Zakaria ECHCHAIR
Zakaria.ECHCHAIR@polytechnique.edu

Lab session # 4
ALTEGRAD 2022

11/29/22

1 Question 1

The number of edges in graph G :
In our case :

$$\frac{100*(100-1)}{2} + 50^2 = 7450 \, edges$$

The number of triangles in complete graph of 100 nodes is :

$$\frac{100*(100-1)*(100-2)}{6} = 161700 \, Triangles$$

The number of triangles in bipartie complete graphs is 0.

2 Question 2

- The maximally possible global clustering coefficient is : 1
- The complete connected Graph achieves the maximally possible global clustering coefficient.

3 Question 3

- the first eigen vector would be [1,1,1,1....,1,1,1] because in the first eigen vector, the node i and j have the same value (1) if they are connected, and by transitivity they will be all ones.

  This is true only when the graph contains only one connected component. However, if it contains more, so the first eigen value will have multiplicity greater than 1 and the first eigen vector won't be all ones.

  Since the first eigen vector contains only ones: so, it gives no idea about the clusters and it doesn't affect the clustering at all, however the second eigen vector will separate the two first clusters (it gives only real number so we should apply sign function).

4 Question 4

- The spectral clustering is a stochastic algorithm:

  Indeed, the first steps of spectral clustering are exact, like the SVD of Laplacian and the first eigen vectors too.

  Nevertheless, the last step which is the k-means isn't deterministic, because running the k-means on the same datasets over and over gives different results (depending on initial centroids).

5 Question 5

We have :

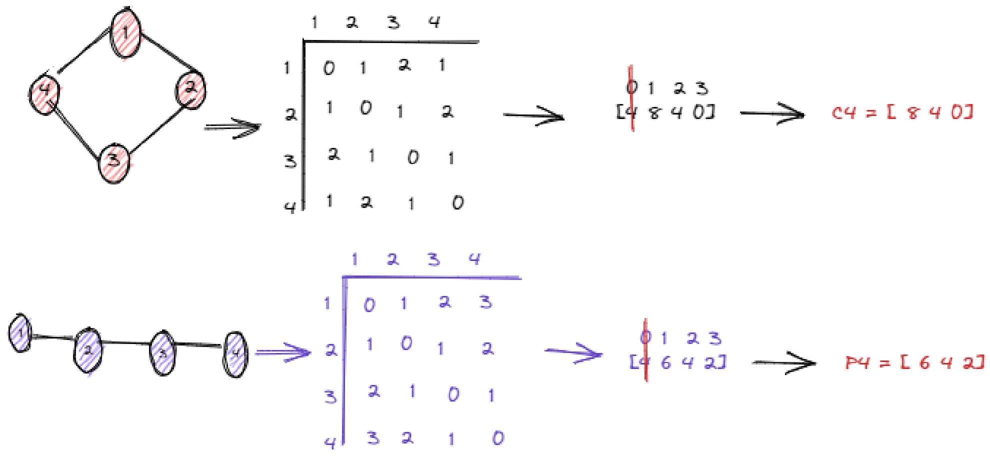
$$Q = \sum^{n_c} \left[\frac{l_c}{m} - \left(\frac{d_c}{2m} \right)^2 \right]$$

$$Q_a = \sum^{2} \left[\frac{l_c}{m} - \left(\frac{d_c}{2m} \right)^2 \right] = 2 * \left[\frac{6}{13} - \left(\frac{13}{2*13} \right)^2 \right] \approx 0.42$$

$$Q_b = \sum^{2} \left[\frac{l_c}{m} - \left(\frac{d_c}{2m} \right)^2 \right] = \left[\frac{2}{13} - \left(\frac{11}{2*13} \right)^2 \right] + \left[\frac{4}{13} - \left(\frac{15}{2*13} \right)^2 \right] \approx -0.05$$

6 Question 6

We have :



And also we have :

$$k\left(x, x^{\prime}\right)=\left\langle\phi(x), \phi\left(x^{\prime}\right)\right\rangle$$

So :

$$\left(C_{4}, C_{4}\right)=80$$

$$\left(C_{4}, P_{4}\right)=64$$

$$\left(P_{4}, P_{4}\right)=56$$