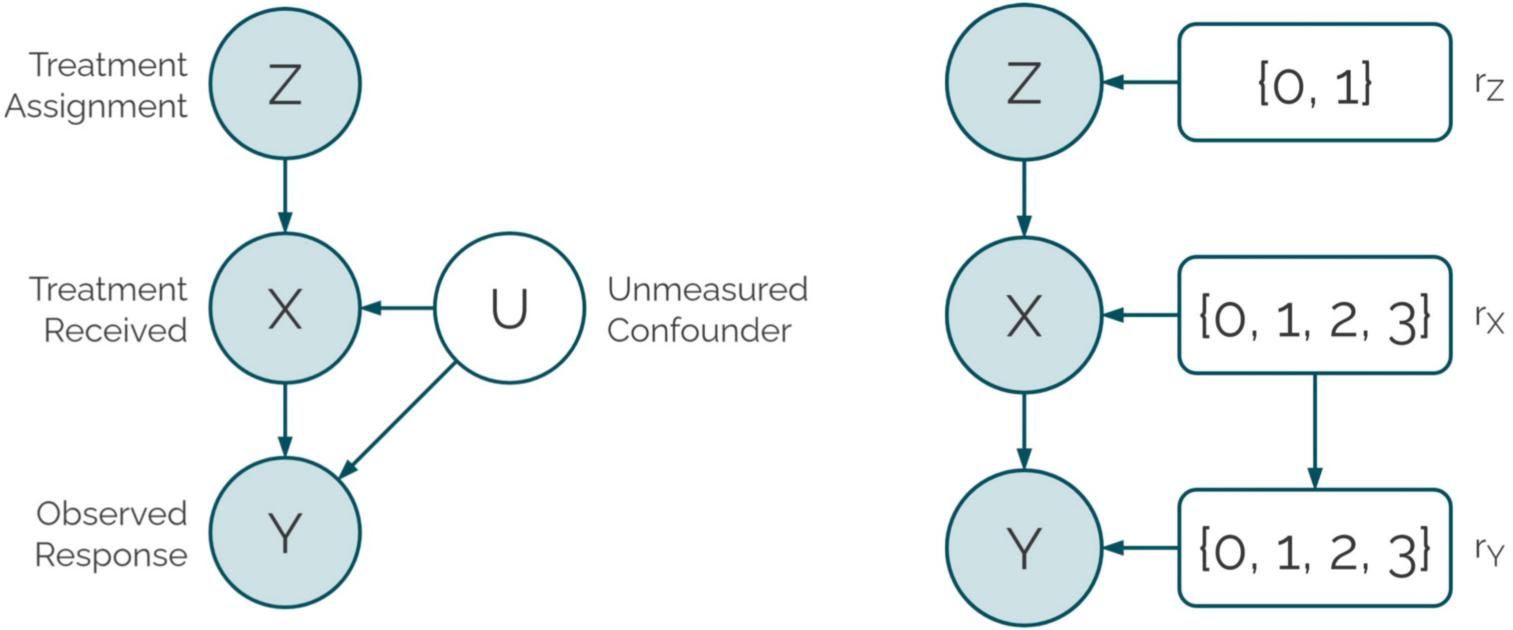
Furthermore, the observational distribution P(V) can be

rewritten as: $\sum_r P(r) \cdot \prod_{V_i \in V} 1(V_i, pa(V_i), r_{V_i})$.

Converting the Exogenous Terms



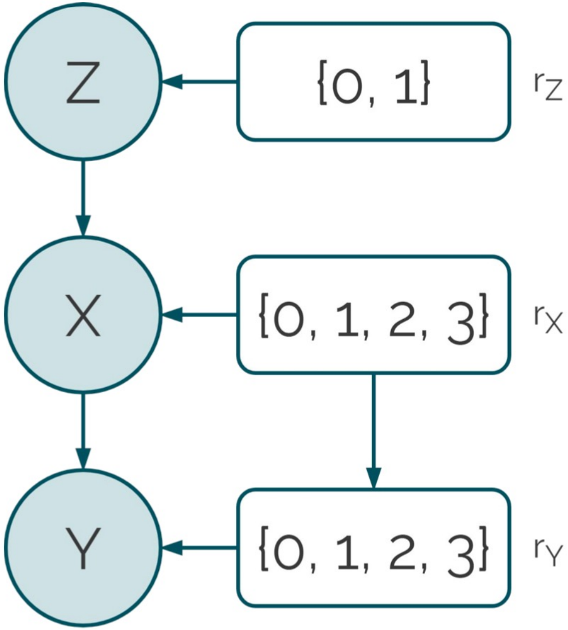
Converting the Exogenous Terms to RFVs



P(Z, X, Y) written as P(R_Z, R_X, R_Y)

$P(\mathbf{v})$

Observed data $P(\mathbf{v})$

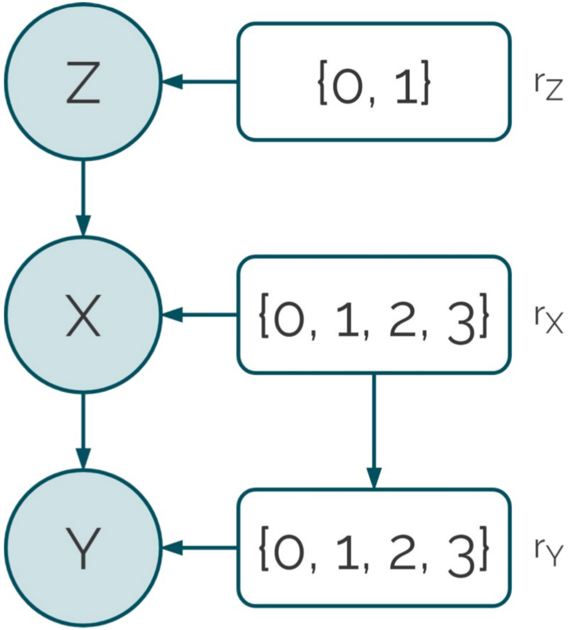


P(Z, X, Y) written as $P(R_Z, R_X, R_Y)$

$$P(\mathbf{v}) = \sum_{\mathbf{r}} P(\mathbf{r}) \prod_{V \in \mathbf{v}} \mathbb{I}(V; \mathsf{pa}_V, r_V)$$

