**MTH 9821 Numerical Methods for Finance**

**Homework 1**

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**(1)**

At time t=0, construct a portfolio by

* Long unit of the underlying asset
* Cash position:

The present value of this portfolio is zero, then according to the generalized law of one price, the future value of this portfolio should always be zero.

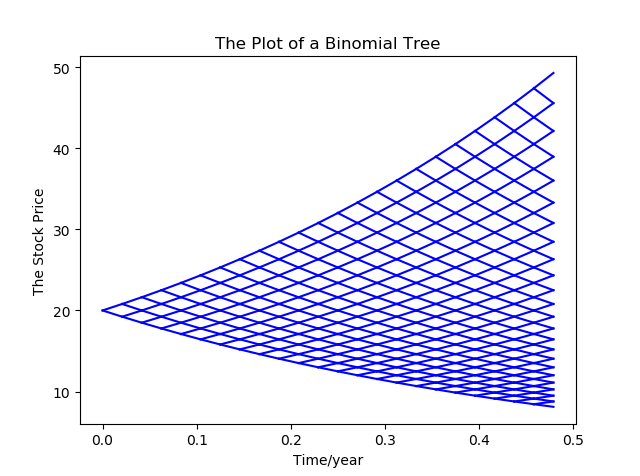
Thus, at time (period 1), it follows that

Since , if , then , thus at period 1, the future value of this portfolio will be non-negative with probability 1, and be positive with a positive probability, which is conflict with no-arbitrage assumption, thus .

Similarly, it’s easy to show that . In summary, we have

Simplify it, we get

**(2)**



**(3)**

View as a function of , then to show the argument, we only need to compute ’s Taylor expansion to the first order, which means we only need to compute the value of , and .

Plug in the value of and , and use L'Hôpital's rule, we have

For ,

Abbreviate the above equation, we get

Apply L'Hôpital's rule to the first term (1), we have

Similarly, we can get

Combine (i) and (ii), we have

Then, using the formula for Taylor expansion, we have

**(4)**

Assume the number of upward move over the time T is , then

So , since , we have , for calculation simplicity, let

and we will calculate the moment generating function for X.

Combine with , and use the formula of binomial expansion, we have

Next, I will plug in the value of p and use Taylor expansion to expand the above terms,

Expand the product of the above two terms and rearrange it according to the order of 1/N, we have

Then, include into the power of N, and again use Taylor expansion to expand it, we have

According to (1), we have (1) = (3) \* (4), plug in the expression we derived above, we get

Now, use the famous limit equation

We have

Recall that , we have

Note that, the RHS of (6) is the moment generating function of normal random variable , then according the one-to-one relationship between moment generating function and distribution function, we have proven that

where Z is the standard normal random variable.