MTH 9821 Numerical Methods for Finance

Fall 2018

Homework 2

Assigned: August 30; Due: September 12

This homework is to be done as a group. Each team will hand in one homework solution, and each member of the team should write at least one problem. On the cover page of the homework, please indicate the members of the team and who wrote each problem.

Trinomial Tree Methods for European Options

Throughout this homework, the following parameterizations will be used for trinomial tree methods:

$$u = e^{\sigma\sqrt{3\delta t}}; \qquad d = e^{-\sigma\sqrt{3\delta t}}; \tag{1}$$

$$p_u = \frac{1}{6} + (r - q - \frac{\sigma^2}{2}) \sqrt{\frac{\delta t}{12\sigma^2}}; \tag{2}$$

$$p_m = \frac{2}{3}; \tag{3}$$

$$p_d = \frac{1}{6} - (r - q - \frac{\sigma^2}{2}) \sqrt{\frac{\delta t}{12\sigma^2}}. \tag{4}$$

Consider a one year European put and a one year American put, both with strike \$39, on an asset with spot price \$41 paying dividends continuously at rate 0.5%, and following a lognormal process with volatility 28.5%. Assume the risk free interest rates are constant at 3%.

Compute the Black–Scholes option value V_{BS}, and the following Greeks: Δ_{BS}, Γ_{BS}, and Θ_{BS}.

Price the European put option using the following tree methods:

- Trinomial Tree with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps;
- Trinomial Black–Scholes with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps;
- Trinomial Black–Scholes with Richardson Extrapolation, with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps.

For each method, record the first six decimals of the following values in the solution template file hw_sol_template-TRINOMIAL-European.xls:

- $V(N)$, the value given by the tree method with N time steps;
- $|V(N) - V_{BS}|$, the approximation error of the tree method;
- $N\ |V(N) - V_{BS}|$ and $N^2\ |V(N) - V_{BS}|$, terms that indicate whether the convergence of the tree method is linear or quadratic;
- the following approximations for the Delta, the Gamma, and the Theta of the option:

$$\Delta_{approx} = \frac{V_{1,0} - V_{1,2}}{S_{1,0} - S_{1,2}}; \tag{5}$$

$$\Gamma_{approx} = \frac{\frac{V_{2,0} - V_{2,2}}{S_{2,0} - S_{2,2}} - \frac{V_{2,2} - V_{2,4}}{S_{2,2} - S_{2,4}}}{S_{1,0} - S_{1,2}}; \tag{6}$$

$$\Theta_{approx} = \frac{V_{1,1} - V_{0,0}}{\delta t}, \tag{7}$$

and the approximation errors $|\Delta_{approx} - \Delta_{BS}|$, $|\Gamma_{approx} - \Gamma_{BS}|$, and $|\Theta_{approx} - \Theta_{BS}|$.

Rank the methods in terms of convergence speed and comment on the order of the convergence.

Trinomial Tree Methods for American Options

Compute the value of an American Put with the same parameters by using an average binomial tree with $10,000$ and $10,001$ time steps and denote it by V_{exact}. *Do not use variance reduction.*

Price the American put option using the following tree methods:

- Trinomial Tree with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps;
- Trinomial Black–Scholes with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps;
- Trinomial Black–Scholes with Richardson Extrapolation, with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps.

For each method, record the first six decimals of the following values in the solution template file hw_sol_template-TRINOMIAL-American.xls:

- $V(N)$, the value given by the tree method with N time steps;
- $|V(N) - V_{exact}|$, the approximation error of the tree method;
- $N\,|V(N)-V_{exact}|$ and $N^2\,|V(N)-V_{exact}|$, terms that indicate whether the convergence of the tree method is linear or quadratic;
- compute the approximations Δ_{approx}, Γ_{approx}, and Θ_{approx}, for the Delta, the Gamma, and the Theta of the option, respectively, and the approximation errors $|\Delta_{approx} - \Delta_{exact}|$, $|\Gamma_{approx} - \Gamma_{exact}|$, and $|\Theta_{approx} - \Theta_{exact}|$, where, e.g.,

$$\Gamma_{exact} \;=\; \frac{\Gamma_{American}(10,000) + \Gamma_{American}(10,001)}{2}.$$

Repeat the process by using Variance Reduction for each method.

Rank the methods in terms of convergence speed and comment on the order of the convergence.