Fall 2018: MTH 9821 Numerical Methods for Finance

Thursday 6:05–8:50pm, Room 9-180

Instructor: Dan Stefanica

Reference Books:

- "A Linear Algebra Primer for Financial Engineering", by Dan Stefanica. Publisher: FE Press, 2014.
- "Numerical Linear Algebra", by Lloyd Trefethen and David Bau. Publisher: Society for Industrial and Applied Mathematics, 1997.
- "The Mathematics of Financial Derivatives: A Student Introduction", by Paul Wilmott, Sam Howison, and Jeff Dewynne. Publisher: Cambridge University Press, 1995.
- "Monte Carlo Methods in Financial Engineering", by Paul Glasserman. Publisher: Springer, 2004.

Detailed Syllabus

Lecture 1

- One-period binomial tree model. Hedging and replication.
- N-period binomial trees.
- Lognormal model for the evolution of the stock price. Calibrating the tree parameters to fit lognormal models for the underlying asset.
- Risk-neutral valuation of European and American plain vanilla options.
- Greeks estimation in binomial trees.
- Variance reduction for pricing American options on binomial trees.
- Implied volatility computations using binomial tree methods.
- Faster binomial tree algorithms: average binomial trees, Black-Scholes Binomial trees (BBS), Black-Scholes Binomial trees with Richardson extrapolation (BBSR).

Lecture 2

- Trinomial trees: one period model, calibration for an asset following a Geometric Brownian motion. Risk-neutral pricing.
- Greeks estimation and implied volatility computations using binomial tree methods.
- Tree methods for pricing barrier options. Sawtooth convergence pattern. Finding the optimal number of time steps for pricing barrier options.
- Pricing barrier options on binomial and trinomial trees.
- Arrow–Debreu tree prices and non–path dependent option pricing using Arrow–Debreu prices.

Lecture 3

- Monte Carlo methods. Convergence and approximation error estimates.
- Advantages and disadvantages of Monte Carlo pricing.
- Monte Carlo simulation for the valuation of non–path–dependent derivative securities and path–dependent derivative securities.
- Greeks computation using Monte Carlo methods for non–path–dependent derivative securities.
- Random number generators. Linear Congruential Generators for the uniform distribution.

- Standard normal random number generators.
- Inverse Transform Method.
- Acceptance–Rejection Method.
- Box–Muller method. Marsaglia–Bray implementation for the Box–Muller method.

Lecture 4

- Variance Reduction Techniques for Monte Carlo simulations.
- Control Variates. Connections to Weighted Monte Carlo.
- Antithetic Variables.
- Moment Matching. Put–Call parity and moment matching simulations.
- Monte Carlo simulations for multi–asset options.
- Monte Carlo pricing of non–path–dependent and path–dependent basket options.
- Monte Carlo simulations for the Heston volatility model.

Lecture 5

- Numerical integration methods
- Derivation of the Black–Scholes PDE. Properties of the Black–Scholes PDE.
- Financial interpretation of the terms from the Black–Scholes PDE.
- Reducing the Black–Scholes PDE to the diffusion equation.
- Boundary conditions for the Black–Scholes PDE and the effect of the change of variables.
- Closed form solution of the diffusion equation.
- Derivation of the Black–Scholes formulas.
- Sharp approximations for ATM-Forward option prices and implied volatilites.
- Explicit implied volatility formulas and approximation properties.

Lecture 6

- Explicit finite difference methods for solving the diffusion PDE: Forward Euler.
- Finite difference discretization and the solution of the diffusion equation.
- Finite difference methods for solving the Black–Scholes PDE. Boundary Conditions.
- Pricing European plain vanilla options using Forward Euler.
- Comparison of domain discretizations for solving the Black–Scholes PDE.
- Finite difference approximations of the Greeks.
- Implied volatility computations using finite difference methods.
- Barrier options. Closed Formulas. Arbitrage pricing.
- Pricing European barrier options using Forward Euler.

Lecture 7

- Iterative Methods for solving Linear Systems. Example: Richardson iteration.
- Convergence criterion. Stopping criteria. General splitting techniques.
- Jacobi, Gauss-Siedel, and SOR methods. Pseudocodes.
- Convergence analysis for the Jacobi, Gauss-Siedel, and SOR iterative methods.
- Comparing the convergence speed of different methods.
- Convergence speed of iterative methods.
- Eigenvalue problems.
- Power Method. Inverse Power Method. QR Algorithm.
- Application: Principal Component Analysis.

Lecture 8 Midterm Exam

Lecture 9 (Backward Euler, and Crank-Nicolson)

- LU decomposition with and without pivoting.

- Cholesky decomposition of symmetric positive definite matrices.
- Implicit finite difference methods for solving the diffusion PDE: Backward Euler and Crank-Nicolson.
- Finite difference discretization and the solution of the diffusion equation.
- Projected SOR.
- Pricing American plain vanilla options using Backward Euler and Crank-Nicolson.
- Implied volatility computations for American options using finite difference methods.
- Domain discretization for finite difference pricing of options on underlying assets paying discrete dividends.
- Forward and backward substitution for bidiagonal matrices.
- LU decomposition with row pivoting for tridiagonal matrices.
- Cholesky decomposition of tridiagonal symmetric positive definite matrices.
- Optimal linear solvers for tridiagonal symmetric positive definite matrices.
- Iterative methods and the convergence of iterative methods for a family of tridiagonal symmetric positive definite matrices.

Lecture 10 Monte Carlo methods for plain vanilla American options.

Lecture 11

- Tian Third Moment Matching binomial trees.
- Comparing the efficiency of tree methods. Truncation, Smoothing, Richardson extrapolation.
- Binomial tree pricers for pricing options on assets paying proportional discrete dividends.
- Binomial tree pricers for pricing options on assets paying fixed discrete dividends.
- Binomial tree methods for path–dependent options.

Lecture 12

- Convergence theory for finite difference methods for options pricing

Lecture 13

- Domain discretization for pricing Bermudan options using finite difference methods.
- Forward Euler, Backward Euler, and Crank-Nicolson for Bermudan options pricing and Greeks approximations.
- Derivation of the Barone-Adesi–Whaley approximate formula for American plain vanilla options.
- Newton's method and the implementation of the Barone-Adesi–Whaley formula.
- Computing an approximate implied volatility for American plain vanilla options using the Barone-Adesi–Whaley formula.

Lecture 14: *The Arrow-Debreu one period market model.*

- Redundant securities and replicable securities.
- Complete markets.
- Arbitrage Pricing Theory. Arbitrage-free markets. Portfolio arbitrage.
- Fundamental Theorem of Asset Pricing.
- State Prices. Elementary insurance contracts.
- Risk-neutral probability and risk-neutral pricing.
- Example of a complete market: the one-period binomial model.
- Binomial tree pricers.
- Matrix setup for a one period market model example.
- Arbitrage for negative insurance contracts prices.
- Incomplete markets.

- Example of an incomplete market: the one-period trinomial model.
- One period index options market models.

Lecture 1xx

- Finite difference approximations. Finite difference discretization and solution of a second order ODE.
- Solutions of ODEs with constant coefficients and with homogeneous coefficients.
- Perpetual options pricing.
- Convergence theory for finite difference methods for solving ODEs.