Observed data $P(\mathbf{v})$

Latent model $P(\mathbf{r})$



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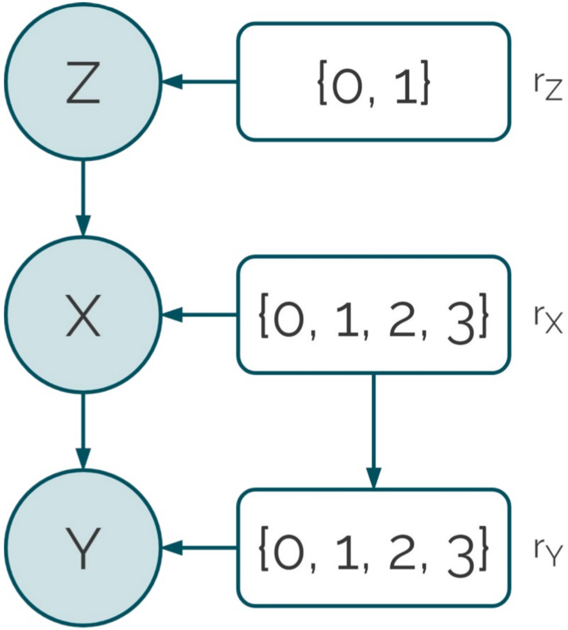
Latent model $P(\mathbf{r})$

E.g.:

$$P(\mathbf{v} = (0,1,1)) = P(\mathbf{r} = (0,1,0)) + P(\mathbf{r} = (0,3,1)) + P(\mathbf{r} = (0,1,0)) + P(\mathbf{r} = (0,3,1))$$

Condition accounts for consistent states



Bound on Causal Effect via **Min-Max** of LP

min/max $\boxed{Objective\ function\ (e.g.\ Average\ Causal\ Effect\ (ACE))}$

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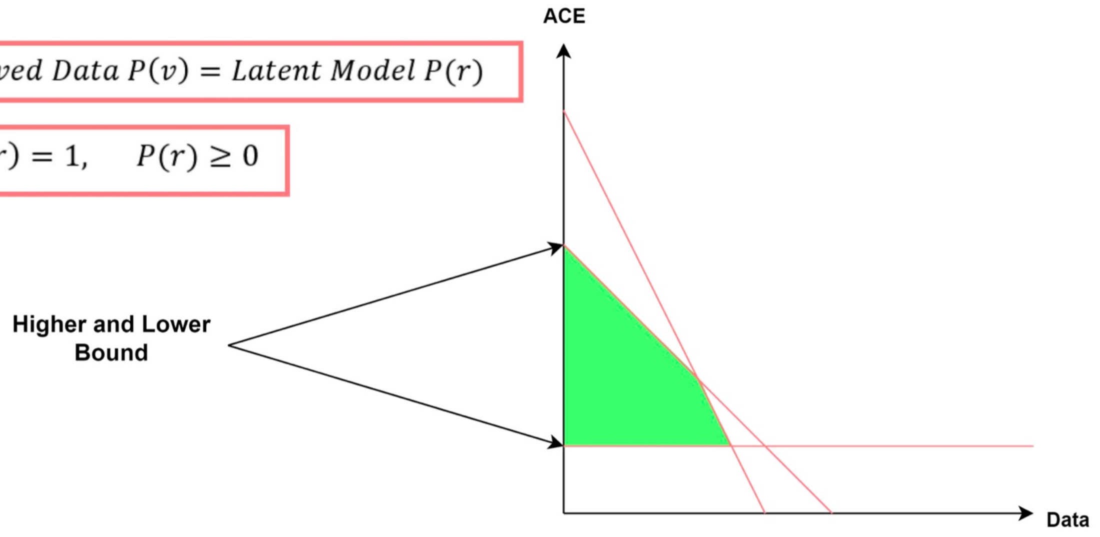
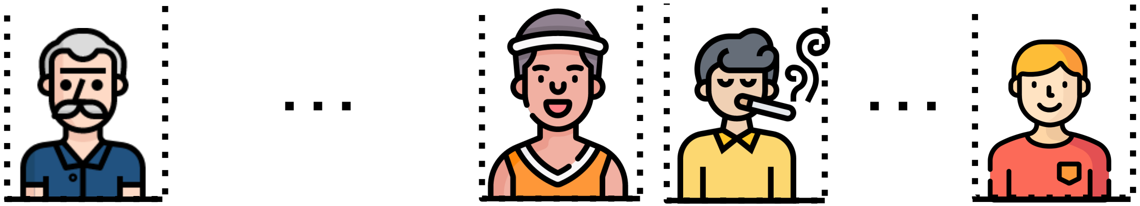


4 Estimation using Machine Learning

Two examples: iSPN & NCM

Individuals (Medical Example: Patient Records)



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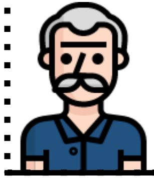
Numerical Representation

$$
\begin{array}{c}
A \\ F \\ H \\ M
\end{array}
\begin{pmatrix}
62 & \dots & \dots & 18 & 24 & \dots & 21 & \dots \\
48 & \dots & \dots & 60 & 20 & \dots & 32 & \dots \\
34 & \dots & \dots & 90 & 40 & \dots & 64 & \dots \\
39 & \dots & \dots & 75 & 37 & \dots & 66 & \dots
\end{pmatrix}
$$



A =Age F =Food Habits (or Nutrition) H =Health M =Mobility

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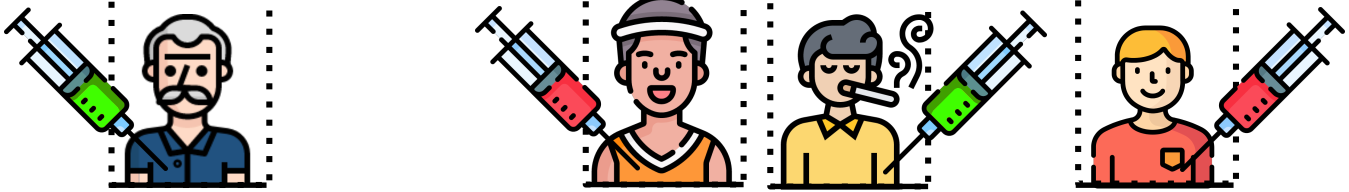
Intervention (e.g. a Vaccine)

$$
\begin{array}{c}
A \\ F \\ H \\ M
\end{array}
\begin{pmatrix}
62 & \dots & \dots & 18 & 24 & \dots & 21 & \dots \\
48 & \dots & \dots & 60 & 20 & \dots & 32 & \dots \\
+ & \dots & \dots & - & + & \dots & - & \dots \\
+ & \dots & \dots & - & + & \dots & - & \dots
\end{pmatrix}
$$



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Structural Causal Model

$$A = U(0, 100)$$

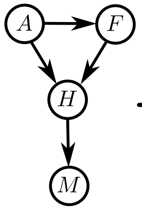
$$F = \frac{1}{2}A + \mathcal{N}(10, 10)$$

$$H = \frac{1}{100}(100 - A^2) + \frac{1}{2}F + \mathcal{N}(40, 30)$$

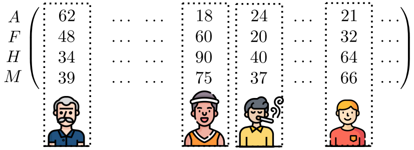
$$M = \frac{1}{2}H + \mathcal{N}(20, 10)$$

(A)ge
(F)ood Habits
(H)ealth
(M)obility

G



D



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$$\color{red}{H = U(0,100)}$$
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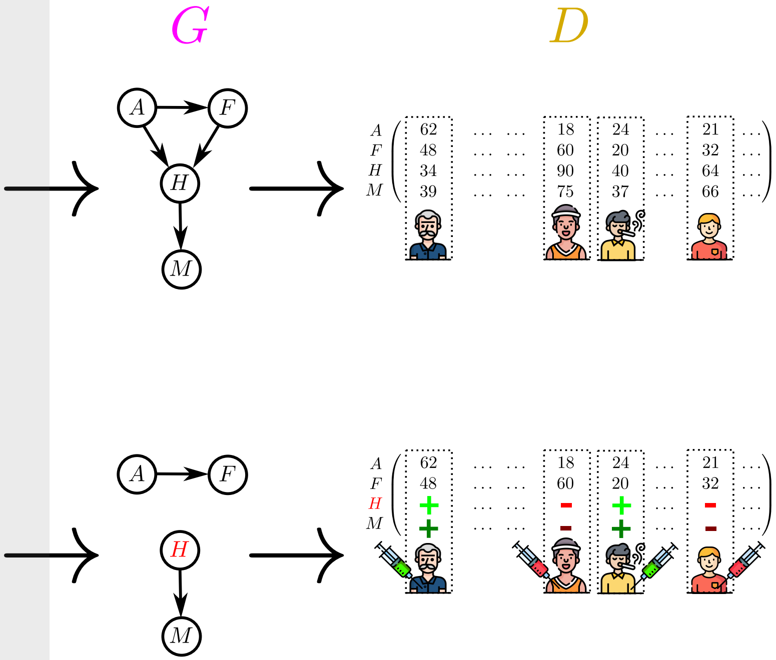

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Interventional Sum-Product Network

Definition 7, iSPN:

A model $M(G, D) = g_{\Theta}(D, f_{\psi}(G))$

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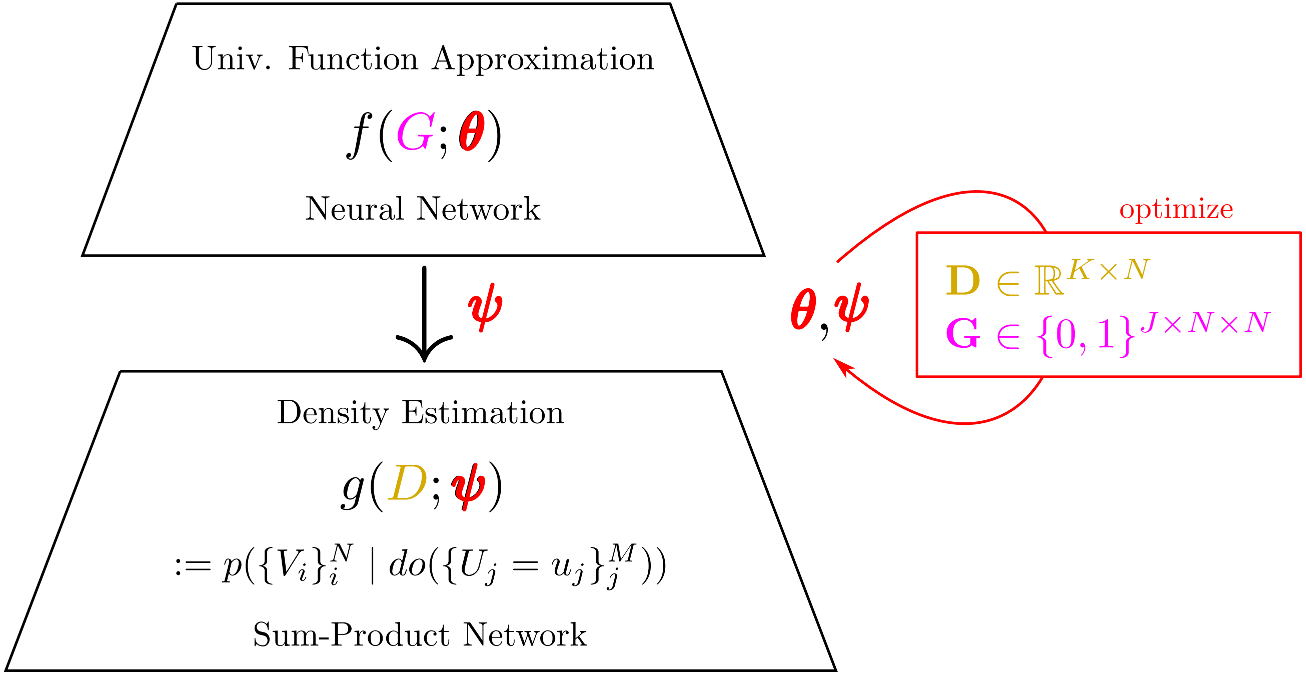
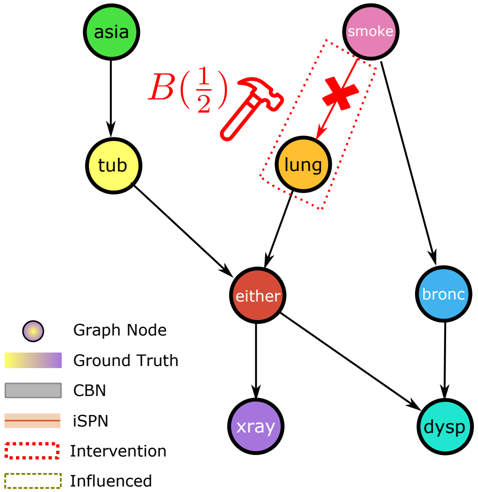
To learn about SPN (without the 'i'), check out the other ESSAI course:

Probabilistic Circuits: Deep Probabilistic Models with Reliable Reasoning
by Robert Peharz and Antonio Vergari

Schematic of the iSPN, a Causal Density Estimator





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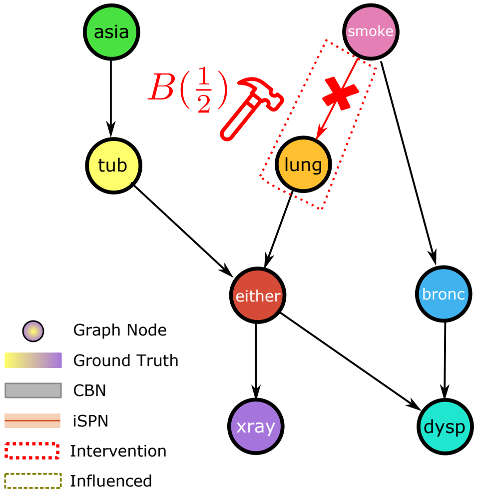
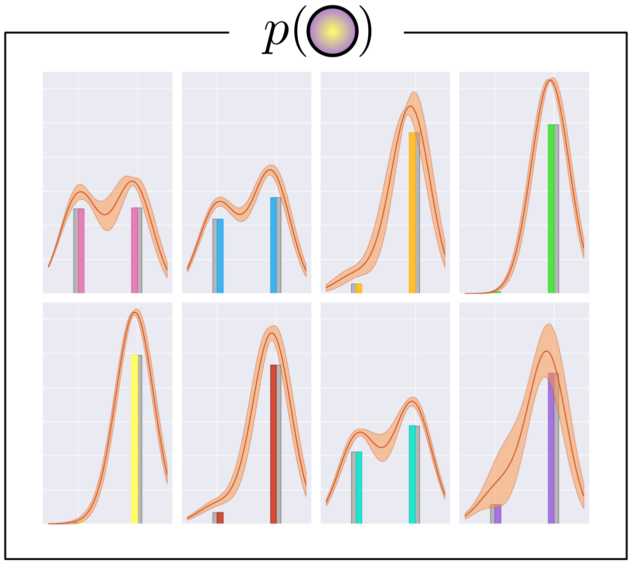
Observational Data





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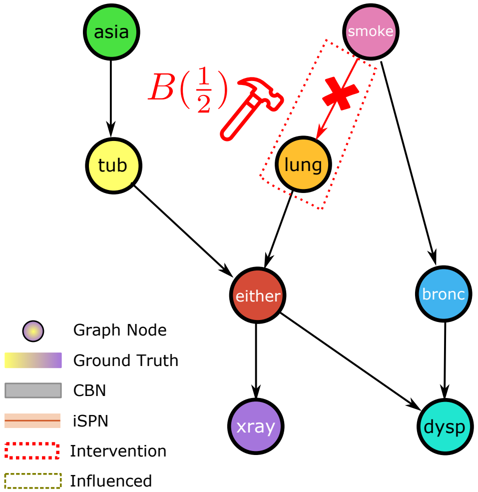
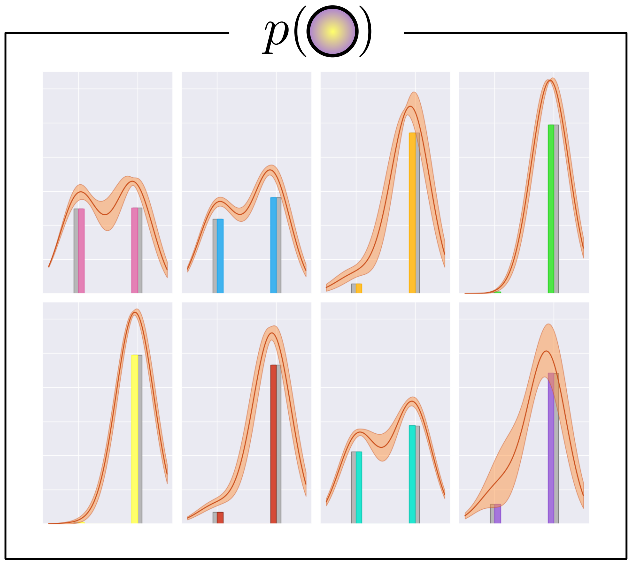
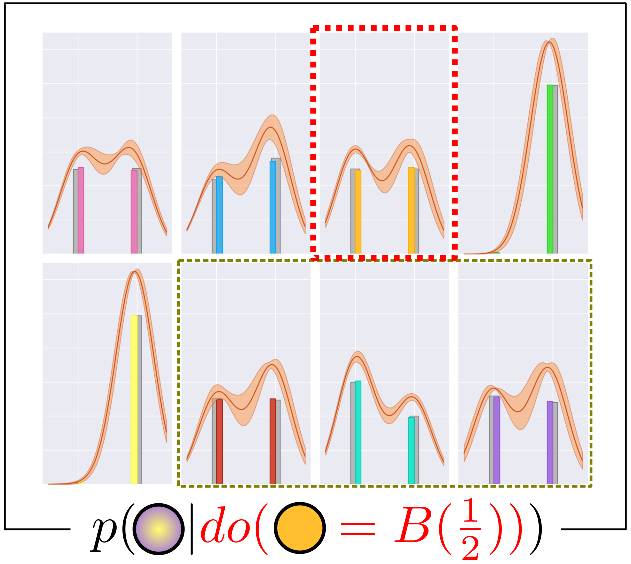


Observational Data



Intervention on 'lung'



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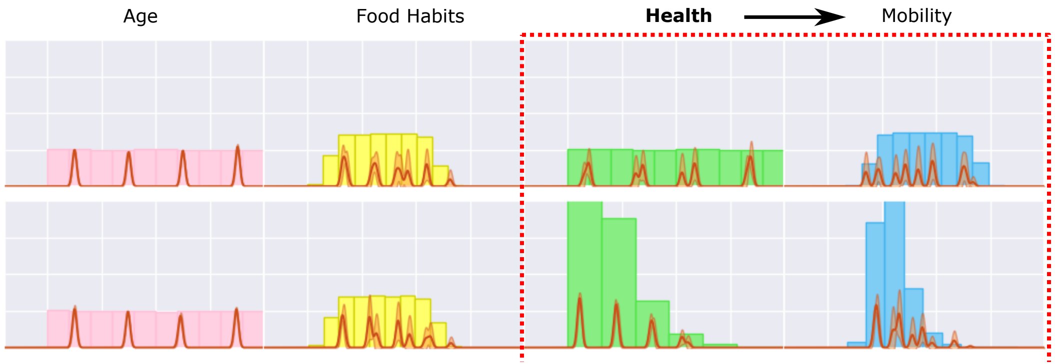
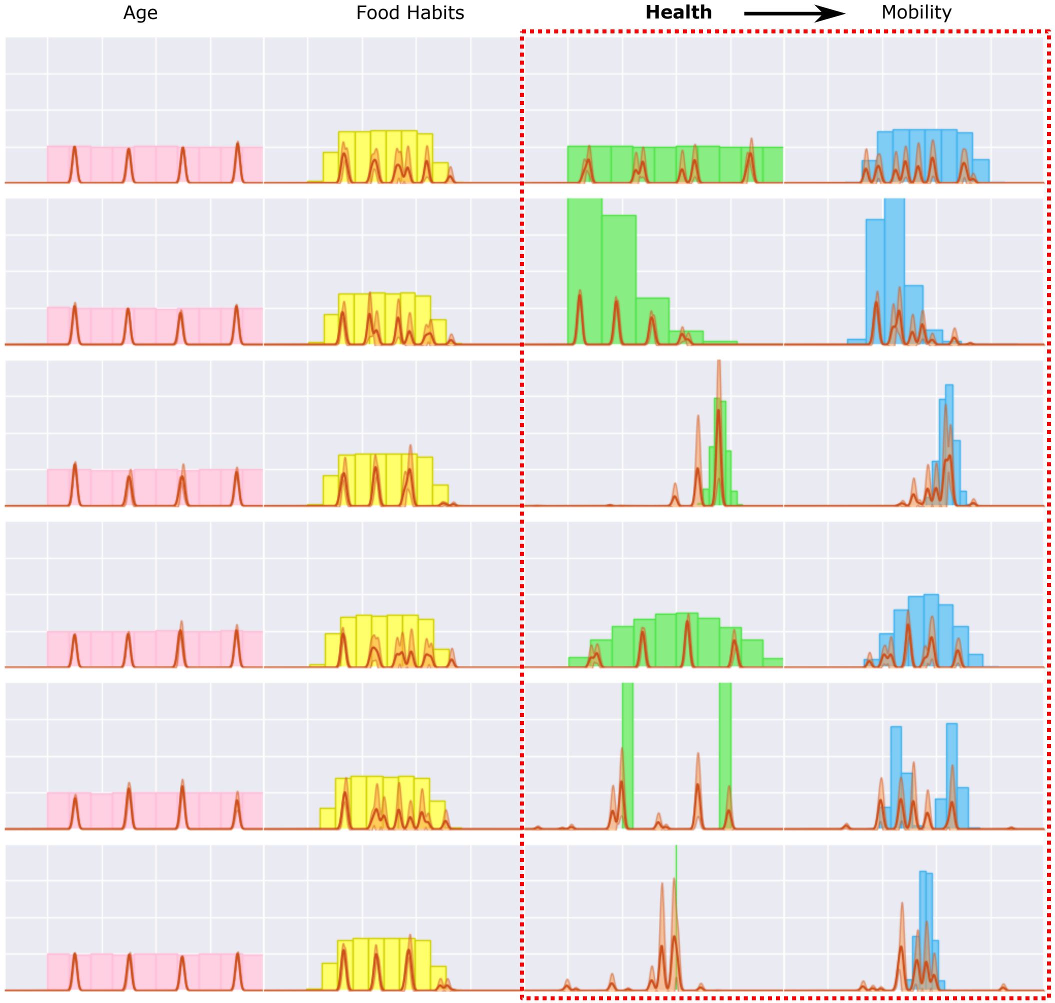
$$do(H = U(0, 100))$$



$$do(H = U(0, 100))$$

$$do(H = \Gamma(2, 10))$$



$do(H = U(0,100))$

$do(H = \Gamma(2,10))$

$do(H = \mathcal{N}(70,10))$

$do(H = 100B(2,2))$

$do(H = 1_{[25,75]})$

$do(H = 50)$

Dealing with different types of inter-ventions

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Tractable Inference, a Plus for iSPN

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- Cooper 1990, Roth 1996:

  Marginal (and conditional) inference

  in Probabilistic Graphical Models (PGM),

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  is *intractable* (**#P-hard**)

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  – linear in the size of the network.

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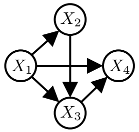
Semantic



n	$O(2^n)$
1	2
2	4
3	8
4	16
…	
100	$1{,}26 \cdot 10^{30}$

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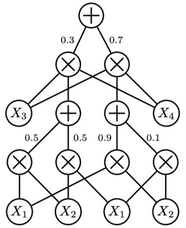
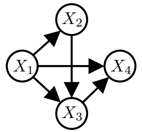
Computational



Semantic



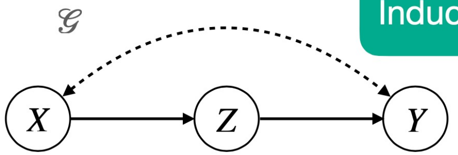
n	O(n)	O(2^n)
1	1	2
2	2	4
3	3	8
4	4	16
…		
100	100	1,26*10^{30}

Negative: no implicit identification!

Next: an alternative without linear time inference but identification

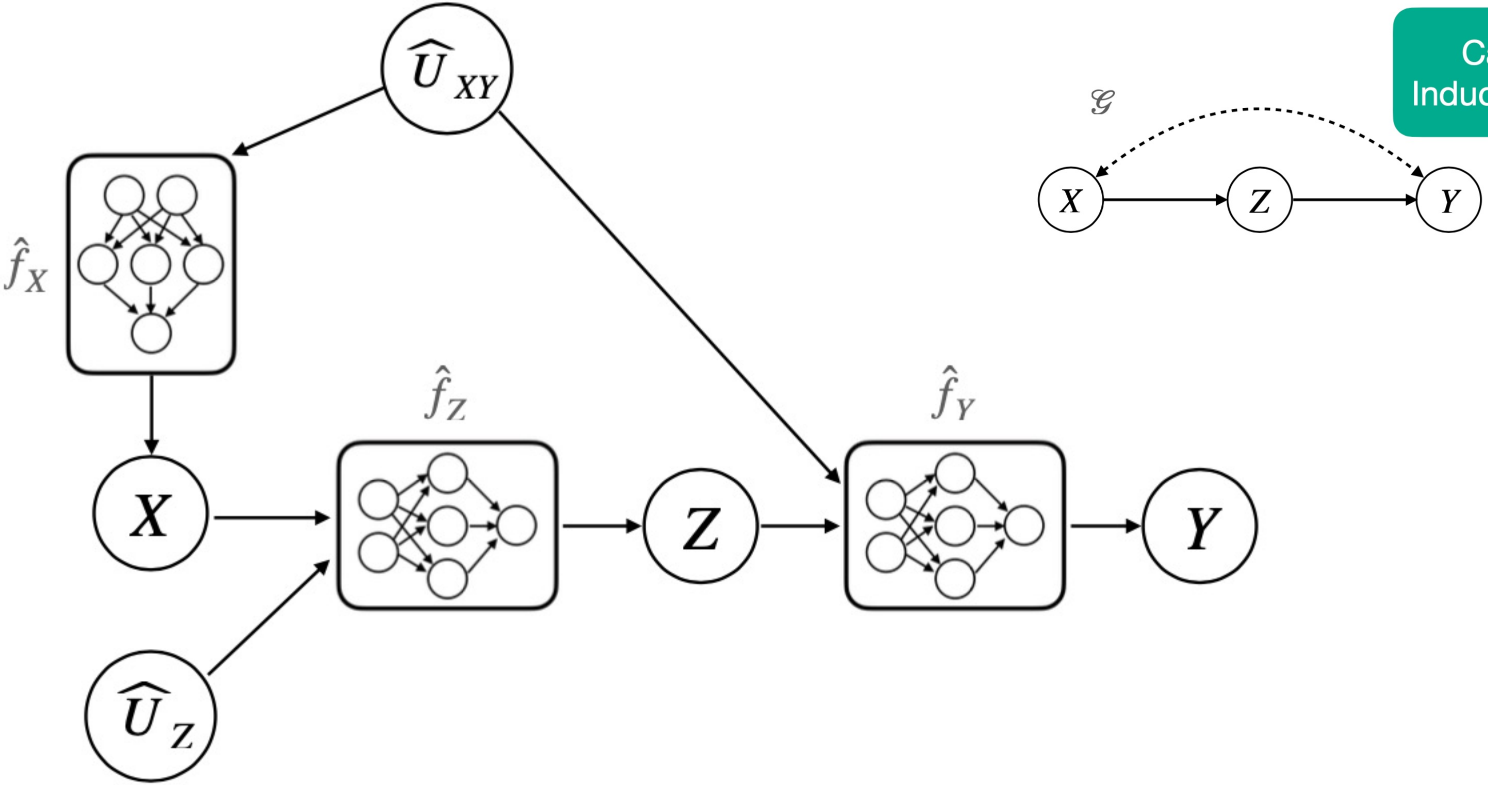
= + Causal Graph

Causal Inductive Bias



Neural Causal Model = Neural Nets + Causal Graph

= parameterized SCM with neural nets mechanisms



Causal Inductive Bias

Algorithm 1: Identifying/estimating queries with NCMs.

Input : causal query $Q = P(\mathbf{y} \mid do(\mathbf{x}))$, L_1 data $P(\mathbf{v})$, and causal diagram $\mathcal{G}$

Output : $P^{\mathcal{M}^*}(\mathbf{y} \mid do(\mathbf{x}))$ if identifiable, `FAIL` otherwise.

$\widehat{M} \leftarrow \texttt{NCM}(\mathbf{V}, \mathcal{G})$ `// from Def. 7`

$\boldsymbol{\theta}^*_{\min} \leftarrow \arg\min_{\boldsymbol{\theta}} P^{\widehat{M}(\boldsymbol{\theta})}(\mathbf{y} \mid do(\mathbf{x}))$ s.t. $L_1(\widehat{M}(\boldsymbol{\theta})) = P(\mathbf{v})$

$\boldsymbol{\theta}^*_{\max} \leftarrow \arg\max_{\boldsymbol{\theta}} P^{\widehat{M}(\boldsymbol{\theta})}(\mathbf{y} \mid do(\mathbf{x}))$ s.t. $L_1(\widehat{M}(\boldsymbol{\theta})) = P(\mathbf{v})$

if $P^{\widehat{M}(\boldsymbol{\theta}^*_{\min})}(\mathbf{y} \mid do(\mathbf{x})) \neq P^{\widehat{M}(\boldsymbol{\theta}^*_{\max})}(\mathbf{y} \mid do(\mathbf{x}))$ **then**

  return `FAIL`

else

  return $P^{\widehat{M}(\boldsymbol{\theta}^*_{\min})}(\mathbf{y} \mid do(\mathbf{x}))$ `// choose min or max arbitrarily`

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Min-Max Optimization subject to **equality** on observational distribution

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Min-Max Optimization subject to **equality** on observational distribution

Query is identifiable **iff. the optimizations agree**

Full Stop!

Now: Summarizing what've learned

Recap of 'New' Formalizations & Results this Lecture

- Definition 1: Causal Effect Identifiability
- Definition 2: Markovian SCM
- Definition 3: Causal Effect
- Definition 4: Confounding
- Definition 5: Hard Intervention
- Theorem 1: Adjustment Sets

Recap of ‘New’ Formalizations & Results this Lecture

- Theorem 1: Adjustment Sets
- Theorem 2: Truncated Factorization
- Theorem 3: do-Calculus
- Definition 6: Response Function Variable
- Theorem 4: RFV Distribution
- Definition 7: iSPN

Recap of ‘New’ Formalizations & Results this Lecture

- And a lot more informally about

  Markovian Identifiability,

  Tractability of Inference,

  NCM / Neural Identification,

  etc.

Z | Announcements

Want to get a picture of who ***the scientists*** are that do causality research?

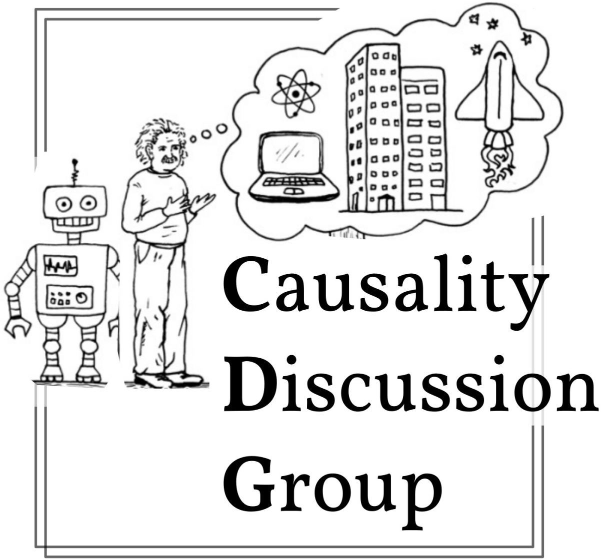
Genealogy of Causality

Access via genealogy.causality.link

Name	Institution	Supervisor	Location	Previous Positions
UCLA				
Judea Pearl	UCLA	?	US	Rutgers, Technion, New
Wesley Salmon	UCLA	Hans Reichenbach	US	?
Hans Reichenbach	UCLA	Paul Hensel, Max Noeth	US	Berlin, Istanbul, Erlange
John Hopkins				
Ilya Shpitser	John Hopkins		US	UCLA, Judea Pearl
Oregon State University				
Karthika Mohan	Oregon State University	Judea Pearl	US	
CMU				
Kun Zhang	CMU		Pittsburgh, US	MPI Tübingen
Clark Glymour	CMU	Wesley Salmon	Pittsburgh, US	
Peter Spirtes	CMU		Pittsburgh, US	
ETH Zürich				
Peter Bühlmann	ETH		Zürich	?
Marloes Maathuis	ETH		Zürich	?
Nicolai Meinshausen	ETH			
LMU Munich				
Stephan Hartmann	LMU		Munich, Germany	
MPI Tübingen				
Bernhard Schölkopf	MPI Tübingen	Vladimir Vapnik	Tübingen, Germ	TU Berlin
Ulrike von Luxburg	MPI Tübingen		Tübingen, Germany	
Michel Besserve				

Want to discuss more *following* this talk?

Every Week with Paper Authors → Discuss LIVE



530 members on Slack

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Past Sessions: [Password: Causa1ity, Direct Access Link]

- Session 01.03.2023 | **Deep Counterfactual Estimation with Categorical Background Variables** | Discussant: Edward De Brouwer
- Session 22.02.2023 | **Information-Theoretic Causal Discovery and Intervention Detection over Multiple Environments** | Discussant: Osman Ali Mian
- Session 08.02.2023 | **CLEAR: Generative Counterfactual Explanations on Graphs** | Discussants: Jing Ma, Ruocheng Guo
- Session 01.02.2023 | **Causal Transformer for Estimating Counterfactual Outcomes** | Discussant: Valentyn Melnychuk
- Session 25.01.2023 | **Abstracting Causal Models** | Discussant: Sander Beckers
- Session 18.01.2023 | **Desiderata for Representation Learning: A Causal Perspective** | Discussant: Yixin Wang
- Session 11.01.2023 | **Causal Feature Selection via Orthogonal Search** | Discussant: Ashkan Soleymani
- Session 14.11.2022 | **Rewind 2022** | Final session of 2022 to simply rewind on what we experienced throughout the year
- Session 07.12.2022 | **Causal Inference Through the Structural Causal Marginal Problem** | Discussant: Luigi Gresele
- Session 30.11.2022 | **Selecting Data Augmentation for Simulating Interventions** | Discussant: Maximilian Ilse
- Session 23.11.2022 | **On Disentangled Representations Learned from Correlated Data** | Discussant: Frederik Träuble
- Session 16.11.2022 | **Causal Curiosity: RL Agents Discovering Self-supervised Experiments for Causal Repr. Learning** | Discussant: Sumedh Sontakke
- Session 09.11.2022 | **Causal Machine Learning: A Survey and Open Problems** | Discussants: Jean Kaddour, Aengus Lynch
- Session 02.11.2022 | **A Critical Look at the Consistency of Causal Estimation with Deep Latent Variable Models** | Discussant: Severi Rissanen
- Session 26.10.2022 | **Nonlinear Invariant Risk Minimization: A Causal Approach** | Discussant: Chaochao Lu
- Session 19.10.2022 | **CausalVAE: Disentangled Representation Learning via Neural Structural Causal Models** | Discussant: Mengyue Yang
- Session 12.10.2022 | **Weakly Supervised Causal Representation Learning** | Discussant: Johann Brehmer
- Session 05.10.2022 | **Towards Causal Representation Learning** | Discussant: Anirudh Goyal
- Session 21.09.2022 | **Selection Collider Bias in Large Language Models** | Discussant: Emily McMilin
- Session 14.09.2022 | **The Causal-Neural Connection: Expressiveness, Learnability, and Inference** | Discussants: Kai-Zhan Lee, Kevin Xia
- Session 07.09.2022 | **Self-Supervised Learning with Data Augmentations Provably Isolates Content from Style** | Discussant: Julius von Kügelgen

⋮

35+ Sessions Completed and All Recorded

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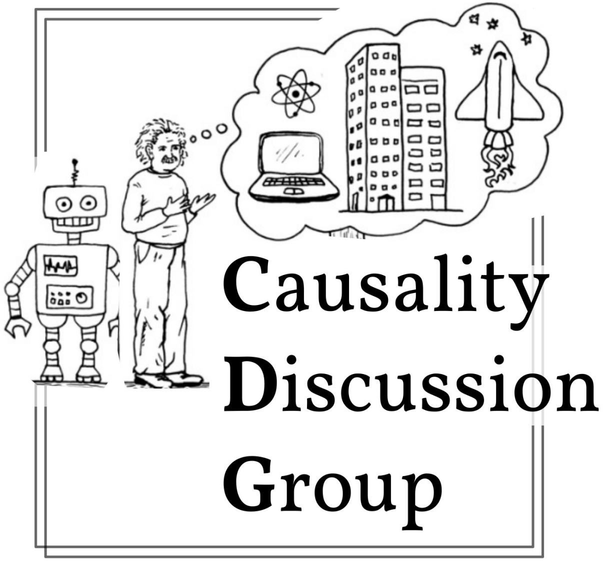
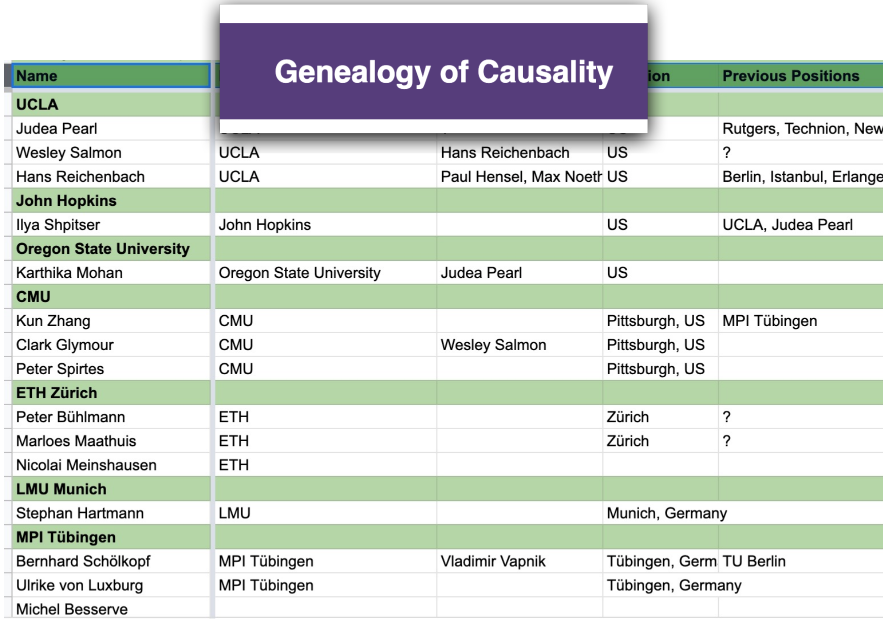


Access the genealogy via genealogy.causality.link



Name			ion	Previous Positions
UCLA				
Judea Pearl				Rutgers, Technion, New
Wesley Salmon	UCLA	Hans Reichenbach	US	?
Hans Reichenbach	UCLA	Paul Hensel, Max Noeth	US	Berlin, Istanbul, Erlange
John Hopkins				
Ilya Shpitser	John Hopkins		US	UCLA, Judea Pearl
Oregon State University				
Karthika Mohan	Oregon State University	Judea Pearl	US	
CMU				
Kun Zhang	CMU		Pittsburgh, US	MPI Tübingen
Clark Glymour	CMU	Wesley Salmon	Pittsburgh, US	
Peter Spirtes	CMU		Pittsburgh, US	
ETH Zürich				
Peter Bühlmann	ETH		Zürich	?
Marloes Maathuis	ETH		Zürich	?
Nicolai Meinshausen	ETH			
LMU Munich				
Stephan Hartmann	LMU		Munich, Germany	
MPI Tübingen				
Bernhard Schölkopf	MPI Tübingen	Vladimir Vapnik	Tübingen, Germ	TU Berlin
Ulrike von Luxburg	MPI Tübingen		Tübingen, Germany	
Michel Besserve				

That's a wrap!

Feel free to reach out:

